A COMPLEAT

Body of Arithmetick,

In Four BOOKS, *Viz:*

Book I.
Part 1. *Integers.*
2. *Fractions.*

Book II.
1. *Geodæticals.*
2. *Figurals.*

Book III.
1. *Decimals.*
2. *Aſtronomicals.*
Part
3. *Logarithmes.*
4. *Coſſicks.*
5. *Surds.*
6. *Algebra.*

Book IV.
1. *Ratio*'s.
2. *Proportions disjunct.*
3. *Proportions continued.*
4. *Æquations*, &c.

WHEREIN THE

Whole Nature of NUMBERS, with their Simple and Comparative Elements in all the Parts of ARITHMETICK, are plainly Declared, and fully Handled.

Every PART Explain'd by

Neceſſary Rules, Caſes, Theorems, Queſtions, Obſervations, and Variety of Operations: Illuſtrated by ſundry Tables, Diagrams, and very many Examples. Together with divers Etymologies, Symboles, Characters, and Abreviations for Artificial Terms, Words, Names, and Denominations: The Whole digeſted into a ſuccinct, and orderly Method; and delivered in a familiar Style.

To every PART is added,

Excellent RULES of PRACTICE; which makes it very Uſeful to Merchants, and all that would underſtand ACCOUNTS, or the Mathematical and Mechanical ARTS and SCIENCES.

WITH

Many Large and Exact TABLES of all Sorts of Coins, Weights, Meaſures, *&c.* Ancient and Modern, in moſt Countries, reduced to our own: The Beſt that has yet been extant.

By *SAMUEL JEAKE*, Merchant.

Non partis Studiis agimur, &c.

LONDON:
Printed for *Tho. Newborough*, at the *Golden Ball*, in St. *Paul*'s Church-yard; and
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MVSEVM
BRITAN
NICVM

6 H.g

To the Honourable Sir *ROBERT SOUTHWELL*, K$^{nt.}$ PRESIDENT OF THE ROYAL SOCIETY.

SIR,

ALthough it was the Author's Pleaſure to prefix my Name to the Enſuing Treatiſe; which I am therefore neceſſitated to inſert: leſt I ſhould be accuſed of altering that Work, the Impreſſion whereof was recommended to me; and whereto I am obliged without any reſpect to Profit. Yet as I thereby received a Right to diſpoſe of the Dedication: So I could not otherwiſe acquieſce, than in the Reſignation of my Intereſt therein, to ſome greater Name;

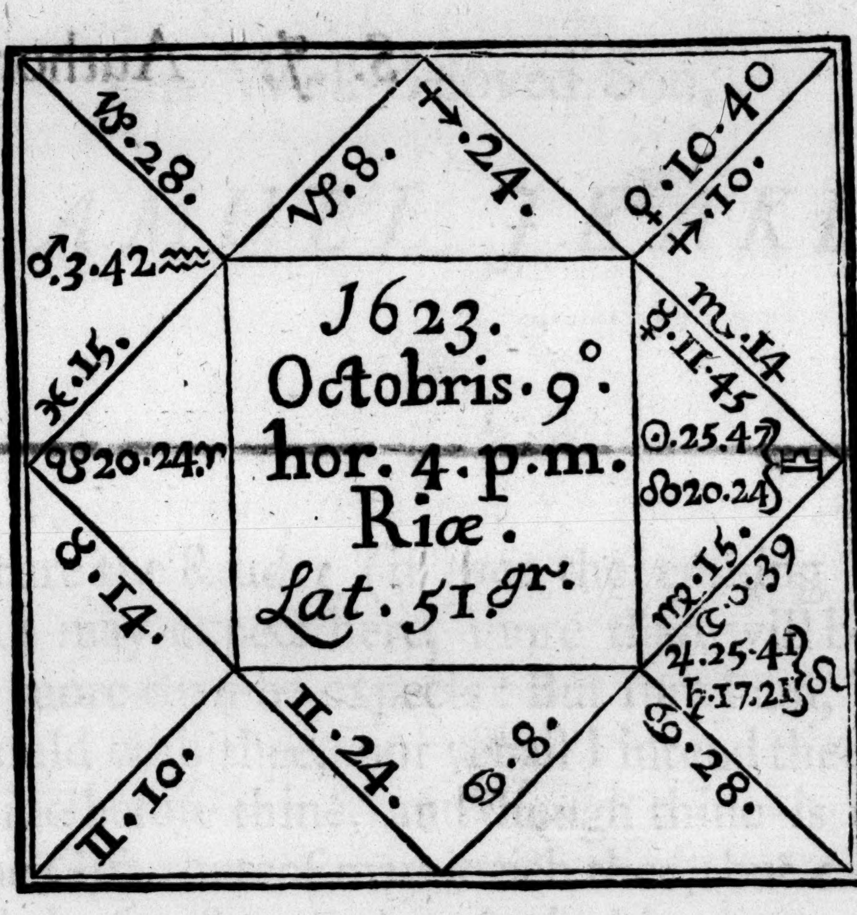
eſteeming my own too minute to ſtand before it. And becauſe I am not under any particular Obligation to whom I ſhould preſent it: I have adventured to ſhroud it under your Honourable Patronage. Preſuming that where no Perſonal Advantages are expected; no imputation of Flattery can be charged: But that the Motives were alone that Merit and Generoſity which muſt always be venerated by

SIR,

Your moſt
Humble Servant,

S. JEAKE.

Defectum Iconiſmi
Placuit ſupplere
Themate Natalitio
Authoris.



NO Planets here by Exaltation proud:
None by a Reſt Supine, in Houſe beſtow'd.
But congruous Heav'n at this Birth diſpos'd,
T'inſpirit a clear Soul in Fleſh enclos'd.
The mildeſt Dodecatemorie ſprings
In beauteous Orient: the encircling Rings
Of her Cœrulean Lord's Quaternion,
By Starry Regulus in Triumph ſhone.
That bright Superior's Domination fixt
In Heav'n's Culmen. Gen'rous Aſpects mixt:
His Fiery Partil Trine to actuate
The Active Houſe to a more Active Fate.
Nor was it vain: the happy Site of this
Æthereal Ruler of the Geneſis,

A Judgment firmly form'd; whoſe Adjutant
Mnemonick pow'r, did by Cœleſtial grant
Of Saturn's ſeminated Beams enſue,
In Platique Synod, with Proportion due.
As when the skilfull Artiſt to compoſe
His mighty Theriaque; Weighs the Critick Doſe
Of Theban Opium; which with Virtue full
Quickens that Brain, it's leaſt Exceſs would dull.
The Wit's Dictator from the brighter Scale
Suits his harmonious Trine, whoſe Rays may fall
On th' Eaſtern Point: Whilſt the Heſperian Face
Reſplendent Venus doth the Ninth Houſe grace.

S. J. Authoris Filius.

THE

THE

Authors Epiſtle.

TO

His Well-Beloved Son,

SAMUEL JEAKE.

My Son,

PEradventure the Reader (if ever the enſuing Piece be made Publick) may expect here, more than will be found; and yet find more than he expects: But ſure I am, Thou wilt not find here all I could wiſh thee, nor yet all I intend thee. What thou findeſt, was mine before thine, and though thine is mine notwithſtanding. The Gift thereof may inrich thee, but cannot impoveriſh me. And the ſureſt way to make it thine is, to make it others too by Publication. By which though others, it will be nevertheleſs ſecured to thee, as being then uncapable of periſhing in Private Papers. And becauſe thy Right to inherit what is mine is indubitable, and thy Duty to defend what I leave thee, (though but a ſmall Patrimony:) I have ſought no other Patron, nor (ſeeing *Vino Vendibili non opus eſt hederâ*) do I want any, or ſhroud it under thy Patronage, thereby to gain the more reſpect or honour to my ſelf or it, (the great cauſe of Dedications) if Art will not Patronize it, I am content to bear the accruing blame, whatever it be.

But ſeeing how good ſoever the Wine of the Book be (to make it the more Vendible) it is now grown Cuſtomary to hang out the Ivy-Buſh of an Epiſtle as an Apology for the Author, or his Work, or both: Or leſt it ſhould be thought ſo uſeleſs or unprofitable, that nothing can be ſaid to commend it or its Publication: I ſhall add a few things, and but a few on that Accompt.

Among the many Authors of Arithmetick that have come to my hands, I have ever obſerved that each pleaſeth himſelf with his own Method, as I my ſelf have done with mine. But this I muſt needs ſay, that I never met with any Single Piece but left me diſſatisfied in ſome or other parts of Arithmetick. Some handling only the Operations in *Whole Numbers* and *Extraction of Roots*, Others *Whole Numbers* and *Fractions*, Some all theſe with ſome *Rules in Proportions*, Others together with them have taught *Decimals*; Some have dealt only with *Logarithmes*, Others with *Coſſicks* and *Algebraical Notes*, *&c.* So as none I have yet ſeen gives a Compleat Accompt of ſome neceſſaries thereto. And beſides, the Accompt given by ſeveral, is ſo diſordered and imperfect, as the Art hath been but a little beholding to them for preſenting her to the World in ſo rural a Dreſs. Wherefore if the labour of a Complete Collection of the Cream of other Authors may be acceptable to any; or the Foundation of a Method large enough to bear all the parts of the Building whereon may be faſtned, and from whence may be drawn, the Reſolution of any Queſtion concerned in Arithmetick; this Piece may as well as others, that want both, be welcome to the Preſs, and crowd in for a place among the multitude of Books now Printed, wherein I hope 'twill neither ſhame the Author, nor be aſhamed of the Title of *Logiſticelogia*, ſeeing all the Concerns and Appurtenances of Arithmetick are therein diſcourſed of, and largely Surveyed and Reviewed.

Perhaps ſome may think, it is but to light a Candle to the Sun, ſince ſo many already have wrote on the Subject; as if *Nihil dictum quod non dictum prius*. To which I may plead with the Lawyers, *Non modo & forma*, and put the Iſſue on the Countrey to try.

True it is, moſt new Models are but the Light that ſometime ſhined in anothers Lamp, with an addition of freſh Oyl out of a new Veſſel, *Et facile eſt inventis addere*. But he that is ſenſible of the charge of buying, and trouble of turning over many Books to learn ſome one thing, will I doubt not excuſe my further plea herein, and plead for me; eſpecially if he knew that I ſpeak not without Experience, of no little time and trouble to glean ſo many Fields for one Griſt, having pickt up the knowledge of *Integers*, *Fractions*, *Figurals*, *Coſſicks*, and *Surdes* principally from *Record*, *Decimals* from *Johnſon*, *Aſtronomicals* from *Blundevile*, *Logarithmes* from *Briggs*, *Species* and *Æquations* from *Oughtred*, with a conference of many others. It follows therefore that each may have his due, what is here may be accompted anothers, yet is it all my own, and ſome things therein ſo far my own, as will be found in none extant that I know of. And becauſe this may prove Beneficial to ſome, unleſs ſuch prove remiſs to themſelves in the peruſal; it may ſerve for a ſecond incouragement to the Publication.

Beſides ſome Pieces are wrote in *Latine*, and of thoſe in *Engliſh*, ſome too ſhort in their Rules, or too long or dark in their Examples, or in the Reaſon and Ground thereof, and worſe than that have Examples inſtead of Rules, or Rules which will not hold generally, nor anſwer to ſeveral Caſes, ſo as ſetting aſide all the Errors of the Preſs, (a great Miſchief to a Learner of theſe things by Book, where the Sence is not guided by the Antecedent or Subſequent Matter) all the Learner reapes after the Expence of his Time, Coſt, and Travel, expecting with the Mower to fill his Lap; is but a handful of the Graſs of the Houſe-top, to wit, the Reſolution of ſome few Queſtions. And therefore the ſpreading this Table with ſuch Varieties of Rules for almoſt all Caſes, and fitting Examples to them (and not the Rules to the Examples) and over and above the Explanation of both in very many places, may I ſuppoſe paſs for a further full and ſufficient plea for it's *Imprimatur*.

As to the Work it ſelf, the occaſion of it's firſt penning was to help an Imperfect Memory, not once then thinking it ſhould ever have ſeen the Sun. Moſt of whoſe rough drawn, and unpoliſht Papers have layn by me above Twenty Years, in which time there have not wanted the often preſſures of Friends for a Tranſcription, which to them yet on this ſide the Grave will I know be grateful. If others undeſervedly ſlight it, it will but give occaſion for the deſerved ſlighting of their own Opinions, and not at all hurt me or it. *Alij quidem neſciunt, neque quidem curant ſcire, & quia neſciunt nolunt ſcire. Nihil enim deſiderabile eſt dum ignotum, nec amatum, niſi cognitum.*

Neverthleſs to take away occaſion from ſuch as ſeek occaſion, (it being too common for ſome to ſeek in Books for advantages againſt their Authors;) and to obviate ſeeming Objections, as alſo that all cauſeleſs Scruples and Calumnies intended to blurr both Author and Work may be wiped off (if poſſible) and utterly to raſe the foundation thereof, and abſterge ſuch rubbiſh, I ſhall add,

1. Wherein I diſſent from others in the Method, placing Continued Proportions after disjunct, and both after Figural Numbers, it is ſufficient there is a neceſſity for it, becauſe without the knowledge of Figural Numbers, and Extraction of their Roots, *Progreſſion* and ſeveral of the Dependants thereof cannot be Learned, nor Doubled nor Tripled *Diſjunct Proportions* wrought, as the newborn *Scioliſt* will eaſily ſee: And whether it be not prepoſterous to teach the more hard and Sublime parts of any Science, before the Introductory; let any *Tyro* judge.

2*ly*. *Fractions* upon the ſame account of neceſſity are here made to precede them all; for otherwiſe it can never be underſtood, how to Square or Cube, *&c.* a *Fraction*, nor how to value the Remain upon any Diviſion in the *Golden Rule*, and other *Proportional Operations*; if it be not known, what a *Fraction* is, and how to work therewith.

3*dly*. For the ſame reaſon, *Æquations* are ſet after *Surdes*, and *Surdes* before *Species*, ſeeing as in *Species* there happen *Surdes*; ſo both *Surdes* and *Species* ariſe in *Æquations*. But if no ſuch neceſſity were for theſe diſagreements; yet the ſymmetry and ſuitable agreement of the ſeveral parts with the whole in the Method purſued, and their concatenate concurrence or dependance one with, or upon another, with the reaſons here and there in places apt for the purpoſe rendred, may put a ſtop to any further diſpute concerning the ſame.

4*ly*. Neither may the placing of *Reduction in Geodæticals* before *Addition*,&c. be any Stone of Stumbling, conſidering *Geodæticals* are now acknowledged to be *Contract Numbers*, as limited to *Denominations*. And in *Contract Numbers* it is no ſtrange thing to ſee *Reduction* an Ortive part of *Numeration* put before the Original. And how helpful *Geodætical Reduction* is to other the Prime parts of *Geodætical Numeration*, nothing can ſo well diſcover as the Survey it ſelf, where it will eaſily be perceived without that Order, eſpecially *Multiplication* and *Diviſion* of thoſe Numbers if placed before, muſt have been imperfect and diſorderly.

5*ly*. If ſome of the Characters in *Coſſicks* and *Surdes* anciently uſed are changed for others: I think it Apology enough to ſay, they are arbitrary, and if for expedition (the reaſon of their alteration) other diſtinct Notes may be found, the Practitioner is at liberty to change all the reſt.

6*ly*. Varieties of working a Queſtion, if objected as troubleſome, are added as a benefit, like multiplicity of words in a Language for the ſame thing; if one be not hit upon, the other may; and where the one is dark, the other helps to Illuſtrate, and each ſerve to prove the other.

7*ly*. Many of the Examples may be thought needleſs; but it may be remembred, that *Profunda luſtrare abſque exemplis arduum*; and oftentimes one Example is not enough to ſhew the ſufficiency of a Rule, wherefore ſeveral Examples are added, where the Rule is dark, or the Work difficult, and ſeveral of them explained, leſt I ſhould ſeem to walk in the Clouds, accompting it for a *Maxime*, That nothing is worth the Writing, which cannot be underſtood when wrote. And ſurely plurality of Examples of ſo general Approbation and Practice with others, cannot come under diſlike here, and if it be a fault, muſt needs be one pardonable.

8ly. When Termes are uſed promiſcuouſly, as *Mixt Numbers*, for *Numbers* and *Fractions*, and ſometimes for Numbers made up of Digits and Articles, or if *Denominator* and *Denomination* be uſed either for other, or *Ratio* and *Proportion*, or ſuch like; Yet will the place where, and Matter about which they are ſo uſed, eaſily diſcover to the Obſervant Reader, which Term is properly intended without further help of an Expoſitor. And thus in the Common Elements of *Addition*, *Subſtraction*, *Multiplication* and *Diviſion*, if any of them be mentioned when *Integers* are in hand, it ſhall be taken for *Addition*, *Subſtraction*, *Multiplication*, or *Diviſion of Integers*: But when *Fractions* or other *Numbers* are in hand, then thoſe *Elements* ſhall accordingly be underſtood to be uſed, and wrought after the manner of *Fractions*, &c. reſpectively, without annexing to every repetition words at length to direct it.

9ly. Where ever any Term or Phraſe hath eſcaped Expoſition, and may ſeem diſcrepant from the more common Road of acceptation in other Writings, yet will not this widen the difference, if ſuch Termes or Phraſes be proper to *Arithmetick*: For all things here are to be taken in congruity thereto. Wherefore a Perfect Number ſhall not be Chymically, but Arithmetically ſo; for with the *Chymiſts* 10 is a Perfect Number. *Chymical Collections*, *p.* 92. but ſeeing the Aliquot parts of 10 will make but 8, Arithmeticians count it Imperfect and Detective.

10ly. It's more than probable, That in the Firſt Chapter of *Geodæticals* ſome of the Diviſions of Foreign or Domeſtick Denominations may have loſt, or in time may loſe much of their Propriety to the preſent State of Affairs, or new Accompts of ſuch things as there declared, they having been adapted to the Elder Laws, Cuſtoms, and Uſages of Kingdoms and Countries, yet mutable and alterable as Reaſons of State or other Contingencies in every Kingdom, or Countrey ſhall or may enforce. Wherefore (this Exception, if not provided againſt by ſufficient Caution and warning thereof given in the Chapter it ſelf) let allowances be made as occaſion ſhall require, either by taking them in the *Præterperfect-Tenſe*, or making all not certainly known, but Suppoſitions, which nevertheleſs will not prejudice the truth of the Concluſions where ſuch Suppoſitions are but Conditionals, ſince it is not neceſſary in the Reſolution of a Queſtion, that the Suppoſitions be true, but that the Concluſion be true according to ſuch a Conditional Suppoſition. For if 120 be counted for an Hundred, then 2 Hundred muſt be 240, but it will not follow that the Hundred muſt alwaies be reckoned at that rate.

11ly. No Man may ſtumble at the uſe of the Word Infinite or endleſs, it being intended only beyond the Power or Skill of Man to count or caſt up, not Infinite in a proper or abſtract Sence, for ſo nothing is

Infinite but God, and it is impoſſible there ſhould be two Infinites.

And laſtly, if beſides all this, the Lawyers will favourably overlook the unintentional miſquoting or repreſenting any Statute or Law and Conſtruction thereof, (if any ſuch be,) And Merchants, Goldſmiths, and other Artificers will bear with the Termes in Queſtions of their particular Concerns, as poſſibly not ſo proper to their Profeſſions as others beſt known to themſelves, And Grammarians add their uſual Grains of Allowance (due on the Score of humane frailty) for miſplacing of Letters, miſſpelling of Syllables, miſpointing of Sentences, Omiſſion of Points, Parentheſes, *&c.* There cannot be much left to need excuſe; but if there be, it muſt now ſtand or fall to it's own Maſter.

Whilſt we all live in the Atmoſphære, no doubt but in a clear day ſome Motes may be ſeen; and conſidering the Imperfections of the Penman, diſcompoſures and diſadvantages under which, moſt was wrote or tranſcribed (enough to have diſtracted a more accurate Accomptant and abler Pen:) 'tis well, if there be no more faults, than the unprejudiced Reader in common Charity can or ought to remit or paſs by. And inaſmuch as none that may be found are wilful or intentional, I will not be ſo uncharitable as to think I ſhall want their Charity or candid Conſtruction thereof.

I never thought *Humanum eſt errare*, was, or ever will be a warrant or plea ſufficient to commit or excuſe the commiſſion of Wilfull Tranſgreſſions; but for involuntary Errors, Miſtakes, and Aberrations, it hath always been accompted ſufficient to cover them when committed.

To pleaſe or diſpleaſe, is an inconſiderable and unprofitable end in Writing, and they that aim at the former commonly miſs. It was ſaid of old, *Ne Jupiter quidem omnibus placet.* I write neither to praiſe or diſpraiſe, flatter or beſpatter any. Some who being dead yet ſpeak by their Writings I have here and there as worthy of Commendation valued, and am yet of Opinion this Nation ſtands much obliged among others to *Record* of old, and *Oughtred* of late, for their labours in this Art; the former for his plainneſs and clearneſs in thoſe things he hath handled, and the latter for his piercing apprehenſion into the more lofty and myſterious parts of the Mathematicks: But none dead or living have I any where wrongfully charged, or mentioned without due reverence to their Perſons and reſpect to their works. So far as I might not contract a guilt of leading or miſleading others into a dark unpleaſant and perplexed Path where one more plain was nearer hand. And many of their Examples (whether corruptly wrought or printed I cannot ſay) I have purpoſely choſen to correct the Errors found therein, that if this

Impreſſion eſcape the *Errata* will need no further amendment.

However whether their Lines or theſe be faulty, it is not ſo much material, if no blemiſh thereby be imputed to the Art it ſelf. *Arithmetick* is a Noble and high born Science, uſeful and profitable in ſeveral things both divine and humane, lends to many, but borrows of few, and hath midwifed into the World divers excellent Atchievements, which without her aid and aſſiſtance would have been Still-born, and ſlept an Everlaſting Sleep. And becauſe a fit Encomium thereof deſerves a moſt clear and unclouded capacity I ſhall deſiſt. Nor ſhall I undertake to exhibit a Narrative of the riſe, growth, production or propagation of *Arithmetick*, nor yet to catalogize ſo much as the *Engliſh* Authors thereof; all I have further to ſay is

Whoever would profit hereby, muſt begin at the beginning, and go through every part gradually, but (if like moſt Scholars a little will content) the Two Firſt Books, and the Second Part of the Fourth, will furniſh the Learner for Merchants Affairs Trade and ordinary Commerce in a good meaſure; the other is written *Non cuivis*, but for thoſe who ſhall not think their labour loſt in ſuch Speculations, to whom though *Studiorum radices amaræ*, yet *fructus erunt ſapidi*.

Thus (my Son) as in ſome Projections of *Geometry* and *Aſtronomy* my thoughts have out-done me, ſo in this of *Arithmetick* I have out-done my thoughts in the length both of the Epiſtle and Book; to ſhorten which I have forborne to exemplifie the Prints of the ſeveral Coines mentioned in the Firſt Chapter of *Geodæticals*, and to purſue my intentions as well about the Addition and Subſtraction of *Surdes*, eſpecially of Univerſals, as the Diviſion of *Surdes* and *Species*, wherein ſomethings might not have been unprofitably added, as likewiſe in *Progreſſion* of both kinds by Inſtances in *Fractions* and *Decimals*: Nevertheleſs what is written may be ſufficient to the ingenuous, and therefore I ſhall ſay no more but conclude, The Lord bleſs thee and it, and make both inſtrumental to his glory, which is and will be the hearty and earneſt Prayer of

Rye, March 25th.
1674.

Thy Loving Father

SA. JEAKE.

Books principally conſulted with in the enſuing Treatiſe, beſides the Sacred Scriptures, divers Hiſtories, Lexicons, Dictionaries, *&c.*

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Richard	*Balam.*	Algebra.	*Lond.*	1653.
Iſaac	*Barrow.*	*Euclide*'s Elements.	*Lond.*	1660.
Thomas	*Blundevile.*	Exerciſes.		
Henry	*Brigges.*	Logarithmica Arithmetica.	*Lond.*	1624.
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Michael	*Dalton.*	The Countrey Juſtice.	*Lond.*	1643.
Michael	*Dary.*	*Dary*'s Diary.	*Lond.*	1650.
John	*Dee.*	Mathematical Preface, *&c.*	*Lond.*	1651.
Richard	*Delamain.*	Grammelogia.	*Lond.*	1630.
Thomas	*Digges.*	Pantometria.	*Lond.*	1591.
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William	*Eldred.*	The Gunners Glaſs.	*Lond.*	1646.
Henry Van	*Etten.*	Mathematical Recreations.	*Lond.*	1633.
Giles	*Fletcher.*	Hiſtory of *Ruſſia.*	*Lond.*	1657.
Thomas	*Fuller.*	Piſgah Sight of *Paleſtine.*	*Lond.*	1650.
Edward	*Grimſtone.*	Eſtates *&c.* of the World.	*Lond.*	1615.
Edmund	*Gunter.*	Works, *&c.* of the *Sector*, *&c.*	*Lond.*	1653.
Peter	*Heylin.*	Coſmography.	*Lond.*	1657.
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John	*Johnſon.*	*Johnſon*'s Arithmetick.	*Lond.*	1657.
Gerard	*Malines.*	Lex Mercatoria.	*Lond.*	1636.
Jonas	*Moore.*	*Moore*'s Arithmetick.	*Lond.*	1650.
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William	*Oughtred.*	Clavis Mathematicæ limata.	*Oxon.*	1652.
		Circles of Proportion.	*Oxford*	1660.
Seth	*Partridge.*	Rabdologia.	*Lond.*	1648.
Fardinando	*Pulton.*	Collection of Statutes.	*Lond.*	1640.
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Robert	*Record.*	Ground of Arts.	*Lond.*	1636.
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John	*Speidel.*	Treatiſe of Sphærical Triangles.	*Lond.*	1627.
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Arithmetick

The Sexagenary Table .

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0. 1	0. 2	0. 3	0. 4	0. 5	0. 6	0. 7	0. 8	0. 9	0. 10	0. 11	0. 12	0. 13	0. 14	0. 15	0. 16	0. 17	0. 18	0. 19	0. 20	0. 21	0. 22	0. 23	0. 24	0. 25	0. 26	0. 27	0. 28	0. 29	0. 30
2	0. 2	0. 4	0. 6	0. 8	0. 10	0. 12	0. 14	0. 16	0. 18	0. 20	0. 22	0. 24	0. 26	0. 28	0. 30	0. 32	0. 34	0. 36	0. 38	0. 40	0. 42	0. 44	0. 46	0. 48	0. 50	0. 52	0. 54	0. 56	0. 58	1. 0
3	0. 3	0. 6	0. 9	0. 12	0. 15	0. 18	0. 21	0. 24	0. 27	0. 30	0. 33	0. 36	0. 39	0. 42	0. 45	0. 48	0. 51	0. 54	0. 57	1. 0	1. 3	1. 6	1. 9	1. 12	1. 15	1. 18	1. 21	1. 24	1. 27	1. 30
4	0. 4	0. 8	0. 12	0. 16	0. 20	0. 24	0. 28	0. 32	0. 36	0. 40	0. 44	0. 48	0. 52	0. 56	1. 0	1. 4	1. 8	1. 12	1. 16	1. 20	1. 24	1. 28	1. 32	1. 36	1. 40	1. 44	1. 48	1. 52	1. 56	2. 0
5	0. 5	0. 10	0. 15	0. 20	0. 25	0. 30	0. 35	0. 40	0. 45	0. 50	0. 55	1. 0	1. 5	1. 10	1. 15	1. 20	1. 25	1. 30	1. 35	1. 40	1. 45	1. 50	1. 55	2. 0	2. 5	2. 10	2. 15	2. 20	2. 25	2. 30
6	0. 6	0. 12	0. 18	0. 24	0. 30	0. 36	0. 42	0. 48	0. 54	1. 0	1. 6	1. 12	1. 18	1. 24	1. 30	1. 36	1. 42	1. 48	1. 54	2. 0	2. 6	2. 12	2. 18	2. 24	2. 30	2. 36	2. 42	2. 48	2. 54	3. 0
7	0. 7	0. 14	0. 21	0. 28	0. 35	0. 42	0. 49	0. 56	1. 3	1. 10	1. 17	1. 24	1. 31	1. 38	1. 45	1. 52	1. 59	2. 6	2. 13	2. 20	2. 27	2. 34	2. 41	2. 48	2. 55	3. 2	3. 9	3. 16	3. 23	3. 30
8	0. 8	0. 16	0. 24	0. 32	0. 40	0. 48	0. 56	1. 4	1. 12	1. 20	1. 28	1. 36	1. 44	1. 52	2. 0	2. 8	2. 16	2. 24	2. 32	2. 40	2. 48	2. 56	3. 4	3. 12	3. 20	3. 28	3. 36	3. 44	3. 52	4. 0
9	0. 9	0. 18	0. 27	0. 36	0. 45	0. 54	1. 3	1. 12	1. 21	1. 30	1. 39	1. 48	1. 57	2. 6	2. 15	2. 24	2. 33	2. 42	2. 51	3. 0	3. 9	3. 18	3. 27	3. 36	3. 45	3. 54	4. 3	4. 12	4. 21	4. 30
10	0. 10	0. 20	0. 30	0. 40	0. 50	1. 0	1. 10	1. 20	1. 30	1. 40	1. 50	2. 0	2. 10	2. 20	2. 30	2. 40	2. 50	3. 0	3. 10	3. 20	3. 30	3. 40	3. 50	4. 0	4. 10	4. 20	4. 30	4. 40	4. 50	5. 0
11	0. 11	0. 22	0. 33	0. 44	0. 55	1. 6	1. 17	1. 28	1. 39	1. 50	2. 1	2. 12	2. 23	2. 34	2. 45	2. 56	3. 7	3. 18	3. 29	3. 40	3. 51	4. 2	4. 13	4. 24	4. 35	4. 46	4. 57	5. 8	5. 19	5. 30
12	0. 12	0. 24	0. 36	0. 48	1. 0	1. 12	1. 24	1. 36	1. 48	2. 0	2. 12	2. 24	2. 36	2. 48	3. 0	3. 12	3. 24	3. 36	3. 48	4. 0	4. 12	4. 24	4. 36	4. 48	5. 0	5. 12	5. 24	5. 36	5. 48	6. 0
13	0. 13	0. 26	0. 39	0. 52	1. 5	1. 18	1. 31	1. 44	1. 57	2. 10	2. 23	2. 36	2. 49	3. 2	3. 15	3. 28	3. 41	3. 54	4. 7	4. 20	4. 33	4. 46	4. 59	5. 12	5. 25	5. 38	5. 51	6. 4	6. 17	6. 30
14	0. 14	0. 28	0. 42	0. 56	1. 10	1. 24	1. 38	1. 52	2. 6	2. 20	2. 34	2. 48	3. 2	3. 16	3. 30	3. 44	3. 58	4. 12	4. 26	4. 40	4. 54	5. 8	5. 22	5. 36	5. 50	6. 4	6. 18	6. 32	6. 46	7. 0
15	0. 15	0. 30	0. 45	1. 0	1. 15	1. 30	1. 45	2. 0	2. 15	2. 30	2. 45	3. 0	3. 15	3. 30	3. 45	4. 0	4. 15	4. 30	4. 45	5. 0	5. 15	5. 30	5. 45	6. 0	6. 15	6. 30	6. 45	7. 0	7. 15	7. 30
16	0. 16	0. 32	0. 48	1. 4	1. 20	1. 36	1. 52	2. 8	2. 24	2. 40	2. 56	3. 12	3. 28	3. 44	4. 0	4. 16	4. 32	4. 48	5. 4	5. 20	5. 36	5. 52	6. 8	6. 24	6. 40	6. 56	7. 12	7. 28	7. 44	8. 0
17	0. 17	0. 34	0. 51	1. 8	1. 25	1. 42	1. 59	2. 16	2. 33	2. 50	3. 7	3. 24	3. 41	3. 58	4. 15	4. 32	4. 49	5. 6	5. 23	5. 40	5. 57	6. 14	6. 31	6. 48	7. 5	7. 22	7. 39	7. 56	8. 13	8. 30
18	0. 18	0. 36	0. 54	1. 12	1. 30	1. 48	2. 6	2. 24	2. 42	3. 0	3. 18	3. 36	3. 54	4. 12	4. 30	4. 48	5. 6	5. 24	5. 42	6. 0	6. 18	6. 36	6. 54	7. 12	7. 30	7. 48	8. 6	8. 24	8. 42	9. 0
19	0. 19	0. 38	0. 57	1. 16	1. 35	1. 54	2. 13	2. 32	2. 51	3. 10	3. 29	3. 48	4. 7	4. 26	4. 45	5. 4	5. 23	5. 42	6. 1	6. 20	6. 39	6. 58	7. 17	7. 36	7. 55	8. 14	8. 33	8. 52	9. 11	9. 30
20	0. 20	0. 40	1. 0	1. 20	1. 40	2. 0	2. 20	2. 40	3. 0	3. 20	3. 40	4. 0	4. 20	4. 40	5. 0	5. 20	5. 40	6. 0	6. 20	6. 40	7. 0	7. 20	7. 40	8. 0	8. 20	8. 40	9. 0	9. 20	9. 40	10. 0
21	0. 21	0. 42	1. 3	1. 24	1. 45	2. 6	2. 27	2. 48	3. 9	3. 30	3. 51	4. 12	4. 33	4. 54	5. 15	5. 36	5. 57	6. 18	6. 39	7. 0	7. 21	7. 42	8. 3	8. 24	8. 45	9. 6	9. 27	9. 48	10. 9	10. 30
22	0. 22	0. 44	1. 6	1. 28	1. 50	2. 12	2. 34	2. 56	3. 18	3. 40	4. 2	4. 24	4. 46	5. 8	5. 30	5. 52	6. 14	6. 36	6. 58	7. 20	7. 42	8. 4	8. 26	8. 48	9. 10	9. 32	9. 54	10. 16	10. 38	11. 0
23	0. 23	0. 46	1. 9	1. 32	1. 55	2. 18	2. 41	3. 4	3. 27	3. 50	4. 13	4. 36	4. 59	5. 22	5. 45	6. 8	6. 31	6. 54	7. 17	7. 40	8. 3	8. 26	8. 49	9. 12	9. 35	9. 58	10. 21	10. 44	11. 7	11. 30
24	0. 24	0. 48	1. 12	1. 36	2. 0	2. 24	2. 48	3. 12	3. 36	4. 0	4. 24	4. 48	5. 12	5. 36	6. 0	6. 24	6. 48	7. 12	7. 36	8. 0	8. 24	8. 48	9. 12	9. 36	10. 0	10. 24	10. 48	11. 12	11. 36	12. 0
25	0. 25	0. 50	1. 15	1. 40	2. 5	2. 30	2. 55	3. 20	3. 45	4. 10	4. 35	5. 0	5. 25	5. 50	6. 15	6. 40	7. 5	7. 30	7. 55	8. 20	8. 45	9. 10	9. 35	10. 0	10. 25	10. 50	11. 15	11. 40	12. 5	12. 30
26	0. 26	0. 52	1. 18	1. 44	2. 10	2. 36	3. 2	3. 28	3. 54	4. 20	4. 46	5. 12	5. 38	6. 4	6. 30	6. 56	7. 22	7. 48	8. 14	8. 40	9. 6	9. 32	9. 58	10. 24	10. 50	11. 16	11. 42	12. 8	12. 34	13. 0
27	0. 27	0. 54	1. 21	1. 48	2. 15	2. 42	3. 9	3. 36	4. 3	4. 30	4. 57	5. 24	5. 51	6. 18	6. 45	7. 12	7. 39	8. 6	8. 33	9. 0	9. 27	9. 54	10. 21	10. 48	11. 15	11. 42	12. 9	12. 36	13. 3	13. 30
28	0. 28	0. 56	1. 24	1. 52	2. 20	2. 48	3. 16	3. 44	4. 12	4. 40	5. 8	5. 36	6. 4	6. 32	7. 0	7. 28	7. 56	8. 24	8. 52	9. 20	9. 48	10. 16	10. 44	11. 12	11. 40	12. 8	12. 36	13. 4	13. 32	14. 0
29	0. 29	0. 58	1. 27	1. 56	2. 25	2. 54	3. 23	3. 52	4. 21	4. 50	5. 19	5. 48	6. 17	6. 46	7. 15	7. 44	8. 13	8. 42	9. 11	9. 40	10. 9	10. 38	11. 7	11. 36	12. 5	12. 34	13. 3	13. 32	14. 1	14. 30
30	0. 30	1. 0	1. 30	2. 0	2. 30	3. 0	3. 30	4. 0	4. 30	5. 0	5. 30	6. 0	6. 30	7. 0	7. 30	8. 0	8. 30	9. 0	9. 30	10. 0	10. 30	11. 0	11. 30	12. 0	12. 30	13. 0	13. 30	14. 0	14. 30	15. 0
31	0. 31	1. 2	1. 33	2. 4	2. 35	3. 6	3. 37	4. 8	4. 39	5. 10	5. 41	6. 12	6. 43	7. 14	7. 45	8. 16	8. 47	9. 18	9. 49	10. 20	10. 51	11. 22	11. 53	12. 24	12. 55	13. 26	13. 57	14. 28	14. 59	15. 30
32	0. 32	1. 4	1. 36	2. 8	2. 40	3. 12	3. 44	4. 16	4. 48	5. 20	5. 52	6. 24	6. 56	7. 28	8. 0	8. 32	9. 4	9. 36	10. 8	10. 40	11. 12	11. 44	12. 16	12. 48	13. 20	13. 52	14. 24	14. 56	15. 28	16. 0
33	0. 33	1. 6	1. 39	2. 12	2. 45	3. 18	3. 51	4. 24	4. 57	5. 30	6. 3	6. 36	7. 9	7. 42	8. 15	8. 48	9. 21	9. 54	10. 27	11. 0	11. 33	12. 6	12. 39	13. 12	13. 45	14. 18	14. 51	15. 24	15. 57	16. 30
34	0. 34	1. 8	1. 42	2. 16	2. 50	3. 24	3. 58	4. 32	5. 6	5. 40	6. 14	6. 48	7. 22	7. 56	8. 30	9. 4	9. 38	10. 12	10. 46	11. 20	11. 54	12. 28	13. 2	13. 36	14. 10	14. 44	15. 18	15. 52	16. 26	17. 0
35	0. 35	1. 10	1. 45	2. 20	2. 55	3. 30	4. 5	4. 40	5. 15	5. 50	6. 25	7. 0	7. 35	8. 10	8. 45	9. 20	9. 55	10. 30	11. 5	11. 40	12. 15	12. 50	13. 25	14. 0	14. 35	15. 10	15. 45	16. 20	16. 55	17. 30
36	0. 36	1. 12	1. 48	2. 24	3. 0	3. 36	4. 12	4. 48	5. 24	6. 0	6. 36	7. 12	7. 48	8. 24	9. 0	9. 36	10. 12	10. 48	11. 24	12. 0	12. 36	13. 12	13. 48	14. 24	15. 0	15. 36	16. 12	16. 48	17. 24	18. 0
37	0. 37	1. 14	1. 51	2. 28	3. 5	3. 42	4. 19	4. 56	5. 33	6. 10	6. 47	7. 24	8. 1	8. 38	9. 15	9. 52	10. 29	11. 6	11. 43	12. 20	12. 57	13. 34	14. 11	14. 48	15. 25	16. 2	16. 39	17. 16	17. 53	18. 30
38	0. 38	1. 16	1. 54	2. 32	3. 10	3. 48	4. 26	5. 4	5. 42	6. 20	6. 58	7. 36	8. 14	8. 52	9. 30	10. 8	10. 46	11. 24	12. 2	12. 40	13. 18	13. 56	14. 34	15. 12	15. 50	16. 28	17. 6	17. 44	18. 22	19. 0
39	0. 39	1. 18	1. 57	2. 36	3. 15	3. 54	4. 33	5. 12	5. 51	6. 30	7. 9	7. 48	8. 27	9. 6	9. 45	10. 24	11. 3	11. 42	12. 21	13. 0	13. 39	14. 18	14. 57	15. 36	16. 15	16. 54	17. 33	18. 12	18. 51	19. 30
40	0. 40	1. 20	2. 0	2. 40	3. 20	4. 0	4. 40	5. 20	6. 0	6. 40	7. 20	8. 0	8. 40	9. 20	10. 0	10. 40	11. 20	12. 0	12. 40	13. 20	14. 0	14. 40	15. 20	16. 0	16. 40	17. 20	18. 0	18. 40	19. 20	20. 0
41	0. 41	1. 22	2. 3	2. 44	3. 25	4. 6	4. 47	5. 28	6. 9	6. 50	7. 31	8. 12	8. 53	9. 34	10. 15	10. 56	11. 37	12. 18	12. 59	13. 40	14. 21	15. 2	15. 43	16. 24	17. 5	17. 46	18. 27	19. 8	19. 49	20. 30
42	0. 42	1. 24	2. 6	2. 48	3. 30	4. 12	4. 54	5. 36	6. 18	7. 0	7. 42	8. 24	9. 6	9. 48	10. 30	11. 12	11. 54	12. 36	13. 18	14. 0	14. 42	15. 24	16. 6	16. 48	17. 30	18. 12	18. 54	19. 36	20. 18	21. 0
43	0. 43	1. 26	2. 9	2. 52	3. 35	4. 18	5. 1	5. 44	6. 27	7. 10	7. 53	8. 36	9. 19	10. 2	10. 45	11. 28	12. 11	12. 54	13. 37	14. 20	15. 3	15. 46	16. 29	17. 12	17. 55	18. 38	19. 21	20. 4	20. 47	21. 30
44	0. 44	1. 28	2. 12	2. 56	3. 40	4. 24	5. 8	5. 52	6. 36	7. 20	8. 4	8. 48	9. 32	10. 16	11. 0	11. 44	12. 28	13. 12	13. 56	14. 40	15. 24	16. 8	16. 52	17. 36	18. 20	19. 4	19. 48	20. 32	21. 16	22. 0
45	0. 45	1. 30	2. 15	3. 0	3. 45	4. 30	5. 15	6. 0	6. 45	7. 30	8. 15	9. 0	9. 45	10. 30	11. 15	12. 0	12. 45	13. 30	14. 15	15. 0	15. 45	16. 30	17. 15	18. 0	18. 45	19. 30	20. 15	21. 0	21. 45	22. 30
46	0. 46	1. 32	2. 18	3. 4	3. 50	4. 36	5. 22	6. 8	6. 54	7. 40	8. 26	9. 12	9. 58	10. 44	11. 30	12. 16	13. 2	13. 48	14. 34	15. 20	16. 6	16. 52	17. 38	18. 24	19. 10	19. 56	20. 42	21. 28	22. 14	23. 0
47	0. 47	1. 34	2. 21	3. 8	3. 55	4. 42	5. 29	6. 16	7. 3	7. 50	8. 37	9. 24	10. 11	10. 58	11. 45	12. 32	13. 19	14. 6	14. 53	15. 40	16. 27	17. 14	18. 1	18. 48	19. 35	20. 22	21. 9	21. 56	22. 43	23. 30
48	0. 48	1. 36	2. 24	3. 12	4. 0	4. 48	5. 36	6. 24	7. 12	8. 0	8. 48	9. 36	10. 24	11. 12	12. 0	12. 48	13. 36	14. 24	15. 12	16. 0	16. 48	17. 36	18. 24	19. 12	20. 0	20. 48	21. 36	22. 24	23. 12	24. 0
49	0. 49	1. 38	2. 27	3. 16	4. 5	4. 54	5. 43	6. 32	7. 21	8. 10	8. 59	9. 48	10. 37	11. 26	12. 15	13. 4	13. 53	14. 42	15. 31	16. 20	17. 9	17. 58	18. 47	19. 36	20. 25	21. 14	22. 3	22. 52	23. 41	24. 30
50	0. 50	1. 40	2. 30	3. 20	4. 10	5. 0	5. 50	6. 40	7. 30	8. 20	9. 10	10. 0	10. 50	11. 40	12. 30	13. 20	14. 10	15. 0	15. 50	16. 40	17. 30	18. 20	19. 10	20. 0	20. 50	21. 40	22. 30	23. 20	24. 10	25. 0
51	0. 51	1. 42	2. 33	3. 24	4. 15	5. 6	5. 57	6. 48	7. 39	8. 30	9. 21	10. 12	11. 3	11. 54	12. 45	13. 36	14. 27	15. 18	16. 9	17. 0	17. 51	18. 42	19. 33	20. 24	21. 15	22. 6	22. 57	23. 48	24. 39	25. 30
52	0. 52	1. 44	2. 36	3. 28	4. 20	5. 12	6. 4	6. 56	7. 48	8. 40	9. 32	10. 24	11. 16	12. 8	13. 0	13. 52	14. 44	15. 36	16. 28	17. 20	18. 12	19. 4	19. 56	20. 48	21. 40	22. 32	23. 24	24. 16	25. 8	26. 0
53	0. 53	1. 46	2. 39	3. 32	4. 25	5. 18	6. 11	7. 4	7. 57	8. 50	9. 43	10. 36	11. 29	12. 22	13. 15	14. 8	15. 1	15. 54	16. 47	17. 40	18. 33	19. 26	20. 19	21. 12	22. 5	22. 58	23. 51	24. 44	25. 37	26. 30
54	0. 54	1. 48	2. 42	3. 36	4. 30	5. 24	6. 18	7. 12	8. 6	9. 0	9. 54	10. 48	11. 42	12. 36	13. 30	14. 24	15. 18	16. 12	17. 6	18. 0	18. 54	19. 48	20. 42	21. 36	22. 30	23. 24	24. 18	25. 12	26. 6	27. 0
55	0. 55	1. 50	2. 45	3. 40	4. 35	5. 30	6. 25	7. 20	8. 15	9. 10	10. 5	11. 0	11. 55	12. 50	13. 45	14. 40	15. 35	16. 30	17. 25	18. 20	19. 15	20. 10	21. 5	22. 0	22. 55	23. 50	24. 45	25. 40	26. 35	27. 30
56	0. 56	1. 52	2. 48	3. 44	4. 40	5. 36	6. 32	7. 28	8. 24	9. 20	10. 16	11. 12	12. 8	13. 4	14. 0	14. 56	15. 52	16. 48	17. 44	18. 40	19. 36	20. 32	21. 28	22. 24	23. 20	24. 16	25. 12	26. 8	27. 4	28. 0
57	0. 57	1. 54	2. 51	3. 48	4. 45	5. 42	6. 39	7. 36	8. 33	9. 30	10. 27	11. 24	12. 21	13. 18	14. 15	15. 12	16. 9	17. 6	18. 3	19. 0	19. 57	20. 54	21. 51	22. 48	23. 45	24. 42	25. 39	26. 36	27. 33	28. 30
58	0. 58	1. 56	2. 54	3. 52	4. 50	5. 48	6. 46	7. 44	8. 42	9. 40	10. 38	11. 36	12. 34	13. 32	14. 30	15. 28	16. 26	17. 24	18. 22	19. 20	20. 18	21. 16	22. 14	23. 12	24. 10	25. 8	26. 6	27. 4	28. 2	29. 0
59	0. 59	1. 58	2. 57	3. 56	4. 55	5. 54	6. 53	7. 52	8. 51	9. 50	10. 49	11. 48	12. 47	13. 46	14. 45	15. 44	16. 43	17. 42	18. 41	19. 40	20. 39	21. 38	22. 37	23. 36	24. 35	25. 34	26. 33	27. 32	28. 31	29. 30
60	1. 0	2. 0	3. 0	4. 0	5. 0	6. 0	7. 0	8. 0	9. 0	10. 0	11. 0	12. 0	13. 0	14. 0	15. 0	16. 0	17. 0	18. 0	19. 0	20. 0	21. 0	22. 0	23. 0	24. 0	25. 0	26. 0	27. 0	28. 0	29. 0	30. 0

The Sexagenary Table.

30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
0. 30	0. 31	0. 32	0. 33	0. 34	0. 35	0. 36	0. 37	0. 38	0. 39	0. 40	0. 41	0. 42	0. 43	0. 44	0. 45	0. 46	0. 47	0. 48	0. 49	0. 50	0. 51	0. 52	0. 53	0. 54	0. 55	0. 56	0. 57	0. 58	0. 59	1. 0
1. 0	1. 2	1. 4	1. 6	1. 8	1. 10	1. 12	1. 14	1. 16	1. 18	1. 20	1. 22	1. 24	1. 26	1. 28	1. 30	1. 32	1. 34	1. 36	1. 38	1. 40	1. 42	1. 44	1. 46	1. 48	1. 50	1. 52	1. 54	1. 56	1. 58	2. 0
1. 30	1. 33	1. 36	1. 39	1. 42	1. 45	1. 48	1. 51	1. 54	1. 57	2. 0	2. 3	2. 6	2. 9	2. 12	2. 15	2. 18	2. 21	2. 24	2. 27	2. 30	2. 33	2. 36	2. 39	2. 42	2. 45	2. 48	2. 51	2. 54	2. 57	3. 0
2. 0	2. 4	2. 8	2. 12	2. 16	2. 20	2. 24	2. 28	2. 32	2. 36	2. 40	2. 44	2. 48	2. 52	2. 56	3. 0	3. 4	3. 8	3. 12	3. 16	3. 20	3. 24	3. 28	3. 32	3. 36	3. 40	3. 44	3. 48	3. 52	3. 56	4. 0
2. 30	2. 35	2. 40	2. 45	2. 50	2. 55	3. 0	3. 5	3. 10	3. 15	3. 20	3. 25	3. 30	3. 35	3. 40	3. 45	3. 50	3. 55	4. 0	4. 5	4. 10	4. 15	4. 20	4. 25	4. 30	4. 35	4. 40	4. 45	4. 50	4. 55	5. 0
3. 0	3. 6	3. 12	3. 18	3. 24	3. 30	3. 36	3. 42	3. 48	3. 54	4. 0	4. 6	4. 12	4. 18	4. 24	4. 30	4. 36	4. 42	4. 48	4. 54	5. 0	5. 6	5. 12	5. 18	5. 24	5. 30	5. 36	5. 42	5. 48	5. 54	6. 0
3. 30	3. 37	3. 44	3. 51	3. 58	4. 5	4. 12	4. 19	4. 26	4. 33	4. 40	4. 47	4. 54	5. 1	5. 8	5. 15	5. 22	5. 29	5. 36	5. 43	5. 50	5. 57	6. 4	6. 11	6. 18	6. 25	6. 32	6. 39	6. 46	6. 53	7. 0
4. 0	4. 8	4. 16	4. 24	4. 32	4. 40	4. 48	4. 56	5. 4	5. 12	5. 20	5. 28	5. 36	5. 44	5. 52	6. 0	6. 8	6. 16	6. 24	6. 32	6. 40	6. 48	6. 56	7. 4	7. 12	7. 20	7. 28	7. 36	7. 44	7. 52	8. 0
4. 30	4. 39	4. 48	4. 57	5. 6	5. 15	5. 24	5. 33	5. 42	5. 51	6. 0	6. 9	6. 18	6. 27	6. 36	6. 45	6. 54	7. 3	7. 12	7. 21	7. 30	7. 39	7. 48	7. 57	8. 6	8. 15	8. 24	8. 33	8. 42	8. 51	9. 0
5. 0	5. 10	5. 20	5. 30	5. 40	5. 50	6. 0	6. 10	6. 20	6. 30	6. 40	6. 50	7. 0	7. 10	7. 20	7. 30	7. 40	7. 50	8. 0	8. 10	8. 20	8. 30	8. 40	8. 50	9. 0	9. 10	9. 20	9. 30	9. 40	9. 50	10. 0
5. 30	5. 41	5. 52	6. 3	6. 14	6. 25	6. 36	6. 47	6. 58	7. 9	7. 20	7. 31	7. 42	7. 53	8. 4	8. 15	8. 26	8. 37	8. 48	8. 59	9. 10	9. 21	9. 32	9. 43	9. 54	10. 5	10. 16	10. 27	10. 38	10. 49	11. 0
6. 0	6. 12	6. 24	6. 36	6. 48	7. 0	7. 12	7. 24	7. 36	7. 48	8. 0	8. 12	8. 24	8. 36	8. 48	9. 0	9. 12	9. 24	9. 36	9. 48	10. 0	10. 12	10. 24	10. 36	10. 48	11. 0	11. 12	11. 24	11. 36	11. 48	12. 0
6. 30	6. 43	6. 56	7. 9	7. 22	7. 35	7. 48	8. 1	8. 14	8. 27	8. 40	8. 53	9. 6	9. 19	9. 32	9. 45	9. 58	10. 11	10. 24	10. 37	10. 50	11. 3	11. 16	11. 29	11. 42	11. 55	12. 8	12. 21	12. 34	12. 47	13. 0
7. 0	7. 14	7. 28	7. 42	7. 56	8. 10	8. 24	8. 38	8. 52	9. 6	9. 20	9. 34	9. 48	10. 2	10. 16	10. 30	10. 44	10. 58	11. 12	11. 26	11. 40	11. 54	12. 8	12. 22	12. 36	12. 50	13. 4	13. 18	13. 32	13. 46	14. 0
7. 30	7. 45	8. 0	8. 15	8. 30	8. 45	9. 0	9. 15	9. 30	9. 45	10. 0	10. 15	10. 30	10. 45	11. 0	11. 15	11. 30	11. 45	12. 0	12. 15	12. 30	12. 45	13. 0	13. 15	13. 30	13. 45	14. 0	14. 15	14. 30	14. 45	15. 0
8. 0	8. 16	8. 32	8. 48	9. 4	9. 20	9. 36	9. 52	10. 8	10. 24	10. 40	10. 56	11. 12	11. 28	11. 44	12. 0	12. 16	12. 32	12. 48	13. 4	13. 20	13. 36	13. 52	14. 8	14. 24	14. 40	14. 56	15. 12	15. 28	15. 44	16. 0
8. 30	8. 47	9. 4	9. 21	9. 38	9. 55	10. 12	10. 29	10. 46	11. 3	11. 20	11. 37	11. 54	12. 11	12. 28	12. 45	13. 2	13. 19	13. 36	13. 53	14. 10	14. 27	14. 44	15. 1	15. 18	15. 35	15. 52	16. 9	16. 26	16. 43	17. 0
9. 0	9. 18	9. 36	9. 54	10. 12	10. 30	10. 48	11. 6	11. 24	11. 42	12. 0	12. 18	12. 36	12. 54	13. 12	13. 30	13. 48	14. 6	14. 24	14. 42	15. 0	15. 18	15. 36	15. 54	16. 12	16. 30	16. 48	17. 6	17. 24	17. 42	18. 0
9. 30	9. 49	10. 8	10. 27	10. 46	11. 5	11. 24	11. 43	12. 2	12. 21	12. 40	12. 59	13. 18	13. 37	13. 56	14. 15	14. 34	14. 53	15. 12	15. 31	15. 50	16. 9	16. 28	16. 47	17. 6	17. 25	17. 44	18. 3	18. 22	18. 41	19. 0
10. 0	10. 20	10. 40	11. 0	11. 20	11. 40	12. 0	12. 20	12. 40	13. 0	13. 20	13. 40	14. 0	14. 20	14. 40	15. 0	15. 20	15. 40	16. 0	16. 20	16. 40	17. 0	17. 20	17. 40	18. 0	18. 20	18. 40	19. 0	19. 20	19. 40	20. 0
10. 30	10. 51	11. 12	11. 33	11. 54	12. 15	12. 36	12. 57	13. 18	13. 39	14. 0	14. 21	14. 42	15. 3	15. 24	15. 45	16. 6	16. 27	16. 48	17. 9	17. 30	17. 51	18. 12	18. 33	18. 54	19. 15	19. 36	19. 57	20. 18	20. 39	21. 0
11. 0	11. 22	11. 44	12. 6	12. 28	12. 50	13. 12	13. 34	13. 56	14. 18	14. 40	15. 2	15. 24	15. 46	16. 8	16. 30	16. 52	17. 14	17. 36	17. 58	18. 20	18. 42	19. 4	19. 26	19. 48	20. 10	20. 32	20. 54	21. 16	21. 38	22. 0
11. 30	11. 53	12. 16	12. 39	13. 2	13. 25	13. 48	14. 11	14. 34	14. 57	15. 20	15. 43	16. 6	16. 29	16. 52	17. 15	17. 38	18. 1	18. 24	18. 47	19. 10	19. 33	19. 56	20. 19	20. 42	21. 5	21. 28	21. 51	22. 14	22. 37	23. 0
12. 0	12. 24	12. 48	13. 12	13. 36	14. 0	14. 24	14. 48	15. 12	15. 36	16. 0	16. 24	16. 48	17. 12	17. 36	18. 0	18. 24	18. 48	19. 12	19. 36	20. 0	20. 24	20. 48	21. 12	21. 36	22. 0	22. 24	22. 48	23. 12	23. 36	24. 0
12. 30	12. 55	13. 20	13. 45	14. 10	14. 35	15. 0	15. 25	15. 50	16. 15	16. 40	17. 5	17. 30	17. 55	18. 20	18. 45	19. 10	19. 35	20. 0	20. 25	20. 50	21. 15	21. 40	22. 5	22. 30	22. 55	23. 20	23. 45	24. 10	24. 35	25. 0
13. 0	13. 26	13. 52	14. 18	14. 44	15. 10	15. 36	16. 2	16. 28	16. 54	17. 20	17. 46	18. 12	18. 38	19. 4	19. 30	19. 56	20. 22	20. 48	21. 14	21. 40	22. 6	22. 32	22. 58	23. 24	23. 50	24. 16	24. 42	25. 8	25. 34	26. 0
13. 30	13. 57	14. 24	14. 51	15. 18	15. 45	16. 12	16. 39	17. 6	17. 33	18. 0	18. 27	18. 54	19. 21	19. 48	20. 15	20. 42	21. 9	21. 36	22. 3	22. 30	22. 57	23. 24	23. 51	24. 18	24. 45	25. 12	25. 39	26. 6	26. 33	27. 0
14. 0	14. 28	14. 56	15. 24	15. 52	16. 20	16. 48	17. 16	17. 44	18. 12	18. 40	19. 8	19. 36	20. 4	20. 32	21. 0	21. 28	21. 56	22. 24	22. 52	23. 20	23. 48	24. 16	24. 44	25. 12	25. 40	26. 8	26. 36	27. 4	27. 32	28. 0
14. 30	14. 59	15. 28	15. 57	16. 26	16. 55	17. 24	17. 53	18. 22	18. 51	19. 20	19. 49	20. 18	20. 47	21. 16	21. 45	22. 14	22. 43	23. 12	23. 41	24. 10	24. 39	25. 8	25. 37	26. 6	26. 35	27. 4	27. 33	28. 2	28. 31	29. 0
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16. 0	16. 32	17. 4	17. 36	18. 8	18. 40	19. 12	19. 44	20. 16	20. 48	21. 20	21. 52	22. 24	22. 56	23. 28	24. 0	24. 32	25. 4	25. 36	26. 8	26. 40	27. 12	27. 44	28. 16	28. 48	29. 20	29. 52	30. 24	30. 56	31. 28	32. 0
16. 30	17. 3	17. 36	18. 9	18. 42	19. 15	19. 48	20. 21	20. 54	21. 27	22. 0	22. 33	23. 6	23. 39	24. 12	24. 45	25. 18	25. 51	26. 24	26. 57	27. 30	28. 3	28. 36	29. 9	29. 42	30. 15	30. 48	31. 21	31. 54	32. 27	33. 0
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17. 30	18. 5	18. 40	19. 15	19. 50	20. 25	21. 0	21. 35	22. 10	22. 45	23. 20	23. 55	24. 30	25. 5	25. 40	26. 15	26. 50	27. 25	28. 0	28. 35	29. 10	29. 45	30. 20	30. 55	31. 30	32. 5	32. 40	33. 15	33. 50	34. 25	35. 0
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18. 30	19. 7	19. 44	20. 21	20. 58	21. 35	22. 12	22. 49	23. 26	24. 3	24. 40	25. 17	25. 54	26. 31	27. 8	27. 45	28. 22	28. 59	29. 36	30. 13	30. 50	31. 27	32. 4	32. 41	33. 18	33. 55	34. 32	35. 9	35. 46	36. 23	37. 0
19. 0	19. 38	20. 16	20. 54	21. 32	22. 10	22. 48	23. 26	24. 4	24. 42	25. 20	25. 58	26. 36	27. 14	27. 52	28. 30	29. 8	29. 46	30. 24	31. 2	31. 40	32. 18	32. 56	33. 34	34. 12	34. 50	35. 28	36. 6	36. 44	37. 22	38. 0
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20. 30	21. 11	21. 52	22. 33	23. 14	23. 55	24. 36	25. 17	25. 58	26. 39	27. 20	28. 1	28. 42	29. 23	30. 4	30. 45	31. 26	32. 7	32. 48	33. 29	34. 10	34. 51	35. 32	36. 13	36. 54	37. 35	38. 16	38. 57	39. 38	40. 19	41. 0
21. 0	21. 42	22. 24	23. 6	23. 48	24. 30	25. 12	25. 54	26. 36	27. 18	28. 0	28. 42	29. 24	30. 6	30. 48	31. 30	32. 12	32. 54	33. 36	34. 18	35. 0	35. 42	36. 24	37. 6	37. 48	38. 30	39. 12	39. 54	40. 36	41. 18	42. 0
21. 30	22. 13	22. 56	23. 39	24. 22	25. 5	25. 48	26. 31	27. 14	27. 57	28. 40	29. 23	30. 6	30. 49	31. 32	32. 15	32. 58	33. 41	34. 24	35. 7	35. 50	36. 33	37. 16	37. 59	38. 42	39. 25	40. 8	40. 51	41. 34	42. 17	43. 0
22. 0	22. 44	23. 28	24. 12	24. 56	25. 40	26. 24	27. 8	27. 52	28. 36	29. 20	30. 4	30. 48	31. 32	32. 16	33. 0	33. 44	34. 28	35. 12	35. 56	36. 40	37. 24	38. 8	38. 52	39. 36	40. 20	41. 4	41. 48	42. 32	43. 16	44. 0
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23. 30	24. 17	25. 4	25. 51	26. 38	27. 25	28. 12	28. 59	29. 46	30. 33	31. 20	32. 7	32. 54	33. 41	34. 28	35. 15	36. 2	36. 49	37. 36	38. 23	39. 10	39. 57	40. 44	41. 31	42. 18	43. 5	43. 52	44. 39	45. 26	46. 13	47. 0
24. 0	24. 48	25. 36	26. 24	27. 12	28. 0	28. 48	29. 36	30. 24	31. 12	32. 0	32. 48	33. 36	34. 24	35. 12	36. 0	36. 48	37. 36	38. 24	39. 12	40. 0	40. 48	41. 36	42. 24	43. 12	44. 0	44. 48	45. 36	46. 24	47. 12	48. 0
24. 30	25. 19	26. 8	26. 57	27. 46	28. 35	29. 24	30. 13	31. 2	31. 51	32. 40	33. 29	34. 18	35. 7	35. 56	36. 45	37. 34	38. 23	39. 12	40. 1	40. 50	41. 39	42. 28	43. 17	44. 6	44. 55	45. 44	46. 33	47. 22	48. 11	49. 0
25. 0	25. 50	26. 40	27. 30	28. 20	29. 10	30. 0	30. 50	31. 40	32. 30	33. 20	34. 10	35. 0	35. 50	36. 40	37. 30	38. 20	39. 10	40. 0	40. 50	41. 40	42. 30	43. 20	44. 10	45. 0	45. 50	46. 40	47. 30	48. 20	49. 10	50. 0
25. 30	26. 21	27. 12	28. 3	28. 54	29. 45	30. 36	31. 27	32. 18	33. 9	34. 0	34. 51	35. 42	36. 33	37. 24	38. 15	39. 6	39. 57	40. 48	41. 39	42. 30	43. 21	44. 12	45. 3	45. 54	46. 45	47. 36	48. 27	49. 18	50. 9	51. 0
26. 0	26. 52	27. 44	28. 36	29. 28	30. 20	31. 12	32. 4	32. 56	33. 48	34. 40	35. 32	36. 24	37. 16	38. 8	39. 0	39. 52	40. 44	41. 36	42. 28	43. 20	44. 12	45. 4	45. 56	46. 48	47. 40	48. 32	49. 24	50. 16	51. 8	52. 0
26. 30	27. 23	28. 16	29. 9	30. 2	30. 55	31. 48	32. 41	33. 34	34. 27	35. 20	36. 13	37. 6	37. 59	38. 52	39. 45	40. 38	41. 31	42. 24	43. 17	44. 10	45. 3	45. 56	46. 49	47. 42	48. 35	49. 28	50. 21	51. 14	52. 7	53. 0
27. 0	27. 54	28. 48	29. 42	30. 36	31. 30	32. 24	33. 18	34. 12	35. 6	36. 0	36. 54	37. 48	38. 42	39. 36	40. 30	41. 24	42. 18	43. 12	44. 6	45. 0	45. 54	46. 48	47. 42	48. 36	49. 30	50. 24	51. 18	52. 12	53. 6	54. 0
27. 30	28. 25	29. 20	30. 15	31. 10	32. 5	33. 0	33. 55	34. 50	35. 45	36. 40	37. 35	38. 30	39. 25	40. 20	41. 15	42. 10	43. 5	44. 0	44. 55	45. 50	46. 45	47. 40	48. 35	49. 30	50. 25	51. 20	52. 15	53. 10	54. 5	55. 0
28. 0	28. 56	29. 52	30. 48	31. 44	32. 40	33. 36	34. 32	35. 28	36. 24	37. 20	38. 16	39. 12	40. 8	41. 4	42. 0	42. 56	43. 52	44. 48	45. 44	46. 40	47. 36	48. 32	49. 28	50. 24	51. 20	52. 16	53. 12	54. 8	55. 4	56. 0
28. 30	29. 27	30. 24	31. 21	32. 18	33. 15	34. 12	35. 9	36. 6	37. 3	38. 0	38. 57	39. 54	40. 51	41. 48	42. 45	43. 42	44. 39	45. 36	46. 33	47. 30	48. 27	49. 24	50. 21	51. 18	52. 15	53. 12	54. 9	55. 6	56. 3	57. 0
29. 0	29. 58	30. 56	31. 54	32. 52	33. 50	34. 48	35. 46	36. 44	37. 42	38. 40	39. 38	40. 36	41. 34	42. 32	43. 30	44. 28	45. 26	46. 24	47. 22	48. 20	49. 18	50. 16	51. 14	52. 12	53. 10	54. 8	55. 6	56. 4	57. 2	58. 0
29. 30	30. 29	31. 28	32. 27	33. 26	34. 25	35. 24	36. 23	37. 22	38. 21	39. 20	40. 19	41. 18	42. 17	43. 16	44. 15	45. 14	46. 13	47. 12	48. 11	49. 10	50. 9	51. 8	52. 7	53. 6	54. 5	55. 4	56. 3	57. 2	58. 1	59. 0
30. 0	31. 0	32. 0	33. 0	34. 0	35. 0	36. 0	37. 0	38. 0	39. 0	40. 0	41. 0	42. 0	43. 0	44. 0	45. 0	46. 0	47. 0	48. 0	49. 0	50. 0	51. 0	52. 0	53. 0	54. 0	55. 0	56. 0	57. 0	58. 0	59. 0	60. 0

ARITHMETICK.

The Firſt BOOK,

CONCERNING
Numbers abſolutely abſtract;

In Two PARTS.

WHEREIN
INTEGERS } are { Examined,
FRACTIONS } are { Explained,
AND THEIR
SIMPLE ELEMENTS.

CHAP. I.

A Brief Introduction to the Parts of Arithmetick *hereafter handled.*

THE Noble Science of *Arithmetick* is the very Foundation of the *Mathematicks*, whoſe *Roots* are *Number* and *Magnitude*. The Original of the one an *Unit*, of the other a *Point*; but the *Products* of either are Infinite. The *Mathematical Science* ſpecifically appropriate to *Number* is *Arithmetick*; to *Magnitude*, *Geometry*. From theſe as Two Principal Springs Mathematical conſidered ſimply or *per ſe*, and mixt or *inter ſe* are derived many other Excellent Arts and Myſteries; uſeful not only in Natural, Subſtantial, Accidental, Viſible, and Inviſible things, but alſo in Supernatural and Divine, as the Learned *John Dee* affirmeth in his *Mathematical Table* inſerted in *John-Henry Alſted* his *Encyclopœdia*, *lib.* 2. *p.* 70. which I have thought fit here to Tranſlate.

Arithmetick *what the ground of.*

The Original an Unit.

Dr. Dee *his Mathematical Table.*

- Sciences and Arts Mathematical are
  - Principal
    - Arithmetick.
      - *Simple*, Which only treateth of *Numbers*, and demonſtrates their Properties. Here Unity is Indiviſible.
      - *Mixt*, Which by the help of *Principal Geometry* demonſtrateth certain Arithmetical Concluſions.
    - *Geometry.*
      - *Simple*, Which treateth only of *Magnitudes*, and demonſtrateth all the Properties and Paſſions of them. Here a Point is Indiviſible.
      - *Mixt*, Which by the help of *Principal Arithmetick* demonſtrateth certain Geometrical things. as *Euclides Elements.*
    - The uſe of theſe Sciences is either in things
      - Supernatural, Eternal and Divine by Application *Aſcending.*
      - Mathematical without any other Application.
      - Natural, Subſtantial, and Accidental Viſible, and Inviſible, by Application *Deſcending.*
    - The like Uſes (only inferiour in one degree) are in the Mathematical Derivative Arts
  - or Derivative ſome of which
    - Retain the Names of the Principal, as
      - Common Arithmetick, which conſidereth
        - Arithmetick of Whole Numbers and their Fractions.
        - Arithmetick of Proportion.
        - Circular Arithmetick.
        - Arithmetick of Rooted Numbers and their Fractions.
        - Arithmetick of Coſſick Numbers and their Fractions.
      - Common Geometry, which teacheth to meaſure a thing.
        - Not diſtant hence
          - *Mecometria.*
          - *Embadometria.*
          - *Stereometria.*
        - Diſtant hence
          - *Apomecometria.*
          - *Hypoſometria.*
          - *Platometria.*
          - from theſe ariſe theſe Arts, *viz.*
            - *Geodeſia.*
            - *Geographia.*
            - *Chorographia.*
            - *Hydrographia.*
            - *Stratarithmetria*
    - Have peculiar Names, As
      - *Perſpectiva.*
      - *Aſtronomia.*
      - *Muſica.*
      - *Coſmographia.*
      - *Aſtrologia.*
      - *Statica.*
      - *Anthropographia.*
      - *Trochilica.*
      - *Helicoſophia.*
      - *Pneumatithmia.*
      - *Menedria.*
      - *Hypodeiodia.*
      - *Hydragogia.*
      - *Horometria.*
      - *Zographia.*
      - *Architectura.*
      - *Navigatio.*
      - *Thaumaturgica.*
      - *Archimagiſterica.*

Number *helpful to ſearch out the Creature Vertues, Natures*, &c.

Number *of Univerſal Uſe.*

And the ſame Doctor *Dee* in his Mathematical Preface to the firſt Six Books of *Euclides* Elements further aſſures us, That *Number* ſeems to be ſo Immaterial and Pure, that thereby we may wind our ſelves into the deep Search and View of all Creatures diſtinct Vertues, Natures, Properties, Forms, *&c.* The Univerſal Uſe of *Number* is Witneſſed to alſo, by the Noble *Picus*, Earl of *Mirandula*, who had ſet up in *Rome* Nine Hundred Concluſions in all kinds of *Sciences* openly to be diſputed of; and in the Eleventh Concluſion ſaith, By *Numbers*, a way is had to the Searching out, and Underſtanding of every thing able to be known.

Without Numbers, *no Knowledge of the Mathematicks.*

Boëtius and others are not far behind them in Commending *Number*; but had it none of their *Encomiums*, yet certain it is, That whoſoever would fit himſelf for the *Mathematicks*, unleſs he begin with the Science of *Numbers*, will quickly find himſelf in a Labyrinth, from which he can never eſcape, nor deliver himſelf from many Inextricable Doubts, without the Aſſiſtance thereof.

Threefold Conſideration of Number.

Great *Mathematicians* and *Philoſophers* have conſidered *Number*: Firſt, In Reſpect of the Creator, Simple, Pure, and Immixt. Secondly, In Reference to Spiritual and Angelical Minds, Including the Soul of Man: And, Thirdly, In relation to every Creature, and their compleat Conſtitution: And in the Firſt and Second reſpect, term it *Number* numbring; but in the Third *Number*, numbred. In the ſence of the Two former I ſhall not intermeddle to treat of *Numbers*, referring it to a more Able and Divine Pen. But in the latter reſpect, and that in the loweſt and moſt groſs conſideration of *Numbers*, *Viz.* In Conference and Coincidence to Viſible, Material, and Corporeal things, I have ſhadowed out this Science of *Numbers* in the Enſuing Treatiſe, under the Name of *Arithmetick Surveyed and Reviewed*: Containing Four Books. The Firſt

Here, under the Third Conſideration to be Viewed in Four Books.

1. Treating of *General Arithmetick*, Examining the *Simple Elements* of *Abſtract Numbers*, in Two Parts, *viz.* *Integers* in the Firſt Part, and *Fractions* in the other Part; although this latter after a Sort may be called Contract, in reſpect to their *Denominators.*

2. The Second and Third Books handle *Special Arithmetick*, Explaining the *Simple Elements*

ments of *Contract Numbers*, more generally as of *Geodeticals* and *Figurals*, in the Second Book, in Two Parts: Thoſe more Specially Contract, are dealt with in the Third Book, in Six Parts; *viz. Decimals*, *Aſtronomicals*, *Logarithmes*, *Coſſicks*, *Surdes*, and *Species*, in each Part diſtinctly. In the Fourth Book is taught the Whole Doctrine of Proportions, *Disjunct*, *Continual*, and *Æquated*. And in one or other of theſe, (eſpecially as to the Practick Part of *Arithmetick*) is included what Others otherwiſe have divided and delivered concerning *Numbers*; under that Name of *Arithmetick*. 3. 4.

Arithmetick derives its Engliſh Name from the Latine *Arithmetica*; and this again from the *Greek* Ἀριθμὸς, ſignifying as *Keckerman* ſaith, *Phyſ. Lib.* 1. both to Number and Meaſure; and ſo may truely include *Arithmetick* mixt with *Geometry*, as before hinted; but with us is generally taken for the Art of Numbring Reſtrictively, intending only its Eſſential Conſiſtence in *Numbers*, and but Collateral Converſe with Magnitudes: In which Notion I am contented likewiſe to take it, reſerving not only a Liberty to underſtand it in a higher Note, when Occaſion ſerves; but alſo to accept it under the Vulgar Terms of *Caſting Accompt* and *Cyphering*, till they that ſo Abaſe this high born *Science* ſhall ſee it worthy of a more Excellent Name than thoſe.

Arithmetick, *whence the Name.*

Is the Science of Numbring generally taken.

Cyphering *too vulgar a Name.*

The Antient Hebrews *and* Greeks *Numbered by their Letters; as Enſueth in theſe Tables.*

Hebrew Account.

Numeral Let. of the Hebrews

א·	One.	יא·	Eleven.	ל·	Thirty.	ר	Two Hundred.	
ב	Two.	יב·	Twelve.	מ·	Forty.	ש·	Three Hundred.	
ג·	Three.	יג·	Thirteeen.	נ	Fifty.	ת·	Four Hundred.	
ד·	Four.	יד·	Foutteen.	ט	Sixty.	תק	Five Hundred.	ך· *Sec. Maſorethas.*
ה·	Five.	טו	Fifteen.	ע·	Seventy.	תר·	Six Hundred.	ם
ו·	Six.	יו	Sixteen.	פ·	Eighty.	תש·	Seven Hundred.	ן·
ז·	Seven.	יז·	Seventeen.	צ	Ninety.	תת	Eight Hundred.	ף·
ח·	Eight.	יח	Eighteen.	ק·	One Hundred.	תתק	Nine Hundred.	ץ·
ט	Nine.	יט·	Nineteen.				One Thouſand.	א·
י·	Ten.	כ·	Twenty.				*&c.*	

Greek Account.

Numeral Let. of the Greeks.

I. α'	One.	ΔΠII. ιζ'.	Seventeen.	𐅅 H. χ'.	Six Hundred.	MMMM. ,μ.	Forty Thouſand.
II. β'.	Two.	ΔΠIII ιή.	Eighteen.	𐅅 HH. ψ'.	Seven Hundred.	IMI. ,ν.	Fifty Thouſand.
III γ'.	Three.	ΔΠIIII. ιϑ'.	Nineteen.	𐅅 HHH. ώ.	Eight Hundred.	IMIM. ,ξ.	Sixty Thouſand.
IIII δ'.	Four.	ΔΔ. κ'.	Twenty.	𐅅 HHHH. ϡ'.	Nine Hundred.	IMIMM. ,ο.	Seventy Thouſand
Π. έ.	Five.	ΔΔΔ. λ'.	Thirty.	X. ,α.	One Thouſand.	IMIMMM. ,π.	Eighty Thouſand
ΠI. ς'	Six.	ΔΔΔΔ. μ'.	Forty.	XX. ,β.	Two Thouſand.	IMIMMMM. ,ς.	Ninety Thouſand.
ΠII ζ'	Seven.	𐅄. ν'.	Fifty.	XXX. ,γ.	Three Thouſand.	,ρ.	One Hundred Thouſand.
ΠIII. ή	Eight.	𐅄 Δ. ξ'.	Sixty.	XXXX ,δ.	Four Thouſand.	,σ.	Two Hundred Thouſand.
ΠIIII. ϑ'.	Nine.	𐅄 ΔΔ. ό.	Seventy.	𐅆. ,ε.	Five Thouſand.	,τ.	Three Hundred Thouſand.
Δ. ί.	Ten.	𐅄 ΔΔΔ. π'.	Eighty.	𐅆X. ,ς.	Six Thouſand.	,υ	Four Hundred Thouſand.
ΔI. ιά	Eleven.	𐅄 ΔΔΔΔ ς.	Ninety.	𐅆XX. ,ζ.	Seven Thouſand.	,φ.	Five Hundred Thouſand.
ΔII. ιβ'.	Twelve.	H. ρ'.	One Hundred.	𐅆XXX. ,η.	Eight Thouſand.	,χ.	Six Hundred Thouſand.
ΔIII. ιγ'	Thirteen.	HH. σ'.	Two Hundred.	𐅆XXXX. ,θ.	Nine Thouſand.	,ψ.	Seven Hundred Thouſand.
ΔIIII. ιδ'.	Fourteen.	HHH. τ'.	Three Hundred.	M ,ι.	Ten Thouſand.	,ω.	Eight Hundred Thouſand.
ΔΠ. ιέ	Fifteen.	HHHH. υ'.	Four Hundred.	MM ,κ.	Twenty Thouſand.	,ϡ.	Nine Hundred Thouſand.
ΔΠI. ις'.	Sixteen.	𐅅. φ'.	Five Hundred.	MMM ,λ.	Thirty Thouſand.		*&c.*

The Latines *made uſe only of Seven of their Letters*: Viz. C. D. I. L. M. V. X.

Latine Account.

Numeral Let. of the Latines.

I.	One.	XII.	Twelve.	L.	Fifty.	DCC.	Seven Hundred.
II.	Two.	XIII.	Thirteen.	LX.	Sixty.	DCCC.	Eight Hundred.
III.	Three.	XIIII. XIV.	Fourteen.	LXX.	Seventy.	DCCCC.	Nine Hundred.
IIII. IV.	Four.	XV.	Fifteen.	LXXX.	Eighty.	M̲. cIↃ. ∞.	One Thouſand.
V.	Five.	XVI.	Sixteen.	XC.	Ninety.	V̄. v∞. IↃↃ.	Five Thouſand.
VI.	Six.	XVII.	Seventeen.	C.	One Hundred.	X̄. ccIↃↃ. ↃMↃ.	Ten Thouſand.
VII.	Seven.	XXII. XIIX.	Eighteen.	CC. ∽.	Two Hundred.	L̲. L∞. IↃↃↃ.	Fifty Thouſand.
VIII.	Eight.	XIX.	Nineteen.	CCC.	Three Hundred.	C̲. CM. c∞. cccIↃↃↃ.	One hundred Thouſand.
VIIII. IX.	Nine.	XX.	Twenty.	CCCC.	Four Hundred.	D̄. IC. D∞. IↃↃↃↃ.	Five hundred Thouſand.
X.	Ten.	XXX.	Thirty.	D. IↃ.	Five Hundred.	ccccIↃↃↃↃ.	One Million.
XI.	Eleven.	XL.	Forty.	DC.	Six Hundred.	*&c.*	

The

Numeral Let. of the Engliſh.

The *Engliſh* following the *Latine* made up their Antient Accompts by the ſame Seven Letters: Accompting I. one. V. five. X. ten. L. fifty. C. a hundred. D. five hundred. and M. a Thouſand, *&c.*

Arithmetick *beſt performed by Figures.*

The Art of *Numbers* being beſt performed by the Pen with the *Arabick* Notes, or Characters, commonly called Figures. I ſhall altogether wave the Practiſe thereof by Counters, and Letters, and all other Inſtrumental Arithmetick (ſave only a little touch of *Nepairs* Bones) and remit thoſe deſirous to learn ſuch kind of Operations to the ſeveral Treatiſes of the reſpective Authours who have dealt therewith.

CHAP. II.

Of the Nature of Numbers.

What Arithmetick *is.*

The Subject thereof.

Number taken reſtrictively.

How an Unit *is a Number.*

Number taken largely.

Parts of an Unite *how Numbers.*

IN purſuance of what hath been ſaid, and hereafter followeth *Arithmetick* may be defined, The *Science* of *Numbring* well. In *Arithmetick* two things are principally to be conſidered; The Subject, in which the Nature of *Numbers* is declared: And the Affection of the Subject, wherein the Elements of *Numbers* are exerciſed. The Subject of *Arithmetick* is *Number*, all the Precepts and Doctrines of the whole Art, having a particular Relation thereunto. *Number* is firſt taken reſtrictively, and ſo *Number* is conſidered only according to it's multitude or capacity of Units; hence *Euclide*, *lib.* 7. *def.* 2. will have *Number* to be a multitude of Units. An Unite is indeed the Original of *Number*, but being in the Prædicament of Quantity, and *in potentia*, may well paſs for a *Number*. And *Euclide* himſelf in the 18, 19, and 20. Definitions of the ſame Book, cannot be excuſed from accompting an Unite for a *Number*. Secondly, *Number* is taken more largely for the Quantity according to which any thing is Numbred, or Accompted; So according to an Unite any ſingular thing is ſaid to be One; as one Man, one Houſe, and ſuch like; and two things, as Stars, Ships, Men, *&c.* are ſaid to be two, according to that *Number*: And in this Metaphyſical Senſe, not only the Unit it ſelf, but the parts of an Unite, as one half of a Day, two third parts of a Pound, *&c.* are reckoned for *Numbers*.

Numbers therefore largely taken, may be better diſcern'd in their proper Natures, as they are capable of being Divided: A brief *Synopſis* whereof followeth.

A Synopſis *of the Nature of Numbers.*

- *Nature of Numbers.*
  - Homogeneal
    - Proper
      - Eſſential
        - Whole
          - Unity or Multitude.
          - Digit, Article, or Mixt.
          - Prime, or Compound.
          - Even, or Odd.
          - Perfect, or Imperfect.
        - Broken
          - Part.
          - Parts
            - Commenſurable.
            - Incommenſurable.
      - Accidental
        - Abſtract
          - More Abſolute. *Integers.*
          - Leſſer Abſolute *Fractions.*
        - Contract
          - More General.
            - *Geodæticals.*
            - *Figurals.*
          - More Special.
            - *Decimals.*
            - *Aſtronomicals.*
            - *Logarithmes.*
            - *Coſſicks.*
            - *Surdes.*
            - *Species.*
    - Common
      - *Ratios.*
      - *Proportions.*
        - *Diſcontinual.*
        - *Continual.*
      - *Æquations.*
  - Heterogeneal
    - Primary. *Integers*, and *Fractions.*
    - Secondary. *Abſtract*, and *Contract.*

For the Explanation whereof obſerve *Numbers* according to their Nature have ſome peculiar properties to themſelves, and ſomething common with others; and are of two ſorts, *Homogeneal*, and *Heterogeneal*; *Homogeneal* are of the ſame Nature or Kind, whether Proper, or Common. Eſſential, or Accidental. Whole, or Broken, Abſtract, or Contract: *Numbers Heterogeneal* are mixt *Numbers* of Whole and Broken, Abſtract and Contract.

Numbers Homogeneal *are proper and common.*

Proper Eſſential, *or* Accidental.

Whole Numbers are called *Integers* from the *Latine*, *Integrum*, and *Broken Numbers*, *Fractions*, from the *Latine*, *Fractio*.

Integers *and* Fractions, *whence the Names.*

A proper *Whole Number* according to his Eſſence, paſſeth under a five fold Conſideration, Firſt, Whither it be Unite or Multitude. Secondly, Digit, Article, or Mixt, Thirdly, Prime, or Compound. Fourthly, Even, or Odd: And Fifthly, Perfect, or Imperfect.

Integers *conſidered five wayes* Eſſentially.

An Unite is the beginning of *Multitude*, and foundation of *Number*; never more than one. *Multitude* is the Collection of Unites, as one, one, one, are equal to three, being Collected together, &c.

1. *One or Many.*

Secondly, *Integers* are again divided into Digits, Articles, and Mixt Numbers. A Digit is any *Integer* under Ten, as One, Two, Three, Four, Five, Six, Seven, Eight, Nine; Digits are ſometimes called *Monades*. An Article is Ten, with all Whole Numbers that may be juſtly divided into Ten parts, as Twenty, Thirty, Forty, &c. Theſe are called Round Numbers, and ſometimes *Decades*. A Mixt Number is compound of an Article, and a Digit, as Eleven, Twelve, &c. Twenty one, Twenty-two, &c.

2. *Digits, Articles, and mixt.*

Digits called Monades.

Articles called Decades.

Thirdly, A Number, Prime, called alſo Simple, and ſometime Uncompound, is a Number made only by *Addition*, or Collection of Units, and not by *Multiplication*, ſo an Unit only can meaſure it; as Two, Three, Five, &c. Compound Numbers are ſuch as are made by *Multiplication* of two Numbers together, and not by *Addition*, though they may ſeem to be made of both: Yet becauſe they may be meaſured only by the Numbers of which they are compounded, they are not to be accounted Prime, as Six made by *Multiplication* of Two and Three, is Compound, becauſe by either of them it may be meaſured, or divided; and though it may be made by *Addition* of Five and One, yet ſhall it not be Prime; for that neither Five nor One, can equally meaſure Six.

3. *Prime or Compound.*

Fourthly, *Numbers* are again conſidered as they are, Even or Odd; Even may be divided into Two Equal Parts, Odd cannot; Even *Numbers* are diſtinguiſhed into Three Sorts. Firſt, Even *Numbers* Evenly, theſe continually may be parted into halves, till you come to an Unite, as Sixteen, into Eight, Four, into Two, One, &c. Secondly, Even *Numbers* Oddly, theſe may be parted into equal halves, but the halves will be odd *Numbers*, as Ten into Five, and Five, &c. Thirdly, Even *Numbers*, Evenly and Oddly, as thoſe which may a while be parted into even halves; but before you come to one, the halves will be odd *Numbers*, as Twelve into Six, Three, &c.

4. *Even or Odd. Even of Three ſorts.*

1.

2.

3.

Fifthly, *Integers* are differenced as they are Perfect or Imperfect. Perfect *Numbers* are ſuch whoſe aliquot or even parts joyned together will exactly return the whole *Number*, as Six, Twenty-Eight, &c. for of Six the half is Three, the Third part Two, and the Sixth part One, which added together, make Six; and it hath no more aliquot parts in whole *Numbers*, for the fourth part, is one and an half, and the fifth part, one and a fifth. So Twenty-Eight, whoſe parts being Fourteen, Seven, Four, Two, and One, exactly return Twenty-Eight, which therefore appears to be a perfect *Number*. Perfect *Numbers* are almoſt as rare as perfect Men; for between One, and One Million of Million there are but Ten; and the Twentieth perfect *Number* exceeds the value of Hundred Thouſand Million of Million of Million. *Mathemat. Recreations. p.* 92. Imperfect *Numbers* on the Contrary, are thoſe whoſe even parts added together, will not return the Primary *Number*, whoſe parts they be: And theſe are either *Abundant*, or *Defective*. *Abundant* called alſo ſuperfluous, whoſe parts added together make more than the whole *Number*; as Twelve, whoſe parts being one, two, three, four, and ſix, together make ſixteen; So the parts of Twenty make Twenty two, &c. *Defective* are ſuch, whoſe parts added together make leſs than the *Integer*; as Eight, whoſe parts being one, two, and four, make but ſeven; likewiſe the parts of Sixteen make but Fifteen, and of Forty Five make but Thirty Three.

5. *Perfect or Imperfect.*

Perfect Numbers very rare.

Imperfect of two ſorts.

Abundant.

Defective.

Fractions or *Broken Numbers* ariſe from the Diviſion of the Unite into parts, and therefore are properly parts of *Numbers*, becauſe every proper *Fraction* is leſs than an Unite; as when a Unite ſignifieth any Denominate Quantity, in reſpect to that Quantity, it may be divided into leſſer parts than that one Quantity; So one Pound parted into Four Equal parts, then ſhall one half, or three Quarters thereof, be leſs than one whole Pound, and theſe, and ſuch parts be called *Fractions* or *Broken Numbers*. Theſe are conſidered as they contain, either one part of the whole, or more parts; as one half,

Fractions, *whence they ariſe.*

Doubly conſidered Eſſentially.

then ſhall the whole be divided but into two parts, but three quarters denote the Unite to be parted into four parts, and the Fraction to contain three of them: Theſe parts are either Commenſurable, or Incommenſurable.

The latter ſort either Commenſurable.

Commenſurable, when the two Termes of the *Fraction* have any common part, that will equally divide them both; as Three being a part of Twelve and Fifteen will divide them both equally: therefore Twelve Fifteenths being the Termes of a Fraction, are parts Commenſurable, and may be meaſured by Three, *&c.*

or,

Incommenſurable.

Incommenſurable on the contrary have no ſuch parts for a common Diviſor, as Eighteen Twenty Fifths, for Twenty Five can be equally divided by no Number but Five, and Five cannot divide Eighteen.

Homogeneal Numbers *conſidered.*
Accidentally as Abſtract and Contract.

The Nature of *Numbers Homogeneal* are further diſcerned in ſome properties accidental to them ſingly, or amongſt themſelves, and that is to be Abſtract, or Contract: That called numbring, or formal; this numbred, or material.

Abſtract more or leſs Abſolute.
More Integers.

Abſtract are ſuch as have no Denomination annexed to them; Theſe are more or leſs Abſolute: The more Abſolute are *Integers*, as One, Two, Three, or any other whole Number, without any denomination, or ſirname, relation, or compariſon, *&c.* thereto belonging, but are free to value Men, Money, Years, or any other quantity. Of *Integers* and their Affections, *Vide plus lib.* 1. *par.* 1.

Leſs Fractions.

The leſs Abſolute are *Fractions*, as one quarter, one half, two thirds, *&c.* though Abſolute without reſpecting Denominations, and therefore left free to be parts of Weight, Meaſure, Time, or any other quantity, or thing whatſoever; yet muſt be conſidered in reſpect of the one number to the other. So that *Fractions* are not ſo Abſolute as *Integers*, but in a ſort, as I ſaid before, may be accompted for a ſort of Denominate Numbers, relation being had between the Numerator, and Denominator. Of theſe *Vide plus. lib.* 1. *par.* 2.

The ſame Numbers Abſtract, *are again to be conſider'd, as they may be* Contract.

Contract.

Contract, called alſo *Concrete Numbers*, proceed from Abſtract, and are ſuch as are refringed by the Annexion of ſome or other Denomination, as Two Groats, Ten Shillings, Two Thirds of a Pound, *&c.* where the Numbers are reſtrained from the liberty of valuing any thing elſe, but Groats, Shillings, *&c.* according to the annexed Denomination. Some *Contract Numbers* are contracted more generally, as *Geodæticals* and *Figurals:* Others more ſpecially, in reſpect to the ſeveral ſorts of Denominations, by which they are contracted.

Generally.

Geodæticals.

Numbers Geodætical, which are conſidered according to thoſe Vulgar Names or Denominations, by which Money, Weights, Meaſures, *&c.* are generally known, or particularly divided by the Laws and Cuſtoms of ſeveral Nations; theſe ſpring from Denominate *Fractions*, and by omitting the Denominators, when the parts are commonly known, and annexing names Artificial or Inartificial, making ſome Mark or Sign to diſtinguiſh them, they paſs into the Catalogue of vulgar Denominate Integral *Numbers*; As becauſe the twentieth part of a pound *ſterling* is well known by the name of a Shilling, we uſe to ſay Three, Four or Five Shillings, *&c* and not Three Twentieths, Four Twentieths, and Five Twentieths of a Pound. The like is done alſo in the Twelve parts of a Shilling; as for One Twelfth, Two Twelfths, Three Twelfths, *&c.* we ſay One penny, Two pence, Three pence, *&c.* The ſame alſo may be underſtood of the parts of Weights, Meaſures, Time, Motion, *&c.* Of theſe more particularly may be ſeen, in the 1st *Part* of the 2d *Book.*

Theſe paſs for vulgar denominate Numbers.

Figurals.

The ſecond ſort of more Generally *Contract Numbers* are Figural, ſo called becauſe they do or may repreſent ſome Figure Geometrical; in relation to which they are ever conſidered, and thereupon by ſome are reckoned abſolutely among Denominate *Numbers.* Figural *Numbers* are either *Lineary*, *Superficial*, or *Solid.*

Theſe of 3. ſorts.

1. Lineary.

Lineary, which have relation to length only, and are conſidered Univerſally or Reſtrictively. Univerſally, and ſo though moſt aptly it be referred to ſuch a Number as will make no other form duely, yet may it alſo be applyed to any Number Abſtract. Reſtrictively, and ſo *Numbers Lineary* are taken for the ſides of all Æquilateral Figures, or Forms, and Metaphorically theſe are called *Roots.*

2. Superficial.

Superficial, Called alſo *Flat Numbers*, are conſidered in the Formes they make by *Progreſſion* or *Multiplication*, whereof there be as many varieties as there be like Figures in *Geometry*; as *Numbers Triangular*, *Quadrangular*, *Quinquangular*, *&c. Numbers Circular*, *Diametral*, *Oblong*, *&c.* all which have length, and breadth, but no depth.

2. *Solids.*

Solids, or *Sound Numbers*; otherwiſe termed Bodily, or Cubical, as by the firſt Multiplication they take length, and breadth like *Flat Numbers*, ſo by the next Multiplication they take depth, or thickneſs; which thickneſs or ſolidity is increaſed according to the Number of Multiplications, and accordingly from thence do they take their Names; as Cubes, Squared Squares, Surſolids, *&c. ad Infinitum.* Theſe take up the 2d *Part* of the 2d *Book.*

Numbers *Specially Contract, of ſix ſorts.*

1. *Decimals.*

Again, *Numbers* Specially Contract are conſiderable, as firſt *Decimals*, which ariſe from the Abbreviation of *Fractions* before mentioned, or rather from the Converſion of one kind of *Fraction* into another; For let the *Integer* be what it will, it ſhall be broken but into Ten parts, and one of theſe Ten parts, into other Ten, and ſo infinitely decreaſing by Ten only, whereas in other *Fractions* the Denominators might be any other Number; and becauſe the Denominators here are ſtill known to be One with Cyphers more by One place than the Figures of the Numerator, the Denominator is alway omitted as needleſs; and the Numerator only ſet down. As is further declared, *Book* 3d *Part* 1ſt.

2. *Aſtronomical.*

Secondly, *Numbers Aſtronomical*, which are converſant about *Aſtronomy*, in Time, and Motion, have their Denominators Sixty certain; and ſo alſo omitted and ordered very like Decimals. As appeareth *Book* 3d *Part* 2d.

3. *Logarithmes.*

Thirdly, *Numbers Logarithmical*, or *Rational*, which have the ſame foundation with *Decimal* and *Aſtronomical Arithmetick*, and like them have their Denominators with the Numbers they come of, alſo omitted. Theſe are of ſuch ſingular uſe, as they will for ever Renown the Honourable Lord *Nepair* who Invented them, and may be further viewed in *Book* 3d. *Part* 3d.

4. *Coſſicks.*

Fourthly, *Numbers Coſſical*, which proceed from Figural Numbers Abſtract, and may be accompted Rational-Contract-Compound-Figural-Numbers, As Two *Roots*, Three *Squares*, Three *Cubes*, and ſuch like. *Coſſicks* are Simple, or Compound, and of either ſort, Whole, and Broken, as is demonſtrated in *Book* 3d. *Part* 4th.

5. *Surdes.*

Fifthly, *Numbers Surde*, or *Irrational*, which are ſuch Numbers ſet for *Roots*, as cannot be expreſſed by any other Number Abſolute, ariſing from Lines, or Figures Inequilateral, whoſe meaſure is a whole Number and a Fraction, and in a ſort are to Figurals like Fractions to Integers. Theſe are alſo Simple, and Compound; and take up the fifth Part of the 3d *Book.*

6. *Species.*

Sixthly, Becauſe the Characters uſed in many Denominations are Arbitrary, and marked with Letters, alſo in Geometrical Figures for brevity and diſtinction ſake one line is noted with one Letter, and another with another; hence the Alphabetical Notes are called *Species*, and are ordered in Arithmetical form in the 6th Part of the 3d *Book*; And are duely inſerted among the reſt of Contract Numbers, becauſe every Magnitude, or Number, *&c.* is at pleaſure denominate A. B. C. *&c.* or by any ſuch other Letter, or Mark.

Homogeneal Numbers conſidered in Common.

Related two wayes.

Ratio.

Homogeneal Numbers, whether Abſtract, or Contract, are conſiderable in their Common Nature that is Relative; as when Five is compared to Ten, it is but one half; ſo Eighteen is tripple to Six; here the Numbers being compared together, are fitly termed Relative. This Relation is taken differently, as when two Numbers only; or more than two are compared together: That called *Ratio*; this *Proportion*, or *Analogy.*

This two-fold Equality and Inequality.

Inequal of two ſorts.

Greater is Simple, or Compound.

Simple 2. fold.

Compound 3. fold.

Leſſer is 2. fold.

Ratio hath a property common with *Fractions*, to be Commenſurable, or Incommenſurable; and peculiar to it ſelf is biſected into Equality, or Inequality. *Ratio* of Equality, is when two Equal Numbers are compared together, as Two to Two, *&c. Ratio* of Inequality, when one Number is compared to another different from him, *viz.* Greater, or Leſſer. Greater, as Six to Two. Leſſer, as Two to Six. The Greater is of two kinds; Prime, or Simple, Conjunct, or Compound. The Simple are again divided into two ſorts. *Firſt*, When the Greater Number containeth the Leſſer, once and a part more, whether one half, one third, *&c.* as Six to Five, *&c. Secondly*, When the Greater containeth the Leſſer with ſome parts of him, as Five to Three; which containeth the Leſſer once, and two Thirds more, *&c. Ratio* of the Greater Inequality Compound, are diſtinguiſhed into three ſorts. *Firſt*, When the Greater Number containeth the Leſſer divers times, whether twice, four times, or ſuch like, as Four to Two, Nine to Three, *&c. Secondly*, When the Greater containeth the Leſſer many times, and a part of him beſides, as Five to Two, Sixteen to Five, *&c. Thirdly*, When the Greater containeth the leſſer many times, and alſo many parts thereof, as Eight to Three, Eleven to Three, *&c. Ratio* of the Leſſer Inequality, like *Fractions*, either contain a part of the Number, as one Third, one Fourth, *&c.* So Two to Six, Two to Eight, *&c.* Or elſe many parts, as Three Quarters, Two Thirds, *&c.* So Four to Six is Two Thirds, and Nine to Twelve, Three Quarters, *&c.* Theſe are further treated of in the 1ſt *Part* of the 4th *Book.*

Proportion 2. fold.

Proportion or *Analogy* when more Numbers than two are compared together, then there may be a conference of the former ſeveral *Ratios* in their ſeveral termes, and this is different either in *Diſcontinual*, or *Continual Proportion*.

Diſcontinual, Direct, and Indirect.

Diſcontinual, Is when in four termes the firſt, and ſecond, third, and fourth termes are compared together, but not the ſecond and third; ſo Five to Fifteen, is as Six to Eighteen: Where the ſecond term Fifteen, and the third term Six are not compared together. *Diſcontinual Proportion* is again divided into *Direct* and *Indirect Proportion*; and either of theſe double. *Indirect*, if the Greater requireth a Leſſer, or the Leſſer, a Greater to be found. *Direct*, when ſometimes the Leſſer requireth the Leſſer, and ſometime the Greater requireth the Greater to be found out.

Continual Arithmetical, Geometrical.

Continual Proportion, is when three or more Numbers bear like Difference, or Proportion in their Progreſſion. This is double, *Firſt*, When between every two Numbers the Difference, or Exceſs is Equal, as between Three, Six, Nine, Twelve, *&c.* the Difference is Three. *Secondly*, When the *Ratio* is equally alike, as Four, Eight, Sixteen, *&c.* the *Ratio* is Two. Theſe are dealt with in the 2d and 3d *Parts* of the 4th *Book*.

Æquations are Compound Proportions or Numbers in the Ratio of Equality.

Again *Homogeneal Numbers* may hold community in Special Denominations, being conſidered in the *Ratio*, or *Proportion* of *Equality*; and ſuch are called *Æquations*, and by ſome *Algebra*; So Numbers Æquational are Numbers equal one to the other, though the Denominations of ſuch Numbers are different, as one Square ſhall be Equal to Three Roots, if the *Root* be Three; wherefore *Æquations* may be rightly placed amongſt Numbers Denominate, the Operations of *Algebra* annexing, and of neceſſity requiring *Coſſical*, *Specioſal*, or other Denominations unto Abſolute and Undenominate Numbers, without which they muſt ſtill remain uncouth and indeterminable. The Myſteries of theſe are unvailed in the 4th *Part* of the 4th *Book*.

Numbers Heterogeneal.

Enough in this place hath been ſaid of *Homogeneal Numbers*, the other ſort are *Heterogeneal*, called alſo Mixt Numbers, ſo that Mixt Numbers admit of a double acceptation as well in difference to Digits and Articles (as before noted) as in oppoſition to *Homogeneal Numbers* which are immixt, or of one ſort.

Integers and Fractions.

Abſtract and Contract.

Briefly, to conclude, Numbers are *Heterogeneal*, or of divers kinds when they are mixed primarily, or ſecundarily. The Firſt are made up of *Integers* and *Fractions*, as One and a half, Two, and Three quarters, *&c.* The other are Mixt of Abſtract, and Contract Numbers, as Three Squares, and Two Thirds, Thirteen Roots, and Four Integers, *&c.* The latter ſort of them may be obſerved in the Three Laſt *Books*, Generally. The Former particularly *Book* 1ſt *Part* 2d, and Occaſionally throughout the whole Volumne.

CHAP. III.

Of the Elements of Numbers.

WHAT hath not been ſeen of the Affections of *Numbers* in the former Chapter of their *Nature*, may be further diſcovered in their *Elements* : A Type whereof followeth.

Affections of Numbers further to be ſeen in their Elements.

A Type of the Elements of Numbers.

- Elements of Numbers.
  - Simple
    - Notation
      - Determinate.
      - Cuſtomary.
      - Arbitrary.
    - Numeration
      - Original
        - Expreſſion
          - by Word
          - by Writing
        - Compoſition
          - Prime, in Addition.
          - Compound, in Multiplication.
        - Diſſolution
          - Prime in Subſtraction.
          - Compound, in Diviſion.
      - Ortive
        - Reduction
          - Multiplication.
          - Diviſion
        - Figuration
          - Production
            - Multiplication.
            - Addition.
          - Extraction
            - Subſtraction.
            - Diviſion.
  - Comparative
    - Deſcription
    - Computation
      - of Ratios.
      - of Proportions.
      - of Æquations.

In this Table the *Elements* of *Numbers* appear double, *viz.* Simple, and Comparative. Simple conſiſteth either in Notation, or Numeration.

Simple Elements, 2. fold.

Notation, which in ſome Authours paſſeth for Numeration teacheth the Notes, Marks, and Characters whereby Numbers, Quantities, Denominations, *&c.* are deſcribed, and accompted as valuable as expreſſed in words at length. And theſe are either Certain, and Determinate; or Uncertain and Arbitrary; or elſe Cuſtomary, nevertheleſs variable, being partly certain, and partly uncertain.

1. Notation and this.

Certain Notation I call the moſt excellent invention of expreſſing all Cardinal Numbers by thoſe Ten Characters; *viz.* 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. Generally called Figures, the Tenth is called properly in Latine, *Circulus*, or *Ciphra*, in Engliſh a Circle, or Cypher, and vulgarly Nought, and of it ſelf ſignifieth nothing, but being joyned to the right hand of the other, helpeth to increaſe their value by accident; the other Nine are called ſignifying Figures, as 1. doth ſignifie One, 2. Two. 3. Three. 4. Four. 5. Five. 6. Six. 7. Seven. 8. Eight. 9. Nine. All Ordinal Numbers are expreſſed by the ſame Figures, only at top inclining to the right hand, is placed one, or two of the final Letters of the Word, as the Firſt, $1^{\text{ſt}}$. Second, 2^{d}. Fourth, 4^{th}. *&c.*

Certain in the uſe of Figures.

Notation Uncertain, and Arbitrary is either in form, or in quantity. In Form, and ſo in Coſſical, and Surde Denominations ſome Authors uſe to write them in one form, and others in another; as ſome write thus, *Z.* for *Zenzike*, others. *Q.* and ſometime, q. for Quadrate: Some thus, ɱ. and others thus, ℨ. at pleaſure. Likewiſe in Surds this w. ſtandeth for a Cube Root in one, for a Squared Square Root in another. Characters both in Coſſicks, and Surdes uſed in this Treatiſe may be ſeen, *Book* 3^{d}. *Part* 4^{th}. and 5^{th}. And ſo from each Authour is to be expected an Explanation of their own Marks, alwayes minding that every man in his private practiſe uſeth thoſe he hath moſt accuſtomed, or beſt fancieth, and may change them for others ſooner made. Secondly, ſome Notes are uncertain in their Quantities, as in Species; for though any Quantity, or Number, whole, or broken, for the time preſent (or during the work of a queſtion) may be noted by any Letter of the Alphabet, *Greek*, or *Roman*, ſmall or Capital; as Α. Β. Γ. Δ. *&c.* α. ϐ. γ. δ. *&c.* A. B. C. D. *&c.* a. b. c. d. *&c.* yet when the Queſtion is ended, and another in working, the ſame Letters ſhall ſignifie other Magnitudes, or Numbers greater or leſſer, at the will of the Arithmetician, as is evident by the practiſe of the 6^{th} *Part* of the 3^{d} *Book.*

Arbitrary in ſeveral Marks and Characters

Cuſtomary Notation is certain in reſpect of the General uſe, yet uncertain in reſpect of the end for which uſed; for if any form of better repreſentation, may be more commodiouſly made, they may be altered for others. And ſuch are all thoſe Symbols, or Characters uſed for Vulgar Denominations with many that are frequented, to avoid prolixity, and the tedious, and often reſcription of ſome Words, and termes of Art: Some of the moſt certain, and chiefly in uſe the following Table will eaſily demonſtrate.

Cuſtomary in ſeveral Symboles and Characters of common uſe.

Vulgar Denominations marked, Words abbreviated, and Termes of Art Characteriſed.

		Latine.		*Engliſh.*		Characters.
Geodæticks.	Money	Libræ		Pounds, Liures, Gilders		l.ł.ł.ℓ.ℓ.
		Solidi		Shillings; Solx, Stivers		s.ſ.ß.ſß.
		Denarij		Pence, Deniers		d.ð.
		Oboli		Half-pence		ob:ob.
		Quadrantes		Farthings		qd.q.
		Coronati		Crownes		Δ.ϖ.
	Weight	Centum		Hundreds		c.C.
		Quadrantes		Quarters		qʒ.qʒ
		Libræ		Pounds		℔.℔.
		Unciæ		Ounces		℥.℥.
		Pondo-denarij		Penny-weights		p.pd.pw^{ts}.
		Drachmæ		Drams		ʒ.ʒ.
		Scrupuli		Scruples		℈.
		Graines		Graines		Gr.gr.
		Caractæ		Karacts		K.Kr.
		Per Centum		By the Hundred		p°/o.
		Continens		Containing		ton.
		Superpondium		Tare		ts.
		Abſque Superpondio		Without Tare		Netto.
Aſtronomicks.	Time and Motion	Anni		Years		A^{i}.
		Anno Domini		In the Year of Our Lord		An.^{o} D.
		Anno Chriſti		In the Year of Chriſt		An.^{o}C.
		Dies		Daies		D.d.
		Horæ		Hours		H.h.
		Signa		Signs		S.
		Gradus		Degrees		G.g.d.deg.
		Minuta, Secunda, *&c.*		Minutes, Seconds, *&c.*		I.II.&c.
		Ante Meridiem		Before Noon		a.m.
		Poſt Meridiem		After Noon		p.m.
	Sexagenæ	Primæ, Secundæ, Tertiæ, *&c.*		Primes, Seconds, Thirds, *&c.*		\.\\.\\\ *videplus*
	Sexageſimæ	Primæ, Secundæ, Tertiæ, *&c.*		Primes, Seconds, Thirds, *&c.*		/.//./// *lib.2.par2.*
	Signes	Aries		Ram	Head and Face	♈.
		Taurus		Bull	Neck and Throat	♉.
		Gemini		Twinns	Arms and Shoulders	♊.
		Cancer		Crab	Breaſt and Stomack	♋.
		Leo		Lion	Heart and Back	♌.
		Virgo		Virgin	Bowels and Belly	♍.♍.
		Libra		Ballance	Reins and Loins	♎.
		Scorpio		Scorpion	Secrets	♏.♏.
		Sagitarius		Archer	Thighs	♐.
		Capricornus		Goat	Knees	♑.♑.
		Aquarius		Waterman	Leggs	♒.
		Piſces		Fiſhes	Feet	♓
	Planets and Mettals.	Saturnus	Plumbum	Saturn	Lead	♄.♄.♄.♄.
		Jupiter	Stannum	Jupiter	Tinn	♃.
		Mars	Ferrum	Mars	Iron	♂.
		Sol	Aurum	Sun	Gold	☉.
		Venus	Ars Cuprum	Venus	Braſs, Copper	♀.
		Mercurius	Argentum vivum	Mercury	Quick-Silver	☿.
		Luna	Argentum	Moon	Silver	☽.
	Nodes	Caput Draconis		Dragons head		☊.
		Cauda Draconis		Dragons tail		☋.
		Pars Fortunæ		Part of Fortune		⊕.
		Antiſcia		Antiſcions		Ant.
		Contrantiſcia		Contrantiſcions		CA.
		Conjunctio		Conjunction		☌.
		Vigintilis		Vigintil		Vg.
	A	B Quindecilis		Quindecil		Qd.

Aſpects

A B

Aſpects	Semiſextilis	Semiſextil	SS.⊻.
	Semiquintilis	Semiquintil, or Decil	Dc.⌓.
	Semiquartilis	Semiquartil	Sq.♖.
	Sextilis	Sextil	*.
	Quintilis	Quintil	Q. ♡.
	Quartilis	Quartil	□.
	Seſquiquintilis	Seſquiquintil or Tredecil	Td. ♡.
	Trigonus	Trine	△.
	Seſquiquartilis	Seſquiquartil	SSQ. ⚼.
	Biquintilis	Byquintil	Bq. 8.
	Quincunx	Quincunx	Vc.
	Oppoſitio	Oppoſition	☍.
Houſes	Horoſcopus, Aſcendens	The Angle of the Eaſt, or 1st houſe	Hor.Aſcen.
	Medium Cœli	Midheaven or 10th houſe	M.C.Med.C.
	Angulus Occidentis	Weſt Angle, or 7th houſe	Ang: Oct.
	Sinum Cœli	The Angle of the Earth, or 4th houſe	IC.
Cardinal Points	Septentrio	North	N.
	Oriens	Eaſt	E.
	Meridies	South	S.
	Occidens	Weſt	W.
Affections	Longitudo	Longitude, or Length	Long.
	Latitudo	Latitude, or Breadth	Lat.
	Altitudo	Altitude, or Height	Alt.
	Recta Aſcentio	Right Aſcention	R.Aſc.
	Recta Deſcentio	Right Deſcention	R. Deſc.
	Obliqua Aſcentio	Oblique Aſcention	ob.Aſc.
	Obliqua Deſcentio	Oblique Deſcention	ob.Deſc.
Logarithmeticks. Canon Artificial	Logarithmus	Logarithme	Log.
	Sinus	Sine	s.
	Sinûs Complementum	Coſine	cos.
	Tangens	Tangent	t.
	Tangentis Complementum	Cotangent	cot.
	Secans	Secant	sec.
	Secantis Complementum	Coſecant	cosec.
Coſſicks	Radix	Root	℞.
Surdes	Radix	Root	√.
	Radix Univerſalis	Univerſal Root	√.
Species	Radix, ſive Latus	Root, or Side	l.
	Quadratus	Square	q.
	Cubus	Cube	c.
	Diameter	Diameter	D.
	Diametri Quadratus	Square of the Diameter	Dq.
	Diametri Cubus	Cube of the Diameter	Dc.
	Radius	Semidiameter, or Radius	R. Rad.
	Radij Quadratus	Square of the Radius	Rq.
	Radij Cubus	Cube of the Radius	Rc.
	Peripheria	Circumference, or Periphery	P.
	Peripheriæ Quadratus	Square of the Periphery	Pq.
	Radix Binomii	Binomial Root	√b.
	Radix Reſidui	Reſidual Root	√r.
	Radix Suppoſititia	Suppoſititious Root	A.
	Duo Numeri	Two Numbers	AE.
	Major	Greater	A.
	Minor	Leſſer	E.
	Summa	Sum	Z.
	Differentia	Difference	X.
	Rectangulum	Rectangle, or Product	Æ.AE.P.
	Summæ Quadratus	Square of the Sum	Zq.
	Differentiæ Quadratus	Square of the Difference	Xq.
	Summæ Cubus	Cube of the Sum	Zc.
	Differentiæ Cubus	Cube of the Difference	Xc.

a b

Vocabula Artis.

Summa Quadratorum	Sum of the Squares	Z.
Summa Cuborum	Sum of the Cubes	Z.
Differentia Quadratorum	Difference of the Squares	X.
Differentia Cuborum	Difference of the Cubes	X.
Tres continuè proportionales	Three Numb. contin. proportional	A, M, E.
Quatuor continuè proportionales	Four Numb. continually proportional	A, M, N. E.
	Arithmetical Progreſſion.	
Primus Terminus minimus	The Firſt Term, or leaſt	α.
Ultimus maximus	The Laſt Term, or greateſt	ω.
Numerus Terminorum	The Number of Termes	T.
Differentia communis	The Difference, or Exceſs common	X.
Summa omnium Terminorum	The Sum of all the Termes	Z.
	Geometrical Progreſſion.	
Terminus primus	The firſt Terme	α.
Terminus ſecundus	The ſecond Terme	β.
Terminus tertius	The third Terme	$\frac{\beta q}{\alpha}$.
Terminus quartus &c.	The fourth Terme &c.	$\frac{\beta c}{\alpha q}$.

Signal Symboles for Terms of Art.

+ Commonly called St. *Georg's* Croſs, is the ſign of *Addition*, and being ſet before a Number, or quantity, ſignifieth *More*, *More by*, *and*, or any ſuch word that may ſhew the ſame Number, or Quantity to be affirmative.

− A *Right Line* is the ſign of *Subſtraction*, and ſet before a number or quantity, imports *leſs*, *leſs by*, *lacking*, *wanting*, or any ſuch word that may declare the ſame number or quantity to be negative.

X Commonly called St. *Andrews's* Croſs, is the ſign of *Multiplication*, and ſet between two Numb. or Quant. implies multiplyed *by*, *in*, *into*, *times*, or any ſuch word, that may denote the numb. or quant. multiplyed one into the other.

) The *Lunular Increſcent* is the ſign of *Diviſion*, and ſet between two numbers or quantities, ſignifyes dividing, and declares the right hand number or quantity of the Two, to be divided by the left.

(The *Lunular Decreſcent* is the ſign of the *Quotient* of any *Diviſion*, and the Number there ſtanding demonſtrates how often the *Diviſor* is contained in the *Dividend.*

•••• *Four Points* in a Right Line, is uſed ſometime in *Reduction of Fractions*, to ſeparate the old Numerators from the new, and the old Denominators from their leaſt termes.

—— A *Right Line* between two numbers, or quantities, one ſtanding above, and the other beneath, commonly denotes a *Fraction*, and that the upper number, or quantity is to be divided by the neather. And ſometimes is uſed to ſeparate one Number from another.

⌊— The *Rectangle* is the *Seperatrix* between *Integers*, and *Decimals*, of which and other Diſtinctions thereof, *Vide plus, lib.* 3. *par.* 1. & 2. Sometime alſo is inſtead of the *Decreſcent Lunular* in *Diviſion.*

, The *Comma*, is the Diſtinction alſo between *Integers* and *Decimals*; and between the *Logarithme* and his *Characteriſtique.*

: The *Colon*, Including *Numbers* or *Quantities*, is a Note of the *Univerſal Root* of ſuch Number, or Quantity.

. The *Period*, is often uſed for Diſtinction-ſake, but in disjunct Proportions, between two Numbers or Quantities, underſtands the word, *To.*

⋮ *Three Pricks* or *Points*, in the middle of four numbers, or quantities, are ſometimes uſed in Disjunct Arithmetical Proportions, for the words, *is as*

∷ *Four Points* thus, in the middle of four numbers or quantities is the ſign of *Analogy*, and frequently uſed in Disjunct Geometrical Proportions for the words, *is as*; or *ſo is.*

= *Parallels*, are the ſign of *Æquality*, and implies the numbers or quantities of the one ſide thereof, are equal to the numbers or quantities of the other ſide thereof.

⊏ Greater. ⊏. Next Greater. ⊐. Leſſer. ⊐. Next Leſſer.

⊏ Not Greater. ⊐. Not Leſſer. ⊑. Equal or Leſs. ⊒. Equal or Greater.

Between

c

• Between two Numbers, or Quantities, the one above, and the other below, is the ſign of a *Ratio.*

$\overline{\cdot\cdot}$ Greater Ratio. $\underline{\cdot\cdot}$ Leſſer Ratio. ∺ Continual Proportionals.

⊓ Commenſurable. ς⊓ Incommenſurable

⊓̲ Commenſurable in Power. ς⊓̲ Incommenſurable in Power.

ɼ Rational. ςɼ Irrational. m∼ Medial.

ϛ A Line cut according to extreme, and mean Proportion.

σ The greater Portion thereof. τ The leſſer Portion thereof.

ní A mean Proportional. ℑ Divided by.

∽ The Difference of the two Magnitudes.

Common Termes.	℘. ꝑ	By.
	lib.	Book.
	Cr.	Creditor.
	Dr.	Debitor.
	fol.	Leafe.
	Nº. N.N	Number.
	pd.	Paid.
	Rd.	Received.
		&c.

Other words not having any proper Symbole are abbreviated when occaſion requires by two, or three of the firſt Letters of the word, as *Duc.* for *Ducats. Mon.* for *Moneths*, &c. *D.* and *Q.* beſides what as before they ſtand for Notes of, may ſometime peradventure be found for *Data*, and *Quæſita*, which termes import the One ſuch *Numbers* or *Magnitudes*, *&c.* as are given for the Reſolution of a Queſtion, the other the *Numbers* or *Magnitudes* ſought.

Nothing need be added for demonſtration of the Table, but that ſome Difference may be obſerved between the Characters uſed in Printed Books, and thoſe practiſed by the Pen, according as the written, and Printed Alphabets differ.

Where denominations have two or more marks any one of them will ſerve, as may be further obſerved in the parts of the Treatiſe to which they belong.

The next part of the *Simple Elements* of *Numbers* is *Numeration*, which is both *Original* and *Ortive*. *Original* is of double uſe; for it ſerveth either rightly to Expreſs, and Accompt the Value of Numbers by their Notes, Symboles, Characters, Places, *&c.* or elſe to find out, and procure Numbers valuable Greater, or Leſſer. This laſt is ſometime called *Algorithme*, though *Algorithme* more properly is *Coſſical Arithmetick*. 2. *Simple Element of Numbers is Numeration, and this either Original, which is of 3 ſorts.*

1. The firſt kind teacheth the Order, which *Arithmeticians* do obſerve, in the uſual expreſſing, and valuing of *Numbers*, *Quantities*, *&c.* either by Word or Writing; and with ſeveral Authours paſſeth for the only Numeration: This for *Integers* is to be ſought in the next Chapter, and for *Fractions*, and all the other ſorts of Numbers, in the firſt Chapter of every Part of the other Books.

2. The ſecond ſort of *Original Numeration*, ſheweth the Method of Increaſing Numbers. The Increaſing of Numbers is called Compoſition, or the Geneſis of Numbers; This is Simple in *Addition*, and Compound in *Multiplication*.

3. The third ſort of *Numeration Original*, is the Diminiſhing of Numbers, or their Diſſolution, which likewiſe is Simple in *Subſtraction*, and Compound in *Diviſion*, and both theſe may be called the *Analyſis* of *Numbers*. So that *Addition* and *Subſtraction*, are both the *Prime*, or *Simple* parts of *Numeration*; and therefore ſhall in order precede *Multiplication*, and *Diviſion*, which are the Conjunct parts thereof: The ſeveral Chapters whereof are to be ſeen for each ſort of Numbers diſtinctly, in the Parts where they are handled. or,

Numeration Ortive, ariſeth from the former Species of *Numeration Original*, and conſiſteth in two things, *Reduction*, and *Figuration*. *Reduction* is uſeful in *Fractions*, and *Contract Numbers*, and performed by *Multiplication*, or *Diviſion*: As will ſufficiently be evident in the ſecond part of this Book, and elſewhere in Contract Numbers. *Ortive, which is 2. fold. Reduction.*

Figuration, ſerveth for Figural Numbers, to produce them, or extract their *Roots*. *Production*, or their *Geneſis*, is effected by *Multiplication*, and *Addition*. The Extraction of their *Roots*, or *Analyſis* is made up principally of *Subſtraction*, and *Diviſion*, occaſionally uſing *Multiplication* and *Addition*; which being further ſet forth in the ſecond Part of the Book may here be ſpared. *Figuration.*

The *Comparative Elements* of *Numbers*, belong to the Work of *Proportions*; The Deſcription, and Computation whereof, both *Arithmetical*, or *Numeral*, and *Geometrical*, or *Menſural*, and of each kind *Disjunct*, and *Continued*, together with *Æquations*, are reſerved for the Fourth Book, and therefore omitted here. *Comparative Elements to be ſeen in the 4th. Book.*

CHAP. IV.

Of INTEGERS.

Integers firſt proceeded with.

THE *Nature*, and *Elements* of *Numbers* ſeen in general, it is now requiſite to deſcend into particulars, and to proceed in Order according to the foregoing Method. The firſt ſort of Numbers that preſent themſelves, are proper, and homogeneal Numbers, called *Integers*, Conſidered abſtractively.

All Numbers expreſſed by 9. Figures and the Cypher.

The Notes, or Characters, frequented both in *Integers*, and *Fractions* abſolute, or of common Denomination, and generally in all Arithmetical Operations, are the forementioned Figures, 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. the which thereby that Arithmeticians may expreſs all Numbers, they have ordered them into certain places, and periods, proceeding alwayes from the right hand towards the left in a Decimal progreſſion, ſo as the aforeſaid Figures may expreſs greater Numbers, or leſſer as neceſſity requireth.

Place of a Figure, what.

A Place is the Seat or Room that a Figure ſtandeth in, and ſo many Figures and Ciphers as there are in one Number or Sum, ſo many places hath that whole Number.

No certain Number of Places.

Of theſe Places there is no certain Number, but that is called the firſt place that is next to the right hand, and reckoning in order towards the left hand, the next is the ſecond, then the third, and ſo *ad infinitum*, as a. b. c. d. thus ſtanding, d. thus ſtandeth in the firſt place, c. in the ſecond, b in the third, and a. in the fourth, *&c.*

Every Place hath ſome Denomination.

Every place hath a certain denomination properly belonging to it. Whereby a figure according to its ſtanding comes to be valued many times more than the Figure ſtanding ſingle would import. For 1. 2. 3. *&c.* ſhall not only ſignifie ſo many entire Unites, or Ones, according to their formes, but may alſo ſignifie 1. 2. 3. *&c.* Tens Hundreds, Thouſands, *&c.* according to the place the figure occupieth, for by ſo much as any figure inclineth towards the left hand, by ſo much is the value thereof increaſed.

Unites Place

The firſt place, hath the denomination of Units, and doth ſignifie that every figure ſtanding there, betokeneth his own ſimple value according to his form, as the figure 1. to ſignifie but One, the figure 2. but Two, and ſo of the reſt.

Tens

The ſecond place, toward the left hand hath the denomination of Tens, and every Figure here ſtanding ſhall betoken his own certain value Ten Times; as 1. if it ſtands in the ſecond place ſhall be one Ten. 2. Twice Ten, or Twenty. 3. Three times Ten, or Thirty, *&c.*

Hundreds.

The third place, to the left hand hath the denomination of Hundreds, and ſo every Figure there ſtanding ſhall betoken his own value a hundred times, as 6. ſtanding there denotes ſix hundred, *&c.*

Thouſands, &c.

In the fourth place, every figure ſtanding, ſignifieth his value a Thouſand times, In the fifth place, Ten thouſand times. In the ſixth place, one hundred thouſand times. In the ſeventh place, one Thouſand Thouſand times, or one Million (which is called by ſome the firſt great Thouſand.) In the Eight place, Ten Millions In the Ninth place, one hundred Millions. In the Tenth place, one Thouſand Millions, (called by ſome the ſecond great Thouſand.) In the Eleventh place, Ten Thouſand Millions. In the Twelfth place, One Hundred Thouſand Millions. In the Thirteenth place One Million of Millions, (or the third great Thouſand.) So infinitely Names may be given to every place, each ſucceeding place exceeding the former Ten times; though in ordinary practiſe we ſeldom need thirteen places, yet if any liſt to exceed, it is but doubling the Millions to begin as at the Eight place, for the fourteenth place is Ten Million of Millions, The Fifteenth like the Ninth is Hundred Million of Millions, and thus proceeding till the Nineteenth place, where tripling the Millions, go on as before to the Twenty fifth place, and then quadruple the Millions, and ſo as before *ad Infinitum.*

Others inſtead of doubling the Millions in the thirteenth place, call it Billion; the nineteenth place Trillion, inſtead of Millions of Millions of Millions; the twenty fifth place Quadrillion, and the Thirty firſt place Quintillian, *&c.*

Denomination of the Places reſpect, only Quantity.

Note here, That a Number though thus denominate according to his place, yet is accompted Abſtract, without another denomination, this denomination only reſpecting the quantity, number, or multitude of the thing or things propounded, and that denomination which truely decyphers a Number Contract, reſpects the quality, or nature of the thing numbred; yet notwithſtanding it bears ſome ſimilitude thereto, for as the denomination of quantity is the laſt recited in the verbal expreſſion of any Number, as to ſay Two Hundred, where Two is the Numerator, or Number, and Hundred the denomination of quantity which is laſt repeated. So is the denomination of quality repeated after the quantity; As in ſaying Two Hundred Pounds, after the Sum of Two Hundred the quantity, is laſt of all named Pounds the denomination of quality; likewiſe Thirty Men, here Men is the Contract Denominator, and Thirty the Numerator, or Valuer, &c.

How different to the denomination of Quality.

Theſe places are diſtinguiſhed into Degrees, and Periods. Degrees are three; Once, Ten times, a Hundred times. A Period is a comprehenſion of Degrees, and is Simple, or Compound. Simple is made up of one Ternary of Degrees, containing three places, as 123. Compound is double, as 12345. Or treble, as 12345678, or fourfold, &c. A further view whereof may be had in the following Table, which may be ſet in ſeveral formes.

Places are diſtinguiſhed into Periods.

Numeration Table.

A.	z. y. x.	w. u. t.	ſ. r q	p. o. n.	m. l. k.	i. h. g.	f. e. d.	c. b. a
5.	6. 7. 8.	9. 0. 1.	2. 3. 4.	5. 6. 7.	8. 9. 0.	1. 2. 3.	4. 5. 6.	7. 8. 9
M	M M M	M M M	M M M	M M M	M M M	M M M	M M M	C X I
M	M M M	M M M	M M M	M M M	M M M	M M M	C X	
M	M M M	M M M	M M M	M M M	M M M	C X		
M	M M M	M M M	M M M	M M M	C X			
M	M M M	M M M	M M M	C X				
M	M M M	M M M	C X					
M	M M M	C X						
M	C X							
&c. ad Infinitum	The eight Period of Thouſands of Millions of Millions of Millions.	The ſeventh Period of Millions of Millions of Millions.	The Sixth Period of Thouſands of Millions of Millions.	The Fifth Period of Millions of Millions.	The Fourth Period of Thouſands of Millions.	The Third Period of Millions.	The Second Period of Thouſands.	The Firſt Period of Units.

Here the order of the places is noted by the *Roman Letters* at top. The Value or denomination by the Numeral Capital Letters. The *Periodical Diviſion* ſhews the thouſandth place of the Number.

Theſe things obſerved it will not be difficult to expreſs the true content of any Number Integral by word or writing: For the former conſider the figures and the places they ſtand in, and call their values ſuch as the figure is augmented by his ſtanding; wherefore if I ſee 98. ſtanding, and would expreſs the right value thereof, I know the firſt figure is but Eight, for it beareth that form, and ſtanding in the Units place, and therefore can ſignifie himſelf but once; but the 9. muſt be Nine times Ten, becauſe it ſtandeth in the ſecond place, and ſo is the value, or content of that quantity Ninety Eight, as if it ſhould ſtand thus 90. 8. and be expreſſed Nine times Ten and Eight. And this 398. Three hundred ninety eight, as if it ſtood thus 300. 90. 8. and after this manner are all quantities valued that are beneath a Thouſand. If the quantity exceed, for the better remembring the periods, and ſo thereby the more readily to expreſs great Sums, or quantities, mark the periodical Diviſion of the Table on the given Number with a prick above or below the figure ſtanding in that place, beginning at the right hand ſet one prick over the firſt Figure, another over the fourth, &c. as to numerate or expreſs the value of this Sum 3479841234. The pricks are placed thus 3479841234. then beginning at the left hand read as it were every pricked parcel apart, with the denomination of the place to it: So at laſt you ſhall have the whole value; as firſt One Thouſand, Two hundred thirty four; Then, Nine Million, eight hundred forty one thouſand, two hundred thirty four; Laſtly, Three Thouſand four hundred ſeventy nine million, eight hundred forty one thouſand, two hundred thirty four. And ſo much is the value, or quantity of that Number; the like is to be done with others.

To expreſs a Number by word.

To expreſs in Writing, any propounded Integer, remember all numbers from one to ten, are expreſſed by a digit in the Units place; All numbers above ten, and under a hundred with two Figures, or one Figure, and one Cipher, to wit Tens in the place

To expreſs a Number by writing.

place of tens, and Units or Cyphers in the place of Units, according to the Numbers to be expreſſed; likewiſe all Numbers under a thouſand, and above a hundred are expreſſed by three Figures, or two Figures and one Cipher, *&c.* Then obſerve the places as before, and begin at the right hand, put down the quantity of Unites propounded, and ſo proceed unto the left.

Void places to be ſupplyed with Cyphers.

If the given Number have any rooms void, they are alwayes to be ſupplyed with Cyphers, as to write down one hundred the places of Units and tens being void are filled up with Cyphers, and one is ſet in the hundreds place thus 100. So one thouſand thus 1000. and nine thouſand and ſeventy million ten thouſand one hundred and one, thus 9070010101. where Cyphers occupy ſeveral places inſtead of Figures, becauſe neither Tens, Thouſands, Hundred Thouſands, Millions, nor Hundred Millions were given in the Number.

CHAP. V.

Addition of Integers.

Having ſeen the due ordering and placing of the Notes for the Expreſſing of Numbers by them, it remaineth to declare the other parts of Numeration, and firſt the Prime Part of

Addition, *what it is.*

Addition is that part of *Arithmetick* whereby divers Numbers are collected, and added together into one total Sum.

Addition of *Integers*, hath reſpect to *Collocation*, *Operation*, and *Probation*.

Addends, what. Total, what.

Collocation, is the due placing the Numbers given to be added, called the *Addends*, and diſtinguiſhing them from the Number found out by *Addition*, called the Sum Total, or Aggregate.

How to place the Addends.

Place the *Addends* in rank and file one directly under another, beginning at the right hand; it matters not whether the greateſt or leaſt be uppermoſt, ſo that Numbers of one quantity, may ſtand under Numbers of the ſame quantity, that is to ſay, Units under Units, Tens under Tens, *&c.* then draw a right line under them, to diſtinguiſh them from the Total.

Operation, hath *Induction*; and *Conſummation*, or *Perfection*.

Induction *what it is, called* Simple Addition.

Induction, teacheth the Sum that any two digits added together make, called ſometime *Simple Addition*; and had need be ready in Memory with the Accomptant; This is declared in the following Table.

Addition Table.

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

The uſe of the Table.

By this Table the ſum of any two digits is eaſily had, for entring with one at the head, and the other at the left hand ſide, the Common Angle is the Sum, as 3. and 7. make 10. and ſo much is found over againſt 7. in the ſide, and 3. in the head, or againſt ſeven in the head and three in the ſide, if the Table be quadrangular as this is; but for that the lower part of the Table beneath the black ſcale is ſufficient, it may be ſet in that Triangular form, for 2. and 1. and 1. and 2. are all alike, *&c.*

By the help of theſe Digits added, the Sum of the Decades or Articles are alſo known, becauſe their ſignifying figures are but digits, as 20. and 20. make 40. becauſe 2. and 2. are 4. the Cyphers being only reſerved to keep place as before, but increaſe not the Sum, for 1. and 0. is but 1.

Perfect Operation called Compound Addition how it is wrought.

Perfect Operation, ſometime called *Compound Addition* finiſheth the Addition of Mixt Numbers thus; begin at the right hand, and take all the figures or digits in the right hand file, as they ſtand one over another, and putting them together conſider what the reſult or total thereof is, and if it be a digit write it under the right line directly in the ſame file, and ſo do in every of the files; but if any file being caſt up amount to an Article write the Cypher under the file, and reſerve in mind the figure of the Article to be added in the next place, and when the Sum of the next file is found add that reſerved Article, to wit, for every ten reſerved, one; as two for 20. three for 30. *&c.* and this reſult or total ſubſcribe accordingly; and if the ſum or total of any file with or without ſuch reſerved Article, if any be, amount to a mixt number, then ſet down the digit of that mixt Number, and reſerve the Article thereof, as before.

Example 1.

Example. There are two Numbers propounded, whereof the one was 234. and the other was 342. What is the Sum of both? The numbers ſet as at A. in the right hand file are found 2. and 4. which put together make 6. to be ſet under the file becauſe a digit, as is repreſented by the work ſtanding at B. Then going forward to the left hand in the next file are 4. and 3. which make 7. to be ſubſcribed as at C. Laſtly in the third place are 3. and 2. which make 5. and the whole work ſtands as at D. where the Total appears to be 576.

Rank

A		B	C	D	
234	File	234	234	234	Addends.
342		342	342	342	
		6	76	576	Total.

Example 2.

Second Example. Suppoſe Two Numbers whereof the Sum is deſired be 40235. and 34973. then are they ſet as before, and in the right hand file 3. and 5 make 8. to be there ſubſcribed. Then 7. and 3. make 10. an Article, the Cypher therefore is ſet down under the line in the ſecond file, and the Article reſerved. Then 1. carried in mind, and 9. is 10, and 2. is 12. which being a mixt number, two the digit thereof is ſubſcribed, and the Article reſerved as before. Again 1. reſerved and 4. is 5. to be ſet down under the 4. not regarding the 0. becauſe the number is not augmented thereby in Addition. Laſtly adding 3. and 4. together they make 7. which is ſet in the laſt place, and the work ſtandeth as at K. where the total 75208. is the number deſired. The ſeveral other Paragraphs ſtand as followeth.

E	F	G	H	I	K
40235	40235	40235	40235	40235	40235
34973	34973	34973	34973	34973	34973
	8	08	208	5208	75208

Proof of Addition. Common.

Probation, is the *Examen* or *Demonſtration* whereby the Operation may be proved true. The vulgar proof is by caſting all the Nines that can be had both out of the Aggregate, and Numbers added, which done will leave like figures if the work be right. To do this begin with the Total, or the Addends, at the left or right hand, not regarding the places of the Numbers, but as though they were all Units add them together, and as the Numbers increaſe above 9. reject 9. and go forward with the reſt, and what remains when all the Numbers are gone over, ſet down at the one ſide or end of a right line; Then do ſo with the other part of the Addition, and this remain place at the other ſide or end of the ſame line: As to inſtance in the firſt Example 4. and 2. make 6 in the right hand file, then 6. and 4. make 10. coming to the next file, from which 9. rejected there reſteth 1. that 1. and 3. make 4. which 4. and 3. make 7. and 2. is 9. which caſt away there remaineth 0. to be ſet on the line as at M. Then in the

Total 6. and 7. is 13. from which 9. caſt there remaineth 4. which 4. and 5. are 9. to be again caſt away, and there reſteth 0. alſo to be ſet under the line as at N.

L $\begin{array}{r}234\\342\\ \hline 576\\ \hline\end{array}$ M 0 N $\frac{0}{0}$ or thus 0—0

This Proof by 9. uncertain.

Although if the work be right, it will never vary from leaving equal remains, yet by reaſon ſome digit or other may happen through inadvertency to be miſplaced, this kind of proof is uncertain, for it is evident, that if the places of 7. and 6. in the Total ſhould be unhappily changed, or of 5. and 7. yet would the remains by rejection of nines be as before, though the Sum much altered in the value. For both 567. and 576. alſo 756. and 765. likewiſe 657. and 675. after the nines are caſt away leave 0. yet is their value greatly different: Which thing may fall out in every other Number.

Beſt Proof by Subſtraction.

Therefore the beſt Proof is by *Subſtraction*, which may be ſeen in the next Chapter.

CHAP. VI.

Subſtraction of Integers.

Subduction or Subſtraction, what it is.

THE firſt part of the Geneſis of Numbers unfolded, The prime part of their *Analyſis* followeth, which is *Subſtraction*, called alſo *Subduction*.

Subſtraction, is that numerative part of *Arithmetick*, which teacheth how to deduct one Number from another, and to ſhew what remaineth.

Subſtraction of *Integers*, like *Addition*, reſpecteth *Collocation*, *Operation*, and *Probation*.

How to place the Numbers.

Subtrahend what.

Remainder what and how called.

Collocation, placeth in the uppermoſt Rank the greater number, (or number from which *Subſtraction* is to be made,) for in *Integers* no greater number can be taken from a leſſer; and underneath the ſame (with or without an interjacent line) the number to be ſubſtracted or deducted called the *Subtrahend*, ſo orderly that every figure may ſtand under his like, as Units under Units, Tens under Tens, &c. Then with an interjacent right line ſeperate theſe two given Numbers from the Number found out by *Subſtraction*, which is called the Remainder, Remain, Reſt, Difference, or Exceſs.

Operation hath *Induction* and *Perfection*.

Induction called Simple Subſtraction.

Induction, ſometime called *Simple Subſtraction* ſheweth the Difference between any two digits ſubſtracted one from the other, and is requiſite to be remembred by the Practitioner, according to the following Table.

Subſtraction *Table.*

0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	1	0	1	2	3	4	5	6	7
3	2	1	0	1	2	3	4	5	6
4	3	2	1	0	1	2	3	4	5
5	4	3	2	1	0	1	2	3	4
6	5	4	3	2	1	0	1	2	3
7	6	5	4	3	2	1	0	1	2
8	7	6	5	4	3	2	1	0	1
9	8	7	6	5	4	3	2	1	0

The Uſe of the Table.

This Table would be ſufficient, if only the upper part above the black Scale be uſed. The difference between two Digits is found thus; enter with the leſſer or ſubſtracting digit at the left ſide, and the other at the head, and in the Common Angle is the difference found, as 2 from 9 leave 7. which ſtands juſt againſt 2 in the ſide, and 9 in the head; the like alſo will be found againſt 2 in the head, and 9 in the left ſide of the Table, if the Table be quadrangular as this is.

By help of the difference of theſe *Monades* the difference of the Articles alſo are had, as 30 from 40 leaves 10, becauſe 3 from 4 leaves 1. the Cypher, only keeping their place as before.

Compound Subſtraction how wrought.

Conſummate Subſtraction ſometime called *Compound* perfecteth the work with mixt Numbers in this manner. It beginneth at the right hand, and withdraweth or abateth the lower Numbers or Figures, particularly one after another, out of the upper ſtanding over them, and ſubſcribeth the Number remaining, if any be, but if nothing remain, 0. except it happen to be the laſt file, and then the Cypher need not be ſet down.

Example without any borrowing.

Example. If 241. were to be taken from 343. and it were deſired to know the Remain. The numbers being placed as at A. then 1. out of 3. leave 2. to be ſubſcribed under the Line in the right hand file, as at B. then 4. from 4. there reſteth 0. as at C. and 2. from 3. there remaineth 1. and the compleat work ſtands as at D. So the whole difference between the two given Numbers is found to be 102.

Rank

A	File.	B	C	D	
343		343	343	343	Greater Number.
241		241	241	241	Subtrahend.
		2	02	102	Remain.

If it happen that the neather figure be greater than the upper, ſo that *Subſtraction* cannot be made, then in imagination borrow Ten, and adding it to the upper Figure make *Subſtraction* from both, and for that borrowed ten account one back in the next file, either by counting the next figure to be ſubſtracted one more than it is, or the next Figure to be ſubſtracted from one leſs than it is.

Example with borrowing.

Example. Suppoſe the difference between 30971. and 12381. be required. The numbers are ſet as at E. and 1. from 1. taken, 0. is left, as at F. but 8. from 7. cannot be taken without borrowing 10. which put to 7. makes 17. from which 8. taken leaves 9. to be ſubſcribed in the ſecond file, as at G. for which 10. reckoning the next 3. for 4. or the 9. but 8. and taking the leſſer from the greater, that is either 4. from 9. or 3. from 8. the remain is 5. ſubſcribed as at H. then 2. from 0. cannot be ſubſtracted, wherefore 2. from 10. as before leaves 8. as at I. and laſtly 1. borrowed, and 1. are 2. abated from 3. leaves 1. as at K. ſo the whole exceſs is found to be 18590.

E	F	G	H	I	K
30971	30971	30971	30971	30971	30971
12381	12381	12381	12381	12381	12381
	0	90	590	8590	18590

When one Number is to be ſubſtracted from many, or many from one, firſt add all the Plural Numbers into one, and therewith proceed to *Subſtraction* as before.

Example where the data are many.

Examples of both. In the firſt 3496001 taken from 3424501. 1042601. and 200000. whoſe total is 4667102. and the Remainder 1171101. as at A. In the ſecond 5423401. 800100. and 10081. are ſubducted from 6245002. and the Remain is 11420. as at M.

L	
3424501 1042601 200000	Numbers from which *Subſtraction* is made.
4667102	Total of the Addends.
3496001	Subtrahend
1171101	Remain.

M	
6245002	Greater Number.
5423401 800100 10081	Subtrahends.
6233582	Total of the Subtrah.
11420	Remain.

Probation

Proof of Subſtraction. Common. *Probation* ſerveth to demonſtrate the Truth or Error of the Work, and is vulgarly performed by caſting away nines from the Number from which *Subſtraction* is made, and keeping the remain thereof equal to the remain left after rejection of nines from the Subtrahend, and Remain; as in the firſt Example above in this Chapter. 9. caſt from 343. leaves 1. So alſo from 241. and 102. to be ſet thus $\frac{1}{1}$, or thus 1 — 1.

This Proof by 9. uncertain. But for the Reaſon ſhewed before in Addition this way of Proof is uncertain. As the beſt proof therefore of *Addition* is by *Subſtraction*; ſo the beſt proof of *Subſtraction* is by *Addition*.

Proof of Addition by Subſtraction 2. waies. *Addition* may be proved by *Subſtraction* two wayes. Firſt by beginning at the left hand, and deducting in order from the particular places of the total, the Sum of the particular Files added, for if the work be right, 0. will remain at laſt.

1. As in the former inſtance where the total of 5423401. 800100. and 10081. was found to be 6233582. here 5. taken from 6 in the total leaves 1. then 8. and 4. in the next File make 12. abated from the 1. left, and 2. in the total, which are 12, leave 0. then 2. and 1. make 3. taken from 3. there reſt 0. and ſo in all the reſt; as at N.

2 Secondly, By cutting off from the Addends any Rank, and adding the Reſidue into one total; then ſubſtracting this total from the firſt total, the remain will be the numbers firſt cut off; if the work be right.

As in the former inſtance, if 5423401. be cut off, and the total of 800100. and 10081. which is 810181. as at O. abated from 6233582. the remain will be 5423401. as at P.

N	O	P	
5423401	5423401	6233582	Firſt Total.
800100	800100	810181	Second Total.
10081	10081		
6233582	810181	5423401	Numbers cut off.
x000000			

Proof of Subſtraction by Addition. *Subſtraction* is proved by *Addition*. For if the Remainder, and Subtrahend be added together, the Number from which *Subſtraction* is made will be returned.

As in the laſt inſtance, if 5423401. be added to 810181. the Total will be 6233582. as at Q.

	6233582	Greater Number.	
Q	810181	Subtrahend	added.
	5423401	Reſidue	
	6233582	Total and Proof.	

Both bottomed on 2. Theorems. The Proof of *Addition*, and *Subſtraction* is bottomed on the two fundamental *Theorems* following, which give life mutually to *Addition*, and *Subſtraction*.

1. 1. In *Addition* the Aggregate is equal to all the Addends, and *contra*, as $3+2=5$. and $5=2+3$.

2. 2. In *Subſtraction* the Number to be ſubducted, and the difference are together equal to the Number from which *Subſtraction* is made, and *contra*, becauſe $3+2=5$. therefore $5-2=3$. and $5-3=2$.

Conſectaries from thence. From hence ſpring theſe two Conſectaries.

1. 1. To know the one Addend, if the Total, and other Addend be given, ſubſtract the given Addend from the Total.

2. 2. To know the Subtrahend, if the Number from which *Subſtraction* is made, and the Remain be given, ſubſtract the Remain from the Number from which *Subſtraction* is made.

Examples in Queſtions of Addition *and* Subſtraction.

Queſtions in Addition.

1. 1. What Number is that to which if 52. be added it makes 397?

Anſwer. Subſtract 52. from 397. and there remaineth 345. the Number ſought.

2. 2. What Numbers are thoſe to which if there be added ſeverally 32. 42. and 52. their Totals will be alike, and the Sum of each will be 130?

Anſwer. Subſtract the given Numbers ſeverally from 130. and the remains are the Numbers queſited, which are 98. 88. 78.

Queſtions in Subſtraction.

3. What Number is that from which if 40. be taken, the remain will be 375? *Anſwer.* Add 50. to 375. and the total is 415. the deſired Number.

1.

4. What Numbers are they from which if 14. 15. or 16. be deducted, yet the remains will be equally alike 226?

2.

Anſwer. Add ſeverally to 226. the ſame Numbers, and the Totals will be 240. 241. 242. the Numbers required.

	1	2			3	4.		
	397	130.	130.	130.	375	226.	226.	226.
Operation	52	32.	42.	52.	40	14.	15.	16.
	345	98.	88.	78.	415	240.	241.	242.

CHAP. VII.

Multiplication of Integers.

Multiplication what it is.

THE Prime, and Uncompound parts of *Numeration*, *Addition*, and *Subſtraction* preceeding, next ſucceed the Compound, which are *Multiplication* and *Diviſion*.

Multiplication is the Compound Geneſis of Numbers, and a Conjunct part of *Numeration*, whereby one Number is led into, or increaſed by another; for the Number given to be multiplyed is ſo often added to it ſelf as there be Units in the multiplying number, wherefore the third number, or number produced thereby, ſhall ſo often contain the firſt number, as there be Units in the ſecond.

Multiplication ſerveth inſtead of many *Additions*, with more ſpeed to augment a leſſer number than *Addition* can. For *Multiplication* effecteth at once, what *Addition* could but do at many times.

Serveth inſtead of many Additions.

Multiplication obſerveth *Collocation*, proceedeth to *Operation*, and concludeth with *Probation*.

Collocation, reſpecteth the *Nomenclator* or Artificial termes, and duely placeth every number thus. Firſt ſet down the greater of the two given numbers for the number to be increaſed, or multiplyed, which is called the *Multiplicand*; and under him beginning at the right hand (obſerving due place, and denomination of Units, Tens, &c.) place the leſſer given number for the multiplying number, called the *Multiplier*, yet as in *Addition* it is not material which ſtandeth uppermoſt, for either may be exchanged for other, but it is moſt orderly to place the greater number at top; then draw a right line under them to ſeperate them from the number found out by their *Multiplication*, which is called the *Multiplee*, and ordinarily the *Product* becauſe produced by the other two, ſometime the *Offcome*, as coming off them, ſometime the *Factus*, or *Fact*, being made by them, in reference to which the two given Numbers, *Viz.* the *Multiplicand* and *Multiplier* are called *Factores*, or *Factors*, and ſometime alſo the *Product* is called the *Rectangle* or *Plain*, which implies that one of the propounded numbers is taken for the length and the other for the breadth of a *Rectangle*, or plain Figure geometrical, in reſpect to which the given numbers are called ſides, and ſo ſhall the *Product* be underſtood for the *Content* or *Area* of that *Rectangle* figure.

How to place the Numbers. Multiplicand what. Multiplier what. Product what, and how called. Factors and Rectangle what.

Operation hath *Induction*, and *Perfection*; That ſometime called *Simple*, and this *Compound Multiplication*.

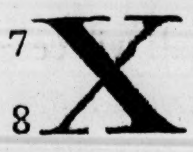
Induction helpeth to know the product of any two digits multiplyed into themſelves; without which knowledge great Numbers cannot be multiplyed; which is commonly thus done at the two ends of a Croſs; on the left ſide place the two digits to be multiplyed, the one over the other; then ſubſtract each digit from 10. and ſet the ſeveral remains reſpectively collateral to the digit from whence it came; again ſubſtract either of theſe differences, it matters not which from the other digit, croſs-wiſe, and this remain place under the digits: Laſtly, Multiply the two firſt differences, and the number amounting place under them to the right hand of the number before ſet down, and this ſhall be the product of the two digits. As to know how much 7 times 8 is, they are placed as at A. then 7 abated from 10 reſt 3, and 8 from 10 reſt 2,

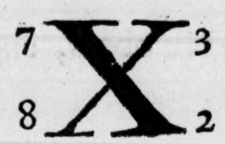
Induction called Simple Multiplication.

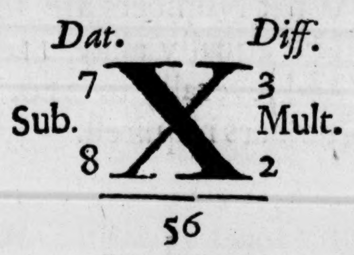
Uſed by Record

Example.

as at B. Laſtly 2 from 7, or 3 from 8 the remain is 5, and 2 times or twice 3 is 6, as at C. So is 7 times 8 found to be 56.

A


B


C


Uſed by Alſted.

This way is uſed both by *Alſted*, our Countrey-man *Record*, and Common School-maſters, yet *Alſted* ſeems not well-pleaſed with it, calls it the Sluggards Rule, and prefers another before it, which is to break one of the digits into ſeveral parts, and multiply thoſe ſmall parts by the other digit, and add all theſe little Products together, as if in the former Example, 8 ſhould be diſected into 2. 2. 2. 2. and each of them multiplyed by 7 make 14. which ſet down 4 times, and added make 56 as before.

Example.

The Rule is undoubtedly true this Authour grounds this Operation on, (which is, If two Numbers one be cut into ſeveral parts, the product of the two Numbers ſhall be equal to the product of the one by the ſeveral parts of the other) but the purpoſe for which both this, and the former are brought, is not without exception; for to produce Rules to teach any thing, which require the knowledge of the thing pretended to be taught by the Rule, is not proper, and ſurely in *Logick* ſuch reaſoning would be accompted begging the Queſtion, as to bring a thing doubtful to prove a thing in diſpute: Yet ſuch is the Miſhap in theſe Rules which inſtead of teaching to know the Sum of any two digits, in order to learn to multiply; ſet the ignorant to multiply to find them out.

Beſt way of Induction.

The better way for the Learner, is to ſet down one digit ſo many times as there be Units in the other digit, and add theſe into one total, as to ſet down 8 ſeven times, and the added total will be 56, as before.

But to avoid the prolixity of this, the beſt way to have *ad Unguem*, the value of any two digits multiplyed together, is to learn by heart the Table commonly called *Multiplication Table*, where they are all expreſſed thus.

Multiplication *Table.*

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Table how far inlarged by Pythagoras. *Uſe of the Table.*

This Table *Pythagoras* had enlarged to 12, others to 10, but 9 being ſufficient, it is contracted here, and becauſe 8 times 7, and 7 times 8, or any ſuch like, is all one, ſome convert it into a Triangular form, uſing only the part above the black Scale.

The Table is thus to be read, 2 times, or twice 2 is 4, twice 3 is 6, twice 4 is 8, *&c.* So 3 times 2 is 6, and 3 times 3 is 9, and 3 times 4 is 12. and ſo of the reſt. So that to find the Product of any two digits multiplyed together, is to find one of the digits in the left ſide, and the other in the head of the Table, and in the Angle common to both, is the product required: As to know how many 5 times 6 is, againſt 5 in the ſide, and 6 in the head, (or alternately contrary, 5 in the head, and 6 in the ſide) is found 30, and ſo much is 5 times 6, or 6 times 5.

The Compound Multiplication how wrought commonly.

Compleat Operation, proceedeth firſt, to multiply by Digits, and Articles, and afterward by mixt Numbers, and in either, the common, and ſelect way of proceeding is obſervable.

The

By Digits.

The Common way to multiply by digits, is after the numbers are placed as before, to begin at the right hand, and multiply every figure of the *Multiplicand* by the multiplying digit, and what doth amount thereof place under the ſame if it be a digit, if an Article ſubſcribe the Cypher, and reſerve the figure of the Article (as in *Addition* was taught) to be added unto the Sum of the next multiplyed figure; and if it be a mixt number, ſet down the digit, and reſerve the Article as before.

Example.

Example. 5291 is to be multiplyed by 2, after the Numbers ſet, as at D. ſay 2 times or twice 1 is 2, which is ſubſcribed, as at E. then twice 9 is 18, whereof 8 is ſubſcribed, as at F. and 1. the Article borne in mind, again twice 2 is 4, and 1 carried is 5, to be ſet in the next place, as at G. and laſtly twice 5 is 10. ſet as at H. So is the product of 5291 doubled or multiplyed by 2, the Sum of 10582.

D	E	F	G	H	
5291	5291	5291	5291	5291	Multiplicand.
2	2	2	2	2	Multiplier.
	2	82	582	10582	Product.

By Articles.

In the ſame manner proceed to multiply by Articles, increaſing the *Multiplicand* by the ſignifying figure of the Article, the Cyphers whereof are beſt placed as at the 10. 11. and 12. Sections of this Chapter, but with inartificial Artiſts are ſet down as in the following inſtance of multiplying 5291 by 20, as at I.

Example.

	5291	Multiplicand.
I	20	Multiplier.
	0000	
	10582	
	105820	Product.

By mixt numbers.

The Common way to multiply by mixt Numbers is, after all the figures of the *Multiplicand* be gone through as before by the firſt multiplying figure of the *Multiplier* then take the ſecond figure of the *Multiplier*, and proceed therewith as before, and ſo with the third, fourth, fifth, &c. multiplying figures, increaſe each figure of the one given number by each figure of the other, till all be gone over, obſerving to ſet the product of the firſt multiplyed digit of each, directly under the multiplying figure; and the reſt gotten thereby orderly in a ſtraight line to the left hand thereof; then under all theſe lines of production ſometime called *Multiplees*, which will ever be as many, as there be figures or places in the *Multiplier*, draw another right line, and add all the particular products into one total product, and this ſhall be the deſired number.

Example.

Example. To increaſe 402 by 349, the numbers placed as at K. after the *Multiplicand* is gone over with 9 the firſt multiplying digit, the work ſtands as at L. then begin with 4, and ſay 4 times 2 is 8, which ſubſcribe under 4, and 4 times 0 is 0, which ſet down to keep place, alſo 4 times 4 is 16, which makes the work as at M, and ſo proceeding do the like with the multiplying 3. Laſtly, Adding thoſe three lines of production together the total product of 402 multiplyed by 349 is found as at N. to be 140298.

K	L	M	N	
402	402	402	402	Multiplicand.
349	349	349	349	Multiplier.
	3618	3618	3618	Multiplees added.
		1608	1608	
			1206	
			140298	Total Product.

Select wayes of Multiplication.

This Select way of *Multiplication* conſiſteth either in certain brief or compendious Rules, or other choice Methods of proceeding, whereby the work of the Common way is abbreviated or meliorated, and theſe are both comprehended in theſe 18 following Sections.

1. If an Unite be in the Multiplier. Becauſe an Unite neither multiplieth nor divideth, whereſoever 1 is found in the *Multiplier*, it is but only to ſubſcribe the *Multiplicand* in its due place among the *Multiplees*, and add them as before; as to multiply 342 by 12, the product is 4104. Set as at O or P.

Example.

O
342 Multiplicand.
684 Double.
342
4104 Product.

P
342
684
4104

342
12 Common way.
684
342
4104

2. If 2 be the Multiplier. To multiply by 2 called *Duplication*, is nothing elſe but to double every figure of the *Multiplicand*, and ſubſcribe the number amounting, if the double be not above a digit, but if an Article or mixt number ſubſcribe the Cypher, or remaining figure, and for the Article account the next figure to the left hand one more than the double; *Example.* as 4372 duplicated or multiplyed by 2, for 2 is ſubſcribed 4, for 7 the next amounting to an Article 4, and go 1. wherefore 3 the next is made 7, that is 1 more than the double, and the double of 4 is 8. Thus,

4372 Multiplicand.
8744 Product.

4372
2 Common way.
8744

3. If 3. be the Multiplier. Example. *Triplication*, or to multiply by 3, is to add the given number to the double of the ſame, as to multiply 4372 by 3 produce 13116, for 4 the double of 2 added to 2 make 6, and 14 the double of 7 added to 7 make 21, of which 1 ſubſcribed, and 2 carried away, which 2, 3 and 6 the double of 3 is 11, whereof 1 ſet down in the next place, and the other 1 reſerved, which at laſt added to 4 and 8 makes 13, and for more eaſe, ſome firſt put down the double: Thus,

4372 Multiplicand.
8744 Double.
13116 Product.

4372
3 Common way.
13116

4. If 4. be the Multiplier. Example. *Reduplication*, *Quadruplication*, or *Multiplication* by 4 is to double the *Duplication*, as to multiply 15 by 4 is to double 30 the double of 15, and the 60 amounting is the Product; ſo in the former Example 17488 is the double of 8744, which was the double of 4372, therefore 17488 is the Product of 4372 multiplyed by 4.

8744 Multiplicand doubled.
17488 Product.

4372
4 Common way.
17488

5. Multiplication by 5. Example. To multiply by 5, called *Quintuplication*, adjoyn a Cypher to the right hand of the *Multiplicand*, and then take the half thereof as to multiply 468 by 5, the Product is 2340.

4680 Cypher adjoyned.
2340 Product.

468
5 Common way.
2340

6. By 6. Example. *Sextuplication*, or to multiply by 6, adjoyn a Cypher to the given number as before, take half thereof, beginning at the right hand, and to the half add the figure ſtanding next before; as to multiply 468 by 6, the Cypher adjoyned makes it 4680, then the half of 0 is 0, but 8 next before is 8, ſo the half of 8 is 4, and 6 next make 10, alſo the half of 6 is 3, to which the next 4 and the 1 reſerved of 10 make 8 for the next place, and at laſt the half of 4 is 2. So the Product is found to be 2808, or the half may be ſet down and added.

468.0 Cypher adjoyned.	468	468
	2340	6 Common way.
2808 Product		
	2808	2808

7. *By* 7.
Example.

Septuplication, or to multiply by 7, add half each figure to the double of the figure next before, a Cypher being firſt adjoyned, as to multiply 468 by 7, the half of 0 is 0. but the double of 8 is 16, of which 6 is ſubſcribed, and 1 reſerved, then the half of 8 is 4, being added to 1 before reſerved, and the double of 6 make together 17, whereof 7 is ſet down, and 1 reſerved, which with the half of 6 and double of 4 make 12, the reſerved 1 of which with the half of 4 make 3 to be ſet at laſt, and ſo the Product is 3276

468.0 Cypher adjoyned.	468
	7 Common way.
3276 Product.	
	3276

8. *By* 8.
Example.

Octuplication, or to multiply by 8. ſubſtract the double of the given number from the ſame increaſed by a Cypher adjoyned to the right hand thereof, as to multiply 468 by 8. the double is 936, which taken from 468.0 leaves 3744 the Product deſired.

468.0 Cypher adjoyned.	468
	8 Common way.
936 Double ſubſtracted.	
	3744
3744 Product.	

9. *By* 9.
Example.

Noncuplication, or to multiply by 9, adjoyn a Cypher to the Right hand of the given Number, as before, and from thence ſubſtract the ſame *Multiplicand*, as to multiply 468 by 9, the Product will be

468 0 Cypher adjoyned.	468
	9 Common way.
468 Multiplicand ſuſtracted.	
	4212
4212 Product.	

10. *By* 10. &c.
Example.

To multiply by 10, 100, 1000, *&c.* adjoyn ſo many Cyphers to the Right hand of the *Multiplicand*, as there be Cyphers in the *Multiplier*, as to multiply 468 by 100 two Cyphers being adjoyned, the *Product* is 46800.

468 Multiplicand.	468
100 Multiplier.	100 Common way.
46800 Product.	000
	000
	468
	46800

11. *By* 20. 30. &c.
Example.

To multiply by 20. 30. 40. *&c.* 200. 300.400.*&c.* multiply the given number by the ſignifying Figure of the *Multiplier*, and to the right hand of the *Product* place ſo many Cyphers as ſhall be in the *Multiplier*, as to multiply 468 by 200, the Product is 93600. for the 468 doubled or multiplyed by 2, is 936, to which the 2 Cyphers in 200 are adjoyned.

468 Multiplicand.	468
200 Multiplier.	200 Common way.
93600 Product.	000
	000
	936
	93600

12. When Cyphers are at the right hand.

Example.

To multiply when Cyphers are at the right hand of the *Multiplier* or both the given Numbers, firſt work with the ſignifying figures, and to the right hand of the Product place ſo many Cyphers as the *Multiplier* and *Multiplicand* had at the right hand places. As to multiply 3400 by 40, firſt 34 multiplyed by 4 produce 136, to which 3 Cyphers adjoyned make the true Product 136000.

```
  3400 Multiplicand.           3400
    40 Multiplier.               40 Common way
------                       ------
136000 Product.                0000
                             13600
                             ------
                             136000
```

13. When Cyphers come between.

Example.

To multiply when one Cypher, or more, fall between the ſignifying figures of the *Multiplier*; inſtead of making a line of Cyphers, place only one Cypher to keep place and then proceed with the next figure of the *Multiplier*. As to multiply 1432 by 204, the *Product* will be 292128.

```
  1432                          1432
   204                           204 Common way.
------                        ------
  5728                          5728
 28640                          0000
------                         2864
292128 Product.               ------
------                        292128
```

14. To multiply by nines.

Example.

To ſquare any number of Nines, that is to multiply them into themſelves, as 4 nines by 4 nines, *&c.* to the right hand of the number of nines to be multiplyed place ſo many Cyphers, as there be nines, then add 1 to the firſt Cypher, and ſubſtract 1 from the firſt nine, ſubſcribe this 1 and 8 in their places, together with the Cyphers between 1 and 8, and the nines beyond 8, ſo is the Product obtained. *Example*, 9999 by 9999, produceth 99980001. Thus,

```
                                        9999
                                        9999 Common way.
                                    --------
        9999.0000                      89991
abated     1    1 added               89991
        ---------                    89991
        99980001 Product            89991
        ---------                   --------
                                    99980001
```

15. To multiply without charging the memory.

Example.

To multiply without charging the memory by carrying the Articles. Multiply as before figure by figure, and what amounteth thereof ſet down if an article or mixt number, but if a digit ſubſcribe the digit and before it a cypher in the place of an article, then multiply the next figure of the *Multiplicand*, and what amounteth ſubſcribe the digit under the Article or Cypher before ſet down, and if there be an article, place it in the 100 place of the Sum, if no article, place a cypher as before inſtead thereof, and ſo going forward for every figure in the *Multiplier* make two lines of production. As to multiply 5142 by 43, becauſe 3 times 2 is but 6 before it is a cypher placed, then 3 times 4 is 12, which is ſet down according to the former directions, the other like proceſs of the work may be plainer diſcerned by the little lines drawn between the Numbers ariſing upon *Multiplication*.

```
    5 1 4 2                    5142
        4 3                      43 Common way.
  -----------                ------
     0,1,0-6                  15426
   1-5 3 2                   20568
    0,1,0-8                  ------
  2-0 4 6                    221106
  -----------                ------
  2 2 1 1 0 6 Product.
  -----------
```

To

16. To multiply and bring the Total into the laſt line of Production.

To multiply and bring the total production in the laſt line of the work, without the ordinary manner of *Addition*, or charging the memory, when the numbers are placed as uſual, multiply figure by figure, as before, and what ariſeth thereof ſet down the digit under the line, and the article above, or a Cypher inſtead thereof, then multiply the next figure, and there add the article before ſet down, cancelling him with a daſh of the Pen, and this amounting Sum place as before, the digit under the line one place nearer to the left hand, and the article if any above, and ſo do till all the figures of the *Multiplicand* be run through, and the product of the laſt figure of the *Multiplicand*, ſet down the article beneath the line as well as the digit; and when the firſt line is done, draw a rectangle line toward the left hand, cutting off the right hand figure, then take the ſecond *Multiplier*, and begin to multiply therewith, and add in their ſeveral places the uncancelled figures within the rectangle line, as well as the articles newly ſet down, and thus renew the rectangle lines, and proceed according to the number of the multiplying figures: So at laſt will all the figures be cancelled except thoſe below the laſt rectangle line, and ſuch right hand figures as were cut off by them, which being pulled down as they ſtand yield the deſired product. *Example.* As to multiply 8546 by 423 when 6 is increaſed by 3 the amounting 18 is placed 8 beneath the line, and 1 above, then 3 times 4 which is 12, and 1 above the line is 13, of which 3 is ſet beneath the line, and 1 above, and the firſt 1 cancelled, then 3 times 5 is 15, and 1 above the line uncancelled is 16, whereof the 6 ſubſcribed and the 1 is ſet at top; again 3 times 8 is 24, and 1 at top make 25, which is ſubſcribed, ſo is the firſt multiplying figure done with, and the work ſtandeth as at Q. Then 8 cut off by the rectangle line, and 2 the ſecond multiplying digit proceedeth with, beginneth with twice 6, which is 12 and 3 above the rectangle make 15, of which 5 ſubſcribed in his place leaveth 1. to be ſet above the Rectangle, ſo twice 4 is 8, with 1 laſt ſet down, and 6 over him in all make 15, which is placed as before, and ſo proceeding to the end of the *Multiplicand* by this digit 2, the work appeareth as at R, and in like manner proceeding with the multiplying 4, the finiſhing Paragraph is repreſented at S, where are found under the line 36149, and cut off by the Rectangle, 5 and 8, which together make the product 3614958.

Q.
8546
423
~~xxx~~
25638

R
8546
423
~~xxx~~
~~2563~~ | 8
~~xxx~~ |
19655

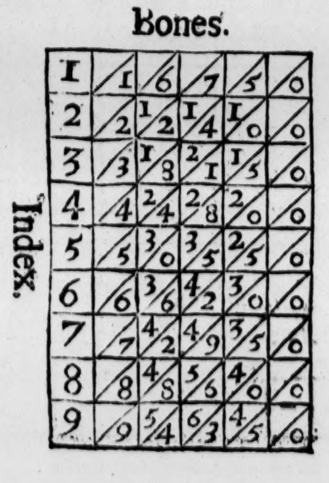
S
8546
423
~~xxx~~
~~2563~~ | 8
~~xxx~~ |
~~1965~~ | 5
~~322~~ |
36149

8546
423 Common way.
25638
17092
34184
3614958

17. To multiply by Nepaires *Bones.*

To multiply by *Nepair*'s Bones: Alwayes Tabulate or Place the *Multiplicand* on the Bones, and by the help of the *Index* taking the Sum of every *Diagonal Square*, or *Rhomboide*, anſwering to the ſeveral multiplying figures in the *Multiplier* you have the ſeveral lines of production which diſpoſe as before taught, *viz.* each line one place nearer to the left hand, then the other according to the multiplying figures, and add them into one total, and you have the product deſired. *Example.* As to multiply 16750 by 258, the *Multiplicand* 16750 being tabulated on the Bones, and the *Multipliees* anſwering to 2. 5. and 8 in the *Index*, taken out are 33500. 83750. and 134000. which added make the product 4321500. See the *Bones* and *Index* at T. the Productions added at V. and as ſome ſet them at W.

T



33500
83750
134000

V
16750
258
134000
83750
33500
4321500

W
16750
258
33500
83750
134000
4321500

To

18. To multiply by the Back-ſide of the Bones.

To multiply by the back-ſide of the Bones, which ſome uſe when they would conceal the numbers they multiply from the preſent Spectators. Firſt tabulate the *Multiplicand* on the Bones as before, then turn up the Bones juſt upſide down, laying that Face which was underneath upward, and work with the number which here appeareth tabulated, as if it were the true number, to the product found thereby add the *Multiplier*, then ſubſtract this total from the *Multiplier*, adding a ſufficient number of Cyphers thereto, and the Remain ſhall be the Product deſired. *Example.* As in the former inſtance 16750 tabulated on the back-ſides, will be tabulated 83249, which multiplyed into 258 produceth 21478242, whereto 258 added, the product and total together are 21478500, this number ſubſtracted from 258, and ſo many Cyphers adjoyned as the *Multiplicand* had Figures, *viz.* 5, leaves the former Product 4321500.

25800000

21478500

4321500

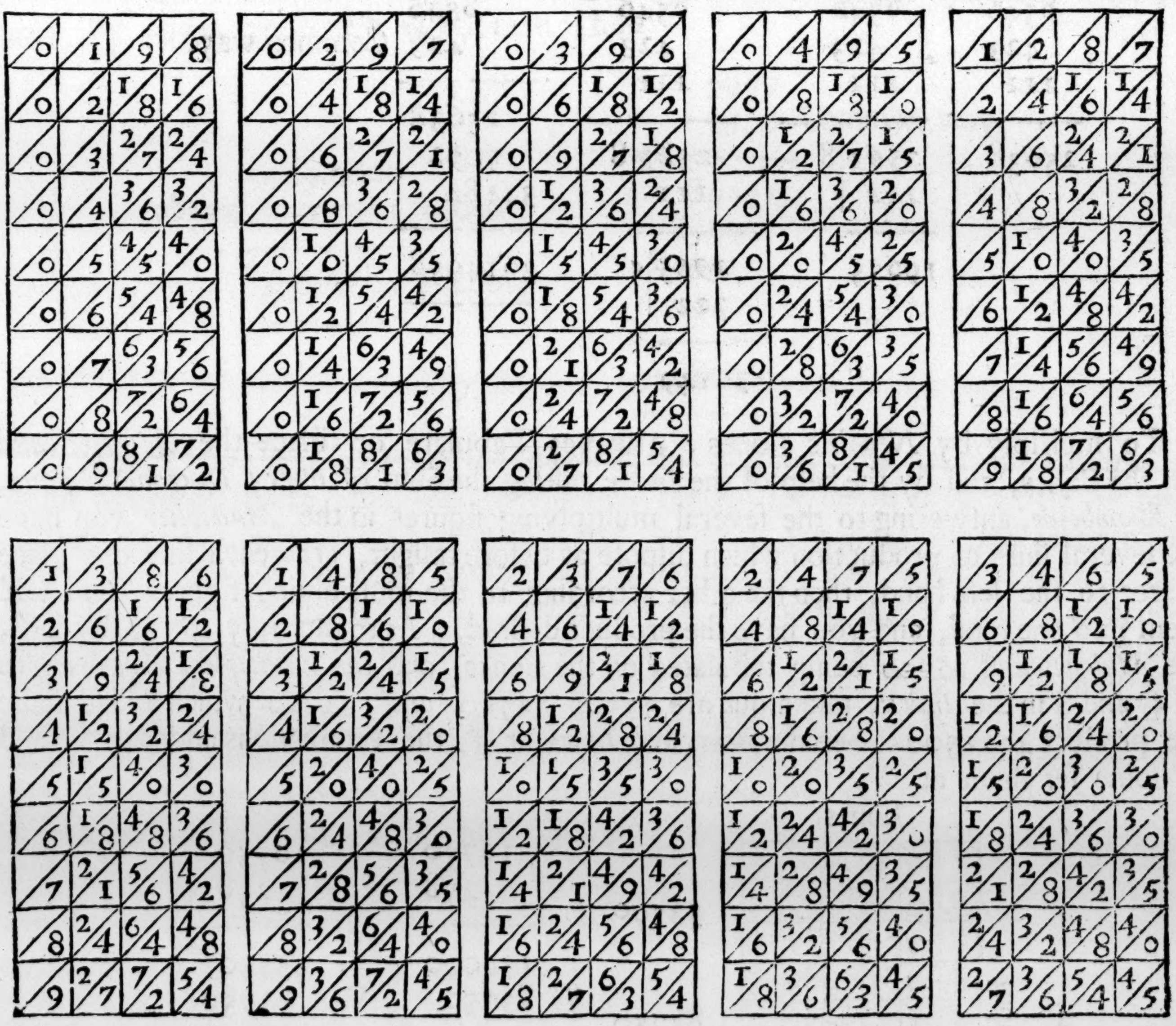
Bones were invented by Nepair.

The uſe of theſe Bones or Rods have been ſufficiently explained by others, it were but *actum agere* to inſiſt upon it here: Only in brief, They are called *Nepair*'s Bones from the Inventer the Honourable Lord of *Marchiſton*, they are beſt made of *Ivory*, or *Wood* 4 ſquare, of an equal thickneſs, about one fifth part of an Inch ſquare, their length about 9 times their breadth, they are in number 10, but to have two or three ſets of them will be convenient, becauſe with one ſet can be tabulated but only 4 figures of one and the ſame *Species*; as 4 Cyphers, 4 Units, &c. but 20 or 30 Rods can tabulate 8 or 12. They have all the digits on them, and their *Multiplees* to 9, being only *Pythagoras*'s Table cut in pieces. There is alſo an *Index* belonging to the Rods, of the ſame length and breadth. See more in the following Table.

They are Multiplication Table cut in pieces.

The Deſcription of all the four faces of every one of the Ten Rods, and the Index.

Figures of the Bones.



The INDEX.

Proof of Multiplication.

Probation is here as before, to try the truth or falſhood of the work. Particularly *Multiplication* performed by any of the Select Rules aforegoing, may be proved by the Common way. But generally the proof of all ſorts of *Multiplication* may be had, either by *Addition*, Caſting away Nines, or *Diviſion.*

By Addition most tedious.

Addition, (eſpecially in great *Multipliers*,) is a moſt tireſome and tedious way; yet it is thus ſet down the *Multiplicand* ſo many times as there be Units in the *Multiplier*, and add all together, the total will be the ſame with the Product if the work be right; As 1343 multiplyed by 3, the Product is 4029, and ſo is the total of that number ſet down 3 times.

1343
1343
1343
4029 Total.

1343
3
4029 Product

Geometry a Theorem that is the Baſis of Multiplication.

The truth of this dependeth on that *Theoreme*, which is the very Baſis of *Multiplication*, *viz.* as an Unit to the *Multiplier*, ſo is the *Multiplicand* to the *Product*, and *contra*, for if 6 times 4 be 24, then 1 to 4 ſhall be as 6 to 24, and 1 to 6, as 4 to 24. But this way of Proof makes the remedy worſe than the Diſeaſe, and therefore is rejected.

Proof by 9 uncertain.

The Proof by caſting away 9, is to caſt 9 as oft as may be from the *Multiplicand* and *Multiplier* ſeverally, and ſet the two remains at the oppoſite Angles of a Croſs; then multiply theſe remains one into another, and if the Product thereof be under 9, ſet it down at another of the Angles of the Croſs, if above 9, caſt away 9, and ſet down the reſidue; Laſtly caſt away 9 from the *Product*, and the Remain thereof ſhall alwayes be equal to the Remain laſt before put down, if the work be right. As in the former Example proved by *Addition*, the nines caſt from 1343 leave 2, which ſet oppoſite to 3 the *Multiplier*, and multiplyed thereby produce 6, and ſo much will be left when 9 is caſt from the *Product* 4029 ſtanding on the Croſs thus,

  6
2 X 3
  6

Proof by Diviſion is beſt.

This kind of Proof upon the ſame accompt it was found faulty before in *Addition*, is fallible alſo here. But the moſt certain and infallible proof of *Multiplication*, is by *Diviſion*; of which more in the next Chapter, wherefore to conclude this; take the two following Obſervations.

Obſervations.

1. If any Number be multiplyed, it is ſo often added to its ſelf as there be Units in the *Multiplier*.

2. If a Number be compounded of two Numbers, and that Number multiply another Number, the *Product* ſhall be equal to the *Product* of that Number multiplyed by thoſe two Numbers. As 6 is compounded of 2 and 3, let 6 then multiply 9, the Product is 54, which is equal to 9, multiplyed into 2, that is 18, and 18 multiplyed into 3, which is 54.

CHAP. VIII.

Diviſion of Integers.

Diviſion what.

Diviſion, called by ſome *Partition*, is the Compound Analyſis of Numbers, and that part of Conjunct Numeration, whereby one Number is ſubſtracted from another ſo often as it is contained in it, and by that means is found how many of the one Number are contained in the other: So that the third Number, or Number artificially obtained by the two propounded Numbers ſhall ſo often contain an Unit, as the greater of the two given Numbers contains the leſſer, and it ſerveth inſtead of many *Subſtractions*, of which and *Multiplication*, it conſiſts, and is compoſed.

Serveth inſtead of many Subſtractions.

Diviſion obſerveth *Collocation*, proceedeth to *Operation*, and concludeth with *Probation*.

How to place the Numbers.
Dividend what.
Diviſor what.
Quotient what and whence it comes.
Parabola what.
Remain if any to be left uncancelled.
To be leſs than the Diviſor.
How to be ſet.
Synonimical termes.

Collocation reſpecteth the *Nomenclature*, and duely placeth every number in manner following. Firſt ſet down the Number to be divided, which is ſometime called the *Dividuum*, but commonly the *Dividend*; and under it at the left hand (contrary to all the former Operations of *Arithmetick*) place the Number by which it is to be parted or divided, called the *Diviſor*, obſerving to ſet the foremoſt figure to the left hand of the *Diviſor*, under the foremoſt left hand figure of the *Dividend*, if the *Diviſor* may be ſubſtracted from thoſe figures of the *Dividend*, which ſtand above over them; but if not, then let the firſt left hand figure of the *Diviſor* ſtand under the ſecond left hand figure of the *Dividend*, and ſo in order proceeding to the right hand place all the other figures of the *Diviſor*, then draw the Decreſcent Lunular, or Seperatrix to diſtinguiſh theſe given Numbers from the Number to be found by *Diviſion*, which is called the *Quotient*, and ſometime *Parabola*; *Quotient* is derived from *Quoties*, becauſe the number is gotten by enquiring how often the *Diviſor* is contained in the *Dividend*. *Parabola* ariſeth, from the application of a plain number, to a given Longitude, that a congruous Latitude may be found out; as will be more demonſtrable in Figural numbers. If after *Diviſion* be ended, any figures be left uncancelled, theſe are called the *Remain*, and denote that the *Dividend* could not be exactly parted by that *Diviſor*; but there will be a number remaining, unleſs the *Dividend* be reduced to another Denomination: As in denominate numbers: And this Remain is alwayes to be leſs than the *Diviſor*; and diſtinguiſhed from the other figures by little Lunular or Rectangle Lines, like the Quotient Line; or ſet over the *Diviſor* with a little Line between; as a *Fraction*. Laſtly may be noted, That to part, meaſure or compare a number, are termes uſed ſynonimically, and ſignifie but to divide that number.

Operation hath both *Induction*, and *Perfection*; The one ſometime called *Simple Diviſion*; and the other *Compound*; and both preſuppoſe the *Dividend* to be greater than the *Diviſor*.

Induction called Simple Diviſion.

Induction by the help of *Multiplication Table*, ſheweth how often any digit is contained in the *Multiplee* of any two digits: And conſequently by ſuffrage of the ſame Table findeth how often the ſame digit is contained in all the intermediate Numbers that happen between ſuch *Multiplees*.

Uſe of Multiplication Table for Induction.

To find how oft any digit is contained in the *Multiplee*, enter the Table with the *Multiplee*, among the *Areal* Numbers juſt againſt the given digit at the left ſide, and the number deſired ſhall be the digit juſt over the *Areal* Number at top; but if the given digit be found in the head of the Table, the deſired number ſhall be the digit at the left ſide juſt againſt the *Areal* number. As to find how often 2 is contained in 10, entring with 10, in the collateral Columne againſt 2 in the ſide, 5 is found at top over 10, as alſo againſt 2 in the head, and 10 in the perpendicular Collumne, which ſheweth that 2 is contained 5 times in 10, and in like manner is found how often any of the other *Multiplees* below or above 10 do contain 2, as 4 containeth 2 twice, 6 three times, &c.

By conſequence alſo may be found how often 2 is contained in all numbers intermediate, between thoſe *Multiplees*, for 3 intermediate between 2 and 4 ſhall contain 2, but once and a half part more, and 5 intermediate between 4 and 6 ſhall contain 2. but twice and a part more, all intermediate numbers never exceeding the precedent *Multiplees* by an Unit, but only by a part or parts: for where there is but one number intermediate there will an Unit remain after the integral content is ſubſtracted; where two intermediate Numbers are, 1. will remain over the firſt intermediate number, and 2 over the ſecond, and ſo accordingly in the reſt.

Compound Diviſion how wrought commonly.
By Digits.

Complete Operation proceedeth firſt to divide by digits and Articles, and afterwards by mixt numbers; and in either the common and ſelect way of proceeding is conſiderable.

The Common way to divide by digits is, after the given numbers are placed as before, begin at the left hand, and ſee how many times the Diviſing digit can be taken out of the figure of the *Dividend* ſtanding over him, or to the left hand thereof, if the Diviſing digit ſtand not under the firſt left hand figure of the *Dividend*, and having found how often this *Diviſor* is contained in ſuch figure or figures of the *Dividend*, ſet beyond the *Quotient* line a figure ſignifying how often; as 1 for once, 2 for twice, &c. then ſubſtract the content of the *Diviſor* ſo many times as is ſet down in the *Quotient* from the *Dividend*, and ſet the *Remain* if any be over the figure of the *Dividend*, where it was left remaining, alwayes cancelling with a daſh of the Pen the *Diviſor* when done with, and thoſe figures of the *Dividend* whence any thing was taken, then ſet down the *Diviſor* one place nearer to the right hand, and thereby get another *Quotient* figure, and ſo proceed as before to the end of the *Dividend*; and ſo many times as the *Diviſor* may be ſet down in the *Diviſion*; ſo many figures or cyphers will be in the *Quotient*.

Example.

Example. To divide 3762 into 3 equal parts, the numbers being ſet as at A. inquire how many times the *Diviſor* may be had out of 3. the *Dividend* figure over him, and finding it may be taken once, place 1 in the *Quotient*, and ſubſtracting once 3 out of 3 there is 0 remaining; then cancelling the 3 at top, and the *Diviſor*, the work ſtands as at B. Then placing the *Diviſor* under 7, and inquiring as before, 2 is gotten and ſet in the *Quotient*, and twice 3 which is 6 ſubſtracted from 7 leaves 1 remaining, as at C. Again placing 3 the *Diviſor* under 6, by like inquiry is found 5 times 3, which is 15 to be contained in 16, therefore 5 is placed in the *Quotient*, and abating 15 from 16. leaves 1 to be ſet over 6, and the work appears as at D. Laſtly, 3 placed under 2 by inquiry as before, 4 is obtained for the *Quotient*, which multiplying the *Diviſor* 3 produceth 12 to be taken from the 12 before left of the *Dividend*. So there remains 0. And the *Quotient* is found to be 1254, which is the third part of the given *Dividend* 3762, and the Compleat work ſtands as at E.

	E	D	C	B	A
	$\cancel{1}\cancel{1}$(0 Remainer	$\cancel{1}$1	1		
Dividend	$\cancel{3}\cancel{7}\cancel{6}\cancel{2}$(1254 Quotient.	$\cancel{3}\cancel{7}\cancel{6}2$(125	$\cancel{3}\cancel{7}62$(12	$\cancel{3}762$(1	3762(
Diviſor	$\cancel{3}\cancel{3}\cancel{3}\cancel{3}$	$\cancel{3}\cancel{3}\cancel{3}$	$\cancel{3}\cancel{3}$	$\cancel{3}$	3

By Articles.

In like manner procede to divide by Articles, dividing the *Dividend* by the ſignifying figure of the Article, for the Cyphers do but only keep place, and are beſt ſet as in the third *Section* of this *Chapter*; though with young Beginners they are ſet down as in the Example following to divide 1762 by 30 where the *Quotient* is 58, and 22 remain, as at F.

Example.

	F
	$\cancel{2}$(2
Dividend	$\cancel{1}\cancel{7}\cancel{6}$(2 (58 Quotient or $58\frac{22}{30}$
Diviſor	$\cancel{3}00$
	$\cancel{3}$

By mixt Numbers.

The Common way to divide by a mixt number is ſomewhat more difficult, yet is the former Method ſtill obſerved, in placing the numbers and finding out the firſt *Quotient* figure only with this difference, that when the *Diviſor* was a ſingle digit or article, the ſame *Quotient* figure is to be as big, as could be ſubſtracted from the figures of the *Dividend*, to the left hand thereof; now the firſt left hand diviſing figure is to be taken no oftner from the figure or figures ſtanding over him than that alſo every following figure of the *Diviſor*, may be taken ſo often out of the figure or figures that ſtand over them, or are to the left hand of them, by borrowing as in *Subſtraction*, to ſupply the deficiency thereof; and though ſometimes the firſt figures of the *Diviſor* will ſeem to leave too much behind, yet the 2^d^ or 3^d^ figure may want: So as it will be convenient to reckon in mind a little before Operation whether the juſt figure be taken, which muſt never be above 9 at moſt, nor under ſo many times as the *Diviſor* is contained, for then will the *Remain* be greater than the *Diviſor*, which is not to be ſuffered.

Example.

Example. If 34633 were to be divided by 12, the numbers placed as before, and ſeeking how often 1 is contained in 3 there is found 3, if 1 were ſingle, becauſe 1 divideth not, but being joyned with 2, if 3 ſhould be cancelled there would be but 4 left, and then thrice 2 which is 6 could not be taken from 4, therefore 1 is to be taken but twice out of 3, and the remaining 1 ſet over 3, and 2 is ſet in the *Quotient* as at G. Then ſeeing 1 is taken but twice out of 3 the other figure of the *Diviſor*, 2 muſt be taken but twice, which is 4, from 4, at top reſteth 0, to be ſet down becauſe of 1 to the left hand as at H. Then ſetting the *Diviſor* down again, and enquiring how many times 1 may be had out of 10 is found but 8, becauſe there will not elſe be enough left to take the other figure of the *Diviſor* ſo often out of the remaining figures to the left hand; therefore taking 8 for the *Quotient* out of 10 at top, reſteth 2 to be ſet over 0, as at I. And 8 times 2 is 16 in like manner taken out of the figure over him, that is out of 6 cannot, but out of 16, borrowing 10 there remains 0, and the borrowed 1 out of 2 reſts 1 as at K, moreover removing the *Diviſor*, and by inquiry as before, 8 is found again for the *Quotient*, and twice 8 taken from 23 leaves 7, as at L. Laſtly, Removing the *Diviſor*, and ſeeking thereby as before, 6 is found for the *Quotient*, taken from 7 leaves 1, and twice 6 is 12 from 13 is left 1 remaining, when the *Diviſion* is ended at M. where the work is compleat.

G	H	I	K	L	M
			1	1	1
		2	2	22	221
1	10	10	100	1007	1007(1
34633(2	34633 (2	34633(28	34633 (28	34633(288	34633(2886.
12	12	122	122	1222	12222
		1	1	11	111

If the Dividend be leſs than the Diviſor. When the *Dividend* is leſſer than the *Diviſor* ſet the numbers in manner of a *Fraction*, with a right line between them; For as no *Subſtraction* in Abſtract Integral Numbers can be made of a greater Number from a leſſer; ſo can no leſſer *Integer* be divided by a greater in Abſtract Numbers, but the further practiſe therewith muſt be referred to Fractionary or Contract Operations; As 15 cannot be divided by 16 in whole Numbers, but may be ſet thus $\frac{15}{16}$(.

Select waies of Diviſion. The Select way of *Diviſion* conſiſteth either in certain brief Rules, or other choice Methods of proceeding, bettering, or abbreviating the Common way; comprehend in the 18 *Sections* following.

1. If the Diviſor be 1. with Cyphers. First, To divide by an Unit with Cyphers, as 10. 100. 1000. *&c.* Cut off from the *Dividend* to the right hand, ſo many figures as there be Cyphers in the *Diviſor*, and place the figures ſo cut off over the *Diviſor* in form of a *Fraction*; *Example.* as 3423 divided by 10, the *Quotient* is $342\frac{3}{10}$, and by 100. $34\frac{23}{100}$.

The Ground thereof. The Reaſon of this Rule, and ſeveral others, is grounded upon that *Theorem.* That an Unit neither multiplyeth, nor divideth, for once 2 is but 2 ſtill; and if application be made of 1 to 2, it may be had out thereof twice; and the like it is with any other Number beſides 2.

2. If alike Number of Cyphers be in the Data. If there be a like Number of Cyphers in the *Dividend*, and *Diviſor*, and an Unit the only figure of the *Diviſor* cut off the Cyphers by a perpendicular ſtroke of the Pen: and the reſidue of the *Dividend* to the left hand ſhall be the *Quotient*. But if the *Diviſor* be more than 1, divide the remaining figures of the one by the Remains of the *Example.* other given number, as 3400 divided by 100 gives 34 in the *Quotient* thus $\frac{34|00}{1|00}$. and 34800 divided by 12 gives 29 thus.

1
10
348 | 00(29
122 | 00
1

3. If Cyphers be at the right hand of the Diviſor. If the *Diviſor* only have one Cypher to the right hand or more, as it happeneth in Articles. Place the Cypher or Cyphers under the right hand figure or figures of the *Dividend*, and divide all the way by the ſignifying figure or figures of the Article unto the Cypher or Cyphers before ſet down, and the *Diviſion* is done ſtriking away the *Example.* Remainer, as in the following Examples, 20 being *Diviſor* to 8401, the Cypher of 20 is ſet under 1, and 840 is divided by 2, and the *Quotient* is $420\frac{1}{20}$, as at N. 800 dividing the ſame *Dividend*, *Quotient* is $10\frac{4\,01}{8\,00}$ as at O, and 150 dividing the ſame *Dividend*, the *Quotient* $56\frac{1}{150}$ as at P.

N		O		P
				3
				39
840|1(420		8|401(10		840|1(56
2220		8800		1550
				1

4. If a Cypher be taken in the Quotient. In *Diviſion*, If a Cypher at any time be taken in the *Quotient* (which happeneth when the *Diviſor* being ſet down cannot be taken once from the numbers ſtanding over him) then forthwith cancel the *Diviſor*, and remove him one place nearer to the right hand, and meddle with none of the Dividend figures, and if this be foreſeen the ſetting *Example.* down the *Diviſor* may alſo be ſpared by placing the *Diviſor* two places nearer to the right hand: Nevertheleſs 0 muſt be ſet in the *Quotient*. As to divide 1340 by 13 at the ſecond ſetting down the *Diviſor*, 13 cannot be taken out of 4, wherefore 0 being ſet in the *Quotient* the *Diviſor* is cancelled, and ſet down again, without altering the *Dividend*, becauſe nought was taken therefrom.

		1(1
1340(1	1340 (10	1340(103 Quotient
13	133	1333
	1	11

When

5. *If an Article ariſe in multiplying 2 digits.*

When it happeneth in *Diviſion* that by multiplying two digits together, there ariſeth an Article, meddle not with the digit of the *Dividend*, ſtanding directly over head, but go to the place of the Article. As in dividing 853 by 24, in the ſecond enquiry twice 5 will be 10, wherefore cancelling the 1 of the 13, the 3 is left, likewiſe 4 times 5 is 20, which taken from the left hand 3 of the 33, leaveth 13 remaining.

Example.

```
1|1
23|
853(35 13/24
244
 2
```

6. *If Cyphers remain on the Dividend uncancelled.*

When in dividing Numbers at the right hand, it happeneth that the *Diviſor* hath cut off all the figures of the *Dividend*, ſo that there remaineth nothing on the *Dividend* but Cyphers; then to the right hand of the *Quotient* adjoyn ſo many Cyphers as are yet remaining to the *Dividend*, without any part of the *Diviſor* ſtanding under them, and the work is finiſhed. As 3654000 divided by 180 or 18, will declare by the following Operations.

Example.

```
3654000(20300        3654000(203000
1888   0             1888
 11                   11
```

7. *To half a Number, or to divide by 2.*

To take the half of any Number called *Mediation*, *Bipartition*, or *Diviſion* by 2, if the given Number conſiſt of even figures begin at the left, or right hand, it matters not which, and take of every figure reſpectively the half, and ſubſcribe under the ſame figures. But if any figure be odd, the beſt way is to begin at the left hand, and take the leaſt part next the half of the odd figure, and augment the ſucceeding figure by 10. As to ſubſtract the half of 42983, thus the half of 4 is 2, of 2 is 1, of 9 is 4, of 18 is 9, and of 3 being the laſt $1\frac{1}{2}$. So is the full half thereof $21491\frac{1}{2}$.

Example.

To take the 3, 4, &c. part of a number.

In like ſort it may be accuſtomed to take the third, fourth, fifth, ſixth, ſeventh, eighth, and ninth parts, of any number, and ſome of theſe alſo may be otherwiſe obtained ſooner than to ſet down the *Diviſor* ſo often as in the Common way of *Diviſion*, for *Tripartition* is but *Diviſion* by 3. *Quadripartition* to divide by 4, *&c.* *Example*, to take the third part of 3687, the fourth part of 5460.

Example.

```
3)3687            2                    4)5460           122
                  3687(1229                             5460(1365 Com way.
1229 Quotient.    3333     Common way   1365 Quotient   4444
```

8. *To divide by 4.*

Quadripartition, or to divide by 4, may alſo be thus performed by half the half of the given number, as in the former inſtance 5460, half is 2730, from whence half abated, leaves 1365 the quarter part as before:

Example.

```
4) 5460
                     122
2) 2730 half.        5460(1365 Com. way.
                     4444
2) 1365 Quotient.
```

9. *To divide by 5.*

Quintipartition, or to divide by 5 may likewiſe be effected thus, double the number given, and cut off the right hand figure or cypher; if any figure be cut off, take half that figure. So 2340 doubled is 4680, from whence 0 cut off, leaves 468 the fifth part thereof. So alſo 4569 parted by 5 gives $913\frac{4}{5}$.

Example.

```
                                        5)4569
5) 2340        2340(468 Com. way.                         1(4
                                        913|8
Quotient. 468|0  555                                      4569(913 Com. way.
                                        Quotient. 913|4/5  555
```

10. *To divide by* 6. *Example.* *Sextipartition*, or to divide by 6, alſo may be thus done; take half the third part of the given number. As to take the ſixth part of 3684, the third part is 1228, half which is 614 the number ſought.

6) 3684

3) 1228 Third part.

2) 614 Half. Quotient.

2
3684(614 Com. way.
666

11. *To divide by* 8. *Example.* *Octipartition*, Or to divide by 8, is but to take half the quarter part of the given Number, as to get the eight part of 5460, the quarter part is 1365, whereof the half is 682, and one half the queſited number.

8) 5460

4) 1365 Quarter part.

2) $682\frac{1}{2}$ Half. Quotient.

62(4
5460(682 Com. way.
888

12. *To divide by* 9. *Example.* *Nonupartition*, or to take the ninth part of a Number, is to take the third part of the third part of the given Number. As to divide 5463 by 9, the *Quotient* will be 607. So will the remain, if the third part of 1821 be taken, which is the third part of 5463.

9) 5463

3) 1821 Third part.

3) 607 Third Part. Quotient.

5463(607 Com. way.
999

13. *To divide by* 20, &c. *Example.* From hence it follows, that to divide by 20. 30. 40. *&c.* 200, 300, 400, *&c.* after the Cyphers are placed, as *Section* 3. above, the half, third, fourth part, *&c.* of the remaining figures of the *Dividend* may be taken. As to divide 45156 by 50, when 0 is ſet under 6, the fifth part of 4515 is taken, which is 903, and ſix fifties are left remaining.

50) 4515|6
        |0

$903\frac{6}{50}$ Quotient.

4515(6 (903 Com. way.
555 0

14. *To divide by Nines.* *Example.* When any ſquared number of nines is given to be divided by the number of nines whereof it was produced, place the *Diviſor* under the right hand places of the *Dividend*, add them together, and from the total cut off all the Cyphers; the Remain ſhall be the *Quotient*. As to return 99980001 being the ſquare of 9999 into the Root, I add 9999 to 99980001, and the total is 99990000, which 4 Cyphers cut off, the Remain is the Root.

99980001
9999

Radix 99990000

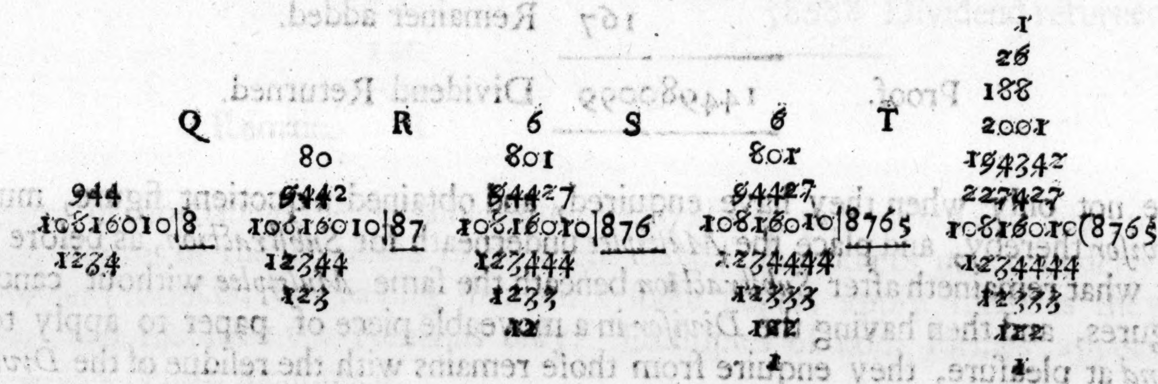
8
198
979
080
18988
99799
008888
1887999
99980001(9999 Common way.
9999999
99999
999
9

15. *To cut ſhort the borrowing, a good and uſual way.*

To cut ſhort the borrowing work in the top figures in *Diviſors* conſiſting of many figures (a moſt commendable practiſe) do thus, after the numbers are placed, and the firſt Quotient figure found out as before, then begin with the right hand figure of the *Diviſor*, and take him ſo many times as the quotient figure denotes from the figure that ſtandeth over him if it may be, if not, borrow in imagination One, or more Tens, as occaſion is, that *Subſtraction* may be made, and place the remain at top as before, then take the next figure of the *Diviſor* and do the like with him, adding in for every 10 borrowed before 1, and ſo proceed toward the left hand with all the diviſing figures, then ſet down the *Diviſor* again, and reiterate this manner of work till the *Diviſion* be ended.

Example.

As to divide 10816010 by 1234 when the numbers are placed, and 8 found for the firſt quotient figure, then beginning at 4, and taking him 8 times, which is 32, becauſe but 6 ſtandeth over 4 cannot wholly be taken without borrowing 3 tens, but 2 may be taken out of 6, and leave 4 at top, or reckoning entire 32 may be taken out of 36 and leave 4, either way 3 is reſerved to be added in the next place. Then coming to 3 in the *Diviſor*, and taking 3 times 8 which is 24, with the former reſerved 3, make 27, which are to be taken out of 1, but cannot without borrowing 3 tens again, and then 27 out of 31 leaves 4 to be ſet at top. Again 2 in the *Diviſor* multiplyed into 8 in the quotient, produce 16, and 3 before borrowed is 19, where but 2 tens need be borrowed, and out of them and 8 in the *Dividend* which are 28 if 19 be taken there will remain 9. Laſtly 1 taken 8 times, and 2 laſt borrowed added make 10, which abated from 10 at top leaves 0, not ſet down becauſe no figure ſtandeth to the left hand thereof. So is the work with the firſt ſetting down the *Diviſor* ended, Then the *Diviſor* removed one place nearer to the right hand, and the like work reiterated till the whole *Diviſion* be ended, the *Quotient* is 8765, and this work thus performed hath 13 figures leſs at top than that wrought the Common way, and much more will it ſhorten great *Diviſions*, and after a little practiſe be as ready and facil as the other. The ſeveral Paragraphs of the work, at Q. R. S. T. are further Exemplary.

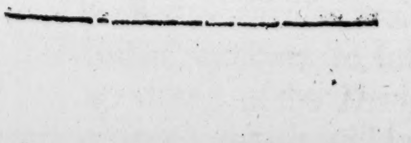


16. *The way called the* Italian *way not chargeable to the memory.*

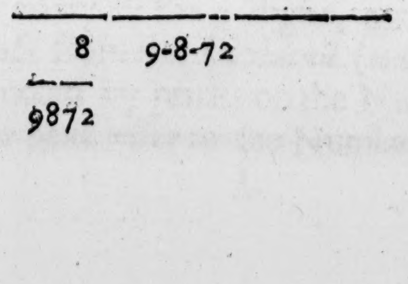
Some to favour the memory practiſe the *Italian* way, to multiply the *Diviſor* by the ſeveral quotient figures as they are found, and accordingly ſubſtract the ſeveral *Multiplees* from the *Dividend* one after another: And they place the numbers moſt conveniently in this form, *viz.* The *Dividend* between two Parallel Lines, to the right hand whereof the *Quotient* beyond the Decreſcent Lunular, as before; and to the left hand beyond an Increſcent Lunular the *Diviſor*; and to repreſent the place of the firſt figure of the *Diviſor*, under the *Dividend* a Cypher, and when a quotient figure is gotten, and the *Diviſor* multiplyed thereby, this *Multiplee* is firſt ſet down under the *Dividend*, and ſubſtracted, and the remainder ſet at top, then another Cypher is placed under the *Dividend* to denote the remove of the *Diviſor*, and ſo another quotient figure obtained, the proceſs is as before, only in ſetting down the ſeveral *Multiplees*, there are two formes; the one as at V. and the other as at W. The firſt of which is the foregoing Example, wherein the ſeveral Paragraphs of the work are diſtinct. The other in dividing 144980099 by 1798 containeth all the difficulties this kind of *Diviſion* can have, and is ſet entire.

Examples:

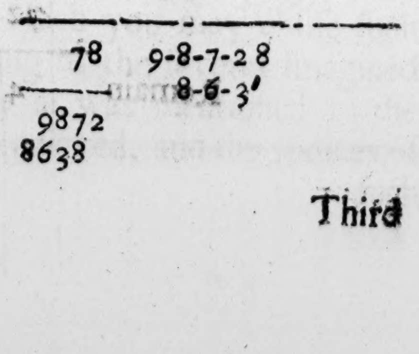
Numbers ready for Work.



Firſt Paragraph.



Second Paragraph.



Third

```
Third          6              and Fourth       6               Paragraphs.
             8 0 1                           8 0 1
            9 4 4 2 7                       9 4 4 2 7
   ________________________         _________________________
   1234) 1 0 8 1 6 0 1 0 (876       1234) 1 0 8 1 6 0 1 0 (8765
           0 0 0                            0 0 0 0
   ________________________         _________________________
      678   9-8-7-2 8 4              5678   9-8-7-2 8 4 0            V
   ______    8-6-3'0'               _____    8-6-3'0'7'
     9872     7-4'                   9872     7-4'1'
    8638                            8638       6'
   7404                            7404      _________
                                   6170     1 0 8 1 6 0 1 0   Proof.
```

In this Example the figures of each *Multiplee*, are marked with a little line for the plainer evidence, though not needful, the following Example is plain without.

```
                              (1
                            673(6    Remain
                          11412 5(7
                  ____________________________
Diviſor.          1798) 144980099 (80634   Quotient.
                        00000
                  ____________________________
                  43608   14384                              W
                  _____    0000
                  14384   10788
                  0000     5394
Multiplees        10788    7192
                  5394   ___________
                  7192    144979932   Total of the Multiplees.
                                167   Remainer added.
                         ___________
             Proof.       144980099   Dividend Returned.
                         ___________
```

17. *To divide and ſet the Remains below.* Some not only when they have enquired, and obtained a quotient figure, multiply the *Diviſor* thereby, and place the *Multiplee* underneath for *Subſtraction*, as before, but alſo ſet what remaineth after *Subſtraction* beneath the ſame *Multiplee* without cancelling any figures, and then having the *Diviſor* in a moveable piece of paper to apply to the *Dividend* at pleaſure, they enquire from thoſe remains with the reſidue of the *Dividend*, for another quotient figure, and ſo continue the work till the *Diviſion* be perfected.

Example. As to divide 34636 by 12, the firſt quotient figure is found to be 2, whereby 12 multiplyed, the *Multiplee* is 24, which ſubſtracted from 34, leaves 10 to be ſubſcribed, then apply the *Diviſor* in the moveable paper to 10 the remain, and 6 in the *Dividend* next to it, and 8 may be found for the quotient, and ſo the like is done with the ſeveral remains after the ſubſtraction of 96, 96 and 72 the Products of the *Diviſor* multiplyed by the Quotient figures, and the laſt remaining 4 is left at the bottom alone thus,

```
Diviſor. Dividend. Quotient.
_____________________________              Proof.
    12) 34636 (2886
_____________________________             24
   6882   24                               96
   ____  ____                               96
     24   10                                 72
    96     96                             _____
   96     _____                           34632  Total of the Multiplees.
  72       10                                 4  Remain added.
            96                            _____
          _____                           34636  Dividend returned.
             7                            _____
             72
          _____
   Remain     4
```

Variety of this work.

In this manner of *Diviſion* there be divers, for the more ready finding the quotient figures multiply the *Diviſor* by all the 9 digits, and ſet a Prick or Cypher under that figure of the *Dividend* which the right hand figure of the *Diviſor* ought to ſtand under; ſo it may be preſently diſcerned which of the ſeveral Multiplications of the *Diviſor* may be ſubſtracted from thoſe figures of the *Dividend* ſtanding from the prick or cypher toward the left hand; and the digit producing that *Multiplee* is to be ſet in the Quotient. As to divide 58688 by 24, the firſt pricked figure is 8 next to 5, becauſe there ſhould ſtand the right hand figure of the *Diviſor*, and amongſt the multiplyed numbers is found 48 next leſſer to 58 the pricked number, which is to be ſubſtracted, and 2 which begot the ſame 48 ſet in the Quotient, the remaining 10 is ſubſcribed, then pricking the next figure of the *Dividend*, and applying the *Diviſor* in the moveable paper thereto, with the remaining 10, which is 106 the next leſſer number thereto among the multiplyed numbers is 96, which therefore is to be ſubſtracted, and 4 ſet in the Quotient, becauſe 96 is 4 times 24 the *Diviſor*, and ſo proceeding the quotient is found to be 2445, the *Remain* 8, and the work ſtandeth thus.

Example.

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

```
Diviſor Dividend Quotient
_________________________
  24)  58688 (2445
        ....
_________________________
        48
       ____
        10
        96
       ____
        10
        96
       ____
        12
        120
       ____
Remain.  8
```

```
Proof.

  48
   96
    96
    120
_____
58680 Total of the Multiplees.
    8 Remain added.
_____
58688 Dividend returned
_____
```

Another Variety.

Others in ſubſcribing the remaines after *Subſtraction*, pull down and adjoyn the remaining figures of the *Dividend* to the right hand: and others never underwrite the multiplyed Products, but having them in a looſe paper, apply them as they do the *Diviſor*, and ſubſcribe the Remains only. Examples of both formes appear in the *Diviſion* laſt before mentioned.

Examples.

```
Remaining numb. of the Dividend
        pulled down.
_________________________
  24 )  58688  (2445
         ....
_________________________
       48
      ____
       10.688
        96
      ____
        10.88
         96
      ____
          12.8
          120
      ____
Remain     8
```

```
  Remains only
   ſubſcribed.
_________________________
  24 )  58688  (2445
         ....
_________________________
         10
        ____
         10
        ____
          12
        ____
Remain     8
```

18. To divide by Nepair's Bones.

To divide by *Nepair*'s Bones, Set the *Diviſor* on the Top of the Bones, then ſhall you have the *Diviſor* multiplyed by all the 9 digits, out of which you may chuſe ſuch convenient numbers to ſubſtract from the *Dividend* (according to the ſeveral imagined ſettings down of the *Diviſor*, noted by pricks on the Number as was mentioned in the laſt variety) which will be the next leſſer to the Numbers ſo pricked, and the Indices of

Example.

ſuch ſubſtracted numbers, are to be ſet in the *Quotient*. As to divide 1396788 by 5678, the *Diviſor* 5678 is tabulated on the Bones, and the ſeveral *Multiplees* taken out, and 7 in the *Dividend* pricked, where the right hand place of the *Diviſor* ſhould ſtand; the next leſſer number to 13967 in the *Dividend*, among the *Multiplees* is 11356, which hath 2 for the *Index*, therefore 2 placed in the *Quotient*, and ſubſtraction made, 8 is pricked in the *Dividend*, and among the *Multiplees* 22712 is found to be the next leſſer to 26118, which remain to the laſt prick of the *Dividend*, and the *Index* of 22712 is 4 to be ſet in the *Quotient*. Laſtly the right hand 8 in the *Dividend* pricked, and finding the next leſſer number to 34068 to be the *Multiplee* of 6, this is therefore ſet in the *Quotient*; and after Subſtraction nothing remaineth. As here appeareth.

1	5	6	7	8	5678
2	1/0	1/2	1/4	1/6	11356
3	1/5	1/8	2/1	2/4	17034
4	2/0	2/4	2/8	3/2	22712
5	2/5	3/0	3/5	4/0	28390
6	3/0	3/6	4/2	4/8	34068
7	3/5	4/2	4/9	5/6	39746
8	4/0	4/8	5/6	6/4	45424
9	4/5	5/4	6/3	7/2	51102

5678) 1396788 (246

~~11356~~

2611

~~22712~~

3406

~~34068~~

General Proof of Diviſion.

Probation is the Proof whether the *Operation* be true or falſe, and is either general or particular.

General Proof is for any kind of *Diviſion*, and this may be effected three wayes.

By Subſtraction moſt tedious.

Firſt by *Subſtraction*. For if the *Quotient* be ſubſtracted from the *Dividend*, ſo many times as there be Units in the *Diviſor*, there will remain at laſt no more than the remainer of the *Diviſion*, if the work be well wrought. As in the former inſtance, *pag*. 31. where 3762 is divided by 3, the *Quotient* is 1254. If therefore 1254 be taken 3 times from 3762, there will 0 remain, as on the *Diviſion*.

3762	Dividend.
1254	Quotient abated
2508	Remain.
1254	Second Subſtraction.
1254	Second Remain.
1254	Third Subſtraction.
0	Laſt Remain.

~~11~~(0
~~3762~~(1254
~~3~~

Grounded on a Theorem that is the baſis of Diviſion.

This is a moſt laborious, and therefore an uſeleſs way of trial, yet the truth of this is grounded on the *Theorem*, which is the foundation of *Diviſion*, *viz*. The *Dividend* to the *Diviſor* is as the *Quotient* to an *Unit*, and *contra*. For if 20 divided by 5 give 4 in the *Quotient*; then 20 to 5 is as 4 to 1, and 1 to 4 is as 5 to 20.

Proof by 9 uncertain.

Secondly, *Diviſion* is commonly proved by caſting away nines thus. Caſt away 9 from the *Diviſor* as oft as may be, and place the remain at one Angle of a Croſs, and the remain, after 9 in like ſort is caſt from the *Quotient*, place at the oppoſite Angle, then multiply both theſe remains together, and to the amounting number add the remain of the *Diviſion* if any, and if the total be under 9, place it at another Angle of the Croſs, but if above 9, caſt away 9 as before, and ſet down the reſidue. Laſtly, caſt away all the nines from the *Dividend*, and ſet this laſt remain oppoſite to the remain laſt before ſet down; which if the work be right will be equal thereto, otherwiſe not. As in the ſecond Example before, the *Diviſor* 30 leaving 3, and the *Quotient* 58 leaving 4 when the nines are rejected; theſe 3 and 4 multiplyed produce 12, which added to 2 and 2 the remain of the *Diviſion* maketh 16, from whence 9 caſt reſteth 7, and ſo much will be left when the nines are caſt from 1762 the *Dividend*.

$$\begin{array}{ccc} & 7 & \\ 3 & \times & 4 \\ & 7 & \end{array}$$

~~2~~(2
~~176~~(2(58
~~33~~ 0

This kind of Proof for the uncertainty before noted in *Addition* is laid by.

Proof of Multiplication by Diviſion.

Thirdly, as *Addition* and *Subſtraction*; So are *Multiplication* and *Diviſion*, mutually the moſt excellent and infallible proofs of each other.

Multiplication is proved by *Diviſion* thus. Divide the *Product* by the *Multiplier*, and the *Quotient* will return the *Multiplicand*; or contrarily the *Product* divided by the *Multiplicand*, the *Quotient* ſhall return the *Multiplier* when the work is duely wrought. As for inſtance in the Example of the 16 Section of the precedent Chapter where the *Product* is 3614958, the *Multiplier* 423, the *Multiplicand* 8546.

```
                          2
                         195
                        22043
Product, Dividend      3614958(8546  Quotient.
Multiplier, Diviſor    4233333       Multiplicand.
                        4222
                         44
```

```
                          258
                         19653
Product, Dividend       3614958(423  Quotient.
Multiplicand, Diviſor   854666       Multiplier.
                         8544
                          85
```

Proof of Diviſion by Multiplication.

Diviſion on the contrary is proved by *Multiplication* thus. Multiply the *Quotient* by the *Diviſor*, and to the *Multiplees* add the Remain of the *Diviſion*, if any be, the total *Product* ſhall be equal to the *Dividend*, if the *Diviſion* be right. As for inſtance in the laſt Example of the 16 Section of this Chapter where the *Dividend* is 144980099, the *Diviſor* 1798, the *Quotient* 80634, and the *Remain* 167.

```
Multiplicand.     80634   Quotient.
Multiplier.        1798   Diviſor.
              -----------
             645072
            7257067
           5644386        Remain added
           806341
              -----------
Product.  144980099   Dividend.
              -----------
```

Conſectaries from hence.

From hence flow theſe two Conſectaries. First the *Multiplicand* may be known if the *Multiplier*, and *Product* be given; or the *Multiplier* if the *Multiplicand* and *Product*: By dividing the *Product* by the other given Number. As above in the Proof of *Multiplication*. 1.

Secondly, If the *Quotient*, and *Diviſor* be given, the *Dividend* may be known by *Multiplication* of the one given number into the other; as above in the proof of *Diviſion*. 2.

And if ſuch numbers were queſited, the queſtions may in like manner be reſolved.

Examples in Queſtions of Multiplication *and* Diviſion.

Queſtions in Multiplication and Diviſion.

1. What number was that which multiplyed by 15, produced 1380?
Anſwer, 92. for 1380 divided by 15 giveth 92 in the *Quotient*.

2. What number being divided by 15 will give in the *Quotient* 92?
Anſwer, 1380. for 92 multiplyed by 15 will produce 1380.

```
              3                       92
Operations.  1380(92 Quotient.        15
             155                     ----
              1                      460
                                     92
                                     ----
                                    1380 Product.
                                     ----
```

Particular Proof of Diviſion.

Particular Proof of *Diviſion* is peculiar to ſome particular methods of Operation; and will not ſerve for all ſorts of *Diviſion*. And ſo the *Diviſions* wrought according to the 16, 17, and 18. Sections of this Chapter may be proved by *Addition*; for the ſeveral *Multiplees* added together with the Remain of the *Diviſion* if any be, will infallibly return the *Dividend*; and all the ſelect wayes of *Diviſion* may be tryed by the Common way; as in the Sections before may be ſeen, and therefore needleſs here to be repeated, only for a Cloſe to this Chapter, and alſo to this firſt part of this Book, concerning *Integers*, take theſe few Obſervations and Theorems following.

Obſervations and Theorems.

1. 1. If a Quantity, either Magnitude or Number, be made of other two Quantities by their Multiplication, the one of them being the *Factor*, will meaſure (that is, evenly divide) the ſame Quantity made by the other, (being the other *Factor.*) As 20 made of 4 into 5, the one of them will meaſure 20 by the other 4, and ſo will 4 meaſure the ſame 20 by 5. For the meaſure of 5 is 4 times in 20, and the meaſure of 4 is 5 times in 20.

2. 2. If a Quantity, either Magnitude or Number, be made of two other Quantities by their Multiplication, it is all one whether any other Number be divided by that one Quantity, or by thoſe other two Quantities. As if 20 made of 4 and 5 as before, divide 60, the Quotient is 3, ſo is it all one if 60 be divided firſt by 4, and the Quotient thereof 15 by 5, for this laſt Quotient will be 3.

3. 3. *Fractions* ariſe from *Diviſion*, when the *Dividend* is leſſer than the *Diviſor*, or cannot be evenly meaſured thereby, a *Fraction* will remain. As ſeveral Examples of *Diviſion*, *pag.* 31. and elſewhere demonſtrate.

4. 4. When the *Quotient* and *Diviſor* are equal, and nothing remain on the *Diviſion*, the *Dividend* is a ſquare number, and the *Diviſor* the Root thereof. As in the inſtance, Sect. 14. of this Chapter 99980001 is a Square Number, and 9999 the Root, *De quibus in ſuo loco.*

Partis primæ Libri primi

FINIS.

THE

THE SECOND PART OF THE FIRST BOOK.

CHAP. I.

Of FRACTIONS.

INtegers diſpatched in the former part of this Book. The next ſort of *Homogeneal* Numbers to be examined are *Fractions*, ſometime called *Fragments*, ſometime *Parts*, ſometime *Broken Numbers*. This ſhall be done generally and particularly.

Fractions next to Integers to be dealt with.

In general, it is requiſite! to know what they are, whence they come, and how to expreſs them.

Fractions what they are.

By what paſſed before *Chap.* 2. *pag.* 5, and 6, *Fractions* may be underſtood to be *Broken Numbers*, , or broken parts of a *Whole Number*, extending infinitely under an Unit as *Integers* do above.

They ariſe from the Diviſion of their *Integers*, as was obſerved in the laſt Chapter before, and in ſumm are expreſſed with the ſame Notes or Characters, as the *Integers* from whence they come. Thoſe ariſing from *Abſtract Integers* are firſt to be treated of; and the proper ſubject of this Part, where *Fractions* without any other note of diſtinction are to be taken for ſuch. To proceed then,

Whence they ariſe.

Abſtract Fractions firſt treated of.

Fractions as they are broken numbers, ſo are they brokenly expreſſed, verbally, or in writing, by two termes or numbers, diſected as it were the one from the other, but *Integers* are uniforme.

Broken Numbers brokenly expreſſed by 2. Terms.

The two termes or numbers expreſſing a *Fraction*, are by ſome ſet one before or over the other, with a *Colon* between them. As Two thirds thus, 2 : 3, or thus $\frac{2}{3}$, but the moſt uſual form is to ſet them one over another, with an interjacent line. As Two Thirds thus, $\frac{2}{3}$.

Of theſe two termes, the neathermoſt under the line, or to the right hand of the *Colon*, denoteth the Unit to be divided into ſo many equal diviſible parts, and is called the *Denominator*. As if it be 2, it noteth the Unit is parted into halfs, if 3 into thirds, if 4 into fourths, *&c.*

Which of them Numerator, and which Denominator.

The uppermoſt, or foremoſt to the left hand, ſheweth how many of thoſe parts into which the Unit is broken, are ſignified to be contained in, or taken from the content or value of the *Fraction*, and this is called the *Numerator*, and ſometime *Nominator*. So that the *Numerator* alway containeth the number of the parts ſignifyed by the *Fraction*; and the *Denominator* the greatneſs of thoſe parts.

As in $\frac{3}{5}$, the *Numerator* is 3, the *Denominator* 5, and the *Fraction* ſignifieth three parts of any one *Integer* divided or broke into five equal parts, which content may be demonſtrated by ſuppoſing the *Unit* or *Integer* to be the line a. e. equally divided into 5 parts, the Diviſion made by o upon the line toward a. ſhall be equal to the *Fraction* $\frac{3}{5}$ of that line, and the reſidue of the line marked with o. e. ſhall be $\frac{2}{5}$, and make up the whole line, *&c.*

Example.

3
2
Numerator.
o
a
e
5
Denominator.

Inlarging the Denominator or Numerator what effect.

From hence it appears. The more the *Denominator* is increaſed, the farther is the value of the *Fraction* from the whole Number or *Integer*, and the more the *Numerator* is augmented, the nearer is the value to the whole. Again the *Numerator* may be inlarged till the value be more than the whole, and contrarywiſe the *Denominator* may be diminiſhed till the *Fraction* become an Unite or more; but the *Denominator* though never ſo great, yet will ever import a part of the *Integer*, though never ſo ſmall. For every *Fraction* if the *Denominator* be increaſed leſs, and if decreaſed is greater in quantity. All this dependeth on that fundamental principle. Such proportion as the *Numerator* beareth to the *Denominator*, the ſame beareth the parts ſignified by the *Fraction* to an Unite as in $\frac{3}{4}$ of a Shilling, it is evident that 3 to 4 is as $\frac{3}{4}$ to 1 ſhilling, or 12 pence, three quarters of which being 9 pence 9 to 12, ſhall in proportion be as 3 to 4.

The Principle on which the effect depends.

Which Termes are Heterologal or Heterogeneal and the contrary.

Numerators, and *Denominators*, compared one to another, are called *Heterologal*, and ſometime *Heterogeneal* Termes; but either of them by themſelves, are called *Homologal*, and ſometime *Homogeneal*.

Numerator is pronounced before the Denominator.

As in *Integers* the Numeration of their Quantities precedeth their *Denomination*, ſo in *Fractions*, the *Numerator* is alwayes pronounced before the *Denominator*.

That by Cardinal, and this by Ordinal Numbers.

The *Numerators* are verbally expreſſed by *Cardinal Numbers*, as One, Two, Three, Four, &c. The *Denominators* are beſt pronounced by the Ordinals, as halves, thirds, fourths or quarters, fifths, &c. As $\frac{1}{2}$. $\frac{3}{4}$. $\frac{6}{7}$. $\frac{17}{20}$. thus, one half, three quarters, ſix ſevenths, ſeventeen twentieths, &c.

Fractions diverſly divided.

In particular, *Fractions* (as well as *Integers*) are capable of being diverſly conſidered, in reference both to their quantity and quality; as they are ſingle or plural, abſtract or contract; generally or ſpecially. A ſhort narrative whereof followeth in the enſuing Table.

A Table of the Nature of Fractions.

- Fractions conſidered in
  - Quantity
    - Numerator
      - Single.
      - Plural
        - Commenſurable.
        - Incommenſurable.
    - Denominator
      - Certain.
      - Uncertain.
  - Quality
    - Abſtract
      - Naturally. Leſs than an Unit.
      - Nominally. Arithmetical.
    - Contract
      - Generally.
        - Geodætical.
        - Figural.
      - Specially.
        - Decimal.
        - Aſtronomical.
        - Logarithmical.
        - Coſſical.
        - Surde.
        - Specioſal.

Quantity of a Fraction how ſignified.

The Quantity of every *Fraction*, as before noted, being ſignified by his *Numerator*, and *Denominator*, both Termes are to be conſidered diſtinctly for the particular knowledge of the *Fraction*.

Numerator notes the content, if Single or Plural.

The *Numerator* denoting the content of the *Fraction*, ſhews whether it be a ſingle, or a plural *Fraction*; that is, whether it contain only a part of the *Integer*. As $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$, &c. or many parts. As $\frac{2}{3}$. $\frac{3}{4}$. $\frac{4}{5}$, &c.

Plural are Commenſurable or Incommenſurable.

Theſe Plural *Fractions* are again diſtinguiſhed into Commenſurable and Incommenſurable.

Commenſurable, called *Compoſiti inter ſe*, or Numbers compound among themſelves, are ſuch as beſides an Unit have ſome Number that will divide exactly both the Termes without leaving any Remain. As $\frac{5}{15}$ is Commenſurable, for both the Termes may be evenly divided by 5.

Common and Greateſt Common Diviſor what.

The Number ſo exactly dividing the Termes, is called the Common Diviſor, or Meaſurer, and becauſe ſome Termes have more Numbers than one that will thus divide them, the greateſt of theſe Numbers is called the greateſt Common Meaſure, or Diviſor, and in Latine *Communis menſura maxima*, or *Communis Diviſor maximus*. As $\frac{6}{12}$ may be exactly divided by 2, but the Greateſt Common Meaſure is 6.

Incommenſurable, called *Primi inter ſe*, or prime among themſelves, are they that cannot be meaſured by any Number but an Unit, which not dividing properly, they have no number that will ſerve for a Common Diviſor evenly to divide both the Terms without leaving ſome remaining number upon the Diviſion of one of them. As $\frac{9}{20}$, where 9 may be exactly divided by no number but 3, and 3 cannot exactly divide 20, but 2 will remain. Such *Fractions* are ſaid to be in their leaſt Terms.

Incommenſurable in their laſt Terms.

The Product of any Incommenſurable Terms multiplied one into another, is called in Latine *Dividuus communis minimus*, and in *Engliſh* the leaſt common Dividend, of which little uſe is made, but the Common Diviſor is very uſeful, as may be further ſeen in the next Chapter of *Reduction*. Here by the way may be ſeen the Difference between Prime and Compound Integers and Prime and Compound Fractions. Uncompound Integers can be meaſured by no number of multitude exactly. As 3. 5. 7. 11. 13, *&c.* But *Fractions* may have either of the termes taken ſingly, or apart, a Compound Integer; yet may be *Primi inter ſe*, conſidered joyntly. As $\frac{8}{9}$ is Incommenſurable, though both 8 and 9 are Compound Integers.

Common Diviſor is uſeful.

Difference between Integers and Fractions.

Fractions are again conſidered in reſpect to their Denominators, which are either Certain or Uncertain.

Denominator certain or uncertain.

The Denominators are certain in *Decimals*, *Aſtronomicals*, and *Logarithmes*, the firſt alwayes increaſing or decreaſing by 10. The ſecond by 60. The third according to the *Radius*, and are omitted; becauſe certainly known.

Where certain, and omitted,

Geodæticals certain in regard their Denominators are known by their Denominations, but various according to ſuch Denominations, and the Denominators of *Fractions* ſometime uſed with them are uncertain.

Geodæticals how certain and uncertain.

All other Denominations beſides theſe are uncertain, that is may be any abſolute Number or Quantity whatſoever; and therefore of neceſſity muſt be expreſſed.

Where uncertain and expreſſed.

As *Fractions* are conſiderable in their quantity, ſo in their quality. As whether they are Abſtract or Contract. Abſtract are free from any reſtraint, but may be a part or parts of any quantity or magnitude. Contract on the contrary have ſome annexed Denomination, or ſpecial Denominator.

Quality of a Fraction to be Abſtract or Contract.

Abſtract abſolute *Fractions*, have community in their nature with others of what ſort ſoever that is to be inferior to an Unit be they ſingle or plural in their Numerators; but they have a name proper to them for diſtinction ſake to difference them from other ſorts, that is *Arithmetical Fractions* ſome time called *Abſtract*, *Vulgar*, or *Common Fractions*. Theſe are ſuch as before in this Chapter are inſtanced, and hereafter in this part at large handled.

Abſtract what.

The names of Abſtract Fractions.

Contract Fractions, like *Integers*, are more generally contract in *Geodæticals* and *Figural Numbers*, and more eſpecially in *Decimals*, *Aſtronomicals*, *Logarithmes*, *Coſſicks*, *Surdes*, and *Species*. In the two firſt, and three laſt principally by reaſon of their Denominations: In the other three by their ſpecial Denominators, as well as in reſpect of their Denominations. The Nature and Operations of which were briefly before touched, *Chap.* 2. but are particularly to be unfolded in the ſeveral parts of the two next Books.

Contract Fractions General or Special.

Denominator and *Denomination*, though ſometime uſed promiſcuouſly, differ in reſpect of Quantity and Quality, the Denominator ſheweth as before the greatneſs or ſmalneſs of the parts the *Integer* is underſtood to be divided into, the *Denomination* declareth the Nature of the *Integer*, of what kind or quality it was, whither Pounds, Ounces, Yards, Ells, Men, Moneths, *&c.*

How Denominator and Denomination differ.

The *Denomination* is ſometime called the Sirname, this annexed makes any abſtract Fraction contract, and thereby makes the number ſeem to be doubly denominate, *Viz.* As to the *Denominator* and *Denomination*, therefore are *Fractions* accompted before leſs abſolute than *Integers*.

Sirname of a Fraction what.

Arithmetical, or *Vulgar Fractions*, in this part dealt with as they ariſe from *Integers*, ſo in ſome things are like them, but in others different. As a greater *Integer* cannot be ſubſtracted from, nor divide a leſſer: But a leſſer *Fraction* may be divided by a greater, though in *Subſtraction* they agree, that the *Subtrahend* ſhall be the leſſer number. They agree to be compoſed, and diſſolved, but their Operations therein are different, and the moſt eaſie in the one, are moſt difficult in the other, though in the prime and compound parts of Original Numeration they take part with *Integers*, to be added, ſubſtracted, multiplyed, and divided, yet becauſe of their *Denominators*, they reſemble the mode of Contract Numbers when their *Denominators* are unlike, and will firſt be reduced before they can be added, or ſubſtracted, and ſo compared to *Integers* in their Simple Elements go in a Retrograde motion, placing Ortive Numeration before Original. They mix well enough with *Integers*, and are as hand-maids to them, whence ariſe the firſt ſort of *Heterogeneal* Numbers, who in this part with them receive their Operations and Reſolutions.

How Abſtract Fractions agree and differ with Integers.

Whence Improper Fractions come.

Reſolutions. In Fine from the mixture of *Integers* and *Fractions*, iſſue *Improper Fractions* in reference to which others are called *Proper Fractions*, between both which by increaſing the Numerators of *Proper Fractions*, or the Denominators of *Improper Fractions*, *Equal Fractions* ſpring forth, and are all eaſily known thus.

Proper Fraction what.

Proper Fractions alwayes have the Numerator leſs than the Denominator, for then the parts ſignified are leſs than an *Vnit* or *Integer*, though the Terms be never ſo great. As $\frac{4}{5}$. $\frac{2}{3}$. *&c.*

Equal Fraction what.

Equal Fractions have the Numerator, and Denominator alwayes equal, and then the *Fraction* is equal to an *Vnit*. As $\frac{4}{4}$. $\frac{5}{5}$. $\frac{2}{2}\frac{1}{1}$. *&c.* and it were better to expreſs ſuch by $\frac{1}{1}$ ſeeing they are but 1. Theſe are improperly called *Fractions*, becauſe they contain one *Integer*, but are thus repreſented for convenience in work.

Improper Fraction what.

Improper Fractions have alwayes the Numerator greater than the Denominator, for then is the *Fraction* greater than the whole. As $\frac{5}{4}$. $\frac{3}{2}\frac{7}{1}$. $\frac{7}{2}$. $\frac{9}{4}$. *&c.*

Proper Fraction

Proper Fractions are of two kinds, Conjunct, and Divided.

Conjunct.

Conjunct, ſometimes called *Fractions of Integers*, are ſuch as are conjoyned together by the Copulative [*and*]. As $\frac{2}{3}$. and $\frac{3}{4}$. *&c.* implying more broken parts than one.

Divided. Theſe are Fractions of Fractions.

Disjunct or divided, are frequently called *Fraction of Fractions* differenced from the other by the intervening [*of*] inſtead of [*and*]. As $\frac{2}{3}$ of $\frac{3}{4}$. *&c.* denoting only a part of a *Fraction*,

Improper Fractions of 2 ſorts.

Improper Fractions alſo are of two ſorts, *Viz.* either they include ſeveral *Integers*. As $\frac{9}{3}$. $\frac{20}{4}$. *&c.* or elſe one or more *Integers* with ſome part or parts of the *Integer*. As $\frac{5}{4}$. $\frac{13}{5}$. *&c.*

Elements of Fractions.

The Simple Elements of *Fractions* in ſum may be concluded under two heads; either to increaſe or decreaſe, their Terms or their Value, theſe as more eſſential are comprehended under *Original Numeration*. Thoſe as accidental under that part of *Ortive Numeration*, called *Reduction*.

CHAP. II.

Reduction of Fractions.

Reduction what and how uſeful.

Reduction in general, is that part of *Ortive Numeration*, whereby one Number, or Magnitude is reduced to another. Uſeful in *Fractions* and *Contract Numbers*, that they may receive a more apt form of Operation, in thoſe to bring them from one Term to another; in theſe from one Denomination to another, yet in both retaining the ſame value or content.

What it conſiſts in.

Reduction conſiſteth principally in *Multiplication*, and *Diviſion*, yet occaſionally converſeth with *Addition* and *Subſtraction*, as neceſſity requireth.

The work thereof to be ſeen in the Sections following.

Reduction ſo far as concerns *Arithmetical Fractions* now in hand will be underſtood in two things, *Operation* and *Probation*.

Operation may be thus methodized.

Reduction of
- Proper Fractions
  - To their leaſt Termes
    - by the great Common Meaſure. §. 1.
    - by Bipartition, Tripartition, *&c.* §. 2.
  - To like Denominators.
    - Conjunct by Multiplication. §. 3.
    - Disjunct by Multiplication. §. 4.
- Improper Fractions
  - To Integers, or mixt Numbers, by Diviſion. §. 5.
  - From mixt Numbers, by Multiplication and Addition. §. 6.
- Integers.
  - To the form of an Improper Fraction, by an Unite. §. 7.
  - To any deſired Denominator, by Multiplication. §. 8.
- Proper or Improper Fractions to any deſired Denominator by Multiplication and Diviſion. §. 9.
- Integers with Proper, and Improper Fractions. §. 10.

§. Abbreviation by the Common Diviſor.

The firſt ſort of *Reduction* belonging to *Proper Fractions* is to reduce them to their leaſt termes. This is commonly called Abbreviation, becauſe it doth abbreviate or cut ſhort the terms of a *Fraction*, and ſo conſequently the work therewith. And becauſe by *Euclide*, *lib.* 7. *Prop.* 17. if one Number multiply two Numbers their Products will retain the ſame proportion either to other the Numbers did before *Multiplication*; of neceſſity if one Number divide two Numbers, the Quotients muſt be ſtill proportional. And hence it is evident that the termes of the ſame *Fraction* may be infinite, and yet bear the ſame proportion one to another. As $\frac{4}{5}$ may be expreſſed by $\frac{8}{10}$. $\frac{16}{20}$. $\frac{32}{40}$, *&c.* multiplying both terms by 2, and by $\frac{12}{15}$. $\frac{36}{45}$ *&c.* multiplying by 3, every of which *Fractions* is ſtill in value, but $\frac{4}{5}$. And as in denominate numbers it is not proper to ſpeak of 100 pence, 500 ſhillings, or ſuch like, when there are other Denominations to abbreviate the numbers. So neither in *Fractions* is it proper to mention any termes ſave the leaſt that they may be reduced into. Wherefore then upon the whole matter, becauſe Abbreviation is but brief Diviſion, it appears that if the Greateſt, or Commenſurable termes of any *Fraction* be divided by any number that will equally meaſure or divide them without leaving any remain, then ſhall the *Fraction* be expreſſed in leſſer terms by help of this Common Diviſor; that is to ſay, by dividing both *Numerator* and *Denominator*, thereby obſerving to let the Numerator of the new Fraction, be the Quotient of the old Numerators Diviſion: And the Denominator his Quotient likewiſe.

Moſt proper to ſet Fractions in their leaſt Termes.

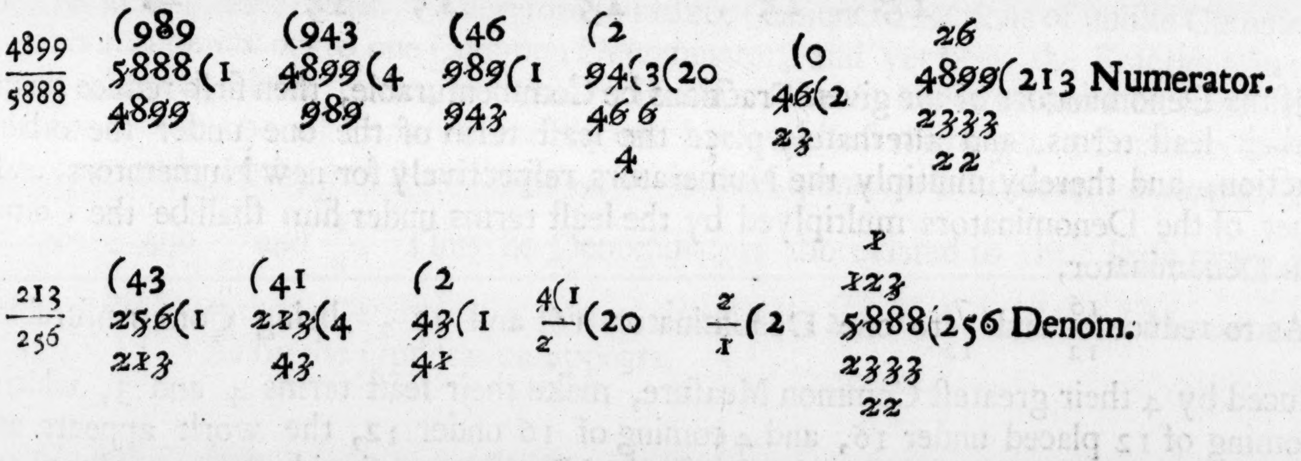
Commenſurable terms ſometimes admit ſeveral Common Diviſors, among which ſome one is the greateſt. As in the foregoing Chapter was obſerved; wherefore it is neceſſary to know how to find this great Common Diviſor, becauſe thereby the *Reduction* is perfected at once, when the leſſer Common Diviſors repeat their operations ſeveral times.

How to find the Greateſt Common Diviſor.

The Greateſt Common Meaſure of two Numbers, by *Euclide, lib.* 7. *prop.* 2. is found by *Subſtraction*, but the far better way is by continual *Diviſion* of the greater by the leſſer, and of the Diviſor by the Remainer till either a Cypher or an Unite remain. For that Diviſor which firſt divideth the Dividend without leaving any remain, is the Greateſt Common Meaſure of both the numbers, and both the numbers thereby are found to be Commenſurable or Compound among themſelves. But if no ſuch Diviſor be found till an Unit remain, the numbers are Incommenſurable, and will not be reduced to leſſer terms.

Example. If $\frac{4899}{5888}$ were the Fraction given, and it were deſired to know if the Numbers were Commenſurable, and if ſo, what is the greateſt Common meaſure, and conſequently their leaſt terms. Then dividing 5888 by 4899, and 4899 by 989 the remain of the firſt Diviſion, and again 989 by 943 the ſecond remain, and ſo continuing this manner of *Diviſion* 23 is found to be the firſt Diviſor, that leaveth 0 remaining on the *Diviſion*, wherefore the Fraction is Commenſurable, and the Great common Diviſor is 23. by which both terms divided, the new Fraction is found to be $\frac{213}{256}$ which are the leaſt terms of $\frac{4899}{5888}$ and numbers Incommenſurable. For dividing $\frac{213}{256}$ as before, no Diviſor will be found evenly to divide them till an Unit remain which as aforeſaid neither multiplyeth nor divideth.

Example.



By frequent practiſe ſome Common Diviſor or other if not the greateſt may quickly be eſpied. To ſpare therefore ſuch multifarious *Diviſion* in getting the Greateſt Common Diviſor. It is common to uſe *Bipartition*, *Tripartition*, or any ſuch ſelect *Diviſion* by the Digits, as in the Chapter of *Diviſion* before was ſet forth, if the given terms of the Fraction will exactly be divided by any of them; placing the Numerators along upon a line, and the Denominators beneath, and ſeparating the ſeveral Quotients in the work, with a down right line or daſh of the Pen. As if $\frac{288}{576}$ were to be reduced into its leaſt termes; firſt mediating, or taking half the Numerator and Denominator, As below till $\frac{9}{18}$ be brought forth, and then the third part of both numbers accordingly. At laſt the Fraction is reduced to $\frac{1}{2}$ thus.

§. 2. How to abbreviate the Common way.

Example.

$$\frac{288}{576} \Big| \frac{144}{288} \Big| \frac{72}{144} \Big| \frac{36}{72} \Big| \frac{18}{36} \Big| \frac{9}{18} \Big| \frac{3}{6} \Big| \frac{1}{2}$$

Where Cyphers are at the right hand.

Alſo where Cyphers are in the terms of the Fraction to the right hand as before in *Diviſion*, a like number of them from both Numerator and Denominator may be cut off by a daſh of the Pen, and the reſidue left, or reduced lower if it may be. As $\frac{300}{400}$ thus $\frac{3|00}{4|00}$ will thereby appear to be but $\frac{3}{4}$. Alſo if from $\frac{250}{500}$ be cut off the two right hand Cyphers, the reſidue $\frac{25}{50}$ ſtraight way is ſeen to be but $\frac{1}{2}$.

But if no ſuch Common Diviſor can be found among the Digits, then the great Common Meaſure, if any is to be ſought, as before ; and the Fraction reduced to its leaſt terms thereby.

§. 3. To bring Conjunct Fractions into one Denominator.

The next ſort of *Reduction* belonging to proper Fractions is to reduce them from Different to like Denominators, that they may be expreſſed in more Commodious numbers. For as in Integers, Units muſt be added to, or ſubſtracted from Units, Tens to or from Tens, *&c.* And as in Denominate numbers Pounds muſt be added with, or taken from Pounds, Shillings, with or from Shillings, *&c.* So in Fractions, muſt halfs be added to halfs, thirds to thirds, *&c.* and ſubſtracted accordingly, or elſe if the Denominators be unlike, both muſt be reduced to like Denominators before *Addition* or *Subſtraction.*

The Reduction of different Denominators into like Denominators is ſometime called Reduction to one Denomination, and the Denominators reduced being all one, the ſame need be ſet down but once to all the given Fractions, which Denominator ſo placed is called the Common Denominator.

Common Denominator what.

This Reduction differs as the Fractions to be reduced are Conjunct, or Disjunct.

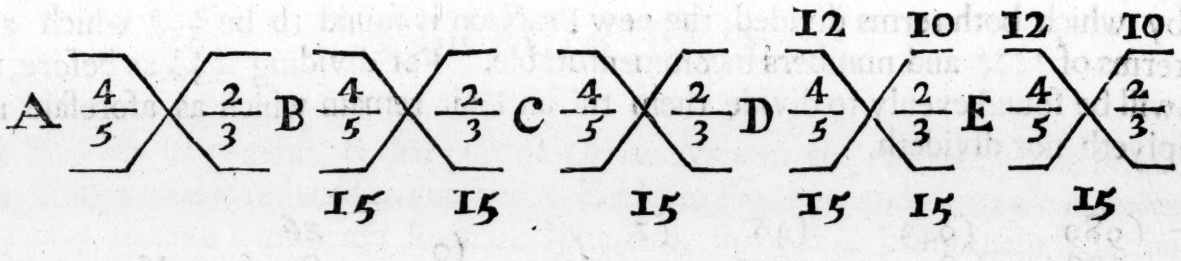
Proper Fractions conjunct are two, or more.

When only 2 Fractions are given, and the Denominator Incommenſurable.

When two Fractions are given to be thus reduced, diſpoſe them for form ſake on either ſide of a Croſs, and if the Denominators be Incommenſurable multiply both the Denominators together, and that Product ſhall be the New or Common Denominator to both the given Fractions; Then for new Numerators multiply Croſwiſe the Numerator of the one Fraction by the Denominator of the other, and place the Product reſpectively overt that Fraction whoſe Numerator was in the Compoſition.

Example.

As to reduce $\frac{4}{5}$ and $\frac{2}{3}$ to one Denomination. They are ſet as at A. and then multiply 5 by 3, the Product 15 is the Common Denominator, ſet as at B. or C. alſo multiplying 4 by 3 the Product 12 is the Numerator to be ſet over 4, and multiplying 2 by 5 the Product 10 is the other Numerator to be ſet over 2, as at D. or E. So are the two reduced Fractions $\frac{12}{15}$ and $\frac{10}{15}$.

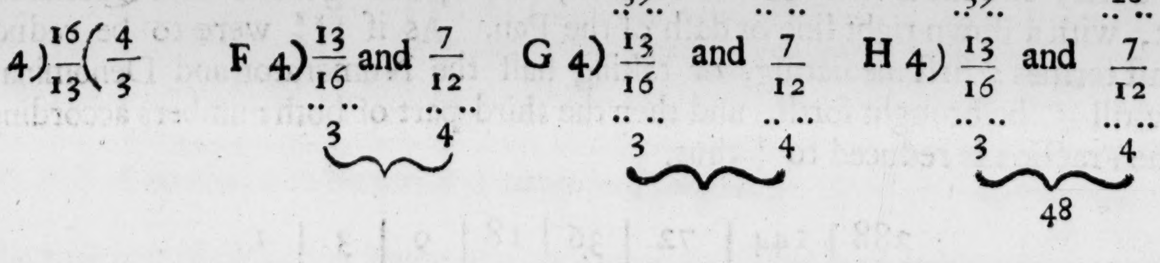


When the Denominators are Commenſurable.

If the Denominators of the given Fractions be Commenſurable, then firſt reduce them to their leaſt terms, and alternately place the leaſt term of the one under the other Fraction, and thereby multiply the Numerators reſpectively for new Numerators, and either of the Denominators multiplyed by the leaſt terms under him ſhall be the Common Denominator,

Example.

As to reduce $\frac{16}{12}$ and $\frac{7}{12}$ to one Denominator 16 and 12 , being Commenſurable reduced by 4 their greateſt Common Meaſure, make their leaſt terms 4 and 3, which 3 coming of 12 placed under 16, and 4 coming of 16 under 12, the work appears as at F. then multiplying 13 by 3, and 7 by 4, the Numerators 39 and 28 are produced at G. and 16 by 3, or 12 by 4 the Common Denominator is found to be 48, and the work ſtands compleat as at H. where is found $\frac{13}{16}=\frac{39}{48}$ and $\frac{7}{12}=\frac{28}{48}$ being reduced.



When more than 2 are given, and the Denominators Incommenſurable.

When more than two Conjunct Fractions are given to be reduced to like Denominators multiply all the Denominators together for the Common Denominator, and to find new Numerators, multiply each Fractions Numerator, into the Denominators of all the other Fractions except its own Denominator.

Example.

As to reduce $\frac{2}{3}$ and $\frac{4}{5}$ and $\frac{6}{7}$ into like Denominators, multiplying 2 the Numerator of the firſt by 5 the Denominator of the ſecond Fraction, and the Product 10 by 7 the Denominator of the third, the amounting Product 70 is the Numerator of the firſt Fraction. For the ſecond Multiply 4 the Numerator of the ſecond by 3 the Denominator of the firſt, and the Product 12 by 7 the Denominator of the third. So the Product 84 is the ſecond Fractions Numerator. Then multiply 6 the Numerator of the third by 5 the Denominator of the ſecond, and the Product 30 by 3 the Denominator of the firſt. So is 90 the Numerator of the third Fraction, unto whom the Common Denominator ſhall be 105. for $3 \times 5 = 15 \times 7 = 105$, and the three reduced Fractions ſet as at I. or K.

Denominators. Numerators.

$$\begin{array}{c}3\\5\\\hline 15\\7\\\hline 105\\\hline\end{array}\qquad\begin{array}{c}2\\5\\\hline 10\\7\\\hline 70\\\hline\end{array}\quad\begin{array}{c}4\\3\\\hline 12\\7\\\hline 84\\\hline\end{array}\quad\begin{array}{c}6\\5\\\hline 30\\3\\\hline 90\\\hline\end{array}\qquad \overset{\text{I}}{\frac{70}{105} \text{ and } \frac{84}{105} \text{ and } \frac{90}{105}}\qquad \overset{\text{K}}{\frac{70 \text{ and } 84 \text{ and } 90}{105}}$$

Variety of work.

Some deliver the Rule thus. Multiply all the Denominators together for a Common Denominator, and divide that Common Denominator by each ſeveral Denominator, and multiply the ſeveral Quotients by their reſpect Numerators. As in the laſt Example.

$$\text{Common}\ \begin{array}{c}3\\5\\\hline 15\\7\\\hline 105\\\hline\end{array}\ \text{Denominator.}\qquad \text{New}\ \begin{array}{r}\cancel{1}\\ \cancel{1}0\cancel{5}(35\\ 2\\ \cancel{3}\ \overline{}\\ 70\\\hline\end{array}\qquad\begin{array}{r}\cancel{1}0\cancel{5}(21\\ 4\\ \cancel{5}\ \overline{}\\ 84\\\hline\end{array}\qquad\begin{array}{r}\cancel{3}\\ 105(15\\ 6\\ \cancel{7}\ \overline{}\\ 90\\\hline\end{array}\ \text{Numerators.}$$

When the Denominators are Commenſurable.

Either of theſe wayes will ſerve if the Denominators be Incommenſurable, but if the Denominators, or any of them be Commenſurable, the Fractions thus reduced will not be in their leaſt terms. Therefore to reduce Conjunct Fractions of unlike Commenſurable Denominators to one Common Denominator, and yet keep the Fractions in the ſame value, and leaſt terms, do thus. Reduce all the Denominators to their leaſt termes, if they be all Commenſurable, and by theſe leaſt terms get a Common Denominator, and thereby new Numerators, as in the Operation laſt before. *Example.* To reduce $\frac{3}{4}$ and $\frac{5}{6}$ and $\frac{7}{8}$. Thus the Denominators abbreviated to their leaſt terms are 2. 3. 4. the Common Denominator gotten thereby is 24, and the ſeveral Numerators 18. 20. 21. As by the Opperation appears.

Example.

$$\begin{array}{ccc}\frac{3}{4} \text{ and} & \frac{5}{6} \text{ and} & \frac{7}{8}\\ \\ 2. & 3. & 4.\end{array}\quad\begin{array}{c}2\\ \underline{3}\\ 6\\ \underline{4}\\ \underline{24}\end{array}\qquad\begin{array}{r}2\cancel{4}\,(6\\ \cancel{4}\ \ 3\\\hline 18\\\hline\end{array}\qquad\begin{array}{r}2\cancel{4}\,(4\\ \cancel{6}\ \ 5\\\hline 20\\\hline\end{array}\qquad\begin{array}{r}2\cancel{4}\,(3\\ \cancel{8}\ \ 7\\\hline 21\\\hline\end{array}\qquad\frac{18 \text{ and } 20 \text{ and } 21}{24}$$

When not all the Denominators are Commenſurable.

When all the Denominators are not Commenſurable but only ſome of them, then either firſt reduce thoſe Commenſurable to one Denominator, and after work with this reduced, and the other remaining Fraction or Fractions, or elſe reject thoſe leſſer Denominators which are even parts of the Greater Compound Denominator, and work with the other Denominators, and the greater Compounds.

Example.

As to reduce $\frac{3}{4}$ and $\frac{5}{8}$ and $\frac{2}{5}$ to like Denominators; becauſe 4 is a part of 8 it may be rejected, and only 8 and 5 multiplyed for the Common Denominator.

Or elſe firſt reducing $\frac{3}{4}$ and $\frac{5}{8}$ the new Fractions are $\frac{6 \text{ and } 5}{8}$ which reduced with $\frac{2}{5}$ make $\frac{30 \text{ and } 25 \text{ and } 16}{40}$, as in the Operations following.

$$\underbrace{4)\ \begin{array}{c}6\\ \cdot\cdot\ \cdot\cdot\\ \frac{3}{4}\\ \cdot\cdot\ \cdot\cdot\\ 2\end{array} \text{ and } \begin{array}{c}5\\ \cdot\cdot\ \cdot\cdot\\ \frac{5}{8}\\ \cdot\cdot\ \cdot\cdot\\ 1\end{array}}_{8} \qquad \begin{array}{c}8\\ 5\\ \hline 40\\ \hline\end{array} \qquad \begin{array}{c}40(5\\ 8\ \ 6\\ \hline 30\\ \hline\end{array} \qquad \begin{array}{c}40(5\\ 8\ \ 5\\ \hline 25\\ \hline\end{array} \qquad \begin{array}{c}40(8\\ 5\ \ 2\\ \hline 16\\ \hline\end{array} \qquad \begin{array}{c}30 \text{ and } 25 \text{ and } 16\\ \hline \frac{3}{4} \text{ and } \frac{5}{8} \text{ and } \frac{2}{5}\\ \hline 40\end{array}$$

§. 4. Fractions of Fractions reduced.
Example.

To reduce Proper Disjunct Fractions; Multiply all the Numerators one into another for a new Numerator, and in like manner all the Denominators together, and the new Denominator is produced. So $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ reduced will be $\frac{15}{64}$.

$$\begin{array}{c}\text{Numerators. } 1\\ 3\\ \hline 3\\ 5\\ \hline 15\end{array} \qquad \begin{array}{c}15\\ \hline \frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{5}{8}\\ \hline 64\end{array} \qquad \begin{array}{c}2 \text{ Denominators.}\\ 4\\ \hline 8\\ 8\\ \hline 64\\ \hline\end{array}$$

When ſome of the terms be Commenſurable.
Example.

If any of the Alternate Heterologal Terms in the Fractions given to be reduced be Commenſurable, the new Fraction gotten as above, will not be in its leaſt terms, unleſs ſuch Heterologal termes be firſt abbreviated to their loweſt, and operation made therewith as before. So $\frac{1}{4}$ of $\frac{2}{5}$ reduced without abbreviation will be $\frac{2}{20}$, but if the Heterologal terms 2 and 4 be firſt reduced to their leaſt terms, and then *Multiplication* made of theſe new Homologal terms as before, the new Fraction will be $\frac{1}{10}$ in its leaſt terms.

$$\begin{array}{c}2\\ \hline \frac{1}{4} \text{ of } \frac{2}{5}\\ \hline 20\end{array} \qquad \begin{array}{c}1\\ \hline 2)\ \begin{array}{c}\\ \frac{1}{4}\\ \cdot\cdot\ \cdot\cdot\\ 2\end{array} \text{ of } \begin{array}{c}1\\ \cdot\cdot\ \cdot\cdot\\ \frac{2}{5}\\ \\ \end{array}\\ \hline 10\end{array}$$

§. 5. Improper Fractions reduced to Integers.
Example.

The next kind of *Reduction* belongeth to Improper Fractions, and is double. Firſt, To reduce Improper Fractions into Integers or mixt numbers. Divide the Numerator by the Denominator, and if any thing remain after Diviſion adjoyn it to the Quotient in form of a Fraction by ſetting the Remain over the old Denominator. As $\frac{15}{3}$ ſhall give 5 Integers, and $\frac{15}{4}$ the mixt number $3\frac{3}{4}$.

Improper Fraction $\frac{\cancel{15}}{\cancel{3}}(5$ Integers $\qquad \overset{(3}{\frac{\cancel{15}}{\cancel{3}}}(3\ \frac{3}{4}$ Mixt Numbers.

§. 6. Integers and Fractions reduced to Improper Fractions
Example.

This *Reduction* is oft-times needful at the end of Operation, that the Reſolution of the Queſtion in hand might be more plain.

Secondly on the contrary, when any mixt number is to be reduced into an Improper Fraction. Multiply the Integer by the Denominator of the Fraction, and to the Product add the Numerator of the Fraction, and this Product ſhall be the Numerator of the Improper Fraction, the old Denominator ſhall ſerve ſtill. As $13\frac{3}{4}$ reduced into an Improper Fraction ſhall be $\frac{55}{4}$. For 13 multiplyed by 4 produceth 52, to which 3 added the Numerator is 55 to the Denominator 4.

This *Reduction* in many times neceſſary is the time of Operation, when mixt Numbers either happen, or are given.

§. 7. Integers ſet as Fractions.

When any Integer, together with *Fractions* are found among the given Numbers, or happen in the work that the terms may be kept diſtinct, it is convenient to ſet ſuch Integers in the form of a Fraction. This is eaſily and ſpeedily done, by placing 1 under the ſame for a Denominator. As 3. 4. *&c.* thus, $\frac{3}{1}$. $\frac{4}{1}$. *&c.*

Example.

§. 8. Integers reduced into a Denomination given.

Sometime it may be requiſite to reduce an *Integer* into a given Denomination, though if it be not of abſolute neceſſity, the former way to place an Unit under it be better for brevity in Operation; but when occaſion requires, it may be thus performed Multiply the Integer given to be reduced by the given Denominator, and this Product ſhall be the Numerator. As to bring 10 into fourths; multiply 10 by 4, the Product 40 ſhall be the Numerator to 4 the Denominator; and ſtand thus; $\frac{40}{4}$.

Example.

§. 9. Fractions reduced into a Denomination given.

Sometime it is deſired to reduce a *Fraction* to ſome given Denomination which is thus effected. Firſt let the *Fractions* given, be they of Integers, or of Fractions, Proper or Improper, be brought to conſiſt but of two termes only as a Simple Fraction doth, Then by the Numerator thereof multiply the deſirred Denominator, and divide this Product by the Denominator of the given Fraction, and the Quotient ſhall be the Numerator to the propounded Denominator; if any thing remain after Diviſion, it ſhall be a Fraction of that Fraction, as to bring $\frac{3}{5}$ into 60 parts, 3 multiplyed by 60, and the Product 180 divided by 5 gives 36 in the Quotient for the Numerator to 60, and nothing remain. But if $\frac{2}{3}$ were to be brought into thouſandth parts; beſides $\frac{666}{1000}$ gotten as before, there will remain $\frac{2}{3}$ which ſhall be a Fragment of a 1000 the given Denominator, *Viz.* $\frac{2}{3}$ of $\frac{1}{1000}$. Or by *Reduction* $\frac{1}{1500}$.

Example.

```
 60         3              1000      22(2 ( 666
  3       180 ( 36            2     ----- (-----   and 2/3 of  1  .
----     ----- (----       -----    2000  (1000        -     ----
180         5  ( 60        2000       3                3     1000
----                       -----
```

§. 10. Integers mixed with Fractions reduced to one Deminator.

In like manner, if *Integers*, *Proper Fractions* conjunct or divided or both, be mixed with *Improper Fractions*, or any other waies, and it be neceſſary to reduce them to one Denominator: The preceeding Rules obſerved, according to the nature of the given *Fractions*, their Reduction will be facil. *Example.* If $\frac{1}{7}$ a proper ſingle Fraction with $\frac{2}{3}$ of $\frac{1}{5}$ be given to be reduced with 2 Integers, and $\frac{7}{2}$ the Improper Fraction, to one Denomination. After $\frac{2}{3}$ of $\frac{1}{5}$ are reduced to one ſingle Fraction, and an Unit placed under the 2 Integers, the Common Denominator is found out as before to be 210, and the ſeveral Numerators 30. 28. 420. 735.

Example.

```
 30          28        420     735      7          6
----        ----      -----   -----    15   210(30    210(14    210(210    210(105
              2                       ----  ---       ---       ---        ---
1            2   1     2        7     105    7   1    15   2     1    2     2    7
- with       - of -  and -  and -       2       ---       ---       ----       ----
7            3   5     1        2     ----      30        28        420        735
            -------                   210       ---       ---       ----       ----
              15                      ----
└──────────────────┬──────────────┘
                  210
```

Proof of Reduction of Fractions.

Reductional Operation ended, Probation follows, that the ſeveral operations may be proved true.

Proper.

This Proof of one kind of Reduction is alternately by another; becauſe the *Fractions* are to keep the ſame equality in value, how ever differently they are expreſſed in greater or leſſer terms; Whence it is that Fractions duely reduced to one Denominator, may be returned to their leaſt terms again by Abbreviation, and Fractions abbreviated to their leaſt terms may be converted to their former greater terms; that by *Diviſion* with the great Common Meaſure, and this by *Multiplication* therewith. As in the laſt Example $\frac{1}{7}$ was reduced with others to $\frac{30}{210}$ which as *termini convertibiles*, may by 30 the great common meaſure or any other common *Diviſor* be reduced from the one to the other. For $\frac{1}{7}=\frac{30}{210}$.

$$30)\ \frac{3\cancel{0}(1}{2\cancel{1}\cancel{0}(7} \qquad \frac{3|0 \mid 1}{21|0 \mid 7}$$

Improper.

Likewiſe Reduction of Improper Fractions reciprocally prove one ſort the other: As *Multiplication*, and *Diviſion* in Integers mutually do, they being performed thereby, as above may be ſeen. For $3\frac{3}{4}$ is but the Quotient of $\frac{15}{4}$, and $\frac{15}{4}$ the Product of $3\times4+3$. Wherefore $3\frac{3}{4}=\frac{15}{4}$.

Integers ſet like Fractions by ſubjecting 1. As in the 7 *Sect.* above are ſoon reverted into their old form by taking away the ſubjected Unite. Integers alſo reduced to given Denominators are returned back by *Diviſion* of the Numerators by the Denominators, like Improper Fractions: And ſo $\frac{40}{4}$ ſhall return 10.

In like manner, *Fractions* reduced to a given Denominator may be abbreviated by Common *Diviſors* till the firſt terms be returned. In the Inſtance above $\frac{3}{5}$ made $\frac{36}{60}$ therefore by the great Common meaſure 12 ſhall $\frac{3}{5}$ be returned.

Proof of the Reduction of Fractions of Fractions, See pag. 53.

Of Contract Fractions, See Geodæticals.

But if a *Fraction* remained; as in turning $\frac{2}{5}$ into thouſandths, the Proof is ſomewhat more difficult than the work becauſe of the divided *Fraction*. Yet neither ſuch nor *Fractions* of *Fractions* are deſtitute of trial. For in thoſe if the Fragment be added to the Quotient, and reduced to their leaſt terms, the former given *Fraction* will be returned. And in *Fractions* of *Fractions* if the *Fraction* of one Denomination be divided by either of the parts multiplyed, the other will be returned in the Quotient, becauſe *Multiplication* and *Diviſion* prove each other. And beſides, in *Contract Fractions* another kind of proof may be had; by finding the value denominate. As in the next Book of *Geodæticks* may be ſeen. But foraſmuch as *Addition* and *Diviſion* of *Fractions* are not yet taught, and both theſe *Reductions* are performed by *Multiplication* or *Diviſion*, or both, as if they were *Integers*: It may ſatisfie as to the truth of theſe *Reductions*, if the *Multiplications*, and *Diviſions* be found right.

CHAP. III.

Addition of Fractions.

Proper Fractions are increaſed by Addition. and Diviſion diminiſhed by Subſtraction & Multiplication. Improper, how they increaſe, or decreaſe. Mixt, how they increaſe, or decreaſe.

HOW to increaſe or decreaſe the terms of a *Fraction* hath been ſeen in *Reduction*; it remains now to ſee how to increaſe or decreaſe their value.

Proper Fractions like to *Integers* have their value increaſed by *Addition* and diminiſhed by *Subſtraction*.

But contrary to *Integers* are leſſened by *Multiplication*, and increaſed by *Diviſion*.

Improper Fractions by *Addition* increaſe, and *Subſtraction* decreaſe their value according to *Integers*, and *Proper Fractions*. In *Multiplication* they are redundant by their value, and increaſe as *Integers*; defective by their *Fractions*, and decreaſe as *Fractions*. But in *Diviſion* are altogether like *Integers*.

Proper Fractions mixt with *Improper*, or *Integers*; are augmented or diminiſhed in *Addition* or *Subſtraction*, , as before. In *Multiplication* they follow the manner of

Improper Fractions; in *Diviſion* if the *Fraction* be the *Dividend* the Quotient is decreaſed, if *Diviſor* the contrary.

Addition of *Fractions,* and mixt Numbers contains Operation and Probation.

Addition of Fractions included under 4 Caſes.

In Operation are four Caſes. The two firſt general, the two laſt ſpecial.

1. Caſe. When the Denominators are alike, add the Numerators together, as *Integers,* and beneath the total ſubſcribe the Common Denominator. As $\frac{2}{7}$ and $\frac{4}{7}$ added together make $\frac{6}{7}$, And ſo $\frac{3}{20}$ and $\frac{13}{20}$ make $\frac{16}{20}$, and by Abbreviation $\frac{4}{5}$ by ſome ſet as at A., by others as at B.

1. Denominators alike. Examples.

$$A\ \frac{2}{7}\overset{6}{\underset{7}{\mathbf{X}}}\frac{4}{7}\qquad \overset{B}{\frac{2}{7}+\frac{4}{7}=\frac{6}{7}}\qquad A\ \frac{3}{20}\overset{16}{\underset{20}{\mathbf{X}}}\frac{13}{20}\qquad \overset{B}{\frac{3}{20}+\frac{13}{20}=\frac{16}{20}\text{ or }\frac{4}{5}}$$

2. Baſe. When the Denominators are unlike, firſt reduce them to one Denomination, then as before add the Numerators, and under the total ſubſcribe the Common Denominator. As $\frac{3}{4}$ and $\frac{4}{5}$ reduced make $\frac{15\text{ and }16}{20}$ then added make $\frac{31}{20}$ the Improper Fraction, or $1\frac{11}{20}$ ſet as at C. or D. So $\frac{2}{3}$ & $\frac{3}{4}$ & $\frac{2}{3}$ firſt reduced make $\frac{1\ \&\ 9\ \&\ 18}{12}$, then added are $2\frac{11}{12}$. As at E. or F.

2. Denominators unlike. Examples.

$$C\ \underset{20}{\overset{\overset{31}{\overline{15\qquad 16}}}{\frac{3}{5}\,\mathbf{X}\,\frac{4}{5}}}\quad \underset{20}{\underline{\overset{\overset{31}{\overline{15\qquad 16}}}{\overline{\frac{3}{4}+\frac{4}{5}}}}}\ D=\frac{31}{20}\text{ or }1\frac{11}{20}\qquad E\ \underset{12}{\underline{\overset{35}{\overline{\overset{8}{\overline{\tfrac{2}{3}}}\text{ and }\overset{9}{\overline{\tfrac{3}{4}}}\text{ and }\overset{18}{\overline{\tfrac{3}{2}}}}}}}\qquad \overset{F}{\frac{2}{3}+\frac{3}{4}+\frac{3}{2}=\frac{35}{12}\text{ or }2\frac{11}{12}}$$

3. Caſe. If Integers or mixt Numbers are to be added with Fractions, either add the Integers after the manner of Integers, and Fractions after the manner of Fractions ſeverally by themſelves; or elſe reduce the mixt numbers into Improper Fractions, and then proceed as above. As if $3\frac{1}{3}$ and $6\frac{1}{4}$ were to be added with $\frac{2}{5}$ either 3 and 6 the Integers added make 9, and the Fractions $\frac{1}{3}\ \frac{1}{4}\ \frac{2}{5}$ make $\frac{59}{60}$ or reducing the mixt numbers $3\frac{1}{3}$ and $6\frac{1}{4}$ into Improper Fractions, and then proceeding the ſame total is at laſt reſulting to $9\frac{59}{60}$ as at G. or H.

3. Integers or mixt Numbers with Fractions. Examples.

$$G\ \underset{9}{\underline{\left.\begin{matrix}3\\6\end{matrix}\right\}}}+\underset{60}{\underline{\overset{59}{\overline{\overset{20}{\overline{\tfrac{1}{3}}}+\overset{15}{\overline{\tfrac{1}{4}}}+\overset{24}{\overline{\tfrac{2}{5}}}}}}}\Bigg\}=9\frac{59}{60}\qquad H\ \underset{60}{\underline{\overset{599}{\overline{\overset{200}{\overline{\tfrac{10}{3}}}+\overset{375}{\overline{\tfrac{25}{4}}}+\overset{24}{\overline{\tfrac{2}{5}}}}}}}\Bigg\}=\frac{599}{60}\text{ or }9\frac{59}{60}.$$

4. Caſe. If Integers, mixt Numbers, Fractions, and Fractions of Fractions, are to be added together, either they may firſt be ſeverally added, and afterwards their ſeveral Totals into one total; or elſe reduced to one Denomination, and then added. *Example.* To add 2 Integers, $3\frac{1}{5}$ a mixt Number, $\frac{1}{3}$ a Fraction, and $\frac{1}{2}$ of $\frac{1}{4}$ a Fraction of a Fraction. As at I. or K.

4. Integers and mixt Numbers with Fractions. Examples.

$$I\ \underset{5}{\underline{\left.\begin{matrix}2\\3\end{matrix}\right\}}}+\underset{120}{\underbrace{\overset{79}{\overline{24\,.\quad 40\,.\quad 15}}\atop{\tfrac{1}{5}+\tfrac{1}{3}+\overset{1}{\overline{\underset{8}{\underline{\tfrac{1}{2}\text{ of }\tfrac{1}{4}}}}}}}}\qquad K\ \underset{120}{\underbrace{\overset{679}{\overline{240\quad \overset{384}{\underline{16}}\quad 40\quad \overset{15}{\overline{1}}}}\atop{\tfrac{2}{1}+3\tfrac{1}{5}+\tfrac{1}{3}+\underset{8}{\underline{\tfrac{1}{2}\text{ of }\tfrac{1}{4}}}}}}$$

Proof of Addition of Fractions.

Addition of Fractions like abſolute Integers is proved by *Subſtraction* and reſerved to the next Chapter. But *Addition* being learned: The Proof of *Reduction* in the 9. *Sect.* of the laſt *Chapter* may be remembred, and exemplified, where after the firſt *Diviſion* remained a Fraction of a Fraction, in reducing of $\frac{2}{3}$ to 1000ths there was $\frac{666}{1000}$ and $\frac{2}{3000}$ both which added by the ſecond caſe above will return $\frac{2}{3}$. As before noted, and prove the *Reduction* right.

$$\begin{array}{ccccc} & \overline{\quad 2|000 \quad} & & & \\ & 1998 & & 2 & \\ & \cdots\cdots & & \cdots\cdots & \\ 1000) & \dfrac{666}{1000} & + & \dfrac{2}{3000} & = \dfrac{2}{3} \\ & \cdots\cdots & & \cdots\cdots & \\ & \underbrace{3 \qquad\qquad\qquad 1}_{3|000} & & & \end{array}$$

CHAP. IV.

Subſtraction of Fractions.

To find which of 2 Fractions given is the greateſt.

Examples.

BEcauſe a greater Fraction cannot be taken out of a leſſer, it is convenient firſt to know how to find which of any two propounded Fractions is the Greater, that ſo the Propoſitions may not be impoſſible. Which to do, Multiply the Numerator of the one into the Denominator of the other, and the Product which is the greateſt ſhall demonſtrate that Fraction biggeſt, whoſe Numerator was one of the Factors. As to know which is the biggeſt Fraction of $\frac{3}{4}$ or $\frac{4}{5}$ multiplying 3 by 5, the Product is 15, and 4, by 4 yieldeth 16, which ſheweth $\frac{4}{5}$ to be greater than $\frac{3}{4}$ and import a bigger part of the Integer. So likewiſe $\frac{2}{3}$ is more than $\frac{3}{5}$, becauſe 10 exceeds 9, and in like manner Equal Fractions may be found, as $\frac{2}{5}$ and $\frac{4}{10}$.

$$\text{Minor}\ \overset{\overline{15}}{\frac{3}{4}}\mathbf{X}\overset{\overline{16}}{\frac{4}{5}}\ \text{Major.}\qquad \overset{\overline{10}}{\frac{2}{3}}\mathbf{X}\overset{\overline{9}}{\frac{3}{5}}\ \text{Minor}\qquad \text{both}\ \overset{\overline{20}}{\frac{2}{5}}\mathbf{X}\overset{\overline{20}}{\frac{4}{10}}\ \text{Equal.}$$

Subſtraction of Fractions included under 8. Caſes.

Subſtraction of Fractions, and mixt Numbers, includes Operation, and Probation. In Operation may happen Eight Caſes the two firſt general, the ſix laſt particular.

1. *Denominators alike.* *Examples.*

1. Caſe. When the Denominators are alike, then ſubſtract the leſſer Numerator from the greater, as in Integers, and place the remain over the Common Denominator. As $\frac{2}{5}$ from $\frac{3}{5}$ leaves $\frac{1}{5}$. So $\frac{3}{8}$ out of $\frac{7}{8}$ there reſteth $\frac{4}{8}$, and by Abbreviation $\frac{1}{2}$ ſet as at A by ſome, as at B by others.

$$\text{A}\ \ \frac{2}{5}\overset{1}{\underset{5}{\mathbf{X}}}\frac{3}{5}\qquad \overset{\text{B}}{\frac{3}{5}-\frac{2}{5}=\frac{1}{5}}\qquad\qquad \text{A}\ \ \frac{3}{8}\overset{4}{\underset{8}{\mathbf{X}}}\frac{7}{8}\qquad \overset{\text{B}}{\frac{7}{8}-\frac{3}{8}=\frac{4}{8}\ \text{or}\ \frac{1}{2}}$$

2. *Denominators unlike.* *Examples.*

2. Caſe. When the Denominators are unlike, firſt reduce them to one Denomination; then abate the leſſer Numerator out of the greater, and under the Remain ſubſcribe the Common Denominator. As in ſubſtracting $\frac{5}{7}$ from $\frac{7}{9}$ being reduced they are $\frac{45\ -\ 49}{63}$

then $\frac{45}{63}$ ſubſtracted from $\frac{49}{63}$ leaves $\frac{4}{63}$ ſet as at C. or D. So $\frac{13}{20}$ out of $\frac{4}{5}$ leaveth remaining $\frac{3}{20}$. As at E. or F.

Examples.

$$\text{C}\quad\begin{array}{ccc}\multicolumn{3}{c}{4}\\ \hline \underline{45} & & \underline{49}\\ \frac{5}{7} & \mathbf{X} & \frac{7}{9}\\ & 63 & \end{array}\qquad \overset{\text{D}}{\frac{7}{9}-\frac{5}{7}=\frac{4}{63}}$$

$$\text{E}\;5)\;\begin{array}{ccc}\multicolumn{3}{c}{3}\\ \hline 16 & & 13\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ \frac{4}{5} & & \frac{13}{20}\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ 4 & & 1\\ \multicolumn{3}{c}{\underbrace{\hphantom{4\qquad\qquad 1}}_{20}}\end{array}\qquad \overset{\text{F}}{\frac{4}{5}-\frac{13}{20}=\frac{3}{20}}$$

3. Caſe. If mixt Numbers conſiſting of Integers and Fractions are given, or happen in the work, then firſt reduce them into Improper Fractions; and afterwards proceed as before. So $1\frac{5}{12}$ taken from $1\frac{13}{16}$ ſhall leave $\frac{19}{48}$ as at G. and $3\frac{1}{4}$ from $7\frac{3}{4}$ leaves $4\frac{1}{2}$ as at H.

3. Mixt Numbers.

Examples.

$$\text{G}\;4)\;\begin{array}{ccccc} & \multicolumn{4}{c}{19}\\ \cline{2-4} & 87 & & 68 & \\ & \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot & \\ & 29 & & 17 & \\ & \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot & \\ 1 & \frac{13}{16} & -1 & \frac{5}{12} & =\frac{19}{48}\\ & \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot & \\ & 3 & & 4 & \\ & \multicolumn{3}{c}{\underbrace{\hphantom{3\qquad\qquad 4}}_{48}} & \end{array}\qquad \text{H}\;\begin{array}{ccc}\multicolumn{3}{c}{18}\\ \hline 31 & & 13\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ 7\frac{3}{4} & - & 3\frac{1}{4}\\ \multicolumn{3}{c}{\underbrace{\hphantom{7\frac{3}{4}\qquad 3\frac{1}{4}}}_{4}}\end{array} = \frac{18}{4}\text{ or }4\frac{1}{2}$$

4. Caſe. If many Fractions are to be ſubſtracted from one, or one from many, then firſt add them that are to be ſubſtracted together, if more than one, into one total, and likewiſe thoſe Subſtraction is to be made from, and afterwards ſubſtract the total of the Subtrahend from the other total, as before. As to take $\frac{7}{8}$ and $\frac{9}{16}$ from $\frac{3}{4}$ and $\frac{5}{6}$, firſt $\frac{7}{8}$ and $\frac{9}{16}$ reduced and added, their Total is $\frac{23}{16}$, then $\frac{3}{4}$ and $\frac{5}{6}$ reduced and added, their Total is $\frac{19}{12}$. Laſtly, $\frac{23}{16}$ ſubducted from $\frac{19}{12}$, there remaineth $\frac{7}{48}$, the Operations appear at I, K. L.

4. Many Fractions giver.

Examples.

$$\text{I}\;8)\;\begin{array}{ccc}\multicolumn{3}{c}{23}\\ \hline 14 & & 9\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ \frac{7}{8} & + & \frac{9}{16}\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ 2 & & 1\\ \multicolumn{3}{c}{\underbrace{\hphantom{2\qquad\qquad 1}}_{16}}\end{array}=\frac{23}{16}\qquad \text{K}\;2)\;\begin{array}{ccc}\multicolumn{3}{c}{19}\\ \hline 9 & & 10\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ \frac{3}{4} & + & \frac{5}{6}\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ 3 & & 2\\ \multicolumn{3}{c}{\underbrace{\hphantom{3\qquad\qquad 2}}_{12}}\end{array}=\frac{19}{12}\qquad \text{L}\;4)\;\begin{array}{ccc}\multicolumn{3}{c}{7}\\ \hline 76 & & 69\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ \frac{19}{12} & - & \frac{23}{16}\\ \cdot\cdot\;\cdot\cdot & & \cdot\cdot\;\cdot\cdot\\ 4 & & 3\\ \multicolumn{3}{c}{\underbrace{\hphantom{4\qquad\qquad 3}}_{48}}\end{array}=\frac{7}{48}$$

5. Caſe. If in two mixt given numbers the leſſer Fraction belong to the Subtrahend, then to work with the Integers ſeverally after the manner of Integers, and with the Fractions by themſelves after their manner, is the beſt way for brevity, becauſe it ſaves many times Reduction and great Multiplications. As to abate $19\frac{1}{5}$ from $48\frac{3}{5}$, the Integers 19 from 48 leave 29, and $\frac{1}{5}$ from $\frac{3}{5}$ leaves $\frac{2}{5}$. So is the whole Remain $29\frac{2}{5}$ as at M. Alſo $40\frac{1}{4}$ from $63\frac{1}{2}$ leave $23\frac{1}{4}$ for 40 withdrawn from 63 leaveth 23, and $\frac{1}{4}$ from $\frac{1}{2}$ leaveth $\frac{1}{4}$ as at N, which as many others ſometime happen are ſo commonly known, that upon ſight, without further work may be diſcerned, but if need be, Operation may be made for the Fraction as before; yet will the work be far ſhorter than if all the Numbers were turned into Improper Fractions.

5. Mixt Numbers and the Leſſer Fraction be in the Subtrahend.

Examples.

$$\begin{array}{rl} 48\frac{3}{5} & \text{Greater Homogeneal.} \\ \hline 19\frac{1}{5} & \text{Subtrahend.} \\ \hline \text{M}\quad 29\frac{2}{5} & \text{Remain.} \\ \hline \end{array}$$

$$\begin{array}{r} 63\frac{1}{2} \\ \hline 40\frac{1}{4} \\ \hline \text{N}\quad 23\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{c} 1 \\ \hline 2 \qquad 1 \\ \cdots\cdots \quad \cdots\cdots \\ 2)\ \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \\ \cdots\cdots \quad \cdots\cdots \\ 2 \qquad 1 \\ \hline 4 \end{array}$$

6. *Mixt Numbers and the Greater Fraction be in the Subtrahend. Example.*

6. Caſe. If the Fractions of the two given mixt Numbers be of one Denomination, and the Fraction belonging to the Subtrahend be greater than the other Fraction , then add the Denominator of the Number from which *Subſtraction* is to be made to the Numerator, and abate an Unit from his Integer, and afterward make *Subſtraction* as before. So if $3\frac{5}{6}$ were to be ſubducted from $9\frac{1}{6}$, becauſe $\frac{5}{6}$ is the major Fraction, 6 is added to 1, and 1 taken from 9. So is the number thus altered $8\frac{7}{6}$, from whence $3\frac{5}{6}$ ſubſtracted there is left $5\frac{2}{6}$ or $5\frac{1}{3}$. For $9\frac{1}{6} = 8\frac{7}{6} - 3\frac{5}{6} = 5\frac{2}{6}$, or $5\frac{1}{3}$

7. *One Fraction, and that in the Subtrahend. Examples.*

7. Caſe. If there be but one Proper Fraction in the two given Numbers, and that belong to the *Subtrahend*, then abate the Numerator from the Denominator, and ſubſcribe under the remain the Denominator, and accompt the Integers in the Subtrahend 1 more, or thoſe in the Greater number 1 leſs. As to take $\frac{2}{3}$ from 4 Integers; firſt 2 taken from 3 leaves 1 to be ſet over 3, then 1 from 4 leaveth the whole remain $3\frac{1}{3}$ As at O. So if $6\frac{3}{7}$ be abated from 9 Integers, 3 taken out of 7 leaves $\frac{4}{7}$, and 6 and 1 out of 9, or 6 out of 8, there remaineth $2\frac{4}{7}$, as at P.

$$\begin{array}{rl} 4 & \text{Integers.} \\ \hline 0\frac{2}{3} & \text{Subtrahend.} \\ \hline \text{O}\quad 3\frac{1}{3} & \text{Remain.} \\ \hline \end{array} \qquad \begin{array}{rl} 9 & \text{Integers.} \\ \hline 6\frac{3}{7} & \text{Subtrahend.} \\ \hline \text{P}\quad 2\frac{4}{7} & \text{Remain.} \\ \hline \end{array}$$

8. *Subtrahend an Integer. Example.*

8. Caſe. If of the two given Numbers, the Subtrahend be Integral, then keep the Fraction intire to the Remain, and make Subſtraction as in Integers. As to take 3 Integers from $5\frac{1}{4}$, take 3 from 5, and the remain ſhall be $2\frac{1}{4}$.

Proof of Subſtraction of Fractions.

Probation of *Subſtraction* and *Addition* , as in Integers, ſo in Fractions is reciprocal, the one by the other; wherefore if one of the Addends be ſubſtracted from the Total, the Additionary work will be proved by the Remain equal to the other Addends; ſo if the Subtrahend and Remain be added, the Subſtractionary work will be proved, for the Sum ſhall amount to the greater Number, from which *Subſtraction* was made, and as in *Subſtraction* of *Integers*, if the Remain be ſubducted from the Greater Number , this Remain ſhall be the Subtrahend *vice-verſâ*. Examples in the Anſwer of theſe 2 Queſtions.

Queſtions in Addition and Subſtraction of Fractions.

1. What Number is that from which if $\frac{1}{20}$ be ſubſtracted, the Remain will be $\frac{3}{4}$?

Anſwer, $\frac{4}{5}$, for ſo much is the Total of $\frac{1}{20}$ and $\frac{3}{4}$ added together.

2. What Number was that to which $\frac{1}{20}$ added the Total was $\frac{4}{5}$?

Anſwer, $\frac{3}{4}$, for ſuch is the Remain after $\frac{1}{20}$ is ſubſtracted from $\frac{4}{5}$.

Proof of Subſtraction.

$$\begin{array}{c} 16 \\ \hline \begin{array}{cc} 1 & 15 \\ \because\because & \because\because \end{array} \\ 4)\ \frac{1}{20} + \frac{3}{4} = \frac{16}{20} \text{ or } \frac{4}{5} \\ \begin{array}{cc} \because\because & \because\because \\ 1 & 5 \end{array} \\ \underbrace{\qquad\qquad}_{20} \end{array}$$

Proof of Addition.

$$\begin{array}{c} 15 \\ \hline \begin{array}{cc} 16 & 1 \\ \because\because & \because\because \end{array} \\ 5)\ \frac{4}{5} - \frac{1}{20} = \frac{15}{20} \text{ or } \frac{3}{4} \\ \begin{array}{cc} \because\because & \because\because \\ 4 & 1 \end{array} \\ \underbrace{\qquad\qquad}_{20.} \end{array}$$

CHAP. V.

Multiplication of Fractions.

BOTH the prime parts of *Fractionary Numeration* ended in *Addition* and *Subſtraction*, their Compound Compoſition and Diſſolution are to follow, in *Multiplication* and *Diviſion*.

Multiplication of *Proper Fractions*, may increaſe the termes but leſſeneth the value, as before obſerved, in this part, *Chap.* 3. and of neceſſity can do no leſs, becauſe being leſs than one, and making another Number ſo many times leſs alſo, muſt needs produce a number as many times leſs as the multiplying Fraction containeth parts in it, wherefore the Elements here may change the names they had in *Integers*, and *Multiplication of Fractions* may be called their Diſſolution, and *Diviſion* their Compoſition, they being increaſed thereby for the moſt part both in termes, and value. Nevertheleſs in *Improper Fractions* it is otherwiſe. As was noted in the ſame 3^d^ *Chap.* before.

The reaſon why the value is leſſened, though the Terms increaſe by Multiplication.

Multiplication of Fractions may be called Diſſolution.

Multiplication of *Fractions* and *mixt Numbers*, retains no difficulty in *Operation* and *Probation*.

Operation comprehendeth Six Caſes, The firſt three *Eſſential*, the laſt 3 *Accidental*.

Multiplication of Fractions included under 6. Caſes.

1.

Alternate Terms Incommenſurable.

Examples.

1. Caſe. If the two Numbers given be *Proper* or *Improper Fractions*, or the one *Proper*, and the other an *Improper*, and the Alternate Heterologal termes Incommenſurable, then multiply as in *Integers* the Homologal Terms, that is Numerator by Numerator, for the Numerator of the Product, and Denominator by Denominator for the Denominator of the Product. As to multiply $\frac{2}{3}$ by $\frac{4}{5}$ the Product is $\frac{8}{15}$ and $\frac{3}{2}$ by $\frac{5}{4}$ produce $\frac{15}{8}$ or $1\frac{7}{8}$. Alſo $\frac{2}{3}$ by $\frac{4}{3}$ will produce $\frac{8}{9}$, every of which have their various Collocations. As at A, or B. C, or D. E, or F.

$$\text{A}\ \begin{array}{c} 8 \\ \hline \frac{2}{3} \times \frac{4}{5} \\ \hline 15 \end{array} \qquad \overset{\text{B}}{\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}} \qquad \text{C}\ \begin{array}{c} 15 \\ \hline \frac{3}{2} \times \frac{5}{4} \\ \hline 8 \end{array} \qquad \overset{\text{D}}{\frac{3}{2} \times \frac{5}{4} = 1\frac{7}{8}} \qquad \text{E}\ \begin{array}{c} 8 \\ \hline \frac{2}{3} \times \frac{4}{3} \\ \hline 9 \end{array} \qquad \overset{\text{F}}{\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}}$$

2.

Alternate Terms Commenſurable.

Examples.

2. Caſe. If the Alternate Heterologal Terms, or either of them be Commenſurable firſt abbreviate them, and then multiply the new Homologal Terms as before; ſo ſhall the Product be alſo in its leaſt terms. As to multiply $\frac{4}{15}$ by $\frac{5}{6}$, here 4 and 6 may be abbreviated to 2 and 3. Alſo 5, and 15. to 1, and 3. then multiplying 2 by 1, the new Numerator is 2 and 3, by 3 the new Denominator is 9. So is $\frac{2}{9}$ the Product Incommenſurable. So $\frac{3}{2}$ by $\frac{6}{5}$ produce $\frac{9}{5}$. And $\frac{5}{7}$ by $\frac{9}{5}$ $\frac{9}{7}$, as at G. H. I.

$$G\quad \begin{matrix}2)\\5)\end{matrix}\;\overset{2}{\overline{\underbrace{\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{15}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}}_{9}}}=\frac{2}{9}\qquad H\quad 2)\;\overset{9}{\overline{\underbrace{\frac{3}{\underset{1}{\cancel{2}}}\times\frac{\overset{3}{\cancel{6}}}{5}}_{5}}}=1\frac{4}{5}\qquad I\quad 5)\;\overset{9}{\overline{\underbrace{\frac{\overset{1}{\cancel{5}}}{7}\times\frac{9}{\underset{1}{\cancel{5}}}}_{7}}}=1\frac{2}{7}$$

3. *Mixt Numbers both, or one a Fraction or Integer. Examples.*

3. Caſe. If Integers or mixt Numbers with a Fraction, or both mixt Numbers be given to be multiplyed, ſubject an Unit as in *Reduction* under the Integers, and reduce the mixt Numbers into Improper Fractions, and then proceed as above. So 3 by $2\frac{2}{7}$ will produce $\frac{48}{7}$, as at K. and 4 by $\frac{2}{3}$ produce $\frac{8}{3}$, as at L. Alſo $4\frac{1}{3}$ by $3\frac{1}{2}$ produce $\frac{91}{6}$, as at M.

$$K\quad \overset{48}{\overline{\underset{7}{\underline{\frac{3}{1}\times \overset{16}{\cancel{2\frac{2}{7}}}}}}}=6\frac{6}{7}\qquad L\quad \overset{8}{\overline{\underset{3}{\underline{\frac{4}{1}\times\frac{2}{3}}}}}=2\frac{2}{3}\qquad M\quad \overset{91}{\overline{\underset{6}{\underline{\overset{\frac{13}{1}}{4\frac{1}{3}}\times\overset{\frac{7}{1}}{3\frac{1}{2}}}}}}=15\frac{1}{6}$$

4. *Integer and Fraction or mixt Number.*

4. Caſe. When an Integer and a Single Fraction, or an Integer and a mixt Number whoſe Fraction is ſingle, the Denominators of the Fractions being digits, be the two given Numbers; then the Integer may be made *Multiplicand*, and the other Number the *Multiplier*, which if a mixt Number, multiply the *Multiplicand* by the Integers of the *Multiplier*, and for the Fraction of the mixt Number take $\frac{1}{2}$. $\frac{1}{3}$. $\frac{1}{4}$, &c. as the Fraction is, of the *Multiplicand*, and add to the former Product, and the total product ſhall be the deſired Number. But if the *Multiplier* be only a ſingle Fraction, then either take the half, third part, fourth part, &c. of the Integer according to the given Fraction, or elſe duplicate, triplicate, quadruplicate, &c. the Fraction according to the given Integer, for the Product deſired: For to multiply any Integer Fraction-wiſe by $\frac{1}{2}$ is Bipartition, by $\frac{1}{3}$ Tripartition, &c. And contrary-wiſe to multiply any Fraction by 2 is but to double the Numerator, or take half the Denominator; by 3 is to triple the Numerator, or take the third part of the Denominator, &c. And hence ſometimes in like ſort *Multiplication* by *Plural Fractions*, if their Denominators be Digits is made uſe of, as ſooner accompliſhed then by *Reduction* into *Improper Fractions*.

Examples. Examples of all theſe varieties, at N. O. P. in the one 48 is multiplyed by $9\frac{1}{3}$ in the other by $\frac{1}{5}$ and in the third by $12\frac{3}{4}$; ſo plainly they need no illuſtration.

N
48 Multiplicand.
$9\frac{1}{3}$ Multiplier
432 Product of the Integers.
16 Third Part added.
448 Total Product.

O
48 Multiplicand.
$\frac{1}{5}$ Multiplier.
$9\frac{3}{5}$ Product.

P
48 Multiplicand.
$12\frac{3}{4}$ Multiplier.
96 Product of 12.
48
36 $\frac{3}{4}$ of 48 added.
612 Total Product.

5. *Both mixt Numbers with Single Fractions, &c.*

5. Caſe. When two mixt Numbers whoſe Fractions are ſingle, and their Denominators Digits, are given to be multiplyed; after *Multiplication* by their Integers, and the the Fraction of the *Multiplier*, as laſt above-mentioned, then multiply the Numerator of the *Multiplicand Fraction* by the Integers in the *Multiplier*, and the Product divide by the Denominator of that Fraction; and add this Quotient to the numbers before ſet down with the Product of the two Fractions multiplyed, the Sum of all theſe ſhall be

Example. the total Product. As to multiply $48\frac{1}{2}$ by $12\frac{3}{4}$, firſt 48 by 12 produce 576, then $\frac{3}{4}$ of 48 is 36, then 12 halfs is 6 whole ones, or 12 multiplying 1, and dividing by 2 is all alike. Wherefore 6 added to the other, and $\frac{3}{8}$ the Product of $\frac{1}{2}$ into $\frac{3}{4}$ make together $618\frac{3}{8}$ for the Total Product; as at Q. examined by the Common way at R. and found alike.

Q

$$\begin{array}{rll} 48\frac{1}{2} & \text{Multiplicand.} & \\ 12\frac{3}{4} & \text{Multiplier.} & \\ \hline 96 & \left.\right\} \text{Product of } 12 & \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{added} \\ 48 & & \\ 36 & \frac{3}{4} \text{ of } 48. & \\ 6 & 12 \text{ halfs.} & \\ \frac{3}{8} & \frac{1}{2} \text{ of } \frac{3}{4}. & \\ \hline 618\frac{3}{8} & \text{Total Product.} & \\ \hline \end{array}$$

R

$$\begin{array}{r} 97 \\ 51 \\ \hline 97 \\ 485 \\ \hline 4947 \\ \hline \end{array} \qquad \begin{array}{c} 4947 \\ \hline \overset{97}{48\frac{1}{2}} \times \overset{51}{12\frac{3}{4}} \\ \hline 8 \end{array} = 618\frac{3}{8} \qquad \begin{array}{l} \cancel{1}\cancel{6}(3 \\ \cancel{4947}(618 \\ \hline 8 \end{array}$$

6. Alternate Terms equal. Examples. 6. Caſe. When the Heterologal Terms either way are equal, cancel them, and let the other Terms ſtand as they are for the Product. But when they are equal both wayes take 1, for the Product ſhall alwayes be an Unit or equal Fraction. As in multiplying $\frac{1}{3}$ by $\frac{3}{4}$, the Product ſhall be $\frac{1}{4}$ cancelling 3, and 3. And in multiplying $\frac{3}{4}$ by $\frac{4}{3}$, the Product ſhall be $\frac{1}{1}$, or 1.

S

$$\begin{array}{cccccc} & \multicolumn{3}{c}{1} & & \\ \hline & & & 1 & & \\ 3) & \dfrac{1}{3} & \times & \dfrac{3}{4} & = & \dfrac{1}{4} \\ & 1 & & & & \\ \hline & \multicolumn{3}{c}{4} & & \end{array}$$

T

$$\begin{array}{cccccc} & \multicolumn{3}{c}{1} & & \\ \hline & 1 & & 1 & & \\ 3) & \dfrac{3}{4} & \times & \dfrac{4}{3} & = & 1 \\ 4) & 1 & & 1 & & \\ \hline & \multicolumn{3}{c}{1} & & \end{array}$$

Proof of Multiplication of Fractions. Beſides the particular Proof of the different *Multiplications* one by another in General; The Proof of *Multiplication* as in *Integers* ſo in *Fractions*; is by *Diviſion*, and to be ſought in the next Chapter.

CHAP. VI.

Diviſion of Fractions.

Diviſion of Fractions when their Compoſition. D*Iviſion of Fractions* may be called their Compoſition as increaſing both their terms and value if the *Fractions* be proper, and Homologal terms Incommenſurable; but Improper and mixt Numbers partly increaſe and partly decreaſe their Quotients as before noted in *Addition of Fractions. Chap.* 3.

How wrought. *Fractions* and *mixt Numbers*, are divided with much facility, both *Operation* and *Probation* being performed by *Multiplication*; that after the manner of *Integers*; this of *Fractions*.

Diviſion of Fractions included under 6. Caſes. *Operation* concludeth with Six Caſes, the three firſt Eſſential, and three laſt Accidental.

1. Homologal Termes Incommenſurable. 1. Caſe. If the two given Numbers be Proper or Improper *Fractions*, or the one Proper, and the other Improper, and the Homologal terms Incommenſurable; then multiply as in *Integers* the Heterologal Terms, that is to ſay the Numerator of the Dividend by the Denominator of the Diviſor, for the Numerator of the Quotient, and the Denominator of the Dividend by the Numerator of the Diviſor, for the Denominator of the Quotient. *Example.* As to divide $\frac{1}{2}$ by $\frac{2}{3}$. and $\frac{3}{2}$ by $\frac{5}{3}$, and $\frac{1}{3}$ by $\frac{3}{2}$, the ſeveral Quotients are $\frac{3}{4}$, $\frac{9}{10}$, $\frac{2}{9}$ variouſly ſet as at A. or B. C. or D. E. or F.

$$\mathrm{A}\ \frac{2}{3}\overset{3}{\underset{4}{\mathbf{X}}}\frac{1}{2}\quad \mathrm{B}\ \frac{2}{3}\Big)\frac{1}{2}\Big(\frac{3}{4}\quad \mathrm{C}\ \frac{5}{3}\overset{9}{\underset{10}{\mathbf{X}}}\frac{3}{2}\quad \mathrm{D}\ \frac{5}{3}\Big)\frac{3}{2}\Big(\frac{9}{10}\quad \mathrm{E}\ \frac{3}{2}\overset{2}{\underset{9}{\mathbf{X}}}\frac{1}{3}\quad \mathrm{F}\ \frac{3}{2}\Big)\frac{1}{3}\Big(\frac{2}{9}$$

2. *Homologal Termes Commenſurable.*

2. Caſe. If the Homologal Terms or either of them be Commenſurable; firſt reduce them to their leaſt Terms, and then multiply the new Heterologal Terms as before, and ſo the Quotient ſhall be alſo kept in its leaſt Terms. *Examples.* As $\frac{2}{3}$ divided by $\frac{4}{5}$, becauſe 2 and 4 may be abbreviated to 1 and 2 the Product of 2 into 3 ſhall be the Denominator of the Quotient; and the Product of 1 into 5 the Numerator, as at G. So the Quotient of $\frac{2}{3}$ divided by $\frac{5}{6}$ ſhall be $\frac{4}{5}$, as at H. and the Quotient of $\frac{5}{8}$ by $\frac{15}{16}$ ſhall be $\frac{2}{3}$, As at I.

$$\mathrm{G}\ 2)\ \overset{2}{\frac{4}{5}}\Big)\overset{1}{\frac{2}{3}}\Big(\frac{5}{6}\qquad \mathrm{H}\ 3)\ \underset{2}{\frac{5}{6}}\Big)\underset{1}{\frac{2}{3}}\Big(\frac{4}{5}\qquad \mathrm{I}\ \begin{matrix}5)\\8)\end{matrix}\ \overset{3}{\underset{2}{\frac{15}{16}}}\Big)\overset{1}{\underset{1}{\frac{5}{8}}}\Big(\frac{2}{3}$$

3. *Mixt Numbers both or one a Fraction or Integer.*

3. Caſe. If *Integers*, or *mixt Numbers*; with *Fractions*, or two mixt Numbers, are given to be divided; Subſcribe an Unit as in *Reduction* under the Integers, and reduce the mixt Numbers into *Improper Fractions*, and then proceed as above. *Examples.* So 4 divided by $\frac{3}{4}$ ſhall give in the Quotient $\frac{16}{3}$ or $5\frac{1}{3}$ as at K. and 4 divided by $\frac{9}{2}$ as at L. ſhall give $\frac{8}{9}$, alſo $4\frac{1}{3}$ dividing $3\frac{1}{2}$ the Quotient ſhall be $\frac{21}{26}$, as at M.

$$\mathrm{K}\ \frac{3}{4}\Big)\frac{4}{1}\Big(\frac{16}{3}\ \text{or}\ 5\frac{1}{3}\qquad \mathrm{L}\ \frac{9}{2}\Big)\frac{4}{1}\Big(\frac{8}{9}\qquad \mathrm{M}\ 4\,\frac{\frac{13}{1}}{3}\Big)3\,\frac{\frac{7}{1}}{2}\Big(\frac{21}{26}$$

4. *Integer and Fraction or mixt Number.*

4. Caſe. When an *Integer* is given to divide an Improper *Fraction*, or a Single *Fraction*, whoſe Denominators are digits, or ſuch a ſingle *Fraction* given to divide an *Integer*, then if the *Integer* be Dividend, double, triple, quadruple, &c. the Dividend according to the Denominator of the *Fraction*. But if the *Integer* be Diviſor, and the Improper *Fraction* or other *Fraction* be Dividend, then either double, triple, quadruple, &c. the Denominator of the Dividend according to the given *Integer*, or elſe accordingly take the half, third part, quarter, &c. of the Numerator, which may beſt be done. *Examples.* As to divide 3 by $\frac{1}{2}$ the 3 doubled ſhall make the Quotient $\frac{6}{1}$. But $\frac{1}{2}$ divided by 3, becauſe the third part of 1 cannot be had, 2 ſhall be tripled, and make the Quotient $\frac{1}{6}$. And in dividing $\frac{3}{2}$ by 3 either 2 may be tripled, or the third part of the Numerator which is 1 taken, and this is beſt, becauſe the Quotient $\frac{1}{2}$ will be in its leaſt Termes, otherwiſe it would be $\frac{3}{6}$, and need Abbreviation. See the Common Operations at N. O. P.

$$\overset{\mathrm{N}}{\frac{1}{2}\Big)\frac{3}{1}\Big(\frac{6}{1}}\qquad\qquad \overset{\mathrm{O}}{\frac{3}{1}\Big)\frac{1}{2}\Big(\frac{1}{6}}\qquad\qquad \overset{\mathrm{P}}{3)\ \overset{1}{\frac{3}{1}}\Big)\overset{1}{\frac{3}{2}}\Big(\frac{1}{2}}$$

5. *Improper Fraction, and Integer,* &c.

5. Caſe. When an Improper *Fraction* is given to divide an *Integer* greater in quantity than the Numerator of the *Fraction*, *Diviſion* may be made after the manner of *Integers* thus; Divide the Dividend by the Numerator of the *Fraction*, and ſubſtract this Quotient from the Dividend, and laſtly divide the Remain by the *Integers* contained in the *Fraction*, and this laſt Quotient ſhall be the queſited Number. As to divide 1480 by $3\frac{1}{12}$, or $\frac{37}{12}$, after 1480 is divided by 37, and the Quotient 40 ſubſtracted, the Remain 1440 is to be divided by 3, and this Quotient 480 is the Number ſought. As at Q. agreeing with the Common way at R.

Example.

$$Q\;3\overset{\overline{37}}{\frac{1}{12}}\qquad \overset{1480}{\frac{1480}{37}}\Big(\underset{\overline{1440}}{40}\qquad \overset{2}{\frac{1440}{3}}\Big(480\qquad R\;37)\;3\,\overset{\overset{1}{\cdot\cdot\;\cdot\cdot}}{\frac{37}{\frac{1}{12}}}\Big)\overset{\overset{40}{\cdot\cdot\;\cdot\cdot}}{\frac{1480}{1}}\Big(\frac{480}{1}$$

6. *Homologal Terms equal.*

6. Caſe. When the Numerators are equal, cancel them, and place the Denominator of the Diviſor for the Numerator of the Quotient, over the Denominator of the Dividend. But when the Denominators are equal reject them, and *vice verſa* place the Numerator of the Diviſor under the Numerator of the Dividend for the Denominator of the Quotient. And if both the given *Fractions* be equal, for the Quotient take 1, for the new *Fraction* in ſuch caſe ſhall alwayes be equal; and generally may be obſerved, if the Dividend be the greater of the two propounded *Fractions*, the Numerator of the Quotient will be greater than the Denominator, but if the Diviſor be the *major Fraction* the contrary. As to divide $\frac{2}{5}$ by $\frac{2}{3}$, the Quotient will be $\frac{3}{5}$. So $\frac{1}{3}$ divided by $\frac{2}{3}$ gives $\frac{1}{2}$ in the Quotient, and $\frac{1}{3}$ by $\frac{1}{3}$ makes the Quotient an Unit. As at S. T. V.

Examples.

Numerators Equal.	Denominators equal.	Fractions equal.
$S\;\frac{2}{3}\Big)\frac{2}{5}\Big(\frac{3}{5}$	$T\;\frac{2}{3}\Big)\frac{1}{3}\Big(\frac{1}{2}$	$V\;\frac{1}{3}\Big)\frac{1}{3}\Big(\frac{1}{1}$

Proof of Diviſion of Fractions.

As in *Integers*, ſo in *Fractions*, *Diviſion* and *Multiplication* alternately prove each other, and though particularly one ſort of work may be tried by another ſort, yet regular and general *Probation* is by diſſolving the Numbers compounded, and compounding the Numbers diſſolved. Wherefore if the Product of any *Fraction* be divided by either of the *Factors*, the other *Factor* ſhall be found in the Quotient, and if the Quotient of any *Fraction* be multiplyed by the Diviſor, the Dividend ſhall be returned. *Example* in the Anſwer of theſe two following Queſtions.

Queſtions in Multiplication and Diviſion of Fractions.

1. What Number is that which being multiplyed by $\frac{1}{5}$ ſhall produce $\frac{3}{10}$?

Anſwer. $1\frac{1}{2}$, for dividing $\frac{3}{10}$ by $\frac{1}{5}$, the Quotient is $\frac{3}{2}$ or $1\frac{1}{2}$.

2. What Number being divided by $\frac{1}{5}$ will give in the Quotient $\frac{3}{2}$?

Anſwer. $\frac{3}{10}$, for ſuch is the Product of $\frac{1}{5}$ multiplyed by $\frac{3}{2}$.

Proof of Diviſion.	Proof of Multiplication.
$\begin{array}{c}3\\ \hline \frac{1}{5}\times\frac{3}{2}=\frac{3}{10}\\ \hline 10\end{array}$	$5)\;\underset{\cdot\cdot\,1\,\cdot\cdot}{\frac{1}{5}}\Big)\;\underset{\cdot\cdot\,2\,\cdot\cdot}{\frac{3}{10}}\;\Big(\frac{3}{2}$

Proof of the Reduction of Fractions of Fractions in p. 48.

Hence it is evident that *Reduction of Fractions of Fractions* may be proved; if the reduced *Fraction*, being but the Product of their *Multiplication*, be divided by any one of the Fragments, for then will the Quotient be the other Fragment, or the Sum of the other Fragments, if more than two were in the Compoſition. As in the former

Inſtances, 2. *Chap.* 4. *Sect.* of *Reduction*, $\frac{1}{4}$ of $\frac{2}{5}$ reduced became $\frac{1}{10}$, if therefore $\frac{1}{10}$ be divided by $\frac{1}{4}$, the Quotient will be $\frac{2}{5}$, or by $\frac{2}{5}$ it will be $\frac{1}{4}$, as at U. and W.

And the Reduction of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ unto $\frac{15}{64}$ will be found true; for if $\frac{15}{64}$ be divided by any of the three Fragments, the Sum of the other two will appear in the Quotient, as at X. Y. Z.

$$\overset{\text{U}}{2)\underset{2}{\frac{1}{4}}\Big)\underset{5}{\frac{1}{10}}\Big(\frac{2}{5}} \qquad \overset{\text{W}}{5)\underset{1}{\frac{2}{5}}\Big)\underset{2}{\frac{1}{10}}\Big(\frac{1}{4}} \qquad \overset{\text{X}}{2)\underset{1}{\frac{1}{2}}\Big)\underset{32}{\frac{15}{64}}\Big(\frac{15}{32}} = \begin{array}{c}15\\ \hline \frac{3}{4}\text{ of }\frac{5}{8}\\ \hline 32\end{array}$$

$$\overset{\text{Y}}{\begin{array}{l}3)\\4)\end{array}\underset{1}{\overset{1}{\frac{3}{4}}}\Big)\underset{16}{\overset{5}{\frac{15}{64}}}\Big(\frac{5}{16}} = \begin{array}{c}5\\ \hline \frac{1}{2}\text{ of }\frac{5}{8}\\ \hline 16\end{array} \qquad \overset{\text{Z}}{\begin{array}{l}5)\\8)\end{array}\underset{1}{\overset{1}{\frac{5}{8}}}\Big)\underset{8}{\overset{3}{\frac{15}{64}}}\Big(\frac{3}{8}} = \begin{array}{c}3\\ \hline \frac{1}{2}\text{ of }\frac{3}{4}\\ \hline 8\end{array}.$$

This kind of Proof was mentioned before in *Reduction*, but reſerved till after *Diviſion* was taught; as not probable before to be underſtood.

Partis ſecundæ, & Libri primi

FINIS.

THE

ARITHMETICK.

The Second BOOK,

CONCERNING

Numbers generally contract;

In Two PARTS.

WHEREIN

GEODÆTICALS } are { Declared.
FIGURALS } are { Demonſtrated.

AND THEIR

SIMPLE ELEMENTS.

CHAP. I.

Of GEODÆTICALS.

Sufficient hath been ſaid of *Abſolute Integers* and *Abſtract Fractions* in the former Book, it is neceſſary now to proceed to *Contract Numbers.*

Contract Numbers were before declared to be Numbers reſtrained by ſome annexed Denomination, or ſpecial Denominator, and *Book* 1. *Part* 1. *Chap.* 2. divided into two ſorts, *General* and *Special.*

After Abſtract Numbers the Contract treated of.

Contract of 2 ſorts.

General are ſuch whoſe Denominations are generally known in moſt Nations, and uſual not only in *Mathematical Sciences*, but alſo in *Mechanical Arts*, and of Common and Vulgar Frequentment in Traffique, Merchandizing, Buying, Selling, *&c.*

Numbers generally Contract are *Geodætical* or *Figural.*

General are Geodæticals and Figurals.

Geodæticals include all Numbers contracted by Vulgar Names or Denominations according to the common and uſual diſtinctions, diviſions, dimenſions, or legal inſtitutions, cuſtoms, or uſages of Nature or Nations, As Men, Women, Horſes, Sheep, Weights, Meaſures, *&c.*

Geodæticals what.

Whence the Word.

The word *Geodætical* comes from *Geodæſia*, and this from the two *Greek* words, γῆ and δαίω, ſignifying a Diviſion, Meaſure or Dimenſion of the Earth, and may ſeem too ſhort to denote what is intended thereby; yet becauſe all Denominations are truly but meaſures of quantities or gravities, accompted according to the Standart of earthly dimenſions inſtituted by the Law of Nature or Nations upon the Terreſtrial Globe; the Term *Geodætical* may fitly ſtand for the purpoſe here uſed till another more fit be found out.

Geodæticals diſtinguiſhed.

Numbers thus concluded may be diſtinguiſhed as in the Order following.

- *Geodæticals*
  - Greater
    - Integral.
    - Fracted.
  - Smaller.
    - Engliſh
      - Meaſure.
      - Weight.
    - Forrain
      - Antient
        - Hebrew.
        - Greek.
        - Latin, and Roman.
      - Modern, *Vide. pag.* 110. to 157.

Greater Geodæticals.

The greater *Geodæticals* are ſuch Denominate Numbers whoſe Names or Denominations admit not ſmaller artificial diviſions, or are the higheſt denominations of that kind or quality, and if they may be divided into ſmaller parts yet cannot be heightned into greater. Of the firſt ſort are Angels, Men, Women, Cities; Towns, Horſes, Sheep, Ships, and a multitude of ſuch other Nounes. Of the other ſort are Leagues, Years, Circles, Bales, Butts, Tons and ſuch like, which are capable of being divided into ſmaller parts or denominations.

Integral or Fracted.

Theſe greater *Geodæticks* are *Integral* or *Fracted.*

Integral what.

Integral, When ſuch denomination is annexed to any *Abſtract Integer*, as 10000 Angels, 36 Men, 800 Ships, *&c.*

Fracted what.

Fracted, When ſuch denomination is annexed to any *Arithmetical Fraction*, as $\frac{1}{2}$ of a Ship, $\frac{3}{4}$ of a Ton, *&c.*

Geodæticals how they come to be reckoned among Integers denominate.

And becauſe ſeveral of theſe greater denominations may be parted into lower and ſubtiller parts, every of which parts having Proper Names, and being thereby known, thoſe parts ſo named become reckoned for *Integers* denominate, and the greater denominations omitted as before, *Chap.* 2. *Of the Nature of Numbers* was declared. For $\frac{3}{4}$ of a Ton, becauſe a Ton is divided into 20 Hundreds ſhall be called 15 Hundreds, which is $\frac{3}{4}$ of 20, and paſs for a denominate *Integer*, and the great denomination [*Ton*] omitted: So becauſe a Pound of Money is divided into Shillings, and Shillings into Pence, parts of a Pound ſhall be accompted by Shillings, and parts of a Shilling by pence, *&c.*

Smaller Geodæticals.

The ſmaller *Geodæticks* ariſe from ſuch of the greater as admit of ſubdiviſions and leſſer parts under particular names and denominations which being abbreviated with the greater denomination may be reduced back again from a ſmaller denominate *Integer* to a fracted *Geodætick* of the greater Contraction; As 10 Shillings abbreviated with 20, the Shillings in one pound ſhall be reduced to $\frac{1}{2}$ *l.*

Engliſh or Foreign.

Geodæticals of the ſmaller ſort may be comprehended under two heads, *Engliſh* and *Foreign.*

Engliſh take in Meaſure and Weight.

Engliſh, Include both *Meaſure* and *Weight.*

Meaſure what.

Meaſure is the accompt of any Quantity or Magnitude taken either in length or breadth, or according to its ſolid thickneſs, or hollow capacity, by ſome Standart approved as a known Meaſure.

Weight what.

Weight is the accompt taken of the gravity of any Quantity or Magnitude according to ſome approved Standart or known Meaſure.

Both doubly conſidered.

Both theſe, *Viz. Weight* and *Meaſure* referring to ſome denominations are pure as conſidered *per ſe*, to others impure as conſidered *inter ſe*, but in Authours found ſomewhat confuſedly, as Money and Bread paſſing only as numbred or accompted, and yet notwithſtanding to have their due proportion of Weight, and Woollen Cloth generally paſſing by Meaſure in length, ought allſo to have due breadth and weight.

Engliſh Meaſures.

Under *Engliſh* Denominations that refer to meaſures of length may be contained not only things once meaſured by the Inch, Foot, Yard, Ell, Rod, *&c.* but allſo ſeveral things accompted by Number, or as commonly called Tale, as Fiſh, Skins, Paper,

per, *&c.* for it is ſufficiently evident that every Number in ſome ſenſe is but a lineary Meaſure or Multitude of Units ſuppoſed to ſtand in length one by another.

Long Meaſures therefore compriſe the Denominations uſed in *Aſtronomy* of *Time* and *Motion.* In *Coſmography* and *Geography* from the meaſure of Leagues down to the length of Barly-Corns. In Merchandiſe the denominations of many ſorts of Silks, Stuffs, Linnen Cloth, Fiſh, Skins, Paper, Parchment, *&c.*

Long.

Broad Meaſures imply things twice meaſured by a line, once in reſpect to length and again to breadth, and contain the Denominations uſed in *Geometry* from *Acres* downward, and in the ſquare meaſures of Glaſs, Plank, Pavements, *&c.* In Merchandize Woollen Cloth, which alſo is to have its proportion of Weight.

Broad.

Meaſures of Thickneſs or *Bodily Capacity* are either Solid, as the Sizes of Fuel, Meaſure of Timber, Stone, *&c.* or Concave which have hollow Capacity. Concave Meaſure is divided into dry and liquid; Dry take in the Meaſures of Corn, Salt, Coals, Lime, *&c.* Liquid, Ale, Beer, Honey, Butter, Fiſh, Oyl, Sope, Wine, *&c.*

Thick.

Among *Engliſh Weights* but two are conſiderable, *Viz. Troy* and *Avoirdupois.*

Engliſh Weights 2. conſiderable.

The Book Intituled *The Path-way to Knowledge* Tranſlated out of *Dutch* into *Engliſh* by *W. P.* 1596. makes mention of three other Weights.

Dutch Book mentions.

1. The *Pound Tower* uſual for Mintage, in the names of its diviſions like the *Pound Troy*, but in quantity leſs, for 16 *Pounds Tower* make but 15 *Pounds Troy.*

Pound Tower.

2. The *Pound Subtil* or *Suttle* ſo termed, for that in ſmall quantity it may be made ratable to repreſent any other greater Weight whatſoever, as *Four Penny weight Troy*, or leſs, to anſwer in due proportion unto the whole *Pound Troy* with all his parts, every part ſenſible and ſeverally to be handled. This Weight is private to Aſſay-Maſters and ſuch as can make trial of *Minerals*, and not known to many other, neither in ordinary Accompts is there any uſe thereof.

Pound Subtil.

3. The *Pound Foile*, is leſſer than the *Pound Troy* by $\frac{1}{5}$ part of the *Pound Troy*, and hath ſmall uſe, ſave only amongſt thoſe that make *Gold Foil* and *Wyre.*

Pound Foile.

But the *Troy* and *Avoirdupois* Weights are of common uſe, the firſt of force by Statute, the other by Cuſtom, yet confirmed by Statute. Why ſo called I could never ſatisfie my ſelf, but crave leave to conjecture that *Troy Weight* was the Weight anciently uſed in Towns and Cities, and *Verſtigan* tells us that *Troy Novant*, is as much as to ſay *New Town*; and *Avoirdupoies*, *Averdupois*, *Averdepois*, or *Avoirdupoys* however written ſeems to ſound like an Over-Weight, from the old *Norman* Language, and was either allowed upon droſſy and coarſe Goods, or probably ſold in the Countrey and the Over-Weight allowed in lieu of Carriage to a Market.

Troy and Avoirdupois Weights of common uſe in England. *Why ſo called probably.*

Troy weight quaſi *Town weight.*

Avoirdupois quaſi *to have Over weight of* 12. *on* 100.

By the *Troy-Weight* is accompted the Aſſize of Bread and ſome Liquid Meaſures conſtituted, alſo Gold and Silver Plate and Bullion, Rings, Jewels, Precious Stones, Musk, Civit, *&c.* are weighed thereby.

What weighed by Troy weight.

By the *Avoirdupois-Weight* are bought and ſold all baſe Mettals, as Copper, Tinn, Steel, Iron, Lead, and ſeveral other Merchandizes, as Allom, Rozin, Pitch, Tar, Wax, Tallow, Hemp, Flax, Beef, Cheeſe, Meal, Corants, Raiſins, Prunes, Figgs, Almonds, Spices, Druggs, Sugar, Tobacco, Wooll, *&c.* So Sope and Butter when ſold by Retail, and many other Retailed Commodities whereof any Garble, Refuſe or Waſte comes; But Butter, Sope, and ſome other things when ſold in groſs, paſs by the Barrel or Firkin where the Content or Meaſure of the Veſſel is alſo conſiderable

What weighed by Avoirdupois weight.

Coyne reſpecting both Weight and Number or Tale which is but long Meaſure as before noted may be placed under either of them, but though vulgarly paſſing by Tale, yet properly belongs to *Troy-weight* according to the diviſions whereof, for the moſt part, *Engliſh Money* is divided, and ought to be valued in Weight and Fineneſs.

Coyne reſpects Weight and Number.

Of all theſe in order, it is convenient ſomething be ſaid, and their Dimenſions and Diviſions ſeen, before Foreign Meaſures and Weights be ſpoken of.

Engliſh Meaſures.

The Meaſures of Time and Motion being properly *Aſtronomicals*, having certain Denominators and ſpecial Operations belonging to them are referred to the ſecond part of the next Book.

Meaſures of Time and Motion. See Book 3. Par. 2.

Coſmographical and *Geographical* Meaſures of length begin at a Barly-Corn, and increaſe upward to a League, in Latin called *Leuca*, which is the greateſt denomination principally in uſe with Mariners, for with Geographers commonly the greateſt deno-

Barley-Corn the beginning of Long Meaſures.

League from the Latin Leuca, *Mile from* Mille.

mination is a Mile, generally thought to be derived from the Latin *Mille*, ſignifying 1000, becauſe a Mile with the Latins contained juſt ſo many Paces, but an Engliſh Mile containeth 1056 Paces.

A Table of Engliſh Long Meaſures for Land.

In 1 League are 3 Miles. In 1 Mile 8 Furlongs, *&c.* As in the following Table.

League.	1	Mile.					
Miles. *. 1.	3	1	Furlong.				
Furlongs. *. 2.	24	8	1	Perch.			
Perches. *. 3.	960	320	40	1	Foot.		
Feet. *. 4.	15840	5280	660	16½	1	Inch.	
Inches.	190080	63360	7920	198	12	1	
Barley-Corns *. 5.	570240	190080	23760	594	36	3	Barley-Corns.

Length of an Engliſh Mile.

*. 1. That an *Engliſh* Mile ſhall contain ſo many Furlongs, Perches and Feet may be ſeen in the Statute. *Anno* 35. *Eliz. Cap.* 6. Intituled, *An Act for the Reſtraint of New Building, &c.* and long before the making of that Statute was a Mile of the ſame content.

Pulton *miſprinted in the number of Furlongs.*

*. 2 The Collection of the Statutes at large by *Ferdinando Pulton*, Printed 1640. *pag.* 1191. in the Statute of 35. *Eliz.* laſt mentioned deſerves Correction, who there makes 1 Mile but 5 Furlongs, whereas it ſhould be 8, as by another Printed Copy by me, and other good Authours may appear.

Perch, ſeveral names thereof. Leſs than a Rod. The Sorts.

*. 3. A Perch or Pearch hath other names, as Pole, Rod, and Lugge, but they are groſly miſtaken who call it a Rood, for a Rood is a quarter of an Acre, a far greater quantity than a Rod.

Perch for the Meaſure of Woodland. Church-land. Foreſts.

The Pole, Rod or Perch, for they are the moſt uſual names, is by the Statute aforeſaid to be but 16½ Feet, yet by the uſage of ſome Countreys the Pole doth vary, for in ſome places it is 18 Feet, in ſome 21 Feet, and in others 24. And Mr. *Osborne* (as witneſſeth *Dalton* in his *Countrey-Juſtice, Chap.* 65.) writeth that the meaſure of 18 Feet to the Perch is commonly called Woodland Meaſure, 21 Feet to the Pole is called Church Meaſure (*Scilt.* of Land which doth or did belong to ſome Church), and 24 Feet to the Pole is called, (and that rightly) Foreſt Meaſure.

Foot, the length thereof. Often uſed Plurally.

*. 4. That a Foot ſhall contain 12 Inches, and one Inch 3 Barley-Corns laid end to end, (or as ſome ſay 4 in thickneſs being dry and round and taken out of the midſt of the Ear) is evident by the Statute made *Anno* 33. *Edw.* 1. *An. Dom.* 1306. *De terris Menſurandis, & De compoſitione Ulnarum & Perticarum.* Foot is often uſed in the Plural as well as Singular; as 2 Foot, 3 Foot, *&c.* for 2 Feet, 3 Feet.

Barley-Corn no meaſure in it ſelf.

*. 5. A Barley-Corn is in it ſelf no Meaſure, but the leaſt thing in a Meaſure, whereof as it were Meaſure is made, and whereby it is rectified by the Ordinance, Intituled, *Compoſitio Ulnarum & Perticarum.* 3 Barley-Corns laid end to end make an Inch, 12 Inches a Foot, *&c.*

Meaſures for depths, &c.

Beſides theſe, ſome meaſure lengths whether depths, heights, or diſtances by Scores, Goads, Fathoms, Paces, Ells, and Yards, which are thus divided.

Yard, the length

In 1 Yard are 3 Feet, or 36 Inches.

Ell, the length.

In 1 Ell are 3 Feet 9 Inches, or 45 Inches.

Both theſe ordained by Statute, and of general uſe in *England*, for the meaſure of Silks, Stuffs, Cloth, Lace, Ribband, and ſeveral other Wares and Merchandize; commonly Woollen Cloth by the Yard, and Linnen Cloth, Silks, *&c.* ſome by the one, and ſome by the other, as is well known to Tradeſmen, but ſeldom uſed for Land. Both Yard and Ell divided equally into halfs, quarters, half quarters and Nails, or Neils. So 1 Yard or Ell containing 16 Nails.

Meaſures of Cloth how much in the

To ſome Cloth and Silks belong greater denominations than Ells and Yards, as Chefs, Bolts, Pieces, Hundreds, Roules, *&c.*

Chef.

In 1 Chef of { Fine Linnen, Silks—————10 } Ells.
{ Fuſtian—————14 }

Bolt.

In 1 Bolt of { Pole Davies————— } 28 Ells.
{ Spruce Elbing————— }

In

In 1 Piece of	Barbers Aprons, or Checks	10 Yards	
	Curle Sipers		
	Borratoes or Bombaſines, Buffins	15 Yards.	
	Moccadoes and Lile Grograins, Buſtians		
	Carrels, Dornix, Strip't or Tufted Canvas		*Piece.*
	Raſhes, Flanders Serges, *&c.*		
	Beaupurs, Frizado, Hounſcot Say	24 Yards.	
	Engliſh Tufted Canvas	30 Yards.	
	Ribband	36 Yards.	
In 1 Piece of	Cambrick	13 Ells.	
	Lawn		
	Lockram called Treagers	106 Ells.	
	Lockram called Broad Dowlaſs		
In 1 Hundred of	Canvas	120 Ells.	
	Sackcloth		
	Soulthwitch		*Hundred.*
	Tiking and Twill of Scotland		
In 1. Roul of	Minſters	1500 Ells.	*Rowle.*
	Ozenbrigs		

In 1 Pace or rather Paſs, from the Latin *Paſſus*, are 5 Feet, called a Pace *Geometrical*, to difference it from other Paces of greater or leſſer content, and is properly a Foreign Meaſure for Land.

Pace or Paſs how much.

In 1 Fathom are 6 Feet, uſed in meaſuring Depths, and Sounding at Sea.

Fathom.

In 1 Goad 1 ½ Yard, or 4 ½ Feet, a Meaſure in ſome places for Land and Cloth received by Cuſtom.

Goad.

In 1 Score 20 Yards or 60 Feet, a Meaſure alſo not ordained by Statute.

Score.

Some Denominations frequent in *Engliſh* Books, or Digits, Palms, Spans and Cubits great and ſmall, are no uſual *Engliſh* Meaſures, but came from other Nations hither, and retaining their Foreign Dimenſions and Diviſions, are to be ſought among the Foreign Accompts.

Digits, Palms, Spans, and Cubits, Foreign not Engliſh Meaſures.

The Meaſure called a Handful uſed in meaſuring the height of Horſes, by 27. *Hen.* 8. *Chap.* 6. is ordained to be 4 Inches.

Handful how much.

In Merchandize under Long Meaſures as before noted, fall ſundry Lengths, commonly called Number or Tale

Many ſmall Wares called Habberdaſhery, and ſome other Commodities are ſold by Dozens, Scores, Shocks, Hundreds, Thouſands, Laſts, Groſſes, *&c.*

Small Wares the Meaſure.

Every dozen contains 12, and of ſome things in ſome places 13. *Dozen.*

Every Score 20, and in ſome places 21 for 20. *Score.*

Every Shock 60. *Shock.*

The Hundred is more or leſs according as the Commodity is, and the Thouſand and Laſt greater or leſſer as the Hundred.

Hundred, 1000 and Laſt.

The Groſs is ſmall or great, the ſmall Groſs is 12 dozen, the great Groſs is 12 ſmall Groſs.

Groſs great and ſmall.

A Breviat of ſome Merchandizes accompted by theſe and ſuch like Denominations.

Alphabets. In 1 Set—24. *Set.*
Balkes
Barrel board } In 1 Hundred —120. *Hundred.*
Bome ſpars.
Bookes. In 1. Maund. 2. Fats. In 1 Fat 4 Bales. *Maund, Fat &c.*
Bowſtaves. In 1 Handred 120. *Hundred.*
Boxes called Sope-Boxes. In 1 Shock 60. *Shock.*
Bracelets or Necklaces of Glaſs. In 1 ſmall Groſs 12 Bundles or Dickers. *Groſs, Bundle.*
Bread, in ſeveral places. In 1 dozen 13 Penny Loaves. *Dozen.*
Buttons. In 1 great Groſs 12 ſmall Groſs. In 1 ſmall 12 ordinary dozen of Buttons. *Groſſes, &c.*
Canes. In 1 Shock 60. *Shock.*
Cantſpars
Capravens } In 1 Hundred 120. *Hundred.*
Clapholt or Clapboard. In 1 Great Hundred 12 Rings. In 1 Ring 2 ſmall Hundred. In 1 ſmall Hundred 120 Boards. So that 1 great Hundred contains 24 ſmall Hundred, or 2880 Boards. *Hundreds, &c.*
Deales. In 1 Hundred, 120.

Length of Fiſh ſold by Tale.

Fiſh, Of the greater ſort barrelled called Countable or Tale-Fiſh, ought to contain in length from the Bone in the Finn to the third joynt of the Tale, 26 Inches at the leaſt, by the 22. *Edw.* 4. *Cap.* 2.

Freſh Fiſh how ſold. Maund thereof. Mary.

Moſt freſh Fiſh is ſold by the Common Dozen or Score, Sometimes by the Maund, if the Fiſh be ſmall; the Maund or Moane, holdeth about a Gallon; and Six Maunds full ſet together, and an heap on them all at top is called a Mary.

Beſides theſe ſome quantities of Fiſh freſh and Salt ſold out of Cask have Common Denominations.

Cod, &c. the 100.

Cod, Alſo Haberdine Ling, and Newland Fiſh. In 1 Hundred 124. Yet the Book of Rates reckons but 120 to the Hundred.

Eeles the Bind, Strike.

Eeles, In 1 Bind, 10 Strikes, In 1 Strike 25 Eeles. So is 1 Bind 250 Eeles.

Herrings.

Herrings Freſh or Salted at Sea, called carn'd or corbed by the Laſt in ſome places thus divided.

A Table of the Laſt of Herrings.

Laſt.	1	Thouſand			
Thouſands.	10	1	Hundred.		
Hundreds.	100	10	1	Warpe.	
Warpes.	3200	320	32	1	
Herrings.	12800	1280	128	4	Herrings.

But by the Statute of Herrings made 31. of *Edw.* 3. *An. Dom.* 1357. *Cap.* 2. there is appointed but Six Score to the Hundred, ſo one Laſt ſhall contain but 12000. Yet at *Yarmouth* they ſell 33. Warpe to the Hundred.

White Herrings. Red Herrings.

Accordingly White Herrings that is Salted in Barrels is ſold by Retail, and Red Herrings that is dryed in the Smoak, in ſome places are accompted by 120 to the Hundred.

A Cade or Carde of Red Herrings ought to contain in 1 Cade 5 Hundred, that is 600 Herrings, and one Laſt 20 Cades.

Hundred. Shrimpes. In 1 Hundred, 120.

Turn. Soles. In 1 Turn 4.

Hundreds. Sprats. The Hundred as Herrings. The Cade of Red Sprats 5000. Yet by the Book of Rates Outwards but 1000.

Stock-fiſh. In 1 Hundred, 120.

Raſtal. 8 Title, Weights and Meaſures, ſaith the Hundred of hard Fiſh muſt be 8 Score.

Ropes. Garlick. In 1 Hundred 15 Ropes. In 1 Rope 15 Heads. So is 1 Hundred, 225 Heads.

Dickers. Gloves. In 1 Dicker, 10 Pair.

Horſhooes. In 1 Dicker, 10 Shooes.

Dozen. Iron. In 1 Dozen 6 pieces. Raſtal 8 Weights and Meaſures.

Bundle, Dicker. Knives. In 1 Bundle 6 Dickers. In 1 Dicker, 10 Knives.

Laſt, Dicker. Leather. In 1 Laſt 20 Dickers. In 1 Dicker 10 Hides. So the Laſt is 200 Hides.

Hundred. Oars. In 1 Hundred, 120.

Paper and Parchment how reckoned.

Paper.

Bale.	1	Ream.		
Reams.	10	1	Quire.	
Quires.	200	20	1	
Sheets.	5000	500	25	Sheets.

Parchment.

Rowle.	1	Dozen.	
Dozens.	5	1	
Skins.	60	12	Skins.

Bind of Skins. Skins. In 1 Bind, 33 Skins.

Timber of Skins.

Skins, or Furrs of

Ermines. Fitches. Grayes. Jennets. Letwis. Martrons. Minkes. Sables. In 1 Timber 40 Skins.

Budge. Cat. Coney. In 1 Hundred 5 Score Skins.

Skins of	Calves.	In 1 Dozen 12.	Skins.
	Goats.	In 1 Kippe, 50.	
	Kidds. Lambs. Sheep.	In 1 Hundred, 5 Score. Yet by the Book of Rates Outward 6 Score.	

Trayes of Wood. In 1 Shock, 60.

Dozen Kippe Hundred } *of Skins.*

Shock.

Meaſures long and broad reach the Denomination uſed about Land Meaſuring, called Acres. It is ſufficiently manifeſt by the Statute *An.* 33. *Edw.* 1. aforeſaid, that an Acre of Land is to contain in length 10 Perches, and in breadth 16. So if the breadth be 1 Perch, the length ſhall be 160; if the breadth be 2, the length ſhall be 80; and ſo proportionably, that is to ſay alwaies 160 Square Perches. The other proportions of the length and breadth of one Acre of Land mentioned in that Statute were obſerved by Record in his Book of *Arithmetick* long ſince, not to be exact. The Acre is thus divided.

Broad or Square Meaſures for Land in England.

Acre how much.

Statute thereof not exact.

Acre.	1	Rood.		
Roods. *.	4	1	Daies work.	
Daies Works.	40	10	1	
Square Perches.	160	40	4	Square Perches.

A Table of Engliſh *Square Meaſures.*

*. A Rood is ſometime called a Farthendele, and ſometime a Yardland, but as to the latter very corruptly, for a Yardland containeth much more than an Acre.

Rood how called Yardland, more than an Acre.

Several Denominations about Land Meaſure beſides a Yardland are found in the Law Books, as Hides, Plowlands, Carves, Carucates, and Oxeganges, but are grown ſo obſolete, That the Lawyers themſelves can hardly agree about the Content thereof; the firſt 4 ſeem to be all one, and are reckoned to contain by ſome 85, by moſt, 100 Acres, yet *Norden* in his *Surveyors Dialogue*, and others, make a difference between a Hide of Land and the other three, and ſay that a Hide of Land containeth 4 Plowlands, and every Plowland, Carve or Carucate, which are all one, 4 Yardlands, and every Yardland 30 Acres. So ſhall one Plowland contain 120 Acres, and one Hide of Land 480 Acres, *Cambden* and *Hollingſhead* will have one Hide of Land to contain 100 Acres, and others ſay, 8 Oxeganges make a Hide or Plowland, and every Oxegange containeth 15 Acres. *Dalton* in his *Countrey-Juſtice* ſaith, that the Common Account in the Eaſt part of *Cambridge-ſhire* of a Yardland is but 24 Acres. And Sir *Edward Coke* in the firſt part of his Inſtitutes under the Title *Eſcuage* (perhaps prudently foreſeeing theſe differences irreconcileable) is of Opinion that a Plowland is of no certain Content, but is rather to be reckoned by the Value than Content, and that more in one place and leſſer in another ſhall be a Plowland according to the quantity that one Plow may till in a Year. But all agree that a Yardland called in Latin *Quatrona terræ* is much more than an Acre, and therefore ought not to be uſed for a Rood, which is but a quarter of an Acre.

Difference in the Accompts of Hides, Plowlands. &c.

Sir Edward Coke*'s Opinion of a Plowland.*

Yardland how called.

Among long and broad Meaſures fall in next, Glaſs, Plank, Pavements, of which Glaſs-Windows are commonly meaſured by the Foot Rule of 12 Inches to the Foot. So one Square Foot ſhall contain 12 Inches in length, and as many in breadth, that is 144 Square Inches. Unwrought-Glaſs ſometime ſold by Weight. *See among Weights.*

Glaſs, &c. *how meaſured.*

Pavements are ſometimes meaſured by the Yard, commonly by the Foot-Rule, and a parcel of Pavement or Tiling of 10 Foot long and 10 Foot broad is ordinarily in theſe parts called One Square. And equivalent to ſuch a Square ſhall be the laying of 100 Gutter Tiles, or Redge-Tiles, though of the latter ſome count but 50.

Pavements, Roofs, how meaſured.

The Square thereof.

Plank or Board is commonly accounted among Square Meaſures, yet more properly belongeth to Timber Meaſure, for that beſides the length and breadth reſpect is had to the thickneſs of the Plank, whether it be Inch, Inch and half, Two Inch, Three Inch Plank or more, all commonly meaſured by the Foot of 12 Inches. The Accompt of the Load of each is referred to Timber Meaſure.

Plank or Board how meaſured.

All Square Meaſures, whether of Acres, Glaſs, Pavements, Plank, *&c.* do properly belong to Figural Numbers treated of in the next part of this Book.

Square Meaſures belong to Figurals.

But Woollen Cloth though as before noted, muſt have length, breadth and weight, yet being commonly accompted only by length, and accordingly by Retail ſold by the Yard keeps place among *Geodæticals*, and as to the making, and Wholeſale thereof duely placed here.

Woollen Cloth.

Statute for the due make thereof.

The due making of Woollen Cloths is declared in a Statute made *An.* 4. *Jac. Chap.* 2. being an Epitome of all former Acts to that purpoſe, in which may be ſeen their Weight, Breadth and Length, Workmens Orders, with the Viewing, Searching and Forfeitures or Abatements of and for the ſame.

One Sack of Wooll (the Content whereof is found among Weights) is accompted to make 4 Standard Clothes of clean Wooll called ſorting Clothes, weighing 60 ℔ the Cloth, and being 24 yards long and 6 quarters ½ broad or thereabouts within the remedy or allowance of 2 ℔ weight upon a Cloth.

Retailed by the Yard and Inch.

The length is preſcribed by the Statute to be meaſured wet within the liſt of the Cloth, by the Yard and Inch, inſtead of which Inch is the allowance of the Thumb in Retailing uſual.

Cloth to be weighed.

In Weighing, which is to be by the *Avoirdupois* Weight is to be obſerved that the Clothes be well ſcoured, thickned, milled, and fully dryed, the Weight of a Cloth ſeems more to be regarded than the meaſure, becauſe the Weight containeth ſubſtance which may be abuſed by ſtretching into Meaſure.

Aſſize of Cloth by Statute.

The Aſſizes of Woollen Cloths by the Statute An. 4. Jacobi. Cap. 2.

	Length. Yards.	Breadth. Quarters.	Weight. Pounds.
Long Broad Cloth, and Clothes of Died Woolls and mingled Colours of *Kent*, *Yorkſhire* and *Reading*, between	30 & 34	$6\frac{1}{2}$	86
Whites of *Worceſter*, *Coventry* and *Hereford*	30 33	7	78
Plunkets, Azures, Blews, and long Whites of *Suffolk*, *Norfolk*, and *Eſſex*.	29 32	$6\frac{1}{2}$	80
Sorting Clothes, *Suffolk*, *Norfolk*, and *Eſſex*	23 26	6	64
Fine ſhort *Suffolks*	23 26	$6\frac{1}{2}$	64
Handiwarpes	29 32	7	76
Plunkets, Azures, *&c.* of *Wilts*, *Somerſet*, &c.	26 28	$6\frac{1}{2}$	68
Yorkſhire ſhort Clothes	23 25	$6\frac{1}{2}$	66
Broad liſted Whites and Reds of *Wilts*, *Gloceſter*, *Oxford* and Eaſt part of *Somerſet*	26 28	$6\frac{1}{2}$	64
Narrow liſted White and Red	26 28	$6\frac{1}{2}$	61 White 60 Red
Fine plain liſted Clothes of the *Shires* laſt mentioned	29 32	$6\frac{1}{2}$	72
Tauntons, *Bridgwaters*, and *Dunſters*	12 13	7	30
Short Cloths of Died Wooll, *&c.*	23 25	$6\frac{1}{2}$	66
Narrow of *Somerſet*, &c.	24 25	4	30
Devon Kerſies called Dozens	12 13	4	13
Check Kerſies, Straights and Plain Graies	17 18	4	24
Ordinary Penniſtone or Foreſt Whites	12 13	$5\frac{1}{2}$	28
Sorting Penniſtones	13 14	$6\frac{1}{2}$	35
Whaſhers of *Lancaſhire* and others	17 18		17
Cogware, Kendal, and Karptmeales at pleaſure	20 at leaſt.		

Cottons by the Goad.

Some accompt *Mancheſter*, *Cheſhire*, and *Welch* Cottons by the Goad, allotting the *Lancaſhire* Cotton to be in length between 20 and 21 Goads, in breadth ¾ within the liſt, and in weight 21 pound. The *Mancheſter* and *Cheſhire* 22 Goads in length, in breadth as the *Lancaſhire*, and in weight 30 Pound.

Meaſures Bodily for Bodies thick, ſolid, or concave.

The next ſort of Meaſures are bodily, whither Solid or Concave, and properly belong to Figural Numbers handled in the next part, yet becauſe many of them are not Rooted Numbers, and in Common Commerce reckoned by Number or Tale rather than in reſpect to their Capacity, they may as to their Denominations ſtand among *Geodæticals*.

Among Solid Meaſures are the Aſſizes of Fuel, Plank, as before noted, Timber, Stone, Laths, Tiles, *&c.*

Fuel the Meaſure.

Fuel contains Billets, Cordwood, Faggots, Talwood, and Coals. But Coals are ſold by Buſhel, and therefore placed among Concave Meaſures.

Billets the Aſſize.

Every Billet by the 7. *Edw.* 6. *Chap.* 7. muſt be 3 Feet 4 Inches in length, and is accompted for 1 or more, according to the bigneſs thereof. For if it be but 7 ½ Inches about, it ſhall be but a ſingle Billet. If it were 10 Inches about it was called a Caſt, and was marked with 1 notch within 4 Inches of the end, and to paſs for 2 Billets. If the two Billets were 14 Inches about it was called a Caſt of two, and marked with 2 notches, within 6 Inches of the middle, and to paſs in Tale for 4 Billets.

But by the 43. *Eliz. Chap.* 14. the Aſſize was altered as to the Caſts, Cleft Billets, aſſized, and proviſion made that no ſingle Billet ſhould be cleft: Thus,

	Round.	Half Round.	Quarter cleft.	Length.
A Single Billet.	7½			40 Inches.
A Caſt marked 1.	11	13	12½	
A Caſt of 2 marked 2.	16	19	18½	

Billets marked with 3, 5, or 7 notches are to paſs for ſo many ſingle Billets, and to be proportional. Billets are commonly ſold by the Hundred, 5 Score to the Hundred.

Cordwood the Load or Cord.

Cordwood is Wood of the bigger ſort of Firewood, meaſured by a Cord or Line, of which there are two Meaſures. That called the Fourteen Foot Cord is to be 14 Feet in length, 3 Feet in breadth, and 3 Feet in height.

The other Cord is to be 8 Feet long, 4 Feet high, and in breadth 4 Feet, yet in ſome places 3 of the 4 Feet high is 4 Foot Wood, and the other Foot but 3 Foot Wood.

Faggots the Load.

Faggots called Two bands by the laſt mentioned Statutes are to be 3 Feet in length, and the band 24 Inches about, beſides the knot. Of ſuch Faggots 50 go to one Load.

Faggots of ſmaller Wood called Bavin and Spray are ſold by the Hundred, and 100 accompted for a Load.

Talſhide the Aſſize.

The Aſſize of Round Talſhide Ordained by 7. *Edw. 6. Chap.* 7. is confirmed by 43. *Eliz. Chap.* 14. and Talſhides half round and quarter cleft aſſized thus.

Number of Notches or Marks.	Round.	Half Round.	Quarter cleft.
1	16	19	18½
2	23	27	26
3	28	33	32
4	33	39	38
5	38	44	43

Every Talſhide named (1–5) muſt be (as above) Inches about within a Foot of the middle.

Board or Plank

Plank or Board cuſtomarily is accompted by the Load according to the thickneſs of the Plank, and to be meaſured by the Foot Rule; the Load thus reckoned.

The Load how much.

	Feet long.	Feet broad.	Inches thick.	
Plank or Board	600	1	1	make 1 Load.
	400	1	1½	
	300	1	2	
	240	1	2½	
	200	1	3	
	171 3/7	1	3½	
	150	1	4	

If the breadth be more than 1 Foot, the length muſt be leſs proportionably.

Timber the Load the Ton.

Timber well hewn and perfectly ſquared, *viz.* 1 Foot broad, and 1 Foot thick 40 Feet long make 1 Ton or Tun. And 50 ſuch Feet 1 Load. If the breadth or thickneſs be more, the length muſt be leſs, if leſs, the length muſt be more.

Stone how meaſured.

Stone ſometimes is meaſured by the Foot after the manner of Timber, and ſometime reckoned by the Ton Weight.

Lath, Tann, and Tile, becauſe in them reſpect is had to their length, breadth and thickneſs may fitly be placed here.

Laths the Load and Bundle, how much.

Lathes are ſold in groſs by the Load bound up in Bundles, every Load 30 Bundles. In Retail by the Bundle, every Bundle 100 Laths. Every Lath ought to be 5 Feet long, 2 Inches broad, and ½ Inch thick, if the Lath be but 4 Feet long, then there muſt be 6 Score to the Hundred.

Tann, the Load how much.

Tann, 1 Load muſt be 60 yards long, 1 yard high, 3 Rinds thick ſet up on each ſide of a Pole laid along to reſt againſt, and 2 Rinds at top. Yet 45 yards thus ſet is a good Waggon Load.

Tiles the Aſſize.

Tiles are of 3 ſorts, 1ſt Plain Tile or Thack Tile. 2ly Gutter Tile, or Corner-Tile. 3dly Roof Tile, Creaſtile or Ridge Tile, commonly ſold by the Hundred in ſome places of 6 Score, and ſome 5 Score to the Hundred, all aſſized by the Statute *An.* 17. *Ed.* 4. *Chap.* 4. thus.

	Inches long.	Thick.	Broad.
Plain Tile	10½	½ and ½ of ¼	6¼
Gutter Tile	10½	with convenient Thickneſs and Breadth.	
Ridge Tile	13	½ and ½ of ¼	with convenient Breadth.

Concave Meaſures.

Concave Meaſures whether Dry or Liquid had their Original from Weight, and therefore ſome place them among Weights, but being formally Bodies, and common uſe reſpecting rather their Content or Quantity than their Weight, they may fitly ſtand here.

Dry.

Dry Meaſures are thoſe by which Dry Goods are meaſured, as Corn, Salt, Coals, Lime, *&c.* And may be diſtinguiſhed into *Wincheſter* Meaſure and Water Meaſure. *Wincheſter* Meaſure is the Standart Meaſure. Water Meaſure greater.

Corn Meaſure.

Corn is meaſured by Troy Weight, and weighed by *Averdupois*, that is the Meaſures for Corn are according to Troy Weight, but Corn ſold by Weight ſhall be weighed by *Averdupois* Weights.

The Aſſize thereof. Wincheſter *Meaſure.* *The Weight.*

By 51 *Hen.* 3. *An.* 1266. 31 *Edw.* 1. *&* 12. *Hen.* 7. *Chap.* 5. The Content of a Gallon of Wheat is to be 8 Pounds or Pints Troy, 8 Gallons 1 Buſhel *London* Meaſure, and 8 Buſhel 1 Quarter. This is called *Wincheſter* Meaſure.

By the Book of Aſſize of Bread ſet forth by *John Powel*, the Buſhel is to contain 56 Pounds or Pints of *Averdupois* Weight, and ſo proportionably for half Buſhels, Pecks, *&c.*

Buſhel above 8 Gallon in ſome places, and by heap.

Cuſtom hath begotten in ſome places greater Buſhels, then 8 Gallons, as 9, 10, *&c.* Alſo greater quantities than the Quarter have their denominations, yet all are to be reckoned according to the meaſure of the Buſhel and Gallon ordained by the Statutes.

Uſage in ſome places hath continued Meaſure by heap, although ſome Statutes order it by Strike, and allowance in ſome places is 21 for 20.

The Diviſions and Subdiviſions of Corn Meaſure called *Wincheſter* Meaſure, may be inſpected in the Table following.

A Table of the Laſt of Corn Meaſure.

Laſt.	1	Load.										
Loads.	2	1	Quarter.									
Quarters or Seams.	10	5	1	Coomb.								
Cornoocks or Coombs.	20	10	2	1	Strike.							
Striks or half Coombs.	40	20	4	2	1	Buſhel.						
Buſhels.	80	40	8	4	2	1	Tovit.					
Tovits or half Buſhels.	160	80	16	8	4	2	1	Peck.				
Pecks.	320	160	32	16	8	4	2	1	Gallon.			
Gallons.	640	320	64	32	16	8	4	2	1	Pottle.		
Pottles.	1280	640	128	64	32	16	8	4	2	1	Quart.	
Quarts.	2560	1280	256	128	64	32	16	8	4	2	1	
Pints or Pounds Troy.	5120	2560	512	256	128	64	32	16	8	4	2	Pints.

Apples, &c. the Laſt, Barrel.

Apples, Nuts, Oatmeal, In 1 Laſt 12 Barrels. 1 Barrel 3 Buſhels.

Charcoal the Load, Sack.

Charcoal is ſold ſometimes by the Load. In 1 Load 80 Buſhels. Sometime by the Sack. In 1 Sack 4 Buſhels, 7. *Edw.* 6. *Cap.* 7. and ſometime they reckon 88 Buſhels, or 22 Sacks *Wincheſter* Meaſure to a Load.

Meal ſold by Meaſure.

Meal in ſome places ſold by Meaſure. In 1 Buſhel 12 Gallons ſtriked.

Lime by Water Meaſure, how much the Buſhel

Lime, Salt, Seacoal are meaſured by Water Meaſure, the Buſhel whereof by the 11. *Hen.* 7. *Cap.* 4. is to contain 10 Gallons of *Wincheſter* Meaſure, nevertheleſs in ſome places is 12. 14. *&c.* Gallons.

Salt, the Hundred Wey.

Salt is reckoned by the Hundred and Wey. In 1 Hundred of Salt 10½ Weyes, in 1 Wey 40 Buſhels. So 1 Hundred contains 420 Buſhels, Water-Meaſure.

Seacoale the Laſt Chaulder.

Seacoale is accompted by the Laſt and Chaulder. In 1 Laſt of Seacoal *Newcaſtle* Meaſure 7½ Chaulders. The Chaulder generally 32 Buſhells, but differs at ſeveral places according to the quantity of the Buſhel. In 1 Chaulder Rye-Meaſure 32 Buſhels, in 1 Buſhel 12 *Wincheſter* Gallons by heap, and the Cop on the Buſhel was equal to 4 Gallons more: Whereupon of late, upon new making the Buſhel, order was given to make a Buſhel that ſhould hold the old Buſhel with the Cop, and this new Buſhel is ſtill in uſe filled up to the brim, but not by heap.

Chaulder at Rye.

Concave Meaſures Liquid.

Liquid Meaſures are Cask that contain Moiſt or Liquid Commodities; As Ale, Beer, Butter, Fiſh, Honey, Oyl, Soap, Wine, *&c.*

In Meaſures of this Nature two things are to be heeded, *viz.* the Content of the Veſſel which is alwaies by the *Wincheſter* Meaſure, and the weight of the empty Cask when the Commodities are ſold by weight.

A Table of Ale Meaſures.

Ale 23. *Hen.* 8 *Cap.* 4.

Laſt.	1	Barrel.						
Barrels.	12	1	Kilderkin.					
Kilderkins.	24	2	1	Firkin.				
Firkins.	48	4	2	1	Gallon.			
Gallons.	384	32	16	8	1	Pottle.		
Pottles.	768	64	32	16	2	1	Quart.	
Quarts.	1536	128	64	32	4	2	1	
Pints.	3072	256	128	64	8	4	2	Pints.

A Table of Beer Meaſures.

Beer 23. *Hen.* 8. *Cap.* 4.

Laſt.	1	Ton										
Tons.	2	1	Pipe.									
Pipes.	4	2	1	Hogſhead.								
Hogſheads.	8	4	2	1	Bunn.							
Barrels or Bunns.	12	6	3	1½	1	Barrel.						
Herring Barrels.	16	8	4	2	1⅓	1	Kilderkin.					
Kilderkins.	24	12	6	3	2	1½	1	Firkin.				
Firkins.	48	24	12	6	4	3	2	1	Gallon.			
Gallons.	432	216	108	54	36	27	18	9	1	Pottle.		
Pottles.	864	432	216	108	72	54	36	18	2	1	Quart.	
Quarts.	1728	864	432	216	144	108	72	36	4	2	1	
Pints.	3456	1728	864	432	288	216	144	72	8	4	2	Pints.

Butter and Soap.

Butter and Soap are of like meaſure, and the Content of the Barrel, Kilderkin, and Firkin of Sope is ordained by 23. *Hen.* 8. *Cap.* 4. to be as the Ale Meaſure, that is the Barrel 32 Gallons, *&c.* And the Weight of the empty Cask as followeth.

Weight of the Cask.

	℔.	
Barrel	26	Averdupois.
Kilderkin or half Barrel	13	
Firkin	6½	

So that of Butter and Sope that is beſides the Weight of the Cask ſhould be ℔. 56 Butter. 60 Soap.

Fiſh, as Eeles and Salmon by the 22. *Edw.* 4. *Cap.* 2. and other Statutes have one and the ſame Meaſure, *Viz.*

A Table of the Meaſure of Cask for Eeles and Salmon.

Laſt.	1	Butt.				
Butts.	6	1	Barrel.			
Barrels.	12	2	1	Half Barrel.		
Half Barrels.	24	4	2	1	Firkin.	
Firkins.	48	8	4	2	1	
Gallons.	504	84	42	21	10½	Gallons.

Herrings in Cask, the Laſt.

Herrings by the ſame Statutes are to be as Ale-Meaſure, the Barrel 32 Gallons, Half Barrel 16, Firkin 8, and 12 Barrels are uſually accompted for a Laſt. But unpacked Herrings by the Book of Rates Outwards, 1 Laſt ſhall be 18 Barrels.

Sturgeon the Firkin.

Sturgeon. In 1 Firkin 2 Caggs.

Other Fiſh, the Laſt.

Other Fiſh barrelled. In 1 Laſt 12 Barrels.

Honey, Oyl, and Wine, the Aſſize.

Honey, Oyl and Wine Meaſures are alike, and in 2 *Hen.* 6. *Cap.* 11. 18. *Hen.* 6. *Cap.* 17. 1 *Ric.* 3. *Cap.* 13. and 28. *Hen.* 8. *Cap.* 14. may be ſeen to contain as in the Table following, but for Honey the Aſſize is altered by 23. *Eliz. Cap.* 8. to 32 Gallons the Barrel, 16 the Kilderkin, *&c.* like the Meaſure for Ale.

A Table of the Meaſure of Wine and Oyl.

Tun or Ton.	1	Pipe.									
Butts or Pipes.	2	1	Puncheon.								
Puncheons or Tertians of a Tun.	3	$1\frac{1}{2}$	1	Hogſhead.							
Hogſheads.	4	2	$1\frac{1}{3}$	1	Tierce,						
Tierces of a But. *. 1.	6	3	2	$1\frac{1}{2}$	1	Barrel.					
Barrels.	8	4	$2\frac{2}{3}$	2	$1\frac{1}{3}$	1	Rundlet.				
Rundlets or Rondlets. *. 2.	14 *ferè.*	7 *ferè.*	$4\frac{20}{37}$	$3\frac{15}{37}$	$2\frac{10}{37}$	$1\frac{26}{37}$	1	Gallon.			
Gallons.	252	126	84	63	42	$31\frac{1}{2}$	$18\frac{1}{2}$	1	Pottle.		
Pottles.	504	252	168	126	84	63	37	2	1	Quart.	
Quarts.	1008	504	336	252	168	126	74	4	2	1	
Pints or Pounds. *. 3.	2016	1008	672	504	336	252	148	8	4	2	Pints.

Tierce reckoned wrong in Pulton.

*. 1. Either an *Erratum* of the Preſs, or an Error of the *Amanuenſis* ſeems to be the Cauſe why *Pulton* in his Collection of the Statutes at large, Imprinted 1640, *pag.* 640. makes the Tierce to contain but 41 Gallons, ſince it is contrary to the Path-way. *John Legat* at the end of *Thomas* his Dictionary, and other good Authours, nor can 41, but 42 be the third part of 126, the Gallons in a Butt.

Rundlet how generally taken.

*. 2. Rundlet is now grown a general name to any ſmall Cask not gage.

Honey the Pound, not a Pint.

*. 3. The Pound of Honey is not a Pint, but the Pound of Wine Troy Weight is reckoned for a Pint 31. *Edw.* 1. and 12. *Hen.* 7. *Cap.* 5, and accordingly the Gallon and other Meaſures ordained.

Engliſh Weights.

Engliſh Weights.

Balances to be exact.

Becauſe Weights cannot be tryed without true Ballances it may not be unprofitable here to conſider the requiſites to the framing of exact Ballances, though it may be thought a digreſſion. Balances require a like proportion between the parts of the Beam, or elſe two unequal Weights may counterpoize one another: For if the Beam be 23 Inches long, and the handle or axle-tree ſo placed that it be diſtant from one end of the Beam 12 Inches, and the other but 11, conditionally that the ſhorter end be as heavy as the longer, then putting in two unequal Weights in ſuch proportion as the parts of the Beam one to another, which is 12 to 11. So that the greater Weight be put into the Scale which hangs on the ſhorter part of the Beam, and the leſſer Weight in the contrary Scale, yet will the Balances hang in *æquilibrio*, as *Ariſtotle* in his *Mechanical Queſtions* fully proves. Wherefore in making of Balances let the Beam be of any convenient length, the Tongue or Point half ſo long as the Beam, and ſtanding upright in the midſt, equidiſtant from both the ends, the Axle-tree three ſquare and ſtraight ſet at right Angles in the midſt of the Beam, ſo that it may alwayes bear upon the edge when the Balances ſhall be charged, the Cheeks very ſtraight and ſomewhat longer than the Tongue, with a pendant point in the middle between them, the Plates or Scales of equal Weight, the Cords of the bigneſs, and equal in length to the Beam.

How to be made.

Troy Weight how much by Statute.

The Statutes aforementioned 51. *Hen.* 3. 31. *Edw.* 1. 12. *Hen.* 7. do ordain that an *Engliſh* Penny round without clipping ſhall weigh 32 Grains of Wheat well dried and taken out of the midſt of the Ear, 20 pence 1 Ounce, 12 Ounces 1 pound Troy, 8 pounds 1 Gallon of Wine, Wheat, *&c.* as before noted, ſo that the pound Troy by the Statutes ſhould contain 7680 Grains. But ſince the making of the Statutes the Penny-weight hath been divided into 24 Grains, and the Grain into ſmaller Diviſions, as in the following Table.

Troy Weight:

A Table of Troy-Weight.

Pound.	1	Ounce.						
Ounces.	12	1	Penny-weight.					
Penny-weights.	240	20	1	Grain.				
Grains.	5760	480	24	1	Mite.			
Mites.	115200	9600	480	20	1	Droite.		
Droites.	2764800	230400	11520	480	24	1	Peroite.	
Peroites.	55296000	4608000	230400	9600	480	20	1	
Blanks.	1327104000	110592000	5529600	230400	11520	480	24	Blanks.

A Maſt how much.

Troy Weight hath ſeldom any greater denomination than the pound, yet ſometime 2 ½ ℔. thereof is called a Maſt allowed for Amber and Gold and Silver Thread of Collen by the Book of Rates.

A Mark how much.

Eight Ounces of the Pound *Troy* make the Weight, called ſometime *Bes*, ſometime the Mark Weight in *Latin*, *Marchus & Marca*. This is beyond Sea more than in *England* uſed to weigh Silver; every Ounce divided into 20 *Engliſh*, and every *Engliſh* into 32 Grains, called by ſome Aſſes or Azes.

Karacts by what Names called, how much, and for what uſe.

The Goldſmiths divide the Ounce *Troy* into 24 parts, which they call Carats, Carrats, Caracts, Characts, Karacts or Karects, and every Karect into 4 Grains. So the pound ſhall have 1152 Grains Karect, and the Ounce 96. They alſo divide the Ounce into 150 Karects, and every of theſe Carrats into 4 Grains. By the firſt ſort of Carrats they try the fineneſs of Bullion, buy and ſell Gold and Silver; by the latter Pearls and Diamonds. An old Authour I have ſeen that parts each of the 24 Karects into 12 Grains, but theſe ſeem *Exotick*, and not of *Engliſh* Extract. *Legat* at the end of *Thomas* his *Dictionary* divides the Ounce *Troy* into 16 parts, which he calls Farthing Gold Weights, and allows to every Charact 20 grains, From hence appear Grains of 3 ſorts, *Viz.*

Farthing Gold-Weights.

Grains the ſorts.

Grains by the Statute. } in 1 ℔. *Troy* { 7680.
Grains of Aſſize. } in 1 ℔. *Troy* { 5760.

Grains Caract in 1 ℔. *Troy.* { 7200.
3456.
1152.

The Grains of Aſſize, and the laſt of the Grains Caract are moſt in uſe with us. Why called Caract, ſee after in the Notes on the *Greek Weight Siliqua*, *pag.* 96.

What to be weighed by Troy Weight.

The Conſtitution of Meaſures by *Troy Weight* hath been already touched, beſides which, the uſe thereof is to weigh Bread, Bullion, Money, Pearles and Precious Stones, and choice Phyſical Druggs, as Ambergreaſe, Bezoar, Civet, Musk, Unicornshorn, *&c.* to all which except Bread and Money, nothing need more to be ſaid.

Bread.

Bread the old Aſſize.

The Statute 51. *Hen.* 3. Intituled the Aſſize of Bread and Ale, Enacteth, That when the price of a Quarter of Wheat is 1 *s.* the Waſtel Bread of a Farthing ſhall weigh 6 *l.* 16 *s.* 00 *d.*

Bread Cocket of the ſame Corn and Bultel ſhall weigh more than the Waſtel by 2 *s.*
Bread Cocket of lower price Corn more than the Waſtel by 5 *s.*
Bread made into a Simnel leſs than the Waſtel by 2 *s.*
Bread made of the Whole Wheat ſhall weigh a Cocket and a half;
So that the Cocket weigh more than a Waſtel by 5 *s.*
Bread of Treet ſhall weigh 2 Waſtels.
Bread of common Wheat ſhall weigh 2 great Cockets. l. s. d.
When a Quarter of Wheat is ſold for 1 *s.* 6 *d.* then Waſtel-Bread of a Farthing white and well baked ſhall weigh———} 4 10 08
When for 2 *s.*——————————— 3 08 00
and ſo proportionably decreaſing at the increaſe of 6 *d.* in every Quarter of Wheat.

Not exact.

But whither through want of knowledge in reverſed proportions, or by the miſtake of ſome *Amanuenſis* is not certain, yet ſure it is that the Aſſize of Bread ſet forth in that Statute for ſome higher prices than 2 *s.* the quarter of Wheat in all the Printed *Latin*, *French* and *Engliſh* Copies extant is erronious, as was long ſince rightly obſerved by Record in his Book of *Arithmetick* called *The Ground of Arts.* Wherefore and for that at the making of the Statute Money by which the Aſſize of Bread is there reckoned was at the old rate, *Viz.* One pound of Silver Money weighed One pound *Troy-Weight*, which is ſince much altered, this old Aſſize grew intricate, and later times have appointed the Aſſize by the *Troy-Weight*, and not by Money, yet grounded on the Aſſize in the Statute, accounting 6 *l.* 16 *s.* Money according to the Standart then to anſwer to 6 ℔. 9 ℥. 12 *pwts. Troy*, as indeed it doth.

Old Aſſize of Bread.

How altered.

Sorts of Bread, in the Statute.

The Statute allows of ſeven ſorts of Bread, though thoſe moſtly now in uſe is White, Wheaten and Houſhold.

Waſtel Bread what.

Waſter Bread ſeems to be Rowles or fine Manchet Bread uſed principally in Victualling Houſes to drink with, perhaps from the old word *Waſſail*, to drink or banquet, as it were a drinking Bread.

Bread Cocket what.

Bread Cocket is White Bread of the beſt ſort drawn through the fineſt Boultel or Laun-Sieve, or of the beſt Wheat; This is the firſt in the latter Aſſize Books.

Fine and Courſe

Bread Cocket of the lower price Corn is the ordinary Wheat-Bread now uſual, drawn through a courſer Cocket or Boultel than the former, or made of worſe Wheat.

Simnel Bread what.

Simnel Bread is a kind of Cake or Bisket-Bread made with ſome Butter and Spice, ſtill in uſe in ſome places.

Bread of the whole Wheat.

Bread made of the whole Wheat is ſometime called Cribble or fine Ravel Bread, but in the Aſſize Books, and in moſt places called Wheaten Bread, made of Flour courſer than the White, but not ſo courſe as the Houſhold Bread.

Bread of Treet.

Bread of Treet ſeems to be Houſhold-Bread of the beſt Wheat unravelled, or ravelled through the courſeſt Boultel, that is only the Husk or Bran taken out of the Meal.

Houſhold Bread

Houſhold-Bread in the Aſſize-Books is aſſized like that in the Statute called Bread of Common Wheat, as being common for the whole Houſhold, which then ſeemed either to be made of the common ſort of VVheat or baked as it came from Mill.

Horſe Bread.

Beſides theſe Horſe Bread though not ordained by Statute, yet hath been of long continuance and allowed, every loaf thereof is alway, at what price ſoever Corn be ſold, to be of the full weight of the Penny white Loaf, and the Baker is to ſell three ſuch Loaves for a Penny, and 13 to the dozen, that is 39 loaves for 1 *s.*

Sea-Bisket.

Sea-Bisket of excellent uſe for the Sea, becauſe baked without Salt, and well dried, is not aſſeſſed by the Statute, therefore weighed and ſold by the hundred of *Averdupois* weight.

Allowance to Bakers.

In the Book of Aſſize of Bread publiſhed towards the latter end of *Q. Eliz.* Countrey Bakers were allowed 4 *s.* upon a Quarter above the middle price of VVheat: But Bakers that dwell in Cities and Towns were allowed 6 *s.* in regard they were ſubject to more Scot and Lot than the Countrey-Bakers were, which 6 *s.* is ſtill generally allowed to Towniſh Bakers; but by the Orders of the Councel Board Dated the laſt of *January*, 1604. the Countrey-Bakers Penny white-Loaf ſhall weigh 2 Ounces more than the Bakers of the Towns, their Penny wheaten loaf 3 Ounces, and their Penny houſhold Loaf 6 Ounces.

Corn Cheap when.

Old Winchelſey *drowned when.*

Former Times had Corn very cheap, either by reaſon of the Plenty thereof, or ſcarcity of Money, for in a Book of Preſidents remaining among the Records of this Town of *Rye*, *pag.* 131. is a *Memorandum* entred, That the Year Old *Winchelſey* was drowned, which is there ſaid to be *An. Dom.* 1287. Corn was at 2 *s.* the Quarter, which at the Rate Money was then, is but 6 *s.* of our Money now. VVhereas the cheapeſt it hath been known of late years was *Annis Dom.* 1654. and 1655, when good VVheat was ſold in ſome places in *England* for 12 *s.* the Quarter.

Farthing Loaves formerly.

Anciently by reaſon of the cheapneſs of Corn, Farthing Loaves of all ſorts of Bread aſſized by the Statute were made; but now none leſs than half-penny Loaves, and theſe but of white and wheaten-bread, for of houſhold bread no Loaf is to be made under a Penny. Nevertheleſs by the Aſſize of the Farthing Loaf is the Aſſize of the half-penny and penny Loaf eaſie to be had by doubling the weight of the leſſer Loaf for the Aſſize of the greater, and contrarywiſe by halving the weight of the greater the aſſize of the leſſer Loaf is had, and having the aſſize of the penny Loaf of any ſort of bread at 1 *s. per* Quarter according to the Statute, the Aſſize of the ſame Loaf by the Reverſed Rule of Three, as hereafter in the 4th Book, *Of Proportions*, may be ſeen, is obtained at any other Rate, but by the Law the Aſſize is not to be altered, but when there is 6 *d.* increaſing or decreaſing in the Price of the Quarter of VVheat.

How the Aſſize at any Rate may be had.

The Statute Aſſize of Farthing, Half-Penny, and Penny-Bread at the price of Twelve pence the Quarter of Wheat, reduced from the Old Standard of Money to Troy-weight, accompting 20 s. *of Money then to be a Pound Troy.*

The Aſſize of Bread by Statute.

Bread.	Loaves.	Money. l.	s.	d.	VVeight. ℔.	℥.	ptws.
Waſtell	Farthing	06	16	00	06	09	12
	Half-penny	13	12	00	13	07	04
	Penny	27	04	00	27	02	08
Fine Cocket.	Farthing	06	18	00	06	10	16
	Half-penny	13	16	00	13	09	12
	Penny	27	12	00	27	07	04
Courſe Cocket	Farthing	07	01	00	07	00	12
	Half-penny	14	02	00	14	01	04
	Penny	28	04	00	28	02	08
Simnel	Farthing	06	14	00	06	08	08
	Half-penny	13	08	00	13	04	16
	Penny	26	16	00	26	09	12
Wheaten	Farthing	10	11	06	10	06	18
	Half-penny	21	03	00	21	01	16
	Penny	42	06	00	42	03	12
Treet	Farthing	13	12	00	13	07	04
	Half-penny	27	04	00	27	02	08
	Penny	54	08	00	54	04	16
Houſhold	Farthing	14	02	00	14	01	04
	Half penny	28	04	00	28	02	08
	Penny	56	08	00	56	04	16

From this Legal Baſis is deduced the following Table containing the Aſſize by the *Troy Weight* of the Penny white, wheaten, and houſhold Loaves being the Bread ſtill in uſe from 12*d.* the Quarter of VVheat unto 2 *l.* the Quarter, omitting ſome of the ſmall Fractions.

The Aſſize of Bread now in uſe.

Price of Wheat.	Penny White.	Penny Wheaten.	Penny Houſhold.	Price of Whear.	Penny Whire.	Penny Wheaten.	Penny Houſhold.
l s d	℔ ℥ p^{wts}	℔ ℥ p^{wts}	℔ ℥ p^{wts}	l s d	℔ ℥ p^{wts}	℔ ℥ p^{wts}	℔ ℥ p^{wts}
— 1—0	28—02—08	42—03—12	56—04—16	1—00—6	1—04—10	2—00—15	2—09—00
— 1—6	18—09—12	28—02—08	37—07—04	1—01—0	1—04—02 $\frac{1}{4}$	2—00—03	2—08—04 $\frac{1}{2}$
— 2—0	14—01—04	21—01—16	28—02—08	1—01—6	1—03—14 $\frac{3}{4}$	1—11—12	2—07—09 $\frac{1}{2}$
— 2—6	11—03—07 $\frac{1}{5}$	16—11—00 $\frac{4}{5}$	22—06—14 $\frac{2}{5}$	1—02—0	1—03—07 $\frac{1}{2}$	1—11—01 $\frac{1}{4}$	2—06—15
— 3—0	9—04—16	14—01—04	18—09—12	1—02—6	1—03—00 $\frac{3}{4}$	1—10—11	2—06—01 $\frac{1}{2}$
— 3—6	8—00—13 $\frac{5}{7}$	12—01—00 $\frac{4}{7}$	16—01—07 $\frac{3}{7}$	1—03—0	1—02—14 $\frac{1}{4}$	1—10—01	2—05—08 $\frac{1}{2}$
— 4—0	7—00—12	10—06—18	14—01—04	1—03—6	1—02—08	1—09—12	2—04—16
— 4—6	6—03—04	9—04—16	12—06—08	1—04—0	1—02—02	1—09—03	2—04—04
— 5—0	5—07—13 $\frac{3}{5}$	8—05—10 $\frac{2}{5}$	11—03—07 $\frac{1}{5}$	1—04—6	1—01—16	1—08—14	2—03—12
— 5—6	5—01—10 $\frac{6}{11}$	7—08—05 $\frac{9}{11}$	10—03—01 $\frac{1}{11}$	1—05—0	1—01—10 $\frac{1}{2}$	1—08—05 $\frac{3}{4}$	2—03—01 $\frac{1}{3}$
— 6—0	4—08—08	7—00—12	9—04—16	1—05—6	1—01—05 $\frac{1}{3}$	1—07—18	2—02—10 $\frac{1}{2}$
— 6—6	4—04—01	6—06—01 $\frac{1}{2}$	8—08—02	1—06—0	1—01—00 $\frac{1}{4}$	1—07—10	2—02—00 $\frac{1}{2}$
— 7—0	4—00—06 $\frac{6}{7}$	6—00—10 $\frac{2}{7}$	8—00—13 $\frac{5}{7}$	1—06—6	1—00—15 $\frac{1}{4}$	1—07—03	2—01—10
— 7—6	3—09—02	5—07—13	7—06—04	1—07—0	1—00—10 $\frac{1}{2}$	1—06—16	2—01—01
— 8—0	3—06—06	5—03—09	7—00—12	1—07—6	1—00—06	1—06—09	2—80—12
— 8—6	3—03—16	4—11—14	6—07—12	1—08—0	1—00—01 $\frac{1}{2}$	1—06—02 $\frac{1}{4}$	2—00—03
— 9—0	3—01—12	4—08—08	6—03—04	1—08—6	—11—17 $\frac{1}{4}$	1—05—15 $\frac{3}{4}$	1—11—14 $\frac{1}{2}$
— 9—6	2—11—12	4—05—08	5—11—04	1—09—0	—11—13 $\frac{1}{4}$	1—05—10	1—11—06 $\frac{1}{2}$
—10—0	2—09—16 $\frac{4}{5}$	4—02—15 $\frac{1}{5}$	5—07—13 $\frac{3}{5}$	1—09—6	—11—09 $\frac{1}{4}$	1—05—04	1—10—18 $\frac{1}{2}$
—10—6	2—08—04 $\frac{1}{2}$	4—00—06	5—04—09	1—10—0	—11—05 $\frac{1}{2}$	1—04—18 $\frac{1}{4}$	1—10—11
—11—0	2—06—15 $\frac{3}{11}$	3—10—02 $\frac{3}{4}$	5—01—10 $\frac{6}{11}$	1—10—6	—11—01 $\frac{3}{4}$	1—04—12	1—10—03 $\frac{1}{2}$
—11—6	2—05—08 $\frac{1}{2}$	3—08—02 $\frac{3}{4}$	4—10—17	1—11—0	—10—18 $\frac{1}{4}$	1—04—07 $\frac{1}{4}$	1—09—16 $\frac{1}{2}$
—12—0	2—04—04	3—06—06	4—08—08	1—11—6	—10—14 $\frac{3}{4}$	1—04—02	1—09—09 $\frac{1}{2}$
—12—6	2—03—01	3—04—11 $\frac{1}{2}$	4—06—02	1—12—0	—10—11 $\frac{1}{2}$	1—03—17 $\frac{1}{4}$	1—09—03
—13—0	2—02—00 $\frac{1}{2}$	3—03—00 $\frac{3}{4}$	4—04—00	1—12—6	—10—08	1—03—12	1—08—16
—13—6	2—01—01	3—01—11 $\frac{1}{4}$	4—02—02	1—13—0	—10—05	1—03—07 $\frac{1}{2}$	1—08—10
—14—0	2—00—03 $\frac{3}{7}$	3—00—05 $\frac{1}{7}$	4—00—06 $\frac{6}{7}$	1—13—6	—10—02	1—03—03	1—08—04
—14—6	1—11—06 $\frac{3}{4}$	2—11—00 $\frac{1}{8}$	3—10—12	1—14—0	—09—19	1—02—18 $\frac{1}{2}$	1—07—18
—15—0	1—10—11	2—00—16 $\frac{1}{2}$	3—09—02	1—14—6	—09—16 $\frac{1}{4}$	1—02—14	1—07—12 $\frac{1}{2}$
—15—6	1—09—16 $\frac{1}{2}$	2—08—14 $\frac{3}{4}$	3—07—13	1—15—0	—09—13 $\frac{1}{4}$	1—[illegible]—09 $\frac{3}{4}$	1—07—06 $\frac{1}{2}$
—16—0	1—09—03	2—07—14 $\frac{1}{2}$	3—06—06	1—15—6	—09—10 $\frac{1}{2}$	1—02—05 $\frac{3}{4}$	1—07—01
—16—6	1—08—10	2—06—15	3—05—00	1—16—0	—09—08	1—02—02	1—06—16
—17—0	1—07—18	2—05—17	3—03—16	1—16—6	—09—05 $\frac{1}{4}$	1—01—17 $\frac{3}{4}$	1—06—10 $\frac{1}{2}$
—17—6	1—07—06 $\frac{1}{2}$	2—04—19 $\frac{3}{4}$	3—02—13	1—17—0	—09—02 $\frac{3}{4}$	1—01—14	1—06—05 $\frac{1}{2}$
—18—0	1—06—16	2—04—04	3—01—12	1—17—6	—09—00 $\frac{1}{4}$	1—01—10	1—06—00 $\frac{1}{2}$
—18—6	1—06—05 $\frac{3}{4}$	2—03—08 $\frac{1}{2}$	3—00—11 $\frac{1}{2}$	1—18—0	—08—18	1—01—07	1—05—16
—19—0	1—05—16	2—02—14	2—11—12	1—18—6	—08—15 $\frac{3}{4}$	1—01—03	1—05—11 $\frac{1}{2}$
—19—6	1—05—07	2—02—00 $\frac{1}{2}$	2—10—14	1—19—0	—08—13 $\frac{1}{2}$	1—01—00 $\frac{1}{4}$	1—05—07
1—00—0	1—04—18 $\frac{2}{5}$	2—01—07 $\frac{1}{2}$	2—09—16 $\frac{4}{5}$	1—19—6	—08—11 $\frac{1}{3}$	1—00—16	1—05—02 $\frac{2}{3}$
				2—00—0	—08—09 $\frac{1}{5}$	1—00—13 $\frac{3}{4}$	1—04—18 $\frac{2}{5}$

Money.

English *Money why called* Sterling.

Engliſh Coine is often called *Sterling* Money, as *Legat* in the end of *Thomas* Dictionary conceives, becauſe there was anciently ſtamped upon it one quarter a litt Bird called a *Star* or *Starling*; although others think it was becauſe Coined firſt at a pla ſo called, or rather by the *Eſterlings*, which Name in the time of the *Saxon Heptarchy England* was given to thoſe *Saxons* which inhabited in the *Eaſtern* parts, as *Cambridg ſhire*, &c.

Sterling *Standard.*

The *Sterling* Standard for Money long before the making of the laſt forementione Statutes even from the time of the *Saxons* had been obſerved to be 20 pence for a Ounce of Silver 11 ℥. 2 p^{wts} fine, and about 22 *s.* and 3 *d.* for an Ounce of Gold of 22 Cara fine, and ſo continued till the Reign of *Henry* 6th and then the Ounce of Silver of the ſam fineneſs was raiſed from 20 pence to 30 pence. *Edw.* 4th raiſed it to 40 pence. *Hen.* 8 to 45 pence, and *Eliz.* to 60 pence, at which rate it yet ſtands; all which was done b

Advanced by Proclamation.

Proclamations according to the Exigencies of Affairs in the Kingdom at the times when was altered, ſo that though both pounds *Sterling* and *Troy* were divided ſomewhat like that into 20 and 12, this into 12 and 20, yet were both pounds then equal, but now thre pound of Money in Accompt is but one pound *Troy*, and ſcarce that, and conſequentl our Shilling and Penny is but the third part of the old Shilling and Penny, and in lik manner was Gold advanced.

Touching Money, Two things are to be noted. 1. The Accompt. 2. The Coin In which latter the Weight and Fineneſs are alſo conſiderable.

The common Accompt of Money is by Pounds, Shillings and Pence. But there are ſeveral other Denominations and Subdiviſions of the Pound *Sterling*, as in the Tabl following.

Sterling Money.

A Table of English *Money.*

	*1	*2	*3	*4	*5											
Pound.	1															
Mark.	1½	1														
Angels.	2	1⅓	1													
Nobles.	3	2	1½	1												
Crowns.	4	2⅔	2	1⅓	1											
Half Crowns.	8	5⅓	4	2⅔	2	1										
Shillings.	20	13⅓	10	6⅔	5	2½	1									
Sixpences.	40	26⅔	20	13⅓	10	5	2	1								
Groats.	60	40	30	20	15	1½	3	1½	1							
Threepences.	80	53⅓	40	26⅔	20	10	4	2	1⅓	1						
Twopences.	120	80	60	40	30	15	6	3	2	1½	1					
Three half pences.	160	106⅔	80	53⅓	40	20	8	4	2⅔	2	1⅓	1				
Pence.	240	160	120	80	60	30	12	6	4	3	2	1½	1			
Three Farthings.	320	213⅓	160	106⅔	80	40	16	8	5⅓	4	2⅔	2	1⅓	1		
Half-pence.	480	320	240	160	120	60	24	12	8	6	4	3	2	1½	1	
Farthings.	960	640	480	320	240	120	48	24	16	12	8	6	4	3	2	1
Mites.	5760	3840	2880	1920	1440	720	288	144	96	72	48	36	24	18	12	6

A Pound why called a Piece.

* 1. A Pound or Twenty Shillings is ſometime called a piece in reference to *xx s.* pieces of Gold; for it is ordinary to ſay 1 piece, 2 pieces, 3 pieces, for 1. 2. or 3. Pounds.

A Mark why ſo called.

* 2. A Mark which is ⅔ of a Pound, and 13 *s.* 4 *d.* deriveth its name from the Mark Weight, becauſe at the old Rate a Mark of Silver Money weighed 8 Ounces *Troy*.

* 3. An Angel is 10 *s.* and ſo called from the Impreſſion of an Angel upon pieces of Gold of that Value. Yet are there Gold Coins called Ship-Angels of more worth; becauſe ſince they were Coined Gold is raiſed.

An Angel why ſo called.

* 4. A Noble is 6 *s.* 8 *d.* half a Mark, or the third part of a Pound, and for the reaſon laſt before-mentioned ſeveral pieces of Gold bearing the name of Nobles, are worth more than 6 *s.* 8 *d.* as the Angel Noble, firſt Coined for a Noble, then advanced to an Angel, and now worth more.

Nobles of old Coine worth more now.

* 5. A Crown is 5 *s.* the Quarter of a Pound, and the greateſt piece of *Engliſh* Silver Coin, ſometime called the *Engliſh* Dollar, from whence down to Farthings are Silver Coins, and beſides theſe ſome Harpers in value 9 *d.* a piece, and half Harpers worth 4*d.* ½ a piece were Coined by Q. *Elizabeth*, but ſeem to be *Iriſh* Money, now almoſt worn out as well as her Three Farthing pieces.

Crown the greateſt Silver Coine in Eng-land *called ſometime Dollar*

Farthings before the advance of the Ounce of Silver to 60 pence were Silver Coins, as appeareth by the Statutes 4 *Hen.* 4. *Cap.* 10. & 14. *Hen.* 8. *Cap.* 12. but ſince grew inconvenient by their ſmallneſs.

Farthings ſometime Silver Coines.

Mites are no pieces of Coines, but a leſſer diviſion uſed about reduction and finding out the Value of Foreign Coines by the *Sterling* Standard; and ſome for their private uſe and more curioſity divide the Farthing into 2 *Ques*, the Q into 2 *Cees*, the C into 2 *Dodkins*, the D into 2 *Mites*, that is 16 Mites in one Farthing.

Mites no Coines.

Farthings how divided by ſome.

The Coines of *England* are of Gold and Silver, ſome of greater value, ſome leſſer, many elder, others later, and ſo different in Fineneſs and Weight, that it is hard for any but thoſe whoſe common Converſe is thereabouts, as Mintmaſters, Goldſmiths, *&c.* to give any perfect accompt thereof; and the rather, becauſe they proceed not by the ſtrict Rules of *Arithmetick*, beſides any ſuch accompt will be ſubject to future alteration by addition of new Coins or advancing the Value of the old.

Mr. *Gerard Malynes* ſometime a Commiſſioner about the Mint Affairs, and a Man expert in Coinage in his Book Intituled *Lex Mercatoria*, hath Calculated the Weight and Fineneſs of ſeveral both *Engliſh* and *Outlandiſh* Coines, and from thence in the ſuccedent Tables much light is borrowed. Nevertheleſs whether becauſe he hath (as he ſaith) omitted the ſmall Fractions as unneceſſary, or becauſe he hath rather inſerted what Weight the Coins ought to be according to the Royal Orders at the times when they firſt iſſued forth then what they are, I know not, but ſure I am, his accompt in ſeveral of the Gold Coines doth not agree with the Book called *Perfect Directions for all Engliſh Gold*, Imprinted 1663, nor others, nor yet with the Common Weights, as by often experience trying ſuch as have come to my hands I have found

Engliſh *Coines of both ſorts different in value and fineneſs.*

Wherefore in the following Table for *Engliſh Gold*, beſides the Weight after *Malynes* and others according to the more regular proportions of *Arithmetick*, is another Collumn containing the Common weight as the pieces were when Coined (for many by uſe are worne much lighter, and muſt have allowance) according to which Weight is the value reckoned; and becauſe the value of Old Coines have ſince the time of their Coinage been increaſed not only by Proclamation, but beyond the Rates limited thereby, to wit, even as Merchants have found them valued in Foreign Exchange, or as Goldſmiths have found them worth to melt down: Let therefore the *Sterling* Silver Coines be underſtood as they anciently were and ſtill are Currant in *England*, but the Gold Coines are differently valued in their reſpective Collumns, *viz.* thoſe of K. *James* and K. *Charles* the Firſt, as in their Proclamations, thoſe elder as they were currant with the Goldſmiths *Anno* 1640, and are now worth ſince the Proclamation of K. *Charles* the Second *Anno* 1660. accompting in the one Penny weight of Gold 22 Carats fine worth 3 *s.* 4 *d.* and in the other 3 *s.* 6 *d.* 2 *q.* 4 *m.* omitting in ſome of the ſmall pieces the odd Farthings and Mites, and proportionably for Gold of other fineneſs: Nevertheleſs according to the Proclamations and vulgar currant Exchange, moſt of the Old Coines were not, nor yet are valued ſo high, for the Old Spurre Royal by the Proclamation of King *James* 1611. is rated at 16 *s.* 6 *d.* yet, with the Goldſmiths long before the Proclamation 1660. was worth 18 *s.* which is 1 *s.* 6 *d.* more than that Proclamation values it, and the like may be obſerved in others. As to the pieces to the *Pound Troy* where the *Fractions* hapned to be ſmall and inconſiderable in ſtead thereof the next neareſt is taken and marked with — or + according as it is too little or too much; other things are perſpicuous enough by the Tables themſelves, and need no explanation.

Old Coinage advanced ſince their Coinage by Proclamation and otherwiſe.

English *Silver Coines their Fineneſs Weight and Value.*

The Table of Sterling-Silver-Coines *now Currant.*

Names of the Pieces.	Kings and Queens.	Fineneſs.		Weight.				Worth.	
		Ounces	*pwts.*	*pwts.*	*gr.*	*m.*	*dr.*	*s.*	*d.*
Crown of	*Edw. 6. & Eliz.*	11	2	℥ 1 or 20	00	00	00	5	0
	James, Charles 1. & 2.	11	0	19	08	10	07$\frac{23}{31}$	5	0
Half Crown of	*Edw. 6. & Eliz.*	11	2	10	00	00	00	2	6
	James, Charles 1. & 2.	11	0	9	16	05	03$\frac{23}{31}$	2	6
Shilling of	*Edw. 6. Phil. & Mary, & Eliz.*	11	2	4	00	00	00	1	0
	James, Charles 1. & 2.	11	0	3	20	18	01$\frac{17}{31}$	1	0
Six pence of	*Edw. 6. Phil. & Mary, & Eliz.*	11	2	2	00	00	00	0	6
	James, Charles 1. & 2.	11	0	1	22	09	00$\frac{24}{31}$	0	6
Old Groat of *Henry* 8.		11	2	1	20	13	00$\frac{24}{43}$	0	4+
Laſt Groat of *Henry* 8.		11	2	1	12	00	00	0	4+
Groat of	*Mary & Eliz.*	11	2	1	08	00	00	0	4
	Charles 1.	11	0	1	06	19	08$\frac{16}{31}$	0	4
Three pence of	*Elizabeth.*	11	2	1	00	00	00	0	3
	Charles 1.	11	0	0	23	04	12$\frac{12}{31}$	0	3
Two pence of	*Henry* 8.	11	2	0	18	00	00	0	2+
	Elizabeth	11	2	0	16	00	00	0	2
	James, Charles 1. & 2.	11	0	0	15	09	16$\frac{8}{31}$	0	2
Three half pence of *Elizabeth.*		11	2	0	12	00	00	0	1$\frac{1}{2}$
Penny of	*Henry* 8. & *Edw.* 6.	11	2	0	09	00	00	0	1+
	Mary, & Eliz.	11	2	0	08	00	00	0	1
	James, Charles 1. & 2.	11	0	0	07	14	20$\frac{4}{31}$	0	1
Three Farthings of *Elizabeth.*		11	2	0	06	00	00	0	0$\frac{3}{4}$
Half penny of	*Elizabeth.*	11	2	0	04	00	00	0	0$\frac{1}{2}$
	James, Charles 1. & 2.	11	0	0	03	17	10$\frac{2}{31}$	0	0$\frac{1}{2}$

English *Gold Coines, their Fineneſs Weight and Value.*

The Table of moſt Sterling Gold Coines *yet Currant.*

Names of the Pieces.	Pieces to the ℔ *Troy.*	Weight by *Malynes, &c.*				Common Weight.				Pieces to the ℔ *Troy.*	Fine		Value 1640.			Value 1660.			Al-low.
		pwts.	*gr.*	*m.*	*dr.*	*pwts.*	*gr.*	*m.*	*dr.*		*Car.*	*gr.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*	*gr.*
Old Double Roſe Noble.	23$\frac{3}{8}$	10	06	08	08$\frac{40}{187}$	10	00	00	00	24	23	3$\frac{3}{4}$	1	16	4	1	18	8	5
Double Roſe Noble of *Henry* 8. *Edw.* 6. *Phil. & Mary Elizabeth.*	24	10	00	00	00	9	22	00	00	24$\frac{1}{5}$ +	23	3$\frac{3}{4}$	1	16	0	1	18	4	5
Great Soveraign of K. *James.*	24	10	00	00	00	9	16	05	03$\frac{27}{31}$	24$\frac{4}{5}$	22	0	1	13	0	1	15	3	4$\frac{1}{2}$
Double Roſe Noble of K. *James.*	26$\frac{2}{3}$	9	00	00	00	8	21	06	16	27	23	3$\frac{3}{4}$	1	13	0	1	15	3	4$\frac{1}{2}$
Double Roſe Royal or Real.	—	—	—	—	—	8	02	03	03	29$\frac{2}{3}$ +	23	3$\frac{1}{2}$	1	10	0	1	12	0	4$\frac{1}{2}$
Double Old Soveraign.	27$\frac{3}{8}$	8	18	08	05$\frac{19}{73}$	8	00	00	00	30	22	0	1	06	8	1	08	5	4
Beſt Double Soveraign of *Henry* Double Soveraign of *Edw.* 6. *Elizabeth.*	30	8	00	00	00	7	04	00	00	33$\frac{1}{2}$ —	22	0	1	03	10	1	05	5	4
Double Soveraign of K. *James* called *Unite* or *Jacobus.*	36	6	16	00	00	6	10	16	18$\frac{18}{31}$	37$\frac{1}{5}$	22	0	1	02	0	1	03	10	3
Laureat or *xx s.* piece of *James.*	39$\frac{3}{5}$	6	01	09	02$\frac{2}{11}$	5	20	09	18$\frac{6}{41}$	41	22	0	1	00	0	1	01	4	3
Twenty Shillings piece of *Charles* 1.	40	6	00	00	00	5	20	09	18$\frac{6}{41}$	41	22	0	1	00	0	1	01	4	3
Old Roſe Noble.	46$\frac{3}{4}$	5	03	04	04$\frac{20}{187}$	5	00	00	00	48	23	3$\frac{3}{4}$	0	18	2	0	19	4	2$\frac{1}{2}$
Spurre Royal of *Henry* 8. *Edward* 6. *Philip & Mary Elizabeth.*	48	5	00	00	00	4	23	00	00	48$\frac{2}{5}$ +	23	3$\frac{3}{4}$	0	18	0	0	19	2	2$\frac{1}{2}$
Spurre Royal of *James*	53$\frac{1}{3}$	4	12	00	00	4	10	13	08	54	23	3$\frac{3}{4}$	0	16	6	0	17	7	2$\frac{1}{2}$
Double Noble of *Elizabeth*	—	—	—	—	—	4	10	06	16	54$\frac{1}{5}$ —	23	3$\frac{1}{2}$	0	16	0	0	17	1	2$\frac{1}{2}$
Old Noble or Noble of *Henry*	53$\frac{3}{4}$	4	11	03	06$\frac{6}{43}$	4	10	00	00	54$\frac{18}{53}$	23	3$\frac{1}{2}$	0	16	0	0	17	1	2$\frac{1}{2}$
Roſe Royal.	—	—	—	—	—	4	01	01	13$\frac{1}{2}$	59$\frac{1}{3}$ +	23	3$\frac{1}{2}$	0	15	0	0	16	0	2$\frac{1}{2}$
Old Soveraign.	54$\frac{3}{4}$	4	09	04	02$\frac{46}{73}$	4	00	00	00	60	22	0	0	13	4	0	14	2	2
Beſt Soveraign of *Henry* Soveraign of *Edward* 6. *Elizabeth.*	60	4	00	00	00	3	14	00	00	66$\frac{42}{43}$	22	0	0	11	11	0	12	8	2
Old Angel Noble or Angel of *Henry*	69	3	11	09	13$\frac{13}{23}$	3	08	00	00	72	23	3$\frac{1}{2}$	0	12	1	0	12	10	2

Laſt

Names of the Pieces.	Pieces to the ℔ *Troy.*	Weight by *Malynes, &c.*				Common Weight.				Pieces to the ℔ *Troy.*	Fine		Value 1640.			Value 1660.			Allowance.
		pwts.	*gr.*	*m.*	*dr.*	*pwts.*	*gr.*	*m.*	*dr.*		*Car.*	*gr.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*	*gr.*
Laſt Angel Noble of *Henry* *Edw. 6.* *Phil. & Mary.* *Elizabeth.* Firſt Angel of *James.*	72	3	08	00	00	3	07	05	00	72$\frac{2}{3}$ +	23	3$\frac{1}{2}$	0	11	11	0	12	8	2
Soveraign of K. *James* called Double-Brittain Crown.	72	3	08	00	00	3	05	08	09$\frac{9}{31}$	74$\frac{2}{5}$	22	0	0	11	0	0	11	9	2
George Noble.						3	00	00	00	80	23	3$\frac{1}{2}$	0	10	10	0	11	6	2
Laſt Angel of *James.*	80	3	00	00	00	2	23	02	05$\frac{1}{3}$	81	23	3$\frac{1}{2}$	0	11	0	0	11	9	2
Half Laureat of *James.*	79$\frac{1}{5}$	3	00	14	13$\frac{1}{11}$	2	22	04	21$\frac{3}{41}$	82	22	0	0	10	0	0	10	8	2
Ten Shilling Piece of *Charles* I.	80	3	00	00	00	2	22	04	21$\frac{3}{41}$	82	22	0	0	10	0	0	10	8	2
Angel of *Charles.*						2	16	14	09	89 +	23	3$\frac{1}{2}$	0	10	0	0	10	8	2
Half Spurre Royal.	96	2	12	00	00	2	11	10	00	96$\frac{4}{5}$ +	23	3$\frac{3}{4}$	0	09	0	0	09	7	2
Firſt Crown of K. *Henry.*	100$\frac{1}{2}$	2	09	06	06$\frac{30}{67}$	2	09	00	00	101$\frac{3}{57}$	22	+	0	08	0	0	08	5	2
Single Noble of *Elizabeth.*						2	05	03	08	108$\frac{2}{5}$	23	3$\frac{1}{2}$	0	08	0	0	08	6	2
Half Old Noble.	107$\frac{1}{2}$	2	05	11	15$\frac{3}{43}$	2	05	00	00	108$\frac{36}{53}$	23	3$\frac{1}{2}$	0	08	0	0	08	6	2
Salute.	108	2	05	06	16	2	05	00	00	108$\frac{36}{53}$	23	3	0	07	11	0	08	5	2
Baſe Crown of K. *Henry*, called Roſe-Crown.	120	2	00	00	00	1	23	00	00	122$\frac{26}{47}$	20	0	0	05	11	0	06	4	1
Crown of *Edward 6.* *Elizabeth.*	120	2	00	00	00	1	19	00	00	133$\frac{41}{43}$	22	0	0	05	11	0	06	4	1
Half Angel Noble of *Henry*	138	1	17	14	18$\frac{18}{23}$	1	16	00	00	144	23	3$\frac{1}{2}$	0	06	00	0	06	5	1
Half Laſt Angel of *Henry.* Half Angel of *Edw. 6.* *Phil. & Mary* *Elizabeth.* Half firſt Angel of *James.*	144	1	16	00	00	1	15	12	12	145$\frac{1}{3}$ +	23	3$\frac{1}{2}$	0	05	11	0	06	4	1
Brittain Crown of *James.*	144	1	16	00	00	1	14	14	04$\frac{20}{31}$	148$\frac{4}{5}$	22	0	0	05	6	0	05	10	1
Half George Noble.						1	12	00	00	160	23	3$\frac{1}{2}$	0	05	5	0	05	9	1
Half Laſt Angel of *James.*	160	1	12	00	00	1	11	11	02$\frac{2}{3}$	162	23	3$\frac{1}{2}$	0	05	6	0	05	10	1
New Crown of *James.*	158$\frac{2}{5}$	1	12	07	06$\frac{6}{11}$	1	11	02	10$\frac{22}{41}$	164	22	0	0	05	0	0	05	4	1
Crown of *Charles* I.	160	1	12	00	00	1	11	02	10$\frac{22}{41}$	164	22	0	0	05	0	0	05	4	1
Two parts of Salute	162	1	11	11	02$\frac{2}{3}$	1	11	00	00	164$\frac{4}{7}$	23	3	0	05	3	0	05	7	1
Half *Henry* firſt Crown	201	1	04	13	03$\frac{15}{67}$	1	04	10	00	202$\frac{6}{57}$	22	+	0	04	0	0	04	2	1
Half Salute	216	1	02	13	08	1	02	10	00	217$\frac{19}{53}$	23	3	0	03	11	0	04	2	1
Half Roſe Crown	240	1	00	00	00	0	23	10	00	245$\frac{5}{47}$	20	0	0	02	11	0	03	2	$\frac{1}{2}$
Half Crown of *Edward 6.* *Elizabeth.*	240	1	00	00	00	0	21	10	00	267$\frac{39}{43}$	22	0	0	02	11	0	03	2	$\frac{1}{2}$
Quarter Old Angel Noble.	276	0	20	17	09$\frac{9}{23}$	0	20	00	00	288	23	3$\frac{1}{2}$	0	03	00	0	03	2	$\frac{1}{2}$
Quarter Laſt Angel of *Henry.* Quart. Angel of *Edw. 6.* *Phil. & Mary.* *Elizabeth.* Quarter Firſt Angel of *James.*	288	0	20	00	00	0	19	16	06	290$\frac{2}{3}$ +	23	3$\frac{1}{2}$	0	02	11	0	03	2	$\frac{1}{2}$
Half Brittain Crown of *James.*	288	0	20	00	00	0	19	07	02$\frac{10}{31}$	297$\frac{3}{5}$	22	0	0	02	09	0	02	11	$\frac{1}{2}$
Quarter Laſt Angel of *James.*						0	17	15	13$\frac{1}{3}$	324	23	3$\frac{1}{2}$	0	02	09	0	02	11	$\frac{1}{2}$

Many of the Phyſical Doſes are weighed by the Pound of 12 Ounces, and every Ounce is divided into 8 Drams, 1 Dram into 3 Scruples, and 1 Scruple into 20 Graines.

Phyſical Doſes how weighed.

Mettals more baſe than Gold or Silver with Courſe Druggs and divers ſorts of Goods and Merchandizes as before noted are bought and ſold by *Avoirdupois* Weight, the ordinary Hundred whereof contains 112 Pounds, and the Pound 16 Ounces, which Ounces are leſs than the Ounce *Troy*, though the Pound be bigger; becauſe that Pound is divided but into 12 Ounces, each of which bear proportion to the Ounce *Avoirdupois* as 1 to 1$\frac{3}{29}$ for 1 Ounce *Troy* makes 1$\frac{3}{29}$ Ounce *Avoirdupois*, and 1 Ounce *Avoirdupois* is but $\frac{29}{32}$ of 1 Ounce *Troy*, as ſaith my *Dutch* Authour, and accordingly he makes the Pound *Troy* to be but $\frac{24}{29}$ of the other Pound, and 1 ℔ *Avoirdupois* to equal 1$\frac{5}{24}$ ℔. *Troy*.

Courſe Druggs and baſe Mettals how weighed.

Avoirdupois and Troy-weight what proportion each beareth to the other.

Herewith alſo agreeth *Dalton* and *Malynes* before named in their making 56 ℔ *Avoirdupois*, and 67 ℔ 8 ℥ *Troy* juſtly accord, though elſewhere both of them unhappily miſtake to count 7 ℔ *Avoirdupois* equal to 102 ℥ *Troy*, which is 8$\frac{1}{2}$ ℔, for then ſhould 56 ℔ *Avoirdupois* be 68 ℔ *Troy*. Others affirm 16 ℥ *Avoirdupois* equal to 14 ℥ 12 *pwts. Troy*, and then ſhall 56 ℔ *Avoirdupois* be equal to 68 ℔ 1 ℥ 12 *pwts. Troy*. Nevertheleſs ſome think anciently the Pounds admitting the like Number of Graines differed no more than the weight of Wheat and Barley one to another, ſeeing 1 ℔. *Avoirdupois* contains 7680 Grains or Barley-Corns, and ſo many Grains of Wheat are found in 1 ℔. *Troy*, if every Pennyweight be multiplyed by 32 Grains, or Wheat Cornes according to the Old Statutes.

By ſome thought to differ no more than Wheat and Barley.

Common

Common greater and ſmaller Diviſions than the Hundred of Avoirdupois *Weight.*

A Table of Avoirdupois-Weight.

Tonn.	1	C. Hundred.							
Hundreds.	20	1	½ C. Half Hundred.						
Half Hundreds.	40	2	1	qr. Quarter.					
Quarters.	80	4	2	1	℔. Pound.				
Pounds.	2240	112	56	28	1	℥. Ounce.			
Ounces.	35840	1792	896	448	16	1	ʒ. Dram.		
Drams.	286720	14336	7168	3584	128	8	1	℈. Scruple.	
Scruples.	860160	43008	21504	10752	384	24	3	1	
Grains.	17203200	860160	430080	215040	7680	480	60	20	Grains.

Cuſtom hath made familiar the uſe and knowledge of Stones, Nails, Cloves, Tods, and ſuch like denominations, though frequented but to weigh ſome ſort of Commodities, and to which of them to allot more Allowance or Tare than 12 on the 100 or leſs, the Experienced Merchant well knows. However for ſatisfaction of the Curious Inquiſitor, take a Breviat of ſuch as have come to hand in peruſal of the Statutes, the Book of Rates (according to which the King receiveth his Cuſtoms), and other approved Authours and Experience.

By Avoirdupois Weight.

Allom the Hundred, Stone.

Allom, 1 Hundred, 13½ Stones, 1 Stone 8 Pounds, by the Ordinance *Compoſitio de Ponderibus*, which allows but 108 ℔. to the Hundred.

Aſhes the Laſt Barrel.

Aſhes, Called *Pot-Aſhes*, alſo *Soap-Aſhes*, 1 Laſt 12 Barrels, 1 Barrel 2 Hundred, by the Rates Inwards.

Barillia the Barrel.

Barillia, Or *Saphora*, 1 Barrel 2 Hundred weight, by the ſame Rates.

Beef, the Nail, Score.

Beef, 1 Nail 8 Pounds of Common uſe. Some places ſell by the Score, each Score 20 Pounds.

Butter the Wey.

Butter in Cask is aſſized by Statute, as is ſeen before in Meaſures, but beſides of *Suffolk* and *Eſſex Butter*, the Wey is uſually reckoned alike to their Wey of *Cheeſe*.

Cheeſe the Wey, Clove.

Cheeſe by the Statute of 9. K. *Hen.6. Chap.* 8. One Wey is to contain 32 Cloves, and 1. Clove 7 Pounds. *Suffolk-Cheeſe* by the Aſſize Book 1597, and uſage ever ſince, notwithſtanding 8 Pounds to the Clove, and *Eſſex-Cheeſe* 10½ Pounds to the Clove, and both 32 Cloves to the Wey. But by ſome the Wey of *Eſſex-Cheeſe* doth contain 42 Cloves, and the Clove but 8 Pounds. Both which agree to make the Wey of *Eſſex-Cheeſe* 336 ℔. of *Suffolk* 256 ℔. whereas by the Statute a Wey is but 224 ℔. as before.

Cinnamon as Allom.

Cinnamon, by the Ordinance *Compoſitio de Ponderibus* is to have the ſame weight as before noted of *Allom*.

Glaſs the Seam, Stone.

Glaſs, by the ſame Ordinance containeth 1 Seam, 24 Stones. 1 Stone 5 Pounds.

Hay, the Load, Truſs.

Hay, by Cuſtom 1 Load 40 Truſſes, 1 Truſs 56 Pounds, which make 20 Hundred weight to the Load, yet moſt times it paſſeth with 18 Hundred.

Hemp the Stone.

Hemp, is commonly ſold by the Stone, which by the Statute of 21. *Hen.* 8. *Cap.* 12. is eſpecially ordained to contain 20 Pounds. Nevertheleſs in *Rye* a Stone of *Hemp* is 32 ℔. and ſo hath been time out of mind.

Lead the Fodder.

The Old Weight

Lead, the Common Accompt 1 Fodder 19½ Hundreds, 1 Hundred 112 Pounds. By the Book of Rates Outwards to 1 Fodder is allowed 20 Hundreds. By the Ordinance aboveſaid, 1 Load 30 Formells, 1 Formel 6 Stones wanting 2 ℔. every Stone 12 ℔. and 1 Pound 25 Shillings *Sterling*. So was the Formel then 70 Pounds, a Weight now grown obſolete.

Meal commonly ſold by Weight, 1 Buſhel 2 Tovits or Half Buſhels, 1 Tovit 2 Pecks, 1 Peck 2 Gallons, 1 Gallon 7 Pounds. So the Buſhel muſt weigh 56 ℔. and hereto agree the old and later Books of Aſſize, yet thereby is the Buſhel more than the Buſhel by Statute, for 56 ℔. or Pints of *Avoirdupois* Weight exactly anſwers to 67 ℔. 8 ℥. *Troy* Weight, whereas the Buſhel by Statute is to contain but 64 ℔. or Pints *Troy*.

Meal how ſold by Weight.

Nutmegs, *Pepper* and *Spice*, as *Allom* and *Cinnamon*, by the Ordinance above mentioned.

Nutmegs, &c. the Old Weight:

Raw Silk, of *China*, *Morea*, Long and Short, 1 Pound, 24 Ounces.

Raw Silk the Pound.

Silk Nubs, or *Husks of Silk*, 1 Pound 21 Ounces, both by the Book of Rates Inwards.

Silk Nubs the Pound.

Wooll, hath the Weight eſtabliſhed by 31. *Edw.* 3. *Cap.* 8. and other Statutes, acding to the *Lunar* Year of 13 Moneths, and 28 Dayes to the Moneth, making one Sack 26 Stones, and 1 Stone 14 Pounds, which makes the Sack 364 ℔. other denominations may be ſeen in the following Table.

Wooll the Weight

The Sack how much.

A Table of Wooll Weight:

Laſt.	1	Sack.					
Sacks.	12	1	Wey.				
Weyes.	24	2	1	Todd.			
Todds.	156	13	$6\frac{1}{2}$	1	Stone.		
Stones.	312	26	13	2	1	Clove.	
Cloves.	624	52	26	4	2	1	
Pounds.	4368	364	182	28	14	7	Pounds.

A Pack of *Wooll* contains but 240 ℔. that is, 2 Hundred Weight, and 16 ℔. over, leſs by 124 ℔. than the Sack of *Wooll* by the Statute.

Difference between the Pack and Sack of Wooll.

Yarne, called *Iriſh Yarne*, by the Book of Rates Inwards is accompted, 1 Pack 4 Hundreds, 1 Hundred 120 ℔.

Yarn the Pack Hundred.

Beſides the Weights allowed as before-mentioned, there hath been allowed at the ings Cuſtom Houſe for Tare (which is the Weight of the Cask or Wrappers, where- Goods are packed up) as followeth.

Tare, What Allowance for it at the Cuſtom-Houſe.

	℔.
Upon Butts of Currance *per Cent.*	14
Caritels of Currance *per Cent.*	16
Quarter Rowles of Currance *per Cent.*	18
Prunes, 6, or 7 C.	84
Prunes 10 C. and upwards	112
Raiſins Solis, *per Cent.*	12
Malaga Raiſins, *per Piece*	$3\frac{1}{2}$
Figgs the Barrel	10
The $\frac{3}{4}$ Barrel	8
The $\frac{1}{2}$ Barrel.	6
The $\frac{1}{4}$ Barrel.	4
Mather the Bale	28
Bales of Raw-Silk from	
Aleppo, with Cotton Legee, the Bale	$34\frac{1}{2}$
Ardus, the Bale	32
Smirna, the Bale.	14
Meſſina, the Bale	8
Bales of Grogran-Yarne from	
Aleppo, the Bale	28
Smirna, the Bale	16
Bales of Silk from	
Naples, the Bale	14
Bologna, the Bale	30
Spaniſh Tobacco the Barrel.	28
The Half Barrel	20
Sugar Cheſts	$\frac{1}{5}$ part.
Sugar in Fatts, 6 C. weight.	84
Goods packed up in Paper.	
For Paper and Packthread, *per Cent.*	2

What allowed by Merchants. *Cloff what.* *Hundred at* Londonderry. *Forraign Geodæticals.*

Moreover there is an Overweight allowed by Merchants called Tret, which is 4 ℔. upon every Hundred of 112 ℔. And alſo 2 ℔. upon every Scale of 3 C. weight, which is called *Cloff*, but in many Places if not conditioned for, will not be allowed. At *Londonderry* in *Ireland* 140 ℔. is reckoned for 1 Hundred weight.

The *Engliſh* Accompt of Meaſures and Weights paſſed, an Eye may be caſt now on the Forrain Accompt of ſmaller *Geodæticals*, whether Ancient, or Modern, and the Credit of both muſt depend on the reſpective Authours out of which they are here Collected.

Of the Hebrews Greeks *and* Latins *to what compared.* Jews *Value of their Gold.*

Ancient Meaſures and Weights to avoid prolixity are referred only to *Hebrew*, *Greek*, and *Latine* as before, moſt of which are here compared to our *Wincheſter* Meaſure, and *Troy* Weight, and the Money valued by our *Sterling* Coine at the rate of 5 *s.* the Ounce of Silver, and 3 *l.* the Ounce of Gold, though ſome ſay the *Jews* valued their Gold but 10 times as much as their Silver. And this is one cauſe why ſome Authours differ in the value they put on their *Hebrew* Coines.

A Table of the Hebrew *Meaſures.*

Hebrew *Meaſures.*

Hebrew *Meaſures.*

- Long
  - (*a*) *Etſbang*, A Fingers Breadth, an Inch.
  - (*b*) *Tophach*, A Palm or Hands Breadth, 4 Fingers, or Inches.
  - (*c*) *Zereth*, A Span.
  - (*d*) *Pagnam*, A Foot, or 12 Inches.
  - (*e*) *Ammah*, A Cubit
    - Common, Half a Yard.
    - Holy, A Yard.
    - Kings, Half a Yard and 3 Fingers.
    - Geometrical, Three Yards.
  - (*f*) *Tſugad*, A Pace, Five Feet.
  - (*g*) *Orgyia*, A Fathom, Six Feet.
  - (*h*) *Chebel*, A Cord, Line, or Rope to meaſure Land with.
  - (*i*) *Kaneh*, A Reed, common 6 Cubits, 6 Cubits and a Palme.
  - (*k*) *Stadium*, A Furlong, 125 Paces.
  - (*l*) *Cibrath haarets*, Half a Dayes Journey, *&c.*
  - (*m*) *Milliarium*, A Mile, 1000 Paces.
  - (*n*) *Paraſanga*, 30 Furlongs.
- Broad
  - (*o*) *Noph*, A Clime, or Tract of Land 60 Feet every way.
  - (*p*) *Maanath*, An Acre, in length 240, in breadth 120 Feet.
- Dry.
  - (*q*) *Kab*, A Quart, *&c.*
  - (*r*) *Omer*, Three Pints and an half.
  - (*s*) *Seah*, A Gallon and an half.
  - (*t*) *Ephah*, Four Gallons and a Pottle.
  - (*u*) *Lethec*, Two Buſhels, 6 Gallons and a Pottle.
  - (*w*) *Homer*, *Cor*, Five Buſhels and Five Gallons.
- Liquid
  - (*x*) *Log*, Half a Pint.
  - (*y*) *Hin*, Three Quarts, *&c.*
  - (*z*) *Bath*, Four Gallons, and an Half.

Hebrews *begin their Dry Meaſures with Barley, and Wet with Eggs.* Shiur *uſed for an Accompt.* Eſtbang, *the Opinions thereof* Zithe *how taken.*

(*a*) *Weemſe* in his *Chriſtian Synagogue*, tells us, the beginning of their Dry Meaſure was Barley, and their Wet Eggs, and therefore ſaith he, An Accompt is called *Shiur*, from *Shiur* Barley. And one *Eſtbang* by him and others, contained the breadth of 6 Barley-Cornes in their greateſt thickneſs. Others again but the ſpace that 2 laid end to end, or 4 laid cloſe ſide by ſide, will lye in. And in round reckoning (though not exactly) paſſed for an Inch. For 4 Fingers make 3 of our Inches, as moſt generally accompt. *Junius* on *Ezek.* 40. 5. and *Jer.* 52. 21. *Holyoke* on the *Latine* word *Pollex*, ſaies an Inch is 1 ½ Fingers breadth, if ſo, then ſhould 3 Inches be 4 ½ digits, or Fingers breadths. Some of the Rabins call a Fingers breadth *Zithe*.

Tophach *how reckoned.* Cith *uſed for what.*

(*b*) The leſſer Palm or Hands breadth, 4. *Eſtbangs*, *Exod.* 28. 16. and 37. 12. *Ezek.* 40. 5. 2 *Chron.* 4. 5. may be accompted 3 Inches *Engliſh* Meaſure. With ſome of the *Rabins*, *Cith*, is uſed for the Meaſure of the Wriſt, to the Roots of the Fingers, which is ſomewhat more than 4 *Eſtbangs*.

Zereth *how much.*

(*c*) The greater Palme 3 *Tophachs*, *Exod.* 28. 16. *Iſa.* 40. 12. properly a Span, and by the 70 rendred Σπιθαμὴ, in *Ezek.* 43. 13. containing the length between the Thumb and the Top of the little Finger ſtretched out.

Pagnam *the length.*

(*d*) *Pagnam*, 4 *Tophachs*, or 16 *Eſtbangs*, *Peter Martyr* in 1 *Kings* 6.

(*e*) A Cubit, ſome ſay, is the length from the Elbow to the Wriſt, others to the top of the longeſt Finger, ſome making it the 4th, others the 6th part of a Man, allowing ſome 2 Foot, moſt 1 ½ Foot, to the Common Cubit or Cubit of a Man ſo called, *Deut.* 3. 11. to which the Great Cubit is reckoned double, 1 *King.* 7. 15. with 2 *Chron.* 3. 15. See it called the Great Cubit, *Ezek.* 41. 8. and the Cubit of the firſt Meaſure, 2 *Chron.* 3. 3. The Kings Cubit in *Herod. lib.* 2. in *Deſcrip. Babyl.* mentioned to be 3 Fingers longer than the Common Cubit. The Geometrical Cubit, *Origen Hom.* 2. *in Gen. Auguſtine de Civit. Dei, lib.* 15. *cap.* 27. take for the Meaſure uſed in Building *Noah's* Ark, And to this Rabbi *Cambi* in his Comment on *Ezek.* 46. 2. cited by *Arias Montanus de Menſuris Sacris*, comes near aſſigning 1000 *Emoth* or Cubits to make a Mile, And ſome ſay, this is the Cubit uſed in *Egypt*. *Gomed*, *Judg.* 3. 16. taken by Interpreters for the ſame with *Ammah* a Cubit.

Ammah *the Sorts of Cubits.*

Gomed *how rendred.*

(*f*) *Tſagad*, or *Tſaad* mentioned often, but becauſe never meaſured in the Text, as to the certainty muſt remain unknown. Several Authours make Two ſorts of Paces, the *Minor* of 2 ½ Feet, a Step, or Half a Remove of the Body; the *Major* of 5 Feet, a Stride or a Pace by removing both Leggs from the Heele at the firſt Stand to the Toe at the laſt.

Tſagad, *uncertain.*

Two ſorts of Paces.

(*g*) A *Fathom* in *Greek* Ὀργυιά, greatly queſtionable if ever any Meaſure with the *Hebrews*, becauſe not once mentioned in the *Old Teſtament*, and but twice in the *New*, in one Verſe, *Viz.* *Acts* 27. 28. is as much of a Rope or Line as a Man can include between the tops of his longeſt Figures, when the Arms are ſtretched out at length in a right line, and ſo uncertain according to the length of the Armes fathoming, but generally taken for 2 Yards or 6 Feet *Engliſh* Meaſure. Some promiſcuouſly taking it for the Pace give it but 5 Feet. Others render it in the *Latine*, *Ulna*, but then muſt not be taken for our Ell, which is but 3 Feet 9 Inches. The *Fathom* is moſtly uſed at Sea to meaſure their Ropes and Soundings, wherein they do not ſtrictly take the length from Finger to Finger, but ſo much as ſhall be included holding the Rope in the Hands extended between the Thumbs and Fore-Fingers.

Fathom *how much.*

When rendred Ulna *muſt not be taken for our Ell.*

Fathom *uſed at Sea.*

(*h*) Moſt confeſs their Ignorance in the length of the *Chebel*, *Pſal.* 16. 6. taken Metonymically for the Inheritance it ſelf.

Chebel, *uncertain.*

(*i.*) Uſed to meaſure Buildings, as the *Chebel*, Lands, expreſſed *Ezek.* 40. 5. to contain 6 Cubits, and an Handsbreadth, but *Tremelius* on the place takes the Reed to contain ſo much of the Kings Cubits, though the Common Reed, by the *Targum* and ſeveral others is accompted but 6 Cubits juſt. *Salel* a *Rabinical* word for a Reed of 6 Cubits, and *Rus* for 70 Reeds, to be found in their Writings.

Kaneh *of what uſe.*

The length.

Salel *and* Rus *how taken.*

(*k*) A *Furlong*, (*quaſi*, *Furrowlong*, becauſe in Champion Countreys their Furrows were uſually very long) not mentioned in the *Old Teſtament*, a Meaſure brought in with the *Græcian* Monarchy as ſeemeth, becauſe firſt met with in 2 *Mac.* 12. 9. continued till the *New Teſtament* Times, and there often the compute of diſtances at Sea and Land, with *Pliny*, *lib.* 2. *cap.* 23. *Iſidore* and others made to be 125 Paces, and ſo muſt be the Eighth part of an *Italian* Mile, which contains 1000 Paces, though the ſame *Pliny* will have 7 ½ Furlongs make a Mile. There is no doubt but *Stadium* hath admitted of different acceptations, as the Controverſie between *Pliny* and *Diodorus Siclus* teſtifies, So alſo *Stadium* becauſe of the different content thereof in ſeveral Countreys with Authours is reſpectively to be taken, beſides the *Italian* which contained as before 125 Paces, or 625 Feet, ſometime for the Olympique at 120 Paces, or 600 Feet, and ſometime for the *Pirhique Stadium*, at 200 Paces, or 1000 Feet.

Stadium *the Furlong from whence.*

The Length.

Difference thereof.

(*l*) Half a Dayes Journey, a Dayes Journey, a Sabbath-Dayes Journey, and a Space leſs than any of them, a Bow-ſhot, all left indeterminate in the Old Teſtament, the New *Acts* 1. 12. *John* 11. 18. accompts the Sabbath-Dayes Journey about 15 Furlongs, which doubled in a Journey forward and backward made about 4 Miles. So *Fuller*, *Piſ. Sight of Paleſtine*, *Book* 1. *p.* 43. 44.

Cibrath, &c. *differently taken.*

(*m*) The *Italian* Mile conſiſting of 1000 Paces, hath ſet the Name *Mile* from the Latine *Mille* in *Engliſh* 1000. Neverthele*ſ*s in ſeveral Countreys more than 1000 Paces go to make up a Mile, and the *German* Mile is 4 times as much as the *Italian*, and though *Buxtorf* renders the word *Cibrath*, *Miliare*, yet we find no Mile mentioned by Tranſlators of the Old Teſtament, *Mat.* 5. 41. argues it rather of *Roman* Extraction than *Jewiſh*, and Kimchi lib. Rad. thinketh that כ in *Cibrath* is but *Servile*, and that the Root is *Barah*, or as ſome write *Berah*, which *Tremelius*, *Gen.* 35. 16. and 48. 7. and 2 *King* 5. 19. readeth it *Exiguum terræ Spatium*, and our Tranſlation, a little way. Nevertheleſs ſome Learned Men conceive *Berah* anſwers to the proportion of a *Roman* Mile, but *Barah* properly ſignifying a Dinner or Meale. Others will when applyed to Journeys take it for ſuch a ſpace of Ground, as uſually is travelled or conveniently may be gone in Half a Day, between Meal and Meal, or Bait and Bait.

Miles *of different length.*

Barah Berah *what.*

(*n*) This

Paraſang *how taken.*

(*n*) This ſeems to be a *Perſian* Word and Meaſure, taken for 30 Furlongs, *Herodot. lib.* 2. *Ramus Geometry* ſet forth by *Bedwell* in *Engliſh*, by others for a *German* Mile. *Elias* in *Thisbi* mentioneth a *Perſah*, which he ſaith was the Great Mile, and contained 4 leſſer.

Noph *how rendred.*

(*o*) *Noph*, rendred by *Buxtorſe*, A Clime, or Tract of Land, *Pſal.* 48 2. *Noph*, alſo the name of a City in Scripture, but the content thereof not there found, other Writings, as *Holyokes Dictionary*, *&c.* reckons it 60 Feet every way, yet may be a Plot of ground big enough for a ſmall Town.

Maanath *diverſly taken.*

(*p*) Uſed 1 *Sam.* 14. 14. and made by ſeveral fitly to correſpond with *Jugerum* in the *Latine*, but differ in the quantity, ſome reckon it 200 Foot every way, others 240, in length, and 120 in breadth. *Quint.* 1. 28. *Iſidor.* 15. 15.

Kab *what it contained.*

(*q*) The fourth part of a *Kab*, 2 *Kings* 6. 25. for want of more accurate Correſpondencies may be taken for our Half Pint, and ſo the *Kab* for our Quart. *Buxtorf* out of *Rab. Alphes. Tract. de Paſchate. cap.* 5. *fol.* 176. with whom divers conclude, that the fourth part of a *Kab*, and a Log are of a like quantity, and that each contained 6 Eggs, *viz.* as much as will fill Six Hen Egg-ſhells of the ordinary, or middle ſize, and thence called by ſome a *Sextary*, and ſo accordingly the *Kab*, 24 Eggs, or 4 Logs.

Called a Sextary. Omer *how much.*

(*r*) An *Omer*, was the Dayly Ordinary of a Man, and the tenth part of an *Ephah*, *Exod.* 16. 16. 36. more than 3 ½ Pints our Meaſure, and not full a Pottle, 43 ½ Eggs.

Seah *the Content.*

(*s*) A *Seah*, noted *Gen.* 18. 6. and 2 *King.* 7. 1. commonly eſtimated by Writers at 6 *Kabs*, that is 2 Hins, or 144 Eggs, or about a Gallon and an Half our meaſure. *Godwin* in *Moſes* and *Aaron*, *lib.* 6. *cap.* 9.

Ephah *the Content.*

(*t*) An *Ephah* contained 3 *Seahs* or 18 *Kabs*, in Eggs 432 about 9 Pottles *Engliſh* Meaſure, it was the tenth part of an *Homer*, *Ruth* 2. 17. *Ezek.* 45. 11.

Lether *its content.*

(*u*) *Lether*, in *Hoſea*, 3. 2. is reckoned to be the Half of an *Homer* or 5 *Ephahs*, and conſequently 90 *Kabs*.

Homer *and* Cor *how much.*

(*w*) An *Homer*, being 10 *Ephahs*, *Ezek.* 45. 11. is thought by ſome to be the ordinary Burden of an Aſs, but with us a good Horſeload, at the Rates aforeſaid 5 Buſhels and 5 Gallons, our Meaſure. The *Cor* was equal to the *Homer*, common to meaſure both Liquids and Dry. *Ezek.* 45. 14. *Luke* 16. 7.

Log *the Content.*

(*x*) The *Log* was the ſmalleſt Meaſure for Liquids, we find mentioned, ſee *Levit.* 14. 12. and before in the *Kab*.

Hin *how much.*

(*y*) The *Hin* often in Scripture, in quantity Three Quarts our Meaſure or thereabouts, 3 *Kabs*, or 72 Eggs. This Meaſure was divided into the Half, Third, Fourth, and Sixth parts, *Numb.* 15. 6, 9. and 28. 5. *Ezek.* 4. 11.

Bath *and* Solomons Sea *their Contents.*

(*z*) The *Bath*, *Ezek.* 45. 14. is made alike to the *Ephah*, ſo muſt the Molten Sea, 2 *Chron.* 4. 5. holding 3000 Baths, contains 210 Quarters, 7 ½ Buſhels, or 13500 Gallons our Meaſure.

Nebel *how tranſlated.*

As for *Nebel*, tranſlated *Jer.* 13. 11. a Bottle, few reckon it a Meaſure except *Epiphanius* who ſaies it contains 150 Sextaries.

A Brief view of moſt of the Hebrew *Meaſures aforeſaid, with their* Engliſh *Content may be taken in the following Table.*

A Table of Hebrew Long Meaſures compared with the Engliſh.

The Table of *Hebrew* Long Meaſures.

	1	8	o	$833\frac{1}{3}$	1000		5000		60000	Milliarium
	Mile.	1	n	$104\frac{1}{6}$	125	h	625		7500	Stadium
	Milliariū.	Furlongs	1	$1\frac{13}{24}$	$1\frac{17}{20}$	lis	$9\frac{1}{4}$	s.	111	Kaneh
Mill.	.1	Stadium	Reeds	1	$1\frac{1}{5}$	ng	6	re	72	Orgya
Stadium.	8	1	Kaneh.	Fathoms	1	E	5	ſu	60	Tſagad
Kaneh.	$540\frac{20}{37}$	$67\frac{21}{37}$	1	Orgya	Paces.	1	$1\frac{1}{2}$	ca	13	Ammah c.
Orgya.	$833\frac{1}{3}$	$104\frac{1}{6}$	$1\frac{13}{24}$	1	Tſagad	Cubits	1	M	12	Pagnam
Tſagad.	1000	125	$1\frac{17}{20}$	$1\frac{1}{5}$	1	Ammah c.	Foot	1	9	Zereth
Ammah Com.	$3333\frac{1}{3}$	$416\frac{2}{3}$	$6\frac{1}{6}$	4	$3\frac{1}{3}$	1	Pangam	Palmes	3	Tophach
Pagnam.	5000	625	$9\frac{1}{4}$	6	5	$1\frac{1}{2}$	1	Zereth.	Inches	
Zereth.	$6666\frac{2}{3}$	$833\frac{1}{3}$	$12\frac{1}{3}$	8	$6\frac{2}{3}$	2	$1\frac{1}{3}$	1	Tophach	
Tophach.	20000	2500	37	24	20	6	4	3	1	
Eſtbang.	80000	10000	148	96	80	24	16	12	4	

A Table of Hebrew Concave Meaſures compared with the Engliſh.

The Table of *Hebrew* Dry and Liquid Meaſures.

	$5\frac{5}{8}$	$11\frac{1}{4}$	$22\frac{1}{2}$	45	90	180	300	720	Homer, Cor.
	Buſhels	$5\frac{5}{8}$	$11\frac{1}{4}$	$22\frac{1}{2}$	45	90	180	360	Lethec.
	HomerCor	Tovits	$2\frac{1}{4}$	$4\frac{1}{2}$	9	18	36	72	Ephah, Bath.
Homer, Cor.	1	Lethec	Pecks	$1\frac{1}{2}$	3	6	12	18	Seah.
Lethec.	2	1	Ephah, Bath	Gallons	$1\frac{1}{2}$	3	6	12	Hin.
Ephah, Bath.	10	5	1	Seah	Pottles	$1\frac{4}{5}$	$3\frac{3}{5}$	$7\frac{1}{5}$	Omer.
Seah.	30	15	3	1	Hin	Quarts	2	4	Kab
Hin.	60	30	6	2	1	Omer	Pints	1	Log, $\frac{1}{4}$ Kab.
Omer.	100	50	10	$3\frac{1}{3}$	$1\frac{2}{3}$	1	Kab	Half Pints	
Kab.	180	90	18	6	3	$1\frac{4}{5}$	1	Log	
Log, $\frac{1}{4}$ Kab.	720	360	72	24	12	$7\frac{1}{5}$	4	1	
Eggs.	4320	2160	432	144	72	$43\frac{1}{5}$	24	6	

Hebrew *Weights.*

Hebrew *Weights.*

The principal Weights in uſe among the *Jews* were Talents, Pounds, Shekels, Drams, and other ſmall diviſions of the Shekel, all which may be further ſeen in the Accompt of their Money.

A Table of the Hebrew *Coines and their Value.*

Hebrew Coines.

					l.	s.	d.
Hebrew	Money	Silver	(*a*) *Shekel*, of the *Sanctuary* in	Weight ½ ℥ *Troy*. Value to *Sterling* Money.	00	02	06
			(*b*) *King's Shekel*, Half the *Sanctuary Shekel*, called *Bekah*.		00	01	03
			(*c*) Third part of a *Shekel*.		00	00	10
			(*d*) *Zuz*, Fourth part of a *Shekel*.		00	00	07½
			(*e*) *Gerah*, *Agorah*, *Keſhitah*.		00	00	01½
		Gold	(*f*) *Zahab* *Golden Siclus*, or *Shekel*	each ½ ℥ *Troy* in value 12 *Silver Shekles* of the *Sanctuary*	01	10	00
			(*g*) *Adarkon.* *Dracmon*, or *Darcon.*	each half ſo much	00	15	00
	Sums of Money		(*h*) *Maneh*, or *Mina*, or a Pound, valuable in	Gold 100 *Shekels.* 25 ℥ *Troy.*	75	00	00
				Silver, 60 *Shekels.* 30 ℥ *Troy.*	07	10	00
			(*i*) *Chichar*, or *Talent*, in Weight 3000 *Shekels* 125 ℔. *Troy* valuable in	Gold	4500	00	00
				Silver	375	00	00

Shekel *how inſcribed and ſtamped.*

Whence the word.

(*a*) This piece of Coin (is often mentioned in Sacred Writ, and looks like the Standard of all the reſt) on the one ſide ſhewed the Veſſel in which the *Manna* was, inſcribed with this Perigraphe *Shekel Iſrael*, on the other ſide, the Rod of *Aaron* that budded, with this Inſcription, *Jeruſalem Kodſhaſh*, in *Engliſh*, *The Holy Jeruſalem*, called *Shekel* from *Shakal* to weigh, for Money at the firſt ſeems to be a Merchandiſe exchanged or given for other Commodities, as *Gen.* 23. 16. after the *Chaldee* called *Silgha*, and commonly with the *Hebrews*, *Keſeph*, *i. e. Silver*, and being put abſolutely rendred a *Silverling*, or piece of Silver, by Expoſitors; as uſually as a *Shekel* when the quality is not mentioned is taken for a Silver *Shekel*.

Silverling *what.*

Bekah *how much, and for what paid.*

(*b*) Firſt mentioned, *Gen.* 24. 22. paid by all as a Yearly tribute, *Exod.* 30. 13. 15. 2 *Chron.* 24. 6. 9. towards the Repair of the Tabernacle firſt, and after of the Temple, between this and the *Sanctuary Shekel*, ſome mention a third ſort of *Shekel*, called the Common *Shekel*, valued at 20 *d.* of our Money, of which I find nothing but uncertainty.

Third part of a Shekel.

(*c*) In *Nehem.* 10. 32. noted as a yearly tribute given of the *Jews* by a Civil Decree to the ſecond Temple.

Zuz, *how much.*

(*d*) 1 *Sam.* 9. 8. ſpoken of, and equal to the Old *Attick* Dram, and *Roman* Peny, as ſeveral ſay, and by ſome called *Zuz*, *Zuza*, *Zur*, and *Zura*.

Gerah, *or* Agorah *the Image of a Lamb thereon.*

(*e*) *Agorah*, rendred *Gerah* by the *Chaldee* Paraphraſe, and alſo *Megna*, or *Megha*, in that Tongue, by the *Arabians*, *Megah*, *Greeks*, *Obolus*, and in *Engliſh*, *a Piece of Silver*, 1 *Sam.* 2. 36. called alſo in the *Hebrew*, *Keſhitah*, becauſe ſigned with the Image of a Lamb, See *Gen.* 33. 19. *Joſhua* 24. 32. Twenty of theſe made a *Shekel*. *Exod.* 30. 13.

Zahab *a Gold Coine how reckoned.*

(*f*) The *Zahab*, and Golden *Shekel* in *Rider*'s Dictionary undervalued at 15 *s.* for though *Hunt* in his *Handmaid to Arithmetick* out of *Breerwood de nummis*, mentioneth the Weight but 2 *Attick* Drams, yet he valueth it at 30 *s.* *Alſted* in his *Encyclopædia of Arithmetick* makes it 4 Drams, which accompted ½ ℥ *Troy*, muſt yield double that value of 15 *s.* and moſt Authours agree that the Gold *Shekel* was equal in weight with the Silver *Shekel*, ſo the difference muſt be only in the value of the Mettal. Some to keep up the Credit of differing Authours conceive there were 2 ſorts of Golden *Shekels* as well as Silver, the one double to the other.

A Darkon *or* Drakmon *ſeems a* Perſian *Coin.*

(*g*) Theſe Golden *Adarkons*, and *Drakmons* ſeem to be *Perſian*, or Coines of ſome other *Nations*, and Currant, not Coined in *Judea*, not read of till after the Captivity, for though the word be uſed 1 *Chron.* 29. 7. yet both Books of the Chronicles, as moſt take it were penned by *Ezra*, after his return from *Babylon*, in whoſe Book mention is made thereof, as in *Ezra* 2. 69. and 8. 27. &c. in *Greek* called *Drachme*, and in *Engliſh*, rendred *Drams*. *Alſted*, in his *Encyclopædia* of *Arithmetick* makes the value equal to a Ducat of *Hungary*. But *Hunt* who throughout his Book hath much loſt himſelf in his Method; forgot himſelf in valuing 2 *Attick* Drams in the *Zahab* at 30 *s.* and 2 *Attick* Drams in the *Adarkon* (which he ſaith is the weight thereof) at 15 *s.*

Maneh, Mina, *how much.*

Difference about it.

(*h*) The *Maneh* in Gold by comparing 1 *King* 10. 17. with 2 *Chron.* 9. 16. is found to be 100 *Shekels*, which *Buxtorf* and others underſtand not of the Holy but the Royal *Shekel*. The *Maneh* in *Ezek.* 45. 12. ſeems to be 60 *Shekels*, and hereto ſeveral agree, but ſome think it was now increaſed 10 *Shekels* more than of old, and call it, The *New Maneh*, valued nevertheleſs with moſt after the *Holy Shekel*, in Silver at 7 ℔. 10 *s. Buxtorfe* tells us alſo of a *Manch* of 25 *Holy Shekels*. The *Aſſemblies Annotations* on *Ezek.* 45. 12. make 3 ſorts of *Manehs*, *viz.* Common 15, the Kings 20, and the Holy 25 *Shekels*, but the word *Maneh* is Singular and not Plural in the *Hebrew* Text there, as noting all thoſe diviſions to make but one *Maneh*.

Talent, *how much.*

(*i*) The *Chichar*, by ſome wrote *Kichar*, and commonly tranſlated *Talent*, contained 3000 *Shekels*, as may be collected from *Exod.* 38. 25, 26. where the Silver Collected is expreſſed to be 100 Talents, and 1775 Shekels, *v.* 25. and the Perſons that paid it at half a Shekel a piece, *v.* 26. numbred to be 603550, whereof the half is 301775, which divided by 3000, the Shekels in one Talent, makes 100 Talents, and leaves the odd Shekels remaining 30|1775(100
3 000

Doubts thereof.

The Talent thus valued after the Holy Shekel, makes the Number of Talents mentioned in ſome Texts of Scripture, eſpecially 1 *Chron.* 22, and 29 *Chapters* amount to ſuch Maſſy Sums, that ſome think the Talents are to be reckoned at the rate of the other Shekel; and others, not improbably that the *Jews* had a piece of Money or Plate of Gold of ſmall value (as may be obſerved anciently in *Homer Iliad*, *lib.* 23.) called a Talent. And *Fuller* in his *Piſgah Sight of Paleſtine*, *Book* 3. *p.* 356, 357. ſhews whereon ſuch an Opinion may be ſtrengthened, and that the Talent mentioned in ſome Scriptures may be rather this than the other.

Talent, *a piece of Coine.*

A Table of Hebrew Gold *and Weights.*

The Accompt both of the Hebrew Weight *and* Money, *with their Value in* Sterling-Money *and* Troy-Weight, *is contracted into the following Table.*

Hebrew Gold *and* Weights.

Chichar, *or* Talent.	1	Maneh.			
Maneh, *or* Pound.	old 60 | new 50	1 new	old	Zahab.	
Zahab, *or* Shekel.	3000	60	50	1	Adarkon.
Adarkon, *or* Dram.	6000	120	100	2	1
Troy-Weight.	125 ℔.	$2\frac{1}{2}$ ℔	2℔ 1℥	$\frac{1}{2}$ ℥	$\frac{1}{4}$ ℥
Sterling-Money.	4500 l.	90 l.	75 l.	1l. 10s.	15 s.

A Table of Hebrew Silver *and Weights.*

Hebrew Silver *and* Weights.

Chichar, *or* Talent.	1	Maneh.						
Maneh, *or* pound.	old 60 | new 50	1 new	old	Shekel.				
Shekel.	3000	60	50	1	Bekah.			
Bekah.	6000	120	100	2	1	Third part.		
Third parts.	9000	180	150	3	$1\frac{1}{2}$	1	Zuz.	
Fourth Parts, Zuzims, *or* Drams.	12000	240	200	4	2	$1\frac{1}{3}$	1	Gerah.
Gerahs.	60000	1200	1000	20	10	$6\frac{2}{3}$	5	1
Troy-Weight.	125 ℔	$2\frac{1}{2}$ ℔	2℔ 1℥	$\frac{1}{2}$ ℥	$\frac{1}{4}$ ℥.	$\frac{1}{6}$ ℥	$\frac{1}{8}$ ℥	$\frac{1}{40}$ ℥
Sterling Money.	375 ℔.	7l 10s.	6l. 5s.	2s. 6d.	1s. 3d.	10d.	$7\frac{1}{2}$ d.	$1\frac{1}{2}$d.

Greeks, *their* Geodæticals.

The like Method may be obſerved in viewing the Meaſures, Weights and Monies of the old *Greeks*, who imitating the *Hebrews* kept ſome like theirs, altered and added others.

A Table of Long Meaſures of the Greeks, and the Engliſh Inches therein.

Græcian *Long Meaſures.*

		Paraſang.	Dolich.	Mile.	Hippacon.	Diaulus.	Furlong.	Plethron.	Fathome.	Pace.	Cubit.	Pygon.	Pygme.	Foot.	Span	Orthodoron.	Licha.	Paleſt.
Schoenes	1																	
Paraſangs	2	1																
Dolichs	5	$2\frac{1}{2}$	1															
Miles	$7\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{1}{2}$	1														
Hippicas	15	$7\frac{1}{2}$	3	2	1													
Diaulus	30	15	6	4	2	1												
Furlongs	60	30	12	8	4	2	1											
Plethras	375	$187\frac{1}{2}$	75	50	25	$12\frac{1}{2}$	$6\frac{1}{4}$	1										
Fathoms	6250	3125	1250	$833\frac{1}{3}$	$416\frac{2}{3}$	$208\frac{1}{3}$	$104\frac{1}{6}$	$16\frac{2}{3}$	1									
Paces	7500	3750	1500	1000	500	250	125	20	$1\frac{1}{5}$	1								
Cubits	25000	12500	5000	$3333\frac{1}{3}$	$1666\frac{2}{3}$	$833\frac{1}{3}$	$416\frac{2}{3}$	$66\frac{2}{3}$	4	$3\frac{1}{3}$	1							
Pygons	30000	15000	6000	4000	2000	1000	500	80	$4\frac{4}{5}$	4	$1\frac{1}{5}$	1						
Pygmes	$33333\frac{1}{3}$	$16666\frac{2}{3}$	$6666\frac{2}{3}$	$4444\frac{4}{9}$	$2222\frac{2}{9}$	$1111\frac{1}{9}$	$555\frac{5}{9}$	$88\frac{8}{9}$	$5\frac{1}{3}$	$4\frac{4}{9}$	$1\frac{1}{3}$	$1\frac{1}{9}$	1					
Feet	37500	18750	7500	5000	2500	1250	625	100	6	5	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{8}$	1				
Spans	50000	25000	10000	$6666\frac{2}{3}$	$3333\frac{1}{3}$	$1666\frac{2}{3}$	$833\frac{1}{3}$	$133\frac{1}{3}$	8	$6\frac{2}{3}$	2	$1\frac{2}{3}$	$1\frac{1}{2}$	$1\frac{1}{3}$	1			
Orthodoron	$54545\frac{5}{11}$	$27272\frac{8}{11}$	$10909\frac{1}{11}$	$7272\frac{8}{11}$	$3636\frac{4}{11}$	$1818\frac{2}{11}$	$909\frac{1}{11}$	$145\frac{5}{11}$	$8\frac{8}{11}$	$7\frac{3}{11}$	$2\frac{2}{11}$	$1\frac{9}{11}$	$1\frac{7}{11}$	$1\frac{5}{11}$	$1\frac{1}{11}$	1		
Lichas	60000	30000	12000	8000	4000	2000	1000	160	$9\frac{3}{5}$	8	$2\frac{2}{5}$	2	$1\frac{4}{5}$	$1\frac{3}{5}$	$1\frac{1}{5}$	$1\frac{1}{10}$	1	
Paleſtes	150000	75000	30000	20000	10000	5000	2500	400	24	20	6	5	$4\frac{1}{2}$	4	3	$2\frac{3}{4}$	$2\frac{1}{2}$	1
Dactyls.	600000	300000	120000	80000	40000	20000	10000	1600	96	80	24	20	18	16	12	11	10	4
Inches *Eng.*	450000	225000	90000	60000	30000	15000	7500	1200	72	60	18	15	$13\frac{1}{2}$	12	9	$8\frac{1}{4}$	$7\frac{1}{2}$	3

Schoen *how different.*

A *Schoen*, in *Greek* Σχοῖνος, generally taken with *Herodot. lib.* 2. to contain 60 Furlongs, *Pliny lib.* 12. *cap.* 14, reckons it 40, ſome ſay 32, others but 30, an *Egyptia* Meaſure as ſome think, *Scapula in Verb.*

Paraſang. *Vide antea.*

A *Paraſang*, See before in the *Hebrew* Meaſures *p.* 84.

Dolich *how taken.*

A *Dolich*, ſome will have 24 Furlongs, but the Common Accompt is but 12 *Scapula in Verb.*

Mile, *Vide antea.*

A *Mile*, called μίλιον, *Mat.* 5. 41. by birth a *Latine*, though in uſe a Word and Meaſure with the *Greeks*, *Hebrews*, *Syrians*, *&c.* See before in *Hebrew* Meaſures, *p.* 84 alſo *Leigh. Crit. Sacra gr.*

Hippicon *the Length, and* Diaulus $\frac{1}{2}$.

An *Hippicon*, commonly taken for 4 Furlongs.

A *Diaulus*, for half an *Hippicon.*

A *Furlong*, See before in the *Hebrew* Meaſures, *Stadium, p.* 83.

Plethron *uncertain.*

A *Plethron*, *Scapula* from *Plutarch* reports an Acre from *Suidas* $\frac{1}{6}$ of a Furlong, o an 100 Feet, from *Heſychius*, a Meaſure of 10000 Feet. of others 100 Furlongs, suc uncertainty there is in the Meaſure or diſcrepancy in the Authours In the Table followed *Suidas*, at 100 Feet, which occaſion the following Numbers in the Table t differ from thoſe in *Alſted* and ſome other Authours.

A *Fathome*, See before in the *Hebrew* Meaſures, *Orgyia*, *p.* 83.

A *Pace*, in *Greek* βῆμα, See before in the *Hebrew* Meaſures, *Tſagad*, *p.* 83.

A *Cubit*, after the meaſure of the Common Cubit with the *Hebrews*: See before in their Meaſures *Ammah*, in *Greek* πῆχυς.

Fathome, Pace *and* Cubit, *Vide*, antea.

A *Pygon*, taken with *Heſychius* to be a Meaſure containing the ſpace from the Elbow to the Fingers bent, called by ſome *Palmipes*, of a Foot, and a Palm, being 20 Fingers breadth, *Scapul. in verb.* and others.

Pygon, *how taken.*

A *Pygme*, uſed *Mark* 7. 3. in meaſure taken for the length from the Elbow to the Fingers cloſed, as the Hand is contracted when it is called a Fiſt, *Leigh Crit. Sacra. Greek.* 2 Fingers breadth ſhorter than the *Pygon*.

Pygme, *the length.*

A *Foot*, in *Greek* πȣ͂ς, the ſame with the *Hebrew Pagnam*, though *Hunt*, upon what Authority I know not, will have the *Greek* πȣ͂ς leſs than the *Roman* Foot, ½ Inch, and greater than the *Hebrew Pagnam* almoſt ¼ Inch. But *James Capel* a Man of far more exactneſs in his Treatiſe *De Menſuris intervallorum*, makes the *Attick* Foot and the *Engliſh* to agree as near as 75 to 76, and the *Roman* and *Engliſh* Foot as 18 to 19. And the Learned *Willebrand*, *Snellius* of *Leiden* in his *Eratoſthenes Batavus* (who ſome think comes nearer the Truth) makes the

Pous, *a Foot, like* Pagnam.

Engliſh	Foot agree as	484.
Roman		500.
Old *Greek*		521.

A *Span*, in *Greek* σπιθαμή, in *Latine*, *Palmus major*, and *Dodrans*, by *Poll. lib.* 2. *Heſych.* and others alwaies accompted 12 Fingers breadth, like the *Hebrew Zereth*, anſwering to 9 of our Inches. One calls this Meaſure a *Graciary*.

Spithama *for how much taken.*

An *Orthodoron*, *Poll. lib.* 2. calls a *Palm*, ſome others a *Span*, ſhorter by a Fingers breadth, than the *Span*, or greater *Palme*.

Orthodoron, *the length.*

A *Lichas* is generally reckoned for the length between the Thumb and the Extent of the Fore-finger, ſhorter than *Orthodoron* by a Fingers breadth. Some make it the ſame with *Dichas*, but *Cooper* in his *Dictionary* makes *Dichas* but 8 Fingers breadths, when moſt agree *Lichas* is a *Span* with the Thumb and Fore-Finger as before.

Lichas, *how taken.*

A *Paleſt*, in *Greek* Παλαίστη, alſo Δῶρον, is the leſs *Palme*, agreeing to the *Hebrew Tophach*, 4 Fingers breadths, anſwering to 3 of our Inches.

Paleſt, *the Length.*

A *Dactyl*, Digit or Fingers breadth: See *Etſbang* in the *Hebrew* Meaſures.

Dactyl, *what.*

The *Greeks* had few if any Land-Meaſures of length and breadth notable, ſave what they borrowed of the *Hebrews* and *Latines*, *Plethron* before ſpoken of, is often rendred *Jugerum* the Old *Latine* word for an *Acre*, which gives occaſion to ſome to think, that the Meaſures aforeſaid, or many of them were conſidered both in breadth and length, as neceſſity ſerved to make uſe of them. Wherefore paſſing over what might be further ſaid of thoſe, the next that come in order to be ſeen are Meaſures of length, breadth, and depth.

Plethron, *oft rendred in Latine* Jugerum.

Whether by confounding the *Attick*, and *Roman* Sextaries, or the Pounds Menſural or Ponderal, or the *Attick* and *Georgick* Meaſures, or by what other occaſions, I know not; but ſure I am, it is hard to reconcile Authors one to another, and ſome to themſelves about the capacious Meaſures of the *Greeks*, and being not willing to ſpare ſo much time, or tumifie theſe Papers, I have given much credit to the Accompt ſet down by *Alſted* out of *Daniel Angelocrator. lib. de Ponderibus.* and from him and others wherein moſt general agreement is to be found, have collected what follows.

Authors hardly reconciled about the Græcian *Meaſures.*

Greek Meaſures of capacity may be conſidered, as *Indigenital* or of moſt uſe and chief note among them. Or, 2. *Exotick*, or uſed but in ſome particular places. Or Thirdly, *Hippiatrical*, or uſed about the Cure of Beaſts.

Capacious Meaſures of the Greeks *of* 3. *ſorts.*

Indigenital, or Proper *Græcian* Meaſures are again conſidered as uſeful for things dry only, or liquid, or both, and thoſe either *Attick* or *Georgick*.

Indigenital, Attick, *or* Georgick.

Græcian *Indigenital Capacious Meaſures.*

Dry	Common	Liquid.
Kypſele.	*Sextary.*	*Metretes.*
Medimnus.	*Kotyle.*	*Amphora.*
Modios.	*Oxybaph.*	*Chous.*
Choenix.	*Kyath.*	*Tetarton.*
	Concha.	
	Myſtrum.	
	Chemes.	
	Cochlear.	

A Table of the Attick *Meaſures of the* Greeks *compared with the* Engliſh.

The Table of Attick Meaſures *compared with the* Engliſh.

Kypſele.	1	Medimnus Metretes											
Medimnus Metretes	6	1	Modion.										
Modios.	36	6	1	Chous.									
Chous.	72	12	2	1	Choenix.								
Choenices.	288	48	8	4	1	Sextary.							
Sextaries.	432	72	12	6	$1\frac{1}{2}$	1	Kotyle.						
Kotyles.	864	144	24	12	3	2	1	Tetarton.					
Tetartons.	1728	288	48	24	6	4	2	1	Oxybaph.				
Oxybaphs.	3456	576	96	48	12	8	4	2	1	Kyath.			
Kyaths.	5184	864	144	72	18	12	6	3	$1\frac{1}{2}$	1	Concha.		
Conchas.	10368	1728	288	144	36	24	12	6	3	2	1	Myſtrum.	
Myſtras.	20736	3456	576	288	72	48	24	12	6	4	2	1	Cheme.
Chemes.	25920	4320	720	360	90	60	30	15	$7\frac{1}{2}$	5	$2\frac{1}{2}$	$1\frac{1}{4}$	1
Cochlears.	51840	8640	1440	720	180	120	60	30	15	10	5	$2\frac{1}{2}$	2
Pints *or* Pounds *Troy.*	648	108	18	9	$2\frac{1}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{40}$

A Table of the Georgick *Meaſures of the* Greeks *compared with the* Attick.

The Table of Georgick Meaſures *compared with the* Attick.

Medimnus. Metretes.	1	Amphora.										
Amphoras.	2	1	Chous.									
Chous.	8	4	1	Choenix.								
Choenices.	48	24	6	1	Sextary.							
Sextaries.	72	36	9	$1\frac{1}{2}$	1	Kotyle.						
Kotyles.	96	48	12	2	$1\frac{1}{3}$	1	Tetarton.					
Tetartons.	192	96	24	4	$2\frac{2}{3}$	2	1	Oxybaph.				
Oxybaphs.	384	192	48	8	$5\frac{1}{3}$	4	2	1	Kyath.			
Kyaths.	576	288	72	12	8	6	3	$1\frac{1}{2}$	1	Concha.		
Conchas.	1152	576	144	24	16	12	6	3	2	1	Myſtrum.	
Myſtras.	2304	1152	288	48	32	24	12	6	4	2	1	Cheme.
Chemes.	2880	1440	360	60	40	30	15	$7\frac{1}{2}$	5	$2\frac{1}{2}$	$1\frac{1}{4}$	1
Cochlears.	5760	2880	720	120	80	60	30	15	10	5	$2\frac{1}{2}$	2
Atticke Cochlears.	12960	6480	1620	270	180	135	$67\frac{1}{2}$	$33\frac{3}{4}$	$22\frac{1}{2}$	$11\frac{1}{4}$	$5\frac{5}{8}$	$4\frac{1}{2}$

Kypſele *the content.*

Kypſele, or after the *Latine*, *Cypſele*, by *Scap.* out of the *Annotat. Schol. Ariſtoph.* is reckoned for a Corn-Meaſure, and by *Hunt* to contain 6 *Atticke Medimnos.*

Medimnus *how reckoned.*

Medimnos, or *Medimnus* both *Attick* and *Georgick*, *Suiadas*, *Scapula*, *Legat*, and ſeveral others agree to contain 48 *Choenices* or 72 *Sextaries*, and *Pollybius lib.* 4. affirms the e *Hemimedimnus* to be 24, *Choenices* accordingly; but becauſe the *Georgick Coenix* as bigger than the *Atticke*, the *Medimnus* was alike proportional.

Metretes, or *Metreta*, a Liquid Meaſure, rendred in Latine ſometime *Cadus*, ſome- me *Amphora*, (but corruptly *Amphora* being another Meaſure) *Legat* and ſome others ake equal with the *Atticke Medimnus*, but that all other *Georgick* Meaſures ſhould e greater than the *Atticke*, and only the *Metretes* equal, ſeems unlikely, uſed *John* . 6. in *Engliſh* there rendred a Firkin, but how juſtly, *quære*.

Metretes *how rendred.*

Amphora, or *Amphoreus*, properly a *Georgick* Liquid Meaſure, and was half the eorgick *Medimnus* or *Metretes*, by *Alſted*, though *Schrevelius* mentions it as an *Atticke* eaſure, and ſaies it contained 3 Urnes, 20 whereof made a *Roman* Cule.

Amphora *the Content.*

Modios, or *Modion*, both Name and Meaſure ſeem borrowed from the Latine *odius*, uſed in *Mat.* 5. 15. *Mark* 4. 21. *Luke* 11. 33. ordinarily Engliſhed a Buſhel, ut in quantity far leſs than the *Engliſh* Buſhel. Neither may the Latine *Modius* and reek *Modios* agree exactly, though each ſhould contain a like number of *Sextaries*, the *Sextaries* be different. *Alſted* reckons it but 8 *Atticke Coenices*, that is 12 *Sex- ries*, as in the Table above. Great Annotations 16 *Sectaries*, and by ſome, a Pint ſs than our Peck.

Modios *whence.*

Different from the Engliſh *Buſhel.*

Chous, wrote often *Chus*, ſometime *Choas*, and confuſedly *Congius* for a *Roman* Mea- re of that Name, was both *Atticke* and *Georgicke*, that contained 6 *Atticke Sextaries*, d this 9 *Georgicke Sextaries*, as moſt agree.

Chous *is corruptly taken for* Congius.

Choenix, mentioned *Rev.* 6. 6. taken for a Meaſure ſerving a Servant with Food ough for a day, whence that ſaying of *Pythagoras*, *Super Choenice non Sedendum*, in- nding the provident care that ſhould be taken for the future. The *Attick Choenix* ntained by ſeveral $1\frac{1}{2}$ *Atticke Sextaries*, the *Georgicke Choenix* $2\frac{1}{4}$ *Atticke xtaries*, whereby the *Georgicke Sextary*, and by conſequence, all the leſſer *Georgicke* eaſures are proportionally greater than the *Atticke*, and accordingly the former Ta- les are computed. But there are others, and of good note too, that reckon the *hoenix Georgicke* to contain but 2 *Sextaries Atticke*. Some mention a Triple *Choenix*, *Bilibral*, *Quadrilibral*, and *Quinquelibral*.

Chœnix *how much.*

By ſome is made 3 *fold.*

Sextary, Sometime *Xeſta*, from the *Greek* ξεστης, the ſixth part of an *Atticke Chous*, made 2 *otyles*, but the *Georgicke Sextarie* though bigger made but $1\frac{1}{3}$ *Georgicke Kotyle*, a Meaſure anſlated a Pot, *Mark* 7.4. and though uſurped ſometimes for the *Roman* Meaſure of that ame, yet upon more exquiſite ſearch, queſtionable; ſince ſome affirm 11 *Atticke xtaries* made 12 *Roman*. *Alſted* and ſeveral others make the *Attick Sextarie* contain lf the *Auguſtane* Meaſure and 2 Ounces over, that is 18 ℥ Menſural. Some reckon 20 ℥, others 24 ℥, that is $1\frac{1}{2}$ Pound, at 16 ℥ to the Pound, as 18 ℥ at 12 ℥ to the ound. Others will have it 13 ℥ 7 *pwts.* 18 *gr. Troy*. Some $1\frac{1}{2}$ Pint our Meaſure. thers but half a Pint, and equal to the *Hebrew Log*.

Sextary *of what Content how Tranſlated.*

Kotyle Atticke is half the *Sextary*. *Scap.* out of *Dioſcorides* & *Heraclitus* make it qual with the *Roman Hemina*, and if ſo, then muſt the *Roman* and *Atticke Sextaries* equal. And with *Thuc. apud. Athen. lib.* 11. is made to contain 9 Menſural unces. Of the *George Kotyle* ſee above, with the *Latines* wrote *Cotyle*.

Kotyle *the Content.*

Tetarton, was properly a Liquid Meaſure in Latine *Quartarius*, being the Quarter the *Atticke Sextary*, but the *Georgicke Sextary* is $2\frac{2}{3}$ *Georgicke Tetartons*.

Tetarton *what and how much.*

Oxybaph, was a Veſſel to pour Vinegar in to dip Meat into at the Table, as the La- e *Acetabulum*, if *Georgicke*, was the Eight part of the *Choenix*, but if *Atticke*, the welfth.

Oxybaph, *what the Content.*

Kyath, in Latine *Cyathus* $1\frac{1}{2}$ whereof whether *Atticke*, or *Georgicke* made 1 *Oxybaph*, vertheleſs in quantity proportioned to the reſpective *Sextaries*. A *Kyath* was uſed *Athens* for a little Drinking Cnp.

Kyath *how much.*

As for the *Concha*, *Myſtrum* and *Cheme*, (meaſures more Minute than the *Kyath*) e Tables follow *Alſted*, *Scapulus*, and others, yet there are not wanting that ſpeak of e uncertainty of them as Meaſures, make 2 ſorts of them a greater and a leſs, and vide the *Kyath* otherwiſe than above.

Differences about the Concha, Myſtrum *and* Cheme.

As *Malines*, in his
Lex Mercatoria, thus.

Kyath {
$\frac{2}{3}$ *Great Concha.*
2 *Small Conchas.*
3 *Great Myſtras.*
4 *Small Myſtras.*
5 *Chemes.*
10 *Dragma*, *Cochlears.*

Legat, at the end of
Thomas his *Dictionary*, thus,

1 *Oxybagh.* = *Greater Concha.*
1 *Kyath.* = *Leſſer Concha.*
1 *Kotyle* = { 16 *Greater Myſtras.* 20 *Leſſer Myſtras.*
1 *Kotyle.* = { 20 *Greater Chemes.* 30 *Leſſer Chemes.*

The loweſt Rank in the Table of *Georgick* Meaſures accompts the Number of *Attick* *Cochlears* or Spoonfuls in every of the ſaid *Georgick* Meaſures at the rate of $2\frac{1}{4}$ of th one for $1\frac{1}{2}$ of the other. And the loweſt rank in the Table of *Atticke* Meaſures, va- lues their Content with the *Engliſh* Pint or Pound of 12 ℥ proportionally to $1\frac{1}{2}$ ℔ for the *Atticke Sextary*, wherein at preſent I am beſt ſatisfied.

A Table of Exotick *Meaſures of the* Greeks *compared with their* Atticke.

Græcian Exoticke Meaſures *compared with the* Atticke.

		Kypſele.	Med.	Mod.	Chous.	Choen.	Sext.	Kot.	Tetar.	Oxyb
1	*Achana Perſica*	7	3							
2	*Metreta Syria*	o	1	4						
3	*Artaba Perſica*	o	1	o	o	3				
4	*Kypros*	o	1	o	o	o				
5	*Ægyptia Artaba*	o	o	5	o	o				
6	*Medimnus Kyprius* *Salam*	o	o	5	o	o				
	Papho	o	o	4	1	o				
7	*Collathum Syrium* *Modios Pondicus*	o	o	2	o	o	1			
8	*Ponticus Cyprus*	o	o	2	o	o	o			
9	*Sabitha Syria*	o	o	1	1	2	1			
10	*Mares Ponticus*	o	o	1	1	1	$\frac{1}{2}$			
11	*Kophinus*	o	o	1	1	o	o			
12	*Modios Kyprios*	o	o	1	o	3	$\frac{1}{2}$			
13	*Kampſaces* *Tetarpe Laconicé*	o	o	1	o	o	o			
14	*Dadix*	o	o	o	1	2	o			
15	*Aphin* *Topium*	o	o	o	o	4	o			
16	*Choenix Syria*	o	o	o	o	2	1			
17	*Capitha* *Mares*	o	o	o	o	2	o			
18	*Inion*	o	o	o	o	o	1			
19	*Elenius*	o	o	o	o	o	o	o	1	
20	*Gabenon*	o	o	o	o	o	o	o	o	
21	*Alabaſtron*									

Achana-Perſica *what.* Metreta *of* Syria *the content.* Artaba-Perſica *the content.*

1. A *Perſian* Corn Meaſure, as *Heſychius* teſtifieth.

2. Expreſſed by *Legat* to contain 120 *Sextaries*, which is all one with 1 *Medimnus* 4 *Modios*.

3. Moſt from the Authority of *Herodotus*, *lib.* 1. *pag.* 49. agree the *Perſian Artab* was 3 *Choenices* greater than the *Atticke Medimnus*, And whence *Hunt* in his Table of *Græcian* Liquid Meaſures ſhould make it leſs then the *Metreta* by 3 *Chous* is to me unknown. *Hierom* on *Iſaiah* 5. *cap.* writes the *Ægyptian Artab* was 20 *Modios*.

Artaba *of Ægypt the content by* Hierome. Kypros *the content.* Artaba *of Ægypt the content by* Fannius. Medimnus *the ſorts and content.*

4. Equal to the *Atticke Medimnus* ſaith *Legat*, after the Latine wrote *Cyprus*.

5. The ſame Author tells us with *Fannius* this Meaſure is but $3\frac{1}{3}$ *Modios*, with *Epiphanius* equal to the *Atticke Medimnus*, as alſo the *Median Artab*.

6. The *Medimnus* at *Papho* is leſs by half a *Modion* than that at *Salaminca*, though both *Kyprian* Meaſures.

Collathum *and* Pontick Modios *how much.* Pontick, Cyprus, Sabitha, Pontick Mares *the content.* Kophinus *the content.*

7. Both the *Collathum Syrium* and the *Pontick Modios* are counted to contain alike 25 *Atticke Sextaries* in our Meaſure, as ſay the Great Annotations a Peck and a Pottle others 12 Ounces a quarter and a half more.

8. The *Pontick Cyprus* after *Epiphanius* is 2 *Modios* as above.

9. *Sabbitha Syria*, is accompted to contain 22 *Attick Sextaries* all one as above

10. *Mares Ponticus*, *Epiphanius* delivers to contain 20 *Alexandrian Sextaries*, which if different from the *Atticke*, the Content above muſt be corrected accordingly.

11. *Kophinus*, a Boetick Meaſure both of Liquid and Dry, according to *Legat*, contains 3 *Congios*, and in our Meaſure 1 Gallon, half a Pint, 3 Ounces $\frac{3}{4}$, but if the *Congius* be 5 Pints $1\frac{1}{4}$ ℥ *Engliſh* as he ſaith *Kophinus* muſt contain more than a Peck by $\frac{1}{2}$ a Pint $3\frac{3}{4}$ Ounces, and near a Peck, if by *Congius* the *Attick Chous* be underſtood. Wherefore of the certainty, *quære*.

Modios Kyprios *the content.*

12. *Modios Kyprios*, or after the Latine *Modius Cyprius*, is reckoned to contain 17 *Atticke Sextaries*; of our Meaſure by the Great Annotations, a Peck and a Pint; by *Legat*, but 14 Pints, 5 Ounces, a quarter and half.

13. *Kampſaces*, and the *Tetarpe Laconicè* are equal, the one being 12 *Sextaries*, the other 24 *Kotyles*, ſeeing 2 *Kotyles* make but 1 *Sextary*, but ſome make the *Kampſaces* but 4 *Sextaries*. The *Tetarpe Laconicè* ſeems to be the quarter of the *Laconian Metretes*.

Kampſaces *and* Tetarpe, *their Contents.*

14. *Dadix*, by *Pollybius* and others containeth 6 *Choenices*. *Malines* calls it a *Boetick* Meaſure.

Dadix *the Content.*

15. *Aphin*, an *Ægyptian* Meaſure, containing 4 *Choenices*, and of the ſame capacity doth *Heſychius* account the *Topium*, but with whom in uſe he ſaith not.

Aphin *and* Topium *their Contents.*

16. The *Syrian Choenix*, is ſuppoſed to be the ſame which with *Fannius* is ſet down at 4 *Sextaries*.

Choenix Syria *the Content.*

17. The *Capitha*, a *Perſian* Meaſure contained 2 *Attick Choenices*, and the *Mares* was equal thereto, containing 6 *Kotyles*, and a *Boetick* Meaſure, as ſome ſay.

Capitha *and* Mares *their Contents.*

18. *Inion*, with the *Ægyptians*, as *Legat* ſaith, was the name of a *Sextarie*, which with the *Alexandrians* contained 2 ℔ of Oyl, as *Epiphanius* hath it. But if by Pound he intend, the *Roman Libra*, or the *Greek Mna*, it muſt not be taken for our Pound; ſince ſome affirm neither of them weighed 11 Ounces *Troy*.

Inion *how taken.*

19. *Elenius*, being the quarter of the *Sextarie* ſeems only another name for the *Tetarton*.

Elenius *the Content.*

20. *Gabenon* was all one with the *Oxybaph*, or *Aretabule*.

21. *Alabaſtron*, contained one of their Pounds of Oyle.

Gabenon *and* Alabaſtron *the Contents.*

Græcian Hippiatrical Meaſures ſeem for the moſt part to keep the Names of the *Atticke* Meaſures, though the Diviſions and quantities differ, in *Alſted* thus found.

12 Ounces hath 2 *Oxybaphs*, 1 *Oxybaph*, 3. *Kyaths*. 1 *Kyath* 4 *Myſtras*. 1 *Myſtrum* 2 *Cochlears*.

In one *Ounce* 8 *Drams*, 1 *Dram* 3 *Scruples*, *&c.*

Hippiatrical *Meaſures of the* Greeks.

Legat, out of *Abſyrtus*, *pag.* 34. and *Hierocles pag.* 35. mentions the *Choe* to contain 10 *Ounces* of Liquid Meaſure, which if Menſural, then was the *Choe* leſſer than the *Hippiatrical Kotyle*, but if Ponderal equal; for he ſaith the *Roman* Menſural Pound (to which the *Hippiatrical Kotyle* was equal) contained ſo much Oyle, as 10 Ponderal Ounces weighed.

Choe *mentioned by* Legat.

In like manner as the Meaſures, ſo the Weights among the *Greeks* are differently to be taken; as they are *Attick*, *Phyſical*, *Hippiatrick*, *Indigenital*, or *Exotick*. Of which ſee further the following Tables and Notes on the ſame.

Weights of the Greeks *as their Meaſures of divers ſorts.*

Græcian Atticke Weights.

A Table of the Atticke Weights.

	Pounds. Minas.	Ounces. Uncias.	Drams. Drachmas.	Scruples. Grammata.	Obolos.	Lupines. Thermos.	Keratias. Siliquas.	Aereolos. Chalkos	Graines. Sitars.	Minutes. Leptas.
Talent Greater.	80	1000	8000	24000	48000	72000	144000	288000	576000	2016000
Talent Leſſer.	60	750	6000	18000	36000	54000	108000	216000	432000	1512000
Mina. New		12½	100	300	600	900	1800	3600	7200	25200
Mina. Old.		9⅜	75	225	450	675	1350	2700	5400	18900
a Uncia. Ounce.			8	24	48	72	144	288	576	2016
b Drachm. Dram.				3	6	9	18	36	72	252
c Gramma. Scruple.					2	3	6	12	24	84
d Obolus.						1½	3	6	12	42
e Therme. Lupine.							2	4	8	28
f Siliqua. Keration.								2	4	14
g Chalkus. Aereolus.									2	7
b Sitar. Graine.										3½
									i	*k*

A Table of Weights uſed by the Græcian *Phyſitians.*

Græcian Phyſical Weights.

	Ounces. Uncias.	Drams. Drachmas.	Scruples. Grammata.	Obolos.	Lupines	Carobſeeds. Keratias. Siliquas.	Aereola. Chalkos.	Graines, Sitars.	Minutes. Leptas.
Mina.	16	128	384	768	1152	2304	4608	9216	32256
Litra.	12	96	288	576	864	1728	3456	6912	24192
aa Uncia. Ounce.		8	24	48	72	144	288	576	2016
Drachm. Dram.			3	6	9	18	36	72	252
Gramma. Scruple.				2	3	6	12	24	84
Obolus.					1½	3	6	12	42
Lupine.						2	4	8	28
Siliqua. Keration. Carobſeed.							2	4	14
Chalkus. *gg* Æreolum.								2	7
Sitar. Graine.									3½

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A Table of Weights uſed by the Græcian *Farriers.*

Græcian Hippiatrical Weights.

	Ounces.	Denarions.	Drams.	Scruples.	Obolos.
Mina.	15	84⅜	112½	337½	675
Litra.	12	67½	90	270	540
aaa Ounce.		5⅝	7½	22½	45
Denarion.			1⅓	4	8
bb Dram.				3	6
cc Scruple.					2

ll

Notes on the Table.

Mna *the ſorts how much.*

a The *Mna* of 100 Drachms is called *Solons*, *Mna*, becauſe thought to be conſtituted by him, ſometime turned into *Latine* by *Mina*, often by *Libra*, though *Libra* be 4 Drachms lighter; the *Roman Libra* being but 96 *Attick* Drachms. The old *Mna* of 75 Drachms now obſolete for Memory ſake hath found room in the Table.

Mna *of the Phyſitian.*

aa. The Phyſitians, as by *Dioſcorides* and *Galen* appears. uſed a *Mna*, or Pound of 16 Ounces, and a *Litra* or other Pound of 12 Ounces, conceived all one with the *Roman Libra* conſiſting of 96 Drachms as this did, and by Interpreters commonly rendred *Libra*, and ſeldom or never *Mna*, and *Mna*, and *Litra*, as alſo *Libra*, commonly Engliſhed a Pound.

Mna *of the* Farriers.

aaa. The *Hippiatricks* had a *Mna* of 15 Ounces, and a *Litra* of 12.

Oungia, Uncia *not the* Engliſh *Ounce.*

b. Oungia, in Latine *Uncia*, muſt not be taken for our Ounce, but for one of their Ounces, ariſing by the diviſion of their Pound into Drams differently according to the quantity of Drams in one Pound.

Denarion *how much.*

bb. Among the *Hippiatrical* Weights there was a *Denarion* of 4 Scruples, 5⅝ whereof made one of their Ounces.

Drams *the Names and ſorts.*

c. Drachme, *Drachma*, and *Dragma*, in *Greek* and *Latine*, in *Engliſh* a *Dram*, is the eight part of their Ounce, whether the Pound had 12 or 16 Ounces therein. By *Alſted* made to equal the *German* Weight *Quintlein*. Some call a Dram *Reſolus*, ſome *Holke*, from the *Greek* ὁλκη.

cc. The Ounce *Hippiatrick*, that divided as well the *Mna* of 15 Ounces, as the *Litra* of 12 Ounces; had but 7½ Drams in it.

Scruples *how called.*

d Drams of all ſorts were parted into 3 Scruples. A Scruple in *Greek* ſometime *Gramma*, ſometime *Grammata*, in *Latine Scripulum*, *Scriptulum*, and *Scrupulum*.

e. Obolus

Obolus, *why our word Half-penny from thence.*

e. Obolus, Sometime a Weight, ſometime a piece of Money commonly rendred an lf-penny, becauſe alwaies was the half of a Scruple.

Lupine, *the Weight of a Seed,*

Lupine, in *Greek Thermos*, was a Weight equal in poiſe to the *Lupine*, which is a ed growing in a Cod like to a Peaſe, and both Plant and Seed bear that name. And ing there are many ſorts as *Parkinſon Theater of Plants*, *pag.* 1073. which ſort of *Lu- e* is meant is uncertain, probably, the Middle White, which are moſt in uſe, big- than the Yellow, and not ſo big as the great Blew, and from the nearneſs in Weight ereto, if not exactneſs might be ſo called.

Siliqua, *how much.*

g.gg. Siliqua, in *Greek Keration*, a Weight alike heavy to the *Carobſeed* or Sweet an, common in many Countreys ſubject to the *Græcian* Empire. Sometime called *rat* or *Caract*, from whence the word ſtill in uſe with us.

Caract, *whence.*

Chalkos, *both Weight and Coine.*

h. Chalkos, in *Latine Aereolus* and *Aereolum*, *Aereolus* was alſo a piece of Braſs Mo- y currant in Antient times among thoſe Countreys of the *Græcian* Dominion.

Sitar, *what.*

Sitar, a Grain of Corn from Σῖτος, *Frumentum*, likely to have been the Original of ir Weight. 2 whereof made 1 *Chalkos.*

Lepton, *what.*

k. Lepton, from *Leptos*, in *Latine Minutum*, and *Minutia*, ſuppoſed to be ſome ſmall le of the Rinde or Bark of ſome Tree, 3 ½ ballanced the *Sitar*

Other Weights mentioned by ſome Authors.

l. Beſides theſe in the Table of Phyſical Weights, ſome Books mention the *Aſſarion*, owed for 2 Drams which is ¼ of an Ounce. Alſo the *Exagion*, wrote ſometime *gion*, ſometime *Agion*, for brevity, which was the *Roman Sextula*, the Sixth part their Ounce, whereof 12 made the *Litra*. Likewiſe *Orobus* which was a graine of a ild Vetch. And *Phaike* a *Lentill*, but whether Weights or no is not worth the quiry.

l l. As the other Weights are divided into leſſer Diviſions than the *Obolus*, ſo no ubt but the *Hippiatrick* alſo were, and may accordingly be done, when occaſion rves. The *Obolus* of all ſorts admitting the like ſmaller Denominations.

Græcian Exotick Weights.

A Table of the Exotick *Weights of the* Greeks.

alents	Mentioned by *Vitrivius*, ſuppoſed to be the *Thracian*, or *Bizantium* Talent	120	*Libras.*
	Several mentioned by *Heſychius*	100 125 165 405 1150	
	Old *Sicilian* (*m*)	24	*Minas.*
	New *Sicilian* (*m*)	12	
alent of	*Alexandria*	12000	*Attick Drams.*
	Aegina *Corinth*	10000	
	Egypt	8000	
	Babylon	7000	
	Rhodium	4500	
	Euboicum	4000	
	Syria	1500	
na	*Alexandria*	20	*Uncias.*
	Ptolemaica	18	
rachma	*Egyptia*	1	*Obolus.*

(*m*) The *Sicilian* Old and New Talent is thus reckoned by *Legat* before-mentioned, t *Rider* and another Author make them pieces of Money, and of a far ſmaller Va- e, ſet afterward among the *Græcian* Coines.

A Table of Græcian Coines and their Value.

					l.	s.	d.
Gracian	Money	Braſs	Mite		00	00	00 $\frac{3}{32}$
			Aereolus		00	00	00 $\frac{5}{24}$
			Quadrans	Oboli	00	00	00 $\frac{5}{16}$
				Aſſis	00	00	00 $\frac{3}{16}$
			Aſſarius		00	00	00 $\frac{3}{8}$
		Silver	*Semiobolus*		00	00	00 $\frac{5}{8}$
			Danaces		00	00	00 $\frac{5}{6}$
			Obolos	*Attick*	00	00	01 $\frac{1}{4}$
				Aeginean	00	00	02 $\frac{1}{12}$
			Diobolus		00	00	02 $\frac{1}{2}$
			Triobolus	*Attick*	00	00	03 $\frac{3}{4}$
				Aeginean	00	00	06 $\frac{1}{4}$
			Ciſtophorus		00	00	04 $\frac{9}{16}$
			Tetrobolus		00	00	05
			Drachma	*Attick*	00	00	07 $\frac{1}{2}$
				Aeginean	00	01	00 $\frac{1}{3}$
			Siglus, Sardinian and *Perſian*		00	00	10
			Didrachma		00	01	03
			Tridrachma		00	01	10 $\frac{1}{2}$
			Stater	*Attick*	00	02	06
				Corinthian	00	01	08 $\frac{3}{4}$
				Macedonian	00	02	09 $\frac{5}{12}$
		Gold	*Semiſtater*	*Attick*	00	07	06
				of *Darius*	00	07	06
			Stater	*Attick*	00	15	00
				of *Darius*	00	15	00
			Stater Macedonian		00	18	04
			Stater Cizycen		01	01	00
			Tetra-Stater		03	00	00
	Sums of Money.	*Mna Attick*			03	02	06
		Talent	Leſſer	*Rhegium*	00	00	03 $\frac{3}{4}$
				Sicilian New	00	01	10 $\frac{1}{2}$
				Sicilian Old	00	03	09
				Neapolitan	00	03	09
			Greater	*Syrian*	046	17	06
				Euboicum	125	00	00
				Rhodium	140	12	06
				Attick Leſs	187	10	00
				Attick Greater	250	00	00
				Babylonian	218	15	00
				Egyptian	250	00	00
				Aeginæan *Corinthian*	312	10	00
				Alexandrian	375	00	00

Mite *how called, the Value.*

A *Mite* in Latine *Minutum* and *Minutia*, in Greek *Lepton* uſed *Mark* 12. 42. and from thence proved to be half the Quadrant, (but not half of our Farthing) by the *Syriack* Interpreter, and *Alſted* reputed $\frac{1}{8}$ of the *Aſſarion*. It weighed $\frac{1}{2}$ Barley Corn as ſome ſay, and was currant for as much as $\frac{3}{32}$ of our Peny, but ſome will have it twice as much.

Aereolus *the Value.*

Aereolus, and *Aereolum*, ſeveral agree to be the weight of 2 Graines, and call it *Chalkos*, and according to the *Attick* Weight, weighed 7 of their *Leptas*, or *Mites*, and was valuable with $\frac{5}{24}$ of our Peny, being the 36th part of their Dram, worth 7 $\frac{1}{2}$ *d. Sterling.*

Quadrans *the Sorts and Value. not a Farthing.*

Aſs *of the* Romans *how much.*

The *Quadrans*, commonly tranſlated a Farthing, *Mat.* 5. 26. *Mark* 12. 42. in *Greek* κοδράντης, by ſome is taken either for the Fourth part of the *Obolus*, or the *Aſs*; and ſo accordingly valued as before. The latter was the double of the *Mite*. The *Aſs* was the 10th part of the *Roman* Peny, which being 7 $\frac{1}{2}$ *d.* made the *Aſs* $\frac{3}{4}$ *d.* and conſequently the 4th part thereof $\frac{3}{16}$.

Aſſarius *the difference about it how called and tranſlated.*

Aſſarius, or *Aſſarium*, *Holyokes* Dictionary makes the 4th part of the *Aſſis* equal to the *Quadrans*; but *Leigh* makes it equal to the *Aſſis* it ſelf; for he ſaith, it is the 10th part of the *Roman* Peny, the 96th part of the *Attick Stater* that is with us but 1 $\frac{1}{2}$ Farthing. Others make it more, and ſay it was 1 Farthing and an Half, which I rather

ther incline to, and ſo have ſet it down, of old called *Aſſar*, and by the *Rabins*, *Iſſor*, *Mat.* 10. 29. tranſlated a Farthing. *Alſted* likewiſe makes *Aſſarium* worth 1 *Cruciat* or *Creutzer*, that is 3 Farthings *Sterling*, all one with the *Roman Aſs*, counting 40 *Creutzers* to a *Stater*, which is 2 *s.* 6 *d.* and ſo 10 to the *Drachmal Denary*.

Semiobolus, is $\frac{1}{12}$ of an *Attick* Dram (of which below) that is 2 $\frac{1}{2}$ Farthings our Money, or $\frac{5}{8}$ of a Peny.

Semiobolus *how much.*

Danaces, in *Greek* Δάνακη. Δάνικη, and Δανᾶη, *Charon*'s Ferriage-Piece, which the *Barbarians* uſed to put into the Mouths of Dead Perſons to pay *Charon* for their Carriage over the River *Styx* into the *Eliſian Fields*. If it be an *Obolus* as *Lucian* calls it, it is worth 1 $\frac{1}{4}$ *d.* our Money. But if of the ſame weight which the *Greek* and *Arabian* Writers call the *Arabian Danich*, weighing $\frac{2}{3}$ of an *Obolus*, is $\frac{5}{6}$ of a Peny, that is 3 *q*, and $\frac{1}{3}$ of a Farthing our Money.

Danaces, *uſed to be put into the Mouthes of Dead Perſons.*

Obolus, is twofold, the *Attick* which is $\frac{1}{6}$ of their Dram in Value with us 1 $\frac{1}{4}$ *d.* the *Aeginean* almoſt double the other, *Viz.* 2 *d.* and $\frac{1}{12}$ of a Peny. *Holyoke* thinks *Obolus* came from *Obelos*, which ſometimes ſignified a *Dart* uſed in War, as being ſtamped with the like Form: Or was ſo called from the Oblong Form thereof, or from the Image of ſome *Obelisk*, or *Spire* coined thereon.

Obolus, *the value, why ſo called.*

Diobolus, was $\frac{1}{3}$ of the *Attick Dram*, or double the *Attick Obolus*, and had on the one ſide *Jupiters* Face, and on the other an *Owle*.

Diobolus, *how ſtamped, the Value.*

Triobolus, was both *Attické* and *Aeginean*, that juſt $\frac{1}{2}$ *Dram*, the other 3 *Aeginean Oboli*, that is 6 $\frac{1}{4}$ *d.* in Value.

Triobolus, *the Value.*

Ciſtophorus, ſo called from the Form of a *Cofer* or *Cheſt* thereon, valued in *Engliſh* Money 4 $\frac{1}{4}$ *d.* and a quarter of a *Farthing*; by *Holyoke* and others who ſet not down the weight thereof.

Ciſtophorus *how ſtamped, the Value.*

Tetrobolus, had *Jupiters* Face on the one ſide, and 2 *Owles* on the other, contained 4 *Oboli*, or $\frac{2}{3}$ of the *Attick Dram*, worth with us 5 *d.*

Tetrobolus *the Value, what Print thereon.*

Drachma, or *Drachme*, uſed *Luke* 15. 8, 9. Sometime *Arguris*, in *Engliſh* a *Dram*, *Acts* 19. 19. a *Silverling*, a Piece of Money common with the *Athenians*, bearing the Image of *Minerva*'s Candle burning, in weight $\frac{1}{8}$ of an Ounce, and accordingly valued at 7 $\frac{1}{2}$ *d.* *Sterling* at the rate of 5 *s.* *Sterling* the Ounce. This was called the *Attick Drachme*, and was all one as very many conceive with the *Roman* Peny. The *Aeginean Dram* was heavier, and ſo worth more the weight 1 $\frac{2}{8}$ ℥ *Attick*, Value 12 $\frac{1}{2}$ *d.* *Engliſh.*

Dram, *the Names, Sorts, and Values.* *The Image thereon.*

Siglus, was of *Exotick* Extract, and weighed 1 $\frac{1}{3}$ ℥ or 4 *Attick Scruples*, may be valued at 10 *d.*

Siglus *the Value.*

Didrachmum, called alſo by the *Athenians Bous*, or *Boos*, becauſe there was an *Ox* ſtamped thereon, whence the Proverb, *Bos in Lingua*, as the *Engliſh*, *The Angels blind their eyes*, applyed to them that are bribed to ſpeak, or blinded in Judgment, equal to 2 *Drams*, or $\frac{1}{2}$ their *Silver Stater*, and was $\frac{1}{4}$ of our *Ounce Troy*, and worth 1 *s.* 3 *d.* all one with the Hebrew *Bekah*, paid by the *Jews* to the Sanctuary, and Temple, till *Cæſar* changed it into Tribute-Money for his own Cofers, *Mat.* 17. 24. and afterwards by Vertue of a Decree made by *Veſpaſian* paid towards the *Roman* Capitol.

Didrachmum *how ſtamped and called.* *Proverbs, whence.* Didrachmum *Exacted by* Cæſar.

Tridrachmum, was 3 *Drams Attick*, and valued with us accordingly.

Tridrachmum *the Value.*

Stater of Silver, was either Attick, having on the one ſide *Minerva*'s Head, and an *Owle* on the other, worth with us 2 *s.* 6 *d.* weighing $\frac{1}{2}$ Ounce; Or *Corinthian*, which was not full 3 *Attick Drams* and worth but 1 *s.* 8 $\frac{3}{4}$ *d.* Or *Macedonian*, which was bigger than either, and worth 2 *s.* 9 $\frac{3}{4}$ *d.* The *Attick Stater* was double the *Didrachmum*, and ſo ſerved for Tribute-Money both for *Chriſt* and *Peter*, *Matth.* 17. 27. and is ſometimes called the *Tetradrachmum*, becauſe it contained 4 *Drams*.

Silver Stater *how ſtamped, the Value and Sorts.* Tetradrachmum.

The *Semiſtater of Gold*, both the Common *Attick*, and that of *Darius* Coine are reckoned equal either of them 1 ℥. valuable with us, 7 *s.* 6 *d.*

Semiſtater *of* Gold *the Value.*

The *Staters* alſo in Weight equal 2 *Drams* of our *Troy Weight*, or $\frac{1}{4}$ ℥ worth 15 *s.* That of *Darius* is reported to have the Image of *Sagitarius* thereon.

Staters *the Sorts and Values.* *Of* Darius.

The *Macedonian Stater* weighed of *Attick* Weights 2 ℥. 2 *ob.* 2 *Siliq.* worth proportionally with us 18 *s.* 4 *d.*

Macedonia.

The *Stater of Cizycen* or *Cizycus*, a City in *Greece* 2 $\frac{4}{5}$ ℥ *Attick*, was valuable in *Sterling* Money at 1 *l.* 1 *s.* 0 *d.*

Cizycen.

The *Tetraſtater*, ſeems to ſome no piece of Coine, but ſignifies only Four *Staters*, worth 3 *l.* *Sterling*.

Tetra Stater.

Mna Attick, containing 100 *Drams*, 96 whereof being equal to the *Pound Troy*, make the whole *Mna* in Silver at the rate as worth with us 3 *l.* 2 *s.* 6 *d.*

Attick Mna *how much in Accompt.*

Talents *the Accompt of them.*

Talents of the Leſſer ſort, and improperly ſo called, ſeem to me rather Pieces, than Sums of Money. That of *Rhegium* a Town in *Italy*, currant in *Greece*, was in Value but $3\frac{3}{4}$ *d.* of the New and Old *Sicilian Talents*, the Old was double the New, and the biggeſt worth no more than that of *Naples*, to wit 3 *s.* 9 *d.* *Engliſh* Money.

The Sorts and Weight.

Talents of the Greater ſort, and indeed deſerving that name, were divers as before noted with their reſpective Value in our Silver Money, according to their weight of the *Attick Drams* to which they are compared, being ſome of them *Exotick* as here followeth.

	Drams Attick	℔. Troy	℥. Troy
Syrian	1500	15	7½
Euboicum	4000	41	8
Rhodes	4500	46	10½
Attick Leſs	6000	62	6
Babylon	7000	72	11
Attick Great *Egyptick*	8000	83	4
Ægina	10000	104	2
Alexandria	12000	125	0

Geodæticals *of the* Latines *and* Romans. *Meaſures of the* Latines.

The Third and laſt ſort of the Ancients whoſe *Geodæticks* are to be ſeen, are the *Latines*, and their Succeſſors the *Romans*.

Alſted fits us with Tables for the Long and Superficial Meaſures, and another which he calls *Geometrical*; wherein the main differences between the other Long Meaſures, and theſe are about the Mile and Furlong. A Fourth Table alſo he hath for diviſion of the Inch, all which here follow.

A Table of the Long Meaſures of the Latines.

Latine Long Meaſures.

		Furlongs	Decempedes.	Paſſes.	Steps.	Cubits.	Palmipedes.	Feet.	Palmes.	Inches.	Digits.
	Mile	8	500	1000	2000	3333⅓	4000	5000	20000	60000	80000
a	Furlong.		62½	125	250	416⅔	500	625	2500	7500	10000
	Decempede.			2	4	6⅔	8	10	40	120	160
b	Paſs.				2	3⅓	4	5	20	60	80
	Step.					1⅔	2	2½	10	30	40
c	Cubit.						1⅕	1½	6	18	24
	Palmipede.							1¼	5	15	20
d	Foot.								4	12	16
e	Palme.									3	4
f	Inch.										1⅓
	g										*h*

A Table of the Meaſures for Land uſed with the Latines.

Latine Superficial Land-Meaſures.

		Centuries.	Jugera.	Modes.	Verſes.	Climes.	Acts.	Feet.
	Saltus.	4	400	800	1152	3200	24000	11520000
i	Centurie.		100	200	288	800	6000	2880000
	Jugerum.			2	2 22/25	8	60	28800
k	Mode.				1 11/25	4	30	14400
l	Verſe.					2 7/9	20⅚	10000
m	Clime.						7½	3600
n	Act.							480

Latine Geometrical-Meaſures.

A Table of the Long Meaſures of the Latines *uſed on ſpecial Occaſions.*

	Miles.	Furlongs.	Cubits.	Feet.	Palmes.	Digits.	Graines.
Paraſang, *or* Schoone.	3	30	12000	18000	72000	288000	1728000
	Mile.	10	4000	6000	24000	96000	576000
aa		Furlong.	400	600	2400	9600	57600
			Cubit.	1½	6	24	144
				Foot.	4	16	96
					Palme.	4	24
						Digit.	6
							hh

The Diviſion of an *Inch.*

A Table of the Diviſion of the Inch *according to* Alſted.

	Drams.	Scruples.	Obolos.	Siliquas.	Points.	Minutes.	Moments.
Inch.	8	24	48	144	288	576	1152
gg	Dram.	3	6	18	36	72	144
		Scruple.	2	6	12	24	48
			Obolus.	3	6	12	24
				Siliqua.	2	4	8
					Point.	2	4
						Minute.	2

Miles of 2 ſorts.

a. aa. Hereby it ſeemeth the *Latines* had 2 ſorts of Miles, *viz.* The Common conſiſting of 8 Furlongs, every Furlong 625 Feet, that is 5000 Feet in the Mile; and a Mile called *Geometrical*, or uſed in accompt upon Special Occaſions, conſiſting of 10 Furlongs, every Furlong 600 Feet which made the Mile 6000 Feet. In the firſt reckoning the Mile was ſhorter, and the Furlong longer than in the ſecond *Milliare*, or *Milliarium* in the *Latine* for a Mile came from *Mille* 1000, as was ſaid before.

Whence the word.

Paraſang *and* Schoen. *Vide antea.*

Of the *Paraſang* or *Schoene*, *Furlong*, *Pace* and *Cubit*, ſee before in the *Hebrew* and *Greek* Meaſures.

Decempede *the length.*

b. A *Decempede*, ſome call a Perch, but becauſe they agree it was but 10 Feet long, and ſo ſignified by the very name; it cannot be taken for our Perch, which is 6½ Feet longer, as before. Some mention a *Decempede* of 12 Feet.

Paces *and* Steps *the ſorts and lengths.*

c. A *Step*, in *Latine*, *Greſſus*, and *Gradus*, here taken for half a Paſs, or 2½ Feet, and not to be Engliſhed a *Degree*, which terme is moſt proper for the 360th part of a Circle. *Alſted* counts upon 3 ſorts of Paces or Paſſes, each of a double difference, thus.

		Feet.		Palmes.
Simple	of the firſt difference	2	or	8.
Double		4		16.
Simple	of the ſecond difference	2½	or	10. This the *Grade*, or *Step*.
Double		5		20. This the Pace *Geometrical*.
Simple	of the third difference	3	or	12.
Double		6		24.

Ulna *the ſorts.*

That double of the firſt difference be called *Ulna Commuis*, or the Common Ell, to difference it from the Cubit of 1½ Feet, which he ſometime calls *Ulna*. The double of the third difference he calls *Ulna agreſtis*, *ſeu Orgyia*, the Countrey Ell or Fathom.

d. A

Palmipes, *the Length.*

d. A *Palmipes* may be ſeen before in the *Greek Pygon*, the *Latine* Name ſhews Content thereof.

Foot *of the* Latines *and* Romans.
how called.
The parts thereof.

e. To what hath been ſaid already on the *Hebrew Pagnam* and *Greek Pous*, may b added, that little difference with any certainty being obſerved the *Roman* or *Latine* P may be parallel'd with the *Engliſh* Foot. The *Romans* called their Foot ſometim a *Pound*, and 2 Foot *Dupondium*, and divided ſeveral of their Land Meaſures in 12 parts called *Vnchiæ*, or *Inches*, of which below at *g*, and ſuch Inch into 24 Scruple uſing like Names as for Weights.

Palmes *of* 2 *ſorts.*

f Palmes are of 2 ſorts, though but one ſet in the Tables, a Greater anſwering t the *Hebrew Zereth*, and *Greek Spithame*, containing 3 Leſſer Palmes or 12 Digits, Th Leſſer Palme which is placed in the Table contains 4 Digits anſwerable to the *Hebre* *Tophach*, and *Greek Paleſte.*

Inch *how much the names thereof.*
Uncia *diverſly rendred.*

g. gg. And Inch in *Latine*, *Pollex*, rendred ſometimes a Thumb, becauſe man times of the ſame breadth, equal to a Digit or Fingers breadth, and a third part of Digit; The parts of which Inch follow in the 4th foregoing Table into Imaginary Moments. *Vncia*, when relating to Meaſure is tranſlated an Inch, when to weight a Ounce, ſometime wrote *Ouncia*, but whether corruptly, or that it contains 3 Digi or 2 Thumbs, making thereby 1 Thumb or Inch, 1½ Digit, as *Malines* and *Thoma* ſay, is further to be quæried. *Vide plus* at *k* on the parts of the *Jugerum.*

Digit, *the Length.*

h. hh. A Digit or Fingers breadth, anſwering to the *Greek Dactyle*, and *Hebrew* *Etſbang* is there ſpoken of, and here in the *Geometrical* Table made to contain 6 Graines; but in the upper Table to be reckoned only the breadth of 4 Graines of Barley.

Saltus *how taken.*

i. Saltus, Sometime taken for a Grove or Foreſt, here for a piece of Land, 4 *Centuries*, or 400 *Jugera*, every *Centurie* being 100 *Jugers.*

Jugerum, *the Content.*
Greater than the Engliſh *Acre.*

k. Jugerum, commonly tranſlated an Acre, muſt alwaies be underſtood for the *Roman* and not *Engliſh* Acre, being far larger as containing 28800 Square Feet in the *Area* thereof ariſing from the Multiplication of 240 Feet in length, and 120 in breadth, when as the *Engliſh* Acre containeth but 2640 Feet, which is the Product of 160 Pecks multiplyed by 16½ the Feet in one Perch as before was declared. *Alſted* divides the *Roman Juger* into 12 parts which he calls Inches, and every Inch into 7 parts, as followeth.

A Table of the Roman Juger *divided by* Alſted.

	Inches.	1 Semiuncias.	2 Siliquas.	3 Sextulas.	4 Drams.	5 Semiſextulas.	6 Scrupl.	7 Obolos.	Longer. Side.	Shorter. Side.	Area. Square Feet.
Juger.	12	24	48	72	96	144	288	576	240	120	28800
Inch.		2	4	6	8	12	24	48	60	40	2400
Semiuncia. ½			2	3	4	6	12	24	40	30	1200
Siliqua. ¼				1½	2	3	6	12	30	20	600
Sextula. ⅛					1½	2	4	8	20	20	400
Dram. ⅛						1½	3	6	20	15	300
Semiſextula. 1/12							2	4	20	10	200
Scruple. 1/24								2	10	10	100
Obolus. 1/48									10	5	50

Modus *how called the Content.*

l. Modus is half a *Juger*, called often *Actus quadratus*, containing 14400 Feet Square, and was ſo called from the Square Form thereof, being every way 120 Feet.

Verſus, *the Content*

m. Verſus, uſed by *Pliny* for a Square Plot of Ground 100 Feet every way.

Clima, *how taken.*

n. Clima, for the *Hebrew Noph.*

Act *how called he Content.*

o. An Act, called *Actus Minimus*, the leaſt or leſſer Act, for diſtinction from *Actus quadratus*, which is 30 times bigger than this leſſer Act, that a Square, and this an Oblong or Long Square, one ſide whereof was 4, and the other 120 Feet, or proportionally 10, that the *Area* might be 480 Feet.

Roman

Roman Capacious Meaſures.

Capacious Meaſures of the Romans.

Dry	Common	Liquid.
Modius.	*Sextary.*	*Cule.*
Modiolus.	*Hemin.*	*Amphora.*
	Acetable.	*Urne.*
	Cyath.	*Congius.*
	Ligula.	*Quartary.*

The Table of *Roman* Dry and Liquid Meaſures.

A Table of the Roman *Concave Meaſures.*

	Amphoras.	Urnes.	Modios.	Congies.	Modiolos.	Sextaries.	Heminas.	Quartaries.	Acetables.	Cyaths.	Ligulas.
Cule. *a*	20	40	60	100	240	960	1920	3840	7680	11520	46080
Amphora. *b*		2	3	8	12	48	96	192	384	576	2304
Urne. *c*			1½	4	6	24	48	96	192	288	1152
Modius *d*				2⅔	4	16	32	64	128	192	768
Congie. *e*					1½	6	12	24	48	72	288
Modiolus. *f*						4	8	16	32	48	192
Sextarie. *g*							2	4	8	12	48
Hemina *h*								2	4	6	24
Quartarie.									2	3	12
Acetable.										1½	6
Cyath.											4
											i. k.

a. Culeus, and ſometime *Coleus*, *Culeum* and *Culleum*, in *Latine* taken alſo for a *Sack*, or ſuch like, wherein *Parricides* were wont to be put, and ſo caſt into the River *Tyber*, by the Old Law of the *Romans.* Some mention *Doleum*, and ſay it contained a *Cule* and an Half.

Cule *how taken.* *Puniſhment of* Parricides. Doleum *the Content.*

b. Amphora, ſome ſay was of a Cubick Form, and therefore called *Quadrantal.*

Amphora *how called.*

c. Urna. Sennertus in his *Inſtitutions of Phyſick, lib.* 5. *par.* 3. *ſect.* 1. *cap.* 4. affirms to be ½ the *Italian*, ⅓ of the *Attick Amphora*, making thereby the *Greek* half as big again as the *Italian Amphora.*

Urna *the Content.*

d. Modius, Engliſhed a Buſhel, was ſpoken to among the *Greek* Meaſures, but whereas there it was made to contain 12 *Attick Sextaries*, here, upon the Authority of *Holyoke*, *Legat*, and others, it is made of 16 *Roman Sextaries.*

Modius *Vide antea.*

e. Congius, was of a like number of *Roman Sextaries* as the *Greek Chous*, of *Attick Sextaries*, which may be the Reaſon why they are ſometime taken the one for the word were needleſs for other.

Congius *the Content.*

f. Modiolus, mentioned in *Plautus*, but without mention of its capacity, yet *Alſted* makes it the Quarter part of their *Modius*, a diminitive of *Modius*, the very word beſpeaks it, and leſs than the *Semi-Modius*, or Half Buſhel very probably, or elſe another the ſame.

Modiolus *the Content leſs than* Semi-Modius.

g. The *Sextary*, and ſo downward to the *Cyath* are divided alike to the *Attick* Meaſures. This *Sextary* was called *Italicus Sextarius*, to difference it from the *Greek Sextarie*, and alſo *Urbicus. q. d.* the City *Sextarie*, with reſpect to *Sextarius Caſtrenſis*, which was a *Sextarie* uſed in the Army, and double to the other.

Sextary *the Sorts.*

h. Hemina is ſometime called *Cotyla*, and *Cotyla Romana.* *Alſted* mentions *Cotyla Italica*, which he ſaith is 12 Menſural Ounces, this ſeems to be ſome New, and not the Old, which himſelf reckons but at 9.

Hemina *how called.*

i. Between the *Cyath* and *Ligula*, *Sennertus* placeth a *Muſtrum*, which he calls a Common little Spoone, containing half a *Cyath*, as the *Greek Concha*, and may not be confounded with the *Greek Myſtrum.*

Muſtrum *of* Sennertus.

k. Ligula, a *Lingel*, as ſome *Engliſh* it, rather a *Spoon* or *Cochlear*, of which *Sennertus* makes 4 ſorts.

Ligula *what.*

The Sorts.

The Leaſt containing—0 $\frac{1}{2}$ ʒ of a thing of a middle Weight.
The next bigger———1 ʒ.
The Great————1 $\frac{1}{2}$ ʒ or 2 ʒ.
The Greateſt————0 $\frac{1}{2}$ ℥.

This *Ligula* then may be reckoned for the *Attick Myſtrum*, for as 4 of them made one *Kyath*, ſo 4 *Ligulas* make one *Roman Cyath*.

Roman *Meaſures their accompt by weight according to* Sennertus.

Sennertus before named accompts the Content of the *Roman* Meaſures by Weight of Oyle, Wine, or Water, and Honey, as followeth, ſave only I have proportioned the Weight of the *Ligula* according to the former Table, at the rate of $\frac{1}{4}$ of the *Cyath*, and have inſerted the *Modius* and *Modiolus*, which being dry Meaſures, *Sennertus* omitteth. *Malines*, *p.* 29. of his *Lex Mercatoria* ſaies the *Romans* did accompt 10 ℥ Ponderal for 12 ℥ Menſural, and ſo the *Sextarie* at 18 ℥ ſhould be 21 $\frac{3}{5}$ and not 21 $\frac{1}{2}$ as is there ſet.

	Oyle.			Wine or Water.				Honey.			
	℔	℥	ʒ	℔	℥	ʒ	℈	℔	℥	ʒ	℈
Cule.	1440	0	0	1600	0	0	0	2160	0	0	0
Amphora.	72	0	0	80	0	0	0	108	0	0	0
Urne.	36	0	0	40	0	0	0	54	0	0	0
Modius.	24	0	0	26	8	0	0	36	0	0	0
Congius.	9	0	0	10	0	0	0	13	6	0	0
Modiolus.	6	0	0	6	8	0	0	9	0	0	0
Sextarie	1	6	0	1	8	0	0	2	3	0	0
Hemina.	0	9	0	0	10	0	0	1	1	4	0
Quartary.	0	4	4	0	5	0	0	0	6	6	0
Acetable.	0	2	2	0	2	4	0	0	3	3	0
Cyath.	0	1	4	0	1	5	1	0	2	2	0
Muſtrum.	0	0	6	0	0	6	2	0	1	1	0
Ligula.	0	0	3	0	0	3	1	0	0	4	$1\frac{1}{2}$

℥ at 12 *per* ℔

A Table of Roman *Weights.*

The Table of *Roman* Weights.

	Minas.	Libras.	Uncias.	Semiun	Duellas.	Sicilicas	Sextulas.	Denarios.	Drams.	Quinars.	Scruples.	Quadr.	Sext.	Obol.	Siliq.	Graines.
Talent.	75	152	1500	3000	4500	6000	9000	10500	12000	21000	36000	42000	63000	72000	216000	864000
Mina.		$1\frac{2}{3}$	20	40	60	80	120	140	160	280	480	560	840	960	2880	11520
Libra. *a*			12	24	36	48	72	84	96	168	288	336	504	576	1728	6912
Uncia.				2	3	4	6	7	8	14	24	28	42	48	144	576
Semiuncia. *b*					$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	4	7	12	14	21	24	72	288
Duella. *c*						$1\frac{1}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$	$4\frac{2}{3}$	8	$9\frac{1}{3}$	14	16	48	192
Sicilicum. *d*							$1\frac{1}{2}$	$1\frac{3}{4}$	2	$3\frac{1}{2}$	6	7	$10\frac{1}{2}$	12	36	144
Sextula. *e*								$1\frac{1}{6}$	$1\frac{1}{3}$	$2\frac{1}{3}$	4	$4\frac{2}{3}$	7	8	24	96
Denarius. *f*									$1\frac{1}{7}$	2	$3\frac{3}{7}$	4	6	$6\frac{6}{7}$	$20\frac{4}{7}$	$82\frac{2}{7}$
Dram.										$1\frac{3}{4}$	3	$3\frac{1}{2}$	$5\frac{1}{4}$	6	18	72
Quinar. *g*											$1\frac{5}{7}$	2	3	$3\frac{3}{7}$	$10\frac{2}{7}$	$41\frac{1}{7}$
Scruple. *												$1\frac{1}{6}$	$1\frac{3}{4}$	2	6	24
Quadrans. *h*													$1\frac{1}{2}$	$1\frac{5}{7}$	$5\frac{1}{7}$	$20\frac{4}{7}$
Sextans. *i*														$1\frac{1}{7}$	$3\frac{3}{7}$	$13\frac{5}{7}$
Obolus. *k*															3	12
Siliqua.																4

a. Libra called alſo *As*, by Tranſlators commonly rendred a Pound, was divided into 12 Ounces, and for every number of Ounces under 12, a Proper Name uſed, As,

Roman Libra *how called and divided.*

Deunx.	11	Ounces.
Dextans and Decunx.	10	
Dodrans	9	
** Bes, Beſſis, and of old Des.	8	
Septunx.	7	
Semis, Semiſſis, Semiſſius, Selibra, and Simbella.	6	
Quincunx.	5	
Triens.	4	
Quadrans, and Triunx.	3	
Sextans	2	
Uncia.	1	

Malines, p. 24. of his *Lex Mercatoria*, divides *Pondus*, which he calls the Old Pound of the *Romans*, into

The Diviſion of Pondus *by* Malines.

64	Denarios.
128	Quinarios.
256	Seſtertios.
640	Aſſes.
1280	Semilibella's
2560	Teruncios.

A Reaſon is wanting why *Legat* makes the *Roman Libra* of 12 ℥ but 10 $\frac{1}{2}$ ℥ *Troy*, ſince if he reckon by the Number of Graines (the Original of Weights) at 5760 Graines of Aſſize in the Pound *Troy*; it can be but 10 ℥ juſt; for 10 times 6912 the Graines in a *Roman* Pound, and 12 times 5760 are equal. But if he count the ℔ *Troy* at 7680 Graines according to the Statute at 32 Graines of Wheat to a Penny Weight, the *Troy* Pounds will be 13 $\frac{1}{3}$ ℥ *Roman*.

Legat *queſtioned.*

** *Bes*, is the Mark Weight, two thirds of the Pound, *Malines p.* 24. aforeſaid makes the *Bes*, or old Mark of the *Romans* to be divided into.

Bes, *how divided.*

16	Loot, *or* Tetradrams.
23$\frac{1}{3}$	Tridrams.
32	Didrams.
64	Drams.
96	Obolos, *or* Treobolas.
128	Triobulos.
384	Obolos.
768	Miobolos.
3840	Moments.

b. Semiuncia, or the Half Ounce is ſometimes called *Aſſarion*, and *Aſſarius*, and by *Alſted*, *Lotho*, anſwering to a *Germaine* Weight of that Name.

Semiuncia *how called.*

c. Duella, being double to the Weight of the *Sextula* is ſometimes called *Binæ Sextulæ*.

Duella *how much.*

d. Sicilicum, or *Sicilicus*, and by Abbreviation *Siclus* is $\frac{1}{4}$ of an Ounce.

Sicilicum *how much.*

e. Sextula, uſed promiſcuouſly with *Sextans*, and underſtood by Import of the Name to be the Sixth part.

Sextula *uſed for* Sextans.

f. Denarius, a Peny-weight, the ſeventh part of the Ounce, whether uſed to weigh any thing but Money as other the Diviſions thereof, ſomewhat queſtionable, See among the Money. *Alſted* compares the *Drachmal Denarius* to the *German* Weight *Quintlein*.

Denarius *the Weight.*

g. Quinar was half the Penyweight, and a piece of Money ſet afterward among the *Roman* Coines.

Quinar *both Weight and Coine.*

*. Between the *Quinar* and *Scruple*, ſome mention a Weight called *Tremiſſis*, containing 32 Graines, being the 18th part of the Ounce.

Tremiſſis *how much.*

h. Quadrans, here is $_4$ of the Peny weight, and ſo called *Quadrans Denarii* to diſtinguiſh it from *Quadrans Libræ*, which was 3 ℥.

Quadrans *what.*

i. Sextans, called *Sextans Denarii* to difference it from *Sextans Libræ*, was the ſixth part of the Peny-weight, and ſometime called *Sextula*.

Sextans *the weight thereof.*

k. Obolus, or Half a *Scruple*, called ſometimes *Simplium*, weigheth 12 Graines. If there be another *Obolus*, as ſome ſay, which was the third part of a *Quinar*, it ſeems:

Obolus *how called, the weight thereof.*

It ſeems to be a Piece of Coine, and muſt weigh 13 $\frac{1}{3}$ Graines, and ſo is all one with the *Sextans*, according to the *Tabulary* Diviſion; yet this ſort of *Obolus*, they make to contain but 10 Graines.

Cerates *how much.*

Between the *Obolus* and the *Siliqua*, ſome mention a *Cerates*, which they ſay contains 6 Graines, and ſo is $\frac{1}{2}$ the *Obolus*, or $\frac{1}{4}$ of the *Scruple*.

A Table of Roman *Coines and their Values, before the tranſlation of the Imperial Seat.*

Roman *Monies and their* Engliſh *Values.*

Roman Coines after the Imperial Seat tranſlated.

Roman Summs of Money.

					l.	s.	d.
Roman	*Money before the tranſlation of the Imperial Seat to* Bizantium.	Braſs	*Leſs than the* As.	Sextula, $\frac{1}{6}$ Unciæ.	00	00	00 $\frac{1}{96}$
				Semiuncia, $\frac{1}{2}$ Unciæ.	00	00	00 $\frac{1}{32}$
				Uncia, $\frac{1}{12}$ Aſſis	00	00	00 $\frac{1}{16}$
				Sextans, $\frac{1}{6}$ Aſſis.	00	00	00 $\frac{1}{8}$
				Quadrans. Triunx. Teruntius. $\frac{1}{4}$ Aſſis.	00	00	00 $\frac{3}{16}$
				Triens, $\frac{1}{3}$ Aſſis	00	00	00 $\frac{1}{4}$
				Semiſſis, $\frac{1}{2}$ Aſſis	00	00	00 $\frac{3}{8}$
				As, *or* Libra, $\frac{1}{10}$ Denarij.	00	00	00 $\frac{3}{4}$
			Greater than the As.	Decuſſis, 10 Aſſes.	00	00	07 $\frac{1}{2}$
				Viceſſis, 20	00	01	03
				Triceſſis, 30	00	01	10 $\frac{1}{2}$
				Quadraceſſis, 40	00	02	06
				Quinquaceſſis, 50	00	03	01 $\frac{1}{2}$
				Sexaceſſis, 60	00	03	09
				Septuaceſſis. 70	00	04	04 $\frac{1}{2}$
				Octaceſſis, 80	00	05	00
				Nonaceſſis, 90	00	05	07 $\frac{1}{2}$
				Centuſſis, 100	00	06	03
		Silver	Nummi *or* Nummuli	Teruntius, $\frac{1}{40}$ Denarij	00	00	00 $\frac{3}{16}$
				Sembella, $\frac{1}{2}$ Libellæ	00	00	00 $\frac{3}{8}$
				Libella, $\frac{1}{10}$ Denarij	00	00	00 $\frac{3}{4}$
				Obolus, $\frac{1}{6}$ Denarij	00	00	01 $\frac{1}{4}$
				Seſtertius, 2 $\frac{1}{2}$ Aſſes	00	00	01 $\frac{7}{8}$
				Victoriatus Quinarius $\frac{1}{2}$ Denarij.	00	00	03 $\frac{3}{4}$
				Bigatus	00	00	07 $\frac{1}{2}$
			Denarius	New, 10 Aſſes, $\frac{1}{8}$ ℥.	00	00	07 $\frac{1}{2}$
				Old, $\frac{1}{7}$ ℥.	00	00	08 $\frac{4}{7}$
		Gold		Tremiſſis, *or* Golden Triens	00	05	00
				Semiſſis, or Golden Drachmal	00	07	06
				Imperatorius	00	15	00
				Amient, *or* Conſularis	00	17	01 $\frac{3}{4}$
	Money after the tranſlation of the Imperial Seat to Byzantium	Braſs		Follis.	00	00	00 $\frac{5}{12}$
		Silver	Siliqua, *or*	Ceratium Simple	00	00	05
				Ceratium Magnum	00	00	07 $\frac{1}{2}$
				Milliariſium	00	01	03
		Gold		Conſtantines Piece	00	08	06 $\frac{6}{7}$
				Valencinians Piece	00	10	00
				Semiſſis, *or* Half Piece.	00	05	00
				Triens, *or* $\frac{1}{3}$ of that Piece	00	03	04
				Scruple, *or* $\frac{1}{4}$ of that Piece	00	02	06

		l.	s.	d.
	Quadrantes. Seſtertios.			
Roman *Sums of Money.*	Sportula, containing 100, *or* 10	00	01	06 $\frac{3}{4}$
	Libra, a Pound of 96 Drams	03	00	00
	Seſtertium (in the *Neuter Gender*) containing 1000 Seſtertios (in the *Maſculine Gender.*)	07	16	03
	Talent, containing 24 Seſtertias, *or* 6000 Denarios.	187	10	00

The Braſs *Uncia*, miſprinted in *Rider*, at $\frac{1}{10}$ *Aſſis*, for $\frac{1}{10}$ part of 3 Farthings cannot be [illegible] of our Peny, counting 4 *Cees* to a Farthing as the doth.

Rider *miſprinted.*

So alſo is *As*, at *ob. q.* for *ob. qa.* for *As* being the 10th part of the *Denarius* muſt be 3 Farthings, 10 times 3 making 30 Farthings which is 7 $\frac{1}{2}$ *d.* the value of the *Denarius*. To the Braſs *As* was the Silver *Libella* equal in value.

Obolus, being [illegible] of the *Roman* Peny, is called by *Celſus*, *Sextans.*

Obolus. *how called.*

Seſtertius, Engliſhed a *Seſtertian* was $\frac{1}{4}$ of the *Roman* Peny, and being of the *Maſculine Gender* was differenced from the other being of the *Neuter Gender*, and in Numbring by theſe *Seſtertias* theſe 3 Rules are to be obſerved.

Seſtertius *the Accompt thereby.*

1. If the Numeral Noun agree in *Caſe*, *Gender* and *Number* with the *Seſtertian*; it ſignifieth barely juſt ſo much as was pronounced, as *Decem Seſtertii* is 10 *Seſtertians.*
2. If the Numeral Noun of another *Caſe* be joyned with the *Genitive Caſe Plural* of *Seſtertius*: It noteth ſo many Thouſands, as *Decem Seſtertiûm* (for *Seſtertiorum*) is Ten Thouſand *Seſtertians.*
3. If an *Adverb* be put without any Numeral joyned, as *Decies*, *Vigeſies*, *&c.* or joyned with *Seſtertiûm* the *Genitive Caſe Plural*; there is underſtood by it ſo many Hundred Thouſand, as *Decies Seſtertiûm*, is Ten Hundred Thouſand *Seſtertians.*

Alſted delivers it thus.

From 1 *Seſtertian* to 1000 in the *Maſculine Gender*, as *Unus Seſtertius*, *Decem Seſtertii*, *&c.* is 1 *Seſtertian*, 10 *Seſtertians.*

From 1000 to 100000 in the *Neuter Gender* and *Plural Number*, as Singula Seſtertia 1000 *Seſtertians*, *Bina* Seſtertia, 2000 *Seſtertians*, *&c.*

From 100000 upward, all expreſſed adverbially and in the *Genitive Plural*, as *Semel Seſtertiûm* 100000, *Decies* Seſtertiûm 1000000, *&c.*

Victoriatus was ſo called, becauſe ſtamped with the Image of *Victory*, and *Quinarius* becauſe equal in value to 5 Braſs *Aſſes*, or Half the *Denarius.*

Victoriatus *how ſtamped.* Quinar, *how much.*

Bigatus, ſome call *Quadratus*, had the Print of a *Cart* or *Chariot* on it, and was of value equal with *Denarius.*

Bigatus, *the Print and Value.*

Denarius, *q. ſ. Dena æris*, becauſe it contained 10 *Aſſes*, rendred a Peny, *Mat.* 18. 28. and 22. 19. at the old rate was $\frac{1}{7}$ of an Ounce, and at the New $\frac{1}{8}$, and at this rate all the other Coines are valued in the Table. This is ſometime called the *Drachmal Denary* for diſtinction ſake. Some make 3 ſorts of Pence, the heavier weighing 1 $\frac{1}{2}$ *Attick* Dram, the Meane of 1 Dram, and the Leaſt lighter than 1 Dram by $\frac{1}{84}$ of an Ounce, or thereabouts. Some ſay one was $\frac{1}{6}$ of the *Roman Uncia*, the Mean $\frac{1}{7}$, and the Lighter $\frac{1}{8}$. *Budæus* makes the *Attick* Dram and *Roman* Peny of the ſame Weight and worth, wherewith moſt agree, and accordingly each in the foregoing Tables are valued at 7 $\frac{1}{2}$ *d.* after 5 *s.* the Ounce.

Denarius, *the Value.* *The Sorts.*

The Golden *Denarius* mentioned in *Holyoke* at 2 *s.* 4 $\frac{1}{2}$ *d. Sterling* I have omitted; as not ſatisfied in the Weight, nor certain of ſuch a Coine.

Golden Denarius.

The Golden *Amient*, ſeems the Eldeſt and Greateſt, a Piece Coined by the *Conſuls*, therefore called *Conſularis*, weighed 2 $\frac{2}{7}$ Drams.

Amient *how called.*

The *Imperatorius*, or Piece of the Emperors Coine 2 Drams.

Imperatorius.

The *Drachmal* 1 Dram, and the *Triens* $\frac{1}{7}$ of the *Imperatorius.*

Drachmal.

After *Conſtantine* removed his Seat to *Bizantium*, now called *Conſtantinople*, a City after his own Name; we read of *Follis* in *Euſebius*, a Braſs Piece, as *Lamprid*, or of *Iron*, as *Euſtathius* ſaith, ſo called becauſe it repreſented a Leaf in Latine *Folium*, and was $\frac{1}{12}$ of the Silver Simple *Siliqua.*

Follis, *what, why ſo called.*

The Silver *Ciliqua* or *Ceratium* was double. The Simple $\frac{1}{3}$ of the *Milliariſium*, valued 5 *d.* The Great called *Cerates*, 1 Dram equal to the Peny 7 $\frac{1}{2}$ *d.*

Cerates *of* 2 *ſorts.*

Milliariſium, weighed 2 Drams.

Milliariſium *the weight.*

Conſtantines Piece of Gold was called *Romanus* Solidus, at the proportion of 7 *s.* 6 *d.* for a Dram of Gold muſt weigh 1 $\frac{1}{7}$ Dram.

Romanus Solidus.

Theſe continued currant till *Valentinian*, who made his Coine ſomewhat heavier.

Valentinian's Piece of Gold by ſome is called Sextula, and being valued at 10 *s.* *Sterling*, muſt weigh 1 $\frac{1}{3}$ Dram.

Valentinian's Sextula.

Of which the	*Semiſſis*	being	$\frac{1}{2}$	was	$\frac{2}{3}$	of a Dram.
	Tremiſſis, or *Triens*		$\frac{1}{3}$		$\frac{4}{9}$	
	Scruple		$\frac{1}{4}$		$\frac{1}{3}$	

Parts thereof.

Sportula, ſay ſome, was a Lawyers Fee, or an Almes diſtributed by Princes among the People. See Selden's *Hiſtory of Tythes*, *chap.* 4. *p.* 37, 38.

Sportula *what.*

Moderne Geodæticals.

Geodæticals of the ſmaller ſort, with the Antient *Hebrews*, *Greeks* and *Latines* Emedullated; it is time to ſay ſomething of others more Moderne. A Perfect Liſt whereof is not yet come to hand, and thoſe that are, very difficult to reconcile one with another to any exact Computation, though ſometimes the difference be inconſiderable, and in round reckoning paſs with Authours one for another, without much ſenſible Error. As the *Engliſh* Mile is often joyned with the *Italian*, yet as aforeſaid this contains but 1000 Paces, and that 1056. So the *Ruſſian Vorſt*, wrote alſo *Verſt*, and ſometime *Worſt*, commonly called a Mile, and counted of *Engliſh* or *Italian* Meaſure; though *Fletcher* in his Hiſtory of *Ruſſia*, *Chap.* 1. tells us it wants a quarter. And ſo *Johnſon* in his *Atlas* accompts *Heylin* without help of the *Printers Errata's* is irreconcileable to himſelf in his *Coſmography*, p. 511. who counts 2260 *Vorſts* to 3690 *Italian* Miles, and not 4 lines of 4400 *Vorſts* to 3300 Miles.

Ruſſian Vorſt.

Some of the Principal, not all of the Modern Geodæticals *follow with the Kingdoms and Countries where uſed.*

Out of the Variety and incertainty of Authours to undertake the diſcovery of the different Meaſures, Weights and Monies of all places; were endleſs. Wherefore amongſt the Moderne, ſome of the Principal may ſuffice. And to ſpare the often Writing the Names of the Kingdoms and Countreys wherein moſt of the Provinces and Cities herein mentioned lye: Take the brief Accompt following; which by help of the figures annexed, will eaſily direct the Reader to find them out when he comes at them.

Parts

Parts of the WORLD.

Parts of the World, and ſome of the Kingdoms and Countries therein.

1. Europe.	2. Aſia.	3. Africa.	4. America.
1 *Denmark*	1 *Anatolia*	1 *Ægypt*	1 *Mexico*
2 *England*	2 *Arabia*	2 *Barbary*	2 *New Spain,* or, *Nova Spagnia*
3 *France*	3 *Armenia*	3 *Iſles*	3 *Peru*
4 *Germany*	4 *Chaldea*	4 *Terra Nigritay*	
5 *Greece*	5 *China*		
6 *Ireland*	6 *India*		
7 *Italy*	7 *Oriental Iſles*		
8 *Low Countries*	8 *Paleſtina*		
9 *Poland*	9 *Perſia*		
10 *Portugal*	10 *Ruſſia*		
11 *Ruſſia*	11 *Syria*		
12 *Scotland*			
13 *Slavonia*			
14 *Spain*			
15 *Sweden*			

Denmark *ſome Places noted therein.*

1. 1. *Denmarke*
- Heylegerhaven.
- Holſtein
  - Hamburgh, *or* Hamborough.
  - Lubecke.
- *Iſlands*
  - Fameren.
  - Scandia-Elbogen, *or* Nellebogh.
  - Seland
    - Coppenhagen, *or* Haffen.
    - Elſinure, *or* Elſingnore
- Juitland —— Ebbeltorfe
- Norway—— Bergen, *or* Barrow.

England, *ſome of the Counties and Cities there.*

1. 2. *England*

Berkſhire	Reading.
Cambridgeſhire	Cambridge
Cheſtire	Weſtcheſter
Devonſhire	Dartmouth. Exceter. Plymouth.
Eſſex	Colcheſter.
Glocesterſhire	Briſtol. Glocester.
Hamſhire	Wincheſter.
Herefordſhire	Hereford.
Kent	Canterbury.
Lancaſhire	Lancaſter. Mancheſter.
Middleſex	London. Weſtminſter.
Norfolke	Great Yarmouth.
Northumberland	Newcaſtle *upon* Tyne.
Oxfordſhire	Oxford.
Somerſetſhire	Bridgewater. Dunſter. Taunton.
Suffolke	Dunwich.
Suſſex	Rye. Winchelſea.
Warwickſhire	Coventry.
Weſtmorland	Kendal.
Wiltſhire	
Worceſterſhire	Worceſter.
Yorkſhire	York.

France
Ireland
Germany
Hungary
Some of the Places in the Book afterward referred to.

- 1. 3. *France.*
  - Aquitaine.
    - Gaſcony
      - Coniac
      - Oleron
    - Guiennæ
      - Baionne
      - Bourdeaux
      - Condet
      - Libourne, *or*
      - Liſborne
    - Xantoigne *or* Santoine
      - Bruage
      - Rochel
      - Santonum, *or*
      - St. Antoine
  - Anjou
    - Angiers
    - Tours
  - Berry
    - Bourges
    - Clermont
  - Bourbon — Tureme
  - Bretagne
    - Morlaix
    - Nantes
    - St. Malo
  - Borgoingue *or* Burgundy — Auxere
  - Champagne — Province
  - Daulphine
    - Valence
    - Vienna
  - Iſle of France
    - Clermont
    - Paris
  - Languedoc
    - Aquiſmort, *or*
    - Aigues Mort
    - Montpellier
    - Narbon
    - Tholouſe
  - La Beauſſe
    - Blois
    - Orleans
  - Lionois — Lyons
  - Normandy
    - Caen, *or*
    - Cane
    - Dieppe
    - Rouen, *or*
    - Roan
  - Nivernois — Nevers
  - Picardy
    - Abbevile
    - Amiens
    - Calais
    - Boulogne
    - Roy
  - Perigort
    - Angoleſme, *or*
    - Engouleſme
  - Poictou
    - Poictiers
    - Mirebeau, *or*
    - Mirabel
  - Provence
    - Aix, *or* Ay
    - Avignon
    - Aurange, *or*
    - Or ange
    - Marſeilles
- 1. 6. Ireland
  - Dublin
  - London-Derry.
- 1. 4 *Germany Hungary or Switzerland*
  - Alſatia
    - Colmar
    - Rufach
    - Strausburgh, *or*
    - Strasburgh
    - Thann
    - Weiſſenbergh
  - Auſtria — Vienna
  - Baden — Baden, Durlach
  - Bavaria
    - Munchen
    - Paſſaw, *or*
    - Patavia
    - Regensburgh, *or*
    - Ratisbone
    - Saltsburgh
  - Bohemia — Prague
  - Brandenburg — Brandenburgh
  - Carinthia — Friſach
  - Cleveland
    - Cleue
    - Weſel, *or*
    - Wiſel
  - Collenland
    - Collen
    - Rhineburgh
  - Eaſt Friſland — Emoden
  - Franconia
    - Bamberg, *or*
    - Babemberg
    - Frankford, *on the* Moene.
    - Nurenburgh
    - Wurtſpurg, *or*
    - Wurtzburgh
  - Gulich
    - Aix, Aken, Aquen, *or* Achon
    - Gulich, Juliers, *or* Guliers.
  - Hungary
    - Pooſen, *or*
    - Presburgh
  - Lorrein, *or* Lorraine.
    - Mets
    - Verdun
  - Luſatia
    - Bautſen, *or*
    - Botſen
  - Lunenbourgh — Lunenbourgh
  - Mecklenburgh *or* Macklenburgh
    - Domyn, *or*
    - Dammin
    - Roſtock
    - Wiſmar
  - Mark *or* March
    - Tremone, *or*
    - Dortmond
    - Werden
  - Mentz
    - Koningsbergh
    - Mentz
  - Meydeburgh
    - Meydeburgh, *or*
    - Magdeburgh, *or*
    - Mageburgh
  - Palatinate
    - Eiſted, *or* Aichſtad
    - Heydelburgh
    - Norenborgh, *or*
    - Norembergh, *or*
    - Nurenbergh
    - Spiers
    - Wormes
  - Pomerania
    - Coſtin
    - Gripſwald
    - Ockermond
    - Stetin
    - Straelſondt

Germany and Switzerland Greece, Italy, ſeveral Places of Note in them afterward referred to.

B

- Saxony
  - Turingia.
    - Erdford
    - Halo, *or* Kala
    - Jene
  - Meiſſen.
    - Friberg
    - Lipſich
    - Meyſen
    - Mansfield
- Sileſia — Breſlaw
- Suevia, *or* Almaine
  - Ausburgh, *or* Auſpurgh
  - Bibrach
  - Briſach, *or* Briſgow
  - Conſtance
  - Friburgh
  - Kempten, *or* Campidona
  - Norlingen
  - Offen, *or* Offner
  - Ravenſpurgh
  - Scaffhauſen, *a* Canton *of the* Switz
  - Ulme
- Switzeeland, *or* Helvetia
  - Upper Baden
  - Baſil
  - Berne
  - Friburg
  - Lauſanna
  - Lucern
  - Soloturn
  - Switz
  - Zurich
- Triers
  - Confluentz
  - Triers
  - Upper Weſel
- Tyrol
  - Inſpruch
  - Trent
- Weſtphalia
  - Breme
  - Homberg
  - Oſenbrigh, *or* Oſenbridg
  - Ravensburgh
- Weteraw
  - Fribergh
  - Naſſaw
  - Weilborough, *or* Wiſſelborough
- Wirtenbergh
  - Tubing
  - Wiberg

1. 5. Greece.

- Achaia — Athens
- Albania — Lavalona
- Arcadia
- Archipelago
- Epirus — Larta
- *Iſlands*
  - Candia, *or* Crete
  - Corfu
  - Euboe — Nigropont
  - Scio
  - Zant
- Laconia — Sparta, *or* Lacedemon
- Lepanto
- Macedon — Salonichi, *or* Salonici, *of* Old Theſſalonica
- Thrace
  - Sapy
  - Conſtantinople.

1. 7. Italy.

- *Church-Lands.*
  - Compagnia di Roma — Rome
  - Ducato Spoletano
    - Narnia,
    - Negropont
  - Eſtate of Urbin
    - Peſara
    - Urbin
  - Marca Anconitana
    - Ancona
    - Rechanati
  - Romandiola, *or* Romagnia
    - Boloignia, *or* Bononia
    - Cervia
    - Ceſena
    - Faenza
    - Forli, *or* Furli
    - Ravenna
    - Rimini, *or* Rimano
  - Territory of Ferrara
    - Carpi
    - Ferrara
- *Common-Wealths.*
  - Genoa — Genoa, *or* Genes
  - Luca — Luca
  - Venice
    - Friuli — Trieſte
    - Hiſtria
      - Cape de Iſtria
      - Piran
    - Marca-Trevigiana
      - Bergamo
      - Breſcia
      - Crema
      - Padua
      - Treviſo
      - Verona
      - Vincentia, *or* Vincenza
    - Venice
- *Dukedoms.*
  - Mantua — Mantua
  - Millan
    - Como
    - Cremona
    - Millan
    - Pavia, *or* Papia
  - Modena — Modena
  - Montferrat
    - Alba
    - Caſal, *or* St. Vas
  - Parma
    - Mirandula
    - Parma
    - Placentia, *or* Plaiſſance
  - Tuſcaine
    - Florence
    - Ligorne
    - Maſſa
    - Prato, *or* Perato
- Corſica
- *Iſlands.*
  - Sardinia
  - Sicilia
    - Marſala, *or* Maſilla
    - Meſſena, *or* Meſſana
    - Nicoſia
    - Palermo
    - Syracuſa
- *Kingdom of Naples.*
  - Abruzzo
    - Aquila
    - Lanciano, *or* Lanſan
  - Apulia, *or* Puglia
  - Calabria — Rhegium
  - Terra de Otranto — Otranto
  - Terra di Lavoro
    - Naples
    - Rivelli
    - St. Fumia, *or* Fiume

- *Principality and Provinces of the Allobroges, &c.*
  - Piedmont — Turin
  - Savoy — Chambery, Lunebourgh
  - Geneva, *or* Jenſer
  - Walſland — Sedun
  - Griſons — Curienſis

- 1. 8. *Low Countries.*
  - Artois — Arras, Perne, St. Omer
  - Brabant *and* Marquiſate *of the Empire* — Antwerp, Arſchot, *or* Aſchot, Barrow, *or* Bergen op Zome, Bolduc, Boiſeleduc, Hertogenboſh, *or* Shertogenboſh, Bruſſels, *or* Bruxels, Dieſt, Lovaine, Maeſtricht, Machlyn, *or* Mechlyn, Malines
  - Cambray — Cambray
  - Flanders — Alſt, Ailſt, *or* Aloſt, Audenarde, *or* Oudenarde, Axele, Bridges, *or* Bruges, Caſel, Cortrycke, *or* Courtray, Damme, Deynſe, *or* Deyſe, Dixmude, Doway, *or* Douay, Dunkerk, Gaunt, Graveling, Honſchotten, Hulſt, Ipre, *or* Ypre, Liſle, *or* Ryſſel, Loo, *or* Lowe, Meanen, *or* Meenen, Newport, Orchies, *or* Orſies, Oſtend, Popering, St. Amand, Scluſe, *or* Sluys, Tournay, *or* Dornir, Walſland, *or* Land Van Waes, Winocksborough, *or* Winocksbergh
  - Groeninghen — Dam, *or* Damme, Groeninghen
  - Guelderland — Arnhem, Bomel, Batenborgh, Ghent, Gueldres, Nimmegent, *or* Nimmeghen
  - Henault — Ach, Halle, Valenciennes, Covyn, *or* Convennes
  - Holland — Amſterdam, Briel, Delfe, Dort, Enckhuyſen, Hague, Harlem, Horne, Leiden, Medenblick, Naerden, Rotterdam, Schoonehaven, Texel, Waſſenar
  - Limbourg — Carpen, Limbourgh
  - Luickland — Aſſelt, Hoey, Hoye, *or* Huy, Liege, Looth
  - Luxenbourg, *or* Lutzenberg — Maſiers, Sedah, *ſet by ſome in* France.
  - Namurs — Namen
  - Overyſſel — Campen, Deventer, Swdoll, *or* Zwoll
  - Utrech — Utrech
  - Weſt-Friſland, *or* Friſland — Harlingen
  - Zeland
    - Scowen — Zuricza
    - South Beverland — Goes, *or* Tergoes, Romerſwal
    - Walcheren — Armuyden, *or* Ramme, Camfere, *or* Vere, Fluſhing, Middleburgh
  - Zutphen — Groll, Zutphen

Low-Countries. *Several of the Places afterward referred to.*

- 1. 9. Poland
  - Liefland, *or* Livonia — Riga
  - Lithuania — Vilna, *or* Wilde
  - Poland — Cracow, Warſaw
  - Pruſſia — Coningsbergh, Dantzick, Melvyn, Thoren

Poland *the like.*

- 1. 10. Portugal
  - Algarve
  - Portugal — Lisbon, St. Tubal, *or* Setubal

Portugal *the like.*

- 1. 11. Ruſſia — Muſcovie
- 1. 12. Scotland — Edenburgh

Ruſſia, *and* Scotland *the like.*

Slavonia, Spain; Sweden; *the places of Note in them and in* Aſia, Africa *alſo in the Book afterward referred to:*

- 1. 13. *Slavonia.*
  - Dalmatia —
    - Raguſa, *or* Raguza
    - Sebenico
    - Spalato
  - Liburnia —
    - Zara
    - Zegna, *or* Sequia
- 1. 14. *Spaine.*
  - Andaluzia —
    - St. Lucar.
    - Sevill
  - Arragon — Saragoſſa
  - Biſcay — Bilboa
  - Caſtile —
    - Burgos
    - Madrid
    - Medinadel Campo
    - Salamanca
  - Catalonia —
    - Barcelona
    - Girone
  - Corduba —
    - Xeres, *or* Sherry
  - Gallicia — Baiome
  - Granada —
    - Almeria
    - Granada
    - Malaga
  - Guipuſcoa —
    - St. Sebaſtians
    - Toloſo
  - Iſlands
    - Gades
      - Cadiz
      - St. Mary
    - Majorca
  - Leon —
    - Leon
    - Villa Vicioſa
  - Murcia — Carthagena
  - Navarre —
    - Viano
    - Victoria, *or* Vellica
  - Toledo —
    - Toledo
    - Medina Cœli
  - Valentia —
    - Allicante
    - Valentia
- 1. 15. *Sweden.*
  - Boden —
  - Finland —
  - Gothland —
    - Oſtrogoth
    - Smalandia
    - Weſtrogoth
  - Lapland —
  - Liefland, *or* Livonia —
    - Revell, *or* Rivallia
    - Narva, *or* Nareca
  - Sweden —
    - Helſingen
    - Suderman
    - Weſtman
    - Upland, *in which is the City* Stockholme

- 2. 1. Anatolia, *or* Aſia *the leſs*
  - Aſia *ſpecially ſo called*
    - Epheſus
    - Smyrna
  - Bithynia — Burſa, *or* Pruſa
  - Cilicia —
    - Alexandria, *or* Scanderoone
  - Iſlands —
    - Cyprus
    - Rhodes
    - Scio
- 2. 4. Chaldea — Babylon
- 2. 6. India —
  - Decan — Goa
  - Malabar — Calicute
  - Narſinga —
    - Maliapur, *or* St. Thomas
- 2. 7. Orient Iſles — Molucco
- 2. 8. Paleſtine — Judea — Rama
- 2. 11. Syria —
  - Comagena —
    - Aleppo
    - Aman
  - Cœle-Syria — Damaſcus
  - Phoenicia —
    - Acon, Achri, *or* Acre
    - Barutti, *or* Berytus
    - Tripoli
  - Syria — Antioch
- 3. 1. Ægypt —
  - Alcario
  - Alexandria
  - Arcadia — Cairo, *or* Caire
  - Forfori
  - Sciba
  - Zaidin
  - Zeroi
- 3. 2. Barbary —
  - Algiers —
    - Algiers
    - Oran
  - Feſſe — Feſſe — Feſſe
  - Moroco —
    - Sus —
      - Capo d' Aguer *or* Capo d' Algier
    - Morocco — Morocco
  - Tunis —
    - Tripoly — Tripoly
    - Tunis — Tunis
  - Una
- 3. 3. Iſles —
  - Canaries
  - Madera
  - St. Thomas
- 3. 4. Terra Nigritarum
  - Cabo Verde
  - Guynea

The *Engliſh* are not alone in the knowledge and uſe of the Inch, Foot, Ell, Pace, Fathom, Furlong, Mile, *&c.* for beſides the *Hebrews*, *Greeks* and *Latines* before ſpoken of; moſt Nations eſpecially of *Europe*, uſe the ſame as occaſion ſerves. Some nearly correſpondent, ſome vaſtly different, and ſome Countries have divers Meaſures of the ſame Names. Hence is obſervable, that although moſt of the *Europæans* reckon diſtances of places by Miles, yet there are no agreement between them. And in *Germany* it ſelf are

Long Meaſures in England *uſed elſewhere, But different.*

German *Miles the ſorts.*

are 3 ſorts. The Common Mile which is 4 *Italian* Miles, and yet the leaſt of the 3. A Mean which is 5, and the Great (called ſometimes the *Great Saxony*) Mile, being 6 *Italian* Miles. And beſides theſe *Alſted* mentions a Mile that containeth 22800 Feet.

Spaniſh *Miles.* *Leagues.*

Spain, by ſeveral hath $\frac{1}{3}$ Mile to a *Dutch* Mile; but their Common Accompt is by Leagues called with them *Leguas*, whereof 17 $\frac{1}{2}$ make 60 *Italian* Miles. *Alſted* ſpeaks of a *Spaniſh* Mile to be 21000 Feet.

French *Leagues.*

France, where they reckon by Leagues, hath no leſs than 4 ſorts, as *Cotgrave* in his *Dictionary* hath well obſerved. As,

The Sorts.

Lieue de Bourgongue, 50 *Portées*, every *Porteé*, 12 *Cords.* Every *Cord* 30 *Feet*, moſt in uſe in *Burgundy*.

Grande Lieue, as the common ſort of *German* Miles, every *Lieue* 4 *Italian* Miles.

Moyennes Lieues, as thoſe of *Dauphine* or *Languedoc*, containing 3 *Italian* Miles.

Petites Lieues, as thoſe of *Italy*, and of this ſort, the *Fiery Collumne*, commonly called the *Dutch Waggoner* in his *Sea Charts* often joynes as one with the *Engliſh* Mile.

French *Mile.*

A *French* Mile ſaith *Cotgrave* is 2 of ours.

Flemiſh *Mile.*

Malines, notes the *Flemiſh* Mile of 2 ſorts. One 1000 Rods of 20 Feet to the Rod, which is alike to the Common *Germane* Mile. And the other of 1400 Rods of 15 Feet to the Rod. And the *Holland* Mile he ſaith is 2000 Roads, and that 5 of them are 4 *Flanders* Miles. *Alſted* mentions the *Belgick* Mile to be 18000 Feet, and by this accompt the Mile of *Flanders* muſt be 2500 Roads, but how long the *Holland* Road is he ſaith not.

Swediſh *Leagues*

Leuca Suevica, after the *Latine*, or the *Swediſh* League called *Scandica* is 5000 Paſſes, or 5 *Italian* Miles.

Florence *Mile.*

Florence, *Malines* reckons 3000 Braces for a Mile, which at the rate of 122 $\frac{1}{2}$ Braces for 60 *Engliſh* Ells comes 5510 Feet, 2 Inches, and ſomewhat over.

Ægyptian *Schoenes.*

In *Ægypt* they reckon by *Schoenes* of various Magnitude, ſome 60, ſome 40, ſome 20 Furlongs. Some will have the *Schoene* equal to a *Spaniſh* Mile. See before in *Greek* Meaſures. The *Ægyptian* Cubit, ſaith *Malines* is 6 of our Cubits.

Ruſſia *Vorſt* vide antea

Of the *Ruſſian Vorſt*, and *Perſian Paraſang* ſee before.

Chinian *Stade.*

The *Fur long* or *Stade* of *China* is almoſt twice as long as ours, for 12 of ours are but $6\frac{1}{4}$ of theirs by *Johnſon* in his *Atlas*; who for the moſt part the following Table, follows.

Long Forain Meaſures compared to a Degree.

Long Outlandiſh Long Meaſures compared.

In	Dalacia	$7\frac{1}{3}$	Miles, make 1 Degree of a Great Circle in the Earth.
	Weſtrogoth	$8\frac{1}{2}$	
	Saxony Great	10	
	Almaine Oſtrogoth Suderman Upland Weſtman	12	
	Germany Smalandia	15	

In	Spain	$17\frac{1}{2}$	Leagues.	make 1 Degree of a Great Circle in the Earth.
	Finland Cajan Livonia	18	Miles	
	France	20	Leagues	
	Helſingen Botnia Lapland	22	Miles	
	Scotland	50		
	Italy England	60		
	Ruſſia	80	Vorſts	
	China	250	Furlongs	

Chinian Diez, *their* Lij, Pu, *and* Icham.

Grimſtone, *p.* 715 tells us that 69516 *Diez* of *China* Meaſure make almoſt 1000 *Spaniſh* Leagues. And that the *Chinois* have 3 Meaſures to Survey withal, which they call *Lij*, *Pu*, and *Icham*. The *Lij* comprehends as much Space as may be aſſigned to a Mans Voice thruſt forth with all his force in a Calme Seaſon upon a fair Plain, 10 of theſe *Lij's* makes 1 *Pu*, which is a great *Spaniſh League*, and 10 *Pu's* make 1 *Icham*, or a whole Journey.

Moderne Furlongs.

The Modern Furlongs, Fathoms, and Paces of the *Europæans* differ little if at all from the Ancients. The Furlong 125 Paces, the Pace 5 Feet, or two of the leſſer Paces.

French *Fathom*

The Fathom in *France* called *Toiſe* ordinarily is as ours 6 Feet, the Kings Fathom 7 Feet 4 Inches. For Woods and Foreſts by the Cuſtom of *Orleans* for 5 ½ Feet. The *Burgonian* Fathom is 7 ½ Feet.

French *Foot.* *The Sorts.*

The Ordinary (called alſo the Royal) *Pied*, or Foot, there is 12 Inches, that of *Clermont* 11, of *Engouleſme* [illegible] longer than the Ordinary. The Foot uſed about *Bourdeaux* to meaſure Land called *Pied de Terræ* is longer than the ordinary by $_{8}$ in $\frac{3}{4}$. *Pied de Ville* there uſed for Timber, Stone, *&c.* longer than the Ordinary by ⅝ of an Inch. The *Petit Pied* is ſhorter than the Ordinary.

Several affirm, that the *Foot* uſed in ſeveral Countries is different, as of 10, 12, 16. *&c.* Inches, and they of different bigneſs. *Capel* and *Snellius* before-mentioned have compared them together, of whom the latter is by ſome reckoned moſt exact. The Tables of both follow.

Capel *his Compariſon of the Foot of* Toledo *with others.*

Capel's *Compariſon of the Foot to the Foot of* Toledo *in* Spain, *ſuppoſed to be divided into 120 parts.*

The Foot of	in	Parts
Heidelberg	*Germany*	137
Tuſcany	*Italy*	138
Sedan	*France*	139
Rome	*Italy*	144
Athens	*Greece*	150
London	*England*	152
Paris	*France*	160
Syria	*Aſia*	166
Egypt	*Africa*	171
Judea	*Paleſtine*	180
Babylon	*Chaldea*	200

Snellius *his Compariſon of the Foot of ſeveral Places with others.*

Snellius *his Compariſon of the* Foot, *to the* Foot *of* Rome *in* Italy, *or* Rheinland, *or* Leiden, *which he ſaith are all one, ſuppoſed to be divided into 1000 parts.*

The Foot of	in	Parts
Toledo	*Spain*	864
Mechlin	*Brabant*	890
Straſburgh	*Germany*	891
Amſterdam	*Holland*	904
Antwerp	*Brabant*	909
Lovain	*Brabant*	
Bavaria	*Germany*	924
Coppenhagen	*Denmark*	934
Goes	*Zeland*	954
Middleburgh	*Zeland*	960
London	*England*	968
Norembergh	*Germany*	974
Zurichzee	*Zeland*	980
The Ancient *Greek*		1042
Dort	*Holland*	1050
Paris	*France*	1055
Briel	*Holland*	1060
Venice	*Italy*	1101
Babylon	*Chaldea*	1172
Alexandria	*Egypt*	1200
Antioch	*Syria*	1360

Palmes, &c. *like the old.* Spaniſh Xeme.

The Palmes greater and leſs Inches and Digits, for the moſt part have the ſame Diviſions as the Ancient. The *Spaniſh Xeme* is half a Foot, or 6 Inches. Their *Cotus* half of their greater Palme, or $4\frac{1}{2}$ Inches, both *Spain* and *France* allow 16 digits to their 12 Inches, called by that *Pulgada*, by this *Poulcee*.

Inch of Spain *and* France. French *Line.* *Superficial Meaſures of the Moderne.* Arpentiers *of* France. *The Sorts.*

France hath a little Meaſure called a *Ligne*, or Line, whereof 12 to 1 Inch.

Superficial Meaſures of Land having length and breadth are ſubject alſo to the different Laws of divers Countries, whether they meaſure by Lines, Cords, Rods or Perches, *&c.* And accompt their Meaſures by Acres, Arpentiers, Bunderen, *&c.*

France reckoning by Arpentiers uſeth not fewer than 10 ſorts, as *Cotgrave* accompts, *Viz.*

Arpent, that is ordinary is 100 Perches Square of 18 Feet to the Perch.

Arpent de Bois is $2\frac{1}{2}$ Roods, 1 Rood 40 Perches, 1 Perch 24 Feet, 1 Foot 24 Inches.

Arpent de Bois, de Bourgoigne, is 440 Perches.

Arpent de Clermont is 100 Verges in moſt places, in ſome but 70, at 26 Feet to the Verge.

Arpent de Dunois, is 100 Perches at 20 Feet to the Perch.

Arpent de Nevers is 4 quarters ſquare, 1 quarter 10 Fathoms, 1 Fathom 6 Feet.

Arpent de Paris, is 100 Perches Square, at 22 to the Perch, and in ſome places about *Paris* it contains 25 Feet, and in others at the ordinary rate 18.

Arpent de la Perch, 100 Perches, 1 Perch 24 Feet, 1 Foot 13 Inches.

Arpent de Poictou is 80 Paces Square.

Arpent Romain, is 240 Feet long, and 120 Feet Broad. This is like to the *Latine Juger*, of which before.

French Septier *of* Land. *Their* Muid *and* Mine.

A *Septier* of Land he ſaith is much about the *Arpent*.

A *Muid* of Land is 12 *Septiers* or *Arpents*.

A *Mine* of Land in *La Chaſtellenie de Bulles*, contains 50 Verges of 24 Feet to the Verge.

The Sorts. Scruple *of Land.* Spaniſh Jugada Faneca Stadale.

A *Mine* of Land in *Clermont*, 60 Verges of 22 Feet to the Verge.

A *Mine* of Land in the *Seigneurie de Remy*, 80 Verges of $22\frac{1}{3}$ Feet to the Verge.

A *Scruple* of an *Arpent* is $\frac{1}{48}$ of an *Arpent* or 10 Feet Square every way.

In *Spain*, 1 *Jugada* is 50 *Faneca's*, 1 *Fanega* is of Land ſowed with Barley 400 Square *Stadales*, of Land with Wheat 600. And 1 *Stadale* is 11 Feet. A Square *Stadale* is 121 Feet.

Moderne Long Meaſures.

Merchants have their *Ells*, *Aulnes*, *Braces*, *Cannes* or *Canes*, *Varras* or *Varas*, *Pitchy*, *&c.* for Commodities proper to be meaſured thereby after the Laws and Uſages of ſeveral Countries. And by Traders is diligently to be ſought out, becauſe in ſeveral Countreys, though ſome Common Meaſures be of moſt general uſe, yet in ſundry places in one and the ſame Countries ſhall meaſures of the ſame Name, Nature and uſe be different, as in *France*, the Common *Aulne*, or that called *Aulne du Roy* is 3 Feet 7 Inches and 8 Lines. But that of *Bourdeaux* $4\frac{1}{2}$ Feet almoſt, that of *Dijon* and *Province*, but $2\frac{1}{2}$. That uſed of *Merchants* for Silks half an Inch ſhorter than the Common, that of *Paris* but 3 Feet and $\frac{2}{3}$ of an Inch, and about $\frac{1}{50}$ of a Foot. Some Cuſtoms beſpeak it formerly 3 Feet 8 Inches and 4 Lines long. And allowing 100 Ells of *Antwerp* to agree with 60 Ells or 75 Yards at *London*, the correſpondency thereof with other places follow out of *Malines* his *Lex Mercatoria*.

Aulnes *of* France *how different.*

100 *Ells of* Antwerp *compared with the Meaſures of other places.*

The 100 Ells of *Antwerp* make at

Townes and Cities	Directions as before		Merchandiſe Meaſured	Contents *or* Quantities	Names of the Meaſures.
Abbevile	1	3		84	Ells
Achrie, *or* Acon	2	11		115	Pichy
Acon, *or* Aken	1	8		$104\frac{1}{2}$	Ells
Adler				$33\frac{1}{2}$	Canes
Aleppo	2	11		108	Pichy
Alexandria	2	1		124	Pichy
Aman	2	11	as *Aleppo*		—
Amſterdam	1	8		$101\frac{1}{4}$	Ells
Ancona	1	7		$107\frac{1}{2}$	Braces
Andaluzia	1	14		$83\frac{1}{2}$	Varras
Antwerp	1	8	for Silks	$98\frac{2}{3}$	Ells
Aragon	1	14		43	Cannes
Archipelago	1	5		100	Pichy
Artois	1	8	all the Province	$98\frac{2}{3}$	Ells

100 *Ells of* Antwerp *compared with the Meaſure of other Places.*

The 100 Ells of *Antwerp* make at

Aſſelt	1	8	as *Acon* in the *Netherlands*		
Audenarde	1	8	as *Antwerp* for Silks	$98\frac{2}{3}$	Ells
Avignon	1	3		60 36	Ells Cannes
Ausburgh, *or* Ausborough	1	4	for Linnen Cloth	125 127	Ells
Barrow, *or* Bergen	1	1	Uncertain, for they meaſure by the bigneſs of your head with a Rope for		1 Ell
Barcelona	1	14	as *Arragon*		
Baſill	1	4		125	Ells
Bautſon	1	4	for Cloth Silks	111 120	Ells
Bergamo	1	7		$101\frac{2}{3}$	Braces
Bolduc, *or* Hertogenboſh	1	8		102	Ells
Bologna	1	7	as *Ancona*		
Brabant	1	8	all the Province (except ſuch Places as are herein excepted) like *Antwerp* for Silks.		
Breme	1	4		$122\frac{1}{2}$	Ells
Breſcia	1	7	as *Bergamo*		
Breſlo	1	4	as *Bautſon*		
Bruges, *or* Bridges	1	8	in the Shops but for Linnen	$98\frac{2}{3}$ $94\frac{1}{4}$	Ells
Bruſſels	1	8	as *Bolduc.*		
Bauti, *or* Baruti	2	11		$111\frac{1}{3}$	Pichy
Burſa	2	1		114	Pichy
Cadiz	1	14	for Cloth Silks	81 108	Varras Ells
Calabria	1	7	as Adler		
Cambray	1	8		96	Ells
Candia	1	5		108	Pichy
Capo d' Algier	3	2		$136\frac{1}{2}$	Covados
			There 1 Cane is 12 Cavados.		
Carpi	1	7	as *Ancona*		
Caſſel	1	8	as *Bolduc*		
Caſtile	1	14	 Some allow but	85 81	Varras
			There 1 Varra is 4 Quarters, 1 Quarto 2 Palmes.		
Ceſena	1	7	as *Ancona*		
Collen	1	4		120	Ells
Conninſbergh	1	9		125	Ells
Conſtantinople	1	5	 For Canvers	113 80	Pichy
Corfu, *an Iſland*	1	5		$116\frac{2}{3}$	Braces
Covin	1	8		70	Ells
Crema Cremona	1	7.	as *Bergamo*		
Damaſcus	2	11	as *Bruti*		
Damme	1	8	as *Antwerp* for Silks		
Dantſick	1	9		122	Ells
Deyſe *and* Dieſt	1	8	as *Bolduc*		
Domyn	1	4	as *Breme*		
Doway	1	8	as *Cambray*		
Dunkirk	1	8	as *Antwerp*	100	Ells
Embden	1	4	as *Breme*		
Erdfurd	1	4		165	Ells
Ferrara	1	7	as *Ancona*		
Flanders	1	8	all the Province, as *Brabant*		
Fluſhing	1	8		104	Ells
Florence	1	7	for Woollen Silks	116 $122\frac{1}{2}$	Ells Braces

France, the Kingdom, except the Places herein excepted. See *Paris.*

100 Ells of Antwerpe compared with the Meaſure of other Places.

The 100 Ells at *Antwerp* make at

Frankford	1	4	as *Ausborough*		
Gaunt	1	8	as *Antwerp* for Silks		
Gelderland	1	8		$104\frac{1}{2}$	Ells
Genes	1	7		122	Braces
			Silks (104, Palmes for $34\frac{2}{3}$ Ells)	288	Palmes
			Woollen Cloth at 9 Palmes the Cane	32	Cannes
			Linnen Cloth at 13 Palmes the Cane	29	
Geneva	1	7		60	Stabb
Goes	1	8		97	Ells
Granada	1	14	as *Andaluzia*		
Gripſwool	1	4	as *Breme*		
Halle	1	4		105	Ells
Hamborough	1	1	as *Breme*		
Harlem	1	8	in the Market for Linnen	$94\frac{1}{2}$	Ells
Henault	1	8	in the Market	$94\frac{1}{2}$	Ells
			in the Shops	$98\frac{1}{2}$	
Hertogenboſh	1	8	See *Bolduc*		
Holland	1	8	in moſt places of the *Province*	$103\frac{1}{2}$	Ells
Honſchotten	1	8	as at *Dunkerk*		
Hoye	1	8	as at *Bolduc*		
Ipre	1	8	as *Antwerp* for Silks		
Iſtrica Iſtria	1	7	for Woollen Cloth	$101\frac{2}{3}$	Braces
			for Silk and Cloth of Gold	108	
Lanſan	1	7	as *Adler*		
Larta, *or* Laarta	1	5	as *Alexandria*		
Lavalona	1	5		111	Pichy
Lepanto	1	5		113	Pichy
Liege	1	8		114	Ells
Lipſich	1	4	for Silks and Linnen	105	Ells
			ſome ſay but $104\frac{1}{2}$		
			Cloth	120	
Lisbon	1	10		62	Varras
			by ſome but 60		
			alſo	83	
			and for Silks	100	Covados
Liſle	1	8	as *Cambray*		
London	1	2	for Linnen with the Palme and Thumb meaſured into it	60	Ells
			Woollen, the Thumb	75	Yards
			Frize at $1\frac{1}{3}$ for a Yard	50	Goads
			ſome ſay	59	
			Roan Canvas, whereof the Centener is 120, being 10 Cords, of 12 Ells to a Cord	61	Ells cords
Loo, *or* Lowe Louvaine	1	8	as *Bolduc*		
Lubeck	1	1	as *Collen*		
Luca	1	7		120	Braces
Lyons	1	3	for Linnen	60	Ells
			for Silks	$94\frac{1}{2}$	
Maeſtricht	1	8	as *Acon* in the *Netherlands*		
Malaca, *or* Malaga	1	14	as *Adler*		
Mantua	1	7	as *Ancona*		
Maroco, *or* Moroco	3	2	as *Capo d' Algier*		
Marſeilles	1	3	for Woollen Cloth	$33\frac{1}{2}$	Cannees
			for Silks	36	
Maſiers	1	8	as *Cambray*		
Maſilla	1	7		$34\frac{1}{2}$	Cannes
Meanen	1	8	as *Cambray*		
Melvyn	1	9	as *Dantſicke*		
Meydeborgh	1	4	as *Halle*		
Meyſen	1	4	as *Lipſich*		

100 Ells of Antwerpe compared with the Measure of other Places.

The 100 Ells of *Antwerp* make at

Middleburgh	1	8	in the Market for Linnen otherwise	94½ 100	Ells
Millan	1	7	for Linnen for Silks	120 141	Braces
Mirandula Modena	1	7	as *Ancona*		
Munster	1	4		65	Ells
Namen	1	8	as *Acon* in the *Netherlands*		
Nantes	1	3	as *Abbeville*		
Naples	1	7		116 33½	Cannes
Narva	1	15		125	Arsins
Negropont	1	7	as *Ancona*		
Nigropont	1	5	as *Lepanto*		
Norenborgh	1	4	as *Lipsich*		
Ockermonde	1	4		106	Ells
Offner	1	4		119 130	Ells
Orsies, *or* Orchis	1	8	as *Cambray*		
Osenbridg, *or* Osenborgh	1	4		63	Ells
Overyssel	1	8	as *Gelderland*		
Padua	1	7	for Cloth for Silks	101⅔ 83½	Braces
Palermo	1	7	as *Masilla*	34½	Canes
			One Cane, 4 Pichy		
Paris			*France*, and most part of all that Kingdom According to others	59 57	Ells Aulnes
Parma	1	7		91 109½	Braces
Perato	1	7	as *Ancona*		
Pesaro	1	7	for Cloth for Silks	107 103	Braces
Picardy	1	3	as *Abbevile*		
Piran	1	7	as *Istrica*		
Prague	1	4	as *Bautson*		
Provence	1	3		36	Cannes
Puglia	1	7	for Cloth for Silks	31 33	Cannes
Raguza	1	13	as *Luca*		
Rama	2	8		115	Pichy
Ravenna	1	7	as *Corfu.* Some say	113	Braces
Rechanati	1	7	as *Bergamo*		
Regenburgh	1	4		78½	Ells
Revel, *or* Rivalle	1	15	as *Coningsbergh*		
Rhode	2	1	as *Adler*		
Riga	1	9	as *Coningsbergh*		
Rochel	1	3	as *Paris*		
Rome	1	7	 for Woollen Cloth	33 105½	Cannes
Romerswal	1	8		99	Ells
Rostock	1	4		119	Ells
Rouen	1	3	The Centener of Ells being 112, that is 28 to a quarter According to some	58 52	Ells Aulnes
Salonici	1	5		109	Pichy
Sapi	1	5	as *Archipelago*		
Saragossa	1	14		33	Cannes
Scio, *or* Sio	2	1	as *Corfu*		
Scotland, most part of that Kingdom, where they reckon 120 to the 100				72	Ells
Sebenico	1	13		112	Braces
Sevil, *or* Sivil	1	14	as *Andaluzia*		

1○○ *Ells of* Antwerp *compared with the Measure of other Places.*

The 100 Ells of *Antwerp* make at

Sicilia	1	7	as *Palermo*		
Sluys	1	8	as *Antwerp* for Silks		
Stetin	1	4	as *Ockermonde*		
Stockholme	1	15	as *Conninsbergh*		
Toledo	1	14	as *Castile.* *Hunt* ſaith	88	Varras
Tournay	1	8		108	Ells
Treviſo	1	7	as *Bergamo*		
Tripoli	3	2	as *Alexandria*		
			1 Cane, 4 Pichy		
Tripoli	2	11		112	Pichy
Valentia	1	14		73	Cannes
Venice	1	7	as *Iſtrica*		
Vere	1	8		$94\frac{1}{2}$	Ells
Verona	1	7	Long Meaſure	86	Braces
			Short Meaſure	$104\frac{1}{2}$	
			For Cloth of Gold	108	
Vincenza	1	7	for Woollen Cloth	$98\frac{3}{4}$	Braces
			Silks	$80\frac{1}{2}$	
Vienna	1	4	for Linnen	$77\frac{1}{2}$	Ells
			Cloth and Silks	$85\frac{1}{2}$	
Ulme	1	4		120	Ells
			Woollen Cloth	96	
Urbin	1	7	as *Bergano*		
Winockxborough	1	8	as *Bolduc*		
Wiſmar	1	4		118	Ells
Yſenghem			as *Antwerp* for Silks		
Zara	1	13	as *Sebenico*		
Zurich	1	4		$116\frac{1}{2}$	Ells

Concave Meaſures.

Forrain Concave Meaſures have had the ſame fate as Long Meaſures to differ in Names and Quantities with moſt Nations; and by the aforeſaid Authour have like Contents and Correſpondencies as follow.

Forreign Meaſures of Wine *and* Oyle.

Fother *of Wine the Content.*

In *Germany* they call the Carriage of the drawing of 2 Horſes, a Fother of Wine, and accompt $2\frac{1}{2}$ Rods for a Fother.

Rod *of Wine how much.*

At *Dort* in *Holland*, they call a great Veſſel 10 Feet Square, and one Foot deep: A Rod of Wine, every ſuch Foot containing $7\frac{1}{2}$ Gallons *Antwerp*, every Gallon called there *Stoop*, weighing 6 ℔.

Ame *of* Dort *the Content.*

An Hogſhead of Wine *Dort* Meaſure, called an *Ame*, contains 100 Gallons or *Stoopen*; and every Gallon 10 *Schreaves*.

Some German *Meaſures.*

Comenius in his *Janua Aurea*, mentions among the *German* Meaſures beſides the Fother, Half *Eymer*, and *Eymer*, 3 ſorts of *Maaſz*, 1 of 24 *Kannes*, 1 of 12, and another of 3, together with the *Noeſel*, Half *Noeſel*, and *Drittentheil*, *&c.* but gives no accompt of the particular Contents thereof.

Malines affirms at *Meyſen* in *Saxony* 20 ℥ Ponderal, make 24 Menſural. And at *Lipſich* 32 ℥ Menſural $26\frac{1}{2}$ Ponderal, but at the rate of 6 Menſural for 5 Ponderal, it ſhould be $26\frac{2}{3}$.

Hogſhead *the Weight and Content.*

Pag. 30. he writes that an Hogſhead of Wine weighs 500 *l.* the Cask 50 *l.* Wine *Netto* 450 ℔. An Hogſhead of Corn 400 ℔. Cask 50 ℔. Corn *Netto* 350 ℔. ſo ſhall the Ton of Wine *Netto* be 1800 ℔. with the Cask 2000 ℔. of Corn but 1600 ℔. with the Cask. Four Hogſheads going to a Ton, and 2 Tons to a Laſt.

Milliar *of Oil.* Ame *of* Antwerpe *how much.*

Pag. 31. A Millier of Oyle at *Antwerp* 1100 ℔. a Butt 152 Stoopes.

One Ame of *Antwerp* containes 300 Stoops, every Stoop weighing 6 ℔. called a Stone. And 6 of theſe Ames of Wine make in

6 Ames *of* Antwerpe *compared with the Meaſure of other Places.*

1	10	*Algarve*	34	Starre
		Anſoy, or *Baſtard Spain*	2	Pipes 16 Stoopes
1	8	*Artois*	$4\frac{2}{3}$	Hogſheads
1	3	*Auxere*	3	Puncheon
1	3	*Ay*, as *Artois*		
1	7	*Bolognia*	13	Corbes
1	3	*Bourdeaux*	$4\frac{1}{2}$	Hogſheads

Ames of Antwerp compared with the Meaſures of other Places.

1 7 *Calabria* ——————————— 8 Salmes
3 3 *Canaries*——————————— 2 Pipes of 150 Stoope, or 1 9/[illegible]

Butt. Every Butt at *Antwerp* 158 Stoopes. They meaſure by the Roove of 30 ℔, which at *Antwerp* is 5 Stoopes. Every Butt containes 30 Rooves. And the Pipe 30 Rooves of 28 ℔ weight.

1 5 *Candia*——————————— 80 Moſtaches
Canado, or *Condado Spain*——————— 2 Butts
1 3 *Coniac* ——————————— 2 Pipes or 4 Hogſheads
1 5 *Conſtantinople*———————————180 Almes

96 $\frac{1}{2}$ Almes of Oyle there is at *Venice* a Milliar.

1 5 *Corfu*——————————— 37 Zare, or Sare.
1 7 *Ferrara* ——————————— 12 Naſtelli, of 8 Seccheio
1 7 *Florence*——————————— 16 $\frac{2}{3}$ Barrels of 20 Fiaſchi

or 18 Stoopes *Antwerpe*, 3 Barrels is 1 Starre, and 1 Starre is 54 Stoops *Antwerp*.

1 7 Iſtria——————————— 15 Venas
1 3 Liborne——————————— 5 $\frac{1}{2}$ Hogſheads
1 10 Liſbon——————————— 37 $\frac{1}{2}$ Almudas.

1 Almuda is 1 $\frac{1}{2}$ Roove of Sevil, accompting 8 Sevil Somers or Covados to 1 Roove, that is 12 Cavados to 1 Almuda. Every Covado 4 Quartils, or Quarts.

Oyl Meaſure is by Alqueri or Canter, 1 Alqueri is 6 Covados, 1 Cantar 4 *Antwerp* Stoopes.

1 2 *London*———————————252 Gallons.

So is 1 Ame of *Antwerp* 42 Gallons at *London*.

3 3 *Madera* ——————————— 2 Pipes, lacking 16 Stoops.
1 3 *Orleans*——————————— 4 Hogſheads, lacking 10

Stoops, or 60 ℔ of *Antwerp*.

1 7 *Padua*———————————1 $\frac{1}{25}$ *Cara*.

Oyl is by the Milliar of 1185 ℔.

1 3 *Paris*, as at *Orleans*.

1 Hogſhead 36 Sextiers, 1 Sextier 4 Quarts, 1 Quart 2 Pints, 1 Pint 2 Choppins or Obles, 1 Choppin 24 Poulceons. *Cotgrave* calls that a Muid of Wine which *Malines* calls an Hogſhead, and ſaies 3 of them go to a Ton by the Cuſtoms of *Clermont*. And between the Muid and Sextier in quantity placeth a Barril to contain 9 Septiers or 72 Pints. The Septier he counts all one with the Sextier or Sextary, and when taken for a Wine Meaſure is 8 *Paris* Pints, and in all theſe Veſſels the Cask ſhall hold ſo much clear Wine beſides the Lees. The Pint is the 288 part of the Muid, almoſt as big as our Quart, weighing 27 Ounces. The Chopine in *Latine Ciopina*, or *Ciopinta*, is called *Oble quaſi Obolus* being half the *Paris* Pint, But in *St. Denis* and ſome other places 3 go to a Pint. The Poulceon, a ſmall meaſure of little uſe, ſave to try the Gage of the Small Sextier of 3 ℥, and Semiſextier of 1 $\frac{1}{2}$ meaſures uſed by the *Apothecaries*. The Poulceon 1 ʒ. *Alſted* mentions a *French* Meaſure called *Arroba*, or *Roove* to contain 2 Sextaries. But *Cotgrave* makes it contain as much as will weigh 25 ℔. Others write of a *Congie* containing about 4 $\frac{1}{2}$ Pints of *Paris*. Some an *Amphora* of about 36 Quarts, but both theſe ſeem *Roman* Meaſures, and perhaps may be in uſe in thoſe parts of *France* that border on *Italy*.

1 3 *Poictou*——————————— 2 $\frac{1}{4}$ Pipes, or 5 Hogſheads.
1 7 *Piran*——————————— 12 Urna.
1 7 *Puglia*, as *Calabria*.

Oyle alſo 8 Salmes, 1 Salme 10 Star, 1 Star 32 Pignatoli.

1 7 *Rome*——————————— 7, Brenten.

1 Brent 96 Pockal, (wrote alſo Bocal), or 13 $\frac{1}{2}$ Rubes or Stones of 10 ℔, of 30 ℥, or 42 Stoops of *Antwerp*.

For Honey the Pound is 44 ℥. The *Spaniards* call Bocal, *Azumbre*. 1 Barrillis 32 Bocals. 1 Bocal 4 Foglietta's, that is 128 in the Barrillis.

1 4 *Seres*

6 Ames of Antwerp compared with the Meaſure of other Places.

1 14 *Seres* or *Sherry*, as *Canaries*.

1 14 *Sevill* —————— $56\frac{1}{4}$ Rooves.

1 Roove, or Arroba, (in *Latine*) 8 Somers or Azumbres, 1 Somer 4 Quartiles or Sextaries. 1 Quartil $\frac{1}{6}$ of a Stoope of *Antwerp*. They deliver 27 and 28 Rooves in a Pipe, but Oyle by 40 and 41 Rooves in a Pipe.

Hunt ſaith that 32 *Spaniſh* Sextaries are equal to 24 *Roman*, and that the *Spaniſh* Sextaries contain $3\frac{1}{2}$ Parillas, which he makes of Oyle $3\frac{3}{4}$ ℥ of Water 4 ℥ *ferè*, of Syrrup 6 ℥ *ferè* accompting $17\frac{1}{4}$ ℥ of Water = $23\frac{1}{2}$ of Syrrup. And mentions the *Modius* or *Moyo* to contain 16 Arroba's or Amphoras. *Heylin p.* 1044 affirms the Arroba of *Spain* to contain 25 Buſhels.

1 7 *Treviſo* —————— 11 Conſi. the 10. one Cara.

3 2 *Tripoli* —————— 45 Metares, of 42 Rotules.

3 2 *Tunis* —————— 60 Matali of 32 Rotules.

1 7 *Venice* —————— 80 Moſtati.

38 make 1 Butt, and 76 an Amphora, $16\frac{4}{5}$ Quarti Beſonts Meaſure. The 4 one Bigontz. Bigonts is a *French* Hogſhead 1 Quart 18 Stoopes of *Antwerp*, $15\frac{2}{5}$ Quarti Meaſure Secchio or ſmall Meaſure of 4 Tiſchauſer.

Amphora is 4 Bigonts or Bigontines, 16 Quarti Bigots Meaſure, $18\frac{1}{2}$ Quarti Secchio. Lagel is a Puncheon, Amphora is 2 Ames. Oyle and Honey ſome meaſure by Amphora, but moſt by the Milliar of 1210 ℔.

Hunt reports the Follieta at *Venice* equal to the *Spaniſh* Sextarie or Quartil, and to contain in Water or Wine 16 ℥.

1 Congitella 4 Bocals, 1 Bocal 2 Medios.

1 Medius 2 Follieta's, that is 16 in the Congitella.

1 7 *Verona* —————— $1\frac{1}{2}$ Cara, or 14 Brents.

1 Brent, 16 Baſſes. Oyle by the Milliar of 1738 ℔, which is 8 Brenten and 11 Baſſes.

1 7 *Vincenza* —————— $1\frac{1}{26}$ *Cara*.

Oyle by the Milliar of *Venice*.

1 5 *Zant*, as *Corfu*.

Beer Meaſures.

Foreign Beer-Meaſures.

English *Barrel the Gallon whereof how many Stoopes.* Holland *Barrel how much.* Lubeck *Barrel how much.* Dantſick *Fat how much.*

The Barrel of Beer in *England* 36 Gallons, is 48 Gallons Wine-Meaſure. Every Beer-Gallon 2 Stoopes in *Flanders*, and at *Amſterdam* $1\frac{2}{3}$ Stoope.

The Barrel of Beer in *Holland* containeth 54 Stoopes, at *Amſterdam* $56\frac{1}{2}$ Stoopes, accompting 60 Stoops there for 64 *Flemiſh*.

The Barrel of Beer of *Lubeck* is juſt 50 Stoopes of *Antwerp*.

Corn Meaſures. The Laſt.

Foreign Corn-Meaſures.

The Laſt is differently reckoned, but with the *Engliſh* juſt 2 Tons or 4000 ℔ in dead Weight, reckoning Barley 5 Score to the Hundred.

The Muid.

In *France* ſeveral parts of the *Netherlands* and other places, they uſe a Meaſure called a Muid, Mudde, Moyo, *&c.* differently according to the Language of the Countrey where uſed; derived as conceived from *Modius*. This Meaſure in *France* as uſed for Land and Wine is ſpoken to before. The more proper uſe thereof is for Corn, Coals, Salt, and dry Commodities.

A Table of Corn Meaſure.

Ordinary French *Corn-Meaſure according to* Cotgrave.

	Septiers.	Mines.	Minots.	Boiſſeaux.	Quarts.	Pints
Muid.	12	24	48	144	576	1152
	Septier.	2	4	12	48	96
		Mine.	2	6	24	48
			Minot.	3	12	24
				Boiſſeau.	4	8
					Quart.	2

A Muid

Muid *of Coales*

A Muid of Coales is 16 Mines.

Hunt, on whoſe Authority I cannot ſay, counts the Common Meaſure of *France* to be 1 Muid, 24 Boiſſeaux, and that the ſame is about 18 Buſhels of our Water Meaſure, and 32 of theſe Muids go to the Hundred: But by another ſort of Meaſure he calls *Oldron*, 1 Hundred is 20 Tons, or 36 Muids.

Septier *of Corn*
The Sorts.

A Septier of Coales and Oates 21 Boiſſeaux, though of Wheat but 12, as above, and is ſaid to weigh 220 ℔ by *Nicot*. Nevertheleſs *Vigenere* will have the Septier in the Table above to be Rye Meaſure, and that of Wheat to weigh 240 ℔. The Septier of *Moulin*'s is 16 Boiſſeaux. The Small Septier is ſpoken of before as a proper Liquid Meaſure.

Mine *of Corn.*

The Mine above is ordinary. The Mine of *Clermont* double, *viz.* as much as the Septier in the Table.

Boiſſeaux *how much.*

The Boiſſeaux is about 3 of our Gallons weighing 20 ℔ by *Cotgrave*, and our 3 Gallons 21 ℔ *Avoirdupois*.

Bichot *and* Mettre.

Some report a Bichot to be a Corn Meaſure uſed in *Burgundy*, and to be 2 Mettres, and 1 Mettre to contain $2\frac{1}{2}$ of that Countrey Boiſſeau.

Chetfrid *of* Ruſſia.

The *Ruſſia* Chetfird is about 3 *Engliſh* Buſhels, as *Fletcher* affirmes.

Laſte *of* Amſterdam *the content how much in ſeveral other places.*

The Laſte, or Laſt of *Amſterdam* is 27 Moyes or Mudden. 1 Mudde, 4 Scheppels. Or 1 Laſt is 29 Sacks, 1 Sack 3 Achtelings, or Archtelings.

This Laſt of *Amſterdam* maketh in the following places, as at

1	8	*Antwerp*———	$37\frac{1}{2}$	Vertules.
1	3	*Bourdeaux*———	38	Boiſſeaux, whereof 33 to the Laſt.
1	8	*Bridges*———	$17\frac{1}{2}$	Hoot.
1	8	*Bruſſels*———	$10\frac{1}{2}$	Mudden, by *Malines* the Mudde or Vertule there is One.
1	3	*Calais*———	18	Raſiers, agree with *England*.
1	9	*Conningsbergh*---	$\frac{6}{7}$	of a Laſt, 6 Laſt there being 7 at *Amſterdam*.
1	1	*Coppenhagen*———	23	Small Barrels, whereof 42 make a Laſt.
2	1	*Cyprus*———	40	Medimnos, 1 Medimnus 2 Cipros.
1	9	*Dantſick*———	56	Scheppels, whereof 60 make a Laſt. There 4 Scheppels are 1 Mudde, which is the Skippound of 340 ℔.
1	8	*Delfe*———	87	Achtelings.
1	8	*Dort*———	28	Sacks.
1	8	*Dunkirk*———	18	Raſiers, Water-Meaſure
1	1	*Ebbeltorff*	23	*Danic* Barrels of 36 to the Laſt. Some allow 42 to a Laſt.
1	4	*Embden*———	55	Werps, whereof 61 make a Laſt, or $15\frac{1}{4}$ of 4 Werps, *Malines* $15\frac{1}{2}$, but then ſhould the Laſt be 62.
1	8	*Enckhuyſen*———	42	Sacks.
1	1	*Fameren*———	78	Scheppels, whereof 96 to the Laſt.
1	8	*Gaunt*———	4	Muddes and 7 Halſters, or 55 Halſters. One Mudde there is 12 Halſters.
1	7	*Genoa*———	$23\frac{1}{2}$	Mina.
2	6	*Goa*———		Their Bharo for Pepper is $3\frac{1}{2}$ Quintals of *Portugal* Weight every Quintal 100 ℔. For Wheat, Rice, and other dry things 1 Candil is 20 Mao's or Hands, 24 Medida's 1 Mao, and 1 Medida 9 ℥.
1	8	*Groeninghen*———	33	Muddes.
1	1	*Hamborough*———	83	Scheppels, 90 whereof make a Laſt.
1	1	*Heyleger-haven*-	80	Scheppels, of 96 to the Laſt.
1	1	*Horne*, as *Enckhuyſen*.		
1	10	*Lisbon*———	225	Alquiers, of which 240 to a Laſt, or 4 Moyo's of 60 Alquiers to the Moyo, and ſo in the *Portugal* Iſlands.
1	2	*London*———	$10\frac{1}{4}$	Quarters, or 82 Buſhels.
1	1	*Lubeck*——— }	85	Scheppels, whereof 96 to the Laſt.
1	4	*Mechelborough* }		
1	8	*Medenblick*, as *Enckhuyſen*.		
1	9	*Melvyn*———	$\frac{17}{21}$	of a Laſt.
1	8	*Middleburgh*———	40	Sacks $41\frac{1}{2}$ to the Laſt in all *Zeland*.
1	1	*Nelleboghe*———	23	Barrels, whereof 42 to the Laſt.
1	7	*Puglia*———	32	Cara, of 36 Timani.
1	9	*Riga*———	42	Loops.
1	3	*Rochell*———	128	Buſhels, 4 to every Seſtier.
1	4	*Roſtock*, as *Lubeck*.		

Laſt *of* Amſterdam *how much in ſeveral other Places.*

1	8	*Rotterdam*, as *Delfe*.		
1	3	*Rouen*	20	until 30 Mines. 1 Mine 4 Boiſſeaux.
1	8	*Schonehaven*	88	Achtelings.
1	14	*Sevill*	54	Hanegas. A Laſt there is 4 Cahis. 1 Cahi is 12 Hanegas.
1	7	*Sicilia*	38	Medimno's, of 6 Moyo's.
1	4	*Stetin*, as *Coningsbergh*.		
1	15	*Sweden*	23	Barrells.
1	8	*Texel*	58	Loops.
1	7	*Venice*	32	Starr.

Salt Meaſures.

Foreign Salt-Meaſures.

Sack of Armuyden, *how much in ſeveral other Places.*

At *Armuyden* in *Zeland* they reckon $8\frac{2}{3}$ weighes for a Hundred, 1 Weigh $11\frac{1}{2}$ Sacks, 1 Sack 4 Meaſures. And 15 Weighes of *Bruwage* Salt make the great Hundred.

The Sack of Salt of *Armuyden*, being 122 ſmall Barrells for the 100, make in the other places, as at

1	8	*Amſterdam*	102	Scheppels
1	8	*Antwerp*	144	Vertels of 24 to the Laſt, and 6 Laſt to the 100.
				But White Salt is meaſured by a Leſs Meaſure of 12 on the 100.
1	8	*Axells*	102	Meaſures.
1	8	*Bruges*	104	Meaſures.
1	3	*Bruwage*	$\frac{4}{7}$	parts of 100 of 28 Moyo's, 1 Moyo, 12 Sacks, and by the Load, 10 Load in the 100, and 48 Moyos or Muys to the laſt, or 21 Barrels.
1	14	*Cadiz*	22	Cays, or Caies.
1	3	*Calais*	130	Barrels 19 to the Laſt, but 20 by Freighting.
1	8	*Damme*, as *Axells*.		
1	1	*Denmark*	$6\frac{2}{3}$	Laſt.
1	8	*Deventer*, as *Amſterdam*.		
1	8	*Dunkirk*	92	Water-Meaſures, but 104 Land-Meaſures.
1	4	*Embden*	100	Barrels of 14 to our Laſt.
1	8	*Gaunt*	108	Sacks, or Barrels.
1	1	*Hamburgh*	7	Laſt, whereof 80 Barrels make the 100.
1	8	*Ipre*	144	Meaſures.
1	10	*Lisbon*	25	Moyo's.
1	2	*London*	$7\frac{1}{2}$	Laſt of 18 Herring Barrels, but by Weys, $11\frac{1}{2}$.
1	1	*Lubeck*	7	Laſt of 18 Barrels.
		Mary Port	28	Moyo's.
1	8	*Oſtend*	98	Meaſures.
1	7	*Piram*	70	Moſe.
1	8	*Rotterdam*	100	Meaſures, whereof 6 make 1 Mudde of 18 to the 100.
1	3	*Rouen*	$16\frac{1}{2}$	Muys, and ſo almoſt all *France*.
1	14	*St. Lucar*	21	Cays, or Caies.
1	10	*St. Tubal*	20	
1	15	*Sweden*	112	Tunnes, or Barrels of 16 to the Laſt.
1	7	*Venice*, as *Piran*.		
1	8	*Utrecht*, as *Amſterdam*.		

Seacoale Meaſures.

Foreign Sea-Coal-Meaſures.

Laſt *of* New-caſtle *how much at ſeveral other Places.*

The Laſt of *Newcaſtle* Sea-coales is $7\frac{1}{2}$ Chalders, which *Malines* ſaith at *London* and *Yarmouth* make 10 Chalders. Some ſay more, beſides the allowance at *London* of 21 for 20.

The ſame Laſt is at

1	8	*Alſt*	200	Muddes.
1	8	*Amſterdam*	$13\frac{1}{2}$	Hoot of 38 Meaſures to an Hoot, or Hoet.
1	8	*Antwerp*	175	Vertels.
1	8	*Bruges*	100	Meaſures for Oates.
1	3	*Condet*	44	Muys, 80 Muys make 1 Cherke
1	8	*Dort*	12	Hoot, alſo by Weighs of 144 ℔. one Weigh 24 Stone, 1 Stone 6 ℔.

1 8 *Gaunt*

Laſte of New-caſtle *how much at ſeveral other Places.*

1 8 *Gaunt*——144 Sacks, or 14 Muys.
1 8 *Middleburgh*, by Weighs of 180 ℔ to a Weigh.
1 8 *Oſtend*, as *Bruges*.
1 3 *Rouen*——100 Barrels, allowing 104 for the 100.
1 8 *Zeland*—— 68 Herring Barrels.

Foreign Weights.

Foreign Weights of 3 ſorts.

Generally 3 ſorts of *Weights* are uſed for Merchandiſe.

1. Weights of great Content, as Hundreds, Kintalls, Centeners, Talents, Thouſands, Weighs, Skippounds, Charges, Liſpounds, Rooves, *&c.*

2. Weights of leſſer Content, as Pounds, Mina's, Manehs, Rotuli, *&c.*

3. Small Weights, as Ounces of 12, 14, 16, 18, 20, 30, *&c.* to the Pound, and the Subdiviſions of the Ounce.

Talents, of the *Hebrews*, *Greeks*, and *Latines* are ſeen before. — Talents,

Cantars, Centeners, or Kintals ſometime wrote Quintals, accompted by Merchants as Hundreds; are of 100, 112, 120, 125, 128, 132, and 140 Pounds. — Cantars, &c.

Weighs, or Weys, are commonly 165 ℔, or 180 ℔, or 200 ½ ℔ for a Charge. — Weighs, &c.

Skippounds, uſed in many places *quaſi Shippound*, or *Shippond*, for as in *Italy* and other Countreys the *Carga*, *Cargo*, or *Charge* is the Loading of an Horſe of 300, or 400 ℔. So is the Skippound taken for the Divident of a Laſt of Corn Laden in a Ship. Skippounds are of 300, 320, 340, and 400 ℔ to the Skippound. *Cargo* is often taken for the whole Lading or Burden of a Ship. — Skippounds. Carga's.

Liſpounds, of 15, 16, and ſometimes 20 ℔ to the Liſpound. — Liſpounds.

Rooves, or Arrobas of 10, 20, 25, 30, and 40 ℔ to the Roove. — Rooves.

Stones, of 6, 8, 10, 14, 16, 20, 21, 24, 32, and 40 ℔. to ſome Stones. — Stones.

Poade, of *Ruſſia* by *Heylin* is 40 ℔. — Poade.

Mixias, is commonly underſtood to be 10000 Drams, of 8 to 1 ℥, and 12 ℥ to 1 ℔. — Mixias.

Seſtertio's of *Cleopatra* in *Ægypt* and other places in *Africa*, were 2 ½ ℔, for 50 Seſtertio's made 125 ℔, but in *Thracia* it was but 2 ⅙ ℔. — Seſtertio of Africa, &c.

Pound is divided into more or leſs Ounces. — Pounds.

Mark Weight, commonly 8 ℥. — Markes.

Mark Pound 16 ℥, that is 2 Markes. — Mark Pound.

Mina Ptolomaica, 1 ½ *Rotuli*, or 18 Ounces, or 144 Drams, and in leſſer Diviſions thus.

A Table of the Mina Ptolomaica.

	Rotuli.	Ounces.	Drams.	Scruples.	Oboli.	Lupines.	Siliquas, *or* Carrats.	Aereoli.
Mina.	1½	18	144	432	864	1296	2592	6912
	Rotulus.	12	96	288	576	864	1728	4608
		Ounce.	8	24	48	72	144	384
			Dram.	3	6	9	18	48
				Scruple.	2	3	6	16
					Obolus.	1½	3	8
						Lupine.	2	5⅓
							Siliqua, *or* Carrat.	2⅔

Maneh of Arabia the Sorts and Names.

Mane, or Maneh, in *Arabia*, double 1 of 16 ℥, and 1 of 20 ℥.

That called *Alialica*, *Baſaria*, *Alanthalica*, and *Aegyptia*.

This *Romana*, and is indeed of *Alexandria*, the Pound there being 20 Ounces.

Rotulus, in *Arabia*, *Syria*, *Aſia minor*, *Ægypt* and *Venice*, reckoned for a Pound is thus divided. — Rotulus.

Rotulus,

A Table of the Arabian Rotulus.

	Sachoſi, *or* Ounces.	Sextaries, *or* Cicles.	Deniers, *or* Aureos.	Darchiny, *or* Drams.	Scruples, *or* Garma.	Obolos, *or* Orloffs.	Danigs, *or* Lupines.	Kirats, *or* Siliquas.	Aereola's, *or* Keſtuffs.
Rotulus, *or* Pound.	12	24	84	96	288	576	864	1728	3456
Sacros Sachos *or* Ounce.		2	7	8	24	48	72	144	288
Sextarie *or* Cicle			$3\frac{1}{2}$	4	12	24	36	72	144
Denier *or* Aureus Aunius Audanakus				$1\frac{1}{7}$	$3\frac{3}{7}$	$6\frac{6}{7}$	$10\frac{2}{7}$	$20\frac{4}{7}$	$41\frac{1}{7}$
Dram *or* Darchiny Alky Oliginat					3	6	9	18	36
Scruple Garme *or* Kenmer						2	3	6	12
Obolus Orloff *or* Onolaffat Onolum							$1\frac{1}{2}$	3	6
Daning *or* Danic Lupine								2	4
Carrat Kirat *or* Siliqua									2

Phyſick Pound at Venice. Lupines *there.* Keſtuff, *how much.* *Pound of* Alexandria. Italian *Pound.*

Some mention the Phyſick Pound at *Veniee* to have but 7 Drams in the Ounce.

The Lupines at *Venice* called Sextula's, becauſe 1 ℥ hath 72, which is 6 times 12.

Every Keſtuff, or Aereolum (or Areolum) is the Weight of 2 Barley-Cornes, ſo is there in the Rotulus 6912 Graines.

The *Alexandrian* Pound 20 ℥, the Ounce 8 ʒ, *&c.*

The *Italian* Pound generally is divided into 12 ℥, 1 ℥ into 2 Staters, and 1 Stater into 4 Drams; ſo hath 1 ℔ 24 Staters, 96 Drams.

But in Phyſick there, and in other Places thus.

A Table of the Italian *Phyſick Pound.*

	Ounces.	Loots.	Sizaynes, *or* Siliqua's.	Drams.	Scruples.	Obolos.	Siliqua's.	Graines.
Pound.	12	24	48	96	288	576	1728	5760
Ounce.		2	4	8	24	48	144	480
Loot.			2	4	12	24	72	240
Sizayne *or* Siliqua				2	6	12	36	120
Dram					3	6	18	60
Scruple						2	6	20
Obolus							3	10
Siliqua								$3\frac{1}{2}$

Spain, ſome ſay hath a *Mina Romana*, which contains 20 ℥. A Common Pound of 16 ℥. and a Phyſick Pound of 12 ℥. each Ounce divid ed into 8 Drams. The Ounce of the *Toletan* Phyſick Pound excepted, which hath 9 ʒ, as ſome affirm.

Spaniſh Weights.

Pound Weights of Spain.

A Table of the Mina Romana *of* Spain.

	Libra.	℥.	Duels.	Quarterns.	Sixths.	ʒ.	Syrian Beans.	℈.	Obolos.	Carats.	Chalcos.	Graines
Mina Romana	1⅔	20	60	80	120	160	240	480	960	2880	5700	11520
	Libra	12	36	48	72	96	144	288	576	1728	3456	6912
		℥	3	4	6	8	12	24	48	144	288	576
			Duel	1⅓	2	2⅔	4	8	16	48	96	192
				Quartern	1½	2	3	6	12	36	72	144
					Sixth	1⅓	2	4	8	24	48	96
						ʒ	1½	3	6	18	36	72
							Syrian Beane	2	4	12	24	48
								℈	2	6	12	24
									Obolus	3	6	12
										Carat	2	4
											Chalcus	2

Tables of the Pounds of Spain.

The Common Pound of Spain.

	Marks.	Ounces.	Drams.	Adarmes *or* Adarames
Pound	2	16	128	256
	Mark	8	64	128
		Ounce	8	16
			Dram	2

The Phyſick Pound of Toledo.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	108	324	6480
	Ounce	9	27	540
		Dram	3	60
			Scruple	20

Pound Weights of France.

Pounds of France.

The Weight uſed by the Merchants for the moſt part is of 16 ℥, called *Liure d' Anuers*, though in ſome places but 14, others 18 ℥. *Cotgrave* writes the *Liure* or *Pound de Lyon* to be 15 ℥. that *de Spaigne* but 14 ℥. And divides the Pound of 16 ℥. into 32 Halfes, 64 Sezaines, 128 Treſeaux, 256 Groſs, 512 Demigroſs. And the Pound uſed by the *Farriers* conſiſting of 12 ℥ into 90 Drams, 270 Scruples, 540 Oboles. After *Malines* the Ordinary, or Pound commonly uſed for *Merchants* is parted thus.

Tables of the Pounds of Paris *and* Lyons.

The Pound Weight of Paris.

	Ounces.	Groſſe.	Scruples.	Graines.
Pound	16	128	384	9216
	Ounce	8	24	576
		Groſſe	3	72
			Scruple	24

The Phyſick Pound of Lyons.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	96	288	5750
	Ounce	8	24	480
		Dram	3	60
			Scruple	20

Cotgrave mentions a Weight called *Sextule* of 4 Scruples, or the Sixth part of 1 ℥.

Sextule, *the Weight.*

Pounds of Germany.

Pound Weights of Germany.

Tables of the Pound of Vienna *and Phyſick Pound.*

The Pound Weight of Vienna *in* Auſtria.

	Ounces.	Loots.	Quints.	Pennings.	Grains.
Pound	16	32	128	512	12800
Ounce		2	8	32	800
Loot			4	16	400
Quint				4	100
Penning					25

The German *Phyſick Pound, by* Alſted.

	Ounces.	Drams.	Scruples.	Graines.
Pound	12	96	288	5760
Ounce		8	24	480
Dram			3	60
Scrupl				20

Pounds of the Low Countries.

In the *Low Countries* they uſe Pounds of 12, 14, 15, *&c.* Ounces.

Pounds of Bridges.

At *Bridges* in *Flanders* they have 1 ℔ of 14 ℥, and 1 ℔ of 16 ℥. The 100 ℔ of 16 ℥ make 108 ℔ of 14 ℥, but the Ounces of 14 to the ℔ are the heavieſt for 100, of theſe are 105 $\frac{1}{3}$ Ounces of 16 to the Pound, This ℔ is thus divided.

Table of the Bridges *Pound.*

	Ounces	Loots	Sizaines	Drams.
Pound	16	32	64	128
Ounce		2	4	8
Loot			2	4
Sizaine				2

Dram, *or* Quint.

Weights at Antwerp.

At *Antwerp* they uſe to weigh by the Hundred Pounds even Weight, called *Suttle*, for which commonly at the Weigh Houſe is allowed 101 ℔. A Stone is 8 ℔. The Skippound 300 ℔. The Weigh 165 ℔. The Carga or Charge 400 ℔. which is two Bales of 200 ℔ each for an Horſe to carry. The Pound there is 16 ℥.

This 100 ℔ of *Antwerp* weigheth in the Places following,

The 100 ℔ *at* Antwerp *how much at divers other Places.*

1	3	*Abbevile*	$94\frac{3}{4}$ ℔.
2	11	*Achri*	$17\frac{1}{4}$ Rotuli. The 100 A Cantar Tambaran.
		Adler	138 ℔ Ordinary Weight. 91 ℔ To weigh Steel, Tinne and Copper.
1	8	*Ailſt*	108 ℔.
3	1	*Alcario*	164 ℔. 78 Minas of 16 ℥ to the Mina. 27 Rotuli of 6 ℔ to the Rotuli. 1 Peſo is $1\frac{1}{2}$ Metallicum, or a Dram. 50 Metallici 1 Mark. Our Mark 42 Metallici. Musk and Amber ſold by this Weight in *Ægypt*.
2	11	*Aleppo*	22 Rotuli of 100 to a Cantar. 1 Rotulus is 60 ℥, or 480 Metecalos or ʒ. 1 ℥ is 8 Metecalos or Dragmes. 1 ʒ or Metecalo is $1\frac{1}{2}$ Peſo. 10 Peſo's are 1 Onga, or Ongia, to weigh Civet.
2	1	*Alexandria*	108 Rotuli, of 100 to a Cantar. 78 Minas of 20 Ounces.
		America Malica	90 ℔, of 12 Ounces to the ℔. 36 Minas Seſtertias of 30 ℥.
2	11	*Aman*, as *Aleppo*.	
1	8	*Amſterdam*	$94\frac{4}{5}$ ℔. And for Silkes they uſe the Weight of *Antwerp*.
1	7	*Aquila*	147 ℔.
1	3	*Aquiſmort*	102 ℔.
2	2	*Arabia*	78 Rotuli. 104 Maires, *or* Minas. 148 Pounds. 936 Ounces, or Sachoſi 12 to 1 Rotulus.

3 1 *Arcadia*

The 100 ℔. *at* Antwerp *how much at divers other Places.*

3	1	*Arcadia*	92 ℔, and 83 ℔ for Mavigetto.
1	5	*Archipelago*	120 ℔.
		Armaria bona	105 ℔. of 16 ℥ to the ℔.
			93 ℔. of 18 ℥ uſed for Silk and Copper.
			54 ℔. of 32 ℥ Fleſh Weight.
2	3	*Armenia*	130 ℔.
1	14	*Arragon.*	106 ℔.
			96 ℔. Great Weight for Wooll.
1	8	*Arſchot*	100 ℔. all one with *Antwerp*
1	8	*Audinarde*	110 ℔.
1	3	*Avignon*	111 ℔. A Centener is 2 Frailes of 56 ℔.
1	4	*Ausburgh*	95 ℔.
1	8	*Barrow Op Zome*	98 ℔.
1	14	*Barcelona*	96 ℔. Wooll Weight.
			106 ℔. Common Weight.
			131 ℔. Saffron Weight.
1	4	*Baſil*	96 ℔. They uſe Centeners of 100 ℔, 120 ℔, and 132 ℔.
1	7	*Bergamo*	137 ℔. and 108 ℔. by the 2 Quintals.
1	1	*Bergen*	96 ℔. but uncertain weighing with a Sling.
1	4	*Bibrach*	92 ℔. of 16 ℥ to 1 ℔ as Conſtance.
2	11	*Barutti*	21 Rotuli
1	7	*Bolognia*	53 ℔. of 30 ℥ to weigh Wax and Wooll by Rooves of 1 ℔.
1	3	*Borgoingne*, as *Abbevile*.	
1	4	*Botſen*	138 ℔. Ordinary Weight.
			91 ℔. To weigh Steel, Tinne, and Copper.
1	3	*Bourdeaux*, as *Abbevile*.	
1	4	*Breſlau*	120 ℔. by the Centener of 24 ℔ to 1 Stone, and 5 Stone to 1 Centener. And 5½ Stone to the Centener of 132 ℔ there alſo uſed.
1	7	*Breſcia*	184 ℔. and for *Venice* Gold 136 ℔.
1	8	*Bridges*	100 ℔.
			93 ℔. for Butter and Cheeſe, The Stone 6 ℔. and 20 Stone 1 Weigh, but Wooll Weight is 108 ℔. weighed by Stones of 6 ℔, called Nailes or Neiles. 18 Neiles to the Hundred, 45 Neiles to the Weigh, 2 Weighs to 1 Pocket of Wooll. *Hunt* ſaies 18 Neiles is 144 ℔ of our Wooll Weight.
1	8	*Bruſſels*, as *Arſchot*.	
		Bucca	44 Ocha's.
1	14	*Burgos*	93 Rotuli.
2	1	*Burſa*	88 Rotuli.
3	4	*Cabo verde*	107½ ℔, or Rotuli, A Quintal is 128 ℔ of 4 Rooves of 32 ℔.
1	7	*Calabria*	147 ℔.
1	3	*Calais*	111 ℔. Ordinary Weight.
			92 ℔. Merchants Weight.
			114 ℔. *Engliſh* Wooll Weight.
2	6	*Calicut*	80 Aracoles. *Malines p.* 18. mentioning the Baccar or Bahar at *Calicut* to be at *Lisbon* 4 great Quintals of 112 ℔ to the Quintal, and that the 4 Quintals are 480 Aracoles, that is 120 Aracoles for 1 Quintal. And again that the Bahar is 20 Faracoles, which is 5 Quintals at *Lisbon* of 32 ℔ *per* Roove, is not well to be underſtood Seeing the great Quintal at *Lisbon* is 128 ℔ or 4 Rooves of 32 ℔ *per* Roove; whereas 4 Quintals of 112 ℔ is but 448 ℔, and 5 Quintals of 128 ℔ is 640 ℔, unleſs there be 2 ſorts of Bahars at *Calicut*, 1 of 48 Aracoles, and another of 20 Faracoles. Or that the Bahar be 5 great Quintals at 129 ℔ the Quintal, that is 645 ℔ for ſo many Pounds or *Portugiſe* Rotuli are in 480 Aracoles for 100 ℔ of *Antwerp*, which anſwer to 107½ ℔ of *Portugal* Weight by his own Conceſſion in the ſame Page a little before.
3	3	*Canary Iſlands*	107 ℔. as Sevill.
1	5	*Candia*	138 ℔. for Gold Thread.
			89 Rotuli, whereof 100 is a Cantar or Quintal.

The 100 ℔ *at* Antwerp *how much at divers other Places.*

1	7	*Carpi*, as *Aquila*.		
1	14	*Caſtile*	102	℔.
		Cataio	87	Rotuli 100 to a Cantar.
1	7	*Ceſena*, as *Bergamo*.		
1	4	*Collen*	93⅓	℔.
1	7	*Como*, as *Aquila*.		
1	9	*Coningsbergh*	125	℔, which is a Centener. A Laſt of Wheat there 5200 ℔. a Stone 40 ℔, a Skippound 10 Stone, that is 400 ℔.
1	4	*Conſtance*	92	℔. of 16 ℥ or 32 Loot. Some by the Centener of 100 ℔, and ſome of 120 ℔.
1	5	*Conſtantinople.*	87½ 39	Rotuli, 100 to a Cantar. Ochaa, *Hunt* writes it *Cohaa*.
			2½	Metallici, which is their Dram, make 3 of ours.
1	1	*Coppenhagen*	96	℔. There the Centener is 112 ℔. A Stone is 10 ℔. A Skippound 32 Stone, or 20 Liſpound of 16 Mark Pound which is a Skippound, or 320 ℔.
1	5	*Corfu*	97 115	℔. Great Weight. ℔. Small Weight.
1	8	*Cortrycke*, as *Audinarde*.		
1	9	*Cracon*	124	℔. The Centener there is 136 ℔.
1	7	*Crema*, as *Aquila*.		
1	7	*Cremona*	143 132 60	℔. of 12 ℥ moſt uſed. ℔ of 12 ℥, being 13 ℥ of the other. ℔. of 28 ℥ to the ℔. uſed for Fleſh.
2	1	*Cyprus*	20¼	Rotuli, 100 to the Cantar.
2	11	*Damaſcus*	26	Rotuli. There 1 Cantar is 5 Zurli, or Stone, and 1 Stone 20 Rotuli, 1 Rivola is 225 ℔. *Antwerp*.
1	9	*Dantſick*	120	℔. There 1 Laſt of Wheat is 4528 ℔. The Laſt of *Rye* 4245 ℔. 1 Skippound 340 ℔. of 10 great Stone. 1 Skippound 320 ℔. of 20 Liſpound. 1 Centener 125 ℔. 1 Stone for Spices 24 ℔. 1 Great Stone for Groſs Wares 34 ℔. 1 Liſpound 16 Mark Pound.
1	3	*Diepe*, as *Abbevile*.		
1	8	*Dixmude*, as *Ailſt*.		
1	8	*Doway*, as *Audinarde*.		
1	6	*Dublin*, and in *Ireland* generally.	91½ 104	℔. by the Great Hundred. ℔. Subtle Weight.
1	12	*Edenburgh* and all *Scotland*.	96	℔. and 103½ ℔. for 112 ℔.
1	4	*Erdford*	85	℔. as at *Vienna*.
1	7	*Faenza*	132	℔.
3	2	*Fez*, or *Feſſe*	96	℔. by *Hunt* wrote *Feas*, and noted as in *Portugal*.
1	7	*Ferrara*, as *Bergamo*.		
		Fio	96	¾ Rotuli, *or* Scrutarij.
1	7	*Fiume*, as *Venice*.		
1	8	*Flanders*	110	℔. for the moſt part, the places herein excepted.
1	7	*Florence*	125	℔. of 12 ℥ to the ℔.
3	1	*Forfori*	65	Rotuli.
1	7	*Forli*, as *Aquila*.		
1	3	*France* generally	111	℔. except herein excepted.
1	4	*Frankford*		as *Baſil*.
1	4	*Friburg*		
1	8	*Gaunt* as *Ailſt*.		
1	8	*Gelderland*	99	℔. The Places herein excepted
1	7	*Genes*, by Rooves to a Quintal of 4 Rooves, and 4 ℔. over.		
			110	℔. a Quintal of Pepper.
			114	℔. a Quintal of Ginger.
1	7	*Geneva*	102 85	℔. Weight for Spices. A Carga is 270 ℔ ſmall Weight. ℔. Great Weight.
1	4	*Germany*, A Centener of the ſmall Weights is 100 ℔, of the great 120 ℔. and 132 ℔. The Centener of 120 ℔. is 5 Stone, of 24 ℔. *per* Stone.		

2 6 *Goa*,

The 100 ℔. at Antwerp how much at divers other Places.

2 6 *Goa*, as *Portugal* by Quintals, Arrobes or Rooves, *&c.* They have alſo another Weight called *Mao*, which ſignifieth the Hand, and weigheth 12 ℔. uſed for Butter, Honey, Sugar, *&c.* in the *Portugal* Dominions.
1 14 *Granada*, as *Armaria bona.*
3 4 *Guynea*, as *Cabo verde.*
1 1 *Hamburgh*——96 ℔. The Centener 120 ℔. of 12 Stone, 1 Stone 10 ℔. A Liſpound 15 ℔, and 20 Liſpound 1 Skippound.
1 4 *Heidelbergh*, as *Baſil.*
1 8 *Hertogenboſh*, as *Arſchot.*
1 8 *Holland*, as *Gelderland.*
1 8 *Hulſt*, as *Ailſt.*
1 8 *Ipre*, as *Ailſt.*
1 7 *Iſtria*, as *Venice.*
1 5 *Laarta*——87 Rotuli, 100 to a Cantar.
1 5 *Laconia*——
138 ℔.
78½ Rotuli.
1 7 *Lanſan*, as *Bergamo*
1 5 *Lavalona*——131 ℔.
1 14 *Leon*——109 ℔.
1 5 *Lepanto*——
156 ℔.
26 Rotuli, 1 Rotulus 6 ℔.
1 4 *Lipſich*, as *Baſil.*
1 10 *Lisbone*, See *Calicut.*
1 8 *Liſle*, as *Audinarde.*
1 2 *London*, and all *England.*——
91½ ℔. Groſs Weight of the Kintal Weight 112 ℔.
104 ℔. Subtle Weight.
189½ Markes of 8 ℥ Troy.
1 8 *Louvaine*, as *Arſchot.*
1 1 *Lubeck*, as *Coppenhagen.*
1 7 *Luca*, as *Aquila.*
1 3 *Lyons*——
111 ℔. ordinary Weight. A Centener is 112 ℔.
102 ℔. Almerick, or Weight of *Geneva* for Spices, abating 8 ℔. *per Cent.*
94¾ ℔. by the Kings Weight to pay Cuſtom by.
A Quintal is 100 ℔. A Charge 300 ℔. A Somme 400 ℔.
3 3 *Madera*, as *Cabo Verde.*
1 8 *Malines*, as *Arſchot.*
1 7 *Mantua*, as *Aquila.*
1 3 *Marſeilles*——111 ℔.
3 2 *Maroco*, or *Moroco*, as *Capo Verde.*
1 14 *Medina del Camporas Caſtile.*
1 9 *Melvin*——124 ℔. The Laſt of Wheat 5200 ℔. The Skippound, and Stone as *Coningsbergh.*
1 4 *Meyſen*——
100 ℔. of 16 ℥ to the ℔. which is the Princes Weight, called *Zigoſtatica.*
96 ℔. Merchants Weight.
148 ℔. of 12 ℥ to the ℔.
1 7 *Millan*, as *Cremona.*
1 3 *Mirabel*, as *Aquiſmort.*
1 7 *Mirandula*, as *Aquila.*
1 7 *Modena*, as *Faenza.*
2 7 *Molucco*——88 Rotuli, 112, a Cantar.
1 3 *Montpelier*, as *Avignon.*
1 4 *Munchen*, as *Ausburgh.*
1 7 *Naples*——120 ℔. and for *Venice* Gold 134 ℔.
1 15 *Narcca*——120 ℔. A Liſpound or Stone is 20 ℔. and 20 Liſpound a Skippound, that is 400 ℔. uſed for Rye, but for Wheat but 350 ℔. to a Skippound.
1 7 *Nicoſia*, or *Nichoſia*, as *Archipelago.*
1 5 *Nigropont*——119 ℔.
1 4 *Norenburgh*, as *Conſtance.*
1 4 *Norlingen*, as *Ausburgh.*
1 4 *Offen*, as *Baſil.*

The 100 ℔. *at* Antwerp, *how much at divers other Places.*

3	2	*Oran*	94	Roturi, 1 Cantar 5 Rooves, 1 Roove 20 Rotuli.
			138	℔. for Spices. 1 Cantar 4 Rooves.
			50	Rotuli for Corne, 1 Cantar 6 Rotuli.
			61	Rotuli, for Cotton Wooll, 1 Cantar 15 Rotuli.
1	7	*Otranto* as *Bergamo.*		
1	7	*Padua*		
1	3	*Paris*	93	℔. accompting 4 Quarters of 25 ℔ to the Hundred.
1	7	*Parma*, as *Aquila.*		
1	4	*Paſſau*	87	℔.
1	7	*Pavia*, as *Cremona.*		
1	7	*Piran*, as *Venice.*		
1	7	*Piedmont* as *Aquila.*		
1	7	*Plaiſſance*		
1	4	*Pooſen*, as *Breſlaw.*		
1	8	*Popering*, as *Ailſt.*		
1	10	*Portugal*	107½	Rotuli or Araters. The great Quintal is 128 ℔. of 4 Rooves, 1 Roove 32 ℔. The Small Quintal is 112 ℔. of 4 Rooves, 1 Roove 28 ℔. The Quintal of Wax 168 ℔, which is 1½ Quintal of 112 ℔. of 4 Rooves of 42 ℔. the Roove.
1	4	*Prague*, as *Paſſau.*		
1	7	*Puglia*, as *Calabria.*		
1	7	*Raguza* as *Faenza.*		
		Raviano		
		Ravenna		
1	7	*Rechanati*	137	℔. but to Gold Thread but 112 ℔.
1	4	*Regensbourgh*, as *Paſſau.*		
1	15	*Revell*	120	℔ which is a Centener. The Skippound there is 400 ℔.
2	1	*Rhodes*	19½	Rotuli, A Cantar is 100.
1	9	*Riga*	120	℔. A Liſpound is 20 ℔. and 20 Liſpound a Skippound.
1	7	*Rimano*, as *Faenza.*		
1	3	*Rochel*	111	℔ and 119 ℔. by the ſmall Weight.
1	7	*Romagna*, as *Naples.*		
1	7	*Rome*	132	℔.
1	3	*Rouen*	91	℔ by the *Viconte*, accounting as at *Paris.*
			94¾	℔. by the ordinary weight, and 4 ℔. *per Cent* over.
1	4	*Saltsburgh*	111	℔. Small Weight.
			83	℔. great Weight.
1	3	*St. Antoine*	127	℔.
1	8	*St. Omar*, as *Audinarde.*		
3	3	*St. Thomas*, as *Cabo Verde.*		
1	14	*Saragoſſa*	112	℔. And the ſmall Quintal 131 ℔.
1	7	*Savoy*	137	℔.
			195	℔. Small Weight.
1	4	*Saxony*, as *Meyſen.*		
3	1	*Sciba*, as *Antwerp*,	320	℔. is there a Skippound.
2	1	*Scio*, as *Fio.*		
1	13	*Sequia*, as *Venice.*		
1	14	*Sevil*	107	℔. The great Quintal is 144 ℔ of 4 Rooves of 36 ℔. The leſſer Quintal is 120 ℔. of 4 Rooves of 30 ℔. The ſmall Quintal is 112 ℔. of 4 Rooves of 28 ℔.
1	7	*Sicilia*	152	℔. of 12 ℥ *per* ℔.
			61	Rotuli of 30 ℥ is a Cantar of 24 Seſtertios.
			54	Rotuli for Fleſh by Talents of 12 Seſtertio's is 30 Rotuli.
1	4	*Sileſia*, as *Breſlaw.*		
1	13	*Spelato*, as *Venice.*		
1	4	*Spiers*, as *Bibrach.*		
1	4	*Stetin*	96	℔. The ſmall Stone 10 ℔. The great Stone 21 ℔. The Centener 112 ℔.
1	15	*Stockholme*	120	℔. The Skippound 320 ℔. and alſo 340 ℔. The Centener 120 ℔. The Stone 10 ℔.
1	4	*Straelſont*	92	℔. The Stone 10 ℔. and the Liſpound 16 ℔.
3	2	*Suus*, or *Sus*, or *Fez.*		

The 100 ℔ *at* Antwerp *how much at divers other Places.*

2 11 *Syria*——156 Minos, 1 Mina 100 Drams.
1 8 *Tergos*——107 ℔.
1 3 *Tholouſe*, as *Avignon.*
3 2 *Thunes*, or *Tunis*— 63 Rotuli.
1 9 *Thoren*——120 ℔. The Stone is 24 ℔.
1 8 *Tournay*, as *Ailſt.*
1 7 *Treviſo*, as *Bergamo.*
1 7 *Trieſte*, as *Venice.*
3 2 *Tripoli*, as *Thunes.*
2 11 *Tripoli*—— 26½ ℔.
1 14 *Valentia*—— { 106 ℔. by Quintals of 4 Rooves of 30 ℔ for Spices.
134 ℔. by Quintals of 4 Rooves of 36 ℔. }
The ſmall Carga is 360 ℔. that is, 3 Quintals of 120 ℔.
The great Carga is 432 ℔. that is 3 Quintals of 144 ℔.
1 7 *Venice*—— { 98⅔ ℔. Great Weight, called *Ala Groſſa*, uſed for Fleſh, Butter, Cheeſe, Leather, Dates, Yarne, Copper, Thread, Iron, Oile, Brimſtone and Wooll.
156 ℔. Small Weight of 12 ℥, called *Ala Sotile*, moſt uſed for all Merchandiſe. }
An Ounce is 6 Saſſi, 1 Saſſi 24 Carrats, 1 Carrat 4 Graines.
They alſo accompt by Thouſands, *&c.* with allowance of 2 ℔. *per Cent.* in the *Cuſtom-Houſe.*
1 Thouſand 40 Mixti, 1 Mixti 25 ℔.
1 Carga 400, ℔ 1 Starre 220 ℔. The Starre is Menſural.
Starres for Corn 130 ℔. Ginger 180 ℔. Raiſins 260 ℔.
The Starre containes 54 Pottles of Wine at *Antwerp.*
1 7 *Veronna*—— 90 ℔. And for Gold Thread 143 ℔.
1 4 *Vienna*—— 85 ℔. as at *Erdford*, where alſo a Summe of Quick-Silver is 275 ℔.
1 14 *Villaco*, or *Vellica*— 80 ℔.
1 4 *Ulme*, as *Baſil.*
3 2 *Una*—— { 65 Rotuli for Cotton.
75 Rotuli for Spices.
94 Rotuli for Corn. }
1 7 *Urbin*, as *Bergamo.*
Wallons Countrey, as *Ailſt.*
1 8 *Walſland*, as *Gelderland.*
1 9 *Wilde*, as *Riga.*
1 4 *Wiſel*, as *Ausburgh.*
3 1 *Zaidin*—— 77 Rotuli.
1 8 *Zeland*, as *Gelderland.*
3 1 *Zeroi*—— 50 Rotuli.
1 8 *Zurich Sea*—— 110 ℔.

Foreign Weights for Money.

Weights for Money

Pounds of Florence *and* Geneva.

In *Florence* they uſe a Weight for Gold and Silver, and at *Geneva* for Silver called a Pound of 12 ℥. 1 ℥ is 24 Deniers, and 1 Denier is 24 Graines. So is there 6912 Graines in the Pound.

Pound of Naples.

In *Naples* their Pound is likewiſe divided into 12 ℥, and every Ounce into 8 Octany, or Octavos.

Mark *of* Antwerp.

The Mark Weight is uſed in many other Places, and at *Antwerp* containeth 8 ℥, and is heavier than their ordinary ℔. by 5 upon the Hundred, as *Malines* ſaith. This Mark is divided in a double manner.

Tables of Antwerpe Mark.

	Ounces.	Engliſh.	Graines.
(1) Mark	8	160	5120
	Ounce	20	640
		Engliſh	32

	Ounces.	Peny-weights.	Graines.
(2) Mark	8	192	4608
	Ounce	24	576
		Peny-weight	24

A Table of the French Mark.

The Mark Weights of ſome other Places ſubdivided.

France.

	Ounce.	Groſſes.	Deniers.	Graines.	Primes, *or* Garobs.	Seconds.	Tercies, *or* Malloquen.
Mark	8	64	192	4608	110592	2654208	63700992
Ounce		8	24	576	13824	331776	7962624
Groſs			3	72	1728	41472	995328
Denier				24	576	13824	331776
Graine					24	576	13824
Garob, *or* Prime						24	576
Second							24

How many Carrats *and* Graines *in their Ounce.*

In *France* the Ounce is alſo divided into 2 Carrats, and every Carrat into 12 Graines.

Tables of Mark *Weights of* Dantſick *and* Geneva.

Dantſick *in* Poland.

	Ounces.	Pence.	Hellers.
Mark	8	256	512
Ounce		32	64
Peny			2

Geneva *for* Gold.

	Ounces.	Deniers.	Graines.
Mark	8	192	4608
Ounce		24	576
Denier			24

Meyſen *and* Norenburgh.

Meyſen *in* Saxony.

	Ounces.	Deniers.	Graines, *or* Momenta.
Mark	8	192	4608
Ounce		24	576
Denier, *or* Peny			24

Norenburgh *in* Germany.

	Ounces.	Loots.	Quints.	Primes.	Seſtertios.
Mark	8	16	64	256	1024
Ounce		2	8	32	128
Loot			4	16	64
Quint				4	16
Prime, Peny, *or* Numulus.					4

Portugal *and* Venice.

Portugal.

	Ounces	Octavos, *or* Oitavos.	Great Grains.
Mark	8	64	288
Ounce		8	36
Oitavo *or* Octavo			4½

Venice.

	Ounces.	Silicos. *or* Quarts.	Siliquas. *or* Carrats.	Graines.
Mark	8	32	1152	4608
Ounce		4	144	576
Quart, *or* Silico,			36	144
				144
Carrat, *or* Siliqua				4

Spain.

Spain.

Spaine.

Gold.

	Ounces.	Caſtellanos.	Tomines.	Graines.
Mark	8	50	400	4800
Ounce		$6\frac{1}{4}$	50	600
Caſtellano			8	96
Tomine				12

Silver.

	Ounces	Drams *or* Octavos.	Graines.
Mark	8	64	4800
Ounce		8	600
Dram, *or* Octavo			75

Rome.

Rome.

	Ounces.	Drams.	Scruples.	Obolos.	Siliquas,	Primi, *or* Graines.
Mark	8	64	192	384	1152	4008
Ounce		8	24	48	144	576
Dram.			3	6	18	72
Scruple				2	6	24
Obolus					3	12
Siliqua						4

Romana Libra, *by* Malines.

A Table of Romana Libra

	Ounces, *or* Guilders.	Denarios.	Victoriatas.	Seſtertio's.	Aſſes.	Quadrantes.	Sexantes.
Libra.	12.	84	168	336	840	3320	5040.

The Ton of Gold in *Latine*, *Tina*, *ſeu Tonna*, by ſome called *Roman*, but by *Alſted*, *German* is thus divided.

Tonne of Gold how called.

A Table of the Tonne of Gold.

	Pounds.	Marks.	Ounces.	Loots.	Drams.
Tonne of Gold.	$781\frac{1}{4}$	$1562\frac{1}{2}$	12500	25000	100000
Pound		2	16	32	128
Mark			8	16	64
Ounce				2	8
Loot					4

Scotland, divides their Pound into 24 Deniers, 1 Denier 24 Primes, 1 Prime 24 Seconds, 1 Second 24 Thirds, 1 Third 24 Fourths, *&c.*

Pound of Scotland.

The Correſpondency of 100 Markes of *Antwerp* to the places following.

The 100 Marks *of* Antwerpe, *how much at ſome other Places.*

		Adler	$76\frac{1}{2}$	lb.
3	1	*Aegypt*	94	Beſſes
3		*Africa*	87	Markes
1	7	*Ancona*	$103\frac{1}{4}$	Markes
1	7	*Aquila*	71	lb.
1	4	*Ausburgh*	$105\frac{2}{19}$	Markes
1	4	*Bambergh*	$103\frac{1}{4}$	Markes
1	4	*Bavaria*		
1	4	*Bohemia*	87	Markes
1	4	*Breſla*	$121\frac{3}{4}$	Markes

A Table of the French Mark.

The Mark Weights of ſome other Places ſubdivided.

France.

	Ounce.	Groſſes.	Deniers.	Graines.	Primes, *or* Garobs.	Seconds.	Tercies, *or* Malloquen.
Mark	8	64	192	4608	110592	2654208	63700992
	Ounce	8	24	576	13824	331776	7962624
		Groſs	3	72	1728	41472	995328
			Denier	24	576	13824	331776
				Graine	24	576	13824
					Garob, *or* Prime	24	576
						Second	24

How many Carrats *and* Graines *in their Ounce.*

In *France* the Ounce is alſo divided into 2 Carrats, and every Carrat into 12 Graines.

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Dantſick *in* Poland.

	Ounces.	Pence.	Hellers.
Mark	8	256	512
	Ounce	32	64
		Peny	2

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	Ounces.	Deniers.	Graines.
Mark	8	192	4608
	Ounce	24	576
		Denier	24

Meyſen *and* Norenburgh.

Meyſen *in* Saxony.

	Ounces.	Deniers.	Graines, *or* Momenta.
Mark	8	192	4608
	Ounce	24	576
		Denier, *or* Peny	24

Norenburgh *in* Germany.

	Ounces.	Loots.	Quints.	Primes.	Seſtertios.
Mark	8	16	64	256	1024
	Ounce	2	8	32	128
		Loot	4	16	64
			Quint	4	16
				Prime, Peny, *or* Numulus.	4

Portugal *and* Venice.

Portugal.

	Ounces	Octavos, *or* Oitavos.	Great Grains.
Mark	8	64	288
	Ounce	8	36
		Oitavo *or* Octavo	4½

Venice.

	Ounces.	Silicos. *or* Quarts.	Siliquas. *or* Carrats.	Graines.
Mark	8	32	1152	4608
	Ounce	4	144	576
		Quart, *or* Silico,	36	144
				144
			Carrat, *or* Siliqua	4

Spain.

Spaine.

Gold.

	Ounces.	Caſtellanos.	Tomines.	Graines.
Mark	8	50	400	4800
	Ounce	$6\frac{1}{4}$	50	600
		Caſtellano	8	96
			Tomine	12

Silver.

	Ounces	Drams *or* Octavos.	Graines.
Mark	8	64	4800
	Ounce	8	600
		Dram, *or* Octavo	75

Rome.

Rome.

	Ounces.	Drams.	Scruples.	Obolos.	Siliquas,	Primi, *or* Graines.
Mark	8	64	192	384	1152	4008
	Ounce	8	24	48	144	576
		Dram.	3	6	18	72
			Scruple	2	6	24
				Obolus	3	12
					Siliqua	4

Romana Libra, *by* Malines.

A Table of Romana Libra

Libra.	12.	84	168	336	840	3320	5040.
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The Ton of Gold in *Latine*, *Tina*, *ſeu Tonna*, by ſome called *Roman*, but by *Alſted*, *German* is thus divided.

Tonne of Gold how called.

A Table of the Tonne of Gold.

	Pounds.	Marks.	Ounces.	Loots.	Drams.
Tonne of Gold.	$781\frac{1}{4}$	$1562\frac{1}{2}$	12500	25000	100000
	Pound	2	16	32	128
		Mark	8	16	64
			Ounce	2	8
				Loot	4

Scotland, divides their Pound into 24 Deniers, 1 Denier 24 Primes, 1 Prime 24 Seconds, 1 Second 24 Thirds, 1 Third 24 Fourths, *&c.*

Pound of Scotland.

The Correſpondency of 100 Markes of *Antwerp* to the places following.

The 100 Marks *of* Antwerpe, *how much at ſome other Places.*

		Adler	$76\frac{1}{2}$	℔.
3	1	*Aegypt*	94	Beſſes
3		*Africa*	87	Markes
1	7	*Ancona*	$103\frac{1}{4}$	Markes
1	7	*Aquila*	71	℔.
1	4	*Ausburgh*	$105\frac{2}{19}$	Markes
1	4	*Bambergh*	$103\frac{1}{4}$	Markes
1	4	*Bavaria*		
1	4	*Bohemia*	87	Markes
1	4	*Breſla*	$121\frac{1}{4}$	Markes

The 100 Marks *at* Antwerp, *how much at some other Places.*

1	14	*Burgas*	116 2/3	Markes
1	7	*Calabria*	76 1/2	℔.
1	14	*Catalonia*	100	Markes
1	4	*Collen*	105 2/19	Markes
1	5	*Constantinople*	87	Markes
1	7	*Crema*	103 1/4	Markes
1	9	*Dantſick*	105 2/19	Markes
1	4	*Erdford*		
1	7	*Florence*	72	℔.
1	4	*Franconia*	103 1/4	Markes
1	4	*Frankford*	105 2/19	Markes
1	4	*Friburgh*	103 1/4	Markes
1	7	*Genes* for Gold	116	Markes
		Silver	77	Markes, *or* ℔.
1	7	*Geneva*, as *Paris* and *Lyons*.		
1	5	*Gracia*	105 2/19	Markes
1	4	*Hungary*	87	Markes
1	4	*Lipſich*	105 2/19	Markes
1	2	*London*	89 8/9	℔.
1	3	*Lyons*	112	Markes, Merchants Weight
			102 1/2	Markes, Merchants Weights The Kings Weight
1	4	*Ments*	105 2/19	Markes
1	4	*Meyſen*		
1	7	*Millan*		
1	7	*Naples*	76 1/2	℔.
2	6	*Narſinga*	87	Markes
1	4	*Norenburg*	103 1/4	Markes
4	2	*Nova Spagnia*	87 1/2	Markes
1	3	*Paris*, as *Lyons*.		
2	9	*Perſia*	87	Mina's
4	3	*Peru*	87 1/2	Markes
1	7	*Piedmont*	99	Markes
1	7	*Puglia*	76 1/2	℔.
1	7	*Rome*	103 1/4	Markes
1	4	*Saxony*	105 2/19	Markes
1	14	*Spain*	107	Markes
1	4	*Trevers*, or *Triers*	105 2/19	Markes
1	7	*Treviſo*	103 1/4	Markes
1	7	*Turin*	99	Markes
2 3		*Turky*	87	Markes
1	7	*Venice*	103 1/4	Markes
1	7	*Verona*		
1	7	*Vicenza*	105 2/19	Markes
1	4	*Vienna*	87	Markes
1	4	*Vlme*	105 2/19	Markes
1	4	*Wiſſilburgh*	103 1/4	Markes

Foreign Monies. To cloſe up the Forain *Geodaticks*, Moneys take their turn, concerning which three things are to be obſerved.

1*st*. Their Diviſions, or greater and leſſer Denominations.

2*ly*. The Accompts thereof, and Exchanges.

3*ly*. The Weight and Worth of the ſeveral Coines.

Accompts and Exchange at ſeveral Places. *Malines*, *p.* 240, 241, 257, 258, 259, and other Authours inform us concerning the former two, as followeth, *viz.* at—

2 11 *Aleppo*, The Exchange is made by *Sultanies* of 120 Aſpers, or Dollers of 80 Aſpers, every Aſper 10 Macherines.

3 1 *Alexandria*, They Accompt by Ducats, either *Ducat de Pargo*, of 120 Maids, Ducat of *Venice* of 40 Maids, or *Italian* Ducat of 35 *Maids*.

1 7 *Ancona*, Exchange is made on the Ducat of 21 Groſs, (which is in Specie 23 Groſs) which Ducat is alſo 14 Carlini, and every Carlini 6 Bollidini, So is the Ducat 84 Bollidini.

Accompts and Exchange at ſeveral Places.

1 4 *Arragon,* The Rial, or Ryal of Plate is 23 Dinero's (*Hunt* ſaith 13) and the Ducat is 12 Ryals, whereon they make Exchange. And they Accompt by Pounds of 20 *s.* and 12 *d.* And the Ducat of 12 Ryals. Every Ryal of 1 *s.* or 12 *d.*

1 8 *Artois,* And in ſeveral other Places they Accompt and Exchange by Pounds or Liures Toirnois of 20 Stivers, or 40 Pence *Flemiſh*, whereof 6 called Guilders or Florius makes the Pound *Flemiſh* in all the 17 Provinces of the *Netherlands.* Which Pound is divided into 20 *s.* and every Shilling into 12 *d.* &c.

Some reckon by the Pound *Paraſis*, which is but 20 Pence, whereof 12 make 1 Pound *Flemiſh*, but their Accompts, as alſo the Finances of the Princes are kept by Pounds Tournois, and both Pounds divided into 20 *s.* and every Shilling into 12 Pence, admitting alſo the Subdiviſions of Obolo's, Maille, Heller, Hallinck, Corte, Mites, Point, Engevin, Poot, and ſuch like Copper Monies.

Alſtead mentions the *Florin* in *Germany* to be 15 Batz, every Batz 2 Albes, every Albe 8 Oboli, or Nummos. So ſhall the Florin be 30 Albes or 240 Oboli.

1 4 *Auguſta,* or *Ausburgh*, Accompts on the Dollar coined at 65 Creutzers, riſen ſince to 72. Exchange is made on the imaginary rate of 65 Creutzers.

Creutzer, how called and ſtamped.

A Creutzer, is ſometime called a Schreikenborger, and in *Latine*, *Crucigerus* and *Cruciatus*, being pieces ſtamped with a Croſs. Their Groſs make 12 Creutzers or 3 Batz, ſo is their Batz 4 Creutzers. Their Lyon Piece half a Creutzer. They have their Snubourgh, Blaphart, or Bohemico's of 3 and $3\frac{1}{2}$ Creutzers. The Rix or Rycks Dollar is 30 Albes of 8 *d.* every Albe, or 72 Creutzers. Every Dollar as before. See the following Table, and afterwards in *Germany.*

A Table of the German Dollar.

	Groſs.	Batz.	Albes.	Creutzers.	Lyon.	Pence.	Black.penys.
Dollar	6	18	30	72	144	240	288
Groſs		3	5	12	24	40	48
Batz			$1\frac{2}{3}$	4	8	$13\frac{1}{3}$	16
Albe				$2\frac{2}{5}$	$4\frac{4}{5}$	8	$9\frac{3}{5}$
Creutzer					2	$3\frac{1}{3}$	4
Lyons						$1\frac{2}{3}$	2
Peny							$1\frac{1}{5}$

3 2 *Barbary,* Generally Accompts are kept, and Commodities ſold by Ducats of 10 ℥, each Ounce divided into 8 parts, which eight part is in Value about 12 *d.* Sterling.

1 14 *Barcelona,* As at *Arragon.*

1 4 *Bavaria,* Accompts and Exchanges both are by Guilders of 7 *s.* and 30 Pence to Shilling.

1 4 *Bohemia,* As in *Germany*, generally by the Dollar of 24 Bohemico's, called alſo White Groſs, each of 3 Creutzers, other Diviſions ſee in the Table following.

A Table of the Bohemian Scoc.

	Marke.	Dollars.	Angſter's.	Bohemico's.	Creutzers.	Pence.
Scoc.	$1\frac{1}{2}$	$2\frac{1}{2}$	30	60	180	600
Marke		$1\frac{2}{3}$	20	40	120	400
Dollar			12	24	72	240
Angſter				2	6	20
Bohemico					3	10
Creutzer						$3\frac{1}{3}$

1 7 *Bolognia,*

1 7 *Bolognia*, They Accompt by Piaſtra, or Pounds (called alſo Piaſtri,) each containing 20 Bologneſi. And exchange on the Ducat of 4 Piaſtri.

1 8 *Brabant*, And in moſt places of the *Low Countries*, Monies, are accompted by the Pound *Flemiſh*, containing 20*s*. *Flemiſh*. And every Shilling 12*d*. or Deniers called Single Stivers, 2 of which make 1 double Stiver. See *Flanders*.

1 4 *Breſlaw*, They reckon by Markes of 32 Groſs, of 12 Heller to the Groſs, And exchange by 30 Florens to have at *Norenbergh* 32 Florins, and at *Vienna* 34 Florins.

1 7 *Calabria*, Exchanges are made by the *Naples* Ducat, of 10 Carlini.

1 14 *Caſtile*, Exchanges are made on the Ducat of 375 *Marvedies*, which they call in the Bill of Exchange *Ducadas d' oro*, or *de Peſo* to be payd out of the Bank is better by 6, or 8 *pro Milliar*. See *Spain*.

1 14 *Catalonia*, as at *Arragon*.

1 4 *Cleves*, *Collen*, Both Accompts and Exchanges are made by Dollars of 72 Creutzers.

A Table of the **Guilder** *of* Cleves *and* Collen.

	Markes.	Morkens.	White pennies.	Shillings, *or* Stivers.
Their Guilder is	4	12	24	48
Marke		3	6	12
Morken			2	4
White Peny				2

1 5 *Conſtantinople*, as *Aleppo*.

1 9 *Dantſick*, They accompt by *Poliſh* Guilders, of 30 Groſs, every Groſs 18*d*. They buy with the Great Mark of 60 Groſs, or the Little Mark of 15 Groſs. Alſo by the Scoc of 3 Great Marks. And exchange upon the *Florin Poliſh*, or the Pound *Flemiſh*. They have Dollars of 35 Groſs of 3 Shillings. And new Dollars of 24, 26, or 30 Groſs. Their Gilden is 80 Groſs. So is

A Table of the Scoc *of* Dantſick.

	Gildens.	Great Markes.	Dollars.	Guilders.	Little Markes.	Groſs.	Pence.
Scoc.	$2\frac{1}{4}$	3	$5\frac{1}{7}$	6	12	180	3240
Gilden		$1\frac{1}{3}$	$2\frac{2}{7}$	$2\frac{2}{3}$	$5\frac{1}{3}$	80	1440
Great Marke			$1\frac{5}{7}$	2	4	60	1080
Dollar				$1\frac{1}{6}$	$2\frac{1}{3}$	35	630
Guilder					2	30	540
Little Mark						15	270
Groſs							18

1 1 *Denmark*, They Accompt by Markes of 20 Shillings. And Exchange upon the Dollar.

1 6 *Dublin*. See *Ireland*.

1 12 *Edenbourgh*. See *Scotland*.

1 4 *Embden*, They reckon by Guilders, and exchange on the Rycks Dollar, but from *London* hither and thither upon the Pound *Sterling*.

1 8 *Flanders*, As before in *Brabant*. See a more particular Diviſion of the *Flemiſh* Money in the following Table.

A *Table of the* Flemiſh *Pound.*

	Gilders.	Shillings.	Double-Stivers.	Single-Stivers.	Groats.	Ortgens.	Negen-Manneken.	Copper-Pence.	Mites.
Flemiſh Pound.	0	20	60	120	240	480	960	1920	2880
Gilder		3⅓	10	20	40	80	160	320	480
Shilling			3	6	12	24	48	96	144
Double-Stiver				2	4	8	16	32	48
Single-Silver					2	4	8	16	24
Groat						2	4	8	12
Ortgen, *or* Ortken							2	4	6
Negen-Manneken								2	3
Copper-Penny.									1½.

Five Single Stivers are Currant in ſeveral Places of the *Low-Countries* for Six pence *Sterling*. Ortkens in ſome places are called Duyts. Mites in ſome places of Flanders are called Cortes, Engcuni, Points, Pites, Pootes.

1 7 *Florence*, They accompt by Crownes of 20 *s.* and 12 *d.* to the Shilling. And exchange by a Ducat called *Largo*, or *Scripto in Banco*. A Florin ther e is 24 Quatrini.

1 3 *France*, Generally they uſe Liures Solx and Deniers, and commonly accompt by them, as the *Engliſh* by Pounds, Shillings and Pence, but by an Edict made 1577, their Accompts are to be kept in *French* Crownes of 60 Sols to the Crown, or 3 Liures, that is Pounds Tournois. And exchange is made thereupon unleſs for ſome places in *Italy*, where they exchange for Number to have ſo many Ducats for ſo many Crownes of the Sum, not in Specie but imaginary, yet reſpecting the Value, or Par. See further in the Table and Notes thereupon.

A *Table of the* French *Crowns*

	Liures.	Sols.	Liarts.	Doubles.	Deniers.
French Crown A	3	60.	240	360	720
Liure B		20	80	120	240
Sols C			4	6	12
Liart D				1½	3
Double E					2

There are alſo Petit Deniers, and Mailles, but not conſiderable.

French *Crown of Gold how much.*

A. This Crown here to make Exchange by is equivalent to the Silver Coines of *Lewis* 13th and 14th, called *Lewiſſes* and imaginary, and not to be accompted for the *French* Gold Crowne, which is a real Coine, and of greater Value now, being worth 8 *s. Sterling*, or thereabouts, but when currant, at 6 *s. Sterling*. The Accompt and Exchange agreed in reality, 10 Sols then and yet commonly reckoned for an *Engliſh* Shilling. Of this Gold Crown was the Cardecue a quarter, and ſo valued in *Sterling* Money at 18 *d.* and ſhould be wrote *Quartid'eſcue*, *Eſcue*, being *French* for a Crown.

Cardecue *what and how to be wrought.*

Liures *and* Sals *how called, whence the word.*
Sorts of Sols.

B, and *C.* The Liures (or Pounds ſometime called Franks) and Sols (wrote ſometime Soulx, ſometime Solx, derived from the *Latine*, *Solidus*, as *Liure* from *Libra* are different. Thoſe commonly uſed are called Tournois, and valued with *Sterling* Money as above. Of the Sols *Barrois* 14 make 20 Sols *Tournois*. The Sols *Mauſais*, is 2 Sols *Tournois*. The Sols *Paris* is 1¼ Sols *Tournois*. The Sols *Bourdelois* is half the Sols *Paris*. And ſo accordingly is the Liure to be accompted.

Liarts, *and* Doubles *not uſed in Accompts.*

D, and *E*, Neither the Liarts nor Doubles, though both Copper Coines are uſed in Common Accompts.

1 4 *Frankford*, Their Guilder or Florin by which they reckon is 60 Creutzers divided by 20 *s.* and every Shilling into 12 Hellers according to the Pound. But they exchange by the Dollar of 65 Creutzers payable in the two Yearly Fairs or Marts, one the Week before *Eaſter*, and the other all the Moneth of *September*.

1 7. *Genoa*, All Accompts, and Exchanges are made by Crowns of 60 *s.* divided into 20 *s.* and every Shilling into 12 Pence.

1 4 *Germany*, Every Batz by which generally they keep Accompts, is 4 Creutzers. They Exchange on the Dollar imaginary at 65 Creutzers, and ſo coined as was noted before at *Auguſta*, though ſince riſen to 72 in Value.

They have Pieces of 3, 6, and 12 Creutzers, and by them, and their Batz they value their own and *Exotick* Coines as the *Hungarian* Ducat is 27 Batz. The Gold Guilder is 18 Batz. The *Poliſh* Guilder or Dollar is 15 Batz, Teſton 5 Batz, *&c.*

Guilder *of the* Romans.

Nummi Dragmi.

A Guilder was the name the Antient *Romans* gave to an Ounce, and 8 ℥ made a Mark, and 12 Ounces or Guilders a Pound. And there were Coined Pieces called *Nummi Dragmi*, or *Groſhen* the 8*th* part of a Dollar. *Anno* 1520. was the Gold Guilder Coined for a General Coine, and valued in *Holland* at 28 Stivers, but now in *Specie* at double the Price. Nevertheleſs Corn brought from *Poland* and the *Eaſt Countries*, is bought and ſold by the ſame at the old value of 28 Stivers.

Angelicies, *what.*

Tribute Peny, abaſed, whence the Name of Batz.

Angelicies was the Sixth part of a Dollar, making 3 Batz, or 12 Creutzers. Theſe Angelici becoming Tribute Pennies were allayed, and ſo being made worſe, did obtain the Name of Batz or Bates (ſometime wrote Batſes) *quaſi* Baſe. And in *Thuringia* they are called Gulielmi, and in *Bohemia*, Bohemici, whereof they have alſo 12 Peeces dividedly, for 12 Pence; which Peny is 2 Hellers in Accompt all over *Germany*.

Pardauue-Xerafin *the Stamp and Value.*

2 6 *Goa*, Their Common Accompt is by their Ordinary Silver Coine a *Pardauue-Xerafin*, having the Image of St. *Sebaſtion* at the one ſide, and 3 or 4 Arrows bound together at the other, which is worth 3 Teſtons, or 300 Res of *Portugal*, but varieth as the Exchange riſeth or falleth: And accordingly their other Coines, and Accompts of which ſome are imaginary, and ſome real. They have alſo ſome good and ſome bad Monies; for 4 good Tanga's, or 5 bad Tanga's are reckoned to value 1 Pardauue-Xerafin. And 1 Tanga is 75 Baſarves. Of theſe Baſarves 375 make 1 Pardauue-Xerafin. And 15 good Baſarves are valued with 18 bad, which are made of Bad Tinne. By theſe other Countrey Coines are rated, as the *Larin* of *Perſia* is worth 105, and 108 Baſarves, as the Exchange goes. A Paradauue of *Larins* is 5 Larins. And the Crowns of *Venice* or *Turkey* are almoſt worth 2 Paradauue-Xeraſins. They have alſo a *Pagode*, or Gold Crown, on which is the Figure of their *Idol*, worth about 8 Tanga's, And Gold Crowns of St. *Thomas* with his Image on them, eſteemed at 7, or 8 Tangas.

Tangas *good and bad.*

Larin *of* Perſia *the Value.*

Pagode *what.*

1 1 *Hamborough*, Their Dollar was firſt Coined at 31 Shillings Lups, and many Years currant for 33, is now inhaunced to 54 *s.* Lups, of 3 White Peny, and every Shilling is 12 *d.* and every Peny 2 Hellers. They Accompt by Markes of 16 *s.* Lubiſh, and 12 *d.* to the Shilling: But Exchange for *London* upon the Pound *Sterling*, and for other Places on the Rycks Dollar of 33 *s.* now by them inhaunced to 54 *s.* Lubiſh, or ſo many Stivers *Flemiſh*.

1 8 *Henault*, As *Artois.*

1 4 *Hungaria*, They accompt by Guilders of 10 *s.* of 30 *d.* to the Shilling, And by *Florins* of 20 *s.* and 12 *d.* to the Shilling: And exchange on the Ducat and Rycks Dollar worth 8 Shillings formerly, but 7 *s.* 7 *d.*

1 6 *Ireland*, They as the *Engliſh* Accompt by Pounds of 20 *s.* *Sterling* and Pence of 12 to the Shilling. Only their Harper valued in *England* but 9 *d.* was with them counted 1 *s.* So as their Pound is but ¾ of ours, or 15 *s.* *Sterling*. And thereon Exchanges are made.

1 4 *Lipſich*, as *Breſta*.

1 10 *Lisbon*, See *Portugal*.

1 8 *Low Countries*, generally as before at *Brabant*.

1 2 *London*, Exchanges are generally made for *Germany* and the *Low Countries*, on the Pound *Sterling*. For *France* on the *French* Crown of 60 Sols *Tournois*. For *Italy*, *Spain*, and other Places on the Ducat Dollar or Florin according to the Cuſtom of the Place

1 7 *Luca*, For divers Places in *Italy* and *Lyons* in *France* Exchanges are made on the Ducat.

1 3 *Lyons*, as before in *France*.

1 14 *Madrid*, See *Spain*.

1 7 *Millan*, Accompts are kept by Ducats Imperial, divided by 20 *s.* and 12 *d.* to the Shilling. And Exchanges made on the ſame, accompting 80 *s.* to the Ducat Imperial: But they buy by a Ducat currant of 120 *s.*

1 7 *Naples*, They Accompt by Ducats, Taries and Graines. The Ducat is 10 Carlini or 5 Taries, for the Tarie is 2 Carlini, or Royals. And hereupon Exchanges are made for moſt places of *Italy*; but for *Lyons*, they Exchange by Number, as 125 Ducats for 100 Crowns.

1 4 *Norenbourgh*, The Exchange is made on the Dollar of 65 Creutzers, and many times on the Guilder of *Florin*, of 60 Creutzers, which they alſo divide into 20 *s.* and every Shilling into 12 *d.* to keep Accompts by; and ſome ſay the Creutzer is 4 *d.* every Peny is 2 Hellers. And 5 *d.* is called a Fynſer, or 5 Pennick.

1 7 *Palermo*, The Ducat is 13 Taries, 1 Tarie 2 Carlini, 5 Ryals of *Spain* are 6 Taries. They accompt by Ounces of 30 Taries, to 20 Graines every Tarie, and every Graine 6 Piccolie. And their Exchanges are made upon *Florins* of 6 Taries, or Tarij.

1 3 *Paris*, as before in *France*.

1 9 *Poland*, They Accompt by Markes and Exchange on the Dollar, and alſo on the Florin of 48 *s.* The Marke is One Third part of it.

1 4 *Pomerania*, They divide their Money as in the next Table following, Accompt by Markes of 16 Snudens, and Exchange upon the Ryckx Dollar of 32 *s.* or 2 Markes Snudens, ſo called to diſtinguiſh them from Markes Lups, and Shillings Lups.

A Table of the Ryckx Dollar *of* Pomeren.

	Marks-Snudens.	Lups-Shillings.	Shillings-Snudens.	Pence.	Hellers.
Ryckx Dollar Marke Lups	2	16	32	384	768
	Marke-Snuden	8	16	192	384
		Shilling-Lups.	2	24	48
			Shilling-Snuden	12	24
				Peny	2

1 10 *Portugal*, They Accompt by *Milrais*, Ducats, or Cruſado's, *&c.* as in the Table following, And Exchange by the ſame Ducat of 400 Raies.

A Table of the Mille Raies.

	Ducats, *or* Cruſado.	Teſtons.	Rials.	Vintaines, *or* Half Rial.	Raies.
Mille Raies	2½	10	25	50	1000
	Ducat, *or* Cruſado.	4	10	20	400
		Teſton.	2½	5	100
			Rial.	2	40
				Vintaine, *or* Half Rial.	20

Of theſe Ducats, Rials, (or Royals) and Raies (wrote alſo Reas, Reyſe, and Res) are moſt in uſe for Accompt. They have Teſtons alſo of 4 Vintaines, 40 Raies are commonly accompted for Six Pence *Sterling*, and ſo accordingly were the other Coines valued, till the late advance whereby the Teſton of 100 Raies were ſtamped, and made currant for 120 Raies, and ſo rated at 1 *s.* 6 *d.* *Sterling*, when before but 1 *s.* 3 *d.*

1 7 ***Puglia***, as ***Calabria.***

1 9 ***Riga***, They buy by Dollars, or Florins *Poliſh* of 18 Farthings, whereof 11 make 10 Dollars, but they Exchange upon the Ryckx Dollar.

1 3 ***Roan***, as before in *France.*

1 7 ***Rome***, Accompts and Exchanges are performed by Ducats *di Camera* of 13 *July*, or *Guili*, every Ducat which they divide into 20 *s.* and every Shilling into 12 *d.*

1 11 } 2 10 } ***Ruſſia***, They have ſmall Coine of 11 ℥. 2 penny weights fine, called *Dengen*, whereof 320 Pieces weigh but a Mark of 8 ℥. They Exchange upon the Dollar of *Germany*; but for *London* upon their Rubble which is valued as a Double Ducat formerly, accompted equal to a Marke Sterling, or 13 *s.* 4 *d.*

1 14 ***Saragoſſa***, as *Arragon.*

Baſe Money of Scotland.

1 12 *Scotland*, They Accompt by Pounds, Shillings, and Pence, as in *England*, but one *Pund Scotch* is but 20 *d.* *Engliſh*. Their Marke is $13\frac{1}{3}$ *s.* *Scotch*, currant in *England* at $13\frac{1}{2}$ *d.* Their Noble, or half Marke with them $6\frac{2}{3}$ *s.* with us $6\frac{3}{4}$*d.* Their half Noble, and third part of their Noble proportionally. They have alſo Turnoners, Pence, and Half-Pence, and baſe Money of Bodles, Achiſons, Babees, Placks, *&c.* accompting 6 Bodles to 1 *d.* *Sterling*, or 12 *d.* *Scotch*, 4 Bodles to 1 Achiſon, 3 to 1 Babee, and 2 to 1 Plack. But they Exchange upon their Marke.

1 14 ***Spaine***, as in *Madrid*, *Sevil*, and other Places their Accompts are all kept by *Malvedies*, or *Marveides* (wrote alſo *Merveides* and *Maravides*) whereof 375 are eſteemed to make a Ducat of 11 Rials, though really every Ryal is but 34 Marveides, and ſo maketh but 374, as in the following Table, and ſo others keep Accompts accordingly. Exchange is made on this Imaginary Ducat of 375 Marveides to be payd in Bank with 5 on the 1000, which is the Salary of the Banker, or without the Bank to be payd without the ſame.

A Table of the Spaniſh Ducat.

	Pieces of Eight.	Rials.	Quartilios.	Marveides.	Carnado's.
Ducat	$1\frac{3}{8}$	11	44	374	2244
Piece of Eight		8	32	272	1632
Rial			4	34	204
Quartilio				$8\frac{1}{2}$	51
Marveide					6

A Rial is about $6\frac{1}{2}$ *d.* *Sterling.*

1 4 *Stratsborough*, or *Strausburgh*, They have Blapharts, Groſs, Bohemico's, all currant for 3 Creutzers a piece, 1 Creutzer at 2 *d.* One Peny at 2 Hellers, and 1 Heller at 2 Orthings.

1 15 *Sweden*, They reckon by Markes, whereof 8 make a Dollar, whereupon they Exchange. And 2 Markes make a Clipping of $9\frac{1}{2}$ Stivers.

1 4 *Tirol*, The Dollar is 72 Creutzers, and the Creutzer 5 Fynfers, or Hellers.

2 11 *Tripoli*, as *Aleppo.*

1 14 *Valentia*, as *Arragon.*

1 7 *Venice*, Thirty Batz make 1 Souldey, and 20 Souldeys 1 Liure of *Venice*. Their Gold Ducat is valued equal to 40 Maides of *Alexandria*. They have alſo Copper Money, 1 Seſſini make 2 Quatrini, and 1 Quatrine 4 Bagatini, and ſo 3 Quatrini, or 12 Bagatini make an Half penny *Sterling*, or thereabouts.

They accompt by Pounds *Flemiſh* of 10 Ducats or 20 *s.* and divide the Ducat into 24 Groſs, and the Shilling into 12 Pence.
And alſo by the Ducat 124 *s.* called *Ducato di Banco*, or Currant, and thereon Exchanges are made.

1 7 *Verona*, Their Accompts are kept by 20 *s.* and 12 *d.* to the Shilling. And they Exchange on the Ducat of 93 *s.*

1 4 *Vienna*, Both Accompts and Exchanges are kept and made by Guilders or Florins of 8 *s.* a piece, 30 *d.* to the Shilling, and 2 Hellers to the Peny.
They eſteem the Ricks Dollar at 8 *s.* and the Ducat at 12 *s.*

1 4 *Vlme*, They reckon by Pounds of 20 *s.* and 12 Heller to the Shilling. And Exchange on the Dollar of 60 Creutzers.

Foreign Coins.

That which remains to finiſh this long and tedious Chapter of *Geodæticals* is only to conſider the weight and worth of the Coines of other Countreys as valued by the *Sterling* Standart, wherein becauſe of the New Coines which may dayly be added by the Laws of the preſent or ſucceeding Governours, and thoſe of different Fineneſs; to obviate the difficulties as well occaſioned thereby, as by the riſe and fall of Exchange, and ſo conſequently of particular Coines, practiſed by Merchants: Every one that would arrive at ſatisfaction beſides what can be here wrote, muſt add his diligent obſervance.

Weight to try the fineneſs of Gold and Silver by in England.

It may be remembred that the *Engliſh* Pound *Troy* is divided for Weight into 12 Ounces, every Ounce into 20 Peny-weights, and every Peny-weight into 24 Graines. And to try the Fineneſs of Silver the ſame diviſions are kept, but for the Fineneſs of Gold, every Ounce is divided into 24 Carrats, and every Carrat into 4 Graines. And the old *Sterling* Standart for Silver is 11 Ounces 2 Peny-weights fine, and for Gold 22 Carrats fine.

Sterling Standart.

Weight beyond Sea to try the Fineneſs by.

They beyond Sea for Weight and Fineneſs of Silver divide their Ounce into 20 *Engliſh*, and every *Engliſh* into 32 Azes? And for weight of their Gold the like; But for the Fineneſs thereof divide their Ounce into 24 Carrats, and every Carrat into 12 Graines.

How the following Coines are valued.

In the following Coines, underſtand the Value according to the *Engliſh* Diviſion, allowing for the Ounce of Silver 11 Ounces 2 Peny-weights fine, 5 *s.* but for the Ounce of Gold 22 Carrats fine 3 *l.* 10 *s.* that is 3 *s.* 6 *d.* for the Peny-weight, and ſo proportionally for Coines of greater or leſſer fineneſs, which Valuation makes the particular Pieces to differ from that found in ſeveral Printed Books, as they one from another, according to the times they were Printrd or Wrote in. Some valuing the Ounce of Silver ſo fine as aforeſaid, and others that of 11 Ounces fine, at 5 *s.* and the Ounce of Gold of the fineneſs aforeſaid but at 55 *s.* others at 3 *l.* ſome at 3 *l.* 6 *s.* and generally not above 3 *l.* 6 *s.* 8 *d.*

All Coines not valued according to their Fineneſs.

Neither are all Coines though of one and the ſame fineneſs alwaies valued alike proportionally. For K. *James*, *May* 14. 1612. by Proclamation ordered the bringers in of Forein Coine might receive at the Mint as followeth.

		Car.	Gr.		*l.*	*s.*	*d.*
	For the Ounce of *Spaniſh* Silver Money of *Sevil*				0	5	0
	The Ounce of *Mexico* Money.				0	4	10
	Ingots of Silver, being 11 ℥. 2 pwts. fine				0	5	0
	And ſo rateably for Silver of other fineneſs.						
For 1 ℥. of	*Spaniſh* Piſtolets being	21	3½	fine	3	6	0
	French Crowns being	22	0	fine	3	6	0
	Milreys, *Cruſado* long and ſhort Croſs.				3	6	2
	Barbary Gold being	23	0½	fine	3	9	0
	Hungary Ducats, being	23	1	fine	3	9	0
	Spaniſh Ducats and Sultaines being	23 *Car.*	1 *Gr.*	fine	3	8	8
	Zechines, or Checkeene of *Venice* being	23	1	fine	3	10	0
	And for the Ounce of all other Gold being	22	0	fine	3	6	0

And in like ſort to this day by the Artifice of *Merchants*, *Goldſmiths*, *Bankers*, &c. Some Coines are valued and currant in Traffique at a value higher, than by a due proportion in reſpect of their fineneſs to the *Sterling* Standart, they ought to be: But in the following Tables, the New Value is equally apportionated, yet without allowance for Coinage, which I take to be about 2 *s.* for the Pound *Troy* of Silver, and for the Pound *Troy* of *Gold* about 15 *s.*

		Foreign Gold.	Fine		Pieces to the ℔	Weight by *Malines.*		Weight by others.		Old Value.			New Value.		
			Car.	*Gr.*	*Troy.*	*pwts.*	*gr.*	*pwts.*	*gr.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*
		Albertines, See Ducats.													
An-gels *the ſorts.*	Angels	with the three Lions	22	0	76	3	$3\frac{15}{19}$	3	$3\frac{3}{4}$	0	8	6	0	11	$0\frac{1}{2}$
	Angels	with *O.*	23	0	72	3	8	3	6	0	9	0	0	12	$2\frac{1}{4}$
	Angels of	*Batenborgh*	21	3	72	3	8	3	6	0	9	0	0	11	$6\frac{1}{4}$
		Flanders, or beſt *Flemiſh* Angel	23	0	72	3	8	3	6	0	9	0	0	12	$2\frac{1}{4}$
		H. M.	17	0	72	3	8	3	6	0	6	9	0	9	0
		Horne	23	$1\frac{1}{2}$	72	3	8	3	6	0	9	6	0	12	$4\frac{3}{4}$
		Thoron	22	$1\frac{1}{2}$	72	3	8	3	6	0	9	0	0	11	$10\frac{1}{4}$
		Vienna	18	3	72	3	8	3	6	0	7	6	0	9	$11\frac{1}{4}$
		Chaſtilion	23	3	$79\frac{3}{4}$	3	$0\frac{72}{319}$	2	23	0	8	10	0	11	$4\frac{1}{4}$
		Croſs Daggers of *Scotland*	22	0	72	3	8	3	6	0	11	0	0	11	8
		The half thereof	22	0	144	1	16	1	15	0	5	6	0	5	10
Cro. *the ſorts.*	Crowns	Flemiſh Crown	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
		Floret Crown of *France*	23	3	$100\frac{1}{2}$	2	$9\frac{21}{67}$	2	9	0	7	0	0	9	$8\frac{1}{4}$
		Charles French Crown	23	3	$100\frac{1}{2}$	2	$9\frac{21}{67}$	2	9	0	7	0	0	9	$0\frac{1}{4}$
		Old French Crown	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
		New French Crown — Some	22	0	$107\frac{1}{2}$	2	$5\frac{25}{43}$	2	$5\frac{1}{2}$	0	6	0	0	7	$9\frac{3}{4}$
		New French Crown — Others	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
		Half Imperial Crown	22	$1\frac{1}{2}$	$107\frac{1}{2}$	2	$5\frac{25}{43}$	2	$5\frac{1}{2}$	0	6	0	0	7	$11\frac{1}{4}$
		Italian Crown	—	—	—	—	—	—	—	0	6	0	0	7	6
		Four Crowns of *Portugal*	—	—	—	—	—	—	—	1	6	2	0	10	0
		K. *Philip*'s Crown of *Spain*	22	$1\frac{1}{2}$	$107\frac{1}{2}$	2	$5\frac{25}{43}$	2	$5\frac{1}{2}$	0	6	0	0	7	$11\frac{1}{4}$
		Scotch Crown	22	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	7	$9\frac{1}{4}$
		Thiſtle Crown	22	0	186	1	$6\frac{30}{31}$	1	$6\frac{3}{4}$	0	4	$4\frac{3}{4}$	0	4	6
Cru-ſadoes *the ſorts.*	Cruſado's	or Ducat with the † of *Portugal*	22	1	105	2	$6\frac{6}{7}$	2	6	0	6	0	0	8	1
		or Ducat with the ✠ of *Portugal*	22	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	2	0	8	$3\frac{1}{4}$
		Great Cruſado, or the *Portuguiſe* of *Emanuel* of *Portugal*	23	3	$10\frac{1}{2}$	22	$20\frac{4}{7}$	22	16	3	8	0	4	6	$4\frac{1}{4}$
		Joannes Great Cruſado	22	3	$10\frac{1}{2}$	22	$20\frac{4}{7}$	22	16	3	5	0	4	2	$8\frac{1}{2}$
		Dublion of *Spain*	—	—	—	—	—	—	—	0	14	6	0	15	2
Du-cats *the ſorts.*	Ducats of	Albertus, *or* Albertines — Single	23	$3\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	8
		Albertus, *or* Albertines — Double	23	$3\frac{1}{2}$	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	17	4
		Albertus of *Auſtria* — Single	23	3	$78\frac{3}{4}$	3	$1\frac{1}{7}$	3	0	0	9	0	0	11	$5\frac{3}{4}$
		Albertus of *Auſtria* — Double	23	3	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	6	0	17	$3\frac{1}{4}$
		$\frac{2}{3}$ parts of the ſame double Duc.	23	3	$70\frac{1}{2}$	3	$9\frac{33}{47}$	3	9	0	10	0	0	12	$10\frac{1}{4}$
		$\frac{1}{3}$ part of the ſame double Duc.	23	3	126	1	$21\frac{5}{7}$	1	$21\frac{1}{2}$	0	5	7	0	7	2
		Arragon	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
		Barbary and elſewhere in *Turkey*. — Some	23	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	4	0	8	$3\frac{1}{2}$
		Barbary and elſewhere in *Turkey*. — Others	23	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	6	0	8	$6\frac{1}{4}$
		Batenborg with the †	19	0	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	5	2	0	6	11
		Biſhops Ducat	23	$0\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	3	0	8	$5\frac{1}{2}$
		Caſtile	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
		Denmark	20	0	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	4	0	7	2
		Emannuel of *Portugal*	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
		Ferdinand of *Batenborg*	19	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	2	0	6	$10\frac{1}{4}$
		Ferdinand and *Carolus* of *Horne*	18	0	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	4	10	0	6	$5\frac{1}{4}$
		Florence	23	1	108	2	$5\frac{1}{3}$	2	5	0	6	4	0	8	$2\frac{1}{2}$
		George Rechem	21	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	1	0	7	$9\frac{1}{2}$
		Guelders	23	1	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	3	0	8	4
		Gulielmus of *Batenborg*	21	3	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	4	0	15	$9\frac{3}{4}$
		Hamborough	—	—	—	—	—	—	—	0	7	2	0	8	9
		Holland	23	2	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	5	0	8	$6\frac{1}{4}$
		Hungary, or Half Noble	23	$3\frac{1}{6}$	$113\frac{1}{2}$	2	$2\frac{170}{227}$	2	$2\frac{1}{2}$	0	6	4	0	8	0
		Other *Hungary* Ducats	23	1	$104\frac{1}{2}$	2	$7\frac{25}{209}$	2	7	0	6	4	0	8	$5\frac{3}{4}$
		Italy — Some as *Venice*													
		Italy — others	23	1	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	3	0	8	4
		Majorca	23	1	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	16	$10\frac{1}{4}$
		Marie of *Batenborgh*	20	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	4	0	7	$2\frac{1}{2}$
		Navarre, and ſome others, as *Majorca*													
		Nimmeghen with *Stephen*	21	1	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	0	0	15	$5\frac{1}{4}$
		Nimmeghen of 1565	18	2	108	2	$5\frac{1}{3}$	2	5	0	4	10	0	6	$6\frac{1}{2}$

	Foreign Gold.	Fine		Pieces to the	Weight by *Malines.*		Weight by others.		Old Value.			New Value.		
		Car.	Gr.	℔ Troy.	pwts.	gr.	pwts.	gr.	l.	s.	d.	l.	s.	d.
Ducats of	*Oswald* Ducat *Cuſa*	19	$0\frac{1}{2}$	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	2	0	6	$10\frac{1}{4}$
	Pancratius Alleb. H. as *Oſwald*													
	Peter Rechem, as *Geo. Rechem*													
	Portugal, ſee *Cruſados, Milreys*													
	Rome Single Some	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
	Others	23	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	6	0	8	$6\frac{1}{4}$
	Double	23	3	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	17	3
	St. Victor Rancratius, as *George Rechem*													
	Spain Single Some	23	1	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$5\frac{1}{4}$
	Others	23	2	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$6\frac{1}{2}$
	Double	23	2	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	13	0	0	17	1
	Great	22	0	24	10	0	10	0	1	10	0	1	15	0
	States of the United Prov. with Letters	22	0	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	4	0	16	0
	The Half thereof	22	0	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	2	0	8	0
	Stephanus of *Batenborg*	19	$0\frac{1}{2}$	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	10	5	0	13	$10\frac{3}{4}$
	Suevia	23	1	$104\frac{1}{2}$	2	$7\frac{25}{209}$	2	7	0	6	4	0	8	$5\frac{3}{4}$
	Valence	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
	Venice	23	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	6	0	8	$6\frac{1}{4}$
	Victor Batenborgh, as *Geo. Rechem*													
	Victor H.B. as *Marie* of *Batenborg*													
	W.B. Margaret Toren	21	3	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	6	1	0	7	$9\frac{1}{2}$
	Water Ducats, as *Marie* of *Batenborg*													
	Zeland Single	23	$0\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	3	0	8	$5\frac{1}{2}$
	Double	23	$0\frac{1}{2}$	$52\frac{1}{2}$	4	$13\frac{5}{7}$	4	13	0	12	6	0	16	11
	Ducats with the *Checquer,* as *Denmark*													
	Floret of *France*	22	0	$100\frac{1}{2}$	2	$9\frac{21}{67}$	2	9	0	6	5	0	8	$4\frac{1}{4}$
	The *New Floret,* See *Gilden St. Andrew*													
	Golden Fleece, or *Toyſon d'or*	23	$3\frac{3}{4}$	$81\frac{1}{6}$	2	$22\frac{470}{487}$	2	$22\frac{3}{4}$	0	9	2	0	11	$3\frac{1}{4}$
	Gold Guilder, or *Guildren*	18	3	$112\frac{1}{2}$	2	$3\frac{1}{5}$	2	6	0	4	9	0	6	$4\frac{1}{4}$
Guilders of	*St. Andrew* *Old*	18	1	108	2	$5\frac{1}{3}$	2	$3\frac{1}{4}$	0	4	10	0	6	$5\frac{1}{4}$
	New, or *Floret*	18	3	108	2	$5\frac{1}{3}$	2	3	0	5	0	0	6	$7\frac{1}{2}$
	Arnoldus	12	0	138	1	$17\frac{17}{23}$	1	$17\frac{1}{2}$	0	2	7	0	3	$3\frac{3}{4}$
	Carolus	14	0	126	1	$21\frac{5}{7}$	1	$21\frac{1}{8}$	0	3	6	0	4	$2\frac{1}{4}$
	Clemmer	13	0	114	2	$2\frac{10}{19}$	2	2	0	3	6	0	4	$4\frac{1}{4}$
	Collen	17	3	114	2	$2\frac{10}{19}$	2	$2\frac{3}{4}$	0	4	8	0	5	$11\frac{1}{4}$
	David of The *Harp*	15	0	114	2	$2\frac{10}{19}$	2	2	0	4	0	0	5	$0\frac{1}{4}$
	Triers	17	2	114	2	$2\frac{10}{19}$	2	2	0	4	8	0	5	$10\frac{1}{4}$
	Utrecht	16	0	114	2	$2\frac{10}{19}$	2	2	0	4	3	0	5	$4\frac{1}{4}$
	Frederick of *Bavaria*	14	0	117	2	$1\frac{3}{13}$	2	1	0	3	8	0	4	$6\frac{3}{4}$
	Gulielmus	18	1	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	6	$5\frac{1}{4}$
	Horne	—	—	$158\frac{3}{4}$	1	$12\frac{36}{127}$	1	$12\frac{1}{4}$	0	4	11	0	6	4
	Joannes	16	0	$109\frac{1}{2}$	2	$4\frac{44}{73}$	2	$4\frac{1}{2}$	0	4	6	0	5	7
	Peter of *Louvaine*	17	$0\frac{1}{2}$	114	2	$2\frac{10}{19}$	2	2	0	4	5	0	5	$8\frac{3}{4}$
	Philip	15	3	111	2	$3\frac{33}{37}$	2	$3\frac{1}{4}$	0	4	2	0	5	5
	The Half thereof	15	3	222	1	$1\frac{35}{37}$	1	$1\frac{5}{8}$	0	2	1	0	2	$8\frac{1}{2}$
	Reniſh Guilder	22	0	$102\frac{3}{4}$	2	$8\frac{24}{411}$	2	8	0	6	9	0	8	2
	Saxon	17	3	113	2	$2\frac{110}{113}$	2	$2\frac{3}{4}$	0	4	8	0	5	$11\frac{3}{4}$
	States of the United Provinces	20	0	$120\frac{3}{4}$	1	$23\frac{113}{161}$	1	$23\frac{1}{2}$	0	4	8	0	6	$3\frac{3}{4}$
	Lions, Golden Lion of *Flanders*	23	3	$89\frac{1}{4}$	2	$16\frac{64}{119}$	2	$16\frac{1}{2}$	0	7	8	0	10	$1\frac{3}{4}$
	$\frac{2}{3}$ parts thereof	23	3	$133\frac{7}{8}$	1	$19\frac{3}{119}$	1	19	0	4	11	0	6	9
	$\frac{1}{3}$ part thereof	23	3	$267\frac{3}{4}$	0	$21\frac{61}{119}$	0	$21\frac{1}{2}$	0	2	5	0	3	$4\frac{1}{2}$
	Louiſſes, of *Louis* 13*th* and 14*th* of *France*	22	0	54	4	$10\frac{2}{3}$	4	8	0	15	0	0	15	$6\frac{1}{2}$
	The Half thereof	22	0	108	2	$5\frac{1}{3}$	2	4	0	7	6	0	7	$9\frac{1}{4}$
Markes and Milreys	Marke of *Bohemia*	—	—	—	—	—	—	—	0	6	0	0	7	6
	20 Markes of *Scotland*	22	0	36	6	16	6	10	1	2	0	1	3	4
	10 Markes of *Scotland*	22	0	72	3	8	3	5	0	11	0	0	11	8
	5 Markes of *Scotland*	22	0	144	1	16	1	14	0	5	6	0	5	10
	6 Markes of *Suevia*	—	—	—	—	—	—	—	0	4	9	0	6	0
	Milreys or Ducat of *Portugal*	22	1	48	5	0	4	20	0	13	4	0	17	$8\frac{1}{4}$
	Halfe Milreys	22	1	96	2	12	2	10	0	6	8	0	8	10
	Conterfeit Milreys	21	0	48	5	0	4	20	0	12	6	0	16	$8\frac{1}{2}$

Ducats.

Guilders the *ſorts.*

Mark *and* Milreys.

Nobles

		Foreign Gold.	Fine		Pieces to the	Weight by *Malines.*		Weight by others.		Old Value.			New Value.		
			Car.	*Gr.*	℔ *Troy.*	*pwts.*	*gr.*	*pwts.*	*gr.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*
Nobles *the ſorts.*	Nobles of	*Bridges*	23	0	$88\frac{1}{2}$	2	$17\frac{5}{59}$	2	17	0	7	4	0	9	11
		Flanders, or *Flemiſh* Noble	23	0	54	4	$10\frac{2}{3}$	4	10	0	12	0	0	16	3
		Half *Flemiſh* Noble	23	0	108	2	$5\frac{1}{3}$	2	5	0	6	0	0	8	$1\frac{1}{2}$
		Gaunt	23	0	54	4	$10\frac{2}{3}$	4	10	0	12	0	0	16	3
		Half Noble, as the *Hungary* Ducat													
		Half Noble with the *Lyon*, as *Bridges*													
		Henry Noble of *France*	22	0	51	4	$16\frac{16}{17}$	4	16	0	13	4	0	16	$5\frac{1}{2}$
		The Half thereof	22	$2\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	6	8	0	7	$7\frac{1}{4}$
		Hollan	23	3	48	5	0	4	20	0	14	3	0	18	$10\frac{1}{4}$
		Overyſſel and *Utrecht*, as *Holland*													
		Zeland, as *Gaunt*													
		Pezo of *Peru*, by *Heylin*, p. 1064								0	6	6	0	8	0
Piſtolets *the ſorts.*	Piſtolets of	*De Lege & Legion*	18	0	108	2	$5\frac{1}{3}$	2	5	0	4	9	0	6	$4\frac{1}{4}$
		Italy Some	21	$2\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	5	9	0	6	$7\frac{1}{2}$
		Others	22	0	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	7	$9\frac{1}{4}$
		Scotland	19	$2\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	5	2	0	6	$11\frac{1}{4}$
		Spaine Single Some	21	$3\frac{1}{2}$	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	7	$9\frac{1}{4}$
		Others	22	0	108	2	$5\frac{1}{3}$	2	5	0	5	10	0	7	$9\frac{1}{4}$
		Double	22	0	54	4	$10\frac{2}{3}$	4	8	0	11	8	0	15	$6\frac{1}{2}$
		Of 26 Ryals	22	0	45	5	8	5	6	0	14	0	0	18	8
		Portuguiſe, See Great Cruſado of *Portugal*													
Poſtulats *the ſorts.*	Poſtulats of	*Bourbon*	12	$0\frac{1}{2}$	$136\frac{1}{2}$	1	$18\frac{18}{91}$	1	18	0	2	7	0	3	$4\frac{1}{4}$
		Cleves	9	$0\frac{1}{2}$	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	9	0	2	$2\frac{1}{4}$
		Dog and Cat, as *Bourbon*													
		Fran. Frier	9	0	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	8	0	2	$2\frac{1}{2}$
		Horne	10	$0\frac{1}{2}$	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	11	0	2	$5\frac{1}{4}$
		Juliers, or *Guliers*	9	3	156	1	$12\frac{12}{13}$	1	$12\frac{3}{4}$	0	1	10	0	2	$4\frac{1}{2}$
	Pounds	6 Pound *Scotch*	22	0	80	3	0	3	0	0	10	0	0	10	6
		12 Pound *Scotch*	22	0	40	6	0	6	0	1	0	0	1	1	0
Riders *the ſorts.*	Ryders of	*Burgundy*	23	$0\frac{1}{2}$	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	3	0	8	$5\frac{1}{2}$
		Campen and *Swoll*	12	3	114	2	$2\frac{10}{19}$	2	2	0	3	2	0	4	$3\frac{1}{4}$
		Deventer, as *Campen* and *Swoll*													
		Flanders	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	6	0	8	$7\frac{1}{2}$
		Frieſland *Guelders* of the Year 1583	21	0	108	2	$5\frac{1}{3}$	2	5	0	5	6	0	7	5
		Guelders Ryder	14	0	114	2	$2\frac{10}{19}$	2	2	0	3	6	0	4	$8\frac{1}{4}$
		Guelders new Ryder	23	3	105	2	$6\frac{6}{7}$	2	$6\frac{1}{2}$	0	6	9	0	8	$7\frac{1}{2}$
		Philip Clincart	14	0	114	2	$2\frac{10}{19}$	2	2	0	3	6	0	4	$8\frac{1}{4}$
		Ryder with the Loaves	10	$0\frac{1}{2}$	114	2	$6\frac{10}{19}$	2	2	0	2	7	0	3	4
		Scotland Some others	19	$2\frac{1}{2}$											
		Others	22	0											
		States of the United Provinces	22	0	36	6	16	6	12	1	0	0	1	3	4
		The Half thereof	22	0	72	3	8	3	6	0	10	0	0	11	8
	Ruble of	*Muſcovie* (by *Heylin* 13 *s.* 4 *d.*)	—	—	—	—	—	—	—	0	10	0	0	13	4
		Poland	—	—	—	—	—	—	—	0	13	4	0	13	4
Royals *the ſorts.*	Royals or Reals of	*Auſtria* Single	23	$3\frac{1}{2}$	$49\frac{1}{2}$	4	$20\frac{4}{11}$	4	20	0	14	4	0	18	5
		Double	23	$3\frac{1}{2}$	$24\frac{3}{4}$	9	$16\frac{8}{11}$	9	16	1	8	8	1	16	10
		Campen and *Swoll*	23	0	$40\frac{1}{2}$	4	$20\frac{4}{11}$	4	20	0	14	2	0	17	9
		The Half thereof	23	0	99	2	$10\frac{2}{11}$	2	10	0	7	1	0	8	$10\frac{1}{2}$
		Flanders, or the Key	23	0	$69\frac{1}{4}$	3	$10\frac{18}{31}$	3	$10\frac{1}{2}$	0	10	0	0	12	$6\frac{1}{4}$
		The Half thereof	23	0	$139\frac{1}{2}$	1	$17\frac{9}{31}$	1	$17\frac{1}{4}$	0	5	0	0	6	$3\frac{1}{4}$
		Imperial Loyal	23	$3\frac{1}{2}$	69	3	$11\frac{11}{23}$	3	11	0	11	0	0	13	$2\frac{1}{2}$
		The Half thereof	18	0	$105\frac{1}{3}$	2	$6\frac{54}{79}$	2	$6\frac{1}{2}$	0	4	11	0	6	$6\frac{1}{4}$
		Philip with the *Spread Eagle*	18	2	$106\frac{1}{2}$	2	$6\frac{6}{71}$	2	6	0	5	0	0	6	$7\frac{1}{2}$
		Philip of *Spain*	23	0	$69\frac{3}{4}$	3	$10\frac{18}{31}$	3	$10\frac{1}{2}$	0	10	0	0	12	$6\frac{1}{4}$
		Ship of *Flanders*, or *Schuytken*	22	1	$109\frac{1}{2}$	2	$4\frac{44}{73}$	2	$4\frac{1}{2}$	0	6	3	0	7	9
		Shock of *Bohemia*, ſome ſay 8 *s.* ſome	—	—	—	—	—	—	—	0	9	0	0	9	0
		Stiver Pieces, 9 Stiv. Pieces of *Batenborg* and *Frize*	7	1	176	1	$8\frac{8}{11}$	1	$8\frac{1}{2}$	0	1	3		6	$6\frac{1}{4}$
		Sultanies of 120 *Aſpers*	23	1	90	2	15	2	16	0	7	6	0	9	$10\frac{1}{4}$

Foreign Gold.	Fine *Car.*	Fine *Gr.*	Pieces to the ℔ *Troy.*	Weight by *Malines.* *pwts.*	Weight by *Malines.* *gr.*	Weight by others. *pwts.*	Weight by others. *gr.*	Old Value. *l.*	Old Value. *s.*	Old Value. *d.*	New Value. *l.*	New Value. *s.*	New Value. *d.*
Tabe of *China*								0	5	$4\frac{1}{2}$	0	7	2
Toman of *Persia*, by *Heylin*, *p.* 839. valued at 20 Crowns, seemeth not to be any Coine, but a denomination used in Accompt								5	0	0	5	0	0
Unicorn of *Scotland*	22	0	$99\frac{1}{4}$	2	$10\frac{14}{397}$	2	10	0	6	0	0	8	$5\frac{1}{2}$
Xeriffe of *Goa* in *India*, by *Heylin* by *Grimstone* p. 197. worth 300 *Res* of *Portugal*								0	6	0	0	7	6
Zechines, or *Checkeenes* of *Venice*	23	1	90	2	16	2	$15\frac{3}{4}$	0	7	6	0	9	$10\frac{1}{4}$

	Foreign Silver.	Fine ℥	Fine *pwts.*	Pieces to the ℔*. Troy.*	Weight of the Pieces ℥.	Weight of the Pieces *pwts.*	Weight of the Pieces *gr.*	*Sterling* Value. *s.*	*Sterling* Value. *d.*	Value by some Authors. *s.*	Value by some Authors. *d.*	*Foreign Silver Coines.*
Albi of	*Bamberg*	4	18	273	0	0	$21\frac{9}{19}$	0	1 +	0	1	
	Collen — Some	5	10	345	0	0	$16\frac{16}{23}$	0	1 +	0	1	Albi, *or* Albes.
	Others	5	10	342	0	0	$16\frac{16}{19}$	0	1 +	0	1	
	Others	5	10	179	0	1	$8\frac{32}{179}$	0	2 –	0	2	
	Frankford, or *Bambergh*											
	Mentz, as *Collen*											
	Norenbergh and *Palatine* of the *Rhine* — as *Bambergh.*											
	Trier, as *Collen*											
	Alteen, or *Atten* of *Muscovia*									0	4 +	
	Altine of *Poland*									0	$4\frac{1}{2}$	
	Angel of *Scrikelborg*	10	$7\frac{1}{2}$	$78\frac{3}{4}$	0	3	$1\frac{1}{7}$	0	$8\frac{1}{2}$ +	0	8	
	Asper of *Turkie*									0	$1\frac{1}{4}$	
	Babee of *Scotland*									0	$0\frac{1}{2}$	
Batz of 4 Creutzes.	*Bavaria* *Brandenburgh* *Colmograve* *Cost.* of 1530. *Friburgh* *Ottinge* *Raynsburgh* *Roy* *Scafhuysen* *Taunte*	5	7	$109\frac{1}{2}$	0	2	$4\frac{44}{73}$	0	3 +	0	3	*Batz of several sorts.*
	Kemptor half *Batz*	4	$12\frac{1}{2}$	$192\frac{1}{2}$	0	1	$5\frac{71}{77}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{2}$	
	Munchien half *Batz*	4	$12\frac{1}{2}$	186	0	1	$6\frac{30}{31}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{2}$	
	3 *Batz*, See *Snaphane*											
	Bemesh, or *Bemish* of *Switz*									0	$2\frac{1}{2}$	
	Bianco, or *Biamco* of *Italy*									0	8	
	Blanks									0	$0\frac{3}{4}$	
	Half *Ruyters* Blank of *Holland*	3	0	144	0	1	16	0	$1\frac{1}{4}$ +	0	$1\frac{1}{4}$	
	Boligneo									0	$0\frac{3}{4}$	
	Carlini of *Italy*									0	6	
	Carolus Guilder, as $\frac{2}{3}$ of the *Philips* Dollar											
3 Carolus of	*Carolus* and *Salsburgh* *Campidona* *Ernestus* *Frankford* *Ottingus* *Patavia* *Reynsborgh*	9	0	$78\frac{1}{4}$	0	3	$1\frac{1}{7}$	0	$7\frac{1}{4}$ +	0	7	*Pieces of 3. Carolus.*
	Causleto, or *Caveleto* of *Italy*									0	$3\frac{1}{4}$	

	Foreign Silver.	Fine ℥.	Fine pwts.	Pieces to the ℔. Troy.	Weight of the Pieces ℥.	Weight of the Pieces pwts.	Weight of the Pieces gr.	*Sterling* Value, s.	*Sterling* Value, d.	Value by others. s.	Value by others. d.
Creutz. *of divers ſorts.*	*Creuciat* of *John* of *Cleve*	8	7	$39\frac{3}{4}$	0	6	$0\frac{48}{53}$	1	$1\frac{1}{2}$+	1	1
Creuciats, Creutzers, or Croſſes.	of *Ausburge* and *Ulme*	5	5	384	0	0	15	0	$0\frac{3}{4}$+	0	$0\frac{3}{4}$
	of *Poland*									0	$0\frac{3}{4}$
	12 *Creutzers* of *Bavaria* *Vienna*	8	$7\frac{1}{2}$	57	0	4	$5\frac{1}{19}$	0	$9\frac{1}{2}$+	0	9
	other 12 *Creutzer* Pieces	8	$7\frac{1}{2}$	$61\frac{1}{2}$	0	3	$21\frac{27}{41}$	0	$8\frac{3}{4}$+	0	9
	And ſome	10	10	ſame		ſame		0	11 +	0	11
	10 *Creutzers* of *Friſe* *Ravenburg* *Salsburg* *Saxony*	8	7	$64\frac{1}{2}$	0	3	$17\frac{13}{43}$	0	$8\frac{1}{4}$+	0	8
	6 *Creutzers* of *Insburgh*	10	10	$124\frac{1}{2}$	0	1	$22\frac{22}{83}$	0	$5\frac{1}{2}$—	0	5
	Vienna	8	$7\frac{1}{2}$	114	0	2	$2\frac{10}{19}$	0	$4\frac{3}{4}$+	0	5
	other 6 *Creutzer* Pieces	10	10	123	0	1	$22\frac{34}{41}$	0	$5\frac{1}{2}$+	0	5
	3 *Creutzers* of *Bavaria*	4	8	375	0	0	$15\frac{9}{17}$	0	$0\frac{3}{4}$+	0	$0\frac{3}{4}$
	Vienna	4	$8\frac{1}{3}$	129	0	1	$20\frac{28}{43}$	0	$2\frac{1}{4}$—	0	2
	other 3 *Creutzer* Pieces	5	10	$136\frac{1}{2}$	0	1	$18\frac{18}{91}$	0	$2\frac{1}{2}$+	0	2
	2 *Croſſes* and *Harpe*	4	0	180	0	1	8	0	$1\frac{1}{4}$+	0	$1\frac{1}{4}$
	Crowns of *France*, See *Louis*										
	Italy									5	0
	Turky									6	0
	Cupſtoke									1	0
	Deghen, or *Denghen* of *Muſcovia* & *de Narde* *Ruſſia*	11	13	$545\frac{1}{3}$	0	0	$10\frac{230}{409}$	0	$1\frac{1}{4}$+	0	$1\frac{1}{4}$
	Denier, *Petit Denier* of *Paris*	1	10	270	0	0	$21\frac{1}{3}$	0	$0\frac{1}{4}$+	not currant in *England*.	
	Tor	1	10	$337\frac{1}{2}$	0	0	$17\frac{1}{15}$	0	$0\frac{1}{4}$+		
	Dicken of a *Wing*									1	4
Dollars *of divers ſorts.* Dollars or Dollers of	*Albania*, or the *Croſs Dollar*									4	8
	Baſil of 6 *Creutzers*	10	$13\frac{1}{2}$	15	0	16	0	3	10+	3	10
	Batenburgh *Bohemia. Ne.Op.* *Bommel*	7	15	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	3	$1\frac{1}{4}$	3	1
	Briſgau	10	15	15	0	16	0	3	$10\frac{1}{2}$—	3	10
	Cambray, The Sixteenth part	6	10	123	0	1	$22\frac{34}{41}$	0	$3\frac{1}{4}$+	0	3
	Chriſtopher, 45.	10	10	$12\frac{3}{4}$	0	18	$19\frac{13}{17}$	4	$5\frac{1}{4}$+	4	5
	Friſland of 1601.	9	0	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	3	$7\frac{1}{4}$—	3	7
	See *Guelders*										
	Ryckx Dollar Ouncia	11	5	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$10\frac{1}{4}$+	5	0
	Germany or *Ryckx Dollar* of 1567	10	12	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	7 +	4	7
	Others	10	13	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$7\frac{1}{4}$+	4	7
	Others	10	14	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$7\frac{1}{2}$+	4	7
	Others	11	0	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	2 +	4	9
	Others	11	3	$12\frac{1}{2}$	0	19	$4\frac{4}{5}$	4	$9\frac{3}{4}$+	4	10
	Guelders and *Friſland* Some	10	4	$14\frac{1}{2}$	0	16	$20\frac{4}{19}$	3	$10\frac{1}{4}$+	3	10
	Others	9	0	$12\frac{3}{64}$	0	19	$22\frac{234}{257}$	4	$0\frac{1}{4}$+	4	0
	Guelders and *Utrecht* Some	9	0	15	0	16	0	3	$2\frac{3}{4}$+	3	3
	Others	10	10	13	0	18	$11\frac{1}{13}$	4	$4\frac{1}{4}$+	4	4
	Others	10	12	13	0	11	$11\frac{1}{13}$	4	$4\frac{3}{4}$+	4	4
	See *Zutphen*										
	Gulielmus of *Sweden*	10	10	$12\frac{3}{4}$	0	18	$19\frac{13}{17}$	4	$5\frac{1}{4}$+	4	5
	Guſtavus of *Liege* the 38*th.*	10	4	$12\frac{3}{4}$	0	18	$19\frac{13}{17}$	4	$3\frac{3}{4}$+	4	4
	Holland	9	0	$13\frac{1}{3}$	0	18	0	3	$7\frac{3}{4}$	3	8
	Holland with the *Crown*	8	0	$13\frac{1}{3}$	0	18	0	3	$2\frac{3}{4}$	3	3
	Iſmenſen, as *Baſil*										
	Luneburgh	10	$16\frac{1}{2}$	15	0	16	0	3	$10\frac{3}{4}$	3	10
	Phillip	10	0	$10\frac{5}{7}$	1	2	$9\frac{3}{5}$	5	$0\frac{1}{2}$+	5	0

Dollar

	Foreign Silver.	Fine ℥	pwts.	Pieces to the ℔ *Troy.*	Weight of the Pieces ℥.	*pwts.*	*gr.*	*Sterling* Value, *s.*	*d.*		Value by others. *s.*	*d.*
Dollar, or Dollers of	Half of the *Phillips Dollar*	10	0	$21\frac{3}{7}$	0	11	$4\frac{4}{5}$	2	$6\frac{1}{4}$	+	2	6
	Fourth Part	10	0	$42\frac{6}{7}$	0	5	$14\frac{2}{5}$	1	3	+	1	3
	Fifth Part	10	0	$53\frac{4}{7}$	0	4	$11\frac{13}{25}$	1	0	+	1	0
	Tenth Part	10	0	$107\frac{1}{7}$	0	2	$5\frac{19}{25}$	0	6	+	0	6
	Twentieth Part	5	0	$107\frac{1}{4}$	0	2	$5\frac{101}{143}$	0	3	+	0	3
	Fortieth Part	5	0	$214\frac{1}{2}$	0	1	$2\frac{122}{143}$	0	$1\frac{1}{2}$	+	0	$1\frac{1}{2}$
	Two third Parts of the ſame *Dollar*	10	0	$16\frac{1}{4}$	0	14	$22\frac{2}{5}$	3	4	+	3	4
	Poland Some as *Batenborgh*											
	Others of 60 *Creutzers*	11	$3\frac{1}{2}$	15	0	16	0	4	$0\frac{1}{4}$	+	4	0
	Prince of Orange or *Lyon Dollar*	9	0	13	0	18	$11\frac{1}{13}$	3	$8\frac{3}{4}$	+	4	0
	Reynsburgh, as *Baſil*											
	Riga	10	$2\frac{1}{2}$	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	4	$0\frac{1}{2}$	+	4	0
	Scotland with the *Croſs Daggers*	11	2	$11\frac{3}{4}$	1	0	$10\frac{10}{47}$	5	$1\frac{1}{4}$	+	5	0
	States General of the United Provinces	9	0	$12\frac{3}{64}$	0	19	$22\frac{34}{257}$	4	$0\frac{1}{4}$	+	4	0
	Succia, or Merchants *Dollar*										3	2
	Ryckx, or *Imperial Dollar*										5	2
	Tremone, as *Briſgau*											
	Utrecht, See *Guilders*											
	Zeland, with the *Eagles*	9	0	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	3	$7\frac{1}{4}$	—	3	7
	Zutphen and *Guilders* of 1586	10	4	$13\frac{2}{3}$	0	17	$10\frac{19}{41}$	4	$0\frac{1}{4}$	+	4	0
	Drier										0	$0\frac{3}{4}$
	Duplus	2	0	324	0	0	$17\frac{7}{9}$	0	$0\frac{1}{4}$	+	not currant	
	Dupli Simple	5	10	882	0	0	$6\frac{26}{49}$	0	$0\frac{1}{4}$	+	in *England*.	
	Dupli Mavi $\frac{1}{9}$ of *Gulicl.* of *Turing*	2	15	440	9	6	$13\frac{1}{11}$	0	$0\frac{1}{4}$	+	nor this.	
Duyts	9 *Duyts Peny* of *Charles* and *Philip*	4	14	129	0	1	$20\frac{28}{43}$	0	$2\frac{1}{4}$	+	0	$2\frac{1}{4}$
	11 *Duyts* of *Charles-Limburg*	4	15	120	0	2	0	0	$2\frac{1}{2}$	+	0	$2\frac{1}{2}$
	Holland	6	0	144	0	1	16	0	$2\frac{1}{2}$	+	0	$2\frac{1}{2}$
	Philip and *Marie*	11	$3\frac{1}{2}$	270	0	0	$21\frac{1}{3}$	0	$2\frac{1}{2}$	+	0	$2\frac{1}{2}$
	17 *Duyts*	10	10	147	0	1	$15\frac{2}{49}$	0	$4\frac{1}{2}$	+	0	$4\frac{1}{2}$
	17 *Duyts* of *Charles* *Guilders* *Liege* *Limburgh* *Lodouicke* *Philip* *Philip* of *Flanders* Two *Standing Lions*	9	10	145	0	1	$15\frac{21}{29}$	0	$4\frac{1}{4}$	—	0	$4\frac{1}{4}$
	17 *Duyts* of *Sluce*	9	5	148	0	1	$14\frac{34}{47}$	0	4	+	0	4
	Flabes in the *Low Countries*										1	4
	Finferkin										0	$0\frac{1}{2}$
	Fleece, See *Stivers.*											
	Florius, by *Heylin*										3	0
	Franks of *Turky*										2	0
	Franks of *France*, 3 to a Crown	10	0	$26\frac{1}{4}$	0	9	$3\frac{3}{7}$	2	$0\frac{1}{2}$	+	2	0
	Gagatta of *Italy*										0	1
	Gnibij of *Rome*										0	6
	Grot, or *Groot*										0	$1\frac{1}{4}$
Groots	3 *Groots* or *Deniers*	5	10	$117\frac{3}{4}$	0	2	$0\frac{144}{157}$	0	3	+	0	3
	5 *Groots* of *Flanders*	10	$6\frac{1}{2}$	$146\frac{1}{2}$	0	1	$15\frac{93}{293}$	0	$4\frac{1}{2}$	+	0	$4\frac{1}{2}$
	Gaunt	5	13	145	0	1	$15\frac{21}{29}$	0	$2\frac{1}{2}$	+	0	$2\frac{1}{2}$
	Philip of *Flanders*	11	3	135	0	1	$18\frac{2}{3}$	0	$5\frac{1}{4}$	+	0	$5\frac{1}{4}$
	Others	10	14	135	0	1	$18\frac{2}{3}$	0	5	+	0	5
	$5\frac{1}{2}$ *Groots* of 1520 *Ma. Flanders*	9	14	120	0	2	0	0	$5\frac{1}{4}$	—	0	$5\frac{1}{4}$

Dollars of ſeveral ſorts.

Duyts of ſeveral ſorts.

Groots of divers ſorts.

Groſh

		Foreign Silver.	Fine ℥	*pwts.*	Pieces to the ℔ *Troy.*	Weight of the Pieces ℥.	*pwts.*	*gr.*	*Sterling* Value, *s.*	*d.*	Value by others. *s.*	*d.*
Gross *of divers sorts.*	Grosh, or Gross of	*Ambaſs*	4	$12\frac{1}{2}$	$94\frac{1}{2}$	0	2	$2\frac{20}{21}$	0	3 +	0	3
		Ausburgh Some	5	7	108	0	2	$5\frac{1}{3}$	0	3 —	0	3
		Ausburgh Others	6	$4\frac{1}{2}$	155	0	1	$13\frac{5}{31}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
		Ausburgh Others of 3 *Batz*									0	9
		Baſil	9	0	$118\frac{1}{2}$	0	2	$0\frac{48}{79}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
		Baſſaw	5	7	$116\frac{1}{2}$	0	2	$6\frac{6}{71}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
		Bohemia, $1\frac{1}{2}$ Silver Groſh	3	$7\frac{1}{2}$	87	0	2	$18\frac{6}{29}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
		Briſau, as *Baſſaw*										
		Briſgrave as *Baſil*										
		Campido as *Baſil*										
		Corinthia as *Baſil*										
		Coningſtein	5	7	108	0	2	$5\frac{1}{3}$	0	3 +	0	3
		Curienſis, as *Baſſaw*										
		Duodena, or the 12. par. of the Sil. Groſh	3	$3\frac{2}{3}$	$874\frac{1}{2}$	0	0	$6\frac{342}{583}$	0	$0\frac{1}{4}$ —	not currant in *England.*	
		Ferdinando of *Dantſicke*	5	0	180	0	1	8	0	$1\frac{3}{4}$ +	0	2
		George and *Wormeſer,* as *Ambaſs*										
		Kempton, as *Baſſaw*										
		Markegrave, as *Ambaſs*										
		Mary									0	$1\frac{1}{4}$
		Melvin 3 *Groſh* of 1340	10	4	138	0	1	$17\frac{17}{23}$	0	$4\frac{3}{4}$ +	0	5
		Melvin 3 *Groſh* Others	10	10	138	0	1	$17\frac{17}{23}$	0	$4\frac{3}{4}$ +	0	5
		Meyſen									0	$2\frac{3}{4}$ or 3d
		Noiling, as *Ambaſs*										
		Poland									0	$1\frac{1}{2}$
		Poland Six *Groſh*	6	0	$13\frac{7}{8}$	0	17	$7\frac{5}{37}$	2	4 +	2	4
		Prague	9	$12\frac{1}{2}$	180	0	1	8	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
		Pruſſia, 3 *Groſh alb.*	10	$10\frac{1}{2}$	138	0	1	$17\frac{17}{23}$	0	$4\frac{3}{4}$ +	0	5
		Reynſburgh	6	$4\frac{1}{2}$	155	0	1	$13\frac{5}{31}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
		Salsburgh Some	6	$2\frac{2}{5}$	$118\frac{1}{2}$	0	2	$0\frac{48}{79}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
		Salsburgh Others	4	$12\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	0	$7\frac{1}{2}$ +	0	$7\frac{1}{2}$
		Saxony, as *Coningſtein*										
		Scafhuyſen, as *Baſil*										
		Sigiſmund of 1532, and 1535	10	4	69	0	3	$11\frac{11}{23}$	0	$9\frac{1}{2}$ +	0	$9\frac{1}{2}$
		Sigiſmund of *Pruſſia* 1534	10	11	69	0	3	$11\frac{11}{23}$	0	$9\frac{3}{4}$ +	0	10
		Others with the Armes of *Dantſick*	10	$0\frac{1}{2}$	69	0	3	$11\frac{11}{23}$	0	$9\frac{1}{4}$ +	0	$9\frac{1}{4}$
		Silver *Groſh* Common									0	2
		Taven, as *Baſil*										
		Vienna	6	4	132	0	1	$19\frac{7}{11}$	0	3 +	0	3
		$1\frac{1}{2}$ Silver *Groſh*	3	$7\frac{1}{2}$	87	0	2	$18\frac{6}{29}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{2}$
		4 *Groſh* Peny	8	0	81	0	2	$23\frac{1}{9}$	0	$6\frac{1}{4}$ +	0	6
Guilders *the ſorts.*	Guilders, or Guldens	*Albertus* Double	10	15	$14\frac{3}{5}$	0	16	$10\frac{38}{73}$	4	0 —	4	0
		Albertus Single	10	15	$29\frac{1}{5}$	0	8	$5\frac{19}{73}$	2	0 —	2	0
		Albertus Half	10	15	$58\frac{2}{5}$	0	4	$2\frac{46}{73}$	1	0 —	1	0
		Albertus Quarter	10	15	$116\frac{4}{5}$	0	2	$1\frac{23}{73}$	0	6 —	0	6
		Carolus, as $\frac{2}{3}$ of the *Philips Doller*										
		Flanders Silver *Guilder*									2	0
		Gulielmus of *Turing*	6	15	129	0	1	$20\frac{28}{43}$	0	$3\frac{1}{4}$ +	0	$3\frac{1}{4}$
		Harp of *Ireland,* or Silver *Harp*	11	0	82	0	2	$22\frac{10}{41}$	0	$8\frac{1}{2}$ +	0	9
		Half *Harp*	11	0	164	0	1	$11\frac{5}{41}$	0	$4\frac{1}{4}$ +	0	$4\frac{1}{2}$
		Baſe *Iriſh Harp*	3	0	82	0	2	$22\frac{10}{41}$	0	$2\frac{1}{4}$ +	0	$2\frac{1}{4}$
		Old *Harp*	9	6	102	0	2	$8\frac{8}{17}$	0	$5\frac{3}{4}$ +	0	6
		Junctine, or *Juſtine* of *Italy*									1	6

Foreign Silver.	Fine ℥.	Fine pwts.	Pieces to the ℔. *Troy.*	Weight of the Pieces ℥.	pwts.	gr.	*Sterling* Value, *s.*	*d.*	Value by others. *s.*	*d.*
Lion of *Gulielmi* — Some	2	5	150	0	1	$14\frac{2}{5}$	0	$0\frac{3}{4}$ +	0	1
Others	2	5	179	0	1	$8\frac{32}{179}$	0	$0\frac{3}{4}$ +	0	1
Lieure of *France*, See Quar. Cro. by *Heylin*									2	0
Louis of *France*	11	2							4	6
Half, Quart. and Eight part accordingly										
Lyarts of *France*, *H.*	3	0							not currant in *England*.	
Lyre of *Geneva*									1	4
Lyre of *Venice*									0	9
Maille, Old *Petit Maille*	1	0	450	0	0	$12\frac{4}{5}$	0	$0\frac{1}{8}$ +	not cur. in *En.*	
Magenburgh, 3 *Armes*	5	$8\frac{1}{3}$	27	0	8	$21\frac{1}{3}$	1	1 +	1	1
Other Piece	11	$3\frac{1}{2}$	51	0	4	$16\frac{16}{17}$	1	2 +	1	2
Mark of *Denmark*									2	2
Mark of *Scotland*	11	2	54	0	4	$10\frac{2}{3}$	1	$1\frac{1}{4}$ +	1	$1\frac{1}{2}$
Half and Quarter accordingly										
Markeſticke of Lady *Mary*, *Lubeck*	10	$16\frac{2}{3}$	27	0	8	$21\frac{1}{3}$	2	2 +	2	2
Medine of *Cairo*									0	$2\frac{1}{4}$
Murſenigo									0	11
Nummi Dragme — Some	6	0	140	0	1	$17\frac{1}{7}$	0	$2\frac{3}{4}$ +	0	3
Others	6	$2\frac{1}{2}$	$118\frac{1}{2}$	0	2	$0\frac{48}{79}$	0	$3\frac{1}{4}$ +	0	3
Peny of *Bohemia* — White	5	7	924	0	0	$6\frac{18}{77}$	0	$0\frac{1}{4}$ +	not currant in *England*.	
Black	2	$13\frac{1}{2}$	990	0	0	$5\frac{9}{11}$	0	$0\frac{2}{55}$ --		
Peny of *Holland*	0	19	518	0	0	$11\frac{31}{259}$	0	$0\frac{1}{9}$ —		
Peny, called the Brass Peny	4	10	120	0	2	0	0	$2\frac{1}{4}$ +	0	2
Half thereof	4	10	240	0	1	0	0	1 +	0	1
Half *Ruyters* Black Peny	4	14	256	0	0	$22\frac{1}{2}$	0	1 +	0	1
Pſound, or *Pſound*									0	$4\frac{3}{4}$
Plappot									0	$2\frac{1}{2}$ or $2\frac{1}{4}$
Poali of *Italy*									0	6
Pound, 3 Pound of *Scotland*	11	2							5	0
Polpate, or *Baldpate* of *Scotland*	11	2							0	$10\frac{1}{2}$
Half thereof	11	2							0	$5\frac{1}{4}$
Quart. Crow. or Quartid'ecue of *France*	10	$6\frac{2}{3}$	39	0	6	$3\frac{9}{13}$	1	5 +	1	6
Lorrain	9	$8\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	1	$3\frac{1}{2}$ +	1	4
Philip, *Savoy*	10	$16\frac{1}{2}$	39	0	6	$3\frac{9}{13}$	1	6 +	1	6
Rappen Muntz									0	$2\frac{1}{2}$
Rouſtick									0	$1\frac{1}{2}$
Ryals of *Albertus* of *Auſtria*	10	15	120	0	2	0	0	$5\frac{3}{4}$ +	0	6
Half and Quarter accordingly										
Pieces of his of 3 *Ryals*	10	15	40	0	6	0	0	$5\frac{1}{4}$ +	1	6
Italy — Some	9	17	108	0	2	$5\frac{1}{3}$	0	$5\frac{3}{4}$ +	0	6
Others	9	14	108	0	2	$5\frac{1}{3}$	0	$5\frac{3}{4}$ +	0	6
Others	9	11	108	0	2	$5\frac{1}{3}$	0	$5\frac{3}{4}$ —	0	6
Mexico, 8 *Ryals*	11	0	$13\frac{2}{3}$	0	17	$13\frac{12}{41}$	4	4 +	4	4
Rome, Courſe *Ryals*	7	0	108	0	2	$5\frac{1}{3}$	0	4 +	0	4
Spain	11	$3\frac{1}{2}$	108	0	2	$5\frac{1}{3}$	0	$6\frac{1}{2}$ +	0	6
Spaniſh 8 *Ryals* called Pieces of 8	11	4	$13\frac{1}{2}$	0	17	$18\frac{2}{3}$	4	$5\frac{3}{4}$ +	4	4
States General of the United Provinces	10	0	$10\frac{5}{7}$	1	2	$9\frac{3}{5}$	5	$0\frac{1}{2}$ +	5	0
The 20*th.* part of the ſame, with the Arrows accordingly										
Venice	11	10	96	0	2	12	0	$7\frac{3}{4}$ +	0	8
Ryder of *Guelders* and *Friſland*	9	0	$12\frac{3}{64}$	0	19	$22\frac{34}{257}$	4	$0\frac{1}{4}$ +	4	0
Salvator of *Venice*	11	10	96	0	2	12	0	$7\frac{3}{4}$ +	0	8

Cardecues *the ſorts.*

Ryals *of ſeveral ſorts.*

Foreign Silver.	Fine ℥	*pwts.*	Pieces to the ℔ *Troy.*	Weight of the Pieces ℥.	*pwts.*	*gr.*	*Sterling* Value, *s.*	*d.*	Value by others. *s.*	*d.*
Saſſenars double	10	$6\frac{1}{2}$	$146\frac{1}{2}$	0	1	$15\frac{93}{293}$	0	$4\frac{1}{2}$ +	0	$4\frac{1}{2}$
Scaby of *Turky*									0	6
Schaneberger									0	$1\frac{3}{4}$
Scya of *Turky*									0	$6\frac{1}{4}$
Senube, or *S'nube* of *Bohemia*	5	7	129	0	1	$20\frac{28}{43}$	0	$2\frac{1}{2}$ +	0	$2\frac{1}{4}$
Half thereof.	5	7	258	0	0	$22\frac{14}{13}$	0	$1\frac{1}{4}$ +	0	$1\frac{1}{4}$
Seſtling									0	$0\frac{3}{4}$
Shillings *of ſeveral ſorts.* — Shillings of										
Bridges of 1582	5	0	57	0	4	$5\frac{1}{19}$	0	$5\frac{1}{2}$ +	0	5
Dantſicke									0	$0\frac{3}{4}$
8 Shilling of *Dantſicke* of 1541	10	12	156	0	1	$12\frac{12}{13}$	0	$4\frac{1}{4}$ +	0	4
Flanders									0	$7\frac{1}{2}$
Friſland of 1586	6	0	57	0	4	$5\frac{1}{19}$	0	$6\frac{3}{4}$ +	0	6
Gaunt of 1583	7	7	54	0	4	$10\frac{2}{3}$	0	$8\frac{3}{4}$ +	0	9
Germany									0	$5\frac{1}{4}$
Guelders, as *Friſland.*										
Hamborough									0	$9\frac{3}{4}$
Lubeck									0	$1\frac{3}{4}$
M. E. and *Philip* of *Flanders*	11	3	135	0	1	$18\frac{2}{3}$	0	$5\frac{1}{4}$ +	0	5
Scotland									0	1
Switz, or *Helvetia*									0	$1\frac{1}{4}$
Utrecht *Zeland* as *Friſland*										
Sicherling									0	$1\frac{1}{4}$
Snaphanen, Coined for 3 *Batz*	7	$7\frac{1}{2}$	$39\frac{3}{4}$	0	6	$0\frac{48}{53}$	1	0 +	1	0
Snaphanen of *Cleve* *Deventer* *Nimmeghen*	7	11	48	0	5	0	0	10 +	0	10
Soldi of *Genoa*									0	$0\frac{3}{4}$
Soli of *Wersburgh*, *Dantſick* and *Pruſſia*	5	$6\frac{1}{4}$	$157\frac{1}{2}$	0	1	$12\frac{4}{7}$	0	2 +	0	2
Soulx, or *Solx* of *France*									0	1
Soulx ſtamped, called *Soulx Marque*									0	$1\frac{1}{4}$
French Sols *of divers ſorts.* — Soulx										
The Old *Soulx* with ☩.	4	5	175	0	1	$8\frac{32}{35}$	0	$1\frac{1}{2}$ +	0	$2\frac{1}{4}$
Ordinary *French Soulx*	3	10	147	0	1	$15\frac{9}{49}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{4}$
Late *French Soulx*	3	$6\frac{1}{2}$	147	0	1	$15\frac{9}{49}$	0	$1\frac{1}{4}$ +	0	1
Double Hand of one *Soulx*	3	15	132	0	1	$19\frac{7}{11}$	0	$1\frac{3}{4}$ +	0	$1\frac{1}{4}$
Two *Soulx* Pieces, or Doubles	6	$6\frac{2}{3}$	117	0	2	$1\frac{3}{13}$	0	$3\frac{1}{2}$ —	0	3
Four *Soulx* Pieces accordingly.										
Styvers *of divers ſorts.* — Styvers of										
Cambray	3	5	135	0	1	$18\frac{2}{3}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{4}$
Embden									0	$1\frac{1}{4}$
Gaunt of 1583	3	0	$175\frac{1}{2}$	0	1	$8\frac{32}{39}$	0	1	0	1
Groeninghen *Liege* as *Cambray*										
States General of the United Provinces	4	0	168	0	1	$10\frac{2}{7}$	0	$1\frac{1}{2}$ +	0	$1\frac{1}{4}$
Utrecht	3	0	167	0	1	$10\frac{82}{167}$	0	1 +	0	1
Old Styver	3	$14\frac{1}{6}$	120	0	2	0	0	2 +	0	2
New Styver	3	$13\frac{1}{3}$	120	0	2	0	0	2 —	0	2
Half Styver	3	10	201	0	1	$4\frac{44}{67}$	0	1 +	0	1
Quarter Styver *Oort*	1	$17\frac{1}{2}$	158	0	1	$12\frac{36}{79}$	0	$0\frac{3}{4}$ +	not currant in *England.*	
Eight part Styver *Duyt*	1	14	474	0	0	$12\frac{12}{79}$	0	$0\frac{1}{4}$		
Old Double Styver	7	$7\frac{1}{2}$	120	0	2	0	0	4 —	0	4
Old Three Styvers	11	$3\frac{3}{4}$	120	0	2	0	0	6 +	0	6
Old Four Styvers with the *Eagle* *Charles* and *Philip*	7	$7\frac{1}{2}$	60	0	4	0	0	8 —	0	8

Styvers *of ſeveral ſorts.*

Teſtons *of divers ſorts.*

	Foreign Silver.	Fine ℥	pwts.	Pieces to the ℔. *Troy.*	Weight of the Pieces ℥.	pwts.	gr.	*Sterling* Value. s.	d.		Value by by others: s.	d.
Styvers	Three Styvers, or Fleece	10	10	108	0	2	5⅓	0	6¼	+	0	6
	Flemiſh Six Styvers	10	0	54	0	4	10⅔	1	0	+	1	0
	The *Bre* 1499 The *Key* and *Joane* — 3 Styvers	10	4	156	0	1	$12\frac{12}{13}$	0	4	+	0	4
	Five Styvers of *Cambray* Some	6	6½	48	0	5	0	0	8½	+	0	8
	Five Styvers of *Cambray* Others	6	6½	51	0	4	$16\frac{16}{17}$	0	8	+	0	8
	Five Styvers of *Guelders*	8	1½	48	0	5	0	0	10¾	+	0	10
	Five Styvers of *Horne*, as *Cambray*											
	Five Styvers of *Liege* Some	7	11	48	0	5	0	0	10	+	0	10
	Five Styvers of *Liege* Others	6	6½	48	0	5	0	0	8½	+	0	8
	Five Styvers of *Liege* Others	6	6½	51	0	4	$16\frac{16}{17}$	0	8	+	0	8
Teaſtons, or Teſtons of	*Baden*, *Chryſoſtome*	10	10½	39	0	6	$3\frac{9}{13}$	1	5½	+	1	5
	Berne — *Ottoman* *Vincent*	11	5⅙	45	0	5	8	1	4	+	1	4
	Caſtile, as *Berne*.											
	Ferrara, *Hercules*, and *Alphonſus*	11	5⅙	45	0	5	8	1	4	+	1	4
	France, *Franciſcus*	10	7	42	0	5	$17\frac{1}{7}$	1	4	—	1	4
	Friburg, *Nicolas*, as *Berne*											
	Geneva	10	4½	42	0	5	$17\frac{1}{7}$	1	3¾	+	1	4
	Lorrain of 1524, and 1529	10	7	42	0	5	$17\frac{1}{7}$	1	4	—	1	4
	Lucerne, *Epiſcopus* *Mantua*, *Francis* — as *Berne*											
	Millan — *Galleacius*, and *Lodovicus*	11	5⅙	45	0	5	8	1	4	+	1	4
	Montſerat, *George* and *Guill.*	10	4½	42	0	5	$17\frac{1}{7}$	1	3¾	+	1	4
	Navarre *Henricus* *Anna* — as *Baden*											
	Portugal, *Io.* *V. L*	10	7	42	0	5	$17\frac{1}{7}$	1	4	—	1	4
	Savoy, *Carolus* Some	11	5⅙	45	0	5	8	1	4	+	1	4
	Savoy, *Carolus* Others	10	10½	39	0	6	$3\frac{9}{13}$	1	5½	+	1	5
	Sedun, *Nicol*, *dan*, *Adrian*	11	5⅙	45	0	5	8	1	4	+	1	4
	Solod, *Vrſus*, as *Berne*											
	Turones of *France*	10	18	26¼	0	9	$3\frac{3}{7}$	2	2¾	+	2	2
	Vieryſers, Double	4	10	138	0	1	$17\frac{17}{23}$	0	2	+	0	2
	Single accordingly											

The Coines that are all Braſs uſed in Foreign Countries are many, and admit of ſeveral Subdiviſions: But (as the *Lawyers* ſay, *de minimus non currat Lex*) they being ſo ſmall and inconſiderable, and few of them being currant in any other place than reſpectively where Coined, are not worth the remembrance here.

Baſe Coines inconſiderable.

Nothing is further needful to finiſh this Chapter, then to ſhew how to ſet down or expreſs any *Geodætical* Number, which to do, place the higheſt Denominate Number to the left hand, and all the reſt in a ſtraight line in order to the right hand, with a little line, or prick or two, or ſome ſuch note of diſtinction between them; and over the head of every Number, or near the ſame, ſet the Character, Symbole, or Note, whereby it may be known of what nature or kind of *Geodætical* the ſame is, as to expreſs Four Pounds Twelve Shillings and Three Pence, ſet them as at *A.* Ten Ounces Three Drams, Two Scruples, and Fifteen Graines, as at *B.*

Geodæticals *how to be placed, or expreſſed.*

Examples.

	l. *s.* *d.*		*l.* *s.* *d.*		*l.* *s.* *d.*
A.	4 : 12 : 3	or thus	4—12—3	or thus	4 + 12 + 3
	℥ ʒ ℈ *gr.*		℥ ʒ ℈ *gr.*		℥ ʒ ℈ *gr.*
B.	10 . 3 . 2 . 15	or thus	10— 3—2—15	or thus	10 + 3 + 2 + 15

And if the *Geodæticals* be fracted in like manner after the Number is ſet down, place the Denomination at or near the head thereof, as $\frac{1}{4}$*d.* $\frac{2}{3}$*s.* $\frac{4}{5}$*l.* $3\frac{17}{20}$ Ton, *&c* which Numbers, though Without thoſe Denominations ſhould have been as Abſtract Fractions; yet now are reſtrained by thoſe Denominations. The firſt to be one Quarter or Fourth part of a Peny, which is a Farthing. The Second to be Two Thirds of a Shilling, which is 8 Pence; for one Shilling broken into Three parts or Groats, two

Example in Fracted Geodæticals.

of them will amount to ſo much. The third Fraction likewiſe now is Four Fifths of a Pound, which is in Value 16 *s.* for if one Pound or 20 *s.* be divided into 5 parts, and 4 of them parts be put together, it maketh 16 *s.* The Fourth Number is 3 Tons, and 17 Twentieth parts of one Ton, that is 17 Hundred, whence alſo is to be obſerved, ☞ That when an Integer and a Fraction is mixt, the Fraction is alwaies ſet to the right hand of the Integer, and is a part of parts of one of thoſe Integers, let the *Geodætical* Denomination be what it will. So $4\frac{1}{4}$ *l.* the $\frac{1}{4}$ ſhall underſtand a quarter of 1 *l.* not a quarter of 4 *l.* and the like of all others.

Engliſh Geodæticals *where to be underſtood.*

Further alſo may be obſerved, to ſave the often repeating the words *Ster ling*, or *Engliſh* in the following Examples; let the ſame be underſtood to thoſe *Geodæticals* of Coin Weight, *&c.* though not expreſſed, where the Denominations or Notes do not expreſs them to be Foreign *Geodæticals*.

CHAP. II.

Reduction of Geodæticals.

Simple Elements of Geodæticals. *How agree and differ to Integers and Faactions.*

THE Nature of *Geodæticals* with their Notes and Denominations declared in the præcedent Chapter, the reſt of their Simple Elements are next to be ſpoken to.

Geodæticals, as they partake of the Nature of Abſtract and Contract Numbers, and ariſe from others; ſo their Numeration is both Original and Ortive; that in *Addition*, *Subſtraction*, *Multiplication* and *Diviſion*; and this in *Reduction*. And (as Fractions) have properly their Ortive Numeration, though but accidental, and for conveniency, fall firſt under conſideration, before the more Eſſential, and Original Numeration. Yet different herein, that *Reduction* of *Fractions* declareth the proportion of one Number to another, or of broken parts to broken parts; but *Reduction* of *Geodæticals*, the Denominations of one Number, leſſer or greater contained in another.

Reduction *of what uſe and how called.*

Reduction of *Geodæticals* bringeth Numbers of one Denomination to another, and ſo ſheweth how to expreſs one and the ſame Number in Value under different Names or Denominations. As 8 *s.* or $\frac{2}{5}$ of a Pound, which is alike valuable. For which reaſon *Reduction* is ſometime called *Æquation*. And ſometime is uſeful to avoid Fractions, ſometime to facilitate thoſe Operations which without *Reduction* are tedious. The Number given to be reduced is called The *Reducend*. The ſeveral Denominations, are *Reducers*. And the Number obtained by *Reduction* is the *Reſult*.

Reducend Reducers Reſult *what*

The Sorts of Reduction.

Reduction of *Geodæticals* is either General, or Special.

$$\text{General Reduction of } \textit{Geodæticals} \text{ is}\left\{\begin{array}{l}\text{Proper}\left\{\begin{array}{l}\text{Synthetical.}\\ \text{or}\\ \text{Analytical.}\end{array}\right.\\ \text{or}\\ \text{Proportional}\left\{\begin{array}{l}\text{Analytical.}\\ \text{and}\\ \text{Synthetical.}\end{array}\right.\end{array}\right.$$

Proper of the Firſt Sort included under 3 Caſes.

Proper Synthetical Reduction ſerveth to reduce Subtiller or Smaller Denominations into Groſſer or Greater, as Pence into Shillings or Pounds, *&c.* and is performed by *Diviſion* under one of theſe 3 Caſes.

1. Data *Single.* Quæſita *Single Rule.*

1. *Caſe.* When the Number to be reduced is Single-Integral, and the Denomination deſired is ſingle.

Then divide the Number to be reduced if it may be, by ſo many as one of the Greater do contain of the Leſſer Denomination; If it cannot be divided, abbreviate it as a Fraction.

1. *Example.*

1. *Example.* To know how many Pounds *Ster ling* are in 81600 Pence. I divide 81600 by 240 the Pence in one Pound, and the Quotient 340 *l.* is the Reſult.

$$\begin{array}{ll} & \ \ \cancel{9} \\ \text{Reducend} & \cancel{8}\cancel{1}\cancel{6}00(340\,l.\ \ \text{Reſult.} \\ \text{Reducer} & \cancel{2}\cancel{4}\cancel{4}\ 0 \\ & \ \ \cancel{2} \end{array}$$

2. *Example.*

2. *Example,* To know what parts of a Pound 8 *s.* or 8 *d.* are, becauſe 8 will not be divided by 20 the Shillings in a Pound, nor yet by 240 the Pence in a Pound; they are both abbreviated as Fractions, and 8 *s.* at *A.* is ſeen to be $\frac{2}{5}$ *l.* and 8 *d.* at *B.* $\frac{1}{30}$ *l.*

$$A.\ \frac{8}{20}\ \Big|\ \frac{4}{10}\ \Big|\ \frac{2\,l.}{5} \qquad B.\ \frac{8}{240}\ \Big|\ \frac{4}{120}\ \Big|\ \frac{2}{6}\ \Big|\ \frac{1\,l.}{30}$$

2. Data, *ſingle,* Quæſita *Plural*

2. *Caſe*, When the *Reducend* is Integral and ſingle, and the Denominations deſired are plural.

Rule.

Then divide the *Reducend*, by ſo many of the Denomination given, as make one of the next Greater deſired, and then ſucceſſively continue the Diviſion of the Quotients reſulting by ſo many as make one of the next Greater Denomination deſired.

1. *Example.*

1. *Example,* To know how many Shillings and Pounds there are in the former Sum of 81600 *d.* Firſt I divide 81600 by 12 the Pence in a Shilling, and the reſulting Quotient is 6800 *s.* which divided by 20, the Shillings in 1 *l.* the Reſult is 340 *l.* as before.

$$\begin{array}{lll}
\cancel{9} & & \\
8\cancel{1}\cancel{6}00 & (\cancel{6}800\,s. & (340\,l. \\
\cancel{1}\cancel{2}\cancel{2} & \;\cancel{2}\;\;0 & \\
\;\cancel{1} & &
\end{array}$$

2. *Example.*

2. *Example,* To know how far 285120 Barley-Cornes laid end to end will reach, dividing by 3 the Barley-Cornes that make 1 Inch, and by 12 the Inches in 1 Foot, and by 5 the Feet in 1 Pace, and by 1056 the Paces in 1 *Engliſh* Mile: The Reſult is 1 $\frac{1}{2}$ Mile. And the ſeveral intermediate Quotients declare the reſpective Inches, Feet, and Paces therein.

$$\begin{array}{lllll}
\cancel{1} & \cancel{1}\cancel{1}\cancel{2} & \cancel{2}4 & (528 & \\
\cancel{2}8\cancel{5}\cancel{1}\cancel{2}0 & (\cancel{9}\cancel{5}\cancel{0}\cancel{4}0 & (\cancel{7}\cancel{9}\cancel{2}\cancel{0} & (\cancel{1}\cancel{5}8\cancel{4} & (1\frac{528}{1056}\ \text{or}\ 1\frac{1}{2}\ \text{Mile.} \\
\cancel{3}\cancel{3}\cancel{3}\cancel{3} & \cancel{1}\cancel{2}\cancel{2}\cancel{2} & \cancel{5}\cancel{5}\cancel{5}\cancel{5} & \cancel{1}\cancel{0}\cancel{5}\cancel{6} & \\
 & \cancel{1}\cancel{1} & & &
\end{array}$$

3. Data *mixt.*

3. *Caſe*, When the *Reducend* hath ſome Fraction annexed thereto.

Rule.

Then reduce the *Reducend* into an Improper Fraction, and divide the ſame after the manner of a Fraction, by ſo many of the given Denomination, as make one of the deſired Denomination, Or elſe divide the Whole Number firſt, and add the Fraction to the Quotient.

Example.

Example. To know how many Shillings are in 196$\frac{1}{2}$ *d.* Firſt I reduce 196$\frac{1}{2}$ into the Improper Fraction $\frac{393}{2}$, and then divide by 12, the Reſult is 16$\frac{3}{8}$ *s.* or dividing 196 by 12, the Reſult is 16 *s.* 4 *d.* to which the half-peny added makes it 16 *s.* 4$\frac{1}{2}$ *d.* all one with 16 $\frac{3}{8}$ *s.*

$$\text{Thus}\ \frac{393}{196\frac{1}{2}}d.\quad \frac{\overset{4}{\dot{1}\dot{2}}}{1}\Big)\frac{\overset{131}{\dot{3}\dot{9}\dot{3}}}{2}\Big(\frac{131}{8} = 16\frac{3}{8}\,s.\quad \text{or thus}\quad \begin{array}{ll} \cancel{7}(4 & \\ \cancel{1}\cancel{9}\cancel{6}(16\,.\,4\frac{1}{2} & \\ \cancel{1}\cancel{2}\cancel{2} & \\ \cancel{1} & 16\,.\,4\frac{1}{2} \end{array}$$

Proper Reduction *of the ſecond ſort included under* 3 *Caſes.*

Proper Analytical Reduction, ſerveth to reduce Groſſer or Greater Denominations into Subtiller, or Smaller, as Pounds into Shillings, or Pence, &c. and is performed by *Multiplication* under one of theſe 3 Caſes.

1. Data, *Single* Quæſita *Single.*

1. *Caſe*, When the Number given to be reduced is of one Denomination and Integral, and the deſired Denomination ſingle.

Rule.

Then multiply the *Reducend* by ſo many as one of the Greater do contain of the Leſſer Denomination.

Example.

Example, To know how many Pence are in 340 *l. Sterling*, I multiply 340 by 240, becauſe ſo many Pence are contained in one Pound, and the Product 81600 is the Reſult.

$$\begin{array}{lrcr}
\text{Operation} & 340\,l. & & 34\ 0\ \text{Reducend} \\
 & 240\,d. & & 24\ 0\ \text{Reducer} \\
\hline
 & 13600 & {}^{\ \ 6}_{6}\!\times\!{}^{7}_{6}\quad \text{or thus} & 136 \\
 & 680 & & 68 \\
\hline
\text{In } 340\,l.\ \textit{Sterl.} & 81600\ \text{pence} & & 81600\ \text{Reſult.}
\end{array}$$

2. Data *Plural.* Quæſita *Plural* 2. *Caſe,* When the *Reducend* is Integral and of divers Denominations, or it is deſired to know how many of the ſeveral intermediate Denominations (if any ſuch be) are between the Denomination given, and that into which it is to be reduced.

Rule. Then multiply the Number to be reduced by ſo many as one of the next Leſſer Denominations to the *Reducend* containeth, and ſucceſſively the Product reſulting by ſo many as one of the next Leſſer Denomination to the Denomination of the Product doth contain. And if any odd Numbers belong to the reſpective Denominations add them to the reſpective Products after the manner of Integers.

1. *Example.* 1. *Example,* In the former Sum of 340 *l.* if it were deſired to know how many Shillings as well as Pence were contained therein. I multiply 340 *l.* by 20, the Shilling in 1 Pound, and the Product 6800 are the Shillings therein, which multiplyed by 12, the Pence in 1 Shilling, produce 81600 *d.* as before. See *C.*

2. *Example.* 2. *Example,* If it be deſired to know how many Shillings, Pence, and Farthings there are in 355 *l.* 15 *s.* 4 *d.* 3 *q. Sterling.* After *Multiplication* by 20, the 15 *s.* are added, and after multiplication by 12, the 4 *d.* are added, and after multiplication by 4 the 3 *q.* are added as at *D,* and the Total Reſult is 341539 *q.*

340 *l.*	355 *l.* 15 *s.* 4 *d.* 3 *q.*
20	20
6800 *s.*	7100
12	15 added
13600	7115 *s.*
6800	12
C. 81600 *d.*	14230
	71154 added
	85384 *d.*
	4
	341536
	3 added
	D. 341539 *q.*

Examples in ſeveral Denominations. In like manner any *Geodætical* of like Nature *Engliſh,* or Foreign may be reduced to a Leſſer Denomination, obſerving to multiply by the Number of Leſſer Denominations contained in the Greater, and adding the odd Numbers if any be.

Examples in Long Meaſure.

Long Meaſure. How many Barley Cornes being laid end to end will reach from *Rye* to *London* being 60 Miles?

Thus 60 Miles	or Thus, 60 Miles
8	1056
480 Furlongs	360
40	300
19200 Perches.	600
$16\frac{1}{2}$	63360 Paces
115200	5
19200	316800 Feet.
9600	12
316800 Feet	633600
12	316800
633600	3801600 Inches.
316800	3
3801600 Inches.	11404800 Early Corns.
3	
11404800 Early Corns	

Example in Square Meaſure.

Square Meaſure

In 1423 Acres, 2 Roods, 30 Perches; How many Perches?

Thus 1423 Acres.
4
5692
2 added
5694 Roods
40
227760
30 added
227790 Perches

or thus, 1423 Acres.
160 Perches in 1 Acre.
85380
1423
110 added 110 Perches.
227790 Perches reſulting.

Example in Weight.

Avoirdupois Weight.

C. qr. ℔.
In 19—2—20 *Avoirdupois Weight.* How many Grains?

Thus 19 C.
4
76
2 added
78 qr.
28
624
156
20 added
2204 ℔.
16
13224
2204
35264 ℥
8
282112 ʒ
3
846336 ℈
20
16926720 gr.

or Thus 19 C.
112 Pounds in 1 C.
38
196 added 76 ℔.
197
2204 ℔.
7680 Graines in 1 ℔.
176320
13224
15428
16926720 Graines reſulting.

Example in Time.

Time.

In 40 Years, 12 Dayes, 10 Hours, How many Minutes?

40 Years.
365
200
240
12022
14622 Dayes
24
58488
29244
10
350938 Hours
60
21056280 Minutes.

Beſides the 12 Daies in the *Reducend*, are added 10 more, (which is 22 in all) becauſe every 4*th.* year being Leap-Year, and that year having 366 Daies in 40 Years is 10 Daies.

3. *Caſe.*

3. Data *mixt.* *Rule.*

3. *Caſe*, When the *Reducend* hath ſome Fraction annexed thereto.

Then Reduce the *Reducend* into an Improper Fraction, aud multiply the ſame after the manner of Fractions, by ſo many of the given Denominations as make one o that deſired.

Example.

Example, To know how many Pence there are in $16\frac{3}{8}\,s$ I reduce them into th Improper Fraction $\frac{131}{8}$, and the multiply by 12, and the Reſult is $196\frac{1}{2}\,d.$

$$\frac{131}{16\frac{3}{8}\,s.} \qquad \frac{131}{\underset{2}{8}} \times \frac{\overset{3}{12}}{1} = \frac{393}{2} = 196\tfrac{1}{2}\,d. \qquad \frac{\overset{\cancel{1}1(1}{\cancel{39}3}}{2}\Big(196$$

Proof of Proper Reduction.

Proper Reduction of both ſorts, Caſe by Caſe is alternately proved by the other; a by the former Examples is evident without further inſtance.

Proportional Reduction *belongs to* Proportions.

Proportional Reduction, is both *Analytical* and *Synthetical*, uſing both *Multiplicatio* and *Diviſion*, and helpeth to reduce Denominations of Meaſure, Weight, Coine, &c. of one Kind or Countrey to another, *Engliſh* to *Foreign*, and *Foreign* to *Engliſh*, and i called *Proportional*, becauſe it properly belongeth to the Doctrine of *Proportions* an *Rule of Three* handled in the *4th. Book*, where more fully thereof may be ſeen. Bu

If an Unit, one of the Data *fall under* 3. Caſes.

becauſe often in that Rule an Unit falls to be one of the Three Numbers, whic neither multiplying nor dividing becomes uſeleſs, and ſo ſuch kind of Reductiv Queſtions become tranſient when the given *Geodæticals* contain in them ſome Par Denomination or Value common to both, and may be tranſlated hither under 3 Caſes

1. Data, *Single.* *Rule.*

1. *Caſe*, When the *Geodætical* given is Integral and Single.

Multiply the given Number by ſuch common Parts or Value, as make one of th Denominate Integers, and divide the Product by the common Parts or Value in th other.

1. *Example.*

1. *Example*, In 300 Ells *Engliſh*, How many Yards?

Here becauſe Ells contain 45 ſuch Inches as Yards 36, or 5 Quarters, as Yards 4 I multiply the 300 by one, and divide the Product by the other, and find 375 Yards.

$$\text{Thus}\quad \begin{array}{r} 300 \\ 45 \\ \hline 1500 \\ 1200 \\ \hline 13500 \\ \hline \end{array} \qquad \begin{array}{l} \cancel{1} \\ \cancel{2}\cancel{7}8 \text{ yards} \\ \cancel{1}\cancel{3}\cancel{5}\cancel{0}\cancel{0}(375 \\ \cancel{3}\cancel{6}\cancel{6}\cancel{6} \\ \cancel{3}\cancel{3} \end{array} \qquad \text{or thus}\quad \begin{array}{r} 300 \\ 5 \\ \hline 1500 \\ \hline \end{array} \qquad \frac{\overset{\cancel{3}2}{\cancel{1}\cancel{5}00}}{\cancel{4}}\Big(375$$

1. *Example.*

2. *Example*, In 108 Pieces (or Ryals) of Eight, at 4 *s.* 4 *d. per* piece: How m ny Crowns *Engliſh* at 5 *s. per* Piece?

Here Pence being the Denomination common to both, the Product of 108 mul plyed by 4 *s.* 4 *d.* or 52 *d.* and divided by 5 *s.* or 60 *d.* giveth $93\frac{3}{5}$ Crowns, t Reſolution, or 93 Crowns and 3 *s. Sterling*.

$$\text{Thus,}\quad \begin{array}{r} 108 \\ 4\frac{1}{3} \\ \hline 432 \\ 36 \\ \hline 468 \\ \hline \end{array} \qquad \frac{\overset{\cancel{1}(3}{\cancel{4}\cancel{6}8}}{\cancel{5}}\Big(\overset{\triangle}{93} \qquad \text{or thus}\quad \begin{array}{r} 108 \\ 52 \\ \hline 216 \\ 540 \\ \hline 5616 \\ \hline \end{array} \qquad \frac{\overset{2(3}{\cancel{5}\cancel{6}1(6}}{\cancel{6}0}\Big(93$$

2. Data *Plural.* *Rule.*

2. *Caſe*. When the given *Geodætical* is Integral and Plural.

Reduce all the Denominations into the loweſt, and then divide or abbreviate before.

1. *Example,* To know what parts of a Pound $7\,s.\ 6\,d.$ or $12\,s.\ 3\,d.\ 2\,q.$ are, Theſe Numbers reduced, the firſt into Pence, and the ſecond into Farthings, and abbreviated becauſe they will not be divided: Declare the former $\frac{3}{8}\,l.$ at *E.* and the other $\frac{59}{96}\,l.$ at *F.* 1. *Example.*

$7\,s.6\,d.$
12
E ———
14
76
———
90
———

$\frac{90}{240}\Big|\frac{9}{24}\Big|\frac{3}{8}\,l.$

F. $12\,s.\ 3\,d.\ 2\,q.$
12
———
24
123
———
147
4
———
590
———

$\frac{590}{960}\Big|\frac{59}{60}\,l.$

2. *Example.* Suppoſe a *Ropemaker* marry his Daughter to a *Sope-maker*, and for her Portion give her Twenty Ropes, and on every Rope 20 Knots, and on every Knot 20 Purſes, and in every Purſe 20 Three Half-pence: How much Money had ſhe for her Portion? 2. *Example.*

To reſolve this Queſtion, the number of Three Half Pence is firſt known by *Multiplication*, and then dividing that Product by 8 the Three Half Pence in one Shilling, and after by 20 the Shillings in one Pound; or elſe by 160 the Three Half-Pence in one Pound: The Portion is found to be 1000 *l.*

20 Ropes
20
———
400 Knots
20
———
8000 Purſes.
20
———
160000 3 Half-Pence
———

$\frac{\cancel{1}\cancel{6}0000}{\cancel{8}}\Big(\frac{2000}{1000}\Big|\frac{0}{l.}$ $\frac{\cancel{1}\cancel{6}000\cancel{0}}{\cancel{1}\cancel{6}\ \ \cancel{0}}\Big(1000\,l.$

3. *Caſe,* When the *Geodætical* Reducend is a mixt Number, or a Fraction. 3. Data *mixt, or* Fractions.

If a mixt Number reduce it into an Improper Fraction, and let the Numerator of the Fraction Proper or Improper be multiplyed by the Parts that make one of the Denominate Integers, and divide that Product by the Denominator with the common parts deſired if any be. And thus the value of any Fraction or Remainer upon a Diviſion may be known.

1. *Example,* To know the value of $\frac{7}{10}\,l.$ The Numerator 7 being multiplyed by the parts of a Pound, which are either 20 Shillings, 240 Pence, 960 Farthings, *&c.* according to that part multiplying, dividing the Product by 10, the Denominator; ſhall the Quotient of that Diviſion be denominate. 1. *Example.*

$\frac{7\,l.}{10}$ $20\,s.$
7
———
140
———

$\frac{\cancel{1}4\cancel{0}}{\cancel{1}\ \cancel{0}}\Big(14\,s.$

$240\,d.$
7
———
1680
———

$\frac{\cancel{1}\cancel{6}8\cancel{0}}{\cancel{1}\ \cancel{0}}\Big(168\,d.$

$\frac{\overset{4}{\cancel{1}\cancel{6}\cancel{8}}}{\cancel{1}\cancel{2}}\Big(14\,s.$

$960\,q.$
7
———
6720
———

$\frac{\cancel{6}72\cancel{0}}{\cancel{1}\ \cancel{0}}\Big(672\,q$

$\frac{\cancel{6}7\cancel{2}}{\cancel{4}\cancel{8}}\Big(14\,s.$

2. *Example,* To know how many *Liures Tournois* at 20 *d. per Liure*, are in $100\frac{1}{2}\,l.$ *Sterling.* The $100\frac{1}{2}\,l.$ reduced is $\frac{201}{2}$ which multiplyed by 240 *d.* the parts of one Integer produce 48240, this divided by 2 the Denominator multiplyed into 20 the common parts of the *Liure* deſired, reſolve the Queſtion into 1206 *Liures.* 2. *Example.*

Thus $\frac{201}{100\frac{1}{2}\,l.}$

201
240
———
8040
402
———
48240
———

$\frac{48240}{2}\ \overset{48240}{\underset{40}{\mathbf{X}}}\ \frac{20}{1}$ $\frac{48\cancel{2}40}{4\ \ \cancel{0}}\Big(1206$

Proof of Proportional Reduction.

Proportional Reduction of each ſort is proved, by reverſing the Queſtion and Work, by making the *Diviſors* and Multipliers in the one Queſtion, the contrary in the other. As in the former Inſtance, If 300 Ells contain 375 yards, and they were given to know how many Ells were therein, Then multiply 375 yards by 4 (the former Diviſor) and that Product 1500 divided by (the former Multiplier) 5, will return 300 Ells, and ſhew the Works right.

Special Reduction what and how done.

Special Reduction conſiſts in ſome Select Rules more brief and commodious than the common way. As——

1. Pence brought into Pounds, &c. at once.

1. To bring Pence into Pounds, Shillings, and Pence at one Work, divide the Sum to be reduced by 24, and from the Quotient cut off the right hand Figure, which is Primes, every Unite in Value 2 *s.* and the Remainer of the Diviſion is Pence.

Example.

Example, If 85390 Pence be divided by 24, I take 12 *d.* for 1 *s.* out of the 22 Pence remaining on the Diviſion, and add to the 14 *s.* or 7 Primes cut off from the Quotient, and with the 10 *d.* left of the 22, I obtain the Total 355 *l.* 15 *s.* 10 *d.*

```
  1   pence
 131(22  l.  primes
85390(355|7
24444 ---------
 222   355:14
       ------------
           1:10
       355:15:10
       ------------
```

Common Way.

```
  (1
 117
8539(0 d. (71(15 s. (355 l.
22222     2220
 111
```

2. Farthings brought into Pounds, &c. at once. Example.

2. To bring Farthings into Pounds, Shillings and Pence, at one work, divide the Sum by 96, and from the Quotient cut off the right hand Figure as before, and for every 48 remaining add a Shilling, the reſt are Farthings.

Example, If 417231 Farthings be divided by 96, the Quotient right hand Figure 6 is 12 *s.* the 15 left on the Diviſion is Farthings, or 3 *d.* 3 *q.* added make the Total 434 *l.* 12 *s.* 3 *d.* 3 *q.*

```
   45(1 q.
  3349(5 l.  primes
417231(434|6
96666 --------
  999  334:12:
            3 3/4
       ------------
       434:12:3 3/4
       ------------
```

Common Way.

```
                      1
     1     (3 q.   812(3d.   1 s.    l.
  417231    104307 ( 869|2 (434
  ------    ------   -----
  444444    12222    2220
             111
```

3. Pounds brought into Shillings. Example.

3. To reduce Pounds into Shillings, double the Number of Pounds, and to the right hand adjoyne a Cypher.

Example, If 30 *l.* be brought into Shillings, 30 doubled is 60, and a Cypher adjoyned makes it 600 *s.*

The Common way.

```
  30 l.        30 l.
   2           20
 ------      ------
 600 s.      600 s.
             ------
```

4. Shillings brought into Pounds. Example.

4. To reduce Shillings into Pounds, cut off the right hand figure, and take half the reſidue, as in the 13th. *Section* of *Diviſion of Integers* was taught to divide by 20.

Example, If 8692 *s.* be brought into Pounds, 2 cut off the half of 869, the reſt is 434 *l.* and 1 remaining makes 2 cut off to be 12 *s.*

```
  (1 s.
 869|2
 ------
 434 l.
 ------
```

Common way.

```
 (1
 869(2 (434 l.
 2220
```

5. *Shillings brought into Pence. Example.*

5. To reduce Shillings into Pence, double the Number given, and placing it one place nearer to the right hand, add it to the given Number.

Example, To bring 16 *s.* into Pence, the double of 16 is 32, placed and added accordingly, make 192 Pence.

16 *s.*
32
192

Common way.

16 *s.*
12
32
16
192 *d.*

6. *Pounds into Primes. Example.*

6. To reduce Pounds into Primes or 2 *s.* adjoyn a Cypher to the right hand of the given Number.

Example, 45 *l.* brought to Primes is 450 *Primes.*

450 *Primes.*

Common way.

45
10
450 *Primes.*

7. *Shillings into Primes. Example.*

7. To reduce Shillings into Primes of Pounds, Take half the given Number.

Example, 346 Shillings reduced make 173 Primes.

$\frac{1}{2}$ 346
173 *Primes*

Common way.

346
222 (173 *Primes*

8. *Reduction by any of the Common Parts. Example.*

8. To reduce any Number to another Denomination, that hath more Common Parts than one, any of thoſe Common Parts may be taken and thoſe are to be choſen, that make the work ſhorteſt.

Example, If one Dollar be worth 4 *s.* 8 *d.* How many Dollars are in 348 *l.* ? Here becauſe Groats as well as Pence are common parts to 4 *s.* 8 *d.* and alſo to Pounds; I accept Groats before Pence, and working thereby find 1491 Dollars and 6 Groats over.

			Common way.		
4 *s.* 8 *d.*	348 *l.*	1	4 *s.* 8 *d.*	348 *l.*	5 (2 Pence
3 Groats	60	622(6 Groats.	12	20	2718(4
		20880(1491 Dollars.			83520(1491 Dollars
12	20880	14444	48	6906	56666
2		111	8	12	555
14 Groats.			56	13920	
				6960	
				83520	

Proof of Special Reduction

Special Reduction may be proved by the General, as in all the laſt 8 Examples is apparent, the Reſult by the Common Way is equal to the other.

Johnſon's Proof of Reduction of Fractions.

Johnſon adviſeth to prove the Fractionary Operations in *Geodæticals* by finding the value of each Fraction, and compare them with their value after *Reduction*, which will be alike if the work be right.

1. *Example*, If $\frac{1}{2}$ *l.* and $\frac{1}{3}$ *l.* be reduced to one Denominator as *Vulgar Abſtract Fractions*, they make $\frac{3}{6}$ and $\frac{2}{6}$, and becauſe $\frac{1}{2}$ *l.* is 10 *s.* and ſo is $\frac{3}{6}$ *l.* and $\frac{1}{3}$ *l.* is 6 *s.* 8 *d.* and ſo is $\frac{2}{6}$ *l.* their Reduction appears right.

$\frac{1}{2}$ *l.* $\frac{20}{2}$(10 *s.* $\frac{3}{6}$ *l.* $\frac{3}{\frac{20}{60}}$ $\frac{60}{6}$(10 *s.* || $\frac{1}{3}$ *l.* (2 $\frac{20}{3}$(6$\frac{2}{3}$ *s.* $\frac{2}{6}$ *l.* $\frac{2}{\frac{20}{40}}$ (4 $\frac{40}{6}$(6$\frac{2}{3}$ *s.*

2. *Example*, If $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ of an old Harper, or Nine-pence be reduced, it will make $\frac{1}{3}$ or Three-pence; and ſo by finding the Value of the Right hand Fraction $\frac{7}{9}$ it is 7 Pence, of which $\frac{6}{7}$ taken is 6 *d.* and $\frac{2}{3}$ of that is 4 *d.* of which $\frac{3}{4}$ is 3 *d.* alſo.

CHAP. III.

Addition of Geodæticals.

Geodæticals *added.*

A*Ddition of Geodæticals* may be conſidered according to the different nature of the *Geodæticals* propoſed to be added.

1. Integers *of one Denomination.*

1. *Caſe.* If the Numbers to be added be *Integral Geodæticals* alone, and of the ſame Denomination,

Rule.

Then add the Numbers as *Abſtract Integers* in *Book* 1. *Part* 1. *Chap.* 5. and to the Total adjoyne or underſtand the Denomination given with the Numbers to be added.

1. *Example.*

1. *Example*, Two Flocks of Sheep, one of 910, and the other of 563 are to be added, the Total ſhall be 1473 Sheep.

2. *Example.*

2. *Example*, Three Men are indebted to another, one 340 *l.* the ſecond 520 *l.* and the third 600 *l.* The Total of thoſe Debts ſhall be 1460 *l.*

Firſt Example.		Second Example.	
	910 563 } Sheep.		340 520 } *l.* 600
Total	1473 Sheep.	Total	1460 *l.*

2. Integers *of ſeveral Denominations.*

2. *Caſe.* If the Addends be *Integral Geodæticals* greater and ſmaller of different Denominations,

To place the Data.

Then place the greater Denomination alwaies to the Left hand, and in order to the Right hand the next Denominations, and alſo Units under Units, and Tens under Tens, *&c.* of every Denomination reſpectively, and Numbers of like Quality or Denomination under Numbers of the ſome Quality or Denomination. And when there are wanting ſome Denominations to fellow with others, ſome ſupply the places of ſuch wanting Denominations with Cyphers.

To add them.

Then begin with the right hand file, and ſmalleſt Denomination firſt, and adding all thoſe ſmaller Numbers together, mark how many of the next Greater Denomination, you can take out of the Total of thoſe ſmaller Numbers ſo reckoned up, and ſo many Units reſerve in your mind, and the Overplus, if there be any, ſubſcribe under the line and file where you are reckoning. And as in *Integers* and the former ſort of Greater *Geodæticals* you reckoned the Article before, ſo reckon them you carry away now into the next Left hand Denomination. And this do if there be 2. 3. or more Denominations in the Numbers given to be added.

Example.

Example, If 4 Men were indebted to me, *viz. A.* 22 *l.* 13 *s. B.* 15 *l.* 10 *s.* 8 *d. C.* 10 *l.* 5 *s.* 2 *d.* and *D.* 3 *l.* 4 *s.* 7 *d.* or any other Sums, and I would know the Total that is owing by them all. Then I ſet them as at *E.* and beginning with the Pence, I ſay 7 and 2 is 9, and 8 is 17, which is 1 Shilling and 5 Pence over, wherefore I ſet down the remaining 5 Pence, and the one Shilling I carry to the Denomination of Shillings, and the Work will ſtand as at *F.* Then the one Shilling I carry and 4 is 5, and 5 is 10, and 3 is 13, and 10 is 23, and 10 is 33 Shillings, out of which I can take but one of the next greater Denomination, which is Pounds, and there reſteth 13 *s.* which I ſet down as at G. Now the one Pound reſerved I add to the next Denomination thus, ſaying 1 and 3 is 4, and 5 is 9, and 2 is 11, where this being the Greater *Geodætical* and higheſt Denomination of that kind, I reſerve the Articles to the next file, and ſubſcribe the digits as before in *Integers* and going forward find the Total to be 51 *l.* 13 *s.* 4 *d.*

	E. l.	E. s.	E. d.	F. l.	F. s.	F. d.	G. l.	G. s.	G. d.	H. l.	H. s.	H. d.	
Debts of *A.*	22	13	00	22	13	00	22	13	00	22	13	00	
Debts of *B.*	15	10	08	15	10	08	15	10	08	15	10	08	
Debts of *C.*	10	05	02	10	05	02	10	05	02	10	05	02	
Debts of *D.*	3	04	07	3	04	07	3	04	07	3	04	07	
						05		13	05	51	13	05	Total.

Pricks used to help the Memory.

Some for the aid of their memory prick that Figure of the leaſt Denomination, which being joyned to the other below it, gives more than one of the next Denomination, and carry the Overplus to the next Figure, where when they exceed one of the next Denomination, they ſet another prick, and ſo ſtill carry the Remaines, and the laſt Remain or Overplus, if any be, ſubſcribe under that File. And then carry ſo many Units as there be pricks unto the Numbers of the next Left hand Denomination, and there proceed in like ſort, and ſo in the following Examples the Numbers will be thus pricked.

Example.

l.	*s.*	*d.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*
398	10	10·	5469	19·	11·	34619	10	06·	2
964	15·	06	2492	08	10·	32418	18·	08	1
813	13·	04·	1983	14·	02	24818	10·	11·	3·
421	11	11	3112	13	03	12498	11	00	1
2598	11	07	13058	16	02	104355	11	02	3

In Addition of Weights, Meaſures, Time, Motion, and all other *Geodæticals Engliſh* and *Forreign* in this Caſe: The like Order may be obſerved as in the *Examples* enſuing.

Examples of

Addition of

Troy and *Avoirdupois-Weight.*

Weight.

℔.	*ounc.*	*pwts.*	*gr.*	*mites.*	*Ton.*	*C.*	*gr.*	℔.	℥	ʒ	℈	*gr.*
30	11	19·	23·	19·	120	19·	3	27·	15·	7·	2	19·
27	10·	16·	20·	16·	89	10·	2·	19·	12·	6·	1·	10·
24	08	12	21	18	30	13	0	10	11	4	0	18
83	07	09	18	13	241	03	3	02	08	2	2	07

Square and *Concave Meaſures.*

Meaſure.

Acres	*Roods*	*Daies-work*	*Perches*	*Feet.*	*Quarters*	*Buſhels*	*Gallons*	*Pints.*
141	3	9·	3·	16·	1944	7·	7·	7·
141	2·	8·	2·	12·	1036	6·	5·	4
191	0	4	1	10	987	1	3	2
474	3	3	0	05	3969	0	0	5

Time and *Motion.*

Time and Motion.

Years	*Dayes*	*Hours*	*Min.*	*Seconds.*	*Signes.*	*Degr.*	*Min.*	*Sec.*	*Thirds.*
234	236·	23·	59·	59·	10	29·	59·	59·	59·
120	142	22·	30·	45	0	10	20	15	11
110	30	10	40	10	0	10	13	12	00
465	173	09	10	54	11	20	33	27	10

3. Fractions or Mixt. Rule.

3 *Caſe.* If the Addends be Fracted *Geodæticals*, whether greater, or leſſer, or Integral when Fracted,

Then proceed in the Addition with the Fractions as Fractions, and the Integers as Integers, according to their reſpective Rules.

Examples. *Example*, To add $\frac{1}{5}$ *l.* with $\frac{3}{5}$ *l.* the Total as at *I*, is $\frac{4}{5}$ *l.* by *Book* 1. *Part* 2. *Chap.* 3. *Caſe* 1. And to add $\frac{1}{2}$ *l.* and $\frac{1}{3}$ *l.* together, the Total as at *K.* is $\frac{5}{6}$ *l.* by *Caſe* 2. there. And to add 40$\frac{3}{4}$ *l.* with 18$\frac{1}{2}$ *l.* the Total as at *L.* is 59$\frac{1}{4}$ *l.* by carrying an Unite from the Fractions to the Integers, and adding them after the manner of Integers in *Book* 1. *Part* 1. *Chap.* 5.

I.

$$\overset{l.}{\frac{1}{5}} \times \frac{3}{5} = \overset{l.}{\frac{4}{5}}$$

K.

$$\frac{5}{\underbrace{\frac{3}{\frac{1}{2}} \times \frac{2}{\frac{1}{3}}}_{6}} = \overset{l.}{\frac{5}{6}}$$

L.

$$\begin{array}{r} l. \\ 40\,\frac{3}{4} \\ 18\,\frac{1}{2} \\ \hline 59\,\frac{1}{4} \\ \hline \end{array}$$

Proof of Geodætical Addition. *Addition of Geodæticals* falling under the firſt Caſe, admits of like Proof with Addition of Integers. And thoſe under the Third Caſe are to be proved, as the Addition of Integers and Fractions, according to the reſpective Operations made uſe of in the Addition: For if Integers be added, the Work is proved as Addition of Integers, if Fractions, as Addition of Fractions. *Addition of Geodæticals* under the ſecond Caſe is to be proved by *Subſtraction*, as is ſhewed in the next Chapter.

Proof by Nines. Addition of *Engliſh* Pounds, Shillings and Pence, in the Second Caſe in this Chapter above ſhewed: Some Schoolmaſters teach to prove thus. Caſt away all the Nines from the Pounds to be added, and what remains double, and bring to the Shillings, and caſt away 9 alſo thence, and what remains treble, and bring to the Pence, and all the Nines being caſt away there, note the laſt Remain. Then reject Nines in like manner from the Total, and if the laſt Remain here be like the former, they approve the Work.

As in the former Example where the Total was 51 *l.* 13 *s.* 5 *d.* caſting away 9 from the Pounds to be added, there remains 5, which doubled is 10, and 9 caſt therefrom, 1 remains, which reckoned to the Shillings, and 9 caſt away as oft as can be there remains 6, this trebled is 18, from which twice Nine caſt, there reſts 0 to be brought to the Pence, and from thence all the Nines rejected there remains at leaſt 8. then in the Total the Pounds make 6 Units, which doubled are 12, and 9 rejected leaves 3 to be reckoned to 4 in the Shillings, which 7 tripled make 21, from which 18, which is twice 9 rejected, there reſts 3 to be added to 5 in the Pence, which alſo make 8 parallel to the former Remain.

The Proof uncertain. But for the Reaſon above remembred in Addition of Integers, all the Proofs by caſting away 9 is uncertain, and the true Proof of *Addition* is by *Subſtraction*, as before taught in *Subſtraction of Integers*, and needs no further repetition here.

Beſt Proof.

Proof of Addition *of* Fractions *by the Value.* *Addition of Geodætical Fractions* beſides the Work above-mentioned may be proved by finding the Value of each Fraction before Addition, and adding them as *Geodætical* Integers, and comparing the Total with the Value of the Total of the added Fractions.

Example, If $\frac{1}{2}$ *l.* be added to $\frac{1}{3}$ *l.* the Total will be $\frac{5}{6}$ *l.* The Value of $\frac{1}{2}$ *l.* is 10 *s.* and $\frac{1}{3}$ *l.* is 6 *s.* 8 *d.* which added together make the Total to be 16 *s.* 8 *d.* and ſo much is $\frac{5}{6}$ *l.*

$$\frac{1}{2}\,l.\ \frac{\cancel{2}0}{\cancel{2}}\Big(10\,s. \;\Big|\Big|\; \frac{1}{3}\ \frac{\cancel{2}4\cancel{0}}{\cancel{3}}\Big(\frac{80}{\cancel{1}2}\Big(6\,s.\ 8d. \quad \begin{array}{rr} s. & d. \\ 10 & 0 \\ 6 & 8 \\ \hline 16 & 8 \\ \hline \end{array} \;\Big|\Big|\; \frac{5}{6}\,l. \quad \begin{array}{r} 240 \\ 5 \\ \hline 1200 \\ \hline \end{array} \quad \frac{1200}{6}\Big(\frac{200}{12}\Big(16s.\ 8d.$$

CHAP. IV.

Subſtraction of Geodæticals.

Geodæticals *Subſtracted.* IN *Subſtraction of Geodæticals*, the different Nature of the propoſed Numbers is to be conſidered.

1. *Caſe.* If the Numbers given be Integral *Geodæticals* only, and of the ſame kind or denomination,

1. Integers of one Denomination. Rule.

Then ſubſtract the Leſſer Number from the Greater, as was taught in Abſtract Integers, *Book* 1. *Part* 1. *Chap.* 6. and underſtand the remain to be of the ſame Denomination with the given Numbers.

1. Example.

1. *Example,* If there were delivered to one 546 *French* Crowns to buy ſome Commodities with, and he did diſburſe but 354 of them; then will remain in his hands 192.

2. Example.

2. *Example,* If one deliver me to keep for him 1006 *Liures Tournois,* and receive again at ſeveral times 821, then will 185 reſt in my hands.

1. *Ex. Fr.* △.		2. *Ex. l. Tour.*	
Delivered	546	Delivered	1006
Diſburſed	354	Received	821
Remaineth	192	Remaineth	185

2. Integers of ſeveral Denominations. To place the Data.

2. *Caſe.* If the given Number be Integral *Geodæticals* of different Denominations, Greater or Smaller, Engliſh or Foreign,

Then place the Greater or higheſt Denomination of the Number from which Subſtraction is to be made alwaies to the left hand, and the other Numbers in order to the right hand, and under the ſame the Subtrahend, ſo as Pounds may ſtand under Pounds, Shillings under Shillings, *&c.* likewiſe the Arithmetical Places of Units under Units, Tens under Tens, *&c.* are to be kept. And where any *Geodætical* Denomination is wanting, the ſame may be ſupplyed with Cyphers to keep place.

To Subſtract them.

Then begin at the right hand and deduct the Lower Numbers or Figures of the Subtrahend and leaſt *Geodætical,* out of the upper ſtanding over them particularly, ſubſcribing the Remain reſpectively under the ſame Files.

When borrowing needful.

If the neather Figure happen to be greater than the upper, then in imagination borrow one of the Denomination next to the left hand, and add to the upper Number, which is too little, and make Subſtraction from both, and ſubſcribe the Remain as before; and for that borrowed accompt one back in the next File, reckoning the next Figure to be ſubſtracted 1 more than it is, or the next Figure to be ſubſtracted from 1 leſs than it is.

Example.

Example, If *A.* lend to *B.* 344*l.* 10*s.* 6*d.* and *B* paid *A.* 124*l.* 6*s.* 9*d.* How much is yet owing? I place the Numbers as at *C.* and becauſe 9*d.* is greater than 6*d.* I borrow 1 Shilling, which is 12*d.* and put to 6*d.* and from the Total 18*d.* I take 9*d.* and there reſteth 9*d.* then coming to the Shillings, I reckon that 1 that I borrowed and 16 is 17, which becauſe I cannot take from 10, the Number over them I borrow 1 Pound, which is 20*s.* and put to the 10 makes 30*s.* from which 17 taken there remaineth 13*s.* to be ſubſcribed, and that 1*l.* borrowed reckoned to the 4*l.* next maketh 5, which I proceed with as in *Subſtraction of Integers,* and finiſh the reſt of the work accordingly, and by the laſt Remain find yet due to *A.* 219*l.* 13*s.* 9*d.* as at *D.*

	l.	*s.*	*d.*	
Lent by *A.*	344—	10—	06	
Paid by *B.*	124—	16—	09	*C.*

	l.	*s.*	*d.*	
Lent by *A.*	344—	10—	06	
Paid by *B.*	124—	16—	09	*D.*
Reſt due to *A.*	219—	13—	09	

When the Data are many.

When many Numbers are given from which Subſtraction is to be made, or many Numbers to be ſubſtracted from one, or many: Let all the Plurals of each ſort be ſeverally added into one, as is taught in *Subſtraction of Integers,* and then proceed in Subſtraction as above.

Examples of Weight, Meaſure, Time, and Motion.

In Subſtraction of Weights, Meaſures, Time, Motion, and other *Geodæticals, Engliſh* and *Foreign* in this Caſe: Let the like order be obſerved, as in the enſuing Examples.

Example,

Weight.					Meaſure.			Time.			Motion.			
Ton.	*C.*	*qrs.*	℔.	℥.	*Acres*	*Roods*	*Rods.*	*Years*	*Daies.*	*Ho.*	*Sig.*	*Deg.*	*Min.*	*Sec.*
40	18	3	24	12	340	2	39	1648	330	19	11	20	40	31
29	19	3	12	14	121	3	36	345	360	20	3	19	50	33
18	19	0	11	14	218	3	03	1302	334	23	8	00	49	58

3. *Fractions or mixt.*

3. *Caſe.* If the given Numbers be Fracted *Geodæticals*, Greater or Leſſer, or Integral mixed with Fracted,

Then proceed in the Subſtraction with the Fractions as Fractions, and Integers as Integers, according to their reſpective Rules.

Examples.

Examples, To ſubſtract $\frac{1}{15}$ *l.* from $\frac{7}{15}$ *l.* the Remain is $\frac{6}{15}$ *l.* as at *E.* by *Book* 1. *Part* 2. *Chap.* 4. *Caſe* 1. And to Subſtract $\frac{1}{4}$ *l.* from $\frac{1}{3}$ *l.* the Remain will be $\frac{1}{12}$ *l.* as at *F.* by *Caſe* 2. there. And if $3\frac{1}{3}$ *l.* be taken from 13 *l.* there will be left $9\frac{2}{3}$ *l.* by borrowing an Unite which is 3 Thirds, to take the $\frac{1}{3}$ from, and for the ſame paying 1 to the 3, which is 4, and ſubſtracting it from 13, after the manner of Integers above-mentioned, as at *G.*

E.

$$\overset{l.}{\frac{7}{15}} - \overset{l.}{\frac{1}{15}} = \frac{6}{15} \text{ or } \frac{2}{5}$$

F.

$$\overset{1}{\underbrace{\overset{4}{\frac{1}{3}}\,l. - \overset{3}{\frac{1}{4}}\,l.}_{12}} = \frac{1}{12}\,l.$$

G.

Lent—13 *l.*
Paid— $3\frac{1}{3}$
Reſt— $9\frac{2}{3}$

Proof of Geodætical Subſtraction *of the First and Third Caſes.*

Subſtraction of Geodæticals falling under the Firſt Caſe is proved like *Subſtraction of Integers* and *Fractions*, And the Subſtractions under the third Caſe, as the *Subſtraction of Integers*, according to the reſpective Operations uſed in the Subſtraction: For if Integers be taken from Integers, the Work is proved as *Subſtraction of Integers*, if Fractions from Fractions, as *Subſtraction of Fractions*, and generally *Addition* by *Subſtraction*, and *Subſtraction* by *Addition*.

Of the Second Caſe and reciprocally Addition *thereby.*

The *Subſtraction of Geodæticals* falling under the Second Caſe, admits of *Geodætical Addition* for Proof. For if the remain be added to the Number ſubſtracted, the Total will be parallel to the Number from which Subſtraction is made when the Work is right. And conſequently when from the Total of any *Geodætical Addition* of this ſort, one or more of the Numbers added, be ſubſtracted; the Remain of that Subſtraction will be equal to the Reſidue of thoſe Numbers added into that Total; and thereby prove that Addition right.

Addition of this ſort may alſo be proved by beginning at the left hand to ſubſtract the reſpective Files from the Figure of the Total under them, and if any thing remain to underwrite, or remember to make allowance of the ſame, in the next right hand Figure of the Total, as was ſhewed before in the Proof of *Subſtraction of Integers*, *Book* 1. *Part* 1. *Chap.* 6. For further Evidence view the Examples of *Addition* at *H.* and *Subſtraction* at *I.*

	Addition		*Proof by Subſtraction.*	
	l. *s.* *d.*	Thus	or	Thus
	22—13—00	22—13—00	22—13—00	51—13—05
	15—10—08	15—10—08		
H.	10—05—02	10—05—02	15—10—08	29—00—05
	03—04—07	03—04—07	10—05—02	
			03—04—07	22—13—00
	51—13—05	51—13—05		
			29—00—05	
		~~11~~—~~1~~—0		

Subſtraction

	l.	*s.*	*d.*	
	51	13	05	Lent by *H.* to *I.* at ſeveral times.
I.	15	10	08	Paid at times to *H.*
	10	05	02	
	03	04	07	
	29	00	05	Total of all the Payments to be ſubſtracted.
	22	13	00	Remaineth due to *H.*
	51	13	05	Proof of the *Subſtraction.*

Proof of Subſtraction of Fractions by the Value.

Subſtraction of Geodatical Fractions; beſides the Proof above mentioned may be proved by finding the Value of each Fraction before Subſtraction, and ſubſtract them as Integers, and compare the Remain with the Value of the Remain of the Subſtracted Fraction.

Example, To take $\frac{1}{4}$ *l.* from $\frac{1}{3}$ *l.* leaves $\frac{1}{12}$ *l.* the Value of $\frac{1}{4}$ *l.* is 5 *s.* and $\frac{1}{3}$ *l.* is 6*s.* 8*d.* from which 5 *s.* ſubſtracted leaves 1 *s.* 8 *d.* and ſo much is $\frac{1}{12}$ *l.*

$$\overset{l.}{\frac{1}{4}}\ \frac{20}{4}\Big(5s. \ \Big|\Big|\ \overset{l.}{\frac{1}{3}}\ \frac{240}{3}\Big(\frac{80}{12}\Big(6\,s.\ 8\,d. \qquad \begin{array}{c} s.\ \ d. \\ 6 - 8 \\ \hline 5 - 0 \\ \hline 1 - 8 \\ \hline \end{array} \qquad \Big|\Big|\ \frac{1}{12}\,l.\ \frac{240}{12}\Big(\frac{20}{12}\Big(1\,s.\ 8\,d.$$

CHAP. V.

Multiplication of Geodæticals.

Geodæticals multiplyed.

TO multiply *Geodaticals*, conſider what Caſe the propoſed Numbers fall under and accordingly proceed in their *Multiplication.*

1. *Single.*

1. *Caſe.* If the Numbers given to be multiplyed be Single *Geodaticals* both Integral, both Fracted, mixt Numbers, or the one an Integer, and the other a Fraction.

Rule.

Then multiply the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as mixt Numbers; of the firſt ſort is to be ſeen, *Book* 1. *Part* 1. *Chap.* 7.

Example 1.

1. *Example*, If 30 Men ſtood in a ſtraight line, every one 3 Feet before the other; and it were deſired to know how many Feet there were between the Front and the Rear. The Anſwer is the Product at *A.* 90 Feet.

Example 2.

2. *Example*, If 40 Planks each of 10½ Feet long were laid end to end; and the Queſtion were, how far they would reach, The Anſwer is the Product at *B*, 420 Feet.

Example 3.

3. *Example*, If a piece of Land contain in length 10 ¼ Yards, and in breadth 5⅓ Yards: And the Queſtion were to know the Area, or Content of the ſame, or how many Square Yards the whole did contain. The Anſwer is the Product at *C.* 54⅔ Square Yards.

Example 4.

4. *Example*, If ½ *l.* be multiplyed by $\frac{1}{6}$ *l.* The Product is $\frac{1}{12}$ *l* as at *D.*

A.

30 Men.
3 Feet.
90

B.

40 Planks.
10½ Feet.
400
20
420 Feet.

C.

$$\frac{656}{\underbrace{\overset{41}{10\frac{1}{4}} \times \overset{16}{5\frac{1}{3}}}_{12}} = \frac{656}{12} \text{ or } 54\tfrac{2}{3}$$

Yards.
Square.

D.

$$\frac{1}{\frac{1}{2}\,l. \times \frac{1}{6}\,l. = \frac{1}{12}\,l.}$$
$$12$$

Weight.					Meaſure.			Time.			Motion.			
Ton.	*C.*	*qrs.*	℔.	℥.	*Acres*	*Roods*	*Rods.*	*Years*	*Daies.*	*Ho.*	*Sig.*	*Deg.*	*Min.*	*Sec.*
40	18	3	24	12	340	2	39	1648	330	19	11	20	40	31
29	19	3	12	14	121	3	36	345	360	20	3	19	50	33
18	19	0	11	14	218	3	03	1302	334	23	8	00	49	58

3.
Fractions or mixt.

3. *Caſe.* If the given Numbers be Fracted *Geodæticals*, Greater or Leſſer, or Integral mixed with Fracted,

Then proceed in the Subſtraction with the Fractions as Fractions, and Integers as Integers, according to their reſpective Rules.

Examples.

Examples, To ſubſtract $\frac{1}{15}$ *l.* from $\frac{7}{15}$ *l.* the Remain is $\frac{6}{15}$ *l.* as at *E.* by *Book* 1. *Part* 2. *Chap.* 4. *Caſe* 1. And to Subſtract $\frac{1}{4}$ *l.* from $\frac{1}{3}$ *l.* the Remain will be $\frac{1}{12}$ *l.* as at *F.* by *Caſe* 2. there. And if $3\frac{1}{3}$ *l.* be taken from 13 *l.* there will be left $9\frac{2}{3}$ *l.* by borrowing an Unite which is 3 Thirds, to take the $\frac{1}{3}$ from, and for the ſame paying 1 to the 3, which is 4, and ſubſtracting it from 13, after the manner of Integers above-mentioned, as at *G.*

E.

$$\overset{l.}{\frac{7}{15}} - \overset{l.}{\frac{1}{15}} = \frac{6}{15} \text{ or } \overset{l.}{\frac{2}{5}}$$

F.

$$\underbrace{\overset{\overset{1}{\overline{4 \qquad\quad 3}}}{\frac{1}{3}l. - \frac{1}{4}l.}}_{12} = \frac{1}{12}l.$$

G.

Lent—— 13 *l.*
Paid—— $3\frac{1}{3}$
Reſt—— $9\frac{2}{3}$

Proof of Geodætical Subſtraction *of the First and Third Caſes.*

Subſtraction of Geodæticals falling under the Firſt Caſe is proved like *Subſtraction of Integers* and *Fractions*, And the Subſtractions under the third Caſe, as the *Subſtraction of Integers*, according to the reſpective Operations uſed in the Subſtraction: For if Integers be taken from Integers, the Work is proved as *Subſtraction of Integers*, if Fractions from Fractions, as *Subſtraction of Fractions*, and generally *Addition* by *Subſtraction*, and *Subſtraction* by *Addition.*

Of the Second Caſe and reciprocally Addition *thereby.*

The *Subſtraction of Geodæticals* falling under the Second Caſe, admits of *Geodætical Addition* for Proof. For if the remain be added to the Number ſubſtracted, the Total will be parallel to the Number from which Subſtraction is made when the Work is right. And conſequently when from the Total of any *Geodætical Addition* of this ſort, one or more of the Numbers added, be ſubſtracted; the Remain of that Subſtraction will be equal to the Reſidue of thoſe Numbers added into that Total; and thereby prove that Addition right.

Addition of this ſort may alſo be proved by beginning at the left hand to ſubſtract the reſpective Files from the Figure of the Total under them, and if any thing remain to underwrite, or remember to make allowance of the ſame, in the next right hand Figure of the Total, as was ſhewed before in the Proof of *Subſtraction of Integers*, *Book* 1. *Part* 1. *Chap.* 6. For further Evidence view the Examples of *Addition* at *H.* and *Subſtraction* at *I.*

	Addition *l.* *s.* *d.*	Thus	*Proof by Subſtraction.* or	Thus
	22—13—00	22—13—00	22—13—00	51—13—05
	15—10—08	15—10—08		
H.	10—05—02	10—05—02	15—10—08	29—00—05
	03—04—07	03—04—07	10—05—02	
			03—04—07	22—13—00
	51—13—05	51—13—05		
			29—00—05	
		xx—x—0		

Subſtraction

l.	s.	d.	
51	13	05	Lent by *H.* to *I.* at ſeveral times.
15	10	08	Paid at times to *H.*
l. 10	05	02	
03	04	07	
29	00	05	Total of all the Payments to be ſubſtracted.
22	13	00	Remaineth due to *H.*
51	13	05	Proof of the *Subſtraction.*

Subſtraction of Geodætical Fractions; beſides the Proof above mentioned may be proved by finding the Value of each Fraction before Subſtraction, and ſubſtract them as Integers, and compare the Remain with the Value of the Remain of the Subſtracted Fraction. *Proof of Subſtraction of Fractions by the Value.*

Example, To take $\frac{1}{4}$ *l.* from $\frac{1}{3}$ *l.* leaves $\frac{1}{12}$ *l.* the Value of $\frac{1}{4}$ *l.* is 5 *s.* and $\frac{1}{3}$ *l.* is 6 *s.* 8 *d.* from which 5 *s.* ſubſtracted leaves 1 *s.* 8 *d.* and ſo much is $\frac{1}{12}$ *l.*

l. $\frac{1}{4}\ \frac{20}{4}($ 5 *s.*

l. $\frac{1}{3}\ \frac{240}{3}(\ \frac{80}{12}($ 6 *s.* 8 *d.*

s. — *d.*
6—8
5—0
1—8

$\frac{1}{12}$ *l.* $\frac{240}{12}(\ \frac{20}{12}($ 1 *s.* 8 *d.*

C H A P. V.

Multiplication of Geodæticals.

TO multiply *Geodæticals*, conſider what Caſe the propoſed Numbers fall under and accordingly proceed in their *Multiplication.* Geodæticals *multiplyed.*

1. *Caſe.* If the Numbers given to be multiplyed be Single *Geodæticals* both Integral, both Fracted, mixt Numbers, or the one an Integer, and the other a Fraction. 1. *Single.*

Then multiply the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as mixt Numbers; of the firſt ſort is to be ſeen, *Book* 1. *Part* 1. *Chap.* 7. *Rule.*

1. *Example*, If 30 Men ſtood in a ſtraight line, every one 3 Feet before the other; and it were deſired to know how many Feet there were between the Front and the Rear. The Anſwer is the Product at *A.* 90 Feet. *Example* 1.

2. *Example*, If 40 Planks each of 10$\frac{1}{2}$ Feet long were laid end to end; and the Queſtion were, how far they would reach, The Anſwer is the Product at *B*, 420 Feet. *Example* 2.

3. *Example*, If a piece of Land contain in length 10$\frac{1}{4}$ Yards, and in breadth 5$\frac{1}{3}$ Yards: And the Queſtion were to know the Area, or Content of the ſame, or how many Square Yards the whole did contain. The Anſwer is the Product at *C.* 54$\frac{2}{3}$ Square Yards. *Example* 3.

4. *Example*, If $\frac{1}{2}$ *l.* be multiplyed by $\frac{1}{6}$ *l.* The Product is $\frac{1}{12}$ *l* as at *D.* *Example* 4.

A.
30 Men.
3 Feet.
90

B.
40 Planks.
10$\frac{1}{2}$ Feet.
400
20
420 Feet.

C.
656
$$\underbrace{\frac{41}{10\frac{1}{4}}\times\frac{16}{5\frac{1}{3}}}_{12}=\frac{656}{12}\text{ or }54\tfrac{2}{3}$$
Yards.
Square.

D.
1
$$\frac{1}{2}\,l.\times\frac{1}{6}\,l.=\frac{1}{12}\,l.$$
12

2. *One of the Data Single, and the other Plural.*

2. *Caſe.* If one of the given Numbers be a Plural *Geodætical*, and the other a Single. Then reduce the Plural Number into the loweſt Denomination, and multiply the Reſult by the other, or if the Single *Geodætical* be a Digit or other ſmall Number; you may eaſily multiply every of the Denominate Numbers ſeverally by that Number, and carry in imagination or otherwiſe to the next greater Denomination, ſo many as the reſpective Products will afford, ſubſcribing the Remaines.

Example.

Example, If a Merchant pay to 20 Porters 1 *l.* 6 *s.* 8 *d.* a piece, and it were deſired to know how much Money in all was paid. Firſt 1 *l.* 6 *s.* 8 *d.* reduced into Pence is 320, which multiplyed by 20, produceth 6400, and reduced back into Shillings and Pounds giveth 26 *l.* 13 *s.* 4 *d.* as at *E*, or otherwiſe 20 Eight-pences which is 40 Groats, or 13 *s.* 4 *d.* and the 13 *s.* carried to the place of Shillings leaveth the 4 *d.* to be ſubſcribed, then 20 times 6 *s.* is 6 times 20 *s.* or 6 *l.* to be carried to the Denomination of Pounds, and the 13 *s.* left to be ſubſcribed. So at laſt the 6 *l.* added to the 20 times 1 *l.* or 20 *l.* makes 26 *l.* 13 *s.* 4 *d.* as at *F.*

```
 1 l. 6 s. 8.          E.
 20
 ——
 20                  4(4 d.
  6                  6400(
 ——                  12  ( 53|3 s.
 26                        ————————
 12   320 pence            26:13:4
 ——   20 Porters           ————————
 52   ————
268   6400
 ——   ————
320
 ——
```

```
   Thus,      or Thus,       or Thus,
l.  s.  d.    l.      F.    l.  s.  d.
1—6—8         1 1/3          1—6—8
     20       20                  20
—————————     ———           ——————————
26—13—4       20                13—4  Product of 20 by 8d.
—————————      6 2/3           6—0    Product of 20 by 6s.
              ——— l.        20        Product of 20 by 1 l.
              26 2/3        ——————————
              ———           26--13—4
                            ——————————
```

3. *Data Plural, and the Product Simple.*

3. *Caſe.* If both the given Numbers be Plural *Geodæticals*, and the Product required Simple, and of the loweſt Denomination,

Rule.

Then reduce the Numbers into their loweſt Denominations, and multiply the Reſults as Integers.

Example.

Example, If 20 *l.* 10 *s.* 3 *d.* be multiplyed by 5 *l.* 4 *s.* 2 *d.* both being reduced make 4923 *d.* and 1250 *d.* which multiplyed produce 6153750 *d.* and they if occaſion be may be reduced into Shillings and Pounds, as at *G*.

```
 l.  s.  d.      l.  s.  d.
 20—10—3          5—4—2        4923
 20              20            1250
 ———             ———          ———————
 400             100          246150                    G.
  10               4          9846        3912(6        s.
 ———             ———          4923       6153750(51281 | 2
 410             104          ———————    2222222———————————
  12              12          6153750     22222  25640—12—6
 ———             ———          ———————            ————————————
 820             208
 4103            1042
 ———             ———
 4923            1250
 ———             ———
```

4. *Data Plural and Product Compound. Variety 1.*

4. *Caſe*, If both the Given Numbers be Plural *Geodæticals*, and the Product required Compound,

Then firſt after Reduction of the Numbers and Multiplication of the Reſults as before, divide the Product by the Product of every Leſſer Denomination contained in one Greater multiplyed together.

Example.

Example, In the Numbers laſt Exemplary, becauſe 240 Pence are contained in 1 Pound, and Pence are the Leſſer Denominations in both the given Numbers, the Product of 240 multiplyed by 240, which is 57600 dividing the 6153750 the Product above obtained, the Quotient will be 106 *l.* and if the Value of the Fraction remaining be gotten, and added thereto; the whole *Geodætical* will be 106 *l.* 16 *s.* 8 $\frac{5}{8}$ *d.* as at *H*.

```
240        (4                 4815   (41           414                       360 | 180 | 90 | 5
240        38(81                20   3874|4         12           (360        ----|-----|----|--
---        61587(5 0(106 l.   -----  96200|0(16 s.  ----          4968(8 d.  576 | 288 |144 | 8
9600       57666 0|0          96300  57866|0        828            576
480        577                -----  57             414
----       5                                        ----
57600                                               4968
```

Or Secondly, Under the Reduced Numbers, Place the Number of Leſſer Denominations contained in one Greater; and multiply as in Fractions. *Variety* 2.

Example, Under the before-reduced Numbers 4923 and 1250, placing 240, and multiplying them as Fractions, the alike Product will ariſe as followeth at *I*. *Example.*

```
     205125       |1           (13            (120
     1641   125   13|60        119|8          1656(8 d.
     4923   1250  20512|5(106l. 3210|0(16 s.  192
I.   ----   ----  19222 0      1922|0
     240    240   199          19             120 | 60 | 30 | 15 | 5
     24     80    1                     138   ----|----|----|----|--
     -----------          1605          12    192 | 96 | 48 | 24 | 8
        1920                20          ---
                          -----         276
                          32100         138
                                        ----
                                        1656
```

Or Thirdly, If the *Geodæticals* be of one Nature, turn all the Leſſer Denominations into Fractionary Parts of the Greater, and multiply as in the 5th. Caſe of *Multiplication of Fractions*, *Book* 1. *Part* 2. *Chap*. 5. or multiplying Number by Number carry out of each Multiplication ſo many of the next Greater Denomination as are therein, and Subſcribe the Remain. *Variety* 3.

Example, Becauſe the *Geodæticals* aforeſaid are both one Nature, *viz*. Sterling Money, the 10*s*. 3 *d*. and 4 *s*. 2 *d*. turned into Fractions, and after Multiplication of the Integers 20 by 5, proceeding to add the parts of the Multiplicand thereto: The like Product appears at *K*. *Example.*

20*l*. 10*s*. 3 *d*. or 20 $\frac{1}{2}$ and $\frac{1}{80}$ × 5 *l*. 4 *s*. 2 *d*. or 5 $\frac{1}{5}$ and $\frac{1}{120}$ produce 106*l*. 16*s*. 8*d*. $\frac{5}{8}$.

	l. *s*. *d*.		*l*.				*l*. *s*. *d*.
Thus	20—10—03		or Thus 20 $\frac{1}{2}$ and $\frac{1}{80}$				
	5— 4— 2		5 $\frac{1}{5}$ and $\frac{1}{120}$				
	100	Product of 5 by 20.	100	The Product of	5	by 20 is 100	100—00—00
	2—10	Five Angels.	2$\frac{1}{2}$		5	by $\frac{1}{2}$ or half pounds	3—10—00
	1— 3	Five Three pences.	0$\frac{1}{16}$		5	by $\frac{1}{80}$ abbreviated is	0—01—03
K.	4—00—00	Fifth part of 20 *l*.	4		$\frac{1}{5}$	of 20 *l*. which is	4—00—00
	2	Fifth part of 10 *s*.	0$\frac{1}{10}$		$\frac{1}{5}$	of $\frac{1}{2}$ *l*. is $\frac{1}{10}$ *l*. or	0—02—00
	00$\frac{3}{5}$	Fifth part of 3 *d*.	0$\frac{1}{400}$		$\frac{1}{5}$	of $\frac{1}{80}$ *l*. or reduced, is	0—00—0$\frac{3}{5}$
	3—04	The 120 part of 20*l*.	0$\frac{1}{6}$		$\frac{1}{120}$	of 20 *l*. abbreviated is	0—03—04
	01	The 120 part of 10*s*	0$\frac{1}{240}$		$\frac{1}{120}$	of $\frac{1}{2}$ *l*. is $\frac{1}{240}$ *l*. or	0—00—01
	00$\frac{1}{40}$	The 120 part of 3*d*.	0$\frac{1}{9600}$		$\frac{1}{120}$	of $\frac{1}{80}$ *l*. reduced is	0—00—0$\frac{1}{40}$
	106—15—08$\frac{5}{8}$	The Total Product.	*l*. 106$\frac{2}{3}$ [illegible]				106—16—8$\frac{5}{8}$

Or, Fourthly, Multiply Number by Number, beginning at the left hand, and to find the Denomination of the ſeveral Products obſerve the firſt line ſhall be denominate, as the Multiplicand, the firſt left hand Figures of the other Lines of Production ſhall be denominate, as the Figures of the Multiplicand under which they ſtand, and all the other Figures in each line reſpectively are parts of one of that denomination, and are Numerators, under which for Denominator you may place or imagine placed as followeth, that is to ſay, under the ſecond placed Figures to the right hand, the number of the next leſſer Denomination to the greater of the Multiplicand, and under the next right hand Figures the number of the next leſſer Denomination, and ſo accordingly under all the Numbers And to add theſe ſeveral Lines of Production into one Total, firſt begin at the left hand, and ſubſcribe the left hand Figures under a line in the place of the greater Denominations as they ſtand, and then collect all the next right hand Collumne Integers together, and ſubſcribe them as in *Addition of Integers*, and ſo do with the *Variety* 4.

Integers in the other Columnes, and then all the Fractions belonging to each line of Production ſeverally, and add them together; and what of the other Denominations are contained in them, place in their Order, and if any thing remain, reduce it to a part of the loweſt Denomination of the Total. And laſtly add all theſe ſubſcribed Numbers into one Total.

Example.

Example, The former Numbers thus multiplyed produce for the Firſt Line 100. 50. 15. for the Second 80. 40. 12. and for the Third 40. 20. 6. as at *L. M. N.* then for 40 in the ſecond Line, becauſe Shillings is the next leſſer Denomination to Pounds, 20 ſhall be the Denominator, and the Denominator to 12 ſhall be 240, the Pence in 1 *l.* and ſo ſhall they be $\frac{40}{20}$ *s.* and $\frac{12}{240}$ *s.* And the like in the third line $\frac{20}{20}$ *d.* and and $\frac{6}{240}$ *d.* Then collecting them as before directed for the $\frac{40}{20}$ *s.* is ſet 2 *s.* and $\frac{12}{240}$ is reduced to $\frac{3}{5}$ of a Peny by Abbreviation, and multiplying the Numerator by 12, and ſo proceeding, the other Fractions make $1\frac{1}{40}$ *d.* and the Numbers ſtand as at *O.* And for plainer Demonſtration another Example is added at *P*, where 2 *l.* 10 *s.* 3 *d.* 2 *q.* is multiplyed by 2 *l.* 5 *s.* 2 *d.* 2 *q.*

Denominations *l.* *s.* *d.*
Multiplicand 20—10—3 *O*
Multiplier 5—4—2

Lines of Production
L. 100—50—15
M. —80—$\frac{40}{20}$—$\frac{12}{240}$
N. ——40—$\frac{20}{20}$—$\frac{6}{240}$

Collection of——100
50 *s.* & 80 *s.*——6—10
15 *d.* & 40 *d.*——4—7
$\frac{40}{20}$ *s.* & $\frac{12}{240}$ *s.*——2—$0\frac{3}{5}$
$\frac{20}{20}$ *d.* & $\frac{6}{240}$ *d.*——$1\frac{1}{40}$
Total Product——106—16—$8\frac{1}{8}$

l. *s.* *d.* *q.*
2—10—3—2 P.
2—5—2—2

4—20—6—4
10--$\frac{50}{20}$--$\frac{15}{240}$—$\frac{10}{960}$
4—$\frac{20}{20}$—$\frac{6}{240}$—$\frac{4}{960}$
4—$\frac{20}{20}$—$\frac{6}{240}$—$\frac{4}{960}$

4
1—10—
10—
2—
2—6—$3\frac{1}{2}$
1—$0\frac{7}{60}$
$1\frac{7}{240}$

5—13—8—$0\frac{3}{4}\frac{1}{8}$

Proof of Geodætical Multiplication.

Multiplication of Geodæticals is proved, if the Numbers be Integral, as *Multiplication of Integers*, if Fracted, as *Multiplication of Fractions*, that is in both by *Diviſion*, as is further noted in the next Chapter.

Proof of Multiplying Fractions by the Value.

Moreover the *Multiplications of Geodætical Fractions* may be examined, by finding the Value of the Fractions to be multiplyed, and the Product thereof, and comparing the ſame with the Product of their Values, multiplyed as Integers; only obſerving to divide this Product by the Number of one of the Leſſer Denominations contained in the Greater.

Example, If $\frac{2}{3}$ *l.* be multiplyed by $\frac{3}{4}$ *l.* the Product will be $\frac{1}{2}$ *l.* And $\frac{2}{3}$ *l.* being 13 *s.* 4 *d.* or 160 Pence, and $\frac{3}{4}$ *l.* being 15 *s.* or 180 Pence, theſe multiplyed produce 28800, which divided by 240, the Pence or Leſſer Denomination contained in one Pound the Greater Denomination, the Quotient will be 120 Pence, or $\frac{1}{2}$ *l.* as before.

$$\underset{2}{\overset{1}{\frac{\overset{1}{2}}{\underset{1}{3}}\times\frac{\overset{1}{3}}{\underset{2}{4}}}} = \frac{1}{2}\,l.\text{ or }10\,s.$$

$\frac{2}{3}$ *l.* = 13 *s.* 4 *d.* or 160 *d.*
$\frac{3}{4}$ *l.* = 15 : 0 or 180 *d.*

12800
160
28800

$$\begin{array}{l} 4 \\ \cancel{2}8800(\cancel{1}\cancel{2}0(10\,s. \\ \cancel{2}\cancel{4}4\ 0\ \cancel{1}\cancel{2} \\ \cancel{2} \end{array}$$

Proof of Multiplication *in the Third and Fourth Caſes.*

And ſo alſo the Operations of multiplying *Geodæticals* in the Third and Fourth Caſes of this Chapter may be compared the one with the other, the Leſſer Denominations being in that Fourth Caſe conſidered as Fractions, and dividing for their Value accordingly.

ly. Becauſe though in multiplying Integers the Multiplicand is increaſed in Figures and Value ſo many times as the Multiplier containeth Units; yet in Fractions, and that ſort of Multiplying *Geodæticals*, though the Figures may be increaſed ſo many times, yet in Value the New Fraction or Product is made ſo much leſs. For a Fraction being properly leſs than one, and making another Number ſo many times leſs alſo; Muſt needs produce a Number ſo much leſs, as the multiplying Fraction containeth parts in it, and the Product is but the Value of a Fraction of a Fraction.

As in the laſt Inſtance, If $\frac{3}{4}$ *l.* be taken for the Multiplier, then $\frac{3}{4}$ of 13 *s.* 4 *d.* or $\frac{2}{3}$ *l.* ſhall be 10 *s.* And ſo is $\frac{2}{3}$ of 15 *s.* which is $\frac{3}{4}$ *l.* If $\frac{2}{3}$ be taken for the Multiplier, the Product $\frac{1}{2}$ *l.* or 10 *s.* is leſs than $\frac{2}{3}$ *l.* or 13 *s.* 4 *d.* by $\frac{1}{4}$ thereof, which is 3 *s.* 4 *d* and leſs than $\frac{3}{4}$ *l.* or 15 *s.* by $\frac{1}{3}$ thereof, which is 5 *s.* But in Integers if they be multiplyed under the Denomination of Shillings, the Product is 20 times 10 *s.* or 200 *s.* if in Pence 28800, as before, which is 240 times as much. And this was the Reaſon for that Diviſion before to equalize their Value. Neverthelеſs this is to be underſtood of Proper Fractions and Leſſer *Geodæticals*, for Improper Fractions are redundant, and increaſe by their Units they contain after the manner of Integers.

CHAP. VI.

Diviſion of Geodæticals.

Geodæticals *divided.*

TO Divide *Geodæticals*, obſerve the Caſe the Numbers propoſed fall under, and proceed accordingly in their Diviſion.

1. *Single.*

1. *Caſe*, The Numbers given to be divided being Single *Geodæticals*, both Integral, both Fracted, or Mixt Numbers, or one an Integer or Mixt Number, and the other a Fraction.

Rule.

Then divide the Integers as Integers, and the Fractions as Fractions, and the Mixt Numbers as Mixt Numbers, Of the firſt ſort, See *Book* 1. *Part* 1. *Chap.* 8. and of the other, *Book* 1. *Part* 2. *Chap.* 6.

Example 1.

1. *Example*, If 41984 *l.* be divided equally among 164 Men, and it be deſired to know how much each ſhould have for his Part, The Anſwer is 256 *l.* the Quotient at *A.*

Example 2.

2. *Example*, If a Lane be 11880 Feet long, and I would know, how many Rods that is, I divide them by $16\frac{1}{2}$ the Feet in a Rod. And the Anſwer is 720 Rods, the Quotient at *B.*

Example 3.

3. *Example*, If a Field be $54\frac{2}{3}$ Square Yards, and 1 ſide thereof be $10\frac{1}{4}$ Yards, and the other ſide be demanded, The Quotient anſwereth at *C.* $5\frac{1}{3}$ Yards.

Example 4.

4. *Example*, If $\frac{1}{4}$ be divided by $\frac{1}{2}$ *l.* The Quotient is $\frac{1}{2}$ *l.* as at *D.*

A.

9
91
41984(256 *l.*
16444
166
1

B.

$16\frac{1}{2}=\frac{\overset{1}{33}}{2}\Big)\frac{\overset{360}{11880}}{1}\Big(720$ Rods

C.

$10\frac{1}{4}=\frac{\overset{1}{41}}{4}\Big)54\frac{2}{3}=\frac{\overset{4}{164}}{3}\Big(\frac{16}{3}=5\frac{1}{3}$ Yards.

D.

$\frac{1}{\underset{1}{2}}\Big)\frac{1}{\underset{2}{4}}\Big(\frac{1}{2}$ *l.*

2. *One of the* Data *Single, and the other Plural.*

2. *Caſe*, One of the given Numbers being a Plural *Geodætical*, and the other a Single,

Rule.

Then reduce the Plural into the Loweſt Denomination, and divide the Reſult by the other. Or if the Single *Geodætical* be a Digit, or other ſmall Number you may eaſie enough divide every one of the Denominate Numbers ſeverally, carrying in imagination from the Greater Denomination to the next Leſſer, the Number remaining on Diviſion of the Greater, if any be, reduced and added to the Leſſer, and make the Diviſion from the Total.

Example.

Example, If 15 *l.* 13 *s.* 1 *d.* be divided equally among 13 Men, to know how much each Man had. Firſt, 15 *l.* 13 *s.* 1 *d.* reduced make 3757, and divided by 13, and after by 12 and 20, gives 1 *l.* 4 *s.* 1 *d.* to every Man, as at *E.* or otherwiſe dividing 15 *l.* by 13, gives 1 *l.* in the Quotient, and 2 *l.* left, which 2 *l.* or 40 *s.* being reduced brought and added imaginary to the 13 *s.* makes 53 *s.* out of which 13 may be had 4 times to be ſet in the Quotient, and 1 *s.* will be left, which being 12 *d.* and added to the 1 *d.* makes 13, out of which the Diviſor may be had once, and the Quotient as at *F.*

```
 l.  s.  d.         E.                                   F.
 15—13—1
 20
 ———
 300                 d.                              2    1
  13           111   4(1   s. l.                  ———————————
 ———           3757(289 (2(4(1                   l.   s.   d.  l.  s.  d.
 313           1333 122  20                  13)15—13— 1(1—4—1
  12            11   1                          ———————————
 626                                             13   13   13
 3131
 ———
 3757
 ———
```

3. Data Plural, and the Quotient Simple. Rule. Example.

3. *Caſe.* Both the given Numbers being Plural *Geodæticals*, and the Quotient deſired Simple, and of the loweſt Denomination,

Then reduce the Numbers into their Loweſt Denominations, and divide the Reſults as Integers.

Example, If 25640 *l.* 12 *s.* 6 *d.* be divided by 5 *l.* 4 *s.* 2 *d.* both being reduced into Pence make 6153750, and 1250, which divided yield in the Quotient 4923 *d.* which if occaſion be, may be reduced into Shillings and Pounds, as at *G.*

```
     l.     s.  d.    l. s. d.
  25640—12—6      5—4—2
      20              20
  ———————          ————
  512800             100                   G.
      12               4               3
  ———————          ————          1 28        1  d.     s.
  512812             104         6153750(4923 (3 (41|0
      12              12         1255550  122    ————————
  ———————          ————           1222    1     20 : 10 : 3
  1025624            208           11            ————————
  5128126           1042
  ———————          ————
  6153750           1250
  ———————          ————
```

4. Data Plural, and the Quotient Compound. Rule by Reduction. Example.

4. *Caſe*, Both the given Numbers being Plural *Geodæticals*, and the Quotient required Compound,

Then after Reduction of the Numbers as above, if the Dividend be the Reduction of the Simple Product, Multiply the Diviſor thus reduced by the Number of the Leſſer Denomination contained in one Greater, and by this Product divide the Dividend.

Example, If 25640 *l.* 12 *s.* 6 *d.* were to be divided by 5 *l.* 4 *s.* 2 *d.* their Reductions are as before 6153750, and 1250, and 1250 the Diviſor multiplyed by 240, the Pence in one Pound, produce for the New Diviſor 300000, by which Diviſion being made, the Quotient is 20 *l.* and if the value of the remaining Fraction be gotten and added thereto, the whole *Geodætical* will be as at *H.* 20 *l.* 10 *s.* 3 *d.* as before.

```
   1250     H.  6|153750(20 l.      30|75 | 00(10 s.      900( 3 d.
    240          ————————            ——  |  ——           300
  ———————        3  00000           3  00 | 00
   50000
   2500           15375              75
  ———————            20              12
  300000         ———————           ————
  ———————        307500             150
                 ———————             75
                                   ————
                                    900
                                   ————
```

Rule without Reduction.

But Secondly, If the Product given to be divided were compound without Reduction. Then firſt reduce the given Numbers into their leaſt Termes, and divide them as Integers or Fractions, as the Caſe happens.

Example. If 106 *l.* 16 *s.* 8 $\frac{5}{8}$ *d.* be divided by 5 *l.* 4 *s.* 2 *d.* both reduced make $\frac{205125}{8}$ and $\frac{1250}{1}$ and after Diviſion the Quotient is $\frac{1641}{80}$, or 20 *l.* 10 *s.* 3 *d.* is found at *I.* as before.

106 *l.*

l. *s.* *d.*
106—16—8$\frac{5}{8}$
20

2120
16

2136
12

4272
21368

25640
8

205125

l. *s.* *d.*
5—4—2
20

100
4

104
12

208
1042

1250

l.
10 1641
1250) 205125 (1641
1 8 80

l.
~~1~~6|41 (20
8 0

41
20

820

s. *d.*
8|20 (10 24(3
8 0 8

2
12

24

Or Secondly, Without *Reduction*, divide the Greater Denomination of the Dividend by the Greater Denomination of the Divisor, and an apt Quotient Figure or Figures found thereby, Multiply the whole Divisor by this Quotient, and placing the Product under the Dividend, ſubſtract it therefrom, and ſet the Remain at top, then by the Greater Denomination of the Divisor inquire out of the Remainer for another Quotient Figure, and thereby multiply the whole Diviſor, and ſubſtract the Product as before, and ſo proceed to the end of the Work. *Variety.*

Example, In the Numbers laſt Exemplary, if 5 *l.* divide 106 *l.* there may be taken for the Quotient 20 *l.* by which 20, if the whole Diviſor be multiplied, the Product will be 104 *l.* 3 *s.* 4 *d.* which ſubſtracted, the Work will ſtand as at *K.* Then out of the 2 *l.* 13 *s.* or 53 *s.* left, inquiring by 5 *l.* for another part of the Quotient 10 *s.* may be had, which multiplying the whole Diviſor as before, and ſubſtracting the Product, the Work will ſtand as at *L.* And likewiſe proceeding for the remaining Numbers, the Work will ſtand as at *M.* *Example.*

K.

2 —13— 4

l. *s.* *d.*) *l.* *s.* *d.* (*l.*
5— 4— 2) ~~1~~0~~6~~—~~1~~6—8$\frac{5}{8}$ (20
20

100—80—40 104— 3— 4

L.

1—3
~~2~~—~~1~~~~3~~—~~4~~

l. *s.* *d.*) *l.* *s.* *d.* (*l.* *s.*
5— 4—2) ~~1~~0~~6~~—~~1~~6—8$\frac{5}{8}$ (20—10
20—10

100—80—40 104— 3—4
50—$\frac{40}{20}$—$\frac{20}{240}$ 2—12—1

M.

~~1~~—~~3~~
~~2~~—~~1~~~~3~~—~~4~~

l. *s.* *d.*) *l.* *s.* *d.* (*l.* *s.* *d.*
5— 4— 2) ~~1~~0~~6~~—~~1~~6—8$\frac{5}{8}$ (20—10—3
20—10— 3

100—80—40 104— 3—4
50—$\frac{40}{20}$—$\frac{20}{240}$ 2—12—1
1—3$\frac{5}{8}$
15—$\frac{12}{20}$—$\frac{6}{240}$

Proof of Geodæticals.

Diviſion of Geodæticals in general is proved by *Multiplication*, and *Multiplication* by *Diviſion* alternately: *viz. Integers* as *Integers*, *Fractions* as *Fractions*, *Simple* as *Simple*, and *Compound* as *Compound*. For by making the Product of any *Multiplication* the Dividend, and the Multiplicand or Multiplier, the Diviſor; the one of them will be returned in the Quotient, if the *Multiplication* be right. So on the contrary, if the Quotient of any Diviſion be made the Multiplicand or Multiplier, and the Diviſor be the other; the Product will return the Dividend, if the *Diviſion* were right. Obſerving only to add to the Product the Remain of the *Diviſion*, if any be.

Diviſion *and reciprocally* Multiplication *thereby.*

Example whereof may be ſeen, *Book* 1. *Part* 1. *Chap.* 8. *Page* 36. and by comparing the *Multiplications* in the former Chapter with the Diviſions in this will be ſufficiently evident without further Example.

Laſt ſort of Diviſion *proved by* Addition.

In particular the laſt ſort of *Diviſion* at *M.* may be proved by *Addition*, for by adding all the Multiplees ſubſtracted in the *Diviſion*, the Total with the Remain, when any is, will return the Dividend.

$$\text{Products}\left\{\begin{array}{r} l.\quad s.\quad d.\\ 104\text{—}03\text{—}04\\ 2\text{—}12\text{—}01\\ 1\text{—}03\tfrac{1}{8}\end{array}\right\}\text{Multiplees}$$

$$\text{Total}\qquad 106\text{—}16\text{—}08\tfrac{1}{8}\qquad \text{Dividend}$$

Proof of Diviſion of Fractions *by the Value.*

Furthermore *Diviſion of Geodætical Fractions* may be tryed by finding the Value of the *Fractions* to be divided, and the Quotient, and comparing the ſame with the Quotient of their Values divided as *Integers*, the Dividend being firſt multiplyed by the Number of one of the Leſſer Denominations contained in the Greater.

Example, If $\frac{1}{2}$ *l.* be divided by $\frac{3}{4}$ *l.* the Quotient ſhall be $\frac{2}{3}$ *l.* and $\frac{1}{2}$ *l.* the Dividend being 10 *s.* or 120 *d.* multiplyed by 240 the Pence, or Leſſer Denomination contained in 1 *l.* the Greater Denomination, make 28800, which divided by 180 *d.* the Value of $\frac{3}{4}$ *l.* gives in the Quotient 160 *d.* which is 13 *s.* 4 *d.* or $\frac{2}{3}$*l.*

$$\frac{3}{4}l. = 15\,s.\ \text{or}\ 180\,d.$$

$$\begin{array}{c}\frac{3}{4}\\ \vdots\\ 2\end{array}\Big)\begin{array}{c}\frac{1}{2}\\ \vdots\\ 1\end{array}\Big(\frac{2}{3}\ \text{or}\ 13\,s.\ 4\,d.\qquad \frac{1}{2}l. = 10\,s.\ \text{or}\ 120\,d.$$

$$\begin{array}{r} 240\\ \hline 4800\\ 240\ \ \\ \hline 28800\\ \hline \end{array}\qquad\qquad \begin{array}{l} \cancel{1}\cancel{0}\qquad\ \ \overset{d.}{\cancel{4}}(4\ s.\\ \cancel{2}8800(\cancel{1}\cancel{6}\cancel{0}(13\\ \cancel{1}88\ \ 0\ \ \cancel{1}\cancel{2}\cancel{2}\\ \cancel{1}\qquad\quad\ \cancel{1}\end{array}$$

In Fractions *a Greater Number may divide a Leſſer.*

Hereby it is evident that in *Fractions* a Greater Number may divide a Leſſer, though in *Integers* it cannot, and that when *Diviſion* is ſaid to make a Sum leſs in Numeration, though the Quotient may be of greater Denomination than the Dividend was: It is to be underſtood of *Integers*, and not of *Fractions* that are Proper *Fractions*, Seeing in this Example $\frac{3}{4}$ of a Pound the Greater Number both in Figures and Value, can divide $\frac{1}{2}$ of a Pound the Leſſer Number both in Figures and Value, and bring forth a Quotient bigger than the Dividend. For 13 *s.* 4 *d.* is greater than 10 *s.* by $\frac{1}{3}$ thereof, and yet divided by $\frac{3}{4}$ or 15 *s.* which is alſo greater than 10 *s.* by $\frac{1}{2}$ thereof.

But in *Integers* and *Improper Fractions* the Caſe is otherwiſe; for in the former alwayes the Numbers will be leſſened, and in the latter ſometime leſſened, and ſometime increaſed, according as the Greater *Improper Fraction* is the Diviſor or the Dividend, which without Example is obvious enough in every Operation.

Partis primæ Libri Secundi

FINIS.

THE SECOND PART OF THE SECOND BOOK.

CHAP. I.

Of FIGURALS.

THE next kind of Numbers to be viewed are called *Figural Numbers*, becauſe they either do or may repreſent ſome Geometrical Figure, and are ever conſidered in relation to thoſe Formes, and from thence borrowing their particular Names or Denominations, are rightly placed among Numbers generally Contract.

Next ſort of Numbers generally contrary, are Figurals, why ſo called.

The knowledge of *Figures*, their various Formes, and how to make and meaſure them, belongs to *Geometry*, and is there to be ſought, but the Numbers contained in the Figures, and how to Order, Increaſe, and Diminiſh them, belong to *Arithmetick*, and thoſe of ſpecial uſe therein, to be found here.

What of them belongs to Geometry, and what to Arithmetick.

Theſe Numbers are thoſe underſtood by the Name of Figural Numbers, and are as various, as the Figures in *Geometry*, from whence they take their Names.

Figural Numbers what.

Geometrical Figures of Chief Note and moſt conveniently fitted to ſerve our turn may be thus aſpected.

A Table of ſome Geometrical Figures.

- Figures
  - Lineary
    - Sides.
    - Roots.
  - Superficial
    - Orbicular
      - Perfect . . . . . Circles.
      - Imperfect . . . . Ovals.
      - &c.
    - Cornered
      - Sided
        - Uniforme
          - Triangles.
          - Quadrangles.
            - Oblongs.
            - Trapezia's
            - &c.
          - Multangles.
        - Omniforme . . . . Like Flats.
      - Rooted . . . . Squares.
  - Solid
    - Globular
      - Perfect . . . . Sphæres.
      - Imperfect
        - Cilinders.
        - Cones
    - Angled
      - Sided
        - Pyramides
        - Priſmes.
      - Rooted
        - Cubes.
        - Long-Cubes.
        - Square Cubes,
        - &c.

Accordingly *Figural Numbers* are of Three ſorts, *viz. Lineary*, *Superficial*, and *Solid*.

Figurals of 3 ſorts.

Lineary, have compariſon and relation to length only, and are therefore ſo called, becauſe Unite is imagined to ſtand by Unit along in a Line, as in the following Figures of 2. 3. 4. 5. which of all others is to be underſtood.

1. Lineary *what.*

Demonſtrated.

This Name, although it be properly referred to ſuch Numbers as will make no other form duely; yet it may alſo be applyed to any Number Abſtract, becauſe all ſuch Numbers may be taken as the ſides of other Figural Numbers.

Side *of a Number, what and how called.* Root *of a Number what, how called, and why.*

The Side (in Latine, *Latus*) is the Length of the Line containing the Side of the Figure: And if the Figure be equal-ſided, or æquilateral, then is the ſide called a *Root* or *Radix* (after the Latine) by a Metaphor, becauſe from thence other Figural Numbers ariſe as Branches from the Root of a Vegetable: Yet are not all equal Sides Roots, unleſs by *Multiplication* they can produce their Figures.

2. *Superficial, what and how called.*

Superficial Numbers, called alſo *Flat*, or *Plain Numbers*, and ſometime *Surfaces* are conſidered according to ſuch Form as they make in *Multiplication* or *Progreſſion*, and have both Length and Breadth.

Sorts of Superficial Numbers. Oval, *whence the Name.*

Of *Superficies*, Some are Orbicular, and of them one ſort only perfectly round, as Circles, and others imperfectly round as Ovals, ſo called from *Ovum*, Latine for an *Egg*, becauſe they bear the reſemblance thereof. Others are partly round, and partly of other Formes. The Figural Numbers of all which Figures I ſhall ſay nothing to, the Circle excepted, and of that but ſparingly.

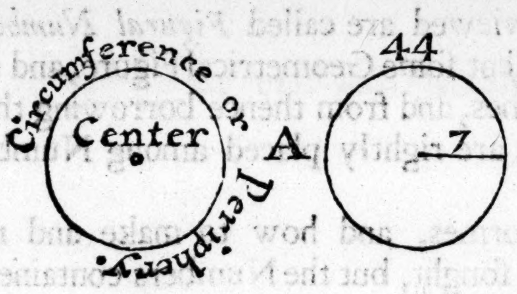
Circle *what,* Circumference, Periphery *what,* Center *what.* Semidiameter *what.* Diameter *what.*

A Circle is a plain Figure, determined with one bowed Line called the Circumference or Periphery, in whoſe midſt is a Point named the Center, From which all right Lines drawn to the Circumference are equal, the Circumference being alwaies æquidiſtant from the Center, as appeareth at *A.* Theſe Lines are called *Semidiameters.* And if the Line be drawn through the Center, and have both its Extremes ending in the Circumference, this is a *Diameter*, and the Longeſt Line a Circle is capable to contain.

Circular Number *what.* Area *of a* Circle.

A *Circular Number* is the Superficial Content, or after the *Latine* the *Area* of a Circle found by multiplying the *Semidiameter* into the *Semicircumference*, as at *B.*

Demonſtration.



Semicircumference——22
Semidiameter ——— 7

Circular Number. 154 or *Area.*

Numbers called Circular *on another accompt.*

Some Numbers ſometime are called Circular, becauſe as a Circle turns to the Point whence it began; ſo they being multiplyed by themſelves, end in themſelves; as 5 and 6, for 5 times 5 is 25, and 6 times 6 is 36, but here Circular Numbers are not taken in that Sence, but to be underſtood as before deſcribed.

Other Sorts of Superficies.

Other *Superficies* are Cornered or Angled, of which ſome are æquilateral, others inæquilateral; but none Rooted ſave the Square. The reſt not Rooted of divers ſorts, if conſidered *per ſe* ſimply are of one Form, if *inter ſe*, comparatively are of divers Formes, and therefore called *Like Flats*, or *Flats* that are alike.

Uniforme.

The *Uniforme Superficies* not Rooted are diſtinguiſhed according to the Number of *Angles* therein: If 3 then called a *Triangle*, if 4 a *Quadrangle*, if more a *Multangle* or *Polygone*.

Triangle, *what*

A *Triangle* is a Figure comprehended of 3 Lines, and containeth as many *Angles*, or *Corners*.

Sorts of Triangles.

Triangles are Plain or Sphærical, according as the Lines are ſtraight or bowed, whereof they are made; and are named both from their *Sides*, and from their *Angles*.

Æquilateral. If the 3 Sides be equal, the *Triangle* is called *Æquilateral.*

Iſoſcheles. If but 2 of the Sides be equal, it is named an *Iſoſcheles*.

Schalenum. If all 3 Sides be unequal, it is called a *Schalenum*.

Right Angled. If it have one *Right Angle*, that is containing 90 Degrees of a *Circle*, then it is called a *Right Angled Triangle*, or *Rectangled Triangle*, and after the *Greek* an *Orthogon.*

Broad Angled. If it have an *Angle* greater than a *Right Angle*, it is called an *Obtuſe* or *Broad Angled Triangle*, or an *Ambligonium.*

Sharp Angled. If it have all the *Angles* leſs than a *Right Angle*, it is called an *Acute*, or *Sharp Angled Triangle*, or an *Oxigonium.*

And accordingly by the mixture of ſuch *Sides* and *Angles* are the *Triangles* known aſunder.

Leggs of a Triangle *how called.*

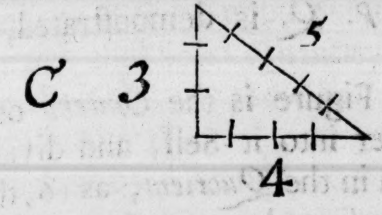
Every of the 3 Lines or Sides ſometimes called the Leggs, and in *Latine*, *Crura*, have their diſtinct Names, as the *Perpendicular*, the *Baſe*, and the *Hypotenuſa*.

Triangular Number *what.*

A *Triangular Number* is the *Area* of a *Triangle*, but becauſe there is no certain Number from which, as from a *Root* ſuch *Area* may be found, but differs according to the

Sides and *Angles* of every *Triangle*; it will not be meet to digreſs further than only to view the Plain Right Angled Triangle at *C*, and the rather, becauſe ſome of the Enſuing Diſcourſe may depend upon the knowledge thereof.

Demonſtration.



3 Perpendicular.
4 Baſe.

12 Product.

6 *Area*, or the *Triangular Number* of this *Triangle*.

Quadrangles what, and their kinds.

The Genus of *Quadrangles* include Squares as well as other Four-corned Figures, but becauſe they are Rooted Figures, they take their Place by themſelves hereafter. The other Species of *Quadrangled Figures* are an *Oblong*, a *Trapezium*, a *Rhombus*, and a *Rhomboide*.

Oblong what.

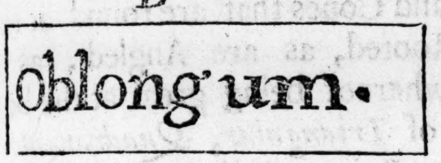
An *Oblong*, called alſo a *Long Square*, and ſometime a *Rectangle Figure* is an *Inæquilateral Paralellogram* conſiſting of 4 *Right Angles*, but only the oppoſite ſides equal, as the Figure at *D*.

Oblong Number what.

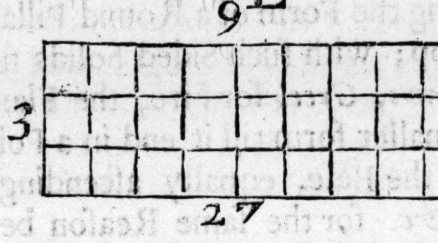
An *Oblong Number* is the *Area* of ſuch Figure, which divided by the Leſſer ſide bringeth the Greater in the Quotient, and if by Greater bringeth forth the Leſſer, as 27 coming of 9 and 3, if divided by 3, giveth 9; if by 9 yieldeth 3 in the Quotient repreſented at *E*. or *F*.

Demonſtration.

D



E



9 F. $\frac{27}{9}$(3
3 IIIIIIIII
IIIIIIIII
IIIIIIIII $\frac{27}{3}$(9
27

Diagonal, by ſome called Diameter, and thence the Name of Diametral Numbers.

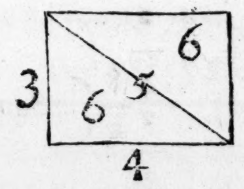
If a Right Line be drawn through the Center of this Figure from one Oppoſite Angle to the other; this Figure will be divided into Two Right Angled Triangles, as the Figure at *G*. ſheweth. This *Diagonal Line* is called by ſome a *Diameter*, and from thence the Name of *Diametral Numbers* came.

Diametral Numbers the uſe.

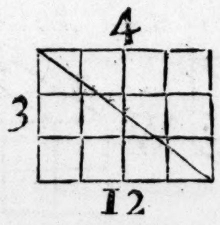
A *Diametral Number* ſerveth to find out the length of this *Diagonal Line*, and hath two parts of that Nature, that if they be multiplyed together, will make the ſaid *Diametral Number*, and the Squares of theſe two parts added together will make a *Square*, whoſe *Root* is the Length of the *Diametral* or *Diogonal Line* to that *Diametral Number*, as 12 is called a *Diametral Number*, becauſe it hath Two parts, *viz.* 3, and 4, which produce it by *Multiplication*, and the Square of 3 is 9, and of 4 is 16, which 9 and 16 make together 25, whoſe *Square Root* is 5, the length of the *Diagonal Line* to the *Platform* 12, and of the *Hypotenuſa* of each *Triangle* as at *G*. or *H*.

Demonſtration.

G.



H



3	3	4	9	5
4	3	4	16	5
12	9	16	25	25

Trapezium, what.

A *Trapezium* hath all Four Sides unequal.

Rhombus what.

A *Rhombus* hath all four ſides equal, but never a *Right Angle*.

Rhomboide, what.

A *Rhomboide* hath the Oppoſite ſides equal, but never a *Right Angle*.

Theſe and all *Multangles* being uncertain in the Meaſure of their Sides, and ſo conſequently in their Figural Numbers are ſet aſide here.

Omniforme Superficies how called.

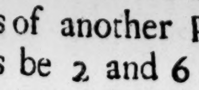
Thoſe *Superficial Figures* called *Like Flats*, whether *Triangular*, *Quadrangular*, or otherwiſe, are ſuch Plain *Homogeneal Superficies*, as bear a certain Proportion in their Sides unto each other, when compared together, as the Figures at *I*. and *K*. declare.

Numbers Like Flats, what.

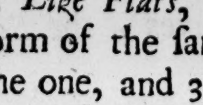
And Numbers called *Like Flats*, have the Sides of one bear like proportion to the Sides of another Platform of the ſame kind. As the Long Squares *L*. and *M*. If the Sides be 2 and 6 of the one, and 3 and 9 of the other, the Figures are *Like Flats*. And ſo the Numbers that expreſs their Quantities, which are 12 and 27, are called *Like Flats*, becauſe 6 is Triple to 2, as 9 is to 3.

Demonſtration.

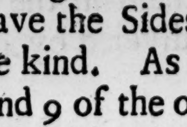
I.



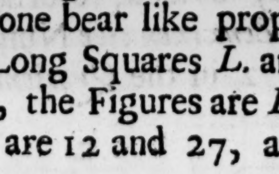
K



L



M



$\frac{6}{2}$(3

9 (3

A Square

Square Figures, *the Names.*

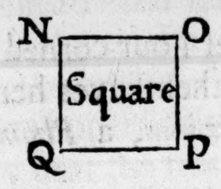
Rooted.

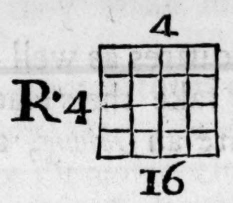
A *Square Figure*, called alſo a *Quadrate*, from the *Latine*, *Quadratus*, and by the *Arabians Zenſus*, is a *Regular Superficies*, or an *Æquilateral Paralellogram* made by 4 *Equal Right Lines*, and as many *Right Angles*; every one of which Line is called the *Side* or *Root* of the *Square*, as by the Figure *N. O. P. Q.* is demonſtrated, whoſe *Sides* are *N. O.* and *O. P.* and *P. Q.* and *Q. N.*

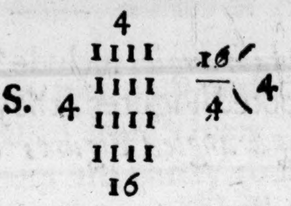
Square Numbers *what.*

A *Square Number* anſwering to the Similitude of this Figure is the *Content* or *Area* thereof, which is produced by multiplying any Number into it Self, and divided by his *Side* or *Root*, bringeth his Root (or *Diviſor*) again in the *Quotient*, as 16, the *Product* of 4 by 4, divided by 4, returneth the *Root* repreſented, as at *R.* or *S.*

Demonſtration.







3\.
Solid *what, and how called.*

Solid Numbers, called ſometime Sound and Corporeal, or Bodily Numbers, are the Third ſort of Figural Numbers, and imploy both Length, Breadth, and Depth or Thickneſs; otherwiſe called Height, according to the Number of *Multiplications* whereof they are produced.

Sorts of Solid Numbers.
Axis *what.*

Of *Solid Numbers*, the only perfect *Globular* is the *Sphare*, or *Globe* repreſented in *Plano* as the Circle, and the *Axis* thereof as the Diameter of the other. This Term being often uſed for that, though the Diameter of a *Circle* is never properly called an *Axis*.

Cilinders *and* Cones, *their Forms.*
Pyramis, *whence the Name.*
Priſmes, *no* Rooted Solids.

Cilinders, repreſenting the Form of a Round Pillar; and Cones that are round at the Baſe, and pointed at Top; with ſuch Sided Solids not Rooted, as are Angled, as a *Pyramis* ſo called from πυς, *Greek* for Fire, the Flame whereof being great at the bottom, aſcendeth into a ſmaller form till it end in a Point) of *Triangular*, *Quadrangular*, or *Multangled* Form at the Baſe, equally aſcending to a Point; And *Priſmes* looking like a Writing-Desk, *&c.* for the ſame Reaſon before noted in ſome of the *Superficies* not Rooted; find no other Room here than to be mentioned.

Rooted Solids.

Among the Regular Rooted Solids, a *Cube* hath the præcedency.

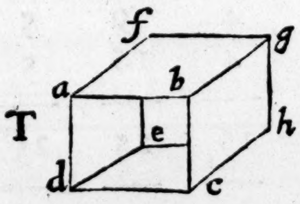
Cube, *what.*

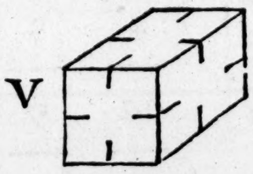
A *Cube*, in *Latine*, *Cubus*, is a Right Angled *Paralelipipedon* æquiangled, and æquilateral, having 6 equal Surfaces, 8 Solid Angles, and 12 Sides, every one of which is the *Root*, repreſented by a Figure like a *Dye*, at *T*, whoſe Surfaces are *a. b. c. d.* and *a. b. f. g.* and *a. d. e. f.* and *b. c. h. g.* and *c. d. e. h.* and *e. f. g. h.* The *Angles a. b. c. d. e. f. g. h.* And the *Sides a. b.* and *a. f.* and *b. g.* and *b. c.* and *c. d.* and *c. h.* and *d. a.* and *d. e.* and *e. f.* and *e. h.* and *f. g.* and *g. h.*

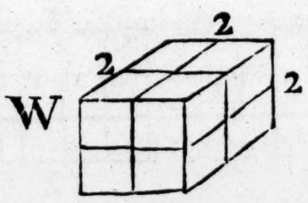
Cube Number *what.*

A *Cube* or *Cubick Number* is the *Solid Content* of ſuch Figure which is the Product of the *Square* multiplyed by the *Root*, and if divided by the *Root* or *Side*, bringeth the *Square* of the *Diviſor* in the *Quotient*, as 8 the *Cube* of the *Root* 2, divided by 2 giveth 4 the *Square* of 2 in the Quotient figured, as at *V.* or *W.*

Demonſtration.







2 Root
2
—
4 $\frac{8}{2}$(4 Square.
2
—
8
—

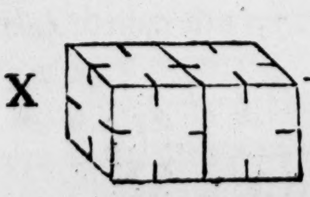
Long Cube, *or* Squared-Square *what.*

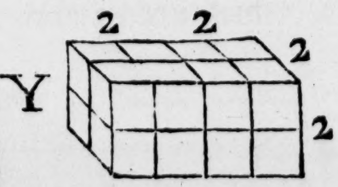
A *Long Cube* (the next Regular Rooted Solid to a *Cube* is a Body of greater Length than Breadth or Depth, yet is the Length in proportion to them; for every Unit in his Depth or Breadth, begets a *Cube* equal to the firſt *Cube* in his Length, as the Figure at *X.* or *Y.* ariſing from the *Root* 2. make evident.

The Number thereof how called.

A *Long Cube Number*, called alſo a *Square of Squares*, ſometime a *Biguadrate*, and commonly a *Zenzizenzike* Number, is the Number expreſſing the Solidity of that Figure; produced by multiplying the *Square* by it ſelf, or the *Cube* by the *Root*, and being accordingly divided, returneth them in the *Quotient*, as 16 in Relation to the Root, 2 is a *Zenzizenzike* Number divided by 4 gives 4 the *Squares*, or by 2 yields 8 the *Cube* of 2 in the *Quotient*.

Demonſtration.





2 Root
2
—
4
2
—
8
2
—
16
—

$\frac{16}{4}$(4 Square

$\frac{16}{2}$(8 Cube.

Squared Cube *or* Surſolide *what.*

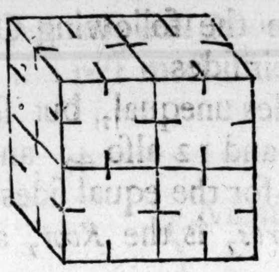
A *Squared Cube* (the next in order) hath Length and Depth alike; but wants equal Breadth, yet ſo as every *Unite* of his Breadth begets a *Cube* both in Length and Depth, as the Figures at *Z*, or *a* from the aforeſaid *Root* 2 make evident.

The Number thereof why called Surſolide.

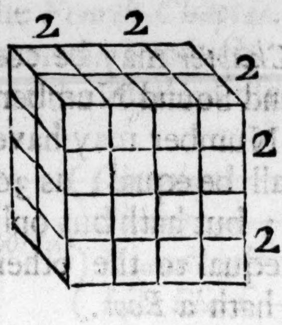
A *Squared Cube Number*, in *Arithmetical Termes* is called a *Surdeſolide*, or *Surſolide*, (perhaps as *Solid* upon *Solid*, *Sur* in *Engliſh* implying as much as *Super* in *Latine*) and contains the Solid Quantity of the Figure, being begotten by multiplying the *Square* into the *Cube*, or the *Squared Square* by the *Root*. And by *Reciprocal Diviſion* returns them in the *Quotient*, as 32 is the *Surſolide* to the *Root* 2. If divided by 4 gives 8 the *Cube*, and by 2 brings 16 the *Squared Square* of 2 in the *Quotient*.

Demonſtration.

Z.



a



2 Root.

$$\begin{array}{r} 2 \\ \underline{2} \\ 4 \\ \underline{2} \\ 8 \\ \underline{2} \\ 16 \\ \underline{2} \\ \underline{32} \end{array}$$

$\frac{32}{4}\Big($ 8 *Cube.*

$\frac{32}{2}\Big($ 16 *Squared Square.*

Cubed Cube *or* Squared Cube *what.*

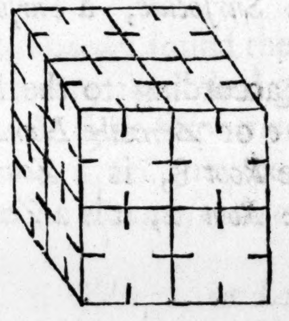
A *Cubed Cube* is the Second Regular Body. *Arithmetick* takes notice of having *Square Solidity*, and is every way increaſed according to the *Units* originally in the *Root* thereof, as the Figures at *b.* or *c.* ariſing from the *Root* 2 demonſtrate.

The Number and Names thereof.

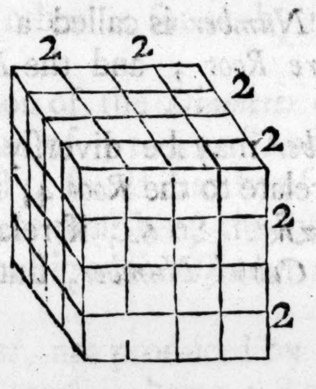
A *Cubick Cube Number* is ſo called in relation to his Form only, otherwiſe the Number which declareth the Solidity of ſuch Figure is called a *Zenzicube*, or a *Squared Cube Number*, made by multiplying the *Surſolide* into the *Root*, or the *Squared Square* by the *Square*, or the *Cube* by it ſelf, and divided accordingly, brings forth the reſpective Numbers in the Quotient, as plain in 64 the *Zenzicube* of 2.

Demonſtration.

b



c



2 Root.

$$\begin{array}{r} 2 \\ \underline{2} \\ 4 \\ \underline{2} \\ 8 \\ \underline{2} \\ 16 \\ \underline{2} \\ 32 \\ \underline{2} \\ \underline{64} \end{array}$$

$\frac{64}{16}\Big($ 4 Square.

$\frac{64}{8}\Big($ 8 Cube.

$\frac{64}{4}\Big($ 16 Squared Square.

$\frac{64}{2}\Big($ 32 Surſolide.

Names Geometrically *and* Arithmetically.

Further Inſtances may be ſpared, ſince you may proceed infinitely on *Rooted Solides*. But it is to be noted that every *Figure* receiveth its name *Geometrically*, according to his Form. And every Number containing the ſolid quantity thereof, being divided by his *Root*, giveth in the Quotient, the Number of the next leſſer quantity. And theſe Numbers ſo containing the ſolidity in *Arithmetick*, have names not ſo much reſpecting the *Geometrical* Forms of the Figures, as the forming or producing of the Numbers themſelves: For in the laſt *Example* 64. according to his Form *Geometrical* is a *Cubed Cube*, but in *Arithmetick* goes by the name of a *Squared Cube*, becauſe the number 64, is formed by ſquaring 8, the *Cube* of 2, the *Root*. And ſo before 16, the *Squared Square* was ſo called, not becauſe of his form, which was a *Long Cube*; but becauſe it was the ſquare of 4, which was the ſquare of the *Root* 2.

Reaſon of the Arithmetical *Names.*

How to name the Higher Powers Geometrically.

And briefly to know how to name *Geometrically* every greater Form or higher Power (as ſome call them) that in order are increaſed by their *Roots*; obſerve that the fifth Body is named like the ſecond, only doubling the *Cubes*, as *Long Cubick Cubes*. And the ſixth Body like the third, as *Square Cubick Cubes*. And ſo the ſeventh Body like the fourth, tripling the *Cubes*. And ſo keeping the words *Long*, *Square*, and *Cube*, to every *Ternary*, the word *Cube* is added, as in the *Table* of *Rooted Numbers* in the next *Chapter* is ſufficiently clear.

How to name them Arithmetically.

But the Terms *Arithmetical* retained to theſe *Rooted Powers*, take their riſe from the original *Rooted Numbers*, *viz.* the *Square* and the *Cube*, except the *Surſolids*, and keep this order in accompt. The firſt *Rooted Number* is called a *Square*, the ſecond a *Cube*, the third a *Squared Square*, the fourth a *Surſolide*, the fifth a *Squared Cube*, the ſixth a

Second Surſolide, the ſeventh a *Square of Squared Squares*, the eighth a *Cube of Cubes*, the ninth a *Square of Surſolides*, the tenth a *Third Surſolide*, the eleventh a *Square of Squared Cubes*, the twelfth a *Fourth Surſolide*, &c. And in *Coſſical Numbers* in the next *Book* ſhall be ſhewn, how to proceed infinitely to name ſuch Numbers by their *Indices*.

Names of all Figurals *by their Quantities.*

Moſt regular to call the Root *the firſt Quantity.*

Yet ſome content themſelves to call theſe *Figural Numbers* neither after their *Geometrical* Forms, nor yet their antient *Arithmetical* Terms; but according to their Content or Quantity. And ſo they call a *Square* the firſt quantity, a *Cube* the ſecond quantity, a *Squared Square* the third quantity, *&c.* And others more regularly call the *Root* a Number of the firſt quantity, a *Square* the ſecond quantity, a *Cube* the third quantity, *&c.*

Obſervations.

All further needful to this *Chapter* may be conſidered in the following Obſervations.

1. Which have ſides.

1. All Angled Superficial and Sound Numbers have their ſides.

2. Unequal ſides many, Equal not more than 2, unleſs all.

2. One and the ſame Plain Number may have many ſides unequal, but ſeldom more than two equal ſides, except all be equal; as 36 hath 3, and 12 alſo 4, and 9, and 2, and 18, for the unequal ſides; but hath but only 6 and 6 for the equal ſides.

3. Which the Root.

3. The one ſide which is equal to the other in *Squares*, is the *Root*, and no *Flat Number*, ſave only a *Square*, hath a *Root*.

4. Which of the Solides *Rooted.*

4. Among *ſolid Numbers* they only have *Roots*, which be made of many Multiplications of ſome one Number by it ſelf, or by that which ariſeth thereof.

5. Which a Surde Number.

5. That Number whoſe Sides cannot be expreſſed by a *Whole Number* is called a *Surde Number*, and is no exact *Square* nor *Cube Number*, and ſuch are all *Prime Numbers*, and (*Squares* only excepted) the moſt part of all *Compound Numbers*. For if any *Whole Number* have a *Root*, that *Root* ſhall be a *Whole Number*.

6. What the Root *is, and how differs from the* Side.

6. The *Root* of a Number is a Number alſo, and is the ſide of the *Figural Number*: But every *Side* is not a *Root*, only the *Equal Side*, as aforeſaid, yet ſometime *Root* and *Side* are uſed *Synonimically*.

7. Roots *are infinite, and are differenced by* Adjectives.

7. *Roots* are as infinite as *Figures*; for any *Number* may be a *Root*, and the *Root* is always denominate according to his *Number*: For the *Root* of a *Square* ſhall be called a *Square Root*; the *Root* of a *Cube Number* is called a *Cubick Root*; ſo the *Root* of a *Squared Square* is a *Squared Square Root*; and the *Root* of a *Surſolide*, a *Surſolide Root*, &c.

8. One Number diverſly called as he ſtands related.

8. One and the ſame *Solide Number* may be diverſly named, according to the *Root* he ſtands related to; as 16, if it relate to the *Root* 4, is a *Square* or *Zenzike Number*, but if to the *Root* 2, is a *Zenzizenzike*. So 64, if related to the *Root* 8, is a *Zenzike Number*, and if to the *Root* 4, is a *Cubick Number*, but if to the *Root* 2, it is a *Zenzicube*, or a *Squared Cube Number*.

CHAP.

CHAP. II.

Production of Figurals.

IN *Figural Numbers* is further to be learned the *Geneſis* and *Analyſis*. The firſt of theſe I call *Production*, and teach in this *Chapter* the manner thereof. The other is to be found in the next. And becauſe *Figural Numbers* principally converſe with *Integers*; *Figurate Fractions* are deferred to the *Fourth Chapter*.

Production of Figurals.

To lay by thoſe *Figural Numbers* of uncertain Product, and proceed orderly in the production of the reſt moſt uſual: Obſerve the Method uſed in the following *Sections*.

Production of *Figural*
- *Numbers* not *Rooted*
  - *Circulars* — §. 1.
  - *Oblongs* — §. 2.
  - *Diametrals* — §. 3.
  - *Like Flats* — §. 4.
- *Rooted Numbers*
  - in General — §. 5.
  - in Particular
    - *Squares* — §. 6.
    - *Cubes* — §. 7.
    - *Higher Powers* — §. 8.

The Figurals *whoſe Production is taught in this Chapter.*

§. 1. The multitude of *Figural Numbers* both *Superficial* and *Solide* wanting *Roots*, whoſe Formes in *Geometry* deſerve inſpection: As they are of leſs concern in *Arithmetick*, ſo they occupy the leſs room here, but only 4 being touched, and that briefly.

Circular Numbers *produced*.

Circular Numbers (as in the next precedent *Chapter*) before noted in the ſenſe here uſed, are but the *Areas* of the *Circles* found out as aforeſaid, by multiplying the *Semidiameter* into the *Semiperiphery*, which are both uſually given to produce them. If but one of them be given, the other is found by the proportion of the one to the other.

Archimedes found the Proportion of the *Diameter* of a *Circle* to the *Circumference*, to be a very ſmall deal greater than of 7 to 22. And of late *Ludoph van Ceulen* inſiſting in the ſame ſteps more preciſely, found it to be of 1 to 3, 141592653589793. for which 3, 1416. may be taken; but moſt keep the former Numbers of 7 to 22, which make the *Circumference* 3 times as big as the *Diameter*, and $\frac{1}{7}$ part more.

Proportion of the Diameter *to the* Periphery, *by* Archimedes *and* Van Ceulen.

§. 2. *Oblongs*, or *Long Squares*, are produced by Multiplying one Side by the other adjoyning. As a *Field* or other *Superficies* being 3 Rods broad, and 8 long, the Form of that *Superficies* is a *Long Square*, and the Content thereof 24 Rods; obtained by Multiplying 3 into 8.

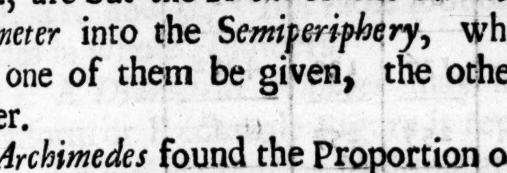
Oblongs *produced*.

If one Side be unknown, having the Content and the other Side, Divide the Content by the known Side, and the Quotient will ſhew the Side unknown. As 24 divided by 3, gives 8; or by 8, gives 3, in the Quotient.

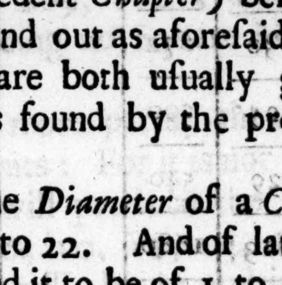
To find a Side unknown.

Thoſe *Long Squares* whoſe Area is a Square Number, may be reduced from an Oblong Form to a perfect Square Figure. As a *Long Square*, whoſe Sides are 3 and 12, or 4 and 9, &c. and conſequently the Area thereof 36, which becauſe it is a Square Number of the Root 6, if each Side of the Platform be reduced to 6, the Figure will be a Regular Square, as here appeareth.

Which may be turned into Squares.



Area 36.



Area 36.

12 Side
3
36

6 Root
6
36

§. 3. *Diametral Numbers* were deſcribed before, and are produced as *Oblongs*, by Multiplying their proper parts together; or one Side of the Rectangle Figure by the other. As 60, produced by the Sides or Parts thereof, 5 and 12. Several others with their reſpective Sides may be ſeen in the *Table* following.

Diametral Numbers *produced.*

The Table of Diametral Numbers *unto the Leſſer Side* 40.

***A Table of* Diametral Numbers.**

Leſſer Side.	Greater Side.	Diameter or Diagone.	Diametral Number.	Leſſer Side.	Greater Side.	Diameter or Diagone.	Diametral Number.
3	4	5	12	25	60	65	1500
5	12	13	60		312	313	7800
6	8	10	48	26	168	170	4368
7	24	25	168	27	36	45	972
8	15	17	120		120	123	3240
9	12	15	108		364	365	9828
	40	41	360	28	96	100	2688
10	24	26	240		195	197	5460
11	60	61	660	29	420	421	12180
12	16	20	192	30	40	50	1200
	35	37	420		72	78	2160
13	84	85	1092	31	480	481	14880
14	48	50	672	32	60	68	1920
15	20	25	300		126	130	4032
	36	39	540		255	257	8160
	112	113	1680	33	44	55	1452
16	30	34	480		180	183	5940
	63	65	1008		544	545	17952
17	144	145	2448	34	288	290	9792
18	24	30	432	35	84	91	2940
	80	82	1440		120	125	4200
19	180	181	3420		612	613	21420
20	48	52	960	36	48	60	1728
	99	101	1980		105	111	3780
21	28	35	588		160	164	5760
	72	75	1512		323	325	11628
	220	221	4620	37	684	685	25308
22	120	122	2640	38	360	362	13680
23	264	265	6072	39	52	65	2028
24	32	40	768		252	255	9828
	45	51	1080		760	761	29640
	70	74	1680	40	75	85	3000
	143	145	3432		96	104	3840
					198	202	7920
					399	401	15960

Touching *Diametral Numbers*, 3 things are further conſiderable.

What is conſiderable in Diametral Numbers.

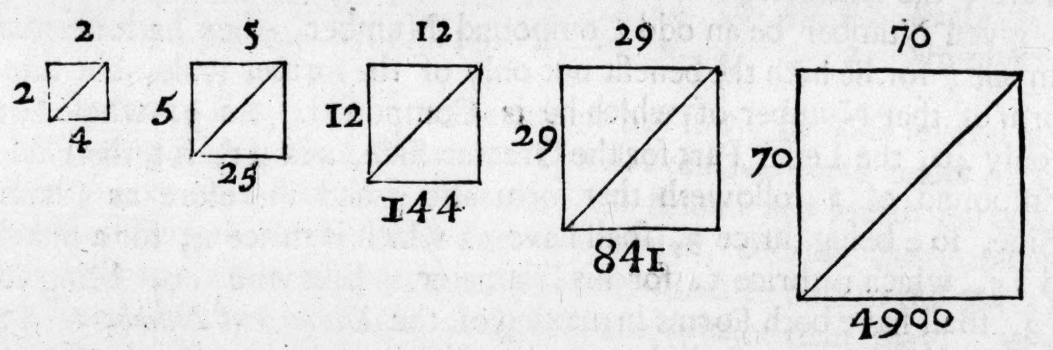
1. Somewhat concerning the knowledge of them in general.
2. If the Leſſer Side of a *Diametral Number* be given, to find out the other.
3. When a Number is propounded, to diſcover if it be a *Diametral Number*, and conſequently to find the Sides.

For the firſt of theſe, let be minded.

Of the knowledge of them.

1. Diametral Numbers *ſet forth a Plain Rightangled Triangle.*

1. All *Diametral Numbers* do ſet forth a Plain Rectangled Triangle, having all 3 Sides known; which as it is rare, and of great uſe in many Geometrical Concluſions, ſo is it to be found in no other Numbers than only in *Diametral Numbers*: For though in Geometrical Figures you may ever infallibly find a Line, that will make a Square equal to the two Squares of any other two Lines; yet the certain meaſure of thoſe Sides are not known in whole Numbers. And though other Numbers may go very nigh, yet it can never be done exactly but with *Diametral Numbers*, as in the following Examples of ſome *Square Numbers*.



Whoſe Doubles I take for the Squares of the Sides unknown, and they make 8, 50, 288, 1682, and 9800. All which differ only by an Unite from being Square Numbers; for 9 is a Square, and ſo are 49, 289, 1681, and 9801. and their Roots 3, 7, 17, 41, and 99. But thoſe Doubles being no Square Numbers, cannot render their Sides in whole Numbers.

2. Diametral Numbers *may have more parts than be apt for the Sides of the Rectangle.*

2. A *Diametral Number* may have more parts, then be apt for the Sides of the Platform or Rectangle Figure it repreſents: For it is not every two parts of the *Diametral Number*, that by Multiplication will produce the Number, that be meet Sides. As 60 hath theſe parts, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30. And beginning with the two extreams, *viz.* 2 and 30, they will, being multiplyed together, produce 60. And ſo will likewiſe any two of thoſe Numbers or Parts equally diſtant from thoſe extreams; as 3 and 20, alſo 4 and 15, likewiſe 5 and 12, and 6 and 10, but none of them ſave 5 and 12 are apt Sides to find the Diameter by. For if the other Sides be multiplyed ſquarely, and the Squares added together, there will ariſe no Square Number.

3. *If the leaſt Side be odd, ſo ſhall the Diameter.*

3. As there are 3 Numbers, to wit, the 2 Sides and the Diameter; ſo alwayes if the firſt or leaſt Side be odd, then ſhall 2 of the 3 Numbers be odd alſo; and the Diameter ſhall be the ſecond and great odd Number.

4. Diametrals *even Numbers.*

4. All *Diametral Numbers* are even, and no odd Number can be a *Diametral Number*.

5. *Odd Numbers above 1, may be the leaſt Side.*

5. All odd Numbers ſave 1, may be the Leſſer Side to *Diametral Numbers*, but even Numbers do not ſerve ſo generally. For if the Leſſer Side be an even Number, he muſt exceed 4; and if the Greater Side be an even Number, he muſt be ſuch an one as may be divided by 4.

6. *Leſſer Side odd what the Square ſhall be.*

6. If the Leſſer Side be an odd Number, then ordinarily the Square of it is juſt equal with the ſumme which amounteth by Addition of the Diameter to the Greater Side. As in the former Platform of 60, where 5 being the Leſſer Side, his Square is 25, and ſo much do the Diameter 13, and the Greater Side 12 make added together.

7. *One Side may ſerve divers* Diametrals.

7. One Side may ſerve to divers *Diametral Numbers*, as 9 to the *Diametral Numbers* 108 and 360, begotten of the Sides 12 and 40, &c.

8. Diametrals *divided by 12.*

8. There is no *Diametral Number*, but it may be evenly divided by 12, wherefore they be all even Numbers, evenly and oddly.

9. *How they end.*

9. There is no *Diametral Number*, but it endeth in 0, in 2, or in 8.

10. *They cannot have more than one Diameter*, &c.

10. There is no *Diametral Number* can have any more Diameters than one, and yet may one Number be the Diameter to divers other. As 25 is the Diameter to 168 and 300, &c.

11. Squares *no* Diametrals.

11. No Square Number can be a *Diametral Number*.

12. *They may be the* Area *of an* Oblong, &c.

12. Though every *Diametral Number* be the Area of a Long Square, yet the Area of every Long Square is not a *Diametral Number*.

Of finding the Side.

Secondly, When the Leſſer Side or Part of a *Diametral Number* is given to find the other thereby. This Leſſer Side will be a—

$$\text{Number}\begin{cases}\text{Odd}\\ \text{or}\\ \text{Even.}\end{cases}\begin{cases}\text{Uncompound}\\ \text{or}\\ \text{Compound.}\end{cases}$$

When the Datum *is Odd and Uncompound.*

If the propounded Number be an odd Uncompound Number, multiply that Number by it ſelf, and the Product (being an odd Square Number) as near the half as may be part into two. The Number leſs than the half by an Unite will be even, and ſhall be the ſecond or Greater Side of the *Diametral Number*. And the other part greater than the half by an Unite ſhall be odd, and the Length of the Diameter. As if 3 be propounded, the Square thereof is 9, the Leſſer Part 4 ſhall be the Greater Side, and the Greater Part 5 the Diameter.

Example.

When the Datum *is Odd and Compound.*

If the given Number be an odd Compound Number, then hath he more Greater Sides than one; for he hath the benefit not only of the former Rule, but alſo he followeth the form of that Number of which he is Compound. As 9, whoſe Square is 81, hath not only 40, the Leſſer Part for the Greater Side, and 41, for the Diameter; but being Compound of 3 followeth that form alſo; and therefore as 3 hath 4 for the Greater Side, ſo 9 being thrice 3, ſhall have 12 which is thrice 4, for a match Side with him, and 15, which is thrice 5, for his Diameter. Likewiſe 15, being compounded of 5 and 3, ſhall have both Forms in making of the *Diametral Numbers*. For as 3 hath 4, ſo 15, being 5 times 3, ſhall have 20, which is 5 times 4 for the ſecond Side, and 25 for his Diameter, which is 5 times 5. Again, as 5 hath 12, ſo ſhall 15, being 3 times 5, have 36, which is 3 times 12 for his ſecond Side, and 39, which is 3 times 13, for his Diameter.

Example.

Proportion between the Diameters *and* Diametrals.

Here by the way may be noted, that though both the *Diameters* and *Diametral Numbers* (as of neceſſity they muſt) vary from the former Numbers; yet is there a marvellous proportion between them. For the proportion of both the Sides in one Figure, to both the Sides in the other, being added together, will be like the proportion between the two *Diametral Numbers*. As if 3 and 4 be the Sides of a *Diametral Number* they make 12, and 9 and 12 being Sides, make 108, that is, 9 times 12. Now 9 to 3 is triple, and ſo is 12 to 4, and both triples added together (Addition of *Ratio's* being as Multiplication of *Fractions*) make the proportion or amounting *Ratio Noncuple*, or ninefold, and ſo are the two *Diametral Numbers*, 12 and 108, in proportion each to other.

When the Datum *is Even.*

If the Leſſer Side propounded be an even Number, then ſquare the Number as before, and of that Square take two Quarters; from one Quarter take an Unit, and put to the other; ſo have you two odd Numbers, the Leſſer of which ſhall be the Greater Side of the *Diametral Number*, and the other the *Diameter*. As 8 ſquared is 64, the Quarter 16, from whence 1 taken leaves 15 for the Greater Side of the Platform, and adding 1 to 16, the total 17 ſhall be the *Diameter*.

Example.

Proportion of the Greater Sides and Diameters.

Such even Numbers as have more Greater Sides than one, yet have they the like Numbers in proportion for their Greater Sides and Diameters, as the Numbers have of which they be Compound. As 20, compound of 4 and 5, ſhall have the Greater Side and Diameter belonging to 5 fourfold, and ſo the Greater Side of 5 is 12, and of 20 is 48, which is 4 times 12, and the Diameter 52, which is 4 times 13, the Diameter to 5.

Of diſcovering Diametrals.

Thirdly, To diſcover if a Number propounded be *Diametral* or not, and conſequently to find the Sides; take theſe 9 Directions.

1. *By the ending.*

1. If it end with any other Figure than 0, 2, or 8, it can be no *Diametral Number*.

2. *Evenly divided by 12.*

2. If it may not be evenly divided by 12, although it end as aboveſaid, it is no *Diametral Number*.

3. *By the Parts of the Number.*

3. If the Number propounded have thoſe two properties, then ſet out all the Parts thereof, ſo as the Leſſer Part ſtand over the Greater Part, which being multiplied together will make the whole Number, and then examine thoſe Parts according to the former Doctrine.

4. *Take the Parts myſt apt.*

4. Obſerve which of the Parts that ſtand for the Sides of the Platform be moſt apt to conſtitute a *Diametral Number*, and make tryal of them, for ſome Parts at firſt ſight appear unapt. For if among the Parts, the Leſſer Number be odd, the Square thereof muſt contain double to that Greater Number that is coupled with it, and 1 more. As in the *Diametral Number* 12, where the Sides are 3 and 4, there 9, the

Example.

Square of 3, is double to 4, and 1 over. And if the Leſſer Number be even, then muſt the Square of it contain the Greater Number that ſtands by it, 4 times and 4 more. As in the *Diametral Number* 48, is 6 coupled with 8, which 6 times 6 is 36, that is 4 more than 4 times 8; and this holds in all Numbers not Compound of other *Diametral Numbers*.

5. *If a Part too great or too little be taken.*

5. When the given Number hath many Parts, to ſave work, gueſs at one which ſeems probable, and making Proof thereby, if he be found too ſmall, aſſay with the reſt of the Parts greater; and if he be too big, refuſe all the Parts above, and examine only the ſmaller Parts till a juſt Part be found. But if thus examining you ſtill find the Part either too great or too little, then is the Number given no *Diametral Number*. As if 120 be the Number propounded, becauſe it ends in 0, and may be evenly divided by 12, it s probable to be a *Diametral Number*. I therefore ſet out the Parts which are theſe.

Example.

Parts of 120 coupled.	2.	3.	4.	5.	6.	8.	10.	12.	15.	20.	24.	30.	40.	60.	Parts.
	60.	40.	30.	24.	20.	15.	12.	10.	8.	6.	5.	4.	3.	2.	Numbers.

Here though I ſee many Parts, yet I need examine but few; becauſe ſeveral have no likelyhood of producing a *Diametral Number*. For all even Numbers under 6, cannot be the Leſſer Side of any ſuch Number, therefore the ſecond and fourth Parts are rejected. Alſo all Numbers above the tenth Part are refuſed, becauſe the Numbers under them are too little to anſwer proportionably for the Greater Side to the Parts ſtanding over, which ſhould be the Leſſer Side, and theſe are greater than they. Again the third Part is ſet aſide, as having under him too great a Number; for under 3 ought to ſtand no other Number than 4, to make a *Diametral Number*. Moreover, under 5 I find 24, but if I ſquare 5, it is 25, which is but 1 more than the Number under 5, when it ſhould be 1 more than the double. Therefore I either pitch upon the ſixth, eighth or tenth Parts for the Sides of the *Diametral Number*, or elſe 120 cannot be *Diametral*; then examining 6, his Square is 36, but this is not 4 times 20, and 4 more; ſo the ſixth Part is laid by alſo as too little. Then I ſquare 10 its 100, but then 4 times 12 and 4 more is but 52, and therefore the tenth Part is too big. So that 8 and 15 muſt be the Sides, or elſe the Number is no *Diametral*. And ſquaring 8 it is 64, which is 4 times 15, and 4 more, whereby 120 is ſeen to be a *Diametral Number*, and hath 8 and 15 for the Sides of the Platform.

On the contrary, proving 72 by his Parts, though he end in 2, and may be divided by 12, yet doth it appear to be no *Diametral Number*.

6. *Observe the Proportion between the Sides.*

6. By obſerving the proportion between the *Diametral Sides* it is eaſie to diſcern, whether the Parts be apt to conſtitute a *Diametral Number* or not: For the two Sides of all *Diametral Numbers* keep a conſtant proportion either to other in the order following, and continue in the ſame accordingly.

The Orders for placing the Sides.

The First Order of Odd Numbers for the Leaſt Sides.

3	5	7	9	11	13	15	17	19	21	23	25	27	
..	..	..	..	..	..	..	..	..	..	..	..	..	&c.
4	12	24	40	60	84	112	144	180	220	264	312	364	

The Second Order of Even Numbers for the Leaſt Sides.

8	12	16	20	24	28	32	36	40	44	48	
..	..	..	..	..	..	..	..	..	..	..	&c.
15	35	63	99	143	195	255	323	399	483	575.	

In both theſe Orders the Leſſer Sides ſtand at top, and the Greater Sides beneath, as Antecedent and Conſequent.

The Order of Stifelius.

Stifelius ſets the Greater Side at top, and the Leſſer Side below, and reduceth the Antecedent into Units, like Integers and Fractions.

The First Order after Stifelius.

$1\frac{1}{3}$ $2\frac{2}{5}$ $3\frac{3}{7}$ $4\frac{4}{9}$ $5\frac{5}{11}$ $6\frac{6}{13}$ $7\frac{7}{15}$ $8\frac{8}{17}$ &c.

The Second Order after Stifelius.

$1\frac{7}{8}$ $2\frac{11}{12}$ $3\frac{15}{16}$ $4\frac{19}{20}$ $5\frac{23}{24}$ $6\frac{27}{28}$ $7\frac{31}{32}$ &c.

And this for obſervance is beſt approved, becauſe in the firſt Order you ſee both in the whole Numbers and Numerators of the Fractions the Natural Order of Numbers, as 1, 2, 3, 4, *&c.* And in the Denominators, the Natural Progreſſion of odd Numbers, as 3, 5, 7, 9, *&c.*

But in the ſecond Order the whole Numbers go in their Natural Order, and the Numerators and Denominators keep an *Arithmetical Progreſſion* by equal diſtance of 4. So that in the Numerators all the Numbers be odd, in the Denominators they be all even.

7. *Parts of the Number may be abbreviated.*

7. The great Parts of any Number to be examined may be abbreviated (like a Fraction) into its leaſt Terms. For if the proportion hold in the leaſt Terms it will hold in the greateſt. As if 540 be propoſed to find whether it be *Diametral*, I ſet down ſo many of the Parts as are neceſſary; thus,

Example.

2	3	4	5	6	10	12	15	18	20	
270	180	135	108	90	54	45	36	30	27	&c.

Where I may abbreviate many of the Parts, as $\frac{6}{90}$, $\frac{10}{54}$, *&c.* but comparing their Proportions with the former, cannot find the Numbers alike proportional; but 15 and 36 are in like proportion, and ſo they continue if abbreviated to 5 and 12, therefore is 540 *Diametral*, and 15 and 36 the Sides thereof.

8. *What Cyphers may be cut off.*

8. From ſuch given Numbers as end in Cyphers, cut off even Cyphers as often as you can, as 2, 4, 6, *&c.* and if the reſt be a *Diametral Number*, ſo was the given Number: For if 540, or 432, *&c* be *Diametrals*, then 54000 and 43200 be the like.

9. *By a Square Diviſor.*

9. If any Number being divided by a Square Number, make the Quotient a *Diametral Number*; then is the Number divided a *Diametral Number* alſo. As 48 divided by 4, (a Square Number) yieldeth 12 in the Quotient, (a *Diametral Number*) therefore is 48 a *Diametral Number* likewiſe.

Like Flats *produced.*

§. 4. *Like Flats*, becauſe of their proportion in their Sides, are thence ſo termed, and therefore no one Number without relation to another can be termed a *Like Flat*. Some call them *Square-like Figures*, becauſe they have ſome properties with *Square Numbers*.

To produce theſe Numbers, Multiply any two Square Numbers by one other Number, and the Products ſhall be *Like Flats*. As 4 and 9, if Multiplyed by 3, give 12 and 27, which be *Like Flats*.

Alſo if 2 Square Numbers will admit of 1 Diviſor, then divide them thereby, and the Quotients ſhall be *Like Flats*. As 36 and 9, divided by 3, give 12 and 3, which are *Like Flats*. And ſo are 4 and 9, being the Quotients of 16 and 36, divided by 4.

Properties of Like Flats.

The 4 following Properties of *Like Flats* are collected out of *Euclid. lib.* 8. *prop.* 18. 20. *&* 26. and *lib.* 9. *prop.* 1. *&* 2.

1.

1. Every two Numbers *Like Flats*, have one mean Number between them in proportion to the *Leſſer Flat*, as the *Greater* is to him. As 4 and 9 have 6 for a mean between them. For as 6 is 1½ to 4, ſo is 9 to 6.

2.

2. One *Flat Number* beareth unto the other double that proportion their Sides do. As 4 and 9, whoſe Sides 3 and 2, or $\frac{3}{2}$ are in proportion *Seſquialter*, and the *Flats* themſelves 9 to 4, are in double *Seſquiquarta* proportion, and ſo will the *Seſquialter Ratio* make doubled.

3.

3. Numbers that be *Like Flats* have ſuch proportion together, as one of the Square Numbers uſed in their Compoſition beareth to the other. For in the former Examples, 12 to 3 is as 16 to 4, or 36 to 9, and if one of them be divided by the other, a Square is brought forth in the Quotient.

4.

4. Any two Numbers being *Like Flats* multiplyed together, will produce a Square Number. As 4 and 9 make 36, ſo 12 and 27 make 324, the Square of 18.

Rooted Numbers *produced Generally.*

§. 5. *Figural Numbers Rooted*, in General are produced by *Multiplication* thus. To Multiply any Number by it ſelf makes a Square Number. Again, that Square Number multiplyed by the Root produceth a Cubick Number. To multiply by the Root that Cubick Number, giveth a Squared Square Number. And to multiply again by the Root yieldeth a Surſolide Number. And ſo multiplying the laſt Product by the firſt Root, bringeth forth a Number of the next greater Quantity, and ſo ſucceſſively. So that to produce a Square, one Multiplication will ſerve (one Number being accounted for the Length, and the other for the Breadth). To make a Cubick Number, two Multiplications are required, by the ſecond whereof the Number taketh Depth, as by the firſt Length and Breadth. Thus to make a Squared Square, 3 Multiplications are requiſite, and then there is made a Line of Cubes. A Surſolide Number produced this way muſt have 4 Multiplications, which make a Square, wherein every Unite is a Cube. So the fifth Multiplication maketh a Cube of Cubes, accounting every Leſſer Cube for an Unite. And then the ſixth Multiplication returneth the multiplyed Numbers to the nature of Lineary Cubes. And the ſeventh to the nature of Squared Cubes. And the eighth to the nature of Cubick Cubes, and ſo forth infinitely as was before expreſſed. Theſe 3 Names of *Long*, *Square*, and *Cubick Cube*, may be reiterated, but a fourth Form can never be deviſed. For further diſcovery inſpection may be made into the *Table* of *Rooted Numbers* procreated after the common way thus.

A Table of Rooted Numbers.

The T*ABLE of* Rooted Numbers, *conſiſting of* Fifteen Figural Formes, *and* Twelve Roots.

	Names Arithmetical.													*Names Geometrical.*
1	Roots.	1	2	3	4	5	6	7	8	9	10	11	12	Roots.
2	Squares.	1	4	9	16	25	36	49	64	81	100	121	144	Squares.
3	Cubes.	1	8	27	64	125	216	343	512	729	1000	1331	1728	Cubes.
4	Squar'd Squar.	1	16	81	256	625	1296	2401	4096	6561	10000	14641	20736	Long Cubes.
5	Surſolides.	1	32	243	1024	3125	7776	16807	32768	59049	100000	161051	248832	Squar'd Cubes
6	Squar'd Cubes.	1	64	729	4096	15625	46656	117649	262144	531441	1000000	1771561	2985984	Cubick Cubes.
7	Second Surſolides.	1	128	2187	16384	78125	279936	823543	2097152	4782969	10000000	19487171	35831808	Long Cubick Cubes.
8	Squares *of* Squar'd Squar.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721	100000000	214358881	429981696	Square Cubick Cubes.
9	Cubes *of* Cubes.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489	1000000000	2357947691	5159780352	Cubes *of* Cubick Cubes.
10	Squares *of* Surſolides.	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401	10000000000	25937424601	61917364224	Long Cubes *of* Cubick Cubes.
11	Third Surſolides.	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609	100000000000	285311670611	743008370688	Square Cubes *of* Cubick Cubes.
12	Squares *of* Squared Cubes.	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481	1000000000000	3138428376721	8916100448256	Cubick Cubes *of* Cubick Cubes.
13	Fourth Surſolides.	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329	10000000000000	34522712143931	106993205379072	Long Cubick Cubes *of* Cubick Cubes.
14	Squares *of* Second Surſolides	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961	100000000000000	379749833583241	1283918464548864	Square Cubick Cubes *of* Cubick Cubes.
15	Cubes *of* Surſolides.	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649	1000000000000000	4177248169415651	15407021574586368	Cubed Cubick Cubes *of* Cubick Cubes.
	&c.													&c.

The Table *explained.*

The *&c.* at the foot of the *Table* denotes it may be increaſed ; but this is large enough to view the orderly production of *Rooted Numbers*, to the 15th *Quantity* and 12th *Root*, where is to be ſeen,

Indices *of* Figurals, *what and why ſo called.*

1. The Numbers of *Quantity* in the firſt Left Hand Column, called *Indices* in *Latin*, becauſe the Power or Quantity of the *Figural Number*, and how far he is removed from the *Root*, is ſhewed thereby.

2. The *Roots* from which each *Figural Number* ariſeth, in the head of the *Table*.

3. The *Arithmetical Names* proper to each Number, at the left ſide.

4. The *Names* according to the Forms of each Number, at the right ſide.

And 5. The Numbers themſelves in the body of the *Table*, which are to be accepted for *Figural Numbers* of that Denomination, which you find written againſt them at the ſides, and have that Number for their *Root*, that ſtandeth over them in the head of the *Table*.

Rooted Numbers *produced particularly. The* Square.

§. 6. *Rooted Numbers*, admit of ſome variety in production, which in particular is ſet forth in this, and the two following *Sections*.

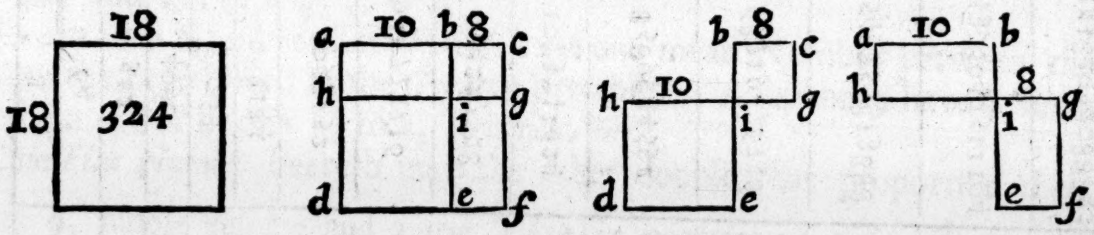
The firſt *Rooted Number* a *Square*, beſides multiplying the *Root* into it ſelf as aforeſaid, may be obtained by breaking the *Root* into Parts, called alſo Segments ; and Squaring the Parts ſeverally, and orderly adding them with the double of the Product produced by Multiplication of the Segments one into another.

Theoreme *of* Euclide.

This Device is grounded on that *Geometrical Theoreme* in *Euclid. lib.* 2. *prop.* 4. That if a Right Line be cut into two Segments, the Square of the whole Line ſhall be equal to the Squares of the Segments, and to the two Right-Angled Figures made of the Segments. As if the Right Line, whoſe Square I would produce, be 18, I cut the Line into two Segments, as ſuppoſe 10 and 8, then ſhall the Squares of 10 and 8, added with the double of 10, multiplyed into 8, be the Square of 18.

Example and Demonſtration where the Root *hath but* 2 *Figures.*

At the Figure following (when the Line or Side of the Square is cut into 2 Segments) there appears 4 plain Figures ; of which 2 are Squares, and 2 Rectangle Figures or Long Squares, all which added together, make the whole Square.

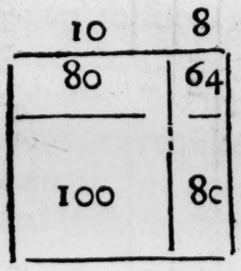


And of theſe 4 plain Figures is the whole Square obtained by the 4 Numbers of their Area ; thus,

1. The Greater Square *h. i. e. d.* is known by multiplying the Line *h. i.* (equal to *a. b.*) into it ſelf, and ſo 10 ſquared is 100.

2. The Leſſer Square *b. c. g. i.* is known by multiplying the Line *b. c.* (equal to *i. g.*) into it ſelf, and ſo 8 multiplyed by 8, gives 64

3. The Long Squares are known by multiplying the 2 Segments one into another, and then doubling the Product. As the Segment *a. b.* 10, into the Segment *b. c.* 8, which gives 80, for the Area of each Rectangle Figure, and added to the Squares are their Complement to the Square of the whole Line 18, as by the ordinary way may be proved.



18 : Radix

10 : 8 . Segments

100 . Square of the Greater Segment
80 }
80 } . Rectangle Figures
64 . Square of the Leſſer Segment

324 Zenſus

Proof.

18 Radix
18

144
18

324 Zenſus

Where the Root *hath more than* 2 *Figures,*

In like manner, if the *Root* conſiſt of many Figures, or the Line be cut into different Segments, yet is the work alike. As if 140 were the ſide of a Square, whoſe Square Number were deſired, I may cut 140, into 100 and 40, or 90 and 50, or 95 and 45, or any other parts.

Example.

140	Radix	140	Radix
00 : 40	Segments	95 : 45	Segments
10000	Great Square	9025	Great Square
4000 4000	Rectangle Figures	4275 4275	Rectangle Figures
1600	Leſſer Square	2025	Leſſer Square
19600	Zenſus	19600	Zenſus

Proof.

140 Radix
140
5600
140
19600 Zenſus

Some inſtead of cutting the Line of the Square in but two Segments, as before, do after a ſort cut the ſame into ſeveral Segments, even as many as the Root hath Figures; and to ſhorten the Multiplying work, accompt every Figure in what place ſo ever, or though never ſo great an Article, but as a Digit, and to ſupply the Cyphers wanting, place the Numbers gotten as aforeſaid, each one place nearer to the Right Hand than the other, and ſome add the two Rectangle Figures together, ere they ſet them down under the Root. As for inſtance, To get the Square of 46, I accompt the Segments not 40 and 6, but 4 and 6, then place under 4 his Square 16, and multiplying the double of 4, which is 8, by 6, or 4 by 6, and double the Product (for it's all as one) the Product 48 is the ſumme of both the Rectangle Figures, which I place under 16, one place nearer to the Right Hand, then under the Segment 6, I ſet his Square 36, and the Total added together, the Square of 46 is found to be 2116, and by the former wayes may be proved true.

Variety of working with 2 Figures.

Example.

4	6	Root
16 4	 8 36	
21	16	Square

Proofes.

46
40 : 6
1600
240
240
36
2116

46 Root
46
276
184
2116 Square

And if the Root propounded conſiſt of many Figures, then after the manner laſt mentioned, when you have gotten the Square of the firſt two Figures to the Left Hand in the ſame ſort proceed to ſeek the Power or Quantity of the reſt. As in ſeeking the Square of 46808, thus;

With many Figures.

Example.

	4	6	8	0	8	Root
	16 4	 8 36				
Square of	21	16 73	 6 64			46
Square of	21	90	24	 0 0		468
Square of	21	90	24 74	00 88	 0 64	4680
Compleat	21	90	98	88	64	Square

Proof.

46808 Root
46808
374464
3744640
280848
187232
2190988864 Square

Touching Square Numbers, and their production, further obſerve.

Obſervations.

1. A Square Number doth never end in 2, 3, 7, 8, or a ſingle 0; but in order terminate thus, 1, 4, 9, 6, 5, 6, 9, 4, 1, 0, 1, 4, 9, 6, &c. beginning again as in the *Table* of *Rooted Numbers* is apparent.

1. Of their Terminations.

2. The

2. *How made of* Diametrals.

2. The Squares of the Parts of a *Diametral Number* added together, make a *Square Number*, as was ſeen before in *Diametral Numbers*.

3. *How made of* Like Flats.

3. Numbers called *Like Flats*, multiplyed together make *Square Numbers*. As 2 and 18 make 36 the Square of 6; ſo 3 and 48 make 144, the Square of 12.

4. *Odd Square half lacking, 1 added makes a Square.*

4. If 1 be taken from any Square Number which is odd, the Square of half the remainer being added to the firſt Square will make a Square Number. As 9 the Square of 3, from which 1 be taken there reſteth 8, the half 4 ſquared is 16, to which if 9 be added the total is 25 the Square of 5.

5. *How made of an Even Number.*

5. The Square of half any even Number, if the even Number be added to it, and 1 more, will make a Square Number. As 10 whoſe half is 5, the Square whereof is 25, to which 10 added, and 1 more, make the Total 36, the Square of 6.

6. *How otherwiſe.*

6. If to the Square of half any even Number 1 be added, and the even Number then ſubſtracted, there will remain a Square Number. As if to 25, the Square of 5, the half of 10, there be 1 added, it will be 26, from which if 10 be ſubſtracted, 16 a Square Number remaineth whoſe Root is 4.

7. *How by Addition of Odd Numbers.*

7. Odd Numbers continually added from an Unit ſucceſſively to the antecedent Squares, make the Totals, Square Numbers. As 1 and 3 is 4, ſo 4 and 5 is 9, and 9 and 7 is 16, &c.

8. *How by Cubes.*

8. Cubick Numbers added ſucceſſively from the Unit, produce Square Numbers. As 1 and 8 is 9, ſo 9 and 27 is 36, &c.

Cube *produced particularly.*

§. 7. The particular conſtruction of the *Cube* (the firſt and leaſt Rooted Body) is next to be ſeen.

A *Cube* by the fifth *Section* of this *Chapter* is produced the common way, by Multiplying the Square Number by the Root. As 2 by 2 makes 4, which 4 again multiplyed by 2 makes 8, the *Cube* of 2.

This is a kind of triple Multiplication, the Root being alwayes valued in himſelf once. For 2 times 2 twice maketh 8, and ſo 3 times 3 thrice yieldeth 27, the *Cube* of 3. And 4 times 4 four times giveth 64, the *Cube* of 4, &c.

Device of Ramus.

Example and Demonſtration where the Root *hath 2 Figures.*

The production of theſe Numbers vary from the ordinary way: If the Root be broken into Parts, and the *Cubes* of the Parts or Segments be added to the two ſolid Figures, comprehended 3 times under the Square of one Segment multiplyed by the other. Which Device *Ramus*, *lib.* 24. *ſect.* 10. imitating that in *Euclide* of the Square, delivereth. As if there be a *Cube*, whoſe Root is 18 Inches, and I would know the *Cube Number* thereof, which ſhall declare how many ſolid Inches there are in that Body; I cut the Root into two Segments, as 10 and 8, and thereby doth the Body at *K.* appear (as much as can be aptly demonſtrated in *Plano*, or by *Flats*) to be parted into 8 Bodies or Solidities, *viz.*

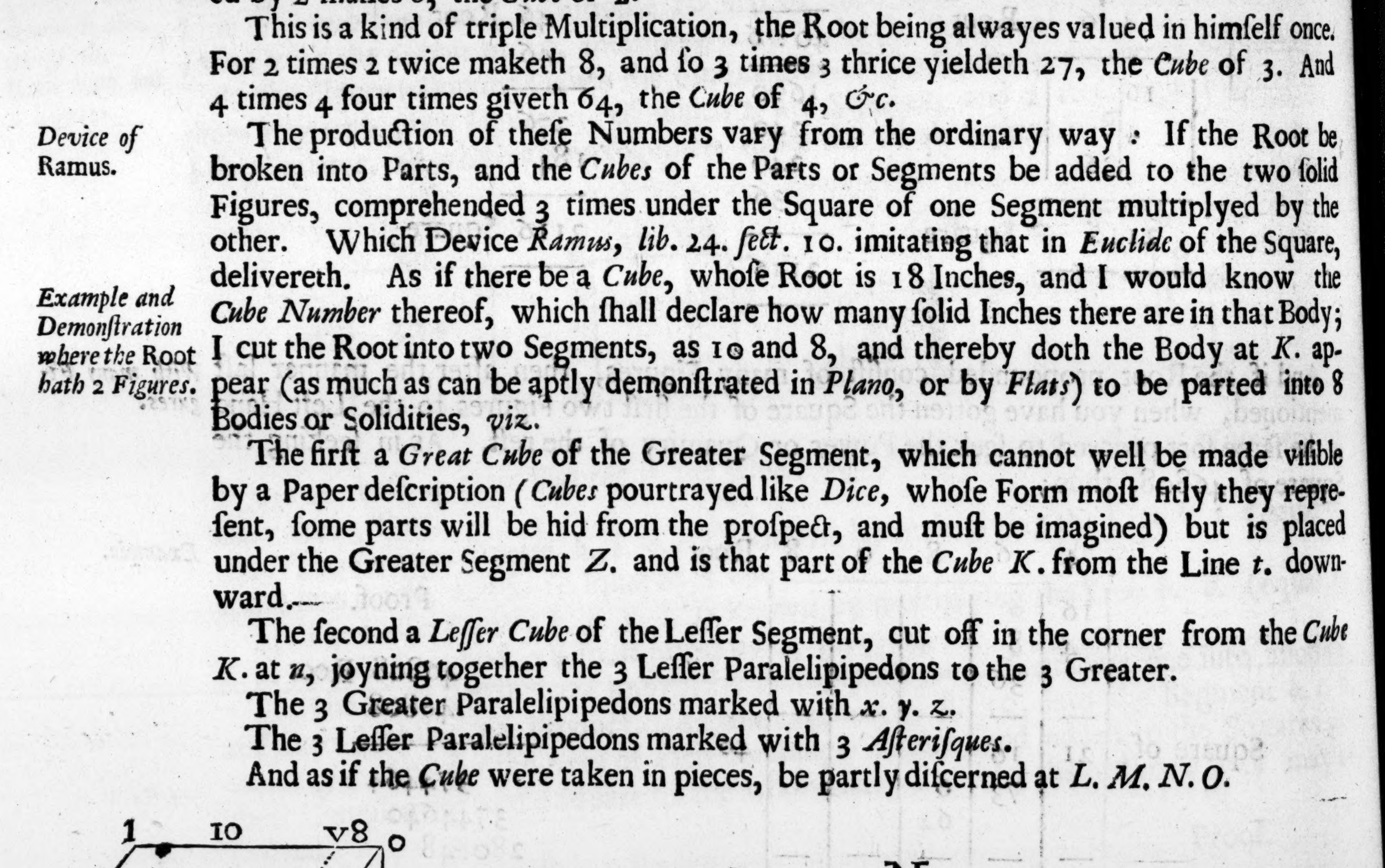
The firſt a *Great Cube* of the Greater Segment, which cannot well be made viſible by a Paper deſcription (*Cubes* pourtrayed like *Dice*, whoſe Form moſt fitly they repreſent, ſome parts will be hid from the proſpect, and muſt be imagined) but is placed under the Greater Segment *Z.* and is that part of the *Cube K.* from the Line *t.* downward.—

The ſecond a *Leſſer Cube* of the Leſſer Segment, cut off in the corner from the *Cube K.* at *n*, joyning together the 3 Leſſer Paralelipipedons to the 3 Greater.

The 3 Greater Paralelipipedons marked with *x. y. z.*

The 3 Leſſer Paralelipipedons marked with 3 *Aſteriſques*.

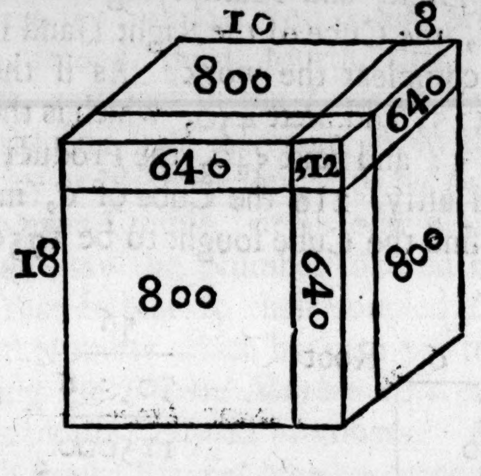
And as if the *Cube* were taken in pieces, be partly diſcerned at *L. M. N. O.*



And as all theſe ſolid Figures joyned together make the *Cube* at *K.* compleat, ſo will their ſeveral ſolid Numbers added together complement the *Cube Number*. For the *Cube* of the Greater Segment is 1000, obtained by multiplying the Greater Segment *Cubically*. And the *Cube* of the Leſſer Segment alſo is gotten by multiplying the ſame

Cubically, and ſo 8 *Cubed* is 512. Then the Greater Paralelipipedons are found by multiplying the Square of the Greater Segment by the Leſſer Segment; that is 100, the Square of 10 by 8, maketh 800, for every one of the 3 Greater Paralelipipedons. Laſtly, the Leſſer Paralelipipedons are known by multiplying the Leſſer Segment ſquared by the Greater Segment. As 64, the Square of the Leſſer by 10 the Greater Segment, give 640 for every of the 3 Leſſer Paralelipipedons, all which Numbers added together render the *Cube Number* 5832. Thus,

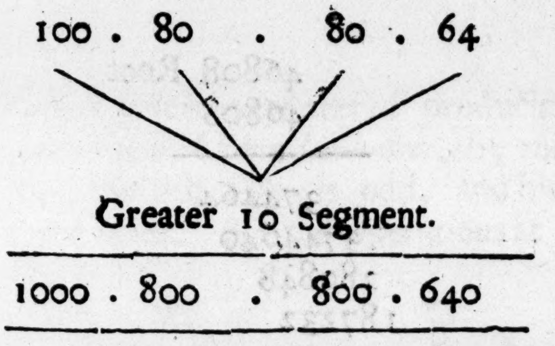
18 Radix
10 : 8 Segments
1000 Cube of the Greater Segment
800
800 } 3 Greater Paralelipipedons
800
640
640 } 3 Leſſer Paralelipipedons
640
512 Cube of the Leſſer Segment
5832 Cubus

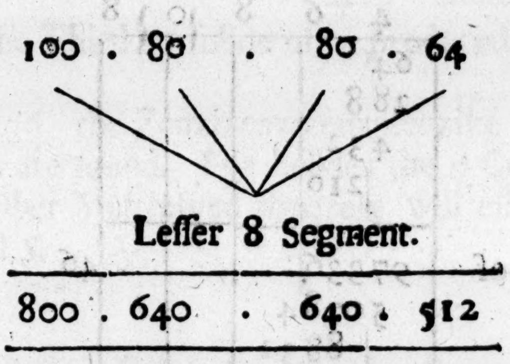


Proof by the Common way.
18 Radix
18
144
18
324 Zenſus
18
2592
324
5832 Cubus

Hereby is the Theoreme of *Ramus* alſo clear, That the *Cube* of the whole Line is equal to the *Cube* of the Segments, and the 2 ſolid Figures comprehended 3 times under the Square of his Segment, and the remaining Segment.

And it may further be noted, that as in the vulgar way of production you firſt get Plain Numbers, and then Multiply thoſe Plains by the Roots, to bring forth the ſolid Numbers: So may you here proceed likewiſe; for if you get the Squares of the Segments, and the Rectangle Figures, as before in the precedent *Section* was taught, and then Multiply thoſe 4 Plain Numbers by the Segments ſeverally, you will produce 8 Solids, the Total whereof will be the *Cube Number*. As in the former Number the 4 Plain Numbers in the Square of 18, were 100, 80, 80, 64, which Multiplyed by 10, give the ſeveral Products of 1000, 800, 800, 640, and when Multiplyed by 8, give 800, 640, 640, 512, all which added together produce the *Cube*, as before, 5832.





Where the Root hath more than 2 Figures.

If there be more Figures in the Root than two, yet this courſe is ſtill kept. As in ſeeking the *Cube* of 468; thus,

Example.

468 Radix
400 : 68 Segments
64000000 Cube of the Greater Segment
10880000
10880000 } 3 Greater Paralelipipedons
10880000
1849600
1849600 } 3 Leſſer Paralelipipedons
1849600
314432 Cube of the Leſſer Segment
102503232 Cubus

Proof.
468 Root
468
3744
2808
1872
219024 Square
468
1752192
1314144
876096
102503232 Cube

Variety of working with 2 Figures.

Some ſhorten the work, accompting the Segments as Digits only, and to ſupply the place of Cyphers (as before in the Square) place the Numbers orderly to the Right Hand. As where the Root hath but 2 Figures, the firſt Right Hand Figure of the Cube of the Greater Segment under the firſt Left Hand Figure of the Root. And next that one place nearer to the Right Hand, the ſumme of the 3 Greater Paralelipipedons gotten by tripling the Square of the Left Hand Digit, and Multiplying that triple by the Right Hand Digit of the Root. And under this one place nearer to the Right Hand is ſet the ſumme of the Leſſer Paralelipipedons obtained by tripling the Square of the Right Hand Digit, and Multiplying that triple by the Left Hand Digit of the Root. And laſtly, the Cube of the Right Hand Digit of the Root (or Leſſer Segment) placed under him compleat the work. As if the Cube of 56 be demanded, I place under 5 his Cube 125, and next 450, which is the Product of 6 into 75, the triple of 25, the Square of 5; and next 540, the Product of 5 into 108, the triple of 36, the Square of 6; and laſtly, 216 the Cube of 6, in their orderly places before directed, and adding them, find the Cube ſought to be 175616.

Example.

```
   5    6    Root
 |125|    |
 | 45|0   |
 |  5|40  |
 |   | 16 |
 175  616    Cube
```

```
    56
  50 : 6
 125000
  15000 }
  15000 }
  15000 }
   1800 }
   1800 }
   1800 }
    216
 ------
 175616
```

```
Proofes.     56  Root
             56
           ----
            336
           280
           ----
           3136  Square
             56
          -----
          18816
         15680
         ------
         175616  Cube
```

With many Figures.

If the Root conſiſt of many Figures, then after in the former manner you have wrought for the Cube of the firſt 2 Figures to the Left Hand, go forward to ſeek the quantity of the reſidue. As in ſeeking the Cube of 46808, this is the work.

Example.

```
           4   6   8   0   8
         |64
         |288
         |  432
         |   |216
Cube of  |97336               |46
         |  5078|4
         |    88|32
         |      |512
Cube of  |102|503|232         |468
         |   |   |   |0
         |   |   |   | 0
         |   |   |   |  0
Cube of  |102|503|232|000     |4680
         |    52|565|760|0
         |       8|985|60
         |            |512
Compleat  102 555 806 746 112  Cube
```

```
Proof.

         46808  Root
         46808
        ------
        374464
      3744640
     280848
    187232
   ----------
   2190988864  Square
        46808
   -----------
     17527910912
   175279109120
  13145933184
  8763955456
  --------------
  102555806746112  Cube
```

How Cubes *are gotten by Addition of Odd Numbers.*

Further in general may be noted, That Cubick Numbers are begotten by Addition of odd Numbers from an Unit ſucceſſively. As 3 and 5 make 8, ſo 7, 9, and 11, the next odd Numbers are 27, the next Cube to 8; alſo 13, 15, 17, 19, make 64, and the like of others; and ſo many Units in the Root, ſo many odd Numbers in the Cube.

How Cube *Numbers end.*

One thing more is remarkable in the termination of the Cube, *viz.* that he may end in any Figure, but he maketh exchange in ſome. For if he have 0, 1, 4, 5, 6, or 9, in the firſt place of his Root, he will have the ſame in the firſt place of the Cube. But if 2 be firſt in the Root he will exchange for 8 in the firſt place of his Cube; and if 8 be the firſt of his Root, 2 ſhall be the firſt of the Cube. And in like ſort do 7 and 3 make exchange in the Cubick Number.

§. 8. The varieties of production of *Higher Powers*, or *Rooted Solides*, greater than the *Cube* ſhall cloſe up this *Chapter*. And becauſe it would be tedious to recite, and conſequently to remember many particulars, take this General Rule to know how many wayes every ſuch *Higher Power* may be produced. Higher Powers *produced.*

Mark their *Indices*, or how many degrees the Number you would produce is removed from the Root, as whether it be the ſecond, third, fourth, fifth, *&c.* Quantity (accompting the Root alwayes for the firſt, as before in the *Table* of *Rooted Numbers*), and couple each two Numbers together, the one next increaſing from the Root, with the other next decreaſing from the Quantity or Number ſought to be produced, and ſo proceeding, at laſt there will be either an even Couple, or an odd Number. If they be all even Couples, then ſo many wayes may the Number deſired be produced by Multiplying each two Numbers together that anſwer to their coupled *Indices*. And if among the coupled *Indices* there be an odd Number which hath none to match him, then may the Number you ſeek, by Multiplying the *Figural Number* anſwering to this odd *Index* into it ſelf, be produced, as well as by the Coupled Numbers. As to know how many wayes the *Third Surſolide*, or 11*th Rooted Figural Number* (accompting the Root for the Prime or Original) may be produced, I couple their *Indices* as at *P*, and ſuppoſing the Root 2, place the correſpondent Figural Numbers as at *Q.* and find he may be produced 5 wayes, by Multiplying, 1. The Root into the Square Surſolide. 2. The Square into the Cubed Cube. 3. The Cube into the Zenzizenzizenzike. 4. The Squared Square into the Second Surſolide. And 5. The Surſolide into the Squared Cube. *By their* Indices. 1. *Example.*

Indices Coupled.

P.					
	10	9	8	7	6
	1	2	3	4	5
	11	11	11	11	11

Index of the Third Surſolide.

Numbers Placed.

Q.					
	1024	512	256	128	64
	2	4	8	16	32
	2048	2048	2048	768	128
				128	192
				2048	2048

The Third Surſolide of 2 produced.

Likewiſe 8 ſeveral varieties of producing 65536, the Zenzizenzizenzizenzike of 2, or the 16*th Figural Rooted Number*, by this way are found. For beſides the 7 Couples of *Indices*, the 8th *Index* is odd, and his Number Multiplyed ſquarely will effect as much as the other. See the Operations at *R.* and *S.* 2. *Example.*

R. Added							
15	14	13	12	11	10	9	8
1	2	3	4	5	6	7	
16	16	16	16	16	16	16	16

S. Doubled							
32768	16384	8192	4096	2048	1024	512	256
2	4	8	16	32	64	128	256
65536	65536	65536	24576	4096	4096	4096	1536
			4096	6144	6144	1024	1280
			65536	65536	65536	512	512
						65536	65536

And further, becauſe this Rule depends upon the proportions between the *Indices* and their Quantities; for as the one increaſe by Addition, ſo the other by Multiplication (as in proceſs of this Treatiſe hereafter may be ſeen); therefore whatever *Indices* added together, will make the Total, the *Index* of the Figural Number ſought, thoſe Figural Numbers anſwering to the added *Indices*, Multiplyed one into another ſhall produce the Figural Number deſired, as in the laſt Example, becauſe 5, 5, and 6, make 16, and alſo 4, 4, and 8, and 11, 4, and 1, and ſeveral other Numbers do the like, the Figurals of thoſe Quantities Multiplyed together, as at *T. V. W.* will produce the Figural Number of 16, and the Root being 2, will be 65536, as before. *Varieties thereof.*

T.		*V.*		*W.*	
32	5	16	4	2048	11
32	5	16	4	16	4
64		96		12288	
96		16		2048	
1024		256		32768	
64	6	256	8	2	1
4096		1536		65536	16
6144		1280			
		512			
65536	16				
		65536	16		

Of the ending of Squared Squares.

Some have obſerved the Terminations of theſe *Higher Powers*, and find,

The Squared Square never endeth in 2, 3, 4, 7, 8, or 9, but either in 0, 1, 5, or 6, and thus according to the Natural Order of Numbers in their Roots, 6, 1, 6, 5, 6, 1, 6, 1, 0, 1, and then begin again.

Surſolides.

The Surſolides imitate their Roots. For if the Root be a Digit, then hath the Surſolide the ſame Digit in his firſt place: But if his Root be an Article, then as the Surſolide is the fifth Quantity, ſo hath he 5 times ſo many Cyphers together in the Right Hand places as the Root had, and the next ſignifying Figure after theſe Cyphers is the firſt Figure ſignificative of his Root. And if the Root be a mixt Number, yet ſtill is the firſt Figure of the Surſolide, the firſt of the Root.

Others higher.

The next Figural Number, as Surſolides follow the manner of their Roots, ſo do they the manner of the Squares. And the next to them the manner of the Cubes. And the next the manner of the Squared Squares. And then they begin again, and are like the Surſolides, *&c.* All which may fully be ſeen in the foregoing *Table* of *Rooted Numbers*.

Proof of Production of Rooted Numbers.

The Proof of Production of Rooted Numbers (beſides the varieties proved one by another) muſt be deferred till Extraction of their Roots be learned, which ſhall be the Subject of the next *Chapter*.

CHAP. III.

Extraction of Roots.

Extraction of Roots.

THE *Geneſis* of *Figural Numbers* now finiſhed, I come to their *Analyſis*, vulgarly called *Extraction of Roots.*

Of what made.

As Production of *Figural Numbers* was made up of Multiplication and Addition, ſo is Extraction of their Roots of Subſtraction and Diviſion. But whereas in Diviſion of Integers, the Diviſor is known, here it is to ſeek, every remove requiring a new Diviſor.

Diviſor to ſeek.

Sides of the Numbers not Rooted, *found by Diviſion.*

The Sides of thoſe *Figural Numbers* not Rooted, mentioned in the former *Chapter*, being ſufficiently known by their Production to be the Factors of ſuch Products by one ſingle Multiplication, render their Invention eaſie by one ſingle Diviſion of thoſe produced Area's by the Side known, and need no further remembrance here. But the finding of the Root of a *Figural Number* made of ſeveral Multiplications, is much more difficult. For the clear underſtanding whereof, and all needful thereto, ſee,

Roots by Extraction.

Extraction of *Roots*
- in General —— §. 1.
- in Particular of
  - *Squares* —— §. 2.
  - *Cubes* —— §. 3.
  - Compounds of both —— §. 4.
  - All Higher Powers, by the *Table* —— §. 5.
  - *Surdes*, and to denominate the Remains —— §. 6.

What neceſſary to Extraction of Roots.

§. 1. To the Extraction of *Roots* in General, 2 things are neceſſary.

1. *To know the* Figural Number *for every Digit.*

1. To have the *Figural Numbers* of every Digit perfectly in mind: For it were ſuperfluous to ſeek Rules for them, ſince they may be eaſier remembred then Rules for their production, and readily found in the *Table of Rooted Numbers*; As for the Square and Cube, thus;

Roots	1	2	3	4	5	6	7	8	9	
Squares	1	4	9	16	25	36	49	64	81	The like of others.
Cubes	1	8	27	64	125	216	343	512	729	

2. *To prick the Number according to the Quantity.*

2. To prick the *Figural Numbers* given whoſe Root is to be extracted, according to their Quantities, beginning at the Right Hand. As the Square, becauſe a Number of the ſecond Quantity, prick every ſecond Figure, and leave one unprickt. The Cube prick every third Figure, and leave two unprickt, and ſo accordingly upward. And let it be noted that ſo many Pricks as the Number will admit of, ſo many Figures muſt be in the Quotient for the Root to conſiſt of.

To Extract the Square Root.

Rules.

§. 2. In Particular, the Square Root of a Number is extracted commonly thus:

1. When the Number is placed with a Crooked or Rectangle Line to ſeparate the Quotient or Root from the Square, and pricked as aboveſaid, then ſeek the greateſt Square Number contained in the Figure or Figures that belong to the laſt Prick to the Left Hand, which Square Number ſet thereunder, and ſubſtract therefrom, and the Root of that Square ſet in the Quotient, cancelling the Figure or Figures ſtanding over this Square, and if there be Remainders ſet them in order at top, and ſo is the work for that Prick ended. 1.

2. Double this Root, and ſet him down for a Diviſor, thus; if he be a Digit, place him under the next unprickt Figure inclining to the Right Hand; and if an Article or Mixt Number, then ſet the firſt Figure or Cypher of this doubled Root in the unprickt place as before, and the other Figure one place nearer to the Left Hand, and diviſe with this Number for a new Quotient Figure, which muſt be no greater than that the Square of the ſame Figure may be alſo ſubſtracted from the remaining Numbers left to the next Right Hand Prick after you have, as in common Diviſion, ſubſtracted the Product of your Diviſor multiplyed into this Quotient Figure, ſtill cancelling the Figures from which any thing is taken, and ſetting down the Remainer if any be. 2.

3. Then ſet down the Square of this Quotient Figure, the firſt Figure thereof, if more than one, under the next Prick inclining to the Right Hand, and the other Figure one place nearer to the Left Hand, and ſubſtract it, cancelling the Figures after Subſtraction, and ſetting the Remainers, if any, at top as before. 3.

4. If your Number admit of more Pricks than 2, then for every Prick exceeding 2 muſt you repeat the work in the ſecond and third Directions, to double the Quotient and divide thereby, and ſubſtract the Square of each Figure in their order. 4.

Example.

For further Explanation; ſuppoſe I would prove whether 46808 be the Square Root of 2190988864, then having placed and pricked the Number, I find 21 to belong to the laſt or Left Hand Prick, I therefore inquire the Greateſt Square in 21, and it is 16, whoſe Root is 4, which placed in the Quotient, and ſubſtracting 16 from 21, there reſts 5 at top, as at *A*.

Then I double the Root 4, and it is 8, which placed under the unprickt Figure, and diviſing thereby, I can take but 6 in the Quotient, or elſe I ſhall not leave enough to the next to make Subſtraction of his Square; for ſhould I take 7, I ſhould leave but 3 to the 0, which would be but 30, and the Square of 7 is 49; therefore I firſt place 6 in the Quotient, and take 6 times 8, which is 48, out of 59, there reſts 11, as at *B*.

Then do I ſquare 6, and it is 36, which I ſet under 110, the Numbers belonging to the Prick, and ſubſtract it, as at *C*.

Then do I double the whole Quotient 46, and the ſumme 92 is the next Diviſor to be placed under 749, and diviſing thereby find the Diviſor may be taken 8 times from 749, and yet leave enough to take the Square of 8, from the remaining Figures, the next pricked Figure being joyned thereto, I therefore ſet 8 in the Quotient, and ſubſtracting the Diviſor 8 times, there reſteth 13; and the Square of 8, which is 64, ſubſtracted leaves 74, uncancelled as at *D*.

Then I again repeat this laſt work, and double the whole Root 468, and place the ſumme 936, in order beginning under the next unprickt Figure, and finding the Diviſor bigger than the uppermoſt Number, I ſet a Cypher in the Quotient, and alſo under the next Prick to the Right Hand; for the Square of 0 is 0, and cancel the Diviſor, as at *E*.

Laſtly, Doubling the whole Quotient 4680, the next Diviſor will be 9360, thereby 8, gotten for the Quotient, and the Diviſor ſubſtracted 8 times, and afterward the Square of 8, as before, there will be nothing left remaining, as at *F.*; whereby 2190988864, is ſeen to be a Square Number, and hath 46808 for his Root. And thus is the production of the Square in the former *Chapter* proved true. And reciprocally the truth of Extraction thereby.

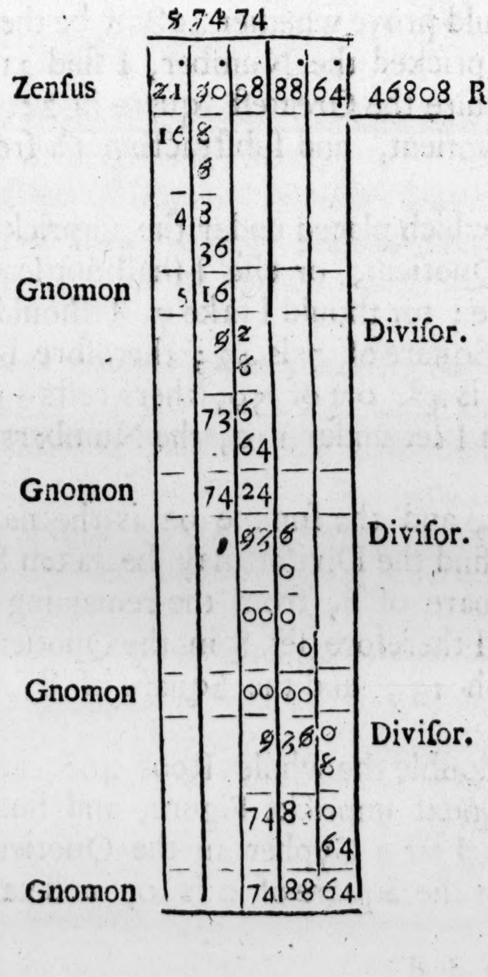
```
                                                   1
        5                                          51
        • • • • •                                  • • • • •
A.    2190988864 | 4                     B.      2190988864 | 46
      16                                         168

      17                                         1717
      514                                        51434
      • • • • •                                  • • • • •
C.    2190988864 | 46                    D.      2190988864 | 468
      1686                                       168624
       3                                          396

      1717                                       1717
      51434                                      51434
      • • • • •                                  • • • • •
E.    2190988864 | 4680                  F.      2190988864 | 46808 Root
      168624600                                  1686246004    46808
       3963                                       39633366    ---------
        9                                           99        374464
                                                             3744640
                                                            280848
                                                           187232
                                                        --------------
                                              Square   2190988864 Proof
```

How to work and prove it by Addition.

Extraction of the Square Root may be proved by Addition: If the ſubſtracted Numbers be orderly placed one under another, and then added, the Total will return the Square whoſe Root was extracted. As in the former Example, after the ſecond Figure is placed in the Quotient, ſet him under the Diviſor, and multiply the Diviſor thereby, and the amounting Product joyned with the Square of the Quotient Figure, makes the ſecond Number to be ſubſtracted, called a *Gnomon*; and ſo continue this work according to the Number of Pricks on the Great Square, and then add all the Gnomons together with the Left Hand Square firſt ſubſtracted.

Example.

Gnomon *what.*



Proof.

```
  16
   516
    7424
      0000
      748864
  ----------
  2190988864
```

Variety of work.

Some omit to place the Diviſors underneath, and others multiply the Root by 20, and take the Product for the Diviſor; but ſet the Cypher under the next Right Hand Prick, and by the new Quotient Figure multiply this Diviſor, to which the Square added ſhall make the Total the Gnomon to be ſubſtracted, and then multiply again by 20, this whole Quotient, to get a new Diviſor, if there be more Pricks, reiterating this work till all be finiſhed; as in the former inſtance. Again thus;

Example.

```
             57474  .           4     4     46     468     4680
             . . . . .          4    20     20      20       20
             2190988864 | 46808 —    ——     ——      ——     ————
             ——————————        16    80    920    9360    93600
             16                 —     6      8       0        8
              516                    ——     ——      ——     ————
               7424                 480   7360    0000   748800
                  0000               36     64       0       64
                  748864             ——     ——      ——     ————
             ——————————             516   7424    0000   748864
 Collection  2190988864 Proof        ——     ——      ——     ————
             ——————————
```

§. 3. The *Cube Root* is extracted with ſome variety, but the moſt received way is thus.

To extract the Cube Root.

Rules.

1. When the Number is placed and pricked as before directed, from the Figures belonging to the laſt Prick to the Left Hand, ſubſtract the Greateſt Cube Number you can have out thereof, and cancelling the ſame Figures, if there be any Remains ſet them at top, and the Root of this Cube ſet in the Quotient, and ſo have you done with this Prick: For this work, as in the Square, is wrought but once 1.

2. Triple the Root, and multiply this triple by the Root, the Number that ariſeth ſhall be the Diviſor, and ſet one place nearer to the Right Hand, by which inquire for a new Quotient Figure from the Figures ſtanding there over, and when found out place in the Quotient; and as in Diviſion of Integers, ſubſtracting the Product of your Diviſor multiplyed by the new Quotient Figure, cancel the Figures from which any thing is ſubſtracted, and ſet the Remainders, if any be, at top. 2.

3. Square the laſt Quotient Figure, and multiply this Square by the triple of the former Quotient Figure, and place the Product one place nearer to the Right Hand, and ſubſtract it from the upper Figures, ſtill cancelling and ſetting the Remains at top, as before. 3.

4. Under the next Right Hand Prick, ſet the Cube of the laſt Quotient Figure, and ſubſtract it likewiſe: For if this Cube, and the Number laſt above mentioned, cannot be ſubſtracted, the laſt Quotient Figure is too big, and a leſs muſt be taken. 4.

5. If there be more Pricks on the given Cube, reiterate the work in the ſecond, third and fourth Directions. 5.

Example.

For *Example*: If I would know what is the Cube Root of 102503232, when I have placed and pricked the Number, I find 102 to belong to the Left Hand Prick, the Greateſt Cube I can have from thence is 64, of which the *Cube Root* is 4, therefore I ſet 4 in the Quotient, and taking 64 from 102 leave 38 over the ſame, as at *G*.

Then 4 tripled is 12, which multiplyed by 4 is 48 for a Diviſor, which ſtanding under 385, I ſee I may take him 6 times from thence, wherefore putting 6 in the Quotient, and ſubſtracting 48 the Diviſor, 6 times, I cancel the Number 385, and ſet the Remainder over the ſame in order as at *H*.

Then I ſquare 6, and it is 36, which multiplyed by 12, the triple of 4, makes 432, placed under 970 and ſubſtracted, leaves remaining 538, and then placing the Cube of 6, which is 216, under 5383, and ſubſtracting it, leave 5167, as at *I*.

Then becauſe there is yet one Prick remaining, I ſquare the Quotient 46, and it is 2116, which I triple, and it is 6348; and this I take for a new Diviſor under 51672, and by it get 8 in the Quotient, and ſubſtracting this new Diviſor 8 times, leave behind 888, as at *K*.

Then do I ſquare 8, and it is 64, which multiplyed by the triple of 46, produceth 8832, this I ſubſtract from the next place to the Right Hand, as at *L*.

Laſtly, the Cube of 8 ſubſtracted, cuts off all the Figures, whereby it appears, 468 is the *Cube Root* of 102503232, and the whole work ſtands as at *M*. And the truth of this Extraction is proved by the Production of the Cube according to the former *Chapter*, as this by Extraction reciprocally.

38

G. 102503232 | 4

64

9

387

H. 102503232 | 46

648

4

51

936

38787

I. 102503232 | 46

648 26

431

42

518

9368

387878

K. 102503232 | 468

648268

4314

423

6

518

93685

3878781

L. 102503232 | 468

6482682

43143

4238

68

518

93685

3878781

M 102503232 | 468

64826822

431431

42385

68

468 Root

3744

2808

1872

219024 Square

468

1752192

1314144

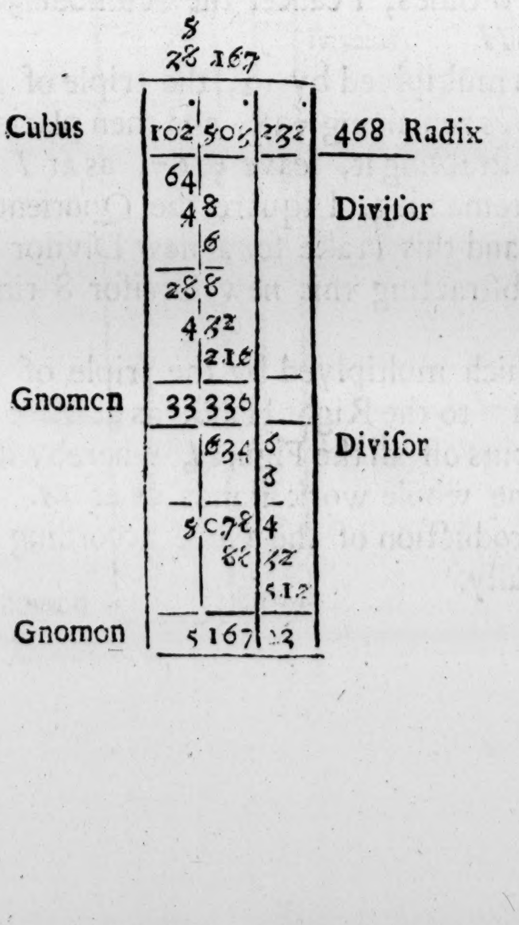
876096

Proof 102503232 Cube

How to work and prove it by Addition.

Cubical Extraction may alſo be proved by Addition: If when you have gotten the ſecond Quotient Figure, you place him under the Diviſor, and multiply the Diviſor thereby, and under this Product, one place nearer to the Right Hand, the Product of the Multiplyer ſquared and multiplyed by the tripled Root, and under this one degree nearer to the Right Hand, the Cube of the Multiplyer, and ſubſtract all theſe Numbers added into one Total Gnomon from the given Cube, cancelling the Figures you make Subſtraction from, and if you pleaſe thoſe underneath, except the Gnomon. And ſo continue this work till all the Pricks be done with, then adding all the uncancelled Gnomons with the firſt ſubſtracted Cube orderly placed, and you will have the Cubical Number returned; as appeareth by the former Example wrought and proved this way.

Example.



Proof.

64

33336

5167232

102503232

Some

Variety of work. Some after pricking the Number, and ſubſtracting the Greateſt Cube out of the laſt Prick, and getting the Diviſor, and thereby a ſecond Figure in the Quotient as before; by this Quotient Figure multiply the Diviſor, and ſubſtract the Product, then triple the firſt Quotient Figure, and to the Right Hand of the triple ſet the laſt Quotient Figure, and the Product of this Number multiplyed by the Square of the laſt Quotient Figure, place under the given Number, ſo that the Right Hand Figure thereof may ſtand under the next Prick to the ſame Hand, and ſubſtract. And ſo reiterate this manner of work till all be finiſhed.

Example. As in the former Number, after 64, as before, is ſubſtracted, the Diviſor 48 I multiply by 6, and the Product 288 ſubſtract from 385, then I triple 4 the firſt Quotient Figure, and by the 12 amounting, I ſet 6 the ſecond Quotient Figure; this 126 I multiply by 36, the Square of 6, and the Product 4536 I withdraw from 9703, and leave at top 5167: Then getting the new Diviſor, as before, 6348, and multiplying it by the new Quotient 8, there is produced 50784, after Subſtraction of which to the triple of 46, which is 138; I adjoyn 8, and increaſe this 1388 by 64, the Square of 8, and the Product 88832, abated from the given Cube, leaveth nothing as before:

```
            5
            9188
            387678
            .  .  .
Cubus       102503232 | 468 Radix
            64  :  :                  126     1388       64
            48  :  :                   36       64       288
             6  :  :                  ----    -----       4536
        {    :  :                     756     5552        50784
Gnomon  {   288 :  :                  378     8328         88832
        {   4536   :                  ----    -----      ---------
            ---------                 4536    88832      102503232
            6348   :                  ----    -----      ---------
               8   :
        {  --------:
Gnomon  {   50784  :
        {    88832
            ---------
```

Proof.

Other variety. Others after the Greateſt Cube out of the Left Hand Prick is ſubſtracted, and by the Diviſor a ſecond Figure placed in the Quotient, as before, triple the whole Quotient, and multiply that triple by the firſt Quotient Figure, and again the Product by the latter Quotient Figure, and to that ſubjoyn the Cube of the ſaid latter Figure one place nearer to the Right Hand, and then deduct the Total out of the given Cube, repeating the like work for every Prick.

Example. As in the former Example thus: After 6 is found for the ſecond Figure of the Quotient, I triple 46, and it's 138, which multiplyed by 4, gives 552, that again by 6 produceth 3312, then 216, the Cube of 6, ſubjoyned makes 33336, for the Gnomon to be ſubſtracted from 38503, ſo reſts 5167 to that Prick; this reiterated for the next leaves 0, as before.

```
            5                          46        468       Proof.
            38167                       3          3
            .  .  .                   ----      -----
Cubus       102503232 | 468 Radix      138       1404       64
            64   :  :                    4         46       33336
Diviſor     48   :  :                 ----      -----        5167232
            -----:  :                  552       8424      ---------
Gnomon      33336   :                    6       5616      102503232
            ------  :                 ----      -----      ---------
Diviſor       6348  :                 3312      64584
            --------:                  216          8
Gnomon      5167232                   ----     ------
            ---------                 33336    516672
                                      ----        512
                                              -------
                                              5167232
                                              -------
```

Tap his way of work. *Tap* in his *Seamans Kalendar* implyes 300 and 30 in the work; thus: After the Number is pricked, and the Cube of the Left Hand Prick ſubſtracted as before, the Square of the Root found multiplyed into 300, ſhall be the Diviſor, to be placed ſo as the Right Hand Cypher thereof ſhall ſtand under the next Prick, and a new Quotient Figure gotten thereby, multiply the Diviſor, and then multiply the firſt Quotient Figure by 30, and that Product by the Square of the ſecond Quotient Figure, and add theſe, with the Cube of this laſt Quotient Figure, into one total Gnomon; and for every Prick do the like.

Example. As in the former Number, firſt 64, the Cube of 4, ſubſtracted from 102, I ſet down 300, and 30, and againſt 300, to the Left Hand the Square of 4, which is 16, and to the Left Hand of 30, the Root it ſelf 4, then multiplying 300 by 16, the Product 4800 is the firſt Diviſor, by which 6 gotten in the Quotient, I place it on the Right Hand of 300, and the Square of 6 on the ſame Hand of 30, and then multiplying all the Numbers in the upper row, I have 28800, and the Product of thoſe in the lower row is 4320, which added with 216, the Cube of 6, make the Gnomon 33336; and ſo proceed to deal with the Numbers remaining to the other Prick, as here appeareth.

```
            5
          38167
         .  .  .
Cubus   102503232 | 468 Radix   16 . 300 : 6           2116 . 300 . 8
         64  :  :                4 . 30 . 36             46 . 30 . 64
        ----  : :
Divisor  4800   :     300      30           |      2116        46
        -----   :      16       4           |       300        30
Gnomon  33336   :    ----     ----   28800  |   -------     -----      5078400
        -----   :    1800      120    4320  |    634800      1380        88320
Divisor    634800     300       36     216  |         8        64          512
        ---------    ----     ----   -----  |   -------     -----     -------
Gnomon   5167232     4800      720   33336  |   5078400      5520     5167232
        ---------       6     360    -----  |   -------      8280     -------
Proof   102503232    ----     ----          |               -----
        ---------    28800    4320          |               88320
```

Adding the uncancelled Gnomons with the Cube firſt ſubſtracted, as they ſtand will return the Great Cube, and prove the work true in this, as the other varieties.

To extract the Root of Compounds. §. 4. Of the Prime or Original Figurate Rooted Numbers, a *Square* and a *Cube*: Several Higher Quantities are Compound, as their Names *Arithmetical* denote. And perhaps this may be the reaſon why they retain Names different from their *Geometrical* Forms, that it might be an eaſie memento to the ſpeedy Extraction of their Roots: For according to the Compoſition ſo ſhall you draw the Root from thence, whether Zenzick or Cubick, and ſo often as the Name is found in the Compoſition, in their order beginning at the Left Hand.

Compounds of 3 ſorts. Theſe two Names *Square* and *Cube*, or *Zenzick* and *Cubick*, make three ſorts of Nominal Compounds, *viz.*

Either Squares with Squares, as Zenzizenzikes, Zenzizenzizenzikes, *&c.*

Or 2. Cubes with Cubes, as Cubicubicks, Cubicubicubicks, *&c.*

Or 3. Squares with Cubes, as Zenzicubes, Zenzizenzicubes, Zenzicubicubes, *&c.* Examples of each ſort follow.

Extraction of the firſt ſort. A Zenzizenzike is the leaſt and firſt Number of the firſt Compoſition, in which, becauſe the Zenzick is twice repeated, I extract the Square Root of the given Number, and from that Root (which alſo will be a Square Number) I extract the Square Root again; and ſo is this laſt Root, the Zenzizenzike, or Squared Square Root of the firſt given Number.

1 Example. As if I would extract the Zenzizenzike Root of 796594176, I firſt extract the Square Root, which is 28224, and from this Square Number extract the Square Root again, and ſo have 168, which I accept for the Squared Square Root deſired.

```
                11225        3
                37235671     1666
                . . . . .    . . .
Zenzizenzike    796594176 | 28224 | 168 Radix
                444644446    12624
                 6556641      336
                    5
```

2 Example. And ſo for the Zenzizenzizenzikes, or of greater Quantity, it is but to reiterate the Zenzick Extraction. As to know the Zenzizenzizenzike Root of 16983563041, I firſt extract the Square Root, which is 130321, and then the Square Root of that, which is 361, and alſo the Square Root thereof, and ſo take 19 for the Root deſired.

```
                      260
                      54716        447      28
                      . . . . .    . . .    . .
Zenzizenzizenzike  16983563041 | 130321 | 361 | 19 Radix
                   12960096441   96621    121
                     2266006       37      8
                      226
```

Numbers of the ſecond Compoſition have the Cubick Root extracted in like manner. As to know the Cubicubike Root of 10604499373, the firſt Extraction gives the Cubick Root 2197, which is alſo a Cube Number, and his Root extracted is 13, the Cubicubike Root of the given Number.

Extraction of the ſecond ſort.

Example.

```
            10132
          13520423
          244376024     122
           . . . .     . .
Cubicube  10604499373 | 2197 | 13 Radix
          8261339333    1377
          113202894      22
            517813
             1432
              3
```

The like is to be done for Numbers of greater Quantity under this Compoſition; as Cubes of Cubick Cubes to extract the Cube Root 3 times, *&c.*

When the Number is of the third Compoſition, as in Zenzicubes, Zenzizenzicubes, Zenzicubicubes, *&c.* ſo often extract the Zenzick Root as that Name is in the Compoſition, and ſo likewiſe the Cubick Root, and in ſuch order as they ſtand compounded.

Extraction of the third ſort.

1 *Example.* For in a Zenzicube, firſt extract the Square Root, this Root ſhall be a Cube, whoſe Cube Root extracted ſhall be the Zenzicube Root of the given Number. As 729 is a Zenzicube, whoſe Square Root is 27, which is a Cube Number, and hath 3 for his Root.

2 *Example.* In a Zenzizenzicube, extract the Square Root twice, and the Cube once at laſt. As in 4096, whoſe Square Root firſt extracted is 64, the Square Root of which is 8, and 8 is the Cube of 2; ſo 2 is the Radix Zenzizenzicubick of 4096.

3 *Example.* But in a Zenzicubicubick extract the Square Root once at firſt, and then the Cube Root twice. As in 262144, the Square Root is 512, the Cube Root whereof is 8, which is alſo a Cube Number, and hath 2 for his Root.

Proof. All theſe Extractions may be proved by common Production, and if the Numbers ſubſtracted in the Work be collected into Gnomons, by Addition, as aforeſaid.

To extract the Roots *of the* Higher Powers *by the* Table *following.*

§. 5. Becauſe the Surſolides are excluded out of the foregoing Compoſitions, whoſe Roots nevertheleſs it is requiſite to know how to extract, when occaſion ſhall require; and becauſe it would be tireſome to ſet down particular Rules for every Quantity of the Higher Powers, and troubleſome to the Memory to retain them, with the varieties of work, ſeeing as a Figural Number may be produced divers wayes, his Root may be many wayes extracted. And beſides the particular Rules for each increaſing according to their Quantities, and ſo in effect are as endleſs and various as the Numbers themſelves, it will be more commodious to perform the Extraction of the Roots of all the Higher Powers by ſome one General Direction, which by the help of the *Table* following is perfectly to be done.

Particular Rules grounded on the Table.

And indeed all the Particular Rules that are given for Extraction of Roots, even thoſe of the Square and Cube, have their ground in the *Table*, and come from thence; as might very eaſily be demonſtrated. And by comparing the Tabulary Numbers and Operation therewith, hereafter in this *Section* ſet forth, with the Particular Rules here ſet down for the Zenzizenzike, Surſolide, and Zenzicube, will be ſufficiently clear without further illuſtration.

Particular Rules for the Zenzizenzike.

Particular Rules for the Squared Square.

1. After the Number is pricked according to his Quantity from the Left Hand Prick, ſubſtract the greateſt Zenzizenzike Number, and ſet the Zenzizenzike Root thereof in the Quotient. 1.

2. Multiply

2. 2. Multiply this Root Cubically, and quadruple the Product for a Diviſor to be placed one place nearer to the Right Hand, and a new Quotient Figure gotten thereby, Multiply the Diviſor, and reſerve this Product to be added with the 3 Numbers in the next Directions to make up the Gnomon.

3. 3. Square the firſt Quotient Figure, ſexcuple the Square, and multiply the Product by the Square of the laſt Quotient Figure.

4. 4. Cube the laſt Quotient Figure, and Multiply the Cube by the quadruple of the firſt Quotient Figure.

5. 5. Take the Zenzizenzike Number of the laſt Quotient Figure, and with the 3 Numbers laſt above-mentioned, add them (duely placed one nearer than another to the Right Hand) into one total Gnomon, and ſubſtract the ſame from the given Number: And if the Number have more Pricks than 2, the work in the ſecond, third, fourth, and fifth Directions is to be repeated.

Example. Thus the former Number 796594176 pricked, and 1 the greateſt Squared Square Number in 7 ſubſtracted, I multiply the Cube of 1, which is 1 by 4, and 4 is Diviſor; which though ſtanding under 69, yet will afford but 6 for the Quotient. This 6 multiplyed into 4, gives 24. Then 1 ſquared is 1, and multiplyed by 6 is 6, and again by 36, the Square of the laſt Quotient Figure, makes 216. And then 216, the Cube of 6, by 4 the quadruple of 1, the Product is 864. Laſtly, 1296, the Zenzizenzick of 6, added with 24, 216, 864, makes the Gnomon for the ſecond Prick.

Then the Cube of 16 quadrupled is 16384, the next Diviſor, whereby 8 is gotten for a new Quotient Figure, which multiplying the Diviſor gives 131072. Then the Square of 16, ſexcupled and multiplyed by the Square of 8, makes 98304. And the Cube of 8, by the quadruple of 16, produceth 32768. And the Zenzizenzike of 8 is 4096, added to the other Numbers make the next Gnomon, which ſubſtracted as the former leave 0 remaining.

```
             1                                   1
             64123                              ----
             .   .   .                          24      }
Zenzizenzike 796594176 | 168 Radix              216     }
             1   :   :   -----                   864    } 55536
Gnomons     {55536   :                            1296  }
            {141234176                          --------
             ---------                          131072    }
Proof        796594176                            98304   }
             ---------                             32768  } 141234176
                                                    4096  }
                                                --------
                                                796594179
                                                ---------
```

Particular Rules for the Surſolide.

Particular Rules for the Surſolide.

1. 1. The Number being pricked according to his Quantity, take from the Figures belonging to the Left Hand Prick, the greateſt Surſolide Number therein, and place the Surſolide Root thereof in the Quotient.

2. 2. Multiply the Zenzizenzike Number of the Root by 5, (or quintuple it) and the Product ſhall be the Diviſor to be ſet one place nearer to the Right Hand, by which get a new Quotient Figure, and Multiply the Diviſor thereby, and reſerve the Product for the firſt Number of the Gnomon.

3. 3. Cube the firſt Quotient Figure, decuple the Cube (or Multiply it by 10) and Multiply the Product by the Square of the laſt Quotient Figure.

4. 4. Square the firſt Quotient Figure, decuple the Square, and Multiply the Product by the Cube of the ſecond Quotient Figure.

5. 5 Quintuple the firſt Quotient Figure, and Multiply the Product by the Zenzizenzike of the next Quotient Figure in the Root.

6. 6. Add theſe 4 laſt mentioned Numbers with the Surſolide of the laſt Quotient Figure, orderly placed every one nearer by one place to the Right Hand than the other, into one Gnomon, and ſubſtract the ſumme from the given Number. And for every other Prick on the given Number, go over again with the work in theſe 5 laſt Directions.

Example. As to extract the Surſolide Root of 28153056843, which marked as directed, it doth appear that 1 is the greateſt Surſolide, and Root alſo of 2. Then the Zenzizenzike of 1 is 1, which quintupled is 5, this 5 is Diviſor to 18, by which I get but 2 for the Quotient, this multiplyed into 2 makes 10. Then the Cube of 1 is but 1, which decupled is 10, and multiplyed by 4 is 40. And the Square of 1 decupled is 10, and multiplyed by 8, the Cube of 2, gives 80. And the quintuple of 1 is 5, which multiplyed by 16, the Zenzizenzike of 2, yields 80. Laſtly, 32, the Surſolide of 2, added with 10, 40, 80, and 80, in their order make up the Gnomoh 148832, for the ſecond Prick. And by reiterating the work for the next, get the Gnomon 3269856843.

```
              12269 8                             1
              .   .   .                         ------
Surſolide     28153056843 | 168 Radix          10      )
              1   :   :                         40     )
Gnomons { 148832      :                          80    ) 148832
        { 3269856843                              80   )
          ---------------                          32  )
Proof     28153056843                          ---------
          ---------------                      311040   )
                                                155520  )
                                                 38880  ) 3269856843
                                                  4860  )
                                                   243  )
                                               ---------
                                               28153056843
```

Particular Rules for the Zenzicube.

Particular Rules for the Squared Cube.

1. The Number pricked according to his Quantity, deduct the greateſt Zenzicube Number from the Left Hand Prick, and put the Zenzicube Root thereof in the Quotient. 1.

2. Sexcuple the Surſolide Number of the Root, this Product ſhall be Diviſor, by which get another Figure for the Quotient, and multiply the Diviſor thereby, reſerving this Number for the Gnomon. 2.

3. Multiply the Zenzizenzike Number of the firſt Figure of the Root by 15, and the Product again by the Square of the next Figure of the Root. 3.

4. Multiply the Cube of the firſt Quotient Figure by 20, and the Product again by the Cube of the next Quotient Figure. 4.

5. Multiply the Square of the firſt by 15, and that again by the Zenzizenzike of the ſecond Quotient Figure. 5.

6. Multiply the firſt Figure of the Root by 6, and the Product by the Surſolide of the ſecond Figure of the Root. 6.

7. Let all theſe five laſt mentioned Numbers be added with the Zenzicube of the ſecond Quotient Figure, orderly placed one nearer to the Right Hand than the other, into one Gnomon, and ſubſtract the ſame from the given Zenzicube; and repeat the work of theſe ſix laſt Directions for every other Prick. 7.

Example. As in extracting the Zenzicube Root of 244140625; firſt out of 244, I take the greateſt Zenzicube I can, which is 64, whoſe Root is 2, the Surſolide of which 32 ſexcupled is 192, for Diviſor, by which 5 is gotten for the Quotient; this 192 multiplyed by 5 is 960. And 16, Zenzizenzike of 2 by 15 is 240, and again by 25, the Square of 5 is 6000. And 8, the Cube of 2, by 20 is 160, which by 125, the Cube of 5, is 20000. And 4, Zenzike of 2, by 15 is 60, which by 625, the Zenzizenzike of 5, is 37500. And 2, the Root by 6 is 12, that by 3125, the Surſolide of 5, is 37500. Laſtly, 15625, the Zenzicube of 5, makes up the Gnomon.

```
              180                       64
               .       .              ------
Zenzicube     244140625 | 25 Radix    960      )
              64       :              6000     )
Gnomon        180140625               20000    ) 180140625
              ----------               37500   )
Proof         244140625                 37500  )
              ----------                 15625 )
                                      ---------
                                      244140625
```

A Table *for* Extraction of Roots.

The Table for Extraction of Roots.

Zenzizenzicube.	12	66	220	495	792	924	792	495	220	66	12
Third Surſolide.	11	55	165	330	462	462	330	165	55	11	
Square Surſolide.	10	45	120	210	252	210	120	45	10		
Cubicube.	9	36	84	126	126	84	36	9			
Zenzizenzizenzike.	8	28	56	70	56	28	8				
Second Surſolide.	7	21	35	35	21	7					
Zenzicube.	6	15	20	15	6						
Surſolide.	5	10	10	5							
Zenzizenzike.	4	6	4								
Cube.	3	3									
Squa.	2										
Root.	1										

The* Table *ex-plained.

The Table Explained.

The little Iſſuants at Top denote the *Table* may be increaſed as occaſion requires, though this reaching to Zenzizenzicubes be large enough for Example.

On the exterior parts on either ſide aſcending from the Root, are Numbers declaring the Quantities of the Powers whoſe Names are placed diſtinctly on the Head of each Quantity.

The Numbers in the Body of the *Table* are thus gotten, after the Numbers of the Quantities are placed in the exterior parts, add the two Numbers belonging to the Cube together, which being 3 and 3, they make 6, to fill up the middle Square belonging to the Zenzizenzike, between the 4 and 4, ſignifying his Quantity which 6 and 4 added make 10, for the two middle Squares of the Surſolide ; and this 10 added to 5 is 15, for the Squares of the Zenzicube next the middle, and for that 10 to 10 gives 20 : And thus adding one Number with another alternately, the other Numbers are found ; and the *Table* may be enlarged *ad infinitum.*

***Uſe of the* Table.**

The Uſe of the Table.

To extract the Root of any Number by help of the *Table* ; after the Number is pricked according to his Quantity as before taught, and the Greater Number of that Quantity whoſe Root you would extract, ſubſtracted out of the Figures belonging to the Left Hand Prick, and the Root thereof ſet in the Quotient, as before ; then ſet apart this Root with his Square, Cube, Zenzizenzike, *&c.* until you come to the Number before ſubſtracted. And in order under them, beginning at the Root, place the Numbers found in the *Table* belonging to the Quantity whoſe Root you are extracting. Then Multiply the Numbers ſtanding one over another, one into another, the Multiplication next the Right Hand ſhall be the Diviſor, which after you have gotten another Figure of the Root by to be ſet in the Quotient, ſet him down under the Diviſor, and his Square under the next Multiplication to the Left Hand, and his Cube under

nder the next, and ſo proceed to the Quantity you are extracting, and ſo will every the former multiplyed Numbers have another Number under them, and one Num-er over; then multiply the one by the other, and add all theſe Products with the Left and Quantity together, in ſuch order that every Number be placed one place nearer the Right Hand than the other; and the Total of theſe Products make up the Gnomon r the next Right Hand Prick to be ſubſtracted, as before: And ſo for every other rick on the given Number, repeat this work.

1. *Example:* To extract the Square Root of 28224. *Example of the Square.*

```
             128                     Rad. Zen.         Rad. Zen.
             . .:.                    1    1            16  &c.
Zenzike      28224 | 168              2                  2  Tabulary Number
             1 : :                   ---               ---
Gnomons { 156  :                      2  Firſt Diviſor.  32  Second Diviſor.
        {  2624                      ---               ---
           -----                 Zen. Rad.         Zen. Rad.
Proof      28224                  36   6            64    8
           -----                 --------          --------
                                  36  12            64  256
                                 --------          --------
                                      12               256
                                      36                64
                                     ---              ----
                                     156 Gnomon.      2624 Gnomon
                                     ---              ----
```

2. *Example.* To extract the Cube Root of 4741632. *Example of the Cube.*

```
          3845                  Rad.  Zen.  Cub.             Rad.  Zen.  Cub.
          .  .  .                1     1     1                16   256   &c.
Cubus     4741632 | 168          3     3                       3     3   Tabulary Numbers.
          1                     ----------                   ---------
Gnomons { 3096                   3     3  Firſt Diviſor       48   768   Second Diviſor.
        {  645632               ----------                   ---------
          -------          Cub.  Zen.  Rad.              Cub.  Zen.  Rad.
Proof     4741632          216   36    6                 512   64    8
          -------          ----------------              ----------------
                           216   108   18                512   3072  6144
                           ----------------              ----------------
                                 108                           3072
                                 216                            512
                                 ----                         ------
                                 3096 Gnomon.                 645632 Gnomon
                                 ----                         ------
```

3. *Example.* To extract the Zenzizenzike Root of 796594176. *Example of the Squared Square.*

```
                       1
                       64123
                       .     .     .
Zenzizenzike           796594176 | 168 Radix
                       1     :     :
Gnomons              { 55536       :
                     { 141234176
                       ---------
Proof                  796594176
                       ---------

Rad. Zen.  Cub. Zenz.                          Rad.   Zen.   Cub.   Zenz.
 1    1     1    1                              16    256    4096   &c.
 4    6     4                                      4      6      4  Tabulary Numbers.
---------------                                ---------------------
 4    6     4  Firſt Diviſor.                   64    1536   16384  Second Diviſor
---------------                                ---------------------
Zenz. Cub.  Zen.  Rad.                  Zenz.  Cub.   Zen.   Rad.
1296  216   36    6                     4096   512    64     8
----------------------                  ---------------------------
1299  864   216   24                    4096   32768  98304  131072
----------------------                  ---------------------------
                  216                                         98304
                   864                                         32768
                   1296                                          4096
                 ------                                  ---------
                  55536 Gnomon.                          141234176 Gnomon.
                 ------
```

4. *Example.*

Example of the Surſolide.

4. ***Example.*** To extract the Surſolide Root of 28153056843.

132698

Surſolide 28153056843 | 123 Radix

Gnomons { 1, 148832, 3269856843

Proof 28153056843

Rad.	Zen.	Cub.	Zenz.	Sur.	
1	1	1	1	1	
5	10	10	5		
5	10	10	5		First Diviſor.

Sur.	Zenz.	Cub.	Zen.	Rad.
32	16	8	4	2
32	80	80	40	10

40
80
80
32

Gnomon 148832

Rad.	Zen.	Cub.	Zenz.	Sur.
12	144	1728	20736	&c.
5	10	10	5	Tabulary Numb
60	1440	17280	103680	Second Diviſ

Sur.	Zenz.	Cub.	Zen.	Rad.
243	81	27	9	3
243	4860	38880	155520	311040

155520
38880
4860
243

Gnomon 3269856843

Example of the Squared Cube.

5. ***Example.*** To extract the Zenzicube Root of 164170508913216.

16
00134619

Zenzicube 164170508913216 | 234 Radix

Gnomons { 64, 84035889, 161346199913216

Proof 164170508913216

Rad.	Zen.	Cub.	Zenz.	Sur.	Zenzic.
2	4	8	16	32	64
6	15	20	15	6	Tabulary Numbers
12	60	160	240	192	First Diviſor

Zenzic.	Sur.	Zenz.	Cub.	Zen.	Rad.
729	243	81	27	9	3
729	2916	4860	4320	2160	576

2160
4320
4860
2916
729

84035889 Gnomon

Rad.	Zen.	Cub.	Zenz.	Sur.	Zenzic.
23	529	12167	279841	6436343	&c.
6	15	20	15	6	Tabulary Numbers
138	7935	243340	4197615	38618058	Second Diviſor

Zenzic.	Sur:	Zenz.	Cub.	Zen.	Rad.
4096	1024	256	64	16	4
4096	141312	2031360	15573760	67161840	154472232

67161840
15573760
2031360
141312
4096

161346199913216 Gnomon

Surdes what.

§. 6. A *Surde Number*, ſometime called *Irrational*, is as much as to ſay a Number from which it is not poſſible to take the Root, but there will remain ſomething, which declares that the Number given was not a perfect Figural Number, and the Side thereof cannot therefore be expreſſed by an Integer. As the Square Root of 12 is 3, and there will remain 3; ſo the Cube Root of 28 is 3, and 1 will remain.

To get the greateſt Root therein.

The Extraction of the Roots of *Surde Numbers* differs nothing from the wayes already ſet forth, and may be proved by the common way of Production, or Addition of the Gnomons, adding in the Remain to the Product or Total. But all the difficulty is to denominate the Remain, to know what part of the whole is ſignified thereby, ſeeing the Diviſors are alwayes uncertain. Some Authors for the Square double the Root; others add 1 to the double for the Denominator; and others double the Remainer for the Numerator, and to the quadruple of the Root add 1 for the Denominator. And for the Cube, triple the Square of the Root, and add thereto the triple of the Root, and 1 more for the Denominator; yet all theſe, and ſeveral other Rules, fail to find out a true Denominator exactly, but that which comes neareſt the truth is thus.

The neareſt Denominator to the Remain.

Adjoyn Cyphers to the Right Hand of the given Number according to his Quantity. As if a Square, as many times two Cyphers, as you pleaſe: If a Cube, as many times three Cyphers, as you pleaſe, *&c.* and continue the Extraction of the Root to the end of the Cyphers, and the more Cyphers are adjoyned the nearer the true Root you come. Then divide the Quotient by an Unit, and half the Numbers of Cyphers you added if it were a Square, and the third part if a Cube, the fourth part if a Squared Square, *&c.*

Example in a Square Plot of Ground.

1. *Example*, in a Square Surde. Suppoſe a Square Plot of Ground were 18 Perches, and I would know the Side thereof: If then I add 2 Cyphers, and extract the Root, the Quotient is 42, which divided by 10, the Root is $4\frac{1}{5}$. And if I add 4 Cyphers, and divide the Quotient 424 by 100, the Root is $4\frac{6}{25}$. But if I add 6 Cyphers, the Quotient will be brought nearer the truth, and be $4\frac{121}{500}$ beſides the ſmall Fractions ſtill left upon the Extractions, which denominate after the other wayes uſed in Authors, and added will ſomewhat increaſe the Number.

```
                                 |2
         2|36                 2 3 6|24
                                                              Proof of both.
          .  .     Radix       .   .   .      Radix
Zen. 1800 |4|2(  4 1/5  Perches  1800 00 |4|24(  4 6/25          42        424
      168 |1 0                    168 :  | 1 00                  42        424
     -----                       -----                          ----      ----
      164                          164                            84      1696
     -----                       -----                          168        848
                                    84                 Remain    36      1696
                                 -----                          ----       224 added
                                  3376                          1800      ----
                                 -----                          ----    180000
                                                                          ----
```

Example in a doubled Cube.

2. *Example*, in a Surde Cube. It is noted of the *Greeks*, that through their great Luxuriousneſs and Riot, they had brought Contagious Diſeaſes upon them, and conſulting their Oracles for redreſs thereof, received anſwer, *That when they would double their Altar* (which was of a Cubick Form) *they ſhould be delivered from thoſe Plagues*; meaning, the beſt Method to deliver Realms from ſuch contagion breeding Vices, was to abate of their Voluptuouſneſs, and apply themſelves to Literature. But now ſuppoſe the Altar was 4 Feet Square every way, and the Altar were doubled, what muſt the Side be? Here if I double the Cube of 4, which is 64, it is 128, from which becauſe I cannot extract the Cube Root without leaving a Remain, I aſſay to come near the Truth, and adjoyn 6 Cyphers, and extracting the Root yet 503 in the Quotient, and 736473 remaineth, then dividing 503 by 1, and 2 Cyphers, which are the third part of 6, I have 5 Feet, and $\frac{3}{100}$ of a Foot for the Root or Side, beſides the odd Remain, which denominated after the common way, will be $\frac{736473}{760537}$ of $\frac{1}{100}$ or $\frac{736473}{76053700}$.

```
                   3|736473          Radix                     503
Root    4            :  :  |                                   503
        4    128 000000|5|03( 5 3/100 Feet                   ------
       ---   125       :  :  1 00                             1509
Square  16     78      :  :                                  25150
        4    -------   :  :                                 --------
       ---     0000       :                                 253009
Cube    64   ----------   :                                    503
       ---      7500      :                                 --------
Double 128   ------------ :                                 759027
       ---    2263527                                     12650450
             -------------                      Remain      736473 added.
                                                           ---------
                                                          128000000
                                                          ---------
```

CHAP. IV.

Figurate Fractions.

To Figurate Fractions,

When a Fraction is given to be multiplyed Figurally, Multiply the Numerator by himself into the Quantity desired, and the Denominator likewise.

And Mixt Numbers.

If an Integer and a Fraction be given, reduce them into an Improper Fraction, and then Multiply Figurally the Numerator by himself, and also the Denominator into the Quantity desired.

Examples.

As {To Square $\frac{3}{4}$ is $\frac{9}{16}$, and $2\frac{3}{4}$ reduced is $\frac{11}{4}$ Squared is $\frac{121}{16}$.
To Cube $\frac{3}{4}$ is $\frac{27}{64}$, and $2\frac{3}{4}$ reduced is $\frac{11}{4}$ Cubed is $\frac{1331}{64}$.

To extract the Roots.

To extract the Root of a Fraction: First, extract the Root of the Numerator, and then the Root of the Denominator of the same Quantity.

Examples.

As to extract the {Square Root of $\frac{9}{16}$ renders $\frac{3}{4}$.
Cube Root of $\frac{27}{64}$ renders $\frac{3}{4}$.

No Roots in Heterogeneal Fractions.

But if the Numerator and Denominator be Heterogeneal, that is, not both of one Nature, though the one be a Square, and the other a Cube, as $\frac{16}{27}$, or the like; yet can the Fraction have no Root extracted, but must remain as a Surde broken Number; of which more hereafter in the next *Book.*

Partis Secundæ & Libri Secundi

FINIS.

ARITHMETICK.

The Third BOOK,

CONCERNING

Numbers ſpecially contract;

In Six PARTS.

WHEREIN

DECIMALS	are	Diſcovered.
ASTRONOMICALS		Anatomized.
LOGARITHMES		Overlooked.
COSSICKS		Characterized.
SURDES		Surveyed.
SPECIES		Inſpected.

AND THEIR

SIMPLE ELEMENTS.

CHAP. I.

Of DECIMALS.

IN the former *Books* enough hath been ſaid of *Abſtract* and *Generally Contract*, or *Vulgar denominate Numbers*, to make repetition here would be tautological. The next ſort of Numbers I ſhall therefore fall upon are *Numbers ſpecially Contract*, which are Numbers challenging to themſelves ſome ſpecial Operations, and reſtrained by ſome Denomination either implyed or expreſſed: Implyed if the Denominators be certain; expreſſed if otherwiſe.

Numbers ſpecially contract follow thoſe generally ſo.

What Numbers ſpecially Contract are.

Denominators are certain in *Decimals*, *Aſtronomicals* and *Logarithmes*, and therefore omitted: Denominations uncertain in *Coſsicks*, *Surdes* and *Species*, and therefore expreſſed. Of theſe in order, and firſt of *Decimals*.

Denominators *where certain where uncertain.*

Decimals their converſe and practice.

Decimals converſe very much with *Geodæticals*, and practice to work both Integers and Fractions together in *Addition*, *Subſtraction*, *Multiplication* and *Diviſion*, thereby facilitating many laborious and intricate works in Common Fractions.

Whence the Name, and what it ſignifies.

Denominator of Decimals what, and how known.

They take their Name from the *Latin* word *Decimus*, ſignifying a Tenth, or Tenth part; becauſe as the Integers above the Unit increaſe by Tens towards the Left Hand, ſo the Decimal Fractions below the Unit decreaſe by Tens towards the Right Hand; the Denominator of every Decimal Fraction being alwayes an Unit with Cyphers: As 10, 100, 1000, &c. that is to ſay, an Unit and ſo many Cyphers as there are Figures in the Numerator. And therefore becauſe the Denominator doth alway conſiſt of one place more than the Numerator, and is thereby certainly known, the Denominator is omitted, and the Numerator only uſed. For if the Numerator conſiſt of 3 Figures or Places, the Denominator ſhall be 1000, which is 3 Cyphers and an Unit; and if the Numerator have 4 Figures, the Denominator ſhall be 10000, &c.

Several wayes of diſtinguiſhing the Decimal.

For diſtinguiſhing of the Decimal Fraction from the Integers, it may truly be ſaid, *Quot Homines, tot Sententiæ*; every one almoſt fancying ſeverally. For ſome call the Tenth Parts, *Primes*; the Hundredth Parts, *Seconds*; the Thouſandth Parts, *Thirds*, &c. and mark them with *Indices* equivalent over their heads. As to expreſs 34 Integers and $\frac{1426}{10000}$ Parts of an Unit, they do it thus, $34.\overset{\prime}{1}.\overset{\prime\prime}{4}.\overset{\prime\prime\prime}{2}.\overset{\prime\prime\prime\prime}{6}.$ Or thus, $34.\overset{(1)}{1}.\overset{(2)}{4}.\overset{(3)}{2}.\overset{(4)}{6}.$ Others thus, $34,1426''''$; or thus, $34,1426^{(4)}$. And ſome thus, 34 . 1 . 4 . 2 . 6 . ſetting the Decimal Parts at little more than ordinary diſtance one from the other, and they read them thus, 34 Integers, 1 Prime, 4 Seconds, 2 Thirds, 6 Fourths: Or 34 Integers 1426 Fourths. Others diſtinguiſh the Integers from the Decimal Parts only by placing a Cŏma before the Decimal Parts thus, 34,1426; a good way, and very uſeful. Others draw a Line under the Decimals thus, $34\underline{\scriptstyle 1426}$, writing them in ſmaller Figures than the Integers. And others, though they uſe the Cŏma in the work for the beſt way of diſtinguiſhing them, yet after the work is done they uſe a Rectangular Line after the place of the Unite, called *Seperatrix*, a ſeparating Line, becauſe it ſeparates the Decimal Parts from the Integers, thus $34|\underline{1426}$. And ſometimes the Cŏma is inverted thus, 34‘1426, contrary to the true Cŏma, and ſet at top. I ſometime uſe the one, and ſometime the other, as cometh to hand.

Cŏma, a good way.

Seperatrix *what.*

For further diſcovery of the orderly progreſſion of *Decimals* behold the following *Tables*, one with *Figures* and *Letters*, and the other with *Letters*, *Figures* and *Cyphers*, ſometime called *Numeration Tables*.

A Table for Numeration of Decimals.

Table with Letters *and* Figures.

			Integers.						Unit.				Decimals.					
9.	8.	7.	6.	5.	4.	3.	2.	1.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
M	M	M	M	M	M	M	C	X	V	X	C	M	M	M	M	M	M	M
M	M	M	M	C	X				The Units place.				X	C	M	M	M	M
M	C	X														X	C	M

Another.

Table with Letters, Figures *and* Cyphers.

			Integers.						Unit.				Decimals.					
I.	H.	G.	F.	E.	D.	C.	B.	A.	V.	a.	b.	c.	d.	e.	f.	g.	h.	i.
9.	8.	7.	6.	5.	4.	3.	2.	1.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
1000000000	100000000	10000000	1000000	100000	10000	1000	100	10	The Units place.	10	100	1000	10000	100000	1000000	10000000	100000000	1000000000

The Tables *explained.*
Indices *what.*

In theſe *Tables* (for both are as one) there are four Progreſſions, two *Arithmetical* in the Figures above, which are called *Indices*, proceeding both wayes from Unity by the difference 1, though contrarily; two *Geometrical*, in the lower continued Proportions from Unity both wayes, by the *Ratio* 10. So that as the Series of the Numbers from the Units place are continued in a decuple proportion from the Right Hand towards the Left, and increaſe in their value, as before was ſaid; ſo their value decreaſeth in a ſubdecuple proportion from Unity towards the Right Hand.

Example, by the latter *Table* ſuppoſe, $4.\ 3.\ 2.\ 1.\ 0.\ _{|}\underline{1.\ 2.\ 3.\ 4.}$ be a Number given, and ſtand with his Letters thus, $\begin{matrix} D. & C. & B. & A. & V. & a. & b. & c. & d. \\ 4. & 3. & 2. & 1. & 0. & _{|}\underline{1.} & \underline{2.} & \underline{3.} & \underline{4.} \end{matrix}$; then ſhall *D.* be equal to 10 *C*, and *C.* equal to 10 *B*, and *B.* equal to 10 *A*, and *A.* equal to 10 *V*, and *V.* equal to 10 *a*, and *a.* equal to 10 *b*, *&c.*

Again, if *A.* be equal to 10 *V*, then ſhall *a.* be equal to $\frac{1}{10}$ of one *V*, and if *B.* be equal to 100 *V*, then ſhall *b.* be equal to $\frac{1}{100}$ of one *V*; and ſo if 1 Prime be $\frac{1}{10}$ of 1 Integer, every Second ſhall be $\frac{1}{100}$ Part, and every Third $\frac{1}{1000}$ Part, *&c.* of that Integer. And hence it is that *Decimals* are ſet down in a retrograde order to *Integers*: As if I were to ſet down Eight Hundred in *Integers*, it is thus 800, but in *Decimals* thus, $_{|}\underline{008}$.

The Unit, or one Integer, is alwayes underſtood to be divided into Parts, bearing denomination or name of the place of the laſt Figure to the Right Hand in the Decimal Fraction.

As $\underset{10}{0,1}$ ſignifieth One Tenth Part, and $\underset{10}{0,2}$ Two Tenth Parts, as if they were writ at large thus, $\frac{1}{10}$, $\frac{2}{10}$, the denomination of the place being Tenths, as the firſt place from the Unit towards the Right Hand, and noted as above with 10 in the *Table*. In like manner ſhall $\underset{100}{0,12}$ ſignifie Twelve Hundred Parts, and $\underset{100}{0,34}$ Thirty Four Hundred Parts, as if written at large thus, $\frac{12}{100}$, $\frac{34}{100}$, the denomination of the laſt place being noted with 100, under the laſt Figures, 2 and 4. The like is to be underſtood of others.

Cyphers *before* Integers *or after* Decimals *ſignifie nothing.*

Note further, That Cyphers before Integers, or after Decimals, that is to the Left Hand of the one, and to the Right Hand of the other, ſignifie nothing at all; but after the Integers and before the Decimals, they are ſignificant, for making up the places whereby the value of the other Figures are eſtimated. As 0001, ſignifieth but 1 in Integers; and ,1000, but $\frac{1}{10}$ in Decimals: Therefore in writing of Decimal Parts, mark well the Units place, by ſome one or other of the diſtinguiſhing Notes before mentioned, and let the void places (if any be) be filled up with Cyphers, and the place of the Unit ſet down, though there be no Integers. As to ſet down 3 Fourths, and 4 Fifths, thus 0,00034.

Units *place to be well noted.*

Removing the Seperatrix *what effect.*

To remove the *Seperatrix* one place nearer to the Right Hand increaſeth the Number ten times in value, and toward the Left diminiſheth it as much: For 0,125 ſhall be ten times leſs than 01,25.

Indices *very conſiderable.*

The *Indices* are very conſiderable, becauſe of ſingular uſe almoſt in all Contract Numbers, as well as Decimals, and ſerve here to find out the true value of the Products in Multiplication, and Quotients in Diviſion, as hereafter may be ſeen.

Indices *how known.*

The *Index* of the Unit is 0, as in the former *Tables*. And the *Index* of any other place may eaſily be known thus:

In Integers, abate one from the number of places from the Units place, and the Remain is the *Index*. As the *Index* of 8 in this Number 8921, is 3, of 9 is 2, of 2 is 1, and 0, the *Index* of 1, becauſe he ſtandeth in the Units place.

In Decimals, the juſt number of the places from Unity is the *Index*. As the *Index* of 8 in this Number 0,8921, is (1), and in this Number 0,1298 is (4), of 9 in the former is (2), in the latter (3), of 2 in the former (3), in the latter (2), of 1 in the former (4), in the latter (1).

How expreſſed by Mr. Oughtred.

Theſe *Indices* are expreſſed by the Learned *Oughtred* in his *Clavis limata*, affirmatively and negatively; that is, of Integers affirmatively by the ſign +, ſignifying *more*;

and of Decimals negatively, by the ſign —, ſignifying *leſs*. And where the ſign — is not ſet, the ſign + is underſtood, though not expreſſed. And ſo 34,1426 ſtands thus, $34,\bar{1}\bar{4}\bar{2}\bar{6}$.

Signs of a Number either Negative or Affirmative.

When a Number is wholly negative or wanting, the ſign *leſs* ſhewing its defect, is ſet at the Left Hand, thus, —34,1426; but if affirmative, the Number is placed without the ſign *more*, as 34,0000, as well as with it thus, +34,0000. And though the Number be not wholly affirmative, but negative in the Decimals, yet it may be ſet without, thus, 34,1426, as well as +34,1426. And Decimals themſelves albeit wholly negative, yet not taken privatively may be marked or not, as 0,1426; or thus, +0,1426.

Theſe Signs helpful.

How auxiliary the right underſtanding of theſe ſigns + and — is to the Simple Elements of Decimals and all Contract Numbers will be ſeen hereafter. It remains therefore, as neceſſary to this *Chapter*, that Decimals be counted Pure, when ſet without Integers. Mixt, when ſet with them. Simple, when affirmative or negative in their Signs. Compound, when their Signs are both affirmative and negative.

Decimals Pure or Mixt, Simple or Compound.

CHAP. II.

Reduction of Decimals.

Decimals reduced, how many wayes, and the uſe thereof.

Reduction of *Decimals* ſerveth to reduce *Fractions* or *Geodæticals* into *Decimals*, or *Decimals* into them, that without impediment they may be wrought together, and Operation being ended their value may be found.

To turn Common Fractions *into* Decimals.

§. 1. To reduce *Common Fractions* into *Decimals*: Adjoyn to the Right Hand of the Numerator of the Fraction as many Cyphers as you pleaſe, and divide by the Denominator, the Quotient ſhall be the Decimal, and denominate according the Number of Cyphers adjoyned; that is to ſay, the Right Hand Figure or Cypher of the Quotient, as the Right Hand Cypher adjoyned to the Number before Diviſion, and ſo the other Figures or Cyphers in order, proceeding towards the Left Hand accordingly. So that if I adjoyn but one Cypher, then the *dexter* Figure of the Quotient ſhall be Primes, if 2 Cyphers, Seconds, if 3 Cyphers, Thirds, *&c.*

Example in a ſingle Fraction *turned into a Perfect* Decimal.

As to ſet $\frac{3}{4}$ in Decimals, I adjoyn to 3 the Numerator, 2 Cyphers, and divide by 4 the Denominator, and the Quotient is 75, which is 7 Primes, 5 Seconds, there being but 2 Cyphers adjoyned, therefore the Quotient can be but Seconds.

$$\frac{3}{4} \qquad \frac{\overset{\cancel{2}}{\cancel{3},\cancel{0}\cancel{0}}}{\cancel{4}}\Big(\overset{(1)(2)}{75}$$

When any Number to be reduced into a Decimal cannot be brought into a perfect Decimal, but there will be ſome Remain upon the Diviſion, then the more Cyphers are adjoyned the nearer the true Decimal you come, but ſeldom above 7 or 8 Cyphers need to be uſed; for that the Quotients of ſuch Diviſions come ſo near the truth that what is wanting is very inconſiderable.

Example in a ſingle Fraction *turned into the Imperfect.*

As to reduce $\frac{2}{3}$ to Decimals, if 5 Cyphers be adjoyned the Quotient is 66666, divided by 3; and if the Diviſion be continued (more Cyphers being adjoined) the ſame Quotienary Numbers will ſtill be increaſed, becauſe the Fraction cannot be reduced into a perfect Decimal.

$$\frac{2}{3} \qquad \frac{\overset{\cancel{2}\cancel{2}\cancel{2}\cancel{2}(2}{\cancel{2},\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}}}{3}\Big(\overset{(1)(2)(3)(4)(5)}{66666}$$

Reaſon of Reduction.

So any other proper ſingle Fraction whatſoever may be turned into a Decimal, the Reaſon being the ſame: For as the Denominator of any Fraction is to the Numerator, ſo ſhall any Denominator be to a Numerator which ſhall have the ſame value to the Denominator given, as the firſt Numerator had to his Denominator, as is evident in

the

he foregoing Examples: For as 4 to 3, ſo is 100 to 75; and as 3 to 2, ſo is 100000 o $66666\frac{2}{3}$.

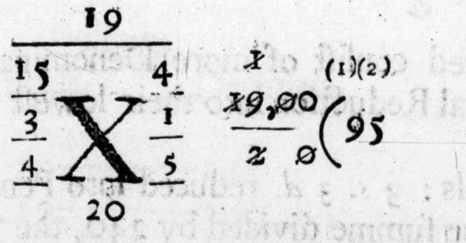
Fractions conjunct and divided reduced into Decimals.

If the Fractions given to be reduced into Decimals be conjunct or divided Fractions, hen firſt by Reduction of Fractions reduce them into one Denomination, and abbreviate them into their leaſt Terms, and if Conjunct add them together, and then turn hem into Decimals, as above. Example of both.

Examples of both.

To reduce $\frac{3}{4}$ and $\frac{1}{5}$ into a Decimal: Firſt by Reduction of Fractions they make being reduced $\frac{15}{20}$ and $\frac{4}{20}$, and added $\frac{19}{20}$, then adjoyning to 19, 2 Cyphers, and dividing by 20, the Quotient 9 Primes, 5 Seconds, is the Decimal.

To reduce $\frac{3}{4}$ of $\frac{1}{5}$ into a Decimal: Firſt by Reduction of Fractions they are brought to $\frac{3}{20}$, and by adjoyning 00, to 3, and the ſumme divided by 20, gives 1 Prime, 5 Seconds, for the Decimal.



$$\begin{array}{l}\cancel{1}\\ \cancel{1}9{,}\cancel{0}0\\ \hline \cancel{2}\ \ \cancel{0}\end{array}\Big(\overset{(1)(2)}{95} \qquad \begin{array}{c}3\\ \hline \frac{3}{4}\text{ of }\frac{1}{5}\\ \hline 20\end{array} \qquad \begin{array}{l}\cancel{1}\\ \cancel{3}{,}\cancel{0}\cancel{0}\\ \hline \cancel{2}\ \ \cancel{0}\end{array}\Big(\overset{(1)(2)}{15}$$

To reduce Decimals into Common Fractions.

§. 2. To reduce *Decimals* into *Common Fractions.* If the Denominator be given to find a Numerator, then multiply the given Decimal by the given Denominator, and the Numbers exceeding the place of Primes ſhall be the Numerator to the given Denominator. And if the Numerator be given to find a Denominator, then by the given Numerator multiply the Denominator of the Decimal, and divide the Product by the given Decimal. Example of both.

Examples.

Suppoſe 0,75 be the given Decimal, and 4 the Denominator given, to which a Numerator is deſired; then multiplying 0,75 by 4, there exceeds the *Seperatrix* 3, which ſhall be the Numerator, and ſo 0,75 reduced to a Common Fraction ſhall be $\frac{3}{4}$. For as 100 to 75, ſo is 4 to 3.

And if 3 the Numerator were given, and 0,75 the Decimal to find a Denominator, then I multiply 100 the Denominator of the Decimal by 3, and the Product 300 divide by 75 the Decimal, the Quotient is 4 the Denominator to 3. For as 75 to 100, ſo is 3 to 4.

$$\begin{array}{r}0{,}75\\ 4\\ \hline 3{,}|\underline{00}\end{array} \qquad \frac{3}{4} \qquad \begin{array}{r}1{,}00\\ 3\\ \hline 3{,}00\end{array} \qquad \begin{array}{l}\cancel{3}{,}\cancel{0}\cancel{0}\\ 7\cancel{5}\end{array}\Big(4 \qquad \frac{3}{4}$$

To reduce Geodæticals into Decimals.

§. 3. To reduce *Geodæticals* into *Decimals*, obſerve whether the *Geodætical* be of one Denomination or more, ſingle or plural.

1. Caſe. Single and next the Great Integer.

If the Geodætical be ſingle, have but one Denomination, and that next the greateſt Denomination of the Integer, then adjoyn to the Right Hand thereof ſo many Cyphers as you pleaſe, and divide the ſumme by the whole Number of Units of that Denomination contained in the Integer, and the Quotient is the Decimal, which ſhall be denominate according to the number of Cyphers adjoyned, as was ſhewed before.

Examples.

As to reduce 9 Shillings, which is the next denomination to Pounds, into Decimals; I adjoyn 2 Cyphers to 9, and this 900 I divide by 20, the whole number of Shillings contained in one Pound, or greateſt Integer, and the Quotient is 45, of which 5 ſhall be Seconds, becauſe 2 Cyphers were adjoyned to 9, and the 4 ſhall be Primes; ſo is $\overset{(1)(2)}{0{,}45}$ the Decimals for 9 *s.*

Alſo to reduce 9 Months into Decimals, 00 adjoyned to 9, and divided by 12, the Months in a Year, the Quotient $\overset{(1)(2)}{0{,}75}$ is the Decimal deſired.

$$\begin{array}{l}\cancel{1}\\ \cancel{9}\cancel{0}\cancel{0}\\ \hline \cancel{2}\ \ \cancel{0}\end{array}\Big(\overset{(\ )(2)}{45}\text{ Decimal for 9 }s. \qquad \begin{array}{l}\cancel{6}\\ \cancel{9}\cancel{0}\cancel{0}\\ \hline \cancel{1}\cancel{2}\end{array}\Big(\overset{(1)(2)}{75}\text{ Decimal for 9 Months.}$$

2. Caſe. Single, and not next the great Integer.

If the ſingle Geodætical given be not next to the greateſt Denomination of the Integer, then the Decimal may be had either working as above, accompting one of the next Denomination for the Integer, or by dividing the given Geodætical increaſed with his Cyphers by the parts of the greateſt Integer.

Example.

As to reduce 9 *d.* into Decimals, becauſe Pence is not the next Denomination to Pounds, the great Integer, I either divide 9 with Cyphers by 12, the parts of one Shilling, which is the Denomination next to Pence, and ſo have I $\overset{(1)(2)}{75}$, the Decimal of a Shilling; or by 240, the Pence in a Pound the greateſt Integer, and thereby get $\overset{(2)(3)(4)}{375}$, which is the Decimal of a Pound for 9 *d.*

$$\begin{array}{l} \cancel{6} \\ \cancel{9},\cancel{0}\cancel{0} \\ \hline 12 \end{array}\Big(\overset{(1)(2)}{75} \qquad\qquad \begin{array}{l} \cancel{1} \\ \cancel{1}8\cancel{2} \\ \cancel{9},\cancel{0}\cancel{0}\cancel{0}\cancel{0} \\ \hline \cancel{2}\ \cancel{4}\cancel{4}\cancel{4}\cancel{0} \\ \ \ \cancel{2}\cancel{2} \end{array}\Big(\overset{(2)(3)(4)}{3\,7\,5}$$

3. Caſe. Plural.

If the given Geodætical to be reduced conſiſt of more Denominations than one, reduce the given Numbers by Geodætical Reduction into their loweſt Denomination, and then work as above.

Examples.

As to reduce 3 *s.* 3 *d.* into Decimals: 3 *s.* 3 *d.* reduced into Pence, make 39 Pence, to which 4 Cyphers adjoyned, and the ſumme divided by 240, the Pence in one Pound, the Quotient is 1625, which is $\overset{(1)(2)(3)(4)}{1\,6\,2\,5}$, becauſe the Right Hand Cypher was of the ſame Denomination, there being 4 Cyphers adjoyned.

So 1 *s.* 4 *d.* 2 *q.* reduced into Decimals make $\overset{(2)(3)(4)(5)}{6\,8\,7\,5}$, being firſt reduced into Farthings make 66, and then divided with 00000 adjoyned by 960, the Farthings in one Pound.

$$\begin{array}{rl} s. & d. \\ 3 & \!\!-\!\!-3 \\ 12 & \\ \hline 36 & \\ 3 & \\ \hline 39 & \\ \hline \end{array} \quad \begin{array}{l} \ \ \cancel{1} \\ \cancel{1}\cancel{5}\ \cancel{6}\cancel{2} \\ \cancel{3}\cancel{9},\cancel{0}\cancel{0}\cancel{0}\cancel{0} \\ \hline \cancel{2}\cancel{4}\ \cancel{4}\cancel{4}\cancel{4}\cancel{0} \\ \ \cancel{2}\ \cancel{2}\cancel{2} \end{array}\Big(\overset{(4)}{1625} \qquad \begin{array}{rll} s. & d. & q. \\ 1 & \!\!-\!\!-4 & \!\!-\!\!-2 \\ 12 & & \\ \hline 16 & & \\ 4 & & \\ \hline 66 & & \\ \hline \end{array} \quad \begin{array}{l} \ \ 7\cancel{4} \\ \cancel{8}\ 4\cancel{2}8 \\ \cancel{6}\cancel{6},\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0} \\ \hline \cancel{9}\ \cancel{6}\cancel{6}\cancel{6}\cancel{6}\cancel{0} \\ \ \ \cancel{9}\cancel{9}\cancel{9} \end{array}\Big(\overset{(5)}{6875}$$

4. Caſe. Many Plural.

If Plural Geodæticals, of one or more Denominations, be given to be reduced into Decimals; firſt add them as Geodæticals are to be added, then reduce them into their loweſt Denomination, and turn them into Decimals, as before.

Example.

As to reduce into Decimals, 3 *s.* 2 *d.*, 2 *s.* 6 *d.* and 1 *s.* 1 *d.* being added the total is 6 *s.* 9 *d.* and in Pence 81, then by adjoyning 4 Cyphers, as above, and dividing, I get in the Quotient $\overset{(4)}{3375}$ for the Decimal of all the 3 given Numbers.

$$\begin{array}{rl} s. & d. \\ 3 & \!\!-\!\!-2 \\ 2 & \!\!-\!\!-6 \\ 1 & \!\!-\!\!-1 \\ \hline 6 & \!\!-\!\!-9 \\ \hline \end{array} \quad \begin{array}{rl} s. & d. \\ 6 & \!\!-\!\!-9 \\ 12 & \\ \hline 12 & \\ 69 & \\ \hline 81 & \\ \hline \end{array} \quad \begin{array}{l} \cancel{1}\ \cancel{1} \\ \cancel{9}\ \cancel{8}\cancel{2} \\ \cancel{8}\cancel{1},\cancel{0}\cancel{0}\cancel{0}\cancel{0} \\ \hline \cancel{2}\cancel{4}\ \cancel{4}\cancel{4}\cancel{4}\cancel{0} \\ \ \cancel{2}\ \cancel{2}\cancel{2} \end{array}\Big(\overset{(1)(2)(3)(4)}{3\,3\,7\,5}$$

The end of Tables.

The like may be underſtood of other Geodæticals of Weight, Meaſure, Time, &c. as well as thoſe inſtanced of Coyn; but ſome to eaſe this continued work, prepare Tables for each Denomination, out of which the Decimals taken for each given Number, may ſoon, by common Addition, be brought into one total. And theſe Tables are eaſily made and underſtood, conſidering (as was before ſhewed in the *Firſt Chapter* of this *Book*) that every Integer is equal to 10 Primes, therefore half that Integer ſhall be equal to 5 Primes, and the quarter of that Integer equal to 2 Primes, 5 Seconds; and ſo the Decimals for the even aliquot parts of the Integer found, the reſt of the in-

Tables eaſily made.

termediate places may be eaſily had by Addition or Subſtraction, &c. as the caſe requireth.

Example.

Example in the *Table* of *Engliſh* Coyn. If 1 Pound or Integer (containing 20 Shillings) in Decimals ſet thus, 1,00, be 10 Primes, then $\frac{1}{2}$ *l.* or 10 *s.* muſt be 5 Primes, and $\frac{1}{4}$ *l.* or 5 *s.* ſhall be 2 Primes, 5 Seconds; and the fifth part of 5 *s.* which is 1 *s.* ſhall give the fifth part of 2′, 5″, which is 5″; then if I ſubſtract 5″, the Decimal of 1 *s.* from 2′, 5″, the Remain, 0,2′, is the Decimal of 4 *s.* the half of which is 2 *s.* and the Decimal thereof 0,1′, being half of the Decimal 0,2′, and ſo alſo if I double 5″, the Decimal of 1 *s.* I have 0,1′, as before. And in like manner all the other intermediate places for the 19 *s.* may be filled with Decimals.

Then taking 0,05″, the Decimal of 1 *s.* and breaking it in two parts, which is 0,025‴, you have the Decimal of 6 *d.*; and again 0,0125⁗, half the latter, the Decimal of 3 *d.*, and ſo of others, as the following *Tables* make conſpicuous.

Decimal Tables *for*

DECIMAL TABLES,

FOR

Sterling-Money.

Sterling *Money.*

Shillings.

Geodæticals.		Decimals.
l.	*s.*	
1, or	20	1,00
	19	0,95
	18	0,90
	17	0,85
	16	0,80
	15	0,75
	14	0,70
	13	0,65
	12	0,60
	11	0,55
	10	0,50
	9	0,45
	8	0,40
	7	0,35
	6	0,30
	5	0,25
	4	0,20
	3	0,15
	2	0,10
	1	0,05

Pence and Farthings.

Geodæticals.		Decimals.
s.	*d.*	
1, or	12	0,050000
	11	0,045833
	10	0,041667
	9	0,037500
	8	0,033333
	7	0,029167
	6	0,025000
	5	0,020833
	4	0,016667
	3	0,012500
	2	0,008333
	1	0,004167
d.	*q.*	
1, or	4	0,004167
	3	0,003125
	2	0,002083
	1	0,001041

Troy-Weight.

Troy-*Weight.*

Ounces.

Geodæticals.		Decimals.
℔.	*Ounces.*	
1, or	12	1,000000
	11	0,916667
	10	0,833333
	9	0,750000
	8	0,666667
	7	0,583333
	6	0,500000
	5	0,416667
	4	0,333333
	3	0,250000
	2	0,166667
	1	0,083333
Pen.w.	*Gra.*	
1, or	24	0,004166
	12	0,002083
	6	0,001041
	3	0,000520
	1	0,000173

Penny-Weights.

Geodæticals.		Decimals.
Ounce.	*Pennyw.*	
1, or	20	0,083333
	19	0,079167
	18	0,075000
	17	0,070833
	16	0,066667
	15	0,062500
	14	0,058333
	13	0,054166
	12	0,050000
	11	0,045833
	10	0,041667
	9	0,037500
	8	0,033333
	7	0,029166
	6	0,025000
	5	0,020833
	4	0,016667
	3	0,012500
	2	0,008333
	1	0,004166

The *Table* of *Troy* Weight for Ounces, may ſerve for Pence, if One Shilling be accompted the Integer: And alſo for Months, One Year being accompted the Integer.

The Decimal Table *for* Avoirdupois *Weight, accompting the Hundred Weight for the Integer.*

A Decimal Table *for* Avoirdupois *Weight.*

Pounds.

Geodæticals. C.	℔.	Decimals.
1, or	112	1,000000
¾, or	84	0,750000
½, or	56	0,500000
¼, or	28	0,250000
	27	0,241071
	26	0,232143
	25	0,223214
	24	0,214286
	23	0,205357
	22	0,196429
	21	0,187500
	20	0,178571
	19	0,169643
	18	0,160714
	17	0,151786
	16	0,142857
	15	0,133929
	14	0,125000
	13	0,116071
	12	0,107143
	11	0,098214
	10	0,089286
	9	0,080357
	8	0,071429
	7	0,062500
	6	0,053571
	5	0,044643
	4	0,035714
	3	0,026786
	2	0,017857
	1	0,008929

Ounces.

Geodæticals. ℔.	℥.	Decimals.
1, or	16	0,008929
	15	0,008371
	14	0,007812
	13	0,007254
	12	0,006696
	11	0,006138
	10	0,005580
	9	0,005022
	8	0,004464
	7	0,003906
	6	0,003348
	5	0,002790
	4	0,002232
	3	0,001674
	2	0,001116
	1	0,000558

Or thus, accompting 1 ℔. for the Integer.

℔.	℥.	
1, or	16	1,0000
	15	0,9375
	14	0,8750
	13	0,8125
	12	0,7500
	11	0,6875
	10	0,6250
	9	0,5625
	8	0,5000
	7	0,4375
	6	0,3750
	5	0,3125
	4	0,2500
	3	0,1875
	2	0,1250
	1	0,0625
	0¾	0,046875
	0½	0,031250
	0¼	0,015625

The Table *for* Ounces *ſerves for* Nails *in a* Yard.

The laſt *Table* for Ounces may ſerve for the Nails in one Yard, which are 16, as well as the Ounces in one Pound, and ſo the Decimals the ſame, accompting one Yard the Integer inſtead of one Pound.

The Table *for the Dayes of one Month, accompting the Integer One Year, or 365 Dayes.*

A Decimal Table for the Dayes in a Month.

Dayes.	Decimals.	Dayes.	Decimals.
1	0,002740	16	0,043836
2	0,005479	17	0,046575
3	0,008219	18	0,049315
4	0,010959	19	0,052055
5	0,013698	20	0,054795
6	0,016438	21	0,057534
7	0,019178	22	0,060274
8	0,021918	23	0,063014
9	0,024658	24	0,065753
10	0,027397	25	0,068493
11	0,030137	26	0,071233
12	0,032877	27	0,073973
13	0,035616	28	0,076712
14	0,038356	29	0,079452
15	0,041096	30	0,082192

To get the Decimals for Hours.

In like manner taking the Decimal for one Day, and breaking it into parts, I may have the Decimals for Hours; as for 12 Hours 0,00137, for 6 Hours 0,000685, *&c.* And in the ſame method Tables may be made for other Geodæticals, and by theſe Tables the Decimals may be ſpeedily had. As if 16 *s.* 9 *d.* were to be turned into Decimals; in the *Table* of *Coyn* againſt 16 *s.* is found 8 Primes, and againſt 9 *d.* is 3 Seconds, 7 Thirds, and 5 Fourths, which added together make the total Decimal 0,8375. So if 17 *C.* 13 ℔. 5 ℥. be repreſented in Decimals, the *Table* of *Avoirdupois* Weight ſheweth the Decimal of 13 ℔. to be 0,116071, and the Decimal of 5 ℥. 0,00279, both which together repreſent the Decimal Number thus, 17,118861.

Uſe of the Tables.

To reduce Decimals *into* Geodæticals.

§. 4. To reduce *Decimals* into *Geodaticals.* If the Decimal be found in the Table, then the correſpondent Number, without further work, effects the deſire: But if not, multiply the Decimal given by the denominate parts contained in the Integer, and what exceeds the Prime Line ſhall be the Number deſired, and ſhall be denominate according to the denomination of the Multiplyer. One Example will make all plain. As if 0,9875 be given, and it is deſired to know how much the ſame is in *Engliſh* Money: Then if I multiply by 20, the Shillings in one Pound, I find 19 exceed the Prime Line or *Seperatrix*, which ſhall be 19 Shillings, becauſe 20 the Multiplyer was Shillings, and the remaining Decimal 0,75, multiplyed by 12 the parts of one Shilling, the Product is 9, which ſhall be Pence, becauſe 12 Pence are the denominate parts of a Shilling that were multiplyed by. But if I multiply the Decimal firſt given by 240, the Pence in one Pound, I find 237 exceed the Prime Line, which 237 ſhall be Pence becauſe the Multiplyer 240 was Pence.

Example.

$$\begin{array}{r|l} \multicolumn{2}{r}{0,9875} \\ & 20 \\ \hline s.\ 19 & 7500 \\ & 12 \\ \hline 1 & 5000 \\ 7 & 500 \\ \hline d.\ 9 & 0000 \\ \hline \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{r}{0,9875} \\ & 240 \\ \hline 39 & 5000 \\ 197 & 50 \\ \hline d.237 & 0000 \\ \hline \end{array} \qquad \begin{array}{l} \ \ \cancel{2} \\ \cancel{1}\cancel{1}(9\ d. \\ \underline{\cancel{2}\cancel{3}7}(19\ s. \\ \ \cancel{1}\cancel{2} \end{array}$$

What to be done with Imperfect Decimals.

Many times in imperfect Decimals, when there is left in the place of Primes near another Integer, and upon a Diviſion almoſt the Diviſor is left remaining, another Unit is commonly taken for the ſame. And ſo is done in many of the Tabellary Numbers before. And when it happeneth may be done in all the Caſes of the Four *Sections* of this *Chapter.*

Proof of Reduction of Decimals.

The Reductions of the firſt and ſecond *Sections* are alternate Proofs of each other; and ſo are they of the third and fourth *Sections.*

CHAP.

CHAP. III.

Indices *Added and Subſtracted.*

Addition *and* Subſtraction *of* Indices.

SO much in the *Firſt Chapter* of this *Third Book* hath been ſaid of *Indices*, as ma be ſufficient to give a right underſtanding of them, that they are Numbers whic as their Name implyeth, do ſhew the number or diſtance of places any Number o Species to which they are annexed is from the Unit, and how many places ſuch Numbe or Species hath ; ſo as there needs no further explanation here.

What ſhewed by their Addition. *What by their* Subſtraction.

Addition of *Indices*, diſcovers the number of places any *Factus* or Product ſhoul conſiſt of, and thereby conſequently ſheweth the true nature of the Product.

Subſtraction of *Indices*, declareth the true nature of any Quotient after Diviſion o Mixt Numbers, or Numbers of ſeveral diſtances from the Unit.

Indices to be added will be either alike, that is, both Integers, or both Decimals or elſe unlike, that is, the one an Integer and the other a Decimal.

How added if alike.

If they be both alike, then add the *Indices* as Integers are added, and the ſumme ſhall be the *Index* of the ſame kind the added *Indices* were.

Examples.

Examples.	Integers.		Decimals.	
Addends	4	5	(4)	(5)
	3	3	(3)	(3)
Totals	7	8	(7)	(8)

How added if unlike.

If the *Indices* be unlike, then ſubſtract the Leſſer Number out of the Greater, th difference ſhall be the *Index* of the ſame kind with that Number which was the greateſt.

Examples.

Examples.	Integers and Decimals.				
Addends	7	2	(2)	(7)	
	(2)	(7)	7	2	
Index	5	(5)	5	(5)	remaining

In both theſe Caſes the Operation and Reaſon thereof is ſo plain they need no explanation.

Indices to be ſubſtracted are to be placed one over the other, as the Numbers to which they belong ought to be placed ; for it matters not though the uppermoſt be the leaſt.

Theſe *Indices* alſo will be either alike or unlike.

How ſubſtracted if alike.

If both the *Indices* be alike, take the difference for the *Index* deſired, and his nature is thus diſcerned : If the Subtrahend or Number to be ſubſtracted be the leaſt, the difference remaining ſhall be an Integer in Integers, and a Decimal in Decimals : But if the greateſt the contrary, to wit, in Integers a Decimal, and in Decimals an Integer.

Examples.

Examples.	Integers.		Decimals.	
	5	4	(5)	(4)
Subtrahends	4	5	(4)	(5)
Remains	1	(1)	(1)	1

How ſubſtracted if unlike.

If the *Indices* be unlike, add them together, and take the ſumme, and as the upper Number from which Subſtraction is to be made, ſo ſhall this *Index* be, whether Integer or Decimal.

Examples.

Examples.	Integers and Decimals.								
	5	(5)	2	(2)	4	(4)	(3)	0	
Subtrahends	(2)	2	(5)	5	(4)	4	0	(3)	
Index	7	(7)	7	(7)	8	(8)	(3)	3	amounting.

In the Examples of Subſtraction, where the *Indices* are alike, the work is plain; for taking the difference between 4 and 5, the Remain 1 is the *Index*, which is either an Integer or a Decimal, according as the Number to be ſubſtracted was greater or leſſer than the other.

And in the latter Examples the Addition of both the *Indices* make the remaining *Index* denominate alwayes as the higher *Index* or Number from which Subſtraction is made. And the Reaſon of both is evident, for that the more is ſubſtracted the leſs of neceſſity muſt remain; and the leſſer Number either in Nature or Figure ſubſtracted, the greater will remain.

In theſe laſt Examples, 0, or the place of the Unit, is ſeen to be accompted as the *Index* of an Integer.

CHAP. IV.

Addition of Decimals.

Decimals *added.*

THE variety of adding Decimals is occaſioned as the Numbers given to be added are Simple or Compound.

Simple.

Simple whether Pure or Mixt, being orderly placed every one under his like Denomination, are added as Integers.

Examples.

Examples.	Pure.		Mixt.		
	0,990625	—0,5875	352,7250	—51,025	Addends.
Simple	0,725000	—0,1250	240,0375	—20,050	
	1,715625	—0,7125	592,7625	—71,075	Totals.

Compound.

Compound Decimals are added by ſubſtracting, as in Integers, the Leſſer Number out of the Greater, and to the Remain which ſhall be the Total, ſubſcribe the ſign of the Greater Number.

Examples.

Examples.	Pure.		Mixt.		
	+0,950	—0,950	+34,125	—34,125	Addends.
Compounds	—0,825	+0,825	— 3,950	+ 3,950	
	+0,825	—0,825	+30,175	—30,175	Totals.

Compounds when many.

Example.

If Compounds given to be added be many, let the Numbers of each ſign be added with their fellows, as Simple Decimals, and then the Totals, as Compounds. As if 34,125 —0,95, were to be added with 1,45 —3,5, the + being added with + and — with —, the Totals are 35,575 —4,45, then taking the — from the +, the remaining Total is 31,125.

34,125	—0,95	+35,575
1,45	—3,5	— 4,45
35,575	—4,45	+31,125

Proof of Decimal Addition.

Beſides the Proof of theſe works by Subſtraction, noted in the next *Chapter*, the truth may be brought to the Teſt by reducing your Decimals into Geodæticals, and adding them in their order, the Totals of both Operations will agree when rightly done. As in the laſt Example.

Decimals.		Geodæticals.										
		l.	s.	d.		l.	s.	d.		l.	s.	d.
34,125	—0,95	34	02	06	lacking	00	19	00	or	33	03	06
1,450	—3,50	1	09	00	lacking	03	10	00		— 2	01	00
35,575	—4,45	35	11	06	wanting	04	09	00		31	02	06
— 4,450		— 4	09	00								
+31,125	Correſpondents	+31	02	06	Proof.							

CHAP. V.

Subſtraction of Decimals.

Decimals Subſtracted. Simple. THE variety in ſubſtracting Decimals, as well as their Addition, is according to the Simplicity or Compoſition of the Numbers given to be ſubſtracted.

Simple Decimals Pure or Mixt, being orderly placed every Denomination under his like, are ſubſtracted by withdrawing the nether Number from the upper, as Integers borrowing 10, if need be, and ſubſcribing the Remain, except the Subtrahend be the greateſt Number, then take the Leſſer Number from the Greater, and change the Sign to the difference.

Examples.

Examples.	Pure.	Mixt.	Pure.	Mixt.	
Simple	0,990625	352,7250	0,725	—1,725	
	0,725000	240,0375	0,950	—1,950	Subtrahends.
	0,265625	112,6875	—0,225	+0,225	Remains.

Compound. Compound Decimals having contrary Signs, for their Subſtraction require Addition; ſo the Totals of the Numbers ſhall be the Remains; to which is to be affixed the upper Numbers ſign, that is, the ſign of the Number from which Subſtraction is to be made.

Examples.

Examples.	Pure.				
Compound	+0,950	—0,950	+0,125	—0,125	
	—0,125	+0,125	—0,950	+0,950	Subtrahends.
	+1,075	—1,075	+1,075	—1,075	Remains.
	Mixt.				
	+34,125	—34,125	+ 3,950	— 3,950	
	— 3,950	+ 3,950	—34,125	+34,125	Subtrahends.
	+38,075	—38,075	+38,075	—38,075	Remains.

Compounds when many. If Compounds given to be ſubſtracted be many, let the Numbers of each ſort be ſubſtracted as Simple Decimals, and then the Remains one from the other as Compounds. *Example.* As if 3,5 —1,45 were to be ſubſtracted from 22,475 —0,8, the + taken from the + leaves +18,975, and the — from the — leaves +0,65, and this Remain added to the other makes the Remain at laſt +19,625. Otherwiſe if I take each — from the reſpective + to which they are joyned, the 2 Numbers will be +21,675 and +2,05, and then Subſtraction made as in Integers, the Remain will be as before.

Proof of Decimal Subſtraction. The Proof of Addition is by Subſtraction, and of Subſtraction by Addition alternately, as before in Integers and Geodæticals hath at large been ſeen, with this difference only here, that Simple Decimals reſerve their Proof by the Simple Operations, and the Compounds by the Compound Operations reſpectively: So as Addition of the Simple ſhall be proved by Simple Subſtraction, and Compound Addition by Compound Subſtraction, and accordingly Subſtraction Simple or Compound by the Simple or Compound Addition.

Beſides this Proof, the truth of Decimal Subſtraction as well as Addition may be oved by turning the Decimals into Geodæticals, and ſubſtracting them in their order; will the Remains of both Operations agree, if done aright. As in the laſt Example.

	Decimals.		Geodæticals. *l. s. d.*		*l. s. d.*		*l. s. d.*
	22,475 —0,80		22—09—06	wanting	0—16—0	or	21—13—6
	3,500 —1,45		3—10—00	wanting	1—09—0		2—01—0
hus	18,975 +0,65		18—19—06	more	0—13—0		19—12—6
	0,65		+ 0—13—00				
	19,625	Correſpondents	19—12—06				
r	+22,475		+22—09—06				
	— 0,800		— c—16—00				
	+21,675		+21—13—06				
Thus	+ 3,50		+ 3—10—00				
	— 1,45		— 1—09—00				
	+ 2,05		+ 2—01—00				
	+19,625	Correſpondents	+19—12—06				

CHAP. VI.

Multiplication of Decimals.

Decimals multiplyed.

THE varieties of multiplying Decimals are ſorted under the like double head of Simple and Compound.

Simple and

Multiplication of Simple Decimals is ſo ordered, that the Product may be procured compleat or contracted, as occaſion ſhall require.

The Product compleat.

To procure the Product compleat, multiply Number by Number, as in Integers, and add the Decimal *Indices* of the Multiplicand and Multiplyer (or the *Indices* of the Right Hand Figures of them) together, and the ſumme ſhall be the *Index* of the Product, and the ſign thereof alwayes +.

Examples.

Examples.		Pure.		Mixt.			
0,9875	(4)	— 0,125	(3)	— 3,625	(3)	13,625	(3)
0,8875	(4)	— 0,025	(3)	— 2,5	(1)	12, 75	(2)
49375	(8)	625	(6)	18125	(4)	68125	(5)
69125		250		7250		95375	
79000						27250	
79000		+0,003125		+9,0625		13625	
0,87640625		Compleat Products.				173,71875	

☞

Not only the Total *Index* of the Product by adding the *Indices* as aboveſaid is found, but the true place in the Product of any two Numbers multiplyed may be had by adding the particular *Indices* of the particular *Factores.* As in the laſt Example; if 5 in the Multiplicand be multiplyed by 1 in the Multiplyer, to know of what place the *Factus* or Product will be, or how far diſtant from Unity, and whether a Decimal or Integer; the *Index* of 5 is (3), and the *Index* of 1 there is 1, added make the Total *Index* (2), and ſo ſhall the Product 5 fall in the place of Decimal Seconds. Alſo the Product of 2 in the Multiplicand, whoſe *Index* is (2), multiplyed into 2 in the Multiplyer, whoſe *Index* is 0, becauſe he ſtandeth in the Units place, makes the Product Decimal Seconds, ſhewed by Addition of their *Indices* as aforeſaid.

Simple and the Product contracted.

Sometime it happens that all the Figures of the Product need not be expreſſed, but only ſome of thoſe towards the Left Hand, becauſe thoſe towards the Right Hand are of ſmall value, and in many Operations inconſiderable, and ſo the Product is contracted into a leſs number of places.

Contraction 3 wayes.

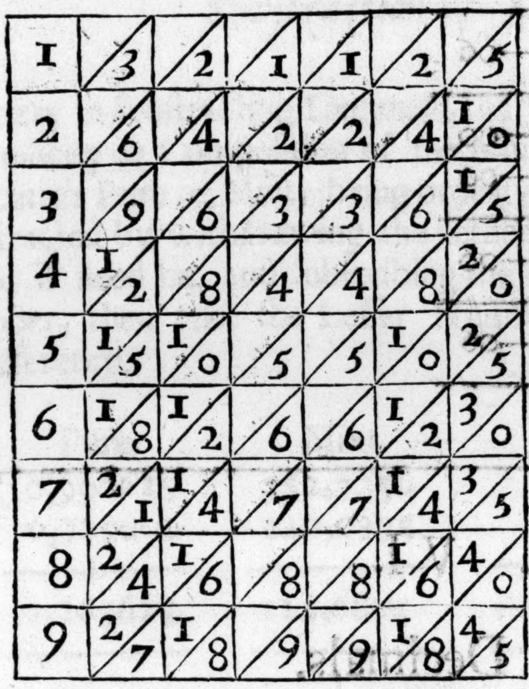
To contract the Product proceed in one of theſe three wayes.

1. *By* Nepaires Bones.

Firſt, by *Nepaires Bones*, having ſet the Multiplicand on the *Bones*, and by the *Index* of the *Bones*, having the ſeveral Products anſwering the Figures of the Multiplyer, take off each *Bone* ſo many Figures as will be ſufficient, having reſpect to the Tens that riſe on the next Right Hand *Bone*, and placing them orderly one under the other, add them into one Total Product.

Example.

As if 321,125 were to be multiplyed by 43,25, where though there be 5 Decimals in both the *Factores*, yet needing but 3 in the Product, I Tabulate 321,125 on the *Bones*, and take out the Multiplees, and in that appertaining to the multiplying 5, omit two places, and one place in the Multiplee of 2, and add the Reſidue in order, as followeth.



1	/3	/2	/1	/1	/2	/5
2	/6	/4	/2	/2	/4	1/0
3	/9	/6	/3	/3	/6	1/5
4	1/2	/8	/4	/4	/8	2/0
5	1/5	1/0	/5	/5	1/0	2/5
6	1/8	1/2	/6	/6	1/2	3/0
7	2/1	1/4	/7	/7	1/4	3/5
8	2/4	1/6	/8	/8	1/6	4/0
9	2/7	1/8	/9	/9	1/8	4/5

```
  64225 .          321,125
 963375              43,25
1284500           ---------
  16056 . .        16056|. .
                   64225|.
                  963375|
                 1284500|
                 ---------
                13888,656
                 ---------
```

2. *By cutting off the Right Hand Figures.*

But Secondly, for that the whole Art of *Arithmetick* may be performed by the Pen, and many times the *Bones* are not at hand, multiply all as Integers, and cut off from the particular Multiplees or Total Product, ſo many of the Right Hand Figures as are uſeleſs, reſpecting the Tens that riſe in the next Right Hand File. And ſo the work of the former Example ſtands.

Example.

```
Thus,   321,125          Or thus,   321,125
          43,25                       43,25
      ----------                  -----------
    16056|25                  16056|25
    64225|0                   64225|0
   963375|                   963375|
  1284500|                  1284500|
  ----------               ---------|---
  13888,656                13888,656|25
  ----------               ---------|---
```

3. *By Mr.* Oughtred's *retrograde way.*

Thirdly, Mr. *William Oughtred* in his *Clavis* proceeds in a retrograde order thus; If the Product be deſired Pure, that is, to be wholly Integral without any Decimal Fraction, then place the Unity of the Multiplyer, or the Leſſer Number, under the Unity of the Multiplicand, or Greater Number, and ſo the Integers in order under the Decimals, and the Decimals under the Integers counterchanged. And if the Product be deſired Mixt, then place the Unity of the leaſt Number under that Decimal which the Product is deſired to be. And if the Product be deſired leſs then Pure, by ſo much remove the Units place of the Leſſer Number towards the Left Hand. And the Numbers thus placed, begin to multiply by each Figure of the leaſt Number or Multiplyer at that Figure of the Multiplicand, or greateſt Number, which ſtands thereover, and the Products of the Left Hand Multiplyers let be ſet firſt, and ſo the reſt in order by a Perpendicular Line.

Example.

As for further inſtance in the former Example thus, I place 3 in the Leſſer Number being in the place of Unity, under the place of Thirds, which is 5 in the Greater

Number, and the other Numbers in order alternately, and then beginning to multiply by 5 in the Leſſer with 1 in the Greater Number which ſtands over him, adding in the Article that will ariſe, if he be multiplyed by 2, the next Right Hand Figure in the Greater Number, I ſubſcribe 6 under the Units place in the Leſſer Number, and ſo proceed towards the Left Hand, and the like is done with the other Multiplying Figures, ſave becauſe the laſt 4 hath no Figure over him, the Product ariſing by him is ſet one place diſtant from the Unity towards the Left Hand : And the work ſtands thus;

```
   321,125
     52|34
  ---------
     16056|
     64225|
    963375|
   1284500.|
  ---------
  13888,656|
```

And if the Product had been deſired purely Integral, or leſs than Pure, the Products would have been found thus by,

```
      The ſecond way.                      Mr. Oughtreds way.
   321,125       321,125              321,125      321,125
     43,25         43,25              52|34        52|34
  ---------     ---------            --------     --------
    16 05625       1|605625              16|           1|
    64|2250        6|42250               64|           6|
   963|375        96|3375               963|          96|
 12845|00       1284|500              12845|        1284|
 -------        ------               ------       ------
 13889|         1389|                 13889|        1389|
 -------        ------               ------       ------
  Pure.      Leſs than Pure.          Pure.     Leſs than Pure.
```

In all theſe is added more than really found in the Right Hand File, an Unit in the pure Products, and two Units in the others, reſpect had to the Tens, or almoſt Tens ariſing from the Figures cut off, as is uſual in ſuch Caſes.

An Unit or more added, and why.

Compound Decimals convert into Simple, if by Addition or Subſtraction they may, and then multiply them as before : Otherwiſe multiply every Number of the Multiplicand by every Number of the Multiplyer, beginning at the Left Hand, as in *Geodæticals, Book 2, Part 1, Chap. 5. Caſe 4.* And for the Signs, when Numbers of like Signs, as + with +, or — with —, are multiplyed together, the Product ſhall be +; but unlike Signs, as + with —, or — with +, ſhall make the Product —.

Compounds multiplyed.

As if 5,9 —2,8 were to be multiplyed by 3,4 —1,2, I either convert them into Simple by ſubſtracting the — out of the +, and then multiply the Remains 3,1 by 2,2; as at *A.* : Or otherwiſe multiply like Geodæticals, as at *B.*; and by adding + with +, and — with — in the Multiplees, and then ſubſtracting the — from the +, the Product of both Operations are alike 6,82.

Example.

```
                 A.                                    B.
 +5,9 —2,8          +3,4 —1,2            5,9   —2,8
 —2,8               —1,2                 3,4   —1,2
 -----              -----               --------------           +20,06
 +3,1               +2,2                 236    112              + 3,36
                                        177     84               -------
        3,1  (1)                        ----------------------   +23,42
        2,2  (1)                        20,06  —9,52             —16,60
        ---  ---                        ----------------------   -------
         62   2                                 118       56     + 6,82
        62                                      59        28     -------
        ----                                   -----    -----
        6,82                                   —7,08    +3,36
        ----                            ----------------------
                                        20,06 —16,60    +3,36
                                        ----------------------
```

Proof of Decimal Multiplication.

Multiplication of Decimals is proved by their Diviſion, as is mentioned in the next *Chapter.* But beſides that Proof, if the Decimals be turned into Geodæticals, and multiplyed, the Products of both will correſpond when the work is without Error: As in the laſt Example at *A.*

```
        Decimals.                        Geodæticals.
                            l.  s.  d.     l.  s.  d.      l.  s.
 5,9 —2,8 or 3,1            5—18—0  leſs 2—16—0   or  3—02
 3,4 —1,2 or 2,2            3—08—0  leſs 1—04—0   or  2—04
             ———                                      ————
              62                                      6—04
             62                                         12—8/20
             ———                                      ————
            6,82          Correſpondents              6—16 2/5
             ———                                      ————
```

Or if the Geodæticals be reduced into Shillings, and their reſults 62, and 44 multiplyed, and then returned again into Pounds by 20 times 20, as in Multiplication of Geodæticals in like Caſe before was taught, there will be obtained 6 *l.* 16 *s.* 2/5 equivalent to 6,82, as before.

```
 l.  s.      s.                   (3
 3—2  or  62         20         27|28( 6 l.
 2—4  or  44         20          400
          ———        ———
          248        400         2(1
         248         ———         65|60( 16 s. 2/5.
         ————                    400
         2728
         ————
```

CHAP. VII.

Diviſion of Decimals.

Decimals *divided.*

THE varieties of dividing Decimals are ordered according to the Simplicity or Compoſition of the given Numbers.

Simple.

Diviſion of Simple Decimals runs into a Bivity of dividing a Greater Number by a Leſſer, or a Leſſer by a Greater.

1. *A Greater by a Leſſer.*

In the firſt Caſe, where a Greater Number is to be divided by a Leſſer, divide Number by Number as in Integers, and to find the true *Index* of the Quotient, ſubſtract the *Indices* Decimal of the Diviſor from the Dividend, or the *Indices* of the Right Hand Figures of them the one from the other, and the Remain ſhall be the *Index* of the Quotient, and the ſign thereof +.

Examples in Pure Decimals.

Examples in Pure.

To divide 0,87640625 by 0,8875, the Quotient will be 9875, which ſhall be Decimal Fourths, becauſe the *Index* of the Dividend is (8), from which (4) the *Index* of the Diviſor ſubſtracted, the Remain is (4).

So —0,003125 divided by —0,125, ſhall make the Quotient Decimal Thirds; for (3) the *Index* of the Diviſor ſubſtracted from (6) the *Index* of the Dividend leaves (3) remaining.

```
 644           Units place ſupplyed by             Places ſupplyed by
 76537         (4)   a Cypher.               6      (3)  Cyphers.
,87640625( 9875, or 0,9875               — 3125( 25, or 0,025+
 ,8875         (8) Index of the Dividend.   —,125    (6)
  ,8875        (4) Index of the Diviſor.     ,125    (3)
   ,8875       ————                                  ———
    ,8875      (4) Index of the Quotient.            (3)
```

Examples

Examples in Mixt Decimals.

Examples in Mixt.

As 173,71875 divided by 13,625, the Quotient 1275, ſhall be 12,75; for by ſubſtracting (3) the Decimal *Index* of the Diviſor from (5) the Decimal *Index* of the Dividend, the Remain is (2).

Alſo if 758 Yards of Cloth coſt 284 *l.* 5 *s.*, or in Decimals 284,250, and the one be divided by the other, the Quotient will be 375, which by ſubſtracting 0, the *Index* of 758, from (3) the *Index* of the Dividend, will be found Decimal Thirds, or 7 *s.* 6 *d.* for the price of one Yard.

Likewiſe if 77448,288 be divided by 0,2864, the Quotient ſhall be 27042, and by ſubſtracting the *Index* (4) of the Diviſor from (3) the *Index* of the Dividend, the Remain 1 ſhews the *Index* of the Right Hand Quotient Figure to be removed one place diſtant from the Unit towards the Left Hand, and therefore Cyphers are placed to fill up the void places.

```
   68
 1021             (2)      37         (3)       25               1
 37 4612                   56 89                20160 72
173,71875(12,75            284,250(375          77448,288(270420,0
13,625            (5)      758        (3)       ,2864           (3)
 13,625           (3)       758       (0)        ,2864          (4)
  13,625         -----       758     -----        ,2864        -----
   13,625         (2)                 (3)          ,2864         1
                                                    ,2864
```

2. *A Leſſer by a Greater, 3 wayes.*

The ſecond Caſe, where a Leſſer Number is to be divided by a Greater, trebly branches it ſelf. As firſt by ſupplying the defect of the Dividend with Cyphers. Secondly, by contracting the Diviſor. Thirdly, by both. So as in Multiplication a Product may be contracted, in Diviſion a Quotient may be increaſed negatively, as far as ſhall be needful, and is ſometime phraſed to divide by an Irrational or Infinite Number.

1. *By adjoyning Cyphers.*

Firſt Branch, adjoyn to the Right Hand of the Dividend as many Cyphers as ſhall be requiſite, and divide as before (accompting the *Index* of the Dividend, as it now ſtands with the Cyphers adjoyned) and ſeparate the Integers, if any, from the Decimals, or ſupply the void places with Cyphers, and the work is done.

Example.

As if 21 *l.* were to be divided among 24 Men, I adjoyn 000 (which are as many as need here) to 21, and divide by 24, the Quotient is 875, and by Subſtraction of the *Indices* appears to be (3) for each Mans Share, which reduced into Godæticals is 17 *s.* 6 *d.*

```
          1
          82     (3)
(3)     21,000(875, or 0,875.
 0        24
---        24
(3)         24
```

2. *By contracting the Diviſor.*

Second Branch in dividing by the *Bones*, ſuppoſeth Cyphers to be adjoyned as before, but otherwiſe taketh as many of the diviſing Figures to the Left Hand as are ſufficient for the firſt Diviſor, and divideth the given Number thereby, then for every particular ſubſequent Diviſion leſſen the Diviſor, cutting off from the Right Hand of the Diviſor each time one Figure, having reſpect to the Tens ariſing in the next Right Hand place (as was taught before in Multiplication) and ſo proceed in the Diviſion till a Quotient be gotten as large as is deſired.

Example.

As to divide 467023 by 357,09264, the firſt Diviſor is but 357093, the next 35709, *&c.* as is plain by the work it ſelf ſubſcribed, and ſo the Quotient 130785 negative.

```
                    317                         Multiplees.
                    2803                        35709264
Diviſor            109930   Quotient            587031
357,09264+)467023(1307,85 -------            -----------
   .. ...          357,093                      357093
                    357,09                      107127
                     357,0                      0000
                      357                       2500
                       35                       286
                        3                       17
```

Third

3. *By both.* Third Branch both adjoyneth Cyphers, and contracteth the Diviſor, as aforeſaid.

Example. As if 34 were to be divided by 4,326481, and 5 Decimals only required in the Quotient: After 5 Cyphers are adjoyned to 34, I cut off 1 from the Diviſor, and divide firſt by 4,32648, and then by 4,3264, &c. The reſt of the work needs no further Explanation.

```
                    | 1
                     36
                    252
                   3713
                  25345                     Multiplees.
  Diviſor         37146 3  Quotient          4,326481
  4,326481 )34,00000( 7,85858                858587
  .  .....   4,32648                        ----------
              4,3264                         3028537
               4,326                         346118
                4,32                         21632
                 4,3                         3461
                   4                         216
                                             35
```

Some in the Diviſions of this and the ſecond Branch, uſe not to ſubſcribe the Diviſor as above, but in ſtead thereof the Multiplees, and ſo make Subſtraction, applying the Diviſor in a Looſe Paper, as was ſhewed in the *Firſt Book*, among the varieties of Diviſion of *Integers*.

Compounds divided. Compound Decimals convert into Simple, by Addition or Subſtraction, if they may, and then divide them as before. Otherwiſe divide the Numbers of the Dividend by the Numbers of the Diviſor, like the Diviſion of *Geodæticals*, in the *Third* and *Fourth Caſes*, *Book* 2. *Part* 1. *Chap.* 6. as they happen. The *Index* of the Quotienary Numbers are got as above. And the ſigns that are like give +, and unlike —, as in the precedent *Chapter* of *Multiplication*.

Example. As if 20,06 —16,60 +3,36 were to be divided by 3,4 —1,2, being turned into Simple Decimals they are 6,82 and 2,2, and Diviſion ended as at *A.* the Quotient is 3,1. If otherwiſe, divided as Geodæticals at *B.* after 34, gotten by the firſt ſetting down of the Diviſor, and the Diviſor multiplyed thereby, the Product is 20,06 —9,52, which ſubſtracted from the Dividend leaves —7,08 +3,36, which is the Product of —12 the next Quotient Figures multiplyed into the Diviſor.

```
                            A.                                      B.
+20,06   +3,4                                                   — 7,08
  3,36   —1,2           2      (1)   +5,9 —2,8 )20,06 —16,60 +3,36( 3,4—1,2
------   ----      (2) 6,82( 3,1     —1,2 +3,4   20,06 — 9,52
+23,42   +2,2      (1) 2,2           ---------              — 7,08 +3,36
—16,60   ----      ---    2,2         236  112
------             (1)                177   84
+ 6,82                               ---------
------                               20,06 —9,52
                                   -------------
                                     118   56
                                      59   28
                                    -----------
                                    —7,08 +3,36
                                    -----------
```

Proof of Decimal Diviſion. Diviſion of Decimals is to be proved by Multiplication, as Multiplication by Diviſion counterchanged; ſo as the Simple Multiplication of Decimals ſhall be proved by their Simple Diviſion, and Compound by Compound accordingly, and on the contrary their Diviſion by their Multiplication.

Alſo thoſe Diviſions that ſet down the Multiplees of the Diviſors underneath to ſubſtract, may be proved by Addition of thoſe Multiplees, which will with the Remains, if any, return the Dividend. As in the laſt Example at *B.* like *Geodaticals* of the fourth Caſe before noted, and the Diviſions of the ſecond and third Branches in this *Chapter*, thus to be placed.

Second Branch.		Third Branch.
(0	Remain added.	(1
17		35
286		216
2500		3461
00000		21632
107127		346118
357093		3028537
467023	Dividend returned.	34,00000

And beſides this let the Decimals be turned into Geodæticals, and accordingly divided, and the Quotients of both works, when done without Error, will clearly correſpond. As in the laſt Example at *A.*

Decimals.		Geodæticals.
		l. *s.*
6,82	Dividend.	6—16 $\frac{2}{5}$
2,2	Diviſor.	2—4
3,1	Quotient.	3—2

$$\begin{array}{l} \qquad\qquad\qquad 4 \\ l.\ \ s.\ )\ l.\ \ s.\ \ (\ l.\ \ s. \\ 2—4\ )\ \cancel{6}—\cancel{1}\cancel{6}\ \tfrac{2}{5}\ (\ 3—2 \\ 3—2 \qquad \cancel{6}—\cancel{1}\cancel{2} \\ 6—12 \qquad \cancel{4}\ \tfrac{8}{20} \text{ or abbreviated } \tfrac{2}{5}. \\ \qquad 4—\tfrac{8}{20} \end{array}$$

Or if the Geodæticals be all reduced into Shillings, then muſt 136$\frac{2}{5}$ *s.* be divided by 44 *s.* the Quotient of which Diviſion will be 3 *l.* and the 4$\frac{2}{5}$ left if reduced into an Improper Fraction multiplyed by 20, and as a Fraction divided by 44, the Quotient will be 2 *s.* as before.

CHAP. VIII.

Figuration of Decimals.

TO produce *Figurate Decimals* hath no difficulty therein; for by multiplying any Decimal Simple or Compound into it ſelf, the Square Decimal is produced. And the Square multiplyed by the Root, produceth the Cube, *&c.* as *Figural Numbers* before in the *Second Part* of the *Second Book* were declared to be commonly produced. And becauſe the ſame is done by Multiplication, the *Indices* of ſuch Figurated Decimals are found by adding together the *Indices* of both the Factors, and the ſigns of the Product known, as in Multiplication of Decimals before was ſhewed.

Figurate Decimals produced.

Examples.

Examples.	Pure Simple } Decimals				{ Mixt. Compound.
(2)	0,15 Root	1,2	—0,5		Root
(2)	0,15	1,2	—0,5		
	075	24	10		
	015	12	5		
		1,44	—,60		
(4)	0,0225 Square		—,60	+,25	
(2)	,15	1,44	—1,20	+,25	Square
	01125	1,2	—0,5		
	00225	288	240	50	
(6)	0,003375 Cube	144	120	25	
	&c.	1728	—1440	+300	
			— 720	+600	— 125
		1,728	—2,160	+,900	—,125 Cube
				&c.	

If

Compound may be turned into Simple.

If the Compound Decimal be turned into a Simple, and Figurated accordingly, and the Number produced compared with the Product of the Compound, they will agree in ſumme when the Operation is right. As if 1,2 —0,5 be turned into a Simple Decimal, by ſubſtracting the 5′ which is lacking out of the 1,2 affirmative, the remaining Root is 7, whoſe Square is 49, and Cube 343. And ſo the ſumme of the Compound Square and Cube Decimal thereof appears by adding + to +, and — to —, and then deducting the — out of the + thus;

```
 + 1,2           + 1,44            + 1,728      — 2,160
 —  ,5           +  ,25            +  ,900      —  ,125
 ______          _______           _______      _______
     7 Root.     + 1,69            + 2,628      — 2,285
    49 Square.   — 1,20
  ,343 Cube.     _______                 + ,343 Cube.
 ______          +  ,49 Square.
                 _______
```

Roots of Simple Decimals how extracted when the Index will be parted.

To extract the Root of a Decimal, if it be Simple, and the Decimal Denomination to the Right Hand be ſuch as will be parted by 2 for the Square, 3 for the Cube, *&c.* then prick the Number, and extract the Root according to the Quantity deſired, as before *Book* 2. *Part* 2. *Chap.* 3. And to find the *Index* of the Root do thus: For the Square take half the *Index* of each pricked Figure, for the *Index* of the ſeveral Figures of the Root. For the Cube take $\frac{1}{3}$ part of the *Index* of the pricked Figures. For the Biquadrate take $\frac{1}{4}$ part, *&c.*

Example in the Square.

As in extracting the Square Root of 0,0225, the Root will be found 15, which is 0,15, becauſe half the *Index* of the pricked 2 being Seconds in the Square ſhall be Primes in the Root, and half the *Index* of 5 in the Square being (4) in the Root, ſhall be Seconds.

Example in the Cube.

So in extracting the Cube Root of 0,003375, the *Index* of the pricked 3 in the Cube being Thirds, ſhall be Primes in the Root. And the *Index* of 5 in the Cube being (6), ſhall make the *Index* of 5 in the Root (2), which is the third part of (6).

```
        2 (4)   (2)                  2 (6)   (2)
        .  .                         .   .
Square  225  ( 15 Root.      Cube    3375  ( 15 Root.
        1  :                         1   :
        10 :                         15  :
         25                           75 :
        ____                          125
        225                          _____
        ____                         3375
                                     _____
```

When the Index will not be parted.

If the Decimal Denomination of the Simple Decimal, whoſe Root you would extract, will not be parted as above, then adjoyn a Cypher or more to the given Number to make the *Index* ſo diviſible, and then extract the Root as before.

Example in the Square.

As if the Square Root were deſired of 0,0015129, becauſe (7), the *Index* of 9, cannot be halfed, I adjoyn a Cypher or 3,5, or more Cyphers proportionably, and extracting the Root with 1 Cypher the Root ſhall be (4), with 3 Cyphers 5, *&c.* according to the Cyphers adjoyned.

Example in the Cube.

Alſo if the Cube Root be deſired of 0,01860867, becauſe (8) the *Index* of 7, cannot be divided by 3, I adjoyn a Cypher, or 4,7, or more Cyphers. But if the given Number had been 0,1860867, then the *Index* of 7 being (7), I muſt adjoyn 2 Cyphers to make the *Index* diviſible by 3, and ſo increaſe the Cyphers proportionably to the Quantity of the Number given; for that the more Cyphers being adjoyned the nearer the truth will the Root be, as before in the *Third Chapter* of *Book* 2. *Part* 2. §. 6. was ſhewn.

```
   |7                       1|208                       |20
  668|46   Root          0032|926  Root            1208|104
  . . . |  (4)             . . . |  (3)           00329 26|551  Root.
(8) 151290 |388        (9) 18608670 |264           . . . .           (4)
   9 : :                    8 : :           (12) 18608670000(2649
   544 :                    9576 :                 8 : : :
    6144                     832744                9576 : :
Remain 746              Remain 208926               823744 :
 ---------               -----------                188821449
 151290 Square            18608670 Cube     Remain  20104551
 ---------               -----------                -----------
                                                    18608670000 Cube.
                                                    -----------
```

To extract the Root of a Compound Decimal; out of the Left Hand Number extract the Root of the Quantity desired, and then by your Diviſor enquire for another Quotient Figure, and ſo proceed according to the inſtructions before given for Extraction of Roots.

Roots *of* Compound Decimals *extracted.*

As to extract the Square Root of 1,44 —1,20 +,25, the Square of 1,44 is 1,2 which doubled is 2,4, the Diviſor, by which 1,20 divided yieldeth 5 in the Quotient, which ſhall be —, becauſe the ſign of 1,20 was —, and of 1,2 the Diviſor was +, and the *Index* of 5 ſhall be Primes; then the Square of 5 is 25, which ſubſtracted leaves 0; and the Root is 1,2 —0,5.

Example in the Square.

Likewiſe to extract the Cube Root of 1,728 —2,160 +,900 —,125, the Cube of 1,728 is 1,2, the Diviſor is 4,32, by which 2,160 divided giveth in the Quotient 5, which ſhall be —, becauſe the Signs were unlike, and the *Index* Primes; then the Square of —0,5, which is +2,5, multiplyed into +3,6, the triple of 1,2, makes +,900 for the next Subſtraction. And laſtly, the Cube of —0,5 is +0,125, which ſubſtracted leaves 0.

Example in the Cube.

```
     Square.                                Cube.
 . :             Root.              . :                       Root.
1,44 —1,20 +,25 | 1,2 —0,5          1,728 —2,160 +,900 —125 | 1,2 —0,5
 .         .    ----------            .                .     ----------

1 :                                 1 :
 4 :                                6 :
 4  +2,4 +25                        12 : +4,32 +,900 —,125
    —0,5                             8   —0,5
   -------                               -------
    —1,20                                —2,160
   -------                               -------
```

Decimal Figurals like others prove their Production by Extraction, and Extraction by Production. Moreover if the Compound Decimals be turned into Simple, the Extractions of their Roots, when the Operations are right, by comparing together will be found to agree.

Proof of Decimal Figuration.

```
     Square.                         Cube.
 +1,44 —1,20 +,25             +1,728 —2,160 +,900 —,125
 +0,25                        +  ,900 —  ,125
 ------                       -------  -------
 +1,69                        +2,628 —2,285
 —1,20
 ------
 + ,49 Root 7 = 1,2 —,5       ,343 Root 7 = 1,2 —,5
```

Partis Primæ & Libri Tertii.

FINIS.

THE

THE SECOND PART OF THE THIRD BOOK.

CHAP. I.

Of ASTRONOMICALS.

THE ſecond ſort of Numbers ſpecially Contract are *Aſtronomicals*, ſometime called *Aſtronomical Fractions*, and other while *Geodæticals* of Time and Motion, which indeed they are; but having ſome Operations more peculiar to them, with other Contract Numbers, then common to other Fractions, I reſerved the handling of them to this place.

Aſtronomicals *the next ſort of Numbers ſpecially Contract. How called.*

Time is meaſured by Years, Dayes, Hours, *&c.* And the Cœleſtial Motions by Circles, Signs, Degrees, *&c.*

Time *and* Motion *how meaſured.*

Annus, or a Year, was by the *Egyptians* in their *Hieroglyphicks*, emblem'd out by a *Snake* biting her Tail in her Mouth, and from *Anguis* therefore ſome conceive proceeded *Annus*; but others, and more truly, from *Annulus*, a Ring or a Circle, which returns again into it ſelf.

Year *how called and Emblemed.*

Annus, *whence the word.*

Neither the beginning of the Year, nor the length thereof, hath had the hap alwayes in all Nations to be alike; nor is the former yet with us in *England* identified. For the *Commonalty* begin the Year the Firſt Day of *January*, the *Lawyers* the Five and Twentieth Day of *March*, and the *Aſtronomers* when the *Sun* enters the firſt Scruple of *Aries*.

Divers beginnings of the Year.

The length of the Year is now generally agreed to be that ſpace of Time in which the *Sun* is running his courſe through the *Zodiack*, beginning at the firſt point of *Aries*, until his revolution or return thither again.

Length of the Year.

This ſpace of Time is divided into Twelve parts, called Moneths, but very unequally in our Calendars or Almanacks, 7 of them having 31 Dayes, 4 but 30, and one but 28. More evenly ordered by *Julius Cæſar*, as ſome ſay, but diſtorted in Honour of *Auguſtus*, to equal the Dayes in the Moneths bearing their Names.

Year *how divided.*

Moneths *now unequal, and by whom ſo made.*

Some think the Year at firſt was, or ought to be divided by the courſe of the *Moon*, who gave being to the Name *Moneth*: And by the Law of *England*, a Moneth is but 28 Dayes, agreeing to the viſibility of the *Moon*, there being ſome time for her both before and after Conjunction with the *Sun* to be hidden under his Beams or Rayes. But neither hath this afforded a *Baſis* for the exact limitation of a Moneth; ſince beſides the time of the *Moons* viſibility, called the *Apparition Moneth*, and the time when moſt proper to adminiſter *Phyſick*, called the *Medicinal Moneth*, containing 26 Dayes, 12 Hours, according to *Johannes de Sacra Boſco*: There is the *Periodical Moneth*, and the *Synodical Moneth*; the former of which called alſo the Moneth of *Paragration*, is the ſpace in which ſhe makes her Revolution, and returns to the place ſhe made her laſt ☌ with ☉, and the latter called by ſome the Moneth of *Conſecution*, the whole ſpace from her laſt Conjunction with the *Sun*, unto her next: Both which by reaſon of the ſometime ſwift, and ſometime ſlow Motion of both Luminaries, take up more or leſs time accordingly, then can be compriſed in any preciſe Number of Dayes without ſurpluſage. The *Periodical* containing 27 Dayes, 7 Hours, 43 Minutes, 7 Seconds; and the *Synodical* 29 Dayes, 12 Hours, 44 Minutes, 4 Seconds, by common Computation.

Moneth *whence the word.*

Sorts of Moneths.

Apparition Moneth.

Medicinal Moneth.

Periodical *or Moneth of* Paragration.

Synodical *or Moneth of* Conſecution.

The length of the Periodical *and* Synodical Moneths.

Moneth *omitted by ſome, by others reckoned at 30 Dayes.*
Moneth *of the* Jews.
Legal Moneth *how much.*
Week *the content.*

Becauſe of theſe differences about the Moneth, ſome omit them among the certain Denominations, others in their Operations accompt 30 Dayes to a Moneth, which ſome conceive to be the Old Accompt of the *Jews*; for comparing *Gen.* 7. 11. with *Gen.* 8. 3, 4 the ſpace from the ſeventeenth day of the ſecond Moneth, to the ſeventeenth day of the ſeventh Moneth, is reckoned 150 Dayes, which being 5 Moneths, allowes 30 Dayes for every Moneth. And others uſe neither them nor the Calendar Moneths, but the Legal Moneth of 28 Dayes, as aforeſaid, which divided into 4 parts called Weeks, makes every Week 7 Dayes, and by this accompt 13 Moneths in the Year.

Legal Moneths *in the* Year.
Year *certainly divided.*
Length of the Tropical Year.
Biſſextil *or* Leap-Year.

The Year then for certainty is ſubdivided into Dayes, and containeth commonly 365 Dayes, and almoſt 6 Hours, but the true Tropical Year by late Writers is not above 365 Dayes, 5 Hours, 49 Minutes, 4 Seconds and 21 Thirds; for which odd time in *Arithmetick*, 6 Hours is uſually taken, which in every Fourth Year called *Biſſextil* or Leap-Year, maketh another Day, and it is added to the end of the Moneth of *February*, which then hath 29 Dayes, or elſe in the Common Years but 28; ſo hath the Leap Year 366 Dayes.

Day *Natural and Artificial.*

Dies, or a Day Natural, is in *Aſtronomers* accompt that ſpace of Time run out while the *Sun* paſſeth from the *Meridian*, ere he can come thither again, and is divided into 24 equal parts called Hours, and comprehendeth both Day and Night. But a Day Artificial is ſometime limited between ☉ and ☉, or *Sun* Riſing and *Sun* Setting, or to a quantity of Hours, as a Working-Day 12 Hours, from 6 at Morning till 6 at Night.

Hours *how ſubdivided.*

Every of theſe Hours is divided into 60 parts, called Minutes, and every Minute into 60 leſſer diviſions, called Seconds; every Second into 60 Thirds, and ſo lower into Fourths, Fifths, Sixths, &c. by 60.

Of Motion.
Circle *how divided.*
Degrees *what part thereof how called.*
How many to a Sign.

Touching Motion, it is to be remembred, That the Circumference of the Heavens, as alſo every Circle, is parted into 360 diviſions, called Degrees or Grades from the Latin *Gradus*, 30 of which make one Sign, ſo 12 Signs make the Circle; for 12 times 30 is 360, anſwerable to the diviſion of Time into 30 Dayes for a Moneth, and 12 Moneths to the Year.

Degrees *ſubdivided.*
Minutes *and* Miles *how agree.*

Every Degree contains in Longitude or Length 60 Minutes, which are not to be taken for Minutes of Time, but Minutes of Meaſure, called alſo Scruples, and ſometime Primes. Every of theſe Minutes of Meaſure in the Heavens is generally accompted to anſwer to one Mile on the Earth. Nevertheleſs Mr. *Oughtred* in his *Circles of Proportion* is of Opinion, that one Degree or 60 Minutes anſwer more truly to 66¼ Miles.

Theſe Minutes, as well as Minutes of Time, are ſubdivided into 60 Seconds, the Second into 60 Thirds, and ſo into Fourths, Fifths, Sixths, &c. of which 60 leſſer are comprehended in one of the next greater.

The reaſon of the Subdiviſions by 60.

That an exact Meaſure both of Time and Motion might be had, the Antients have thought fit to divide one Hour of Time, and one Degree of Motion into 60, and ſo downwards, unto very ſmall ſubdiviſions, as aforeſaid. Foraſmuch as alwayes an whole Year, Moneth or Day, &c. is not the ſubject of the Queſtion, neither the moving of the Cœleſtial Bodies to be alwayes meaſured by whole Circles, Signs, Degrees, &c. but ſometimes Parts or Fractures of the whole are uſeful. And the rather have they choſen 60 for the Integer or Denominator, becauſe it is a Number that may receive more equal diviſions than any Number under 100; for it may be parted by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30.

Denominator 60 certain and omitted.
Difference of Aſtronomicals *from* Decimals.

The Denominator 60 being certainly known is alway omitted, as the Decimal Denominators are, and the given Numerators only uſed. And as in Decimals the Numbers exceeding the Units place to the Left Hand paſs for Integers; ſo theſe. Nevertheleſs here ſeems to be ſome difference in that beyond the Units place to the Left Hand (1 Day in Time, and 1 Degree in Motion being generally accompted the Integers) here are Fractions beſides thoſe at the Right Hand, as Signs in Motion, and Moneths in Operations of Time; for the utmoſt Integer or greateſt Geodætical of Time is the whole Year, and of Motion the whole Circle, notwithſtanding although ſometime between the Unit and utmoſt Integer ſuch Fractions be found, yet their Denominators being certain they are omitted, and accompted as Integers.

Aſtronomicals *why ſo called.*
Sometime called Phyſical Fractions.
Phyſical Signs what.
Signs with Aſtronomers.

Becauſe the uſe of theſe Diviſions is moſt converſant about *Aſtronomy*, they are called *Aſtronomicals*, or *Aſtronomical Fractions*: Some Writers call them *Phyſical Fractions*, and with *Alphonſus* divide the Circle into 6 diviſions, and then one Sign containeth 60 Degrees, and the other diviſions are as before, which is certainly the moſt expedient way for Multiplication and Diviſion; but *Aſtronomers* in all their Calculations generally reſerve the Sign of 30 Degrees, and accordingly reckon all the Aſpects both Old and New, Siniſter and Dexter.

Aſpects Old and New, how many Degrees therein.

Old					Aſpects	New				
Names.	*Marks.*	*Signs.*		*Degrees.*	*Names.*	*Marks.*	*Signs.*		*Degrees.*
Sextile	✳	2		60	Semiſextile	SS.or ⚺	1		30
Quartile	□	3	or	90	Semiquintile	♀.	$1\frac{1}{5}$		36
Trine	△	4		120	Semiquartile	⚼.	$1\frac{1}{2}$		45
Oppoſition	☍	6		180	Quintile	Q.or ♡	$2\frac{2}{5}$		72
					Seſquiquintile	Td.or ♡	$3\frac{3}{5}$	or	108
					Seſquiquadrate	⚼.	$4\frac{1}{2}$		135
					Byquintile	Bq.or ∞	$4\frac{4}{5}$		144
					Quincunx	Vc.or ⚻	5		150

Denominations of late uſe.

Late Writers generally uſe two kinds of Names or Denominations, the one called *Sexagenæ*, and the other *Sexageſima*, both from the *Latin* importing Sixties, becauſe of the general Denominator 60.

Sexagenæ *what.*

Sexagenæ are the collection of Degrees all under 60, ſtanding in the place of the Unit, and having a Cypher to their *Index*, then by Collection all above 60, and under 3600, fall in *Sexagenæ primis*, and have 1 for their *Index*; 2 is the *Index* of *Sexagenæ* Seconds, 3 of *Sexagenæ* Thirds, and ſo of other Numbers reſpecting Motion.

The Collection of Dayes under 60, reſembling the former ſtand in the place of the Unit, and all above 60, and under 3600, poſſeſs the ſecond place from the Unit to the Left Hand, and are called *Sexagenæ Primes*, and 60 times 60 Dayes, which is almoſt 10 Years, make one *Sexagenæ* Second, and ſo they are marſhalled to Thirds and Fourths, *&c.* And that Hours may yield the place of Unity to Dayes, they are to be reduced to Minutes of a Day, as in the next *Chapter*. Nevertheleſs where there are no *Sexagenæ*, Hours may keep the place of Unity without Reduction.

Sexageſimæ *what.*

Sexageſimæ, both of Motion and Time, are ſet on the Right Hand of their Integers (Degrees or Dayes) like the Decimals on theirs, and are to the *Sexagenæ*, as Decimals to their Integers, and are the diviſion of Minutes, Seconds, Thirds, *&c.* by 60, as aforeſaid like the Decimals by Tens.

How both marked.

Both the *Sexagenæ* and *Sexageſimæ* are marked with exponent *Indices*, as the Decimals are, which accordingly ſhew the power of the Numerator either affirmatively or negatively, that is, either above or below the Unit.

Indices *differently expreſſed.*

Theſe *Indices* are found differently expreſſed, and ſometime the one, and ſometime the other Form are uſed, which is not at all material whether.

Examples.

		Sexagena's.							*Sexageſima's.*							
Thus,	&c.	6	5	4	3	2	1	0	(1)	(2)	(3)	(4)	(5)	(6)	&c.	Common with
	&c.	6	5	4	3	2	1	0	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	*&c.*	Decimals.
or																
Thus,	&c	vj	v	‵‵‵‵	‵‵‵	‵‵	‵	0	′	″	‴	⁗	v	vj	&c.	Proper to
	&c.	æ6	æ5	æ4	æ3	æ2	æ1	0	′	″	‴	⁗	v	vj	*&c.*	Aſtronomicals

So if I ſee this Number 3‵‵‵, 1‵‵, 12‵, 15^{Dayes}, 16′, 10″, 12‴, I read it thus, 3 *Sexagenæ* Thirds, 1 *Sexagenæ* Second, 12 *Sexagenæ* Primes, 15 Dayes, 16 Minutes or Primes, 10 Seconds, and 12 Thirds of a Day; and ſo of others.

Aſtronomicals *Simple and Compound.*

Moreover if the Signs + and — be uſed, let the Aſtronomicals be counted Compound, but if not Simple.

Cyphers ſupply intermediate places.

If any intermediate Denominations in Numbers given be omitted, let the places be ſupplyed with Cyphers, as 2‵‵ and 2′, thus, 2‵‵, 0‵, 0°, 12′.

CHAP. II.

Reduction of Aſtronomicals.

Aſtronomicals reduced how many wayes.

Reduction, called alſo Converſion of Aſtronomicals, is threefold, *viz.*

Either to convert { Geodæticals / Decimals } into Aſtronomicals, or the contrary.
Aſtronomicals of one ſort into another.

The

1.

*To reduce Common Signs into Phyſical and Years into Sex*agenæ, *or contrary.*

The firſt ſort ſerveth either to turn Common Signs into *Phyſical*, and Years or Moneths into *Sexagenas*, or the contrary.

To reduce Common Signs into Phyſical.

To turn Common Signs into Phyſical, half them, or reduce Geodætically by 30, the Signs into Degrees, and add the odd Degrees (if any) to them, then divide by 60, what Degrees remain place in the Units place, and the Numbers in the Quotient, if under 60, are Phyſical Signs, and to be ſet in the place of *Sexagenæ* Primes; if above 60, divide the Quotient again by 60, and place the Remainers as *Sexagenæ* Primes, and the Quotient of this Diviſion for *Sexagenæ* Seconds, *&c.*

To reduce Phyſical Signs into Common.

On the contrary to turn Phyſical Signs into Common, or *Sexagenæ* of Motion into Circles and Signs; double or multiply the Phyſical Signs by 2, or reduce Geodætically all the *Sexagenæ* into Degrees, multiplying by 60, and then divide the ſumme by 30, and the Quotient, if under 12, ſhall be Signs; if above 12, divide by 12, and the Quotient of this Diviſion ſhall be Circles.

Example of both.

As if 9 Signs, 12 Degrees, be given to be converted into Phyſical, reduced into Degrees they are 282, which divided by 60, gives 4 Phyſical Signs or *Sexagenæ* Primes, and 42 Degrees to be ſet in the place of Unity; which if on the contrary had been given to have been turned into Common Signs, the 4 Signs multiplyed by 60, make 240, to which the 42 Degrees added make 282 Degrees, which divided by 30 gives 9 Signs in the Quotient, and 12 Degrees remaining.

Common Signs. Degrees.
9 —— 12
30
270
12
282 Degrees.

Degrees. Signs Phyſical.
| 4
28|2 (4
60

′ °
4 —— 42
60
240
42
282

Degrees. Signs Common.
(1
28(2 (9
30

To reduce Years &c. into Sexagenæ.

To turn Years or Moneths into *Sexagenæ*, reduce all Geodætically into Dayes accompting in, as in Reduction of Geodæticals was obſerved, the Dayes ſupernumerary for the Leap-Years, and adding in alſo the odd Dayes given in the Number, if any be, then divide by 60, and ſo Quotient after Quotient as far as you can, the Remain of the firſt Diviſion being Dayes is to be placed as Integers in the Units place, the other Remains and laſt Quotient in their places orderly to the Left Hand, as *Sexagenæ* Primes, Seconds, Thirds, *&c.*

To reduce Sexagenæ into Years.

On the contrary to turn *Sexagenæ* of Dayes into Years, multiply Geodætically all the *Sexagenæ* into Dayes by 60, and then divide by the Dayes in one Year, and from the Remain ſubſtract the Dayes for the Leap-Years of the Quotienary Number, and the reſt of the Remain ſhall be the odd Dayes, which if occaſion be may be turned into Moneths by Diviſion with the Dayes of one Moneth.

Example of both.

As if 1000 Years, 20 Dayes, were to be converted into *Sexagenæ*, the Common Dayes in 1000 Years by Geodætical are found to be 365000, to which 250 Dayes added, becauſe there are ſo many Leap-Years in 1000, and the 20 Dayes given alſo added, make the Total 365270; then divided by 60, there is 1‴, 41″, 27′, 50°; that is, 1 *Sexagenæ* Third, 41 *Sexagenæ* Seconds, 27 *Sexagenæ* Primes, 50 Dayes; which if on the contrary were to be turned into Years, after Reduction by 60, and divided by 365, the Quotient will be 1000, and from the Remain 270, if 250 Dayes be ſubſtracted for the Leap-Years, there will remain 20 odd Dayes.

2
1000 (250 } Leap Years
4

1000 Years.
365
365000
250
20
365270 Dayes.

4|5 ° | 2 ′ | 4 ″ ‴
36527|0 (608|7 (10|1 (1
6 0 6 0 60

‴ ″ ′ °
1 —— 41 —— 27 —— 50
60
60
41
101 ″
60
6060
27
6087 ′
60
365220
50
365270 °

Dayes. Years.
365 | 270 (1000
365 250
365 20
365
365

2. *To reduce* Aſtronomicals *into* Decimals, *or the contrary.* Sexagenæ *into* Integers, Decimals *into* Sexageſimæ.

The ſecond ſort of Aſtronomical Reduction is to turn Aſtronomicals into Decimals, or Decimals into them, as *Sexagenæ* into Integers, and *Sexageſimæ* into Decimals, or the contrary.

To turn *Sexagenæ* into Integers, or Decimals into *Sexageſimæ*, multiply continually by 6, every time removing the *Seperatrix* one place, but on the contrary to turn Integers into *Sexagenæ*, or *Sexageſimæ* into Decimals, removing the *Seperatrix*, as before, divide continually by 6, proceeding both wayes till you come to the Units place: For Diviſion by 60 removeth the *Seperatrix* one place towards the Left Hand, and divideth by 6; and Multiplication by 60, promoteth the *Seperatrix* one place towards the Right Hand, and multiplyeth by 6; and ſo the Cypher being cut off by the *Seperatrix*, the work is ſhort and eaſie.

Variety.

Otherwiſe by common Geodætical Reduction bring all the Circles and Signs, or *Sexagenæ* thereof, into Degrees, alſo Years and Moneths, or *Sexagenæ* thereof, into Dayes by Multiplication, and them reſerve for the Integers; then reduce by Multiplication with 60, all the *Sexageſimæ* into the loweſt Denomination, and the laſt Product thereof adjoyning Cyphers if need be, divide by 60, Figurated to a Power of the Second, Third, Fourth, *&c.* Quantity, according to the Right Hand Denomination of the *Sexageſimæ*, and the Quotient ſhall be the Decimals. And to convert Decimals given into the other, multiply the Decimals as aforeſaid, and divide the Integers by their proper Denominators.

Example of both.

As if 6‶, 30‵, 20°, 15′, 45″, were to be reduced into Integers and Decimals; by the firſt way I place them with the ſeperating Lines, as at *A.* and beginning at top, multiply the 6‶ by 6, the 36 produced is ſubſcribed under the upper *Seperatrix*, as at *B.* which with the 30‵ multiplyed by 6, produceth 234, adjoyned to the 20°, makes in all 23420 Integers, as at *C.* then beginning with the *Sexageſimæ* at bottom, I divide 45″ by 6, (ſuppoſing 00 adjoyned) I ſuperſcribe over the lower *Seperatrix* the Quotient 75, as a Decimal to 15′, at *D.* And laſtly, I divide 1575 (with 2 Cyphers alſo ſuppoſed to be adjoyned) and ſuperſcribe the Quotient 2625, as the Decimals deſired to be adjoyned to the Integers; and the whole work ſtands compleat at *E.*

A.	*B.*	*C.*	*D.*	*E.*
6‶	6‶	6‶	6‶	6‶ ×6
30‵	36	36	36	36
20°	30‵	30‵	30‵	30‵
,15	20°	23420°	23420°	6) 23420,2625 ×6
″45	,15	,15	,15 75	,15 75
	″45	″45	″45	6) ″45

Where there are many Species, this is much the ſhorter and more ingenuous way; the other for the Integers hath had its like often exemplified in Geodæticals, as at *F.*; for the 15′, 45″, after Reduction into 245″, I adjoyn 0000, and divide by 3600, which is the Square of 60, or the Power of the Second Quantity, the Number being ″ and the Quotient is 2625 at *G.* So is the Number as before 23420,2625.

```
 ‶   ‵   °   ′   ″                 (4)
 6  30  20  15  45   Reduced is 23420,2625.
 60         60
 ---        ---      (1) 60  Root.              1
 360        900      (1) 60                  2298  (6)    (4)
  30         45      -----------             9450000 (2625   G.
F. ---       ---     (2) 3600 Square.        36   00
 390        945″     -----------              36
  60        ---                                36
 ----                                           36
 23400
    20
 ----
 23420 Integers.
 ----
```

And if 23420,2625 were given to be converted into Aſtronomicals this way, I continue Diviſion of 23420, and Multiplication of ,2625 by 60, and obtain my deſire.

```
      Sexagenæ.                          Sexageſima.
  5     °    |3  '   "                        ,2625
 234|20(   39|0(  6                      |       60
 ------    -----                    -----|---------
  6    0    60                       ' 15|     7500
                                         |       60
                                    -----|---------
                                     "45 |     0000
                                    ---------------
```

But by the other way proceed, as before, by 6, to multiply the Decimals, and divide the Integers, cancelling, as ſome do, the Figures divided or multiplyed.

```
Numbers placed.        6"|  Sexagenæ.          6"|  Sexagenæ.
                       ---                     ---
                        3 '                     3 '
                       390|0                   390|0
            1          ----                    ----
23420,|2625         23420,|2625             23420,|2625
                                                ,15|75
                                  Sexageſimæ   " 45
```

3. *Astronomicals reduced one into another.* The third ſort of Aſtronomical Reduction is uſed to convert one Aſtronomical into another.

As { Degrees into { Decimals of a Day —— / Hours and Decimals of an Hour } or the contrary.
Hours into { Decimals of a Day / Minutes of a Day } or the contrary.

To reduce Degrees, &c. into Decimals of a Day. Example. To turn Degrees with or without Decimals annexed to them, into Decimals of a Day, divide by 360, that is 6 × 60. And contrarily to turn Decimals of a Day into Degrees, multiply by 360, that is 6 × 60.

As to convert 236,4276 Deg. into Decimals of a Day, I firſt take the ſixth part of the Number, or divide by 6, and ſubſcribe, which is 39,4046, of which the 60th part is 0,6567433 for the Decimals of a Day deſired. And if this Number had been given to procure Degrees, aud Decimals of a Degree, I had multiplyed firſt by 60, and the Number ſuperſcribed multiplyed again by 6, as at *H*, but by the common way, the one and the other is at *I.* and *K.* with Cyphers adjoyned, as needful, in the Diviſion at *I*, and ſupply for the defect of the Decimal at *K*, making the 588(6), to be 6(4), or accompting them ſo, as moſt uſual in Imperfect Decimals.

```
Diviſors.     H.                          I.                       K.
{6 )   236,|4276   Multiplyers.      2  xxx|1                 0,6567433
{60)    39|4046        × 6 }        20 26522|2      (7)           |  360
         0|6567433     × 60}       236,427600|0(6567433       ----|------
                                   36         0                39|4045980
                           (7)      36                        197|02299
                            0        36                       ----|------
                           ---        36                      236,427588
                           (7)         36                     ----------
                                        36
                                         36
```

To reduce Degrees, &c. into Decimals of an Hour. Example. To turn Degrees with or without Decimals annexed to them, into Hours and Decimals of an Hour, divide by 15, that is 3 × 5. And on the contrary to turn them into Degrees, multiply by 15, that is 3 × 5.

As to convert the former Number of Degrees, and Decimals of a Degree 236,4276 into Hours, and Decimals of an Hour, I firſt divide by 3, and the Number 78,8092 ſubſcribed, divide by 5, and the ſubſcribed 15,76184, as at *L.* is the deſire; which if given by Multiplication firſt by 5, and the Product ſuperſcribed multiplyed by 3, would have produced 236,4276, as by Common Diviſion and Multiplication by 15, at *M.* and *N.* appears.

Diviſors

```
Diviſors.      L.
{3)   236|4276    Multiplyers.
{5)    78|8092        × 3}
       15|76184       × 5}
```

```
            M.
    1
    81 911 (5)        (5)
    236,42760 ( 15,76184
    15
     15
(5)   15
 0     15
 ——     15
(5)      15
          15
```

```
      N.
   15,76184
      |   15
   ———+——————
    78|80920
   157|6184
   ——————————
   236,4276
```

To reduce Hours, &c. into Decimals of a Day.

To turn Hours with or without Decimals annexed to them, into Decimals of a Day, divide by 24, that is 4 × 6. And to turn them into the other, multiply by 24, that is 4 × 6.

Example.

As to convert the aforeſaid 15 Hours, and 76184 Decimals of an Hour into Decimals of a Day, I firſt divide by 4, and ſubſcribe the Quotient 3,94046, the which I divide by 6, and the Number ſubſcribed is 0,6567433, the Decimals of a Day deſired as at *O.* Contrariwiſe, if that ſame Number had been given, I would have multiplyed firſt by 6, and the Numbers ſuperſcribed on the *Seperatrix* would have been 3,94046 as before, which multiplyed by 4, would have produced the 15,76184. And ſo by Common Diviſion or Multiplication with 24, the Numbers agree as at *P.* and *Q.* allowance being made for the Imperfect Decimal at *Q.*

```
Diviſors.      O.
{4)   15|76184     Multiplyers.
{6)    3|94046         × 4}
       0|6567433       × 6}
```

```
            P.
      1 1
      1 317088|8
      15,7618400 ( 6567433
      24
       24
(7)     24
 0       24
 ——       24
(7)        24
            24
```

```
      Q.
  (7) 0,6567433
        |   24
    ————+————————
       2|6269732
      13|134866
    —————————————
    15,7618392
```

To reduce Hours, &c. into Minutes of a Day.

To turn Hours, with or without Minutes and other ſmaller parts of an Hour, into Minutes of a Day, that is, into Sixtieth parts of a Day, which Hours are now, and ſo into other ſmaller parts of a Day, multiply every Number by 5, and half the Product, that is multiply by 2½. And on the contrary to turn the parts of a Day into Hours, and parts of an Hour, double every Number, and divide by 5, that is divide by 2½, becauſe one Hour anſwers to 2′ 30″ of a Day, to make the Denominator 60, there being twice 24 and 12 therein.

Example.

As to convert 12 Hours and 40 Minutes of an Hour into parts of a Day: If I multiply by 5, the Products 60, 200, halfed are 30′, 100″, which 100″, becauſe above 60, I carry a Minute to 30′, and ſubſcribe the 40″ remaining, ſo the reſult 31′, 40″, of a Day, as at *R.* which if given to be turned into Hours, I firſt double them, and they are 62′, 80″; then dividing 62 by 5, I get 12 Hours, and the 2′, 80″, remaining turned into ″ make 200, which divided by 5 give 40′ of an Hour. The Multiplications and Diviſions by 2½ are at *S.* and *T.*

```
Hours.  ′    R.
  12   40    ″
5 × Day 60  200  ) 5
2)      30  100  × 2
       ———— ————
        31   40
```

```
      S.
   12 . 40
      2½
   ————————
   24 | 80
    6 | 20
   ————————
   30 . 100
   ————————
   31 . 40
```

```
      T.
    ′    ″
   31 . 40
      2
   ————————   Hours ′
   62 . 80 ( 12 . 40
   ————————
      5
```

Proof of Aſtronomical Reduction.

The Reductions of this *Chapter* being reciprocal ſerve for Proof each to other without further illuſtration.

CHAP.

CHAP. III.

Addition of Aſtronomicals.

Aſtronomicals *added. Simple.*

ADdition of Aſtronomicals is either Simple or Compound.

Simple Addition differeth nothing from Geodætical Addition, for you begin at the Right Hand, and add the Numbers of like Denomination to their fellows: And if at any time the Numbers of that Denomination you are adding exceed 60, for every 60 carry an Unit to the next Left Hand Denomination, and ſubſcribe the overplus: And ſo proceed in order to the Left Hand.

Example.

As to add 13‶, 26‵, 59°, 30′, 45″, to 2‶, 5‵, 17°, 15′, 30″, the Total will be 15‶, 32‵, 16°, 46′, 15″; which is ſo plain, explanation is needleſs.

	‶	‵	°	′	″
Addends	13	26	59	30	45
	2	5	17	15	30
Total	15	32	16	46	15

Compound.

Compound Aſtronomical Addition is like Compound Decimal Addition; for taking the Leſſer Numbers out of the Greater, to the Remaining Total ſubſcribe the Sign of the Greater.

Example.

As to add 13‶, 26‵, 59°, 30′, 45″, with —2‶, 5‵, 17°, 15′, 30″; theſe muſt be ſubſtracted from the other, and the Total remaining will be +11‶, 21‵, 42°, 15′, 15″. And if the Signs be intermixt, as to add 4‵+10° —40′, to 3‵—20° +10′, the Total will be +7‵—10° —30′. As the Examples ſhew ſufficiently without further explanation.

	‶	‵	°	′	″		‵	°	′
Addends	+13	26	59	30	45		+4	+10	—40
	—2	5	17	15	30		+3	—20	+10
Totals	+11	21	42	15	15		+7	—10	—30

When the Signs are intermixed and Units carried to the next.

If where the Signs are intermingled in adding the Numbers of one Denomination together there amount to more than 60, for every of which an Unit be carried over to the next Left Hand Denomination, if the Sign there be changed, then ſubſtract an Unit for every 60 ſo carried, and ſet down the Total of the reſt.

Examples.

As to 3′—50″, add 2′—40″, or —3′+50″ to —2′+40″; in both caſes the 50″ and 40″ make 1′ and 30″, in the firſt inſtance —, in the ſecond +, which 1′ carried over to the Left Hand ′, being of a contrary Sign, I therefore ſubſtract an Unit from the Total 5′, and ſubſcribe the remaining 4′, as followeth;

	′	″		′	″
Addends	+3	—50		—3	+50
	+2	—40		—2	+40
Totals	+4	—30		—4	+30

Proof of Aſtronomical Addition.

Aſtronomical Addition hath the ſame benefit of being proved by Aſtronomical Subſtraction, as other Additions by their reſpective Subſtractions. And as Decimals may be alſo proved by reducing the Numbers into Geodæticals, and comparing the Totals of both Additions together, as equal in value, when the Operations are right after the manner uſed in Decimals.

As to inſtance in the 2 laſt Examples, thus;

′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
+3 —50	+2 —40	+4 —30	—3 +50	—2 +40	—4 +30
60	60	60	60	60	60
+180	+120	+240	—180	—120	—240
— 50	— 40	— 30	+ 50	+ 40	+ 30
+130	+ 80		—130	— 80	
Totals +210		Equal +210	Totals —210		Equal —210

CHAP. IV.

Subſtraction of Aſtronomicals.

Aſtronomicals *ſubſtracted. Simple.*

SUbſtraction of Aſtronomicals is either Simple or Compound.

Simple Subſtraction differeth nothing from Geodætical Subſtraction; for you begin at the Right Hand, and withdraw the under Number from the uppermoſt of like Denomination, and ſubſcribe the Remain: And if the Number beneath be the greateſt then borrow 60, and ſuppoſing the ſame to be added to the upper, make Subſtraction from the Total, and for every 60 borrowed pay an Unit in the next Left Hand Denomination; except where the Subtrahend is the greateſt, and there, as in Decimals, the difference ſhall be taken with the contrary Signs; or elſe proceeding as before till the Left Hand Denomination, and the Sign there changed to the difference, the Signs of all the other Remains ſhall be as the given Numbers.

Example.

As to ſubſtract 13‶, 26‵, 59°, 30′, 45″, from 15‶, 32‵, 16°, 46′, 15″, the Remain ſhall be 2‶, 5‵, 17°, 15′, 30″, which needs no explanation.

	‶	‵	°	′	″
Greater Number	15	32	16	46	15
Subtrahend	13	26	59	30	45
Remain	2	5	17	15	30

But if 15‶, 32‵, 16°, 46′, 15″, were to be ſubſtracted from 13‶, 26‵, 59°, 30′, 45″; here becauſe the Subtrahend is the greateſt Number in the ′, I borrow 60, which I pay again, by counting 16° one more than it is, or 59° an Unit leſs than it is. And in the ‵ I borrow 60 again, for which I count the 15‶ to be 16‶, or the 13‶ at top but 12‶; but now becauſe I cannot take 16‶ from 13‶, or 15‶ from 12‶, but ſhall want 3‶, I ſet down the difference 3‶ with the contrary Sign, as at *A.* Otherwiſe, as in Decimals, I change the Sign to the difference of every Number in the Subtrahend too great to be ſubſtracted from the Numbers that ſtand over him reſpectively at top, as at *B.*

	‶	‵	°	′	″		‶	‵	°	′	″	
	+13	26	59	30	45		+13	26	59	30	45	Upper Numbers
A.	+15	32	16	46	15	*B.*	+15	32	16	46	15	Subtrahends
	— 3	+54	+42	+44	+30		— 2	— 6	+43	—16	+30	Remains

Compound.

Compound Aſtronomical Subſtraction is like Compound Decimal Subſtraction; for where Numbers of unlike Signs are to be ſubſtracted one from the other, the Numbers muſt be added together, and their Totals ſhall be the particular Remains, and their Signs ſhall be the upper Numbers Sign.

Example.

As if the Remain above at *B.* were to be ſubſtracted from the upper Number from which Subſtraction there is made, then muſt the Subtrahend there be the Remain here; as followeth;

	‶	‵	°	′	″	
Upper Number	+13	+26	+59	+30	+45	Upper Number
Subtrahend here	— 2	— 6	+43	—16	+30	Remain above
Remain here	+15	+32	+16	+46	+15	Subtrahend above.

When in contrary Signs an Unit is carried to the next.

If where the Signs are intermixt in adding up the Numbers of contrary Signs, the ſumme exceed 60, then the overplus is to be ſubſcribed under that Denomination where the ſumme ariſeth, and for every 60 an Unit is to be carried over to the next Left Hand Denomination, which Unit ſhall have the Sign of the upper Number. And if the Number of the next Left Hand Denomination annexed to him be of a like Sign, then this Unit or Units ſo carried over ſhall be added thereto, but if of a contrary Sign ſubſtracted therefrom.

Examples.

As if —2′+50″, be ſubſtracted from +3′ —10″, the 50″ and 10″ of contrary Signs added make 60, for which 1, that is —1′, becauſe 10″ the upper Number is — is carried to the +3′, which being contrary is ſubſtracted therefrom, and ſo leaves but +2′ to be added with —2′, which make +4 for the Remain, as at *C*.

But if +2′ —50″ be ſubſtracted from —3′ +10″, the 50″ and 10″ added make 60, as before, but + becauſe 10″ the upper Number is +; for which 60 is +1′ carried to the —3′, and being contrary and ſubſtracted therefrom, leaves but —2′ to be added with +2′, which make the Remain —4′, as at *D*.

Contrarywiſe, if —2′ —50″ be ſubſtracted from +3′ +10″, or +2′ +50″ from —3′ —10″, in the former the Unit carried over is +1′, and in the latter —1′, and to be added accordingly to the 5′ amounting of —2′ and +3′, or +2′ and —3′, making the Remains +6′ in the one, and —6′ in the other, as at *E*. and *F*.

	C.	*D.*	*E.*	*F.*
	′ ″	′ ″	′ ″	′ ″
Upper Numbers	+3 —10	—3 +10	+3 +10	—3 —10
Subtrahends	—2 +50	+2 —50	—2 —50	+2 +50
Remains	+4 —00	—4 +00	+6 +00	—6 —00

When Signs are intermixt, and the Subtrahend greateſt.

If among Numbers of intermixt Signs, ſome of the reſpective Species or Denominations, both in the Subtrahend and Number from which Subſtraction is to be made, be of like Signs, and it happen that the Number in the Subtrahend is the greater, then I am left at liberty, whether for the Remain I will change the Sign to the difference, or elſe as in Simple Subſtraction borrow 60. But if ſo, this muſt be remembred, that the Unit to be paid (for the 60 borrowed) in the next Left Hand Denomination, if the Sign thereof be contrary to the Sign of that Denomination where the 60 was borrowed, this Unit payable muſt be contrary; that is + with —, and — with +; and although in both Remains the Numbers and Signs differ, they agree in value.

Examples.

As to deduct —2′ +50″ from —3′ +10″, there after the former way, the Remain will be —1′ —40″; but after the latter way —2′ +20″; for to take 50″ from 10″, and 60″ borrowed to put thereto, the Remain will be +20″, for which 60, the Unit payable is +1′, and being affirmative therefore leſſens the —2′ in the Subtrahend being of a contrary Sign, and makes it but —1′, which deducted from —3′ leaves —2′, as at *G*.

But to take +2′ —50″ from +3′ —10″, there after the former way, the Remain will be +1′ +40″, but after the latter +2′ —20″, becauſe the 60 borrowed there is negative, and ſo being contrary to the +2′ in the Subtrahend is to be taken therefrom, and the remaining +1′ taken from +3′, leaves for the Remain +2′, as at *H*.

	G.	*H.*	
	′ ″	′ ″	
Upper Numbers, or Numbers from which Subſtraction is made	—3 +10	+3 —10	
Subtrahends	—2 +50	+2 —50	
Remains	—1 —40	+1 +40 by the former way	Equal.
Remains	—2 +20	+2 —20 by the latter way	

Proof of Aſtronomical Subſtraction.

Aſtronomical Subſtraction will be proved by Aſtronomical Addition, as well as other Subſtractions by their reſpective Additions; and together with Decimals will abide the tryal by being turned into Geodæticals.

As in the laſt inſtance at *H.* will be plain.

′ ″	′ ″	′ ″	′ ″
+3 —10	+2 —50	+ 1 +40	+ 2 —20
60	60	60	60
180	120	60	120
— 10	—50	+ 40	— 20
+170 lacking	+70 is by the firſt Remain	+100 by the other	+100

CHAP.

CHAP. V.

Multiplication of Aſtronomicals.

Multiplication of Aſtronomicals is either Simple or Compound.

Aſtronomicals multiplyed. Simple.

Simple Multiplication is like the fourth variety of the fourth Caſe of Geodætical Multiplication; for every Number of the Multiplicand is to be multiplyed by every Number of the Multiplyer. And to know the Denomination of the Products add the *Indices* together, as in Decimals, then collect the ſeveral Products or Multiplees into one Total Product. And in collection, if any file of Multiplees exceed 60, for every 60 carry 1 to the next Left Hand Denomination, and ſubſcribe the overplus.

Example in the Moons *Diurnal Motion.*

As ſuppoſe *Luna* in her ſwift Motion run in one Day 14 Degrees, 30′, and I would know according to that Diurnal Motion, how far ſhe will run in 3 Dayes, 6 Hours, and 40′. The 6 Hours, 40′, being reduced into Minutes and Seconds of a Day, according to the laſt kind of Aſtronomical Reduction mentioned in the *Second Chapter* before, make 16′, 40″. Then multiplying, as aforeſaid, Number by Number, the ſeveral Multiplees appear as at *A*, and collecting the ſumme, find 47 Degrees, 31′, 40″, or 1 Common Sign, 17 Degrees, 31 Minutes and 40 Seconds.

		°	′	″	‴	
	Multiplicand	14	30			*Index* ′
	Multiplyer	3	16	40		*Index* ″
A.	Multiplees	42	90			
			224	480		
				560	1200	*Index* ‴
	Total Product	47	31	40		

1200/60 (20″

480
560
4(4
1060/60 (17′

90
224
331/60 (5°

Compound.

Compound Aſtronomical Multiplication is like Compound Decimal Multiplication; for every Number of the Multiplicand is to be multiplyed by every Number of the Multiplyer; and the Numbers of like Signs ſhall produce +, and unlike Signs —. And in collection of the Multiplees, by taking the + from the —, or — from the +, the Product may be contracted.

Example.

As to multiply 3′—20″, by 2′—30″, the Product ſhall be at large +6″—130‴+600⁗, as at *B*, which may be contracted by taking the 600⁗, that is +10‴ out of the —130‴, the Remain —120‴, that is —2″ taken out of +6″, will leave but 4″ at laſt.

		′	″	‴	⁗	
	Multiplicand	+3	—20			*Index* ″
	Multiplyer	+2	—30			*Index* ″
B.	Multiplees		+ 6	— 40		
				— 90	+600	*Index* ⁗
	Total Product		+ 6	—130	+600	
	Product contracted		+ 4″			

+600⁗/60 (10‴

—130‴
+ 10
—120/60 (2″

+6″
—2
+4

Proof of Aſtronomical Multiplication

Aſtronomical Multiplication is to be proved by Aſtronomical Diviſion, as well as other Multiplications by their reſpective Diviſions; and together with Decimals will endure the tryal, if reduced into Geodæticals: As in the laſt Operation thus,

′	″		′	″
+ 3	—20		+ 2	—30
60			60	
+180			+120	
— 20			— 30	
+160″			+ 90″	

160″
90″
14400⁗

14400⁗/60 (240‴/60 (4″

CHAP.

CHAP. VI.

Diviſion of Aſtronomicals.

Aſtronomicals divided. Simple.

DIviſion of Aſtronomicals is either Simple or Compound.

Simple Diviſion is like the ſecond variety of the fourth Caſe of Geodætical Diviſion; for by the Number of the higheſt Denomination of the Diviſor, the greateſt Denomination of the Dividend is to be divided, and thereby an apt Quotient Figure gotten, by which multiply the Diviſor, and ſubſtract the Product from the Dividend, and ſet the Remains at top, which when to be brought to the Right Hand multiply by 60, or ſuppoſe it ſo done, and ſo continue the Diviſion to the end of the work, or till a Quotient be gotten large enough for uſe. And to know the Denomination of the Quotient, ſubſtract the *Indices* as in Decimals.

Simple Diviſion in 3 Caſes.

This kind of Diviſion may be throughly underſtood under the varieties in theſe three following Caſes.

1. Caſe, When both Dividend and Diviſor are ſingle Numbers, though of a different *Index*.

2. Caſe, When the one of them is Single, and the other Plural.

3. Caſe, When both the given Numbers are Plural.

1. Data *ſingle. Dividend greateſt and evenly.* &c. *Example.*

Firſt, Both the given Numbers Single may admit of two varieties; that is, either the Dividend greater than the Diviſor, or leſs.

If the Dividend be greater, and will be evenly divided by the Diviſor, then the Numbers are divided as Integers, and the *Index* found as in Decimals.

As to divide 45″ by 5′, the Quotient will be 9, and ſubſtracting 1 the *Index* of 5, from (2) the *Index* of 45, the Remain (3) is the *Index* of the Quotient 9, as below at *A*.

Dividend greateſt and not evenly, &c.

If the Dividend be greater than the Diviſor, and will not be evenly divided thereby, then after the firſt Quotient Figure gotten by dividing, as above, multiply the Remain by 60, and divide the Product by the Diviſor, and add this Quotient to the former, and ſo proceed to the end of the work, or a Quotient large enough for your occaſion.

Example.

As to divide 45″ by 12′, the firſt Quotient Figure gotten will be 3‴, the 9 which is left remaining multiplyed by 60 produceth 540, which divided by 12, giveth 45⁗, as below at *B*.

Diviſor greateſt.

The other variety of this Caſe is when the Diviſor is greater than the Dividend, and then multiply the Dividend by 60, and divide the Product by the Diviſor; and if any thing remain, proceed as laſt abovementioned.

As to divide 9 Degrees by 10 Degrees. 9 Degrees multiplyed by 60 produce 540′, which divided by 10 give in the Quotient 54′, as at *C*.

$$\begin{array}{c} \textit{A.} \\ \overset{\prime}{5}\Big)\,\overset{\prime\prime}{\cancel{45}}\,\Big(\overset{\prime\prime\prime}{9} \\ (2) \\ \underline{1} \\ \textit{Index}\ (3) \end{array}
\qquad
\begin{array}{c} \textit{B.} \\ \overline{9\qquad} \\ \overset{\prime}{12}\Big)\,\overset{\prime\prime}{\cancel{45}}\ \ \overset{\prime\prime\prime}{00}\,\Big(\overset{\prime\prime\prime}{3}\ \ \overset{\prime\prime\prime\prime}{45} \\ (2)\ \ (3) \\ \underline{1}\ \ \ \underline{1} \\ \textit{Indices}\ (3)\ \ (4) \end{array}
\qquad
\begin{array}{c} 9'' \\ \underline{60'} \\ \overset{\prime}{12}\Big)\,\overset{\prime\prime\prime}{\cancel{540}}\,\Big(\overset{\prime\prime\prime\prime}{45} \end{array}
\qquad
\begin{array}{c} \textit{C.} \\ \overset{\circ}{10}\Big)\,\overset{\prime}{\cancel{540}}\,\Big(\overset{\prime}{54} \\ (1) \\ \underline{0} \\ \textit{Index}\ (1) \end{array}$$

2. *One Single and the other Plural. Diviſor Single.*

2. The one of the given Numbers Single, and the other Plural may alſo admit of a double variety, that is, either the Diviſor Single, or the Dividend.

If the Diviſor be Single, then thereby divide every Number of the Dividend, and carry over the Product of the Remains, if any be, reduced by 60, to the next Right Denomination, as above at *B*.

Example in the Hourly Motion of the Moon.

As ſuppoſe in 1 Day Natural, or 24 Hours, ☽ in her ſwift Motion runneth 14 Degrees, 58′, 20″, and I would know her Hourly Motion; becauſe 14 is leſs than 24, I multiply it by 60, and thereto add the 58′, and the Total 898′ divided by 24 giveth in the Quotient 37′, and the 10′ remaining multiplyed by 60, and carried to the 20″, make 620″, which divided by 24 gives 25″ in the Quotient, and there remaineth 20, which multiplyed as before, and the Product divided, addeth to the Quotient 50‴, and the whole Quotient is 37′, 25″, 50‴, for the *Moons* Hourly Motion at the courſe aforeſaid.

```
         Diviſor            Dividend                  Quotient
            24°) 14°   58'   20''   00''' (00°  37'  25''  50'''
                             24
         14°                 10'                 20''
         60'                 60'                 60'
        ----                ----                ------
        840'                600''        24°) 1200''' (50'''
         58                  20                  00
  24°) 898 (37'      24°) 620 (25''
        178                 140
         10                  20
```

Dividend Single. If the Dividend be ſingle, as in the other varieties of this Caſe, then place Cyphers to the Right Hand as far as ſhall be needful, and then like the work of the former Examples, inquire with the firſt Figures of the Diviſor for a Quotient Figure out of the Left Hand Numbers of the Dividend, and thereby multiply all the Diviſor, and ſubſtract the Total of theſe Products from the Dividend, leaving the Remains at top, and then removing the Diviſor inquire for another Quotient Figure, and ſo repeat this work till the Diviſion be ended, or a Quotient large enough obtained.

Example in the Revolution of Lunæ. As ſuppoſe the mean Motion of the *Moon* from the *Sun* daily be 13 Degrees, 10', 35'', and I would know when, according to that courſe, ſhe will make her Revolution, or come to that Point of the *Zodiack* where ſhe made her laſt ☌ with ☉. Then I divide 360 Degrees, which make the whole Circle, having adjoyned a convenient number of Cyphers, by 13°, 10', 35''; and finding at firſt 27 times 13 may be had out of 360, I multiply the Diviſor by 27, and the Products 355°, 45', 45'', ſubſtracting leave remaining 4°, 14', 15''; and then removing the Diviſor, 13 may be had 19 times out of 4°, 14', or 254', the Diviſor then multiplyed by 19, the Products to be ſubſtracted will be 4°, 10', 21'', 5'''. And ſo in like manner proceeding to Thirds, which is far enough, the Revolution is 27 Dayes, 19', 17'', 45''' of a Day, and reduced into Hours is 27 Dayes, 7 Hours, 43', 6'' of an Hour. And if occaſion were, the Diviſion might be continued, becauſe there remaineth 2''', 08'''', 45^{v}.

```
                                         | 2    8    45
Index                           9    55    5
(5)                       3    53    55
(2)              4       14    15
---
(3)  13Deg.  10'  35'') 360°  00'  00''  00'''  00''''  00v (27Dayes  19'  17''  45'''  &c.
                        355   45   45
                          4   10   21    5
                               3   43   59    55
                                    9   52    56    15
```

3. *Data Plural.* When both Dividend and Diviſor are Plural, the Operation is like the laſt aforegoing; for the Cyphers adjoyned repreſent the Dividend there to be of Plural Denominations.

Example in ☌ *of the* Moon *with the* Bulls Eye. As in Caſe ☽ want 2 Degrees, 12', 40'', 13''', 20'''', of *Aldebaran*, or ſome other fixed *Star*, and her Hourly Motion be 37', 40'', I would know when ſhe will be in Conjunction with the ſame *Star*: Then inquiring with 37', out of 2°, 12', that is reduced 132', I find 3 may be taken in the Quotient, and multiplying the Diviſor thereby ſubſtract the Total Product, and ſo continue the Diviſion till I get 3 Hours, 31', 20'', and nothing left remaining.

```
Index                        12   33
(4)                    19    40
(2)
        37'  40'') 2°  12'   40''   13'''   20'''' (3Hours  31'  20''
---
(2)                1   53    00
                       19    27     40
                             12     33      20
```

Compound. Compound Aſtronomical Diviſion is like Compound Decimal Diviſion ; for Number is to be divided by Number. The *Index* of the Quotienary Numbers are got as above; and the Signs, as in Decimals, that is + with like Signs, and — with unlike.

Example. As if 6'' —130''' +600'''', the Product of the Compound Multiplication in the laſt *Chapter*, were to be divided by 2' —30'', there diviſing with 2' out of 6'', I get 3' for the Quotient with the Sign +, becauſe 2 and 6 were both +, by which 3', multiplying the Diviſor, the Product to be ſubſtracted is 6'' —90''', which ſubſtracted leaves —40''' +600''''. Then diviſing again by +2' out of —40''', I get 20'' for the Quotient with the Sign —, becauſe 2' was + but 40''' —, and the Diviſor multiplyed by this 20'', makes the Product equal to the Remain before, as is here to be ſeen.

$$\begin{array}{llllll}
 & & & -\cancel{4}\cancel{0} & & \\
2'\ -30'' & \big) & +\cancel{6}'' & -\cancel{1}\cancel{3}\cancel{0}''' & +\cancel{6}\cancel{0}\cancel{0}'''' & \big(\ 3'\ -20'' \\
3\ -20 & & +\cancel{6} & -\cancel{9}\cancel{0} & & \\
\hline
6\ -90 & & & -\cancel{4}\cancel{0} & +\cancel{6}\cancel{0}\cancel{0} & \\
-40\ +600 & & & & &
\end{array}$$

Proof of Aſtronomical Diviſion. Aſtronomical Diviſion may be proved by Aſtronomical Multiplication, like other Diviſions by their reſpective Multiplications ; and like Decimals may be turned into Geodæticals, and tryed thereby.

As in the laſt Example thus ;

$$\begin{array}{r}
+\overset{''}{6} - \overset{'''}{130} + \overset{''''}{600} \\
60 \\
\hline
+360''' \\
-130 \\
\hline
+230 \\
60 \\
\hline
+13800'''' \\
+\ \ 600 \\
\hline
+14400 \\
\hline
\end{array}
\qquad
\begin{array}{c}
+\overset{'}{2} - \overset{''}{30} \\
60 \\
\hline
+120'' \\
-\ 30 \\
\hline
+\ 90 \\
\hline
\end{array}
\qquad
\begin{array}{c}
+\overset{'}{3} - \overset{''}{20} \\
60 \\
\hline
+180 \\
-\ 20 \\
\hline
+160 \\
\hline
\end{array}$$

$$\begin{array}{c}
\cancel{5} \\
\cancel{1}\cancel{4}\cancel{4}\cancel{0}\cancel{0} \\
\hline
\cancel{9}\quad\cancel{0}
\end{array}\Bigg(
\begin{array}{c}
4'' \\
\cancel{1}\cancel{6}|0 \\
\hline
6\ 0
\end{array}\Bigg(\ 2'
\qquad\qquad
\begin{array}{c}
4'' \\
\cancel{1}\cancel{6}|0 \\
\hline
\cancel{6}\ 0
\end{array}\Bigg(\ 2'$$

C H A P. VII.

Figuration of Aſtronomicals.

Figurate Aſtronomicals produced. TO produce Figurate Aſtronomicals, is no other than to multiply any Aſtronomical Simple or Compound into it ſelf, for the Square and the Square multiplyed by the Root produceth the Cube, *&c.* as other Figural Numbers are produced : And the ſame being done by Multiplication, the *Indices* and Signs of the Product are found as before declared.

Examples.

Examples	°	′	″	‴	⁗	v	vj
Root	1	10	9				
	1	10	9				
	1	10	9				
		10	100	90			
			9	90	81		
Square	1	22	1	1	21		
	1	10	9				
	1	22	1	1	21		
		10	220	10	10	210	
			9	198	9	9	189
Cube	1	35	53	29	43	42	9

Simple

Compound

Root 1′ —10″

1′ —10″

1″ —10‴

—10‴ +100⁗

Square 1″ —20‴ +100⁗

1′ —10″

1‴ —20⁗ $+100^{v}$

—10⁗ $+200^{v}$ -1000^{vj}

Cube 1‴ —30⁗ $+300^{v}$ -1000^{vj}

Roots of Simple Aſtronomicals extracted.

To extract the Root of a Simple Aſtronomical, prick the Number, and proceed in the ſame as in Extraction of Roots before taught, only obſerving, as before in Decimals, that when the Right Hand Denomination will not be parted evenly by the *Index* of the Quantity whoſe Root you would extract, you muſt adjoyn one or more Cyphers, that ſo it may be diviſible accordingly, that is by 2 for the Square, 3 for the Cube, 4 for the Squared Square, *&c.*

Example in the Square.

As to extract the Square Root of 1°, 22′, 1″, 1‴, 21⁗, the Numbers pricked will be 21⁗, 1″, 1°, the Greateſt Square in 1° is 1°, and the Root thereof 1°, the double whereof is 2, the Diviſor to 22′, by which 10′ gotten for the next Quotient Figure, and 10 times 2 taken from 22, leaves 2′ behind. Then the Square of 10′ is 100″, that is 1′, 40″, which ſubſtracted in order from the next pricked Number, or added into a Gnomon, with the Multiplication of the Diviſor, and ſubſtracted leaves 21″. Then doubling the Quotient 1°, 10′, the next Diviſor will be 2°, 20′, by which 21″, 1‴, divided, 9″ will be gotten for the Quotient, and the Gnomon to be ſubſtracted cut off all the Remain.

~~21~~

Square) ~~1°~~ ~~22′~~ ~~1″~~ ~~1‴~~ ~~21⁗~~ (1° 10′ 9″ Root

1

Gnomon { 20
1 40

Gnomon { 21
—1 21

Example in the Cube.

Alſo to extract the Cube Root of 1°, 35′, 53″, 29‴, 43⁗, 42^{v}, 9^{vj}, the pricked Numbers will be 9^{vj}, 29‴, 1°, the Greateſt Cube in 1° is 1°, and the Root thereof 1°, the treble whereof, becauſe 1 doth not multiply, is the Diviſor to 35′, and thereby 10′ is gotten for the next Quotient Number, multiplyed by 3 makes 30, which with the Square of 10 increaſed by the triple of 1, and the Cube added into a Total, make the Gnomon 35′, 16″, 40‴; then will the next Diviſor be 4°, 5′, 0″, and the Gnomon to be ſubſtracted cut off all the Remain.

$$\text{Cube}\ \Big)\ \cancel{1}^{\circ}\ \ \cancel{35}'\ \ \overset{\cancel{36}}{\cancel{53}''}\ \ \overset{\cancel{49}}{\cancel{29}'''}\ \ \cancel{43}''''\ \ \cancel{42}^{v}\ \ \cancel{9}^{vi}\ \Big(\ 1^{\circ}\ \ 10'\ \ 9''\ \text{Root}$$

1

Gnomon { 30′
5 00
16 40

Gnomon { 36 45 00
4 43 30
12 9

Roots of Compound Aſtronomicals *extracted.*

To extract the Root of a Compound Aſtronomical, prick the Number as before according to the Quantity, and out of the pricked Numbers to the Left Hand, having taken the greateſt Figurate Number, whoſe Root you would extract, and placed the Root in the Quotient, you get the Diviſor as before, and differ in nothing from the Extractions of the Simple, ſave as Compound Multiplication or Diviſion differs from the Simple.

Example in the

As to extract the Square Root of $1'' - 20''' + 100''''$; and the Cube Root of $1''' - 30'''' + 300^{v} - 1000^{vi}$, the Numbers pricked in the firſt are $100''''$ and $1''$, in the other 1000^{vi} and $1'''$, in both the Greateſt Square and Cube of 1 is but 1, and the Root of both is but 1, which in the Square will be ′ the half of ″, and in the Cube ′ the third part of ‴, this 1 doubled is the Diviſor to $20'''$ in the Square, and tripled, becauſe 1 doth not multiply, makes 3 the Diviſor to $30''''$ in the Cube, whereby $10''$ is gotten for the Quotient to both, and ſo proceeding the reſt of the work is plain in the Operations following.

Square.

$$\text{Square}\ \Big)\ \cancel{1}''\ -\cancel{20}'''\ +\cancel{100}''''\ \Big(\ 1' - 10''\ \text{Root}$$

1
—20
+100

+ 2 Diviſor		— 10
—10		— 10
—20		+100

Cube.

$$\text{Cube}\ \Big)\ 1''' - 30'''' + 300^{v} - 1000^{vi}\ \Big(\ 1' - 10''\ \text{Root}$$

1
—30
+300
—1000

	— 10	— 10
+ 3 Diviſor	— 10	— 10
—10	+100	+100
—30	+ 3	— 10
	+300	—1000

Theſe Numbers above Exemplary being of ſuch Denominations as will be equally divided by the *Indices* of their Quantities need no Cyphers to be adjoyned, ſuch as do having no other difference in their Extraction than to continue the Extraction to the end of the adjoyned Cyphers, need not be made exemplary here.

Proof of Aſtronomical Figuration.

Beſides the Proof of Production by Extraction, and Extraction by Production, as in other Numbers; if the Aſtronomicals be reduced into Geodæticals of the Loweſt Denomination, and the Compound turned into Simple, and their Figurations compared with the Figurations of the Numbers ſo reduced as Integers, the works will equally agree.

Examples in the Numbers above-mentioned in this *Chapter.*

Simple.

Root $\overset{0}{1}\ \overset{\prime}{10}\ \overset{\prime\prime}{9}$
60
70′
60
Reduced 4209″
4209
37881
84180
16836
Square 17715681⁗
4209″
159441129
354313620
70862724
Cube 74565301329 vj

Square $\overset{0}{1}\ \overset{\prime}{22}\ \overset{\prime\prime}{1}\ \overset{\prime\prime\prime}{1}\ \overset{\prime\prime\prime\prime}{21}$
60
82′
60
4921″
60
295261‴
60
17715681⁗

Cube $\overset{0}{1}\ \overset{\prime}{35}\ \overset{\prime\prime}{53}\ \overset{\prime\prime\prime}{29}\ \overset{\prime\prime\prime\prime}{43}\ \overset{v}{42}\ \overset{vj}{9}$
60
95′
60
5753″
60
345209‴
60
20712583⁗
60
1242755022 v
60
74565301329 vj

1 7
. . . . Root
Square 17715681 | 4209
16 : : :
Gnomon 164 : :
Gnomon 000 :
Gnomon 75681

10477
. . . . Root
Cube 74565301329 | 4209
64 : : :
Gnomon 10088 : :
Gnomon 0000 :
Gnomon 477301329

53 3 | 2 ⁗ 52 ‴ 1 ″ | 2 ′ 0
1771568 | 1 | 29526 | 1 | 492 | 1 8 | 2 | 1 Square
6 0 6 0 6 0 6 0

12143 1 vj 13 2 | 4 v 23 | 4 ⁗ 3 | 2 ‴ 3 ″ | 3 ′ 0
74565301332 | 9 | 124275502 | 2 | 2071258 | 3 | 34520 | 9 | 57 | 53 | 9 | 5 | 1 Cube
6 0 6 0 6 0 6 0 6 0 6 0

Compound.

Root $\overset{\prime}{1} - \overset{\prime\prime}{10}$
60
60″
—10
Reduced +50″
50″
Square 2500⁗
50″
Cube 125000 vj

Square $\overset{\prime\prime}{1} - \overset{\prime\prime\prime}{20} + \overset{\prime\prime\prime\prime}{100}$
60
60‴
—20
+40‴
60
2400⁗
+ 100
2500⁗

Cube $\overset{\prime\prime\prime}{1} - \overset{\prime\prime\prime\prime}{30} + \overset{v}{300} - \overset{vj}{1000}$
60
60⁗
—30
+30⁗
60
1800 v
+ 300
2100 v
60
126000 vj
— 1000
125000 vj

Square $\overset{\cdot\ \cdot}{2500} \mid \underline{50}$ Root   Cube $\overset{\cdot\ \ \cdot}{\cancel{1}\cancel{2}\cancel{5}000} \mid \underline{50}$ Root

$\cancel{2}\cancel{5}$ $\cancel{1}\cancel{2}\cancel{5}$

00 000

$\overset{\cancel{1}|\underline{4}}{\cancel{2}\cancel{5}0}|\overset{''''}{0}\ |\overset{'''}{41}$ Square $= \overset{''}{1} - \overset{'''}{20} + \overset{''''}{100} = \overset{''}{1} - \overset{'''}{19} + \overset{''''}{40}$

$\cancel{6}\ \ \cancel{0}$

$\overset{\cancel{2}|\underline{2}}{\cancel{1}\cancel{2}\cancel{5}\cancel{0}\cancel{0}}|\overset{vj}{0}\ |\overset{\cancel{2}|\underline{4}}{\cancel{2}\cancel{0}8}|\overset{v}{3}\ |\overset{''''}{34}$ Cube $= \overset{'''}{1} - \overset{''''}{30} + \overset{v}{300} - \overset{vj}{1000} = \overset{'''}{1} - \overset{''''}{26} + \overset{v}{44} - \overset{vj}{40}$

$\cancel{6}\ \ \cancel{0}\ \ \cancel{6}\ \ \cancel{0}$

CHAP. VIII.

Of the Sexagenary Table.

Sexagenary Table, *its uſe, and why ſo called.*

ENough hath been ſaid of the Simple Elements of *Aſtronomicals*, to underſtand them; yet it is convenient to ſay ſomething of the *Sexagenary Table*, becauſe in Multiplication, Diviſion and Extraction of Roots, before ſhewed, adding up the ſeveral Multiplees in Multiplication, and the Remains in Diviſion and Extraction oftentimes need Reduction by 60, whereby the work is more tedious; therefore is the *Table* made to contain all the Products of any two Numbers under 60, multiplyed together and reduced into the next Denomination, where the Products will bear the ſame, and from thence called *The Sexagenary Table*.

Triangular Form ſufficient.

The *Table* conſiſts of a Quadrangular Form, yet if it were Triangular, as ſome make it, were ſufficient: As may be ſeen by the upper part thereof above the Black Scale; for that 9 times 10, and 10 times 9, make the Product alike, *viz.* both 90.

Table *explained.*

The *Table* on each ſide hath 60 Columns, the outermoſt on the Left Hand, and the other on the Head, ſignifie ſometimes Multiplicands and Multiplyers, ſometimes Quotients and Roots. The other Columns ſerve to find out the Products of any two Numbers under 60, being multiplyed one by another, and ſignifie Dividends and Square Numbers, as occaſion requireth, among which the Square Numbers are eaſily diſcernable, being only thoſe that ſtand next above the Black Scale.

Thoſe Angles that have 2 Numbers in them, imply that next the Right Hand to be the Number of the loweſt Denomination the Product will afford, and the Number next the Left Hand one place higher.

As if I enter with 10'' at the Left Hand, and 20'' at the Head, the Common Angle is 3, 20, which 20 ſhall be '''', and the 3 ſhall be one place higher, that is '''.

And if I had entred with 10'' and 20''', then ſhould the Angular 3, 20, be 3'''', 20v, the like is to be underſtood of all others in finding the true *Index*.

The

Place the *Sexagenary Table* here.

The Uſe of the Table.

Uſe of the Table *in* Multiplication.

In Multiplication, enter the *Table* with the two Factors or multiplying Numbers, the one at the Head, and the other at the Left Side of the *Table*, and the Numbers or Number in the Common Angle is the deſired Product. And if there be 2 Numbers, ſubſcribe the Right Hand Number, and carry the Left Hand Number in mind to be added to the next Product, and ſo proceed till the Multiplicand be run through; and laſt of all ſet down the Left Hand Number, if there be any, ordering the *Indices* as above was directed, and adding together all the Multiplees, you have the Total Product.

In Diviſion.

In Diviſion enter with the Diviſor at the Head, or the Left Side of the *Table*, and run along with your Eye in the ſame Column, till you eſpy a Number equal, or near to your Numbers in the Dividend ſtanding over the firſt Left Hand Figures of your Diviſor entred with: If they be not found exactly, take the leſſer, alwayes having reſpect to the greatneſs or ſmallneſs of the other Figures in the Diviſor, and the other outermoſt Number anſwering to the Common Angle, ſhall be the Quotient Figure. For if the Diviſor be entred with at the Side, the Quotient is found at the Head, and if at the Head, the contrary. The Quotient Figure found, the Diviſor is to be multiplyed thereby, and the Product ſubſtracted from the Dividend, as before; and then inquire for another Quotient Figure, and ſo proceed to the end of the work.

Example in Multiplication

As if 06, 05, 20, be multiplyed by 18‵, 15°, 10′. I enter the *Table* with 18 and 20, and find in the Common Angle 6, 0; ſubſcribing the 0, I carry the 6 in mind. Then entring the *Table* with 18 and 5, I find 1, 30; to which 30 I add the 6 in mind, and ſubſcribe the amounting 36, and bear the 1 in mind. Again, I enter the *Table* with 18 and 6, and find 1, 48; to which 1 added is 1, 49, to be ſet down; and ſo I have done with the firſt Multiplyer. And thus proceeding with the reſt, and adding all the Multiplees together, the Product at laſt will be 1‷, 51‶, 8‵, 20°, 53′, 20″, as at *A*.

Example in Diviſion.

And if I divide this Product by one of the Factors, as ſuppoſe by 18‵, 15°, 10′, firſt I enter the *Table* with 18, and running along in the Column, I look for the Dividend Numbers 1, 51, which I find not in the Column, but the next leſſer Number is 1, 48, and over againſt the ſame, in the other outermoſt Column, is 6, which is to be ſet in the Quotient, and multiplying all the Diviſor thereby, the Product 1, 49, 31, 0, I ſubſtract and leave remaining 1, 37 for the next inquiry. Then againſt the diviſing 18, I look in the *Table* for 1, 37, and find the next leſſer Number to be 1, 30, and in the other outermoſt Column, anſwering to 18, is found 5, which ſet in the Quotient, and the Diviſor multiplyed thereby, produceth 1, 31, 15, 50′; this ſubſtracted leaveth 6, 5, for the next inquiry, which in the Column againſt 18 is not found in the *Table*, but 6, 0, the next leſſer, which hath 20 for the Quotient anſwering thereto, the Product of which multiplyed into the Diviſor, and ſubſtracted leaveth 0 behind, as at *B*.

	Multiplicand		6‵	5°	20′			(1)
	Multiplyer		18‵	15°	10′			(1)
		1	49	36	00			
A	Multiplees		1	31	20	00		
				1	00	53	20	(2)
	Total Product	1‷	51‶	08‵	20°	53′	20″	

				6				
	Diviſor		~~1~~	~~37~~	~~5~~	~~3~~		Quotient
	18‵ 15° 10′)	~~1‷~~	~~51‶~~	~~08‵~~	~~20°~~	~~53′~~	~~20″~~	(6‵ 5° 20′
B		1	49	31	00			
			1	31	15	50		(2)
				6	5	3	20	(1)
		1	51	08	20	53	20	(1)

Table *uſeful in Extraction of the* Square Root. *What to be done for Higher Roots.*

In Extraction of Roots, foraſmuch as Diviſion and Multiplication is uſed therein, the *Table* is not a little helpful to Aſtronomical Extraction: Only if a Root higher than the Square be extracted, thoſe higher Quantities ſubſtracted at firſt out of the Numbers belonging to the Left Hand Prick, muſt be gotten by ordinary Multiplication, no other Figural Numbers but Squares being in the *Table*.

Partis Secundæ Libri Tertii

FINIS.

THE

THE THIRD PART OF THE THIRD BOOK.

CHAP. I.

Of LOGARITHMES.

I Have with all poſſible brevity tranſited *Decimals* and *Aſtronomicals*, and ſhall now apply my ſelf to overlook *Logarithmes*, martialled in the beginning of this *Book*, in the third rank of Numbers ſpecially Contract.

Logarithmes *next ranked in Numbers ſpecially Contract.*

Logarithmes, are Numbers artificially prepared for other Numbers, firſt invented by the Honourable *John Nepeir* Baron of *Marchiſton* in *Scotland*, and afterward transformed, and their foundation and uſe illuſtrated by the truly Ingenuous Mr. *Henry Briggs*, (from whoſe Labours I acquainted my ſelf with them) but need ſay the leſs of them here, becauſe their excellent uſe in the *Mathematicks* hath made them familiar to many; for by them, and with much expedition, all troubleſome Multiplications and Diviſions in *Arithmetick* are avoided, and performed only by Addition inſtead of Multiplication, and by Subſtraction inſtead of Diviſion. The Curious and Laborious Extractions of Roots, are alſo performed with great Eaſe, as hereafter ſhall be ſhewed. Proportions Disjunct and Continued, Double, Triple, and what elſe, are thereby made more facil than otherwiſe can be poſſible. All Triangles, of what kind ſoever, with facility reſolved. Alſo not only in *Arithmetick*, but generally in *Geometry*, *Geography*, *Navigation*, *Aſtronomy*, *&c.* their uſe is ſuch, as a Volumn of it ſelf is little enough to give Example.

Logarithmes *by whom firſt invented and illuſtrated.*

How excellently uſeful.

They are called *Logarithmes* from the *Greek* word λόγος, which ſignifieth Reaſon or Proportion, and ἀριθμὸς, another *Greek* word ſignifying Numbers. So as the word *Logarithmes* implyeth Rational or Proportional Numbers.

Whence the word, and what implyed thereby.

They have the ſame Foundation with Decimal and Aſtronomical *Arithmetick*, as the *Table* in the *Firſt Chapter* of *Decimals* well underſtood will clearly teſtifie, becauſe as was there hinted, the uppermoſt Numbers are in Arithmetical Proportion, and are *Indices* or *Logarithmes*, and the lower in Geometrical Progreſſion or Proportion, and do perform by Multiplication and Diviſion, what the other by Addition and Subſtraction. For if $3+2=5$. *Ergo* $1000 \times 100 = 100000$; 3 being the *Index* of 1000, and 2 of 100, the Total of both 5, ſhall be the *Index* of the Product of 100 multiplyed into 1000, that is 100000; and ſo of others.

Their foundation one with the Decimals, &c.

The *Indices* of Numbers being thus uſeful, gave riſe to the Invention of Logarithmes, which are indeed nothing elſe but the *Indices* or Numbers of places in the Higher Powers of Figural Numbers. For if all Integers be advanced into one and the ſame Quantity of a very High Power, as ſuppoſe to the Ten Thouſandth Million, then the number of places contained in thoſe Figural Quantities ſhall be the ſeveral Logarithmes for thoſe Integers ſo advanced.

Their riſe from Decimal Indices.
Logarithmes *what they are, and how made.*

To verifie this, you may make tryal with any Number, as ſuppoſe 100, the *Index* of which is 2: Let then 100 be multiplyed Figuratively to the 10th Power, the Figural Number thereof will be 100000000000000000000, that is an Unit and 20 Cyphers. Then ſhall the 100th Power be 1 and 200 Cyphers; the 1000th Power 1, and 2000 Cyphers: And ſo conſequently the 10000000000th Power 1, and 20000000000 Cyphers. The Number of places then being ſo many lacking 1, becauſe the *Index* of

To make the Log. *of* 100.

the Units place is 0, that Number of Cyphers ſhall be the Logarithme of 100, *viz.* 20000000000. All this is plain by the following Operation.

Example.

		Number of Quantities.	
Root	100	1	2 Decimal Indices
Square	10000	2	4
Cube	1000000	3	6
&c.	100000000	4	8
	10000000000	5	10
	1000000000000	6	12
	100000000000000	7	14
	10000000000000000	8	16
	1000000000000000000	9	18
	100000000000000000000	10	20
I.	40	20	40
I.	80	40	80
I.	120	60	120
I.	160	80	160
I.	200	100	200
I.	2000	1000	2000
I.	20000	10000	20000
I.	200000	100000	200000
I.	2000000	1000000	2000000
I.	20000000	10000000	20000000
I.	200000000	100000000	200000000
I.	2000000000	1000000000	2000000000
I.	20000000000	10000000000	20000000000
	Figural Quantities.	Figural Indices.	Logarithmes.

What being certain is omitted in Logarithmes.

In the uſe of Logarithmes both the Figural Numbers themſelves, and the Figural Indices which declare the Number of their Quantities or Figurate Multiplications, are omitted, and only the Decimal Indices or number of places in ſuch Figural Numbers uſed. And becauſe theſe Indices uſed for Logarithmes have reference to ſome certain Quantity or Power to which all abſolute Integers are to be contracted, therefore they are rightly placed among Contract Numbers. And though the Figural Quantities, and their Figural Indices be omitted, yet they are certain, and may certainly be known by the number of places in the Logarithme.

Explained.

As becauſe the Logarithme of 100 is 20000000000, in which there is 11 places; I know that 100 was multiplyed Figurately 10 Thouſand Million of times; for that 10 Thouſand of Millions is the 11th place in Numeration. And if I uſe the Logarithme of 100 multiplyed to ſuch an High Quantity, all the other Logarithmes I uſe muſt be equivalent; that is, the other Numbers muſt be multiplyed to the ſame Power or Quantity of 10 Thouſand of Millions, and the number of places, or Decimal Indices, taken for their ſeveral Logarithmes.

Tables of Logarithmes *to be gotten ready for uſe. What called the* Canon of Logarithmes. *Tables of Mr.* Briggs *large. Large* Tables *beſt. The* Table *following tranſcribed from his. The* Table *explained.*

To have Logarithmes to ſeek when they ſhould be uſed, is inconvenient; to make them as above is more tedious than to reſolve the Queſtion otherwiſe without them. To eaſe the Artiſt therefore in his work with Logarithmes, *Tables* are to be prepared, which ſome call, *The Canon of Logarithmes.* Mr. *Briggs* hath fitted Logarithmes for all Whole Numbers from 1 to 100000, which are ſufficient enough to ſerve for any uſe, ſeeing thereby the Logarithme of any Number betwixt 100000 and 10000000000 may be found out. And every Practitioner will find a large *Table* moſt beneficial. But leſt thoſe *Tables* may not be in hand, that the Learner may not be altogether unfurniſhed wherewith to make experiment, and prove the truth of the Exemplary Operations following, I have therefore in the end of this *Chapter* tranſcribed from Mr. *Briggs*, a *Table* of Logarithmes for all Integers, from an Unit to 1000, which will be ſufficient to give Example by.

The *Table* conſiſts of two Columns, in that towards the Left Hand under the Title Numbers, you have the Abſolute Numbers from 1 to 1000, in the Right Hand Column, over againſt the ſeveral Numbers you have their Logarithmes.

The Logarithme of 1 is 0; of 10 is 1, with Cyphers; of 100 is 2, with Cyphers; of 1000 is 3, with Cyphers, and ſo the *Table* may be increaſed; for the Logarithme of 10000 is 4, with Cyphers; of 100000 is 5, with Cyphers, and ſo infinitely.

Characteriſtique *or* Index *of the* Logarithme *what, and how made.*

The Left Hand Figure or Cypher of every Logarithme, may fitly be called the *Characteriſtique* or *Index* of the Logarithme; for it ſhews how many places the Abſolute Number doth conſiſt of, becauſe it is alwayes leſs by an Unit than the places of the Abſolute Number. As the *Index* of all Abſolute Numbers under 10 ſhall be 0, for they have but 1 place, and Unit ſubſtracted from Unit leaves 0. But from 10 to 100, the *Index* ſhall be 1, becauſe the Numbers are but 1 place diſtant from the Unit. And for the like Reaſon Numbers from 100 to 1000, ſhall have 2 for their *Index*; and Numbers that have 4 places, as 1000 hath, and all Numbers from 1000 to 10000 ſhall have 3 for their *Index*, and ſo infinitely; as in the *Tables* in the *Firſt Chapter* of *Decimals* is plainly to be ſeen: Wherefore in ſome *Tables* the *Characteriſtique* being ſo generally known is omitted.

The like is to be obſerved in Decimal Fractions; from 1 to Tenth Parts the *Index* is 0; from Tenth Parts to 100 Parts, it's 1; from Hundred Parts to 1000 Parts, it's 2. And ſo decreaſing infinitely.

Cõma, *of what uſe in* Logarithmes.

The *Characteriſtique* is commonly ſeperated from the reſt of the Logarithme by a Cõma, as alſo the next 5 places from the Remainder by another Cõma, and every 10 Logarithmes in the *Table* from the enſuing by a Line. All which is for no other ſervice, than to help the Eye more readily to diſcern and find them out. Some omit 2 or 3 of the Right Hand Figures, which breeds no material Error.

Logarithme *of a* Fraction *how known from the* Logarithme *of an* Integer.

For diſtinction between the Logarithme of Integers, and the Logarithme of Fractions; it is to be noted, that as the Logarithme of a Proper Fraction is defective or negative, ſo it is to be marked with — the ſign of Subſtraction, either over the Head, or at the Left Hand of the *Index*. And yet the Logarithme of an Improper Fraction, or Mixt Number, that conſiſts of an Whole Number and a Fraction, is not defective, but affirmative or abundant.

Numbers

Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
1	0,00000,00000	51	1,70757,01761	101	2,00432,13738	151	2,17897,69473
2	0,30102,99957	52	1,71600,33436	102	2,00860,01718	152	2,18184,35879
3	0,47712,12547	53	1,72427,58696	103	2,01283,72247	153	2,18469,14308
4	0,60205,99913	54	1,73239,37598	104	2,01703,33393	154	2,18752,07208
5	0,69897,00043	55	1,74036,26895	105	2,02118,92991	155	2,19033,16982
6	0,77815,12504	56	1,74818,80270	106	2,02530,58653	156	2,19312,45984
7	0,84509,80400	57	1,75587,48557	107	2,02938,37777	157	2,19589,96524
8	0,90308,99870	58	1,76342,79936	108	2,03342,37555	158	2,19865,70870
9	0,95424,25094	59	1,77085,20116	109	2,03742,64979	159	2,20139,71243
10	1,00000,00000	60	1,77815,12504	110	2,04139,26852	160	2,20411,99827
11	1,04139,26852	61	1,78532,98350	111	2,04532,29788	161	2,20682,58760
12	1,07918,12460	62	1,79239,16895	112	2,04921,80227	162	2,20951,50145
13	1,11394,33523	63	1,79934,05495	113	2,05307,84435	163	2,21218,76044
14	1,14612,80357	64	1,80617,99740	114	2,05690,48513	164	2,21484,38480
15	1,17609,12591	65	1,81291,33566	115	2,06069,78404	165	2,21748,39442
16	1,20411,99827	66	1,81954,39355	116	2,06445,79892	166	2,22010,80880
17	1,23044,89214	67	1,82607,48027	117	2,06818,58617	167	2,22271,64711
18	1,25527,25051	68	1,83250,89127	118	2,07188,20073	168	2,22530,92817
19	1,27875,36010	69	1,83884,90907	119	2,07554,69614	169	2,22788,67046
20	1,30102,99957	70	1,84509,80400	120	2,07918,12460	170	2,23044,89214
21	1,32221,92947	71	1,85125,83487	121	2,08278,53703	171	2,23299,61104
22	1,34242,26808	72	1,85733,24964	122	2,08635,98307	172	2,23552,84469
23	1,36172,78360	73	1,86332,28601	123	2,08990,51114	173	2,23804,61031
24	1,38021,12417	74	1,86923,17197	124	2,09342,16852	174	2,24054,92483
25	1,39794,00087	75	1,87506,12634	125	2,09691,00130	175	2,24303,80487
26	1,41497,33480	76	1,88081,35923	126	2,10037,05451	176	2,24551,26678
27	1,43136,37642	77	1,88649,07252	127	2,10380,37210	177	2,24797,32664
28	1,44715,80313	78	1,89209,46027	128	2,10720,99696	178	2,25042,00023
29	1,46239,79979	79	1,89762,70913	129	2,11058,97103	179	2,25285,30310
30	1,47712,12547	80	1,90308,99870	130	2,11394,33523	180	2,25527,25051
31	1,49136,16938	81	1,90848,50189	131	2,11727,12957	181	2,25767,85749
32	1,50514,99783	82	1,91381,38524	132	2,12057,39312	182	2,26007,13880
33	1,51851,39399	83	1,91907,80924	133	2,12385,16410	183	2,26245,10897
34	1,53147,89170	84	1,92427,92861	134	2,12710,47984	184	2,26481,78230
35	1,54406,80444	85	1,92941,89257	135	2,13033,37685	185	2,26717,17284
36	1,55630,25008	86	1,93449,84512	136	2,13353,89084	186	2,26951,29442
37	1,56820,17241	87	1,93951,92526	137	2,13672,05672	187	2,27184,16065
38	1,57978,35966	88	1,94448,26722	138	2,13987,90864	188	2,27415,78493
39	1,59106,46070	89	1,94939,00066	139	2,14301,48003	189	2,27646,18042
40	1,60205,99913	90	1,95424,25094	140	2,14612,80357	190	2,27875,36010
41	1,61278,38567	91	1,95904,13923	141	2,14921,91127	191	2,28103,33672
42	1,62324,92904	92	1,96378,78273	142	2,15228,83444	192	2,28330,12287
43	1,63346,84556	93	1,96848,29486	143	2,15533,60375	193	2,28555,73090
44	1,64345,26765	94	1,97312,78536	144	2,15836,24921	194	2,28780,17299
45	1,65321,25138	95	1,97772,36053	145	2,16136,80022	195	2,29003,46114
46	1,66275,58317	96	1,98227,12330	146	2,16435,28558	196	2,29225,60714
47	1,67209,78579	97	1,98677,17343	147	2,16731,73347	197	2,29446,62262
48	1,68124,12374	98	1,99122,60757	148	2,17026,17154	198	2,29666,51903
49	1,69019,60800	99	1,99563,51946	149	2,17318,62684	199	2,29885,30764
50	1,69897,00043	100	2,00000,00000	150	2,17609,12591	200	2,30102,99957

Numb.

Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
201	2,30319,60574	251	2,39967,37215	301	2,47856,64956	351	2,54530,71165
202	2,30535,13694	252	2,40140,05408	302	2,48000,69430	352	2,54654,26635
203	2,30749,60379	253	2,40312,05212	303	2,48144,26285	353	2,54777,47054
204	2,30963,01674	254	2,40483,37166	304	2,48287,35836	354	2,54900,32620
205	2,31175,38611	255	2,40654,01804	305	2,48429,98393	355	2,55022,83531
206	2,31386,72204	256	2,40823,99653	306	2,48572,14265	356	2,55144,99980
207	2,31597,03455	257	2,40993,31233	307	2,48713,83755	357	2,55266,82161
208	2,31806,33350	258	2,41161,97060	308	2,48855,07165	358	2,55388,30266
209	2,32014,62861	259	2,41329,97641	309	2,48995,84794	359	2,55509,44486
210	2,32221,92947	260	2,41497,33480	310	2,49136,16938	360	2,55630,25008
211	2,32428,24553	261	2,41664,05073	311	2,49276,03890	361	2,55750,72019
212	2,32633,58609	262	2,41830,12913	312	2,49415,45940	362	2,55870,85705
213	2,32837,96034	263	2,41995,57485	313	2,49554,43376	363	2,55990,66250
214	2,33041,37733	264	2,42160,39269	314	2,49692,96481	364	2,56110,13836
215	2,33243,84599	265	2,42324,58739	315	2,49831,05538	365	2,56229,28645
216	2,33445,37512	266	2,42488,16366	316	2,49968,70826	366	2,56348,10854
217	2,33645,97338	267	2,42651,12614	317	2,50105,92622	367	2,56466,60643
218	2,33845,64936	268	2,42813,47940	318	2,50242,71200	368	2,56584,78187
219	2,34044,41148	269	2,42975,22800	319	2,50379,06831	369	2,56702,63662
220	2,34242,26808	270	2,43136,37642	320	2,50514,99783	370	2,56820,17241
221	2,34439,22737	271	2,43296,92909	321	2,50650,50324	371	2,56937,39096
222	2,34635,29745	272	2,43456,89040	322	2,50785,58717	372	2,57054,29399
223	2,34830,48630	273	2,43616,26470	323	2,50920,25223	373	2,57170,88318
224	2,35024,80183	274	2,43775,05628	324	2,51054,50102	374	2,57287,16022
225	2,35218,25181	275	2,43933,26938	325	2,51188,33610	375	2,57403,12677
226	2,35410,84391	276	2,44090,90821	326	2,51321,76001	376	2,57518,78449
227	2,35602,48572	277	2,44247,97691	327	2,51454,77527	377	2,57634,13502
228	2,35793,48470	278	2,44404,47959	328	2,51587,38437	378	2,57749,17998
229	2,35983,54823	279	2,44560,42033	329	2,51719,58979	379	2,57863,92100
230	2,36172,78360	280	2,44715,80313	330	2,51851,39399	380	2,57978,35966
231	2,36361,19799	281	2,44870,63199	331	2,51982,79938	381	2,58092,49757
232	2,36548,79849	282	2,45024,91083	332	2,52113,80837	382	2,58206,33629
233	2,36735,59210	283	2,45178,64355	333	2,52244,42335	383	2,58319,87740
234	2,36921,58574	284	2,45331,83400	334	2,52374,64668	384	2,58433,12244
235	2,37106,78623	285	2,45484,48600	335	2,52504,48070	385	2,58546,07295
236	2,37291,20030	286	2,45636,60331	336	2,52633,92774	386	2,58658,73047
237	2,37474,83460	287	2,45788,18967	337	2,52762,99009	387	2,58771,09650
238	2,37657,69571	288	2,45939,24878	338	2,52891,67003	388	2,58883,17256
239	2,37839,79009	289	2,46089,78428	339	2,53019,96982	389	2,58994,96013
240	2,38021,12417	290	2,46239,79979	340	2,53147,89170	390	2,59106,46070
241	2,38201,70426	291	2,46389,29890	341	2,53275,43790	391	2,59217,67574
242	2,38381,53660	292	2,46538,28514	342	2,53402,61061	392	2,59328,60670
243	2,38560,62736	293	2,46686,76204	343	2,53529,41200	393	2,59439,25504
244	2,38738,98263	294	2,46834,73304	344	2,53655,84426	394	2,59549,62218
245	2,38916,60844	295	2,46982,20160	345	2,53781,90951	395	2,59659,70956
246	2,39093,51071	296	2,47129,17111	346	2,53907,60988	396	2,59769,51859
247	2,39269,69533	297	2,47275,64493	347	2,54032,94748	397	2,59879,05068
248	2,39445,16808	298	2,47421,62641	348	2,54157,92439	398	2,59988,30721
249	2,39619,93471	299	2,47567,11883	349	2,54282,54270	399	2,60097,28957
250	2,39794,00087	300	2,47712,12547	350	2,54406,80444	400	2,60205,99913

Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
401	2,60314,43726	451	2,65417,65419	501	2,69983,77259	551	2,74115,15989
402	2,60422,60531	452	2,65513,84348	502	2,70970,37171	552	2,74193,90777
403	2,60530,50461	453	2,65609,82020	503	2,70156,79851	553	2,74272,51313
404	3,60638,13651	454	2,65705,58529	504	2,70243,05364	554	2,74350,97647
405	2,60745,50232	455	2,65801,13967	505	2,70329,13781	555	2,74429,29831
406	2,60852,60336	456	2,65896,48427	506	2,70415,05168	556	2,74507,47916
407	2,60959,44092	457	2,65991,62001	507	2,70500,79593	557	2,74585,51952
408	2,61066,01631	458	2,66086,54780	508	2,70586,37123	558	2,74663,41990
409	2,61172,33080	459	2,66181,26855	509	2,70671,77823	559	2,74741,18079
410	2,61278,38567	460	2,66275,78317	510	2,70757,01761	560	2,74818,80270
411	2,61384,18219	461	2,66370,09254	511	2,70842,09001	561	2,74896,28613
412	2,61489,72160	462	2,66464,19756	512	2,70926,99610	562	2,74973,63156
413	2,61595,00517	463	2,66558,09910	513	2,71011,73651	563	2,75050,83949
414	2,61700,03411	464	2,66651,79806	514	2,71096,31190	564	2,75127,91040
415	2,61804,80967	465	2,66745,29529	515	2,71180,72290	565	2,75204,84478
416	2,61909,33306	466	2,66838,59167	516	2,71264,97016	566	2,75281,64312
417	2,62013,60550	467	2,66931,68806	517	2,71349,05431	567	2,75358,30589
418	2,62117,62818	468	2,67024,58531	518	2,71432,97597	568	2,75434,83357
419	2,62221,40230	469	1,67117,28427	519	2,71516,73578	569	2,75511,22664
420	2,62324,92904	470	2,67209,78579	520	2,71600,33436	570	2,75587,48557
421	2,62428,20958	471	2,67302,09071	521	2,71683,77233	571	2,75663,61082
422	2,62531,24510	472	2,67394,19986	522	2,71767,05030	572	2,75739,60288
423	2,62634,03674	573	2,67486,11407	523	2,71850,16889	573	2,75815,46220
424	2,62736,58566	474	2,67577,83417	524	2,71933,12870	574	2,75891,18924
425	2,62838,89301	475	2,67669,36096	525	2,72015,93034	575	2,75966,78447
426	2,62940,95991	476	2,67760,69527	526	2,72098,57442	576	2,76042,24834
427	2,63042,78750	477	2,67851,83750	527	2,72181,06152	577	2,76117,58132
428	2,63144,37690	478	2,67942,78966	528	2,72263,39225	578	2,76192,78384
429	2,63245,72922	479	2,68033,55134	529	2,72345,56720	579	2,76267,85637
430	2,63346,84556	480	2,68124,12374	530	2,72427,58696	580	2,76342,79936
431	2,63447,72702	481	2,68214,50764	531	2,72509,45211	581	2,76417,61324
432	2,63548,37468	482	2,68304,70382	532	2,72591,16323	582	2,76492,29846
433	2,63648,78964	483	2,68394,71308	533	2,72672,72090	583	2,76566,85548
434	2,63748,97295	484	2,68484,53616	534	2,72754,12570	584	2,76641,28471
435	2,63848,92570	485	2,68574,17386	535	2,72835,37820	585	2,76715,58661
436	2,63948,64893	486	2,68663,62693	536	2,72916,47897	586	2,76789,76160
437	2,64048,14370	487	2,68752,89612	537	2,72997,42857	587	2,76863,81012
438	2,64147,41105	488	2,68841,98220	538	2,73078,22757	588	2,76937,73261
439	2,64246,45202	489	2,68930,88591	539	2,73158,87652	589	2,77011,52948
440	2,64345,26765	490	2,69019,60800	540	2,73239,37598	590	2,77085,20116
441	2,64443,85895	491	2,69108,14921	541	2,73319,72651	591	2,77158,74809
442	2,64542,22693	492	2,69196,51028	542	2,73399,92865	592	2,77232,17067
443	2,64640,37262	493	2,69284,69193	543	2,73479,98296	593	2,77305,46934
444	2,64738,27901	494	2,69372,69489	544	2,73559,88997	594	2,77378,64450
445	2,64836,00110	495	2,69460,51989	545	2,73639,65023	595	2,77451,69657
446	2,64933,48587	496	2,69548,16765	546	2,73719,26427	596	2,77524,62597
447	2,65030,75231	497	2,69635,63887	547	2,73798,73263	597	2,77597,43311
448	2,65127,80140	498	2,69722,93428	548	2,73878,05585	598	2,77670,11840
449	2,65224,63410	499	2,69810,05456	549	2,73957,23445	599	2,77742,68224
450	2,65321,25138	500	2,69897,00043	550	2,74036,26895	600	2,77815,12504

Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
601	2,77887,44720	651	2,81358,09886	701	2,84571,80180	751	2,87563,99370
602	2,77959,64913	652	2,81424,75957	702	2,84633,71121	752	2,87621,78406
603	2,78031,73121	653	2,81491,31813	703	2,84695,53250	753	2,87679,49762
604	2,78103,69386	654	2,81557,77483	704	2,84757,26591	754	2,87737,13459
605	2,78175,53747	655	2,81624,13000	705	2,84818,91170	755	2,87794,69516
606	2,78247,26242	656	2,81690,38394	706	2,84880,47011	756	2,87852,17955
607	2,78318,86911	657	2,81756,53696	707	2,84941,94138	757	2,87909,58795
608	2,78390,35793	658	2,81822,58936	708	2,85003,32577	758	2,87966,92056
609	2,78461,72926	659	2,81888,54146	709	2,85064,62352	759	2,88024,17759
610	2,78532,98350	660	2,81954,39355	710	2,85125,83487	760	2,88081,35923
611	2,78604,12102	661	2,82020,14595	711	2,85186,96007	761	2,88138,46568
612	2,78675,14221	662	2,82085,79894	712	2,85247,99936	762	2,88195,49713
613	2,78746,04745	663	2,82151,35284	713	2,85308,95299	763	2,88252,45380
614	2,78816,83711	664	2,82216,80794	714	2,85369,82118	764	2,88309,33586
615	2,78887,51158	665	2,82282,16453	715	2,85430,60418	765	2,88366,14352
616	2,78958,07122	666	2,82347,42292	716	2,85491,30223	766	2,88422,87696
617	2,79028,51640	667	2,82412,58339	717	2,85551,91557	767	2,88479,53639
618	2,79098,84751	668	2,82477,64625	718	2,85612,44442	768	2,88536,12200
619	2,79169,05490	669	2,82542,61178	719	2,85672,88904	769	2,88592,63398
620	2,79239,16895	670	2,82607,48027	720	2,85733,24964	770	2,88649,07252
621	2,79309,16002	671	2,82672,25202	721	2,85793,52647	771	2,88705,43781
622	2,79379,03847	672	2,82736,92731	722	2,85853,71976	772	2,88761,73003
623	2,79448,80467	673	2,82801,50642	723	2,85913,82973	773	2,88817,94939
624	2,79518,45897	674	2,82865,98965	724	2,85973,85662	774	2,88874,09607
625	2,79588,00173	675	2,82930,37728	725	2,86033,80066	775	2,88930,17025
626	2,79657,43332	676	2,82994,66959	726	2,86093,66207	776	2,88986,17213
627	2,79726,75408	677	2,83058,86687	727	2,86153,44109	777	2,89042,10188
628	2,79795,96437	678	2,83122,96939	728	2,86213,13793	778	2,89097,95970
629	2,79865,06454	679	2,83186,97743	729	2,86272,75283	779	2,89153,74577
630	2,79934,05495	680	2,83250,89127	730	2,86332,28601	780	2,89209,46027
631	2,80002,93592	681	2,83314,71119	731	2,86391,73770	781	2,89265,10339
632	2,80071,70783	682	2,83378,43747	732	2,86451,10811	782	2,89320,67531
633	2,80140,37100	683	2,83442,07037	733	2,86510,39746	783	2,89376,17621
634	2,80208,92579	684	2,83505,61017	734	2,86569,60599	784	2,89431,60627
635	2,80277,37253	685	2,83569,05715	735	2,86628,73391	785	2,89486,96567
636	2,80345,71156	686	2,83632,41157	736	2,86687,78143	786	2,89542,25460
637	2,80413,94323	687	2,83695,67371	737	2,86746,74879	787	2,89597,47324
638	2,80482,06787	688	2,83758,84382	738	2,86805,63618	788	2,89652,62175
639	2,80550,08582	689	2,83821,92219	739	2,86864,44384	789	2,89707,70032
640	2,80617,99740	690	2,83884,90907	740	2,86923,17197	790	2,89762,70913
641	2,80685,80295	691	2,83947,80474	741	2,86981,82080	791	2,89817,64835
642	2,80753,50281	692	2,84010,60945	742	2,87040,39053	792	2,89872,51816
643	2,80821,09729	693	2,84073,32346	743	2,87098,88138	793	2,89927,31873
644	2,80888,58674	694	2,84135,94705	744	2,87157,29355	794	2,89982,05024
645	2,80955,97146	695	2,84198,48046	745	2,87215,62727	795	2,90036,71287
646	2,81023,25180	696	2,84260,92396	746	2,87273,88275	796	2,90091,30677
647	2,81090,42807	697	2,84323,27781	747	2,87332,06018	797	2,90145,83214
648	2,81157,50059	698	2,84385,54226	748	2,87390,15979	798	2,90200,28914
649	2,81224,46968	699	2,84447,71757	749	2,17448,18177	799	2,90254,67793
650	2,81291,33566	700	2,84509,80400	750	2,87506,12634	800	2,90308,99870

Nu.	Logarithmes.	Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
801	2,90363,25161	851	2,92992,95601	901	2,95472,47910	951	2,97818,05169
802	2,90417,43683	852	2,93043,95948	902	2,95520,65375	952	2,97863,69484
803	2,90471,55453	853	2,93094,90312	903	2,95568,77503	953	2,97909,29006
804	2,90525,60487	854	2,93145,78707	904	2,95616,84305	954	2,97954,83747
805	2,90579,58804	855	2,93196,61147	905	2,95664,85792	955	2,98000,33716
806	2,90633,50418	856	2,93247,37647	906	2,95712,81977	956	2,98045,78923
807	2,90687,35347	857	2,93298,08219	907	2,95760,72871	957	2,98091,19378
808	2,90741,13608	858	2,93348,72878	908	2,95808,58485	958	2,98136,55091
809	2,90794,85216	859	2,93399,31638	909	2,95856,38832	959	2,98181,86072
810	2,90848,50189	860	2,93449,84512	910	2,95904,13923	960	2,98227,12330
811	2,90902,08542	861	2,93500,31515	911	2,95951,83770	961	2,98272,33877
812	2,90955,60292	862	2,93550,72658	912	2,95999,48383	962	2,98317,50720
813	2,91009,05456	863	2,93601,07958	913	2,96047,07775	963	2,98362,62871
814	2,91062,44049	864	2,93651,37425	914	2,96094,61957	964	2,98407,70339
815	2,91115,76087	865	2,93701,61075	915	2,96142,10941	965	2,98452,73133
816	2,91169,01588	866	2,93751,78920	916	2,96189,54737	966	2,98497,71264
817	2,91222,20565	867	2,93801,90975	917	2,96236,93357	967	2,98542,64741
818	2,91275,33037	868	2,93851,97252	918	2,96284,26812	968	2,98587,53573
819	2,91328,39018	869	2,93901,97765	919	2,96331,55114	969	2,98632,37771
820	2,91381,38524	870	2,93951,92526	920	2,96378,78273	970	2,98677,17343
821	2,91434,31571	871	2,94001,81550	921	2,96425,96302	971	2,98721,92299
822	2,91487,18175	872	2,94051,64849	922	2,96473,09211	972	2,98766,62649
823	2,91539,98352	873	2,94101,42437	923	2,96520,17010	973	2,98811,28403
824	2,91592,72117	874	2,94151,14326	924	2,96567,19712	974	2,98855,89569
825	2,91645,39485	875	2,94200,80530	925	2,96614,17327	975	2,98900,46157
826	2,91698,00473	876	2,94250,41062	926	2,96661,09867	976	2,98944,98177
827	2,91750,55096	877	2,94299,95934	927	2,96707,97341	977	2,98989,45637
828	2,91803,03368	878	2,94349,45159	928	2,96754,79762	978	2,99033,88548
829	2,91855,45306	879	2,94399,88751	929	2,96801,57140	979	2,99078,26918
830	2,91907,80924	880	2,94448,26722	930	2,96848,29486	980	2,99122,60757
831	2,91960,10238	881	2,94497,59084	931	2,96894,96810	981	2,99166,90074
832	2,92012,33263	882	2,94546,85851	932	2,96941,59124	982	2,99211,14878
833	2,92064,50014	883	2,94596,07036	933	2,96988,16437	983	2,99255,35178
834	2,92116,60506	884	2,94645,22650	934	2,97034,68762	984	2,99299,50984
835	2,92168,64755	885	2,94694,32707	935	2,97081,16109	985	2,99343,62305
836	2,92220,62774	886	2,94743,37219	936	2,97127,58487	986	2,99387,69149
837	2,92272,54580	887	2,94792,36198	937	2,97173,95909	987	2,99431,71527
838	2,92324,40186	888	2,94841,29658	938	2,97220,28384	988	2,99475,69446
839	2,92376,19608	889	2,94890,17610	939	2,97266,55923	989	2,99519,62916
840	2,92427,92861	890	2,94939,00066	940	2,97312,78536	990	2,99563,51946
841	2,92479,59958	891	2,94987,77040	941	2,97358,96234	991	2,99607,36545
842	2,92531,20915	892	2,95036,48544	942	2,97405,09028	992	2,99651,16722
843	2,92582,75746	893	2,95085,14589	943	2,97451,16927	993	2,99694,92485
844	2,92634,24466	894	2,95133,75188	944	2,97497,19943	994	2,99738,63844
845	2,92685,67089	895	2,95182,30353	945	2,97543,18085	995	2,99782,30807
846	2,92737,03630	896	2,95230,80097	946	2,97589,11364	996	2,99825,93384
847	2,92788,34103	897	2,95279,24430	947	2,97634,99790	997	2,99869,51583
848	2,92839,58523	898	2,95327,63367	948	2,97680,83373	998	2,99913,05413
849	2,92890,76902	899	2,95375,96917	949	2,97726,62124	999	2,99956,54882
850	2,92941,89257	900	2,95424,25094	950	2,97772,36053	1000	3,00000,00000

CHAP.

CHAP. II.

Reduction of Logarithmes.

TO operate by Logarithmes, two things are neceſſary.

What neceſſary to work with Logarithmes.

1.

Firſt, That for every Integer from an Unit upward, and for every Fraction from an Unit downward, *ad infinitum*, we know how to fit a Logarithme thereto; only an Unit, which neither multiplyeth nor divideth, needeth no Logarithme.

2.

Reduction *ſometime called* Invention of Logarithmes.

Secondly, That for every Logarithme we be able to find out the Integer, or Fraction, Common or Decimal, that anſwers thereto. Both theſe are to be found under the *Propoſitions* following in this *Chapter* of *Reduction*, called therefore by ſome, *Invention of Logarithmes.*

Prop. 1. *To find a* Logarithme *for a Number under* 1000.

Propoſition I. *To find a Logarithme for any Abſolute Number under* 1000, *expreſſed in the* Table.

Seek in the Left Hand Column of the *Table*, under the Title Numbers, for the Number whoſe Logarithme is deſired; and over againſt the ſame, in the Right Hand Column, you ſhall find the Logarithme anſwering thereto.

Example.

As if the Logarithme of 12 be deſired; over againſt 12 ſtands in the *Table*, 1,07918,12460, which is the Logarithme thereof. So the Logarithme of 340 is found to be 2,53147,89170.

Prop. 2. *To find a* Logarithme *for a Number above* 1000.

Prop. II. *To find a Logarithme for any Abſolute Number above* 1000.

Where the Tables of Logarithmes are large enough, the Logarithme is to be found, as the Logarithme of any Number under 1000 is to be found in the *Table* foregoing.

But where the Tables are too ſhort, proceed thus;

1. According to the firſt Propoſition find the firſt 3, or more Left Hand Figures of your given Number, as far as your Tables will ſerve.

2. Inſtead of the *Index* of the Logarithme found, place another which ſhall fit the Number given.

3. Take the difference between the Logarithme found, and the next enſuing, and take the reſt of the Figures that remain to the Number given, after the 3 or more Left Hand Figures be cut off.

4. By theſe get a proportional part thus: As an Unit and ſo many Cyphers, as there be Figures remaining, to the ſame Figures, ſo ſhall the difference between the Logarithme found, and that which follows, be to another Number, which found is to be added to the Logarithme before found; and the ſumme you may take for the queſited Logarithme. And though there will be ſome little alteration from the true Logarithme, yet the difference being inconſiderable makes the Error immaterial.

Example.

Example. If the Logarithme of 99945, be ſought, then by the *Table* the Logarithme of 999, the firſt 3 Left Hand Figures is ſeen to be 2,99956,54882. The *Index* of 99945, muſt be 4, therefore I alter the *Index*, and make the Logarithme 4,99956,54882. The difference between the Logarithme of 999, and the Logarithme of 1000, the next is 43,45118. The Figures that remain beſides 999 are 45. Then the Analogy is thus: As 100. 45 :: 4345118. 1955303; for by multiplying 45 into 4345118, and dividing the Product by 100, that Number of 1955303 is gotten, which added to the Logarithme before found makes 4,99976,10185 the Logarithme deſired; and differs but little with the true Logarithme in the Tables of Mr. *Briggs*, found there to be 4,99976,10723.

Prop. 3. *To find the* Logarithme *of a* Common *Fraction.*

Prop. III. *To find the Logarithme of a Common Fraction, or Integer and Fraction Mixt.*

Vulgar Fractions in the *Second Part* of the *Firſt Book*, were conſidered, as Proper, Equal, and Improper.

Equal.

Equal Fractions being alwayes an Unit, have 0 for their Logarithme.

Proper or Improper.

The Logarithme of Proper or Improper Fractions are found by ſubſtracting the Logarithme of the Leſſer Term out of the Logarithme of the Greater. The remain ſhall be the Logarithme of the Fraction, which ſhall be affirmative if the Fraction be Improper, but negative if the Fraction be Proper.

Examples.

As to find the Logarithme of $\frac{3}{4}$ or $\frac{4}{3}$, the Logarithme of each is 0,12493,87366, but of the Proper Fraction negative, of the other affirmative.

Proper Fraction $\frac{3}{4}$.		Improper Fraction $\frac{4}{3}$.	
0,60205,99913	Log. of 4.	0,60205,99913	Log. of 4.
0,47712,12547	Log. of 3.	0,47712,12547	Log. of 3.
—0,12493,87366	Log. of $\frac{3}{4}$ differ.	0,12493,87366	Log. of $\frac{4}{3}$.

Mixt Numbers firſt to be reduced into an Improper Fraction. Example.

If the Logarithme of a Mixt Number be ſought, reduce the ſame into an Improper Fraction, and ſeek the Logarithme thereof as before.

As to get the Logarithme of 2½, I reduce it into $\frac{5}{2}$, and taking the Logarithme of 2 from the Logarithme of 5, find the Log. deſired of $\frac{5}{2}$ or 2½ to be 0,39794,00086.

Arithmetical Complement what. Logarithme *of the Decimal Fraction how got. Example.*

If the Logarithme of the Denominator of any Common Fraction be ſubſtracted out of the Logarithme of the Numerator, the Remain will differ nothing, ſave in the *Index*, from the Arithmetical Complement (which is the Remainder of any Logarithme ſubſtracted out of 10 with Cyphers); and this Complement, with the true *Index*, may be taken for the Logarithme of the Fraction, but is properly the Logarithme of the Decimal Fraction; for ſo the Logarithme of a Decimal Fraction may be gotten.

As in the former inſtance of $\frac{3}{4}$, taking the Logarithme of 4 from the Logarithme of 3, in the *Index* there will lack an Unit, therefore marked negative. The Logarithme by this way is found to be —1,87506,12634, which Logarithme in the *Table* anſwers to 75, being here negative ſhall be the Logarithme of '75, or $\frac{75}{100}$, which is $\frac{3}{4}$: For 75 is $\frac{3}{4}$ of 100. And this Logarithme and the former make up the Logarithme of 1, as at *A.* may be ſeen, by adding them together, and differs not from the Arithmetical Complement, except in the *Index*, as following may be ſeen at *B.*

Proper Fraction $\frac{3}{4}$.

0,47712,12547 Log. of 3.
0,60205,99913 Log. of 4.
—1,87506,12634 Log. of $\frac{3}{4}$ of 100, or '75.

A.		*B.*	
—1,87506,12634	Log. of '75.	10,00000,00000	10, and Cyphers.
—0,12493,87366	Log. of $\frac{3}{4}$.	0,12493,87366	Log. of $\frac{3}{4}$.
0,00000,00000	Log. of 1.	9,87506,12634	Arith. Compl.

Notice to be taken how the Logarithme *of the Fraction is gotten.*

But great notice is to be taken, whether the Logarithme of the Fraction be gotten this or the other way: For that in Addition and Subſtraction, Multiplication and Diviſion of Logarithmes hereafter, the Caſe will differ between the Logarithme of the Decimal, and the Logarithme of the Common Fraction.

Prop. 4. *To find the* Logarithme *of a Decimal Pure or Mixt.*

Prop. IV. *To find the Logarithme of a Decimal, or Integers mixt with Decimals.*

Beſides the way of getting the Decimal Logarithme of a Fraction laſt mentioned, take this General Rule.

Suppoſe the Numbers whoſe Logarithme is ſought to be an Integer, and find the Logarithme thereof accordingly, then prefix an *Index* thereto according to the diſtance of the firſt Left Hand Figure of the given Number from unity: For the Characteriſtique alwayes differs according to the nature of the Number, though the reſt of the Logarithme may be the ſame. Examples.

Examples.

	Numbers.	Logarithmes.
Integers	485	2,68574,17386.
Mixt	48,5	1,68574,17386.
	4,85	0,68574,17386.
Decimals	,485	—1,68574,17386.
	,0485	—2,68574,17386.

Prop. 5. *To find the Number for a* Logarithme *given.*

Prop. V. *To find the Abſolute Number correſponding to a Logarithme given, be it integral or mixed.*

Thoſe Logarithmes that anſwer to Integers or Mixt Numbers may be found in the *Tables.*

1. Either the Logarithme with the *Index*. Or,
2. The Logarithme with another *Index* Greater or Leſſer. Or,
3. The *Index* with another Logarithme. Or elſe,
4. Neither *Index* nor Logarithme exactly. And therefore,

If the Logarithme *be in the* Tables.

If the Logarithme be expreſſed in the Tables, then by the orderly increaſe or decreaſe of the Logarithmes, ſeeking in the Column under the Title Logarithmes, you will ſoon find the Logarithme ſought, and juſt againſt it under the Title Numbers in the Left Hand Column, you find the Abſolute Number that anſwers thereto.

Example.

As if the Logarithme 2,15836,24921 be given, and the Abſolute Number belonging thereto be deſired, I look in the *Table* and find the Logarithme, and in the Left Hand Column 144 to be the correſponding Number.

If the Logarithme *have an* Index *Greater or Leſſer.*

When found with a Leſſer.

If the Logarithme given have a greater *Index* than is to be found in the *Tables*, then conſidering that the Logarithme of 2 is the Logarithme of 20, only altering the *Index*; and ſo the Logarithme of 3 the ſame with 30, of 4 with 40, *&c.* the Logarithme of 11 the ſame with 110; the Logarithme of 12 with 120, *&c.* It is eaſie having found the Logarithme with the leaſt *Index* in the *Tables* to produce the true Number correſponding to the Greateſt *Index*, by adding to the Right Hand of the Number anſwering the Logarithme found with the leaſt *Index*, ſo many Cyphers as there are Units in the *Index* of the given Logarithme more than in the *Index* of the Logarithme found.

Example.

As if 4,30102,99957, be the Logarithme given, neglecting the *Index* I look in the *Tables*, and find the Logarithme againſt 2, 20, 200, *&c.* but all of a different *Index*. I take that of 2, being the leaſt, and adjoyn to 2 the Number correſponding 4 Cyphers, becauſe the *Index* of the Logarithme given was 4, and the *Index* of 2 was 0. So is 20000 the Number anſwering to the Logarithme 4,30102,99957.

When found with a Greater Index.

If the Logarithme given be not preciſely found in the *Table*, in the proper place, according to the *Index* thereof, or with a leſſer *Index*, then the ſame may be ſought for among the Logarithmes of a greater *Index*. And if found there you ſhall have the abſolute Number thereof in more Figures than the *Index* of the given Logarithme requires. Wherefore cut off ſo many of the Right Hand Figures as are ſuperfluous; for the Numerator of a Fraction, whoſe Denominator ſhall be an Unit with ſo many Cyphers as there be Figures in the Numerator, or it may be ſet as a Decimal.

Example.

Example. Let the Logarithme given be 1,09691,00130, which ſought for among the Logarithmes whoſe *Index* is 1, cannot be found exactly, but is found among the Logarithmes that have 2 for their *Index*, and over againſt the ſame the abſolute Number 125, which conſiſts of 3 Figures, whereas the *Index* of the given Logarithme being but 1, required but 2 Figures in the abſolute Number, therefore I cut off the laſt Right Hand Figure of 125, and leave 2, *viz.* 12, and the 5 is Numerator of a Fraction to 10 the Denominator; ſo ſhall the abſolute Number be $12\frac{5}{10}$, or 12,5.

☞ And if the given Logarithme had been found with a greater *Index* than 2, as it happeneth oftentimes in large Tables, ſuch as thoſe of Mr. *Briggs* are: As ſuppoſe 1,04328,36656, the Logarithme given and found under the *Index* 4, and the correſponding Number 11048, then ſhould the Number be 11,048, or $11\frac{048}{1000}$. And if the *Index* of the given Logarithme had been 2, then $110\frac{48}{100}$. If 3, then $1104\frac{8}{10}$, *&c.*

If the Index *be found with another* Logarithme.

If the *Index* be found with another Logarithme than that given, and the *Table* not large enough to find it with a greater *Index*, then enter the *Table* with the *Index* of the Logarithme given, and find the next leſſer Logarithme to the given Logarithme, and you have the Integer anſwering thereto, to which a Fraction is to be adjoyned, which is thus gotten.

Subſtract the Logarithme found from the Logarithme given, the Remain ſhall be the Numerator, and the difference between the Logarithme found, and that which next follows in the *Table* ſhall be the Denominator of the Fraction.

Example.

As in the former inſtance; if 1,09691,00130, be the given Logarithme, the next leſſer Logarithme found in the *Table* with the *Index* 1, is 1,07918,12460, and the abſolute Number anſwering thereto is 12; then ſubſtracting the Logarithme of 12 from the Logarithme given, the Remain is 1772,87670, which ſhall be the Numerator to 3476,21063, the difference between the Logarithme of 12 and the Logarithme of 13. And being near $\frac{1}{2}$, by cutting off many of the Right Hand Figures, (without ſenſible Error) may be reduced to $\frac{17}{34}$ equivalent to $\frac{5}{10}$ or $\frac{1}{2}$, as before.

Beſt to be turned into a Decimal.

Or rather, turn it into a Decimal Fraction, which is thus done; adjoyn Cyphers to the Right Hand of the Difference or Remain after the next leſſer Logarithme in the *Table* is ſubſtracted from the given Logarithme: And divide this Number with the Cyphers ſo adjoyned by the Difference between the next Leſſer and next Greater Logarithmes found in the *Table*; wherefore if to 1772,87670, there be but 2 Cyphers

Example.

phers adjoyned, and the ſame be divided by 3476,21063, the Quotient will be 51, to be added to 12 as a Decimal, and ſo the Logarithme given, ſhall be the Logarithme for 12,51, and by adjoyning more Cyphers, and continuing the Diviſion, the Decimal will be greater.

| 9278
347 1385|7
1772,8767000 (51
3476,210633
3476,2106

If Index *nor* Logarithme *be ſound.*

If neither *Index* nor Logarithme be found exactly in the *Table*, proceed thus; under the Greateſt *Index* your Tables will afford, find the next leſſer Logarithme to the Logarithme given, neglecting the *Indices* of both, and reſerve the Number anſwering to the Logarithme found apart, and note the true *Index* of that Number. Then ſubſtract the Logarithme found from the Logarithme given, and with the Difference between the Logarithme found and that which next follows in the *Table*, you may get a proportional part by this Analogy. As the Difference between the Logarithme found and the following, to the Difference between the Logarithme found, and the Logarithme given; ſo is an Unit with ſo many Cyphers as there are Abſolute Numbers wanting in the Number found to make up the *Index* of the Logarithme given, to the ſame Numbers wanting, which Numbers gotten adjoyn to the Right Hand.

Example.

As if the Logarithme given be 4,99976,10185, under the greateſt *Index* in the foregoing *Table* 2, I find 99956,54882, the next Logarithme to the Logarithme given, and the correſponding Number to be 999, the true *Index* whereof is 2, the Logarithme of 999 taken from the given Logarithme leaves 19,55303, the Difference between the Logarithme of 999, and the Logarithme of 1000 is 43,45118, the *Index* of the given Logarithme is 4, and of 999 but 2, therefore 2 places are wanting: Then I ſay, As 43,45118, to 19,55303, ſo is 100 to 45, almoſt; which 45 adjoyned to 999, makes 99945, for the Number correſponding to the given Logarithme.

| 434510
2 1725588|8
19,5530300 (44 and above.
43,451188
43,4511

And in like manner the Abſolute Number anſwering to Logarithmes given that have 6, 7, 8, or more Units in their *Index* may be had exact enough for Operation.

Prop. 6. *To find a Fraction for a* Logarithme.

Prop. VI. *To find a Fraction for a given Logarithme.*

Fractions deſired for a given Logarithme may be either Common or Decimal. The Decimal much the eaſier to be found, and therefore in common uſe the Logarithmes of them are beſt to be choſen; yet ſometime the Logarithme of a Vulgar Fraction may be neceſſary: The way to get the Logarithmes of both, ſee in the *Sections* following.

Logarithme *of a Common Fraction found with the* Index *and deſired* Common.
Example.

§. 1. When a Common Fraction is deſired for the Logarithme thereof given. If the given Logarithme with the proper *Index* be found in the *Table*, then the abſolute Number correſponding ſhall be the Denominator of the Fraction, to which 1 ſhall be Numerator.

As if —0,30102,99957, be the Logarithme given, the abſolute Number anſwering thereto is 2, which ſhall be the Denominator; ſo ſhall $\frac{1}{2}$ be the Fraction of the given Logarithme.

Logarithme *of a Common Fraction found, and deſired to be a Decimal.*

§. 2. When the Logarithme of a Common Fraction with the given *Index* is found in the *Table*, and the Fraction is deſired to be a Decimal; ſubſtract the given Logarithme from the Logarithme of the Decimal Denominator given, the correſponding Number to the remaining Logarithme ſhall be the Numerator to the given Denominator.

Example.

As if —0,30102,99957, be given, and the Decimal thereof be ſought in Primes or Tenths, then I take the Logarithme given from the Logarithme of 10, the Remain is 0,69897,00043, which is the Logarithme of 5; ſo ſhall 5 be the Numerator to 10, and the Decimal Fraction $\frac{5}{10}$.

If the Denominator given had been 100, and ſo the Decimal ſet in Seconds, then the Logarithme given taken from the Logarithme of 100, leaves 1,69897,00043, the Logarithme of 50, which ſhall be the Numerator to 100, and the Fraction $\frac{50}{100}$, or $\frac{5}{10}$ as before.

The like is to be done with others: Nevertheleſs it often happens, the greater the Denominator given, the more eaſie to find the correſponding Number to the Logarithme of the Remain, to be an Integer.

The Greater the Denominator the more eaſie to find.

§. 3. When the given Logarithme of a Common Fraction is not found in the *Table*, yet if the Fraction be deſired to be a Decimal, the work is the ſame with the laſt above-mentioned.

Logarithme *of a Common Fraction not found, and deſired to be a Decimal.*

As if —0,12493,87366, be the Logarithme given, and I deſire to know the Decimal Fraction ſignified thereby, whoſe Denominator ſhall be 100; or Seconds, after Subſtraction of the given Logarithme from the Logarithme of 100, the Remain is 1,87506,12634, the Logarithme of 75, which ſhall be the Numerator to 100, as was ſeen before.

Example.

Here may be noted what was mentioned above, that the greater the Denominator, the more exact the Whole Number may be found. For if to this Logarithme 0,12493,87366, the Denominator had been given 10, then would the Remain have been 0,87506,12634, the neareſt Integer anſwering which is 7, which is 7 Primes or Tenths; but then for that the Remain is greater than the Logarithme of 7, the Numerator is not exact, but I loſe the 5 Seconds, unleſs I work for them by ſome of the varieties in the *Fifth Propoſition.*

The Greater the Denominator *the nearer the truth.*

§. 4. When the Logarithme of a Common Fraction is given, which is not to be found in the *Table*, and the Fraction thereof deſired Vulgar and not Decimal: Then after the Decimal Fraction thereof is found as above, reduce the ſame to the leaſt Termes.

Logarithme *of a Common Fraction not found, and deſired* Common.

As if $\frac{75}{100}$ be found as above, for the Logarithme of —1,87506,12634, I abbreviate $\frac{75}{100}$ to $\frac{3}{4}$ the Common Fraction, whoſe Log. as above was —0,12493,87366.

Example.

§. 5. When the Logarithme of a Decimal Fraction is given, look out the correſponding Number: As if the given Logarithme were the Logarithme of an Integer, and then place the *Seperatrix* according to the Units contained in the *Index*, or which is all one, prefix before the Number found ſo many Cyphers lacking one, as there be Units in the *Index* of the Logarithme given.

Decimal found for the Logarithme *thereof.*

As if —2,68574,17386, be the Logarithme of a Decimal, which I find in the *Table* againſt the Integers 485, then I place Unity two places before the Left Hand Figure thereof, for that the *Index* here is 2 defective, ſo is the Decimal 0,0485.

Examples.

But if the Logarithme given were —1,68574,17386, then ſhould the Decimal be 0,485. If the Logarithme were 0,68574,17386, the Integers and Decimal would be thus 4,85, *&c.* as was ſeen in the *Fourth Propoſition* of this Chapter, and the like is to be underſtood of others.

Nothing need be added to prove the Invention of Logarithmes for Numbers, or Numbers for Logarithmes, ſeeing both are to be examined and proved by the foregoing *Table* of *Logarithmes.*

Proof by the Tables.

CHAP. III.

Addition of Logarithmes.

ADdition of Logarithmes is equivalent to Multiplication of other Numbers. If therefore the Logarithme of the Multiplyer be added to the Logarithme of the Multiplicand, the Total ſhall be the Logarithme of the Product.

Logarithmes *added what equivalent to.*

To add two Logarithmes, conſider whether the given Logarithmes be both of one Nature or not, that is, Affirmative, Negative or Mixt; for Affirmative will produce Affirmative, and Negative Negative; but if they be Mixt, the Product will be ſometime

The Key of Addition *of* Logarithmes.

time the one, and ſometime the other; for one of them is in nature ſubſtractive from the other.

As if $4+2=6$, then is $10000 \times 100 = 1000000$. And as $\overline{4}+\overline{2}=\overline{6}$, ſo is $,0001 \times ,01 = ,000001$. But $3+\overline{2}=1$, or $1000 \times ,01 = 10$, is as $1000 \mathfrak{D} 100 = 10$. And as $\overline{3}+2=\overline{1}$, or $,001 \times 100 = 0,1$, ſo is $100 \mathfrak{D} 1000 = 0,1$. This Key may unlock Addition of Logarithmes, but the particular Caſes following make all plain.

1.
If both be Affirmative.

1. *Caſe.* If the given Logarithmes be both affirmative, then add the Numbers, as if they were Integers, and the ſumme ſhall be a Logarithme of the ſame kind, and his Abſolute Number the Product.

Example.

As to add the Logarithme of 90 to the Logarithme of 9, the Total is the Logarithme of 810, the Product of both.

Multiplicand	1,95424,25094	Log. of 90
Multiplyer	0,95424,25094	Log. of 9
Product	2,90848,50188	Log. of 810

2.
If both be Negative.

2. *Caſe.* If the given Logarithmes be both Negative, as before, add the Numbers as Integers, and the ſumme ſhall be a Logarithme of the ſame kind; only in the Logarithmes of Decimals, if from the Left Hand place next the *Index*, any Tens be carried onward to the *Index*, they are alwayes Affirmative, although the *Index* be Negative: And then the Affirmative muſt be ſubſtracted out of the Negative *Indices*, as was taught in Addition of Decimal *Indices*.

Example in a Fraction.

As to add —0,30102,99957, the Logarithme of $\frac{1}{2}$ to it ſelf, the Total ſhall be —0,60205,99914, the Logarithme of $\frac{1}{4}$, which is the Product of $\frac{1}{2}$ multiplyed by it ſelf, and herein is no difficulty.

Example in a Decimal.

But if the Fraction had been a Decimal, that is 0,5, or $\frac{50}{100}$, the Logarithme of which is —1,69897,00043, then in adding them, when I come to the Figures next the *Indices*, an Unit is to be carried over to the *Indices* for the 10 which is there, and this Affirmative I take from the ſumme of both the Negative *Indices*, and the Remain is 1 Negative; ſo is the Total —1,39794,00086, which is the Logarithme of 0,25, that is the Product of 5′ by 5′, and agrees to the other in value, though in other Terms, becauſe 25 is $\frac{1}{4}$ of 100.

Multiplicand	—0,30102,99957	Log. of $\frac{1}{2}$	—1,69897,00043	Log. of 0,5′
Multiplyer	—0,30102,99957	Log. of $\frac{1}{2}$	—1,69897,00043	Log. of 0,5′
Product	—0,60205,99914	Log. of $\frac{1}{4}$	—1,39794,00086	Log. of ,25″

3.
Data *of divers kinds, and the* Logarithme *defective be the* Logarithme *of a Fraction. Examples.*

3. *Caſe.* If the given Logarithmes be of different kinds, that is, the one Affirmative and the other Negative, and the defective Logarithme be the Logarithme of a Common Fraction; then ſubſtract the Leſſer Logarithme out of the Greater, the Remainder ſhall be the Logarithme of the Product required, and ſhall alwayes be of the ſame kind with the Greater Logarithme.

As if 5 were to be multiplyed by $\frac{1}{2}$, then —0,30102,99957, the Logarithme of $\frac{1}{2}$ taken from 0,69897,00043, the Logarithme of 5, leaveth 0,39794,00086, the Logarithme of $\frac{5}{2}$ or $2\frac{1}{2}$, the Product not defective becauſe the Logarithme of 5 the Greater Logarithme was abundant.

But if 20 be multiplyed by $\frac{1}{40}$, then 1,30102,99957, the Logarithme of 20 being the Leſſer, is to be ſubſtracted from —1,60205,99913, the Logarithme of $\frac{1}{40}$, and the Remain will be —0,30102,99956, the Logarithme of the Product $\frac{20}{40}$ or $\frac{1}{2}$ defective, becauſe the Greater Logarithme is Negative.

Multiplicand	0,69897,00043	Log. of 5	1,30102,99957	Log. of 20
Multiplyer	—0,30102,99957	Log. of $\frac{1}{2}$	—1,60205,99913	Log. of $\frac{1}{40}$
Product	0,39794,00086	Log. of $\frac{5}{2}$	—0,30102,99956	Log. of $\frac{20}{40}$

4. *Caſe.* If the given Logarithmes be of divers kinds, and the Negative Logarithme be the Logarithme of a Decimal Fraction, then add the Logarithmes together till you come to the *Index*, and there ſubſtract the Leſſer *Index* from the Greater, remembring the 10 carried over, if any be, is Affirmative, as before noted in the *Second Caſe*, and to be ordered accordingly.

4. Data *of divers kinds, and the* Logarithme *defective be the* Logarithme *of a Decimal. Examples.*

As if —1,69897,00043, the Logarithme of $\frac{50}{100}$, or 0,5, which is $\frac{1}{2}$, be added to the Logarithme of 5, that is 0,69897,00043, the Total Product is 0,39794,00086, the Logarithme of 2,5 and Affirmative; for there the Unit carried for the 10 next the *Index* being Affirmative, ſubſtracted from the Negative *Index* 1, leaves 0 to the Product abundant.

But if the defective Logarithme of the Decimal 0,025, that is —2,39794,00087 be added to the Logarithme of 20, the Total will be —1,69897,00044, the Logarithme of 5 Primes or ,50″; for the *Index* 1 Affirmative taken from 2 Negative, leaves the Remain defective, becauſe the *Index* of that kind was the Greater.

Multiplicand	—1,69897,00043	Log. of	0,5′	1,30102,99957	Log. of	20
Multiplyer	0,69897,00043	Log. of	5	—2,39794,00087	Log. of	,025‴
Product	0,39794,00086	Log. of	2,5′	—1,69897,00044	Log. of	,5′

5. *Caſe.* If ſeveral Numbers be given to be multiplyed one into another, add all their Logarithmes together, according to the Directions foregoing, and the Total thereof ſhall be the Product deſired, whereby in diſpatch of great Multiplications, a wonderful expedition is attained. View the Examples following.

5. Data *many Numbers.*

Examples.

Integers.

0,47712,12547	Log. of	3	0,30102,99957	Log. of	2	Numbers multi-
1,04139,26852	Log. of	11	0,69897,00043	Log. of	5	plyed one into
1,23044,89214	Log. of	17	1,90308,99870	Log. of	80	another.
2,74896,28613	Log. of	561	2,90308,99870	Log. of	800	Product.

Common Fractions.

—0,60205,99913	Log. of	$\frac{1}{4}$	—0,17609,12590	Log. of	$\frac{2}{3}$	Fractions mul-
—0,60205,99913	Log. of	$\frac{1}{4}$	—0,12493,87366	Log. of	$\frac{3}{4}$	tiplyed toge-
—0,30102,99957	Log. of	$\frac{1}{2}$	—0,07918,12461	Log. of	$\frac{5}{6}$	ther.
—1,50514,99783	Log. of	$\frac{1}{32}$	—0,38021,12417	Log. of	$\frac{5}{12}$	Product.

Decimals.

—1,00000,00000	Log. of	0,1′	—2,47712,12547	Log. of	0,03″
—1,17609,12591	Log. of	0,15″	—3,90308,99870	Log. of	0,008‴
—1,95424,25094	Log. of	0,9′	—3,60205,99913	Log. of	0,004‴
—2,13033,37685	Log. of	0,0135⁗	—7,98227,12330	Log. of	0,00000096$^{\text{viij}}$

Mixt Numbers.

0,30102,99957	Log. of	2	1,22530,92817	Log. of	16,8′
2,15836,24921	Log. of	144	—1,87506,12634	Log. of	0,75″
—1,87506,12634	Log. of	0,75″	—2,69897,00043	Log. of	0,05″
2,33445,37512	Log. of	216,00″	—1,79934,05494	Log. of	0,63000$^{\text{v}}$

The Product of the Numbers multiplyed anſwering to the Number correſponding to the Total Logarithme of the Logarithmes added, is Proof ſufficient of Logarithmical Addition.

Proof of Addition *of* Logarithmes.

CHAP.

CHAP. IV.

Subſtraction of Logarithmes.

Logarithmes *ſubſtracted what it performeth.* *The Key of* Subſtraction *of* Logarithmes.

Subſtraction of Logarithmes performeth as much as Diviſion of other Numbers, and therefore to ſubſtract the Logarithme of the Diviſor from the Logarithme of the Dividend, the Remain ſhall be the Logarithme of the Quotient.

But as before in Addition, the Logarithmes being ſome defective, and others abundant, conſideration muſt be had in the Subſtraction, that the Remain or Quotient be rightly Denominate, Affirmative or Negative, according as the Natures of the given Logarithmes will admit.

For as in *Indices* $4 - 2 = 2$: And in multiplyed Numbers correſponding 10000 𝔛 100 = 100. And $\overline{4} - \overline{2} = \overline{2}$; that is in Decimals thus, ,0001 𝔛 ,01 = ,01. So if $0 - \overline{2}$ be taken from $0 + 3$, the Remain ſhall be $= 5$, that is in Numbers. If 1 𝔛 ,01 be taken from 1 × 1000, the Remain ſhall be = 100000. For there as the *Indices* are added, 1000 ſhall be multiplyed by 100, the Quotient of the Diviſion. But if $0 + 2$ be taken from $0 - \overline{3}$, the Remain ſhall be $= \overline{5}$, which in Numbers is as 1 × 100 taken from 1 𝔛 ,001, the Remain ſhall be = ,00001. This is the Key of Subſtraction, and may be fully underſtood in the Caſes following.

1. *If both be Affirmative.*

1. *Caſe.* If the given Logarithmes be both affirmative, and that of the Diviſor leſs than that of the Dividend, then ſubſtract the leſſer Logarithme from the greater, as if they were Integers; and the Remain ſhall be a Logarithme of the ſame kind, and the abſolute Number anſwering thereto, the Quotient.

Example.

As to divide 810 by 9, the Quotient is 90; ſo to ſubſtract 0,95424,25094, the Logarithme of 9 from 2,90848,50189, the Logarithme of 810, the Remain is 1,95424,25095, the Logarithme of 90, the Quotient.

Dividend	2,90848,50189	Log. of	810
Diviſor	0,95424,25094	Log. of	9
Quotient	1,95424,25095	Log. of	90

2. *If both be Affirmative, and the Diviſor greateſt.*

2. *Caſe.* If both the given Logarithmes are affirmative, and that of the Diviſor greater than that of the Dividend, then after Subſtraction of all the reſt of the Logarithme of the Diviſor from the Logarithme of the Dividend, except the *Index*, take the leſſer *Index* from the greater, and change the Sign of the Remain. And in ſubſtracting, if in the next Figure to the *Index* you have occaſion to borrow 10, then accompt the *Index* of the Dividend 1 leſs than it is; as if 3, accompt it 2, if 2 but 1, *&c.* Examples of both.

Examples.

As if 9 be divided by 90, the Quotient will be $\frac{1}{10}$ or 0,1; ſo to ſubſtract 1,95424,25094 the Logarithme of 90, from 0,95424,25094, the Logarithme of 9, the Remain will be —1,00000,00000, the Logarithme of $\frac{1}{10}$ or 0,1, as at *A*.

And if 36 be divided by 90, the Quotient by abbreviation will be $\frac{2}{5}$: So 1,95424,25094 the Logarithme of 90, taken from the Logarithme of 36, which is 1,55630,25008, the Remain will be the Logarithme of 0,4′, or abbreviated $\frac{2}{5}$; where 1 for the 10 borrowed next the *Index* abates the Characteriſtique 1 of the upper Logarithme; and ſo 0 being left for the leſſer *Index* taken from the lower, changes the Sign, as at *B*.

	A.			*B.*		
Dividend	0,95424,25094	Log. of	9	1,55630,25008	Log. of	36
Diviſor	1,95424,25094	Log. of	90	1,95424,25094	Log. of	90
Quotient	—1,00000,00000	Log. of	0,1′	—1,60205,99914	Log. of	0,4′

3. *Caſe.* If the given Logarithmes are both Negative, of Vulgar Fractions, and that of the Diviſor the leſſer; then as if they were Integers, ſubſtract the Logarithme of the Diviſor from the Logarithme of the Dividend, and the Remain ſhall be a defective Logarithme of the ſame kind. But if the Logarithme of the Diviſor be the greater, take the leſſer Logarithme from the greater, and change the Sign of the Remain, for in this caſe it ſhall be abundant. 3. *If both be Negative of Fractions.*

As if $\frac{1}{32}$ be divided by $\frac{1}{4}$, the Quotient will be $\frac{1}{8}$; ſo if the Logarithme of $\frac{1}{4}$, that is —0,60205,99913, be taken from —1,50514,99783, the Logarithme of $\frac{1}{32}$, the Remain will be —0,90308,99870, the Logarithme of $\frac{1}{8}$, as at *C*. *Examples.*

But if $\frac{1}{2}$ be divided by $\frac{1}{4}$ the Quotient will be 2; for the Diviſor having the greater Logarithme, the *Index* which was negative is now changed, and the Remain affirmative, as at *D*.

	C.		*D.*	
Dividend	—1,50514,99783	Log. of $\frac{1}{32}$	—0,30102,99957	Log. of $\frac{1}{2}$
Diviſor	—0,60205,99913	Log. of $\frac{1}{4}$	—0,60205,99913	Log. of $\frac{1}{4}$
	—0,90308,99870	Log. of $\frac{1}{8}$	0,30102,99956	Log. of 2

4. *Caſe.* If the Logarithmes given are both of Decimals, then ſubſtract them as Integers till you come to the *Index*; and there if in the next place to the *Index* you borrow 10, accompt the *Index* of the Dividend 1 leſs than it is; as if —1, accompt it —2, if —2 then —3, *&c.* and then take the leſſer *Index* from the greater: And if the upper *Index*, which is of the Dividend, be the leaſt, then change the Sign of the Remain, which ſhall be the Logarithme of the Quotient. 4. Data *Decimals.*

As to divide 0,0135⁗ by 0,1′, the Quotient will be 0,135‴: So if the Logarithme of 0,1′ be taken from —2,13033,37685, the Logarithme of 0,0135⁗, the Remain will be —1,13033,37685, the Logarithme of 0,135‴, as at *E*. *Examples.*

And if 0,0135⁗ be divided by 0,9′, the Quotient will be 0,015‴: So if the Logarithme of 0,9′, which is —1,95424,25094, be taken from —2,13033,37685, the Logarithme of 0,0135⁗, there will be left —2,17609,12591, the Logarithme of 0,015‴, as at *F.*; where by borrowing the 10, next the *Index*, the upper *Index* is accompted —3.

	E.		*F.*	
Dividend	—2,13033,37685	Log. of 0,0135⁗	—2,13033,37685	Log. of 0,0135⁗
Diviſor	—1,00000,00000	Log. of 0,1′	—1,95424,25094	Log. of 0,9′
Quotient	—1,13033,37685	Log. of 0,135‴	—2,17609,12591	Log. of 0,015‴

Other Examples of the varieties that may happen under this Caſe here follow.

Dividend	—1,87506,12634	Log. of 0,75″	—1,65321,25138	Log. of 0,45″
Diviſor	—2,69897,00043	Log. of 0,05″	—1,95424,25094	Log. of 0,9′
Quotient	—1,17609,12591	Log. of 15,0′	—1,69897,00044	Log. of 0,5′
Dividend	—2,13033,37685	Log. of 0,0135⁗	—1,30102,99957	Log. of 0,2′
Diviſor	—2,69897,00043	Log. of 0,05″	—2,60205,99913	Log. of 0,04″
Quotient	—1,43136,37642	Log. of 0,27″	0,69897,00044	Log. of 5,0′
Dividend	—1,95424,25094	Log. of 0,9′	—2,07918,12460	Log. of 0,012‴
Diviſor	—1,17609,12591	Log. of 0,15″	—1,17609,12591	Log. of 0,15″
Quotient	—0,77815,12503	Log. of 6,0	—2,90308,99869	Log. of 0,08″

5. *Data of divers kinds.*

5. *Caſe.* If the Logarithmes given are of divers kinds; that is one affirmative and the other negative, and the defective Logarithme be the Logarithme of a Common Fraction; add them together, and the ſumme ſhall be the Logarithme of the Quotient, and of the ſame kind with the Dividend. But if the defective Logarithme be the Logarithme of a Decimal Fraction, then ſubſtract the Logarithme of the Diviſor out of the Logarithme of the Dividend till you come to the *Index*, and then add the *Indices* together: And if 10 be borrowed, accompt the *Index* of the Dividend one leſs, as before.

Examples.

Examples of both ſorts follow ſo plainly that illuſtration thereof is needleſs.

Integers with Common Fractions.

Dividend	0,77815,12504	Log. of	6	—0,12493,87366	Log. of	$\frac{3}{4}$
Diviſor	—0,12493,87366	Log. of	$\frac{3}{4}$	0,77815,12504	Log. of	6
Quotient	0,90308,99870	Log. of	8	—0,90308,99870	Log. of	$\frac{1}{8}$
Dividend	1,20411,99827	Log. of	16	—1,90308,99870	Log. of	$\frac{1}{80}$
Diviſor	—1,90308,99870	Log. of	$\frac{1}{80}$	1,20411,99827	Log. of	16
Quotient	3,10720,99697	Log. of	1280	—3,10720,99697	Log. of	$\frac{1}{1280}$

Integers with Decimals.

Dividend	0,60205,99913	Log. of	4	—3,90308,99870	Log. of	0,008‴
Diviſor	—3,90308,99870	Log. of	0,008‴	0,60205,99913	Log. of	4
Quotient	2,69897,00043	Log. of	500	—3,30102,99957	Log. of	0,002‴
Dividend	0,69897,00043	Log. of	5	—1,39794,00087	Log. of	0,25″
Diviſor	—1,39794,00087	Log. of	0,25″	0,69897,00043	Log. of	5
Quotient	1,30102,99956	Log. of	20	—2,69897,00044	Log. of	0,05″

Mixt Numbers.

Dividend	0,68574,17386	Log. of	4,85″	0,69897,00043	Log. of	5
Diviſor	—1,79588,00173	Log. of	0,625‴	—2,90308,99870	Log. of	0,08″
Quotient	0,88986,17213	Log. of	7,76″	1,79588,00173	Log. of	62,5′

Proof of Subſtraction *of* Logarithmes.

The Quotient of the Numbers divided, anſwering to the Number of the Remaining Logarithme of the Logarithmes ſubſtracted, is Proof enough of the truth of Logarithmical Subſtraction.

CHAP. V.

Multiplication of Logarithmes.

Logarithmes *multiplyed produceth Figurals.*

MUltiplication of Logarithmes reſembleth Multiplication of Ratio's, hereafter treated of; for that it maketh the Product a Figurate Number. As to multiply a Logarithme by 2, produceth the Logarithme of the Square. And to multiply by 3 the Logarithme of the Cube, *&c.* according to the Figural *Index* of the Quantity uſed for Multiplyer.

1. *Caſe.*

1. *If the* Index *be affirmative, or the defective* Logarithme *be the* Logarithme *of a Fraction. Examples.*

1. *Caſe.* There is no difficulty therein, if the *Index* of the given Logarithme be affirmative, or the defective Logarithme be the Logarithme of a Common Fraction; for then the Multiplication is as in Integers.

For as 9 multiplyed by 9 giveth 81; ſo the Logarithme of 9 multiplyed by 2 produceth the Logarithme of 81. And the Fraction $\frac{2}{9}$ ſquared is $\frac{4}{81}$; ſo the Logarithme of $\frac{2}{9}$ multiplyed by 2 ſhall give the Logarithme of $\frac{4}{81}$. And the like may be done for any other power.

	Integers.		Common Fractions.	
Root	0,95424,25094	Log. of 9	—0,65321,25137	Log. of $\frac{2}{9}$
	2		2	
Square	1,90848,50188	Log. of 81	—1,30642,50274	Log. of $\frac{4}{81}$
Root	0,69897,00043	Log. of 5	—0,17609,12590	Log. of $\frac{2}{3}$
	3		3	
Cube	2,09691,00129	Log. of 125	—0,52827,37770	Log. of $\frac{8}{27}$
Root	0,60205,99913	Log. of 4	—0,69897,00043	Log. of $\frac{1}{5}$
	4		4	
Sq. Squa.	2,40823,99652	Log. of 256	—2,79588,000172	Log. of $\frac{1}{625}$

2. *If the* Logarithme *be the* Logarithme *of a Decimal. Examples.*

2. *Caſe.* If the given Logarithme be the Logarithme of a Decimal, there is no difference between the Multiplication thereof and others, ſave when in multiplying the Figure next the *Index* if any Tens ariſe, the Units carried over for them are affirmative, and to be ſubſtracted from the Product of the negative *Index*.

As in ſquaring 0,05″, the 10 carried from the Multiplication of 6 in the Logarithme, ſhall abate 1 from the Product of the *Index*, and leave but —3. The like alſo happeneth in other Examples.

Root	—2,69897,00043	Log. of 0,05″	—1,30102,99957	Log. of 0,2′
	2		2	
Square	—3,39794,00086	Log. of 0,0025''''	—2,60205,99914	Log. of 0,04″
Root	—2,69897,00043	Log. of 0,05″	—2,30102,99957	Log. of 0,02″
	3		3	
Cube	—4,09691,000129	Log. of 0,000125^{vj}	—6,90308,99871	Log. of 0,000008^{vj}
Root	—1,69897,00043	Log. of 0,5′	—1,30102,99957	Log. of 0,2′
	4		4	
Sq. Squ.	—2,79588,00172	Log. of 0,0625''''	—3,20411,99828	Log. of 0,0016''''

The Product of the Numbers multiplyed Figurately anſwering to the Number of the produced Logarithmes, ſerveth for a ſufficient Proof of the truth of Logarithmical Multiplication. *Proof of* Multiplication *of* Logarithmes.

CHAP. VI.

Diviſion of Logarithmes.

Logarithmes *divided, is Extraction of Roots.*

AS Multiplication of Logarithmes produceth Figural Numbers, ſo Diviſion of Logarithmes anſwereth to Extraction of Roots, and is much like Diviſion of Ratio's hereafter ſpoken to in the *Fourth Book* : For the Quotient is the Ratio of the Figurate Number whoſe Logarithme is the Dividend, according to the Ratio in the Diviſor. So as if the Diviſor be 2, the Quotient is the Logarithme of the Square Root ; if 3, the Logarithme of the Cube Root, *&c.* according to the Figural *Index* of the Quantity uſed for Diviſor.

1. *If the* Index *be affirmative, or the defective* Logarithme *be the* Logarithme *of a Fraction.*

1. *Caſe.* Diviſion is eaſie, if the *Index* of the given Logarithme be affirmative, or the defective Logarithme be the Logarithme of a Common Fraction ; for then the Diviſion is performed as in Integers. As in the Examples of the former *Chapter* may be ſeen.

Examples.

	Integers.		Fractions.	
Square	1,90848,50188	Log. of 81	—1,30642,50274	Log. of $\frac{4}{81}$
	2		2	
Root	0,95424,25094	Log. of 9	—0,65321,25137	Log. of $\frac{2}{9}$
Cube	2,09691,00129	Log. of 125	—0,52827,37770	Log. of $\frac{8}{27}$
	3		3	
Root	0,69897,00043	Log. of 5	—0,17609,12590	Log. of $\frac{2}{3}$

2. *If the* Logarithme *be the* Logarithme *of a Decimal.*

2. *Caſe.* If the Logarithme given to be divided be the Logarithme of a Decimal, and the *Index* will be evenly divided by the Diviſor, then there is no difference between the Diviſion thereof and others : But when the *Index* of the Logarithme will not be evenly divided by the Diviſor, then add to the *Index* ſo many Units till it may be evenly divided thereby, and ſetting the Quotient down for a new *Index*, keep the Units added in mind, multiply them by 10, and add the Product thereof to the next Right Hand Figure of the Logarithme, and then divide the reſt of the Logarithme, as others, where nothing was borrowed.

Examples.

As among the former Examples in the laſt *Chapter* foregoing, where —6,90308,99871, was found to be the Logarithme of the Cube $0,000008^{vi}$; becauſe —6 the Characteriſtique will be evenly divided by 3, the Diviſion is as in Integers, at *A*. But in dividing —4,09691,00129, the Logarithme of the Cube $0,000125^{vi}$, becauſe —4 the *Index*, will not be evenly divided by 3, I add in mind 2 Units to 4, to make it ſo diviſible, and ſo get —2 for the new *Index* ; which added 2, multiplyed by, make 20, and to this ſhould have been added the next Figure of the Logarithme, but there being 0, leaves 20, only to be divided by 3, which gives 6, and ſo continuing the Diviſion, the Logarithme of $0,05''$ is gotten, as at *B*.

	A.		*B.*	
Cube	—6,90308,99871	Log. of $0,000008^{vi}$	—4,09691,00129	Log. of $0,000125''$
	3		3	
Root	—2,30102,99957	Log. of $0,02''$	—2,69897,00043	Log. of $0,05''$

For the better underſtanding of the Diviſion of Decimal Logarithmes whoſe Characteriſtiques will not be evenly divided, Mr. *Oughtred* at the end of the *Reſolution of Affected Æquations*, hath preſented us with part of a *Table* to be increaſed at pleaſure ; which though the foregoing Rule may ſupply the uſe of, yet becauſe ſome by

Occular inſpection in a *Table* receive a firmer impreſſion in their Minds of the Numbers uſed therein, I have here tranſcribed it a little altered, with the Explanation following.

A Table *for Diviſion of* Decimal Logarithmes.

Diviſors						Indices.
2)				1	2	1
				3	4	2
				5	6	3
				7	8	4
				9	10	5
					&c.	
3)			1	2	3	1
			4	5	6	2
			7	8	9	3
					&c.	
4)		1	2	3	4	1
		5	6	7	8	2
					&c.	
5)	1	2	3	4	5	1
	6	7	8	9	10	2
					&c.	
&c.	40	30	20	10	0	

The Table *explained.*

In this *Table* the Numbers that ſtand to the Left Hand within a bowed Line are Diviſors, or the Numbers of Quantity, as 2, 3, 4, 5, and may be increaſed as the *&c.* denotes.

The Numbers next the Right Hand are the *Indices* of the Quotient, anſwering collaterally to ſuch Diviſors with ſuch *Indices* of the Dividends as ſtand along with them in the ſame Line from the Right Hand to the Left.

The Numbers next the *Indices* of the Quotient are ſuch *Indices* of the Dividends as will be evenly divided by the Diviſors ſtanding againſt them, and ſo need no Units to be added to them. Theſe may alſo be increaſed at pleaſure, as the ſeveral *&c.* ſignifie.

But the Numbers next them, if they happen to be the Characteriſtiques of the Logarithmes given to be divided, muſt have 1 Unit added, and the next File of Numbers muſt have 2 Units added, the next File 3, and the next 4; and ſo of others, if the *Table* be enlarged. Every of which multiplyed by 10, as before noted in the Rule, makes the 10, 20, 30, 40, at the bottom of the *Table*, to be added to the Figure next the *Index* of the Logarithme to be divided.

So if I were to divide a Logarithme whoſe *Index* is —7 by 2, I need add but 1 to it, and the *Index* of the Quotient ſhall be —4; but if a Logarithme whoſe *Index* is —7 by 3, I muſt add 2, and the *Index* of the Quotient ſhall be —3. The like is to be ſeen of others, and all to be underſtood of Decimal Logarithmes.

Proof of Diviſion *of* Logarithmes.

The Root of any Number extracted agreeing with the Number anſwering to the Quotient of the divided Logarithme, will be Proof enough of the truth of Logarithmical Diviſion.

Partis Tertiæ Libri Tertii

FINIS.

Ocular inſpection in a *Table* receive a former impreſſion in their Minds of the Numbers uſed therein, I have here tranſcribed it a little altered, with the Explanation following.

A Table for Diviſion of Decimal Logarithms.

2)	1				2	1
	3				4	2
	5				6	3
	7				8	4
	9				10	5
					&c.	
	1	2			3	1
Diviſors 3)	4	5			6	2 Indices
	7	8			9	3
					&c.	
4)	1	2	3		4	1
	5	6	7		8	2
					&c.	
5)	1	2	3	4	5	1
	6	7	8	9	10	2
					&c.	
0	10	20	30	40	&c.	

The Table explained.

In this *Table* the Numbers that ſtand to the Left Hand within a bowed Line are Diviſors, or the Numbers of Quantity, as 2, 3, 4, 5, and may be increaſed as the &c. denotes.

The Numbers next the Right Hand are the *Indices* of the Quotient, anſwering collaterally to ſuch Diviſors with ſuch *Indices* of the Dividends as ſtand along with them in the ſame Line from the Right Hand to the Left.

The Numbers next the *Indices* of the Quotient are ſuch *Indices* of the Dividends as will be evenly divided by the Diviſors ſtanding againſt them, and ſo need no Units to be added to them. Theſe may alſo be increaſed at pleaſure, as the ſeveral &c. ſignifie.

But the Numbers next them, if they happen to be the Characteriſtiques of the Logarithmes given to be divided, muſt have 1 Unit added, and the next File of Numbers muſt have 2 Units added, the next File 3, and the next 4, and ſo of others, if the Table be enlarged. Every of which multiplyed by 10, as before noted in the Rule, makes the 10, 20, 30, 40, at the bottom of the Lines, to be added to the Figure next the *Index* of the Logarithme to be divided.

So if I were to divide a Logarithme whoſe *Index* is —7 by 2, I need add but 1 to 7, and the *Index* of the Quotient ſhall be —4; but if a Logarithme whoſe *Index* is —7 by 3, I muſt add 2, and the *Index* of the Quotient ſhall be —3. The like is to be ſeen of others, and all to be underſtood of Decimal Logarithmes.

The Proof of Diviſion of Logarithmes.

The Root of any Number extracted agreeing with the Number anſwering to the Quotient of the divided Logarithme, will be Proof enough of the truth of Logarithmical Diviſion.

THE FOURTH PART OF THE THIRD BOOK.

CHAP. I.

Of COSSICKS.

Numbers ſpecially contract whoſe Denominators are uncertain.

NUmbers ſpecially Contract, that have their Denominators implyed, and becauſe of their certainty omitted, are already handled in the three former Parts of this *Third Book*. The next in order are thoſe ſpecial Contract Numbers, whoſe Denominations are uncertain, and therefore needful to be expreſſed: As *Coſſicks*, *Surdes*, and *Species*.

Coſſicks *what.* *Compound* per ſe.

Coſſicks, are Figural-Numbers-Compound, either *per ſe*, or *inter ſe*.

Thoſe Compound *per ſe*, or by themſelves, are ſuch Figural Numbers of ſome ſingle Species as have annexed to them ſome Abſolute Number. As 3 Roots, 8 Squares, 5 Cubes, or the like.

Compound inter ſe.

Thoſe Compound *inter ſe*, or among themſelves, are ſuch Figural Numbers of different Species as are annexed one to another, and have to each of them ſome Abſolute Numbers alſo adjoyned. As 2 Roots and 3 Squares, or 4 Cubes lacking 2 Roots, or the like.

Abſtract Numbers uſed with Coſſicks *are marked diſtinct*

So as Coſſicks are a Number of Figural Numbers, or a Quantity of Quantities; and in both ſorts the Numbers are Contract to the Figural Quantities whereto they are adjoyned. And when any Abſtract Number is uſed with them beſides the Number of the Quantities, this Abſtract Number is kept diſtinct and marked accordingly.

Characters uſed for what end.

Whence the Name.

Characters uncertain.

The Figural Names or Denominations of the Quantities, as well to avoid prolixity in the often reſcription, as for conveniency in working, are uſually expreſſed by Marks or Characters, under the cover whereof the Numbers got the Name of *Coſſicks*, derived, as is thought, from the *Hebrew* כָּסָה, ſignifying to cover, hide or conceal: And from thence both the *Latin*, *Coſſa* and *Coſſicus*, and the *Italian*, *Coſſica*, whence it ſeems the Name came to us. But as the Denominations are various, and therefore muſt be expreſt; ſo the Stenographical Mantles in which they are wrapt up, are not certain, but arbitrary at the pleaſure of the Operator, as he conceiveth moſt expeditious or commodious for his uſe, and therefore are to be ſought in the reſpective *Nomenclatura* of every Author. In the enſuing Operations let the following Characters be thus underſtood.

A Table of the Coſſical Characters uſed in this Book.

Indices.	*Characters.*	*Signification of the Characters.*
0	N	An Abſolute Number, as if it had no Mark.
1	ƶ	The Root of any Number.
2	ʒ	A Square.
3	φ	A Cube.
4	ʒʒ	A Squared Square, or Zenzizenzike.
5	ſβ	A Surſolide.
6	ʒφ	A Squared Cube, or Zenzicube.
7	Bſβ	A Second Surſolide.
8	ʒʒʒ	A Zenzizenzizenzike, or Square of Squared Square.
9	φφ	A Cubed Cube.
10	ʒſβ	A Square of Surſolids.
11	Cſβ	A Third Surſolide.
12	ʒʒφ	A Zenzizenzicube, or Square of Squared Cubes.
13	Dſβ	A Fourth Surſolide.
14	ʒBſβ	A Square of Second Surſolids.
15	φſβ	A Cube of Surſolids.
16	ʒʒʒʒ	A Zenzizenzizenzizenzike, or Square of Squares Squaredly (Squared.
17	Eſβ	A Fifth Surſolide.
18	ʒφφ	A Zenzicubicube, or Square of Cubick Cubes.
19	Fſβ	A Sixth Surſolide.
20	ʒʒſβ	A Square of Squared Surſolids.
21	φBſβ	A Cube of Second Surſolids.
22	ʒCſβ	A Square of Third Surſolids.
23	Gſβ	A Seventh Surſolide.
24	ʒʒʒφ	A Square of Squares of Squared Cubes, or a Zenzizenzizenzicube.
&c.	&c.	

What Indices the Indices of Coſſicks are.

The Reader may not take theſe *Indices* either for Decimal *Indices*, or the *Indices* of Logarithmes, nor confound the one with the other, for theſe are no other than the Figural *Indices*, or Numbers of the Quantities as they exceed in Power, mentioned before in the *Second Part* of the *Second Book* among Simple Figural Numbers.

How to increaſe the Characters of Coſſicks.

As the Powers may be increaſed, ſo may the Coſſical Characters be formed higher, though it will be rare that ſo many as above ſet down need be uſed. But if any deſire to know how to form the Characters of Higher Powers than theſe; let him firſt ſet down the Figural *Indices* of the Powers in their Natural Order or Progreſſion as far as he pleaſeth, then obſerve, that all the Characters after that of the Root, are but of 3 ſorts, ʒ, φ, ſβ, *viz.* thoſe for the Square, Cube and Surſolide. For whereas B. C. D. *&c.* are uſed with the Surſolids of different Powers, it is but Numerically according as they ſtand in the Alphabet, to ſhew they are the ſecond, third, fourth, *&c.* of that ſort; ſo ſhall the Eighth Surſolide be Hſβ, the Ninth Iſβ, the Tenth Kſβ, and ſo on. This being obſerved it remains then, that as the Figural *Indices* are Numbers Uncompound or Compound, ſo are the Characters, and conſequently the Arithmetical Names of theſe Higher Powers ſignified by the Coſſical Characters.

For Indices Uncompound.

In the *Firſt Book*, *Chap.* 2. it was ſhewed that Uncompound Numbers are procreated by collection of Units, but cannot be made by Multiplication, as 5, 7, 11, 13, 17, 19, 23, 29, 31, *&c.* under all which Uncompound *Indices* after 3, fall Surſolids; therefore under the firſt of them is to be placed the Surſolide Character; under the ſecond, the ſame Character, only with B. the ſecond Letter in the Alphabet added to denote it is a Second Surſolide. Wherefore (as above) againſt 11, the third of theſe uncompound *Indices* is Cſβ, betokening the Third Surſolide, againſt 13, Dſβ, againſt 17, Eſβ, *&c.* And if the *Table* were enlarged, againſt 29 would be Hſβ, and 31, Iſβ, *&c.*

For Indices Compound.

Numbers Compound muſt be compound of 2 or 3, or of 2 and 3, or of 2 or 3, and ſome other Number.

Of 2.

If the *Index* be compound of 2, ſet down ʒ ſo often as 2 is in the compoſition. As 16 being compound of 2 four times, ſhall have ʒʒʒʒ for its Character. So 32 having 2 five times in the compoſition, ſhall have ʒ five times; and 64, ſix times, *&c.*

Of 3.

If the *Index* be compound of 3 only, then ſet down the Character of the Cube as often as 3 is uſed to compound the *Index*. As 9, compounded of 3, twice ſhall be φφ; ſo 27, becauſe it is compounded of 3, thrice ſhall have φφφ for the Character, *&c.*

Of 2 and 3.

If the *Index* be compounded of 2 and 3, then for every time that 2 is multiplyed in the compoſition, ſet down ʒ; and for every time 3 is multiplyed ſet down φ, minding ſtill to ſet ʒ before φ to the Left Hand, becauſe ʒ is the foremoſt Power. Thus againſt 6, made of 2 and 3, is ʒφ; and againſt 12, compounded of 2 by 2, which makes 4, and then by 3, the Character ʒʒφ; ſo 18 hath ʒφφ, becauſe twice 3 is 6, and thrice 6 is 18.

Of 2 or 3, and ſome other.

If the *Index* be compound of 2 or 3, and ſome other Number, then joyn their Characters together in ſuch order, as the Leſſer Figural Powers may precede the Greater. As 10, compounded of 2 and 5, is ʒſδ, where the Mark of the ſecond Quantity being the Leſſer, ſtands before ſδ, the Mark of the fifth Quantity; 15 likewiſe compounded of 3 and 5, is φſδ.

In like manner for Higher *Indices*; as 20, compounded of 4 and 5, ſhall be a ʒʒſδ; 50, of 5 and 10, ſhall be a ʒſδſδ, *&c.*

How ſome ſhorten the Names.

According to the Characters ſo are their Arithmetical Names, ſave that ſome to ſhorten and facilitate the long Names of ſuch Higher Powers, as have the Square or Cube often ingeminated, borrow ſome Names bordering on the *Latin*, and call ʒʒ a Biquadrate, ʒʒʒ a Triquadrate, ʒʒʒʒ a Quaquadrate, ʒʒʒʒʒ a Quinquadrate, *&c.* Alſo φφ a Bicube, φφφ a Tricube, *&c.* and their Compounds accordingly; as ʒʒφ a Biquadratcube, ʒʒʒφ a Triquadratcube, *&c.* But having kept the older Names in the former Treatiſe of Figural Numbers, I have here retained them to their Coſſical Characters.

Nature of Coſſicks.

As to the Nature of Coſſicks, they are either Whole or Broken, and both either Simple or Compound: For though all Coſſicks are Compound Figural Numbers (as aforeſaid), yet are none counted Compound Coſſicks, unleſs they admit of various Denominations, and have their Characters connexed by the Signs + or —.

Of Broken Coſſicks, ſee the 6, 7, 8, 9, 10 and 11th *Chapters* following.

Simple Whole Coſſicks.

Simple Coſſicks then that are Whole or Integral, are Homogeneal, or Quantities of one ſort: As 4 ℨ, or 10 ʒ, or 8 φ, *&c.* to be read, 4 Roots, 10 Squares, 8 Cubes, and ſhall be underſtood with no more relation one to another, than 4 Inches, 10 Planks, 8 Trees, becauſe they are not knit together by any Sign to bring them under an obligation of relation to one and the ſame Root, from whence they ſhould all ſpring: Yet if Simple Coſſicks are to be added or ſubſtracted, multiplyed or divided with other Simple Coſſicks, then they are underſtood to have all one Root.

Compound Whole Coſſicks

Compound Whole Coſſicks, connexing together divers Simple Coſſicks, are Heterogeneal. As 8 φ + 4 ℨ, read thus, 8 Cubes more [or and] 4 Roots: So 10 ʒ — 4 ℨ, that is 10 Squares lacking 4 Roots, whoſe Quantities are ever conſidered in ſumme, according to the mixture of the Signs + or —: For they are increaſed by +, and diminiſhed by —.

Compound of 3 ſorts. Binomials. Reſiduals *or* Apotomes. Medials, *how otherwiſe called.*

Compound Coſſicks are of 3 ſorts.

Thoſe connexed by the Additional Sign +, are called *Binomials.*

Thoſe connexed by the Sign of Subſtraction —, are called *Reſiduals*, and ſometime *Apotomes.*

Thoſe knit together by both Signs are called *Medials*, and by ſome *Multinomials*, or *Polynomials*, that is, many named. And yet more Coſſicks than 2 joyned with + or —, deſerve as well as others the name of a *Polynomial*, and are improperly called *Binomials* or *Reſiduals.*

Examples.

Examples of Binomials.

6 ʒ + 4 ℨ that is, 6 Squares and 4 Roots.
5 φ + 3 ʒ + 10 ℨ is 5 Cubes and 3 Squares and 10 Roots.

Examples of Reſiduals.

12 φ — 5 ʒ that is, 12 Cubes lacking 5 Squares.
10 ſδ — 3 φ — 2 ʒ is 10 Surſolids lacking 3 Cubes and 2 Squares.

Examples of Medials or Polynomials.

8 ʒʒ + 2 φ — 4 ʒ that is, 8 Squared Squares and 2 Cubes, lacking 4 Squares.
5 φ — 4 ʒ + 5 ℨ is 5 Cubes and 5 Roots, lacking 4 Squares.

Difference of Simple and Compound Coſſicks.

Coſſicks Simple and Compound, being all Rooted Numbers, admit of this difference, that the Simple having not relation one to another, may have different Roots indefinitely;

nitely; but Compound Coſſicks in one Queſtion alwayes imply one and the ſame definite Root to all the Quantities joyned together. Nevertheleſs ſeveral Compound Coſſicks, or the ſame Coſſicks in ſeveral Queſtions or Operations may have ſeveral Roots. For 3 ℨ, 10ʒ, the Simple Coſſicks; ſuppoſe one Root be 3, the 3 ℨ ſhall be 9, in Abſolute Number: But the 10ʒ are not tyed to the Root 3, but may have any other Number for the Root: Except it were 10ʒ + 3 ℨ, or 10ʒ — 3 ℨ; then if the ℨ be 3, the Root of the ʒ ſhall alſo be 3, and the 10ʒ in Abſolute Numbers 90. Yet in another Operation the 10ʒ + 3 ℨ are capable of any other Abſolute Number for the Root thereof.

Sign + where underſtood. Coſſicks *how placed.*

In Coſſicks, where the ſign — is not, the ſign + is underſtood, and on the outmoſt to the Left Hand of Compound Coſſicks commonly omitted. And in all Polynomials it is moſt uſual, though not eſſential except in Multiplication and Diviſion, to ſet all the Coſſicks connexed with + to the Left Hand of thoſe connexed with —, though theſe are Quantities of Higher Powers than the other. Some promiſcuouſly uſe the word Sign, as well for the Coſſick Quantity, as for the ſign + or —. But it is moſt orderly to reſerve the name of Sign only to theſe. And this is to be remembred both in *Surdes* and *Species*. And alſo that theſe ſigns + and — be uſed, where a Simple Quantity or Magnitude is affirmative or negative to another Simple Quantity or Magnitude: But when an Aſteriſque is ſet over either of them, then the Quantity or Magnitude-Compound is affirmed or denyed of a Simple, or a Simple of a Compound. As 5φ $\overset{*}{-}$ 4ʒ + 5 ℨ ſhall ſignifie, that the 5 Cubes ſhall want 4ʒ and 5 ℨ, which without the Aſteriſque, the 5 ℨ affirmative and the 5φ, ſhall be added together, and from the ſumme only the 4ʒ ſhall be deducted.

Word Sign how uſed.

Signs uſed with or without Aſteriſques *what noted thereby.*

Abſolute Numbers uſed with Coſſicks, *ſometime not marked.*

Coſſicks Simple and Compound, Whole and Broken, take into their ſociety Abſolute Numbers, marked as before with N. and ſometime uſed without any Character.

CHAP. II.

Addition of Whole Coſſicks.

Addition *of Whole* Coſſicks *Simple and Homogeneal.*

TO add Simple Coſſicks, if they be Homogeneal, add the Numbers together as Integers, and to the Total adjoyn the Coſſical Character common to the given Numbers.

As 5 ℨ added to 10 ℨ, make 15 ℨ. So 15ʒ to 40ʒ are 55ʒ.

Examples.

		Homogeneal.		
Simple	5 ℨ 10 ℨ	15ʒ 40ʒ	30φ 10φ	} Addends.
	15 ℨ	55ʒ	40φ	Totals.

Simple and Heterogeneal.

If the Simple Coſſicks to be added are Heterogeneal, then place the Higheſt Power to the Left Hand, and connex the other thereto by the ſign of Addition +.

As to add 14ʒ to 10φ; or 30φ to 4ʒ and 15ʒʒ; they are ſet thus,

Examples.

		Heterogeneal.	
Simple	10φ 14ʒ	15ʒʒ 30φ 4ʒ	} Addends.
	10φ + 14ʒ	15ʒʒ + 30φ + 4ʒ	Totals

Compound of like Signs.

To add Compound Coſſicks, add together like Coſſicks with like, as ℨ with ℨ, and ʒ with ʒ, *&c.* alſo + with +, and — with —. But if any Coſſick be odd, or Heterogeneal to the others given to be added, adjoyn him to the Total with his proper Sign.

Examples. As to add 12ʒ + 10ℨ to 5ʒ + 6ℨ: Or 12ℨ — 10ℨ to 5ʒ — 6ℨ; the Total of the Binomials will be 17ʒ + 16ℨ, and of the Reſiduals 17ʒ — 16ℨ. But if 10φ + 10ʒ were added to 4φ + 3ʒ + 2ℨ; or to 4φ + 3ʒ — 2ℨ, there the Totals ſhall be 14φ + 13ʒ + 2ℨ, or 14φ + 13ʒ — 2ℨ.

Binomials.	Reſiduals.	Medials or Polynomials.	
12ʒ + 10ℨ	12ʒ — 10ℨ	10φ + 10ʒ	10φ + 10ʒ
5ʒ + 6ℨ	5ʒ — 6ℨ	4φ + 3ʒ + 2ℨ	4φ + 3ʒ — 2ℨ
17ʒ + 16ℨ	17ʒ — 16ℨ	14φ + 13ʒ + 2ℨ	14φ + 13ʒ — 2ℨ

Compound of unlike Signs. If the Compound Coſſicks have their Signs unlike, then take the Leſſer Number out of the Greater, and to the Remain, which ſhall be the Total, ſubſcribe the Sign that belongeth to the Greater Number, whether it be + or — accordingly.

Examples. As to add 12ʒ + 10ℨ to 5ʒ — 6ℨ, or 12ʒ — 10ℨ with 5ʒ + 6ℨ, the Total of the former will be 17ʒ + 4ℨ, of the latter 17ʒ — 4ℨ, as at *A.* and *B.* Other Examples of Polynomials follow at *C. D. E.*

	Binomials and	Reſiduals.		Polynomials.
A.	12ʒ + 10ℨ	*B.*	12ʒ — 10ℨ	*C.* 46ʒʒ + 10φ + 2ʒ
	5ʒ — 6ℨ		5ʒ + 6ℨ	16ʒʒ + 8φ — 5ʒ
	17ʒ + 4ℨ		17ʒ — 4ℨ	62ʒʒ + 18φ — 3ʒ

D.	18φ + 16ʒ — 9N.	*E.*	4ß + 16ʒʒ + 19ʒ
	4φ — 10ʒ + 4N.		3ß — 13φ — 14ʒ
	22φ + 6ʒ — 5N.		7ß + 16ʒʒ — 13φ + 5ʒ

Proof of Coſſical Addition. To prove Coſſical Addition, beſides the tryal by Coſſical Subſtraction, reſolve your Coſſicks into Abſtract Numbers, by ſuppoſing 2, 3, or ſome other Number for a Root, and ſo accordingly getting the ſumme of the ʒ, φ, *&c.* of the other Coſſicks, and then compare the Numbers to be added with the Total of the Addition, and the ſummes will be parallel when the Operation is right.

As in the former inſtance at *A*, ſuppoſe 2 be a Root, then is 10ℨ 20, and 4 being the Square of 2, the 12ʒ ſhall be 48; which 48 and 20 make 68; then 5ʒ more is 20 lacking 6ℨ which is 12, leave 8 remaining, this added to 68 makes the Total 76. And ſo much is 17ʒ and 4ℨ, ſuppoſing 2 for a Root.

12ʒ + 10ℨ	48 + 20 = 68
5ʒ — 6ℨ	20 — 12 = 8
17ʒ + 4ℨ	68 + 8 = 76

CHAP. III.

Subſtraction of Whole Coſſicks.

Subſtraction of Whole Coſſicks. *Simple and Homogeneal.* To ſubſtract Simple Coſſicks, if they be Homogeneal, take the Leſſer Number out of the Greater, and to the Remain ſubſcribe the Coſſical Character common to both the given Numbers, when the Subtrahend is the leaſt, but if it be the greateſt of the two, then change the Sign to the Remain.

Examples. As to abate 3φ from 10φ, there will remain 7φ: But if 10φ were to be taken from 3φ, there will lack 7φ, therefore the Sign ſhall be — to the Remain.

	Homogeneal.		
Simple	10φ	3φ	Numbers from which Subſtraction is made.
	3φ	10φ	Subtrahends.
	7φ	—7φ	Remains.

Simple and Heterogeneal.

If the Simple Coſſicks to be ſubſtracted are Heterogeneal, then place the Higheſt Power to the Left Hand, and connex the other thereto with the Sign of Subſtraction —.

Examples.

As to take 4 ℨ out of 10 ʒ, or 5φ from 20ʒʒ + 2 ℨ, they are ſet thus;

	Heterogeneal.		
Simple	10ʒ	20ʒʒ + 2 ℨ	Numb. from which Subſtraction is made.
	4 ℨ	5φ	Subtrahends.
	10ʒ — 4 ℨ	20ʒʒ — 5φ + 2 ℨ	Remain.

Compound of like Signs.

To ſubſtract Compound Coſſicks, take like Coſſicks from like, as ℨ from ℨ, and ʒ from ʒ, *&c.* alſo + from +, and — from —, and to the Remain ſubſcribe the ſame Sign, except the Number to be ſubſtracted be the greater, then change the Sign, and ſet down thereto the difference of the Numbers. And if any ſingle Coſſick have none to fellow him, adjoyn him to the Remain with the contrary Sign if he belonged to the Subtrahend, but with the ſame Sign he hath if he belonged to the Number from which Subſtraction is made.

Examples.

As to take 5ʒ + 6 ℨ from 12ʒ + 10 ℨ, the Remain ſhall be 7ʒ + 4 ℨ, as at *A.* So 5ʒ — 6 ℨ from 12ʒ — 10 ℨ, ſhall leave 7ʒ — 4 ℨ, as at *B.* But to ſubſtract 5ʒ + 10 ℨ from 12ʒ + 6 ℨ, the Remain ſhall be 7ʒ — 4 ℨ, as at *C.* And 5ʒ — 10 ℨ from 12ʒ — 6 ℨ, ſhall leave 7ʒ + 4 ℨ, as at *D.* For in both theſe latter the Roots to be ſubſtracted being the Greater Number changeth the Sign to the difference of the Numbers.

Alſo if 3φ + 4ʒ + 10N, be taken from 16φ + 18ʒ, the Remain ſhall change the Sign to N, as at *E.* But if N had not been in the Subtrahend, he ſhall keep his Sign as at *F.*

	Binomials.		Reſiduals.		Medials or Polynomials.
A.	12ʒ + 10 ℨ	*B.*	12ʒ — 10 ℨ	*E.*	16φ + 18ʒ
	5ʒ + 6 ℨ		5ʒ — 6 ℨ		3φ + 4ʒ + 10N
	7ʒ + 4 ℨ		7ʒ — 4 ℨ		13φ + 14ʒ — 10N
C.	12ʒ + 6 ℨ	*D.*	12ʒ — 6 ℨ	*F.*	16φ + 18ʒ + 10N
	5ʒ + 10 ℨ		5ʒ — 10 ℨ		3φ + 4ʒ
	7ʒ — 4 ℨ		7ʒ + 4 ℨ		14φ + 14ʒ + 10N

Compound of unlike Signs.

If the Compound Coſſicks have contrary Signs, add the Coſſicks with their fellows of unlike Signs, and to the Total, which is the Remain, adjoyn the Sign of the upper Number, that is that from which Subſtraction is to be made.

Examples.

As to take 5ʒ — 6 ℨ from 12ʒ + 10 ℨ, or 5ʒ — 10 ℨ from 12ʒ + 6 ℨ, in both the Remain ſhall be 7ʒ + 16 ℨ, as at *G.* and *H.* which 16 ℨ is the ſumme of — 6 ℨ and + 10 ℨ, or — 10 ℨ and + 6 ℨ. But if 5ʒ + 6 ℨ be taken from 12ʒ — 10 ℨ, or 5ʒ + 10 ℨ from 12ʒ — 6 ℨ, in both theſe Caſes the Remain ſhall be 7ʒ — 16 ℨ, as at *I.* and *K.* becauſe the Sign of the upper Number was —. Other Examples of Polynomials follow at *L.* and *M.*

	Binomials and Reſiduals.				Polynomials.
G.	12ʒ + 10ℨ	*I.*	12ʒ − 10ℨ	*L.*	3φ + 10ʒ + 100N
	5ʒ − 6ℨ		5ʒ + 6ℨ		9φ + 10ʒ − 30N
	7ʒ + 16ℨ		7ʒ − 16ℨ		130N − 6φ
H.	12ʒ + 6ℨ	*K.*	12ʒ − 6ℨ	*M*	3ʒ + 19ℨ − 10N
	5ʒ − 10ℨ		5ʒ + 10ℨ		18ℨ − 3ʒ + 4N
	7ʒ + 16ℨ		7ʒ − 16ℨ		6ʒ + 1ℨ − 14N

To prove Coſſical Subſtraction, as before in Addition, by ſome fit Root convert the Coſſicks into Abſtract Numbers, and making Subſtraction as in Integers, the Remains will be left equal, if the Operations be right.

Proof of Coſſical Subſtraction.

As in the Example above at *G*, ſuppoſing the Root 2, then ſhall 10ℨ be 20, and 12ʒ be 48, in all 68. And the Subtrahend 5ʒ ſhall be 20 lacking 12, which is 6ℨ, that is 8, which 8 taken from 68, leaves 60 equal to 7ʒ, that is 28, and 16ℨ which is 32.

12ʒ + 10ℨ	48 + 20 = 68
5ʒ − 6ℨ	20 − 12 = 8
7ʒ + 16ℨ	28 + 32 = 60

Beſides this Proof, if you add 7ʒ + 16ℨ to 5ʒ − 6ℨ, the Total will be 12ʒ + 10ℨ, as before, and ſhew the alternate Proof of Coſſical Addition by Subſtraction, and Subſtraction by Addition.

CHAP. IV.

Multiplication of Whole Coſſicks.

TO multiply Simple Coſſicks, multiply Number by Number as in Integers, and add the reſpective *Indices* of the Coſſical Quantities, and the Total ſhall be the *Index* of the Product, whoſe Character is to be affixed thereto.

Multiplication of Whole Coſſicks. Simple. Examples.

As to multiply 3φ by 9ʒ, the Numbers 3 and 9 multiplyed produce 27, and 2 the *Index* of ʒ, added to 3 the *Index* of φ, make 5, which is the *Index* of ſβ; ſo is the Product 27ſβ.

So 10ʒʒ multiplyed by 8φ, produce 80Bſβ; for the *Index* of φ 3, added to 4 the *Index* of ʒʒ, make together 7, which hath Bſβ for his Character.

Multiplicands	3φ	3	10ʒʒ	4	30ſβ	5	Indices.
Multiplyers	9ʒ	2	8φ	3	10ʒʒ	4	
Product	27ſβ	5	80Bſβ	7	300φφ	9	

If Compound Coſſicks be multiplyed, let the Coſſicks be placed according to their Quantities, and every Number in the Multiplicand be multiplyed by every Number in the Multiplyer, and the reſpective *Indices* thereof gotten, as if they were Simple Coſſicks. And for the Signs, as before in other Contract Numbers, obſerve like Signs multiplyed together produce +, and unlike —.

Compound.

As to multiply 3φ + 2ʒ − 4ℨ by 2ʒ + 3ℨ − 8N, the Product will be 6ſβ + 13ʒʒ − 26φ − 28ʒ + 32ℨ, as by the Operation it ſelf beginning at the Left Hand, and after Multiplication adding the ſeveral Lines of Production, or Multiplees together, appeareth plainly thus,

Example.

Multiplicand	3φ + 2ʒ — 4ℨ
Multiplyer	2ʒ + 3ℨ — 8N
Multiplees	6ß + 4ʒʒ — 8φ 9ʒʒ + 6φ — 12ʒ — 24φ — 16ʒ + 32ℨ
Product	6ß + 13ʒʒ — 26φ — 28ʒ + 32ℨ

Proof of Coſſical Multiplication. To prove Coſſical Multiplication, beſides the tryal by Coſſical Diviſion, turn the Coſſicks into Abſtract Numbers, taking at pleaſure ſome fit Number for a Root, and after Multiplication of the Numbers as Integers, compare the Product with the ſumme of the Coſſical Product, for without Error they exactly agree.

As if 3ʒ + 4ℨ be multiplyed by 2ʒ — 3ℨ, the Product will be 6ʒʒ — 1φ — 12ʒ. To prove which I ſuppoſe 2 the ℨ, then ſhall 4ℨ be 8, and 3ʒ, 12, in ſumme 20. And 2ʒ, 8, lacking 3ℨ, 6, makes the ſumme of the Multiplyer but 2, which multiplying 20, produceth 40, and ſo much is the ſumme of 6ʒʒ — 1φ — 12ʒ.

3ʒ + 4ℨ	12 + 8 ═ 20
2ʒ — 3ℨ	8 — 6 ═ 2
6ʒʒ + 8φ	96 + 64 ═ 160
— 9φ — 12ʒ	— 72 — 48 ═ — 120
6ʒʒ — 1φ — 12ʒ	96 — 8 — 48 = 40

CHAP. V.

Diviſion of Whole Coſſicks.

Diviſion *of Whole* Coſſicks *Simple.* TO divide Simple Coſſicks, divide Number by Number, as in Integers, and ſubſtract the *Index* of the Diviſor from the *Index* of the Dividend, and the Character belonging to the Remaining *Index* adjoyn to the Quotient. And if the Numbers will not evenly be divided without a Remainer, or the Diviſor have the Greater Coſſick, then are they to be ſet, as Coſſical Fractions.

Examples. As to divide 40ʒʒ by 10φ, the Quotient will be 4ℨ: For if 3, the *Index* of φ, be abated from 4, the ʒʒ *Index*, the Remain will be 1, the *Index* of ℨ.

So 60ß, divided by 5ʒ, gives in the Quotient 12φ.

	Indices.		*Indices.*		
Dividends	4	40ʒʒ (4ℨ	5	60ß (12φ	Quotients
Diviſors	3	10φ	2	5ʒ	
	1		3		

Compound. If Compound Coſſicks be divided, then as before in Compound Decimal and Aſtronomical Diviſion, let the Numbers in the Dividend be divided by the Numbers in the Diviſor. The *Indices* of the Quotienary Numbers are got as in Diviſion of Simple Coſſicks. And the Signs alike give +, and unlike —, as in Multiplication of Compound Coſſicks and other Contract Numbers. If any Coſſical Quantity be omitted in the given Numbers, expreſs the ſame with Cyphers, and the Sign +, and place all Coſſicks according to their Powers, whether their Signs be + or —. And if the Numbers will not be evenly divided, or the Diviſor have the Greater Coſſicks, then the Diviſor is placed beneath to repreſent it as a Fraction.

Example. As to divide 6ß + 13ʒʒ — 26φ — 28ʒ + 32ℨ by 2ʒ + 3ℨ — 8N, after by the firſt application of the Diviſor to the Dividend 3φ is gotten in the Quotient, the Diviſor is multiplyed thereby, and the Product 6ß + 9ʒʒ — 24φ ſubſtracted from the Dividend; by the ſecond application of the Diviſor 2ʒ is gotten in the Quotient, the Product of the Diviſor multiplyed thereby is 4ʒʒ + 6φ — 16ʒ. And after this ſubſtracted the laſt application of the Diviſor gives 4ℨ in the Quotient, and the Diviſor multiplyed thereby produceth a Coſſick Quantity equal to what was left on the Dividend after the former Subſtractions.

Diviſor **Dividend** **Quotient**

$$\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad - 8φ \\
\qquad\qquad\qquad\qquad \cancel{4}ʒʒ - \cancel{2}φ - \cancel{1}\cancel{2}ʒ \\
2ʒ + 3ℨ - 8N)\ \overline{\cancel{6}ſβ + \cancel{1}\cancel{3}ʒʒ - \cancel{2}\cancel{6}φ - \cancel{2}8ʒ + \cancel{3}\cancel{2}ℨ}\ (3φ + 2ʒ - 4ℨ \\
\qquad\qquad\qquad 6ſβ + 9ʒʒ - 24φ \\
\qquad\qquad\qquad\qquad 4ʒʒ + 6φ - 16ʒ \\
\qquad\qquad\qquad\qquad\qquad - 8φ - 12ʒ + 32ℨ
\end{array}$$

To prove Coſſical Diviſion as before in the other Elementary Operations, let the Coſſicks be exchanged for Abſtract Numbers, at pleaſure taking ſome fit Number for a Root, and the Diviſion as in Integers performed, the Quotient will parallel the ſumme of the Coſſical Quotient ſo exchanged, when the Diviſion is well wrought. *Proof of* Coſſical Diviſion.

As if 8φ + 64N, be divided by 2ℨ + 4N, the Quotient will be thus, 4ʒ — 8ℨ + 16N. I then ſuppoſe the Root 2, the Dividend then ſhall be 128, for ſo much is 8φ and 64N. The Diviſor is 8, that is 2ℨ which are 4, and 4 Numbers over. And the Quotient of 128 divided by 8 is 16, which agreeth exactly with the Coſſical Quotient: For 4ʒ is 16, and 16N make 32, from which 8ℨ taken which are 16, there is left but 16 for the ſumme of the Quotient.

$$\begin{array}{l}
\qquad\qquad\qquad - \cancel{1}\cancel{6}ʒ + \cancel{3}\cancel{2}ℨ \\
2ℨ + 4N)\ \overline{8φ + \cancel{0}ʒ + \cancel{0}ℨ + \cancel{6}\cancel{4}N}\ (4ʒ - 8ℨ + 16N \\
\qquad\qquad 8φ + 16ʒ \\
\qquad\qquad\qquad - 16ʒ - 32ℨ \\
\qquad\qquad\qquad\qquad + 32ℨ + 64N
\end{array}$$

$$\begin{array}{l}
\qquad\qquad - \cancel{6}\cancel{4} + \cancel{6}\cancel{4} \\
4 + 4)\ \overline{\cancel{6}\cancel{4} + \cancel{0} + \cancel{0} + \cancel{6}\cancel{4}}\ (16 - 16 + 16 \qquad \frac{\cancel{1}\cancel{2}8}{8}(16 \\
\qquad\quad 64 + 64 \\
\qquad\qquad - 64 - 64 \\
\qquad\qquad\qquad + 64 + 64
\end{array}$$

Beſides this Proof, if you multiply 4ʒ — 8ℨ + 16N by 2ℨ + 4N, the Product will be the Dividend 8φ + 0ʒ + 0ℨ + 64N, as before, and thereby ſhew the alternate Proof of Multiplication by Diviſion, and Diviſion by Multiplication in Coſſicks, as well as other Numbers.

CHAP. VI.

Of Broken Coſſicks.

As Whole Coſſicks (of which the foregoing Five *Chapters* have ſufficiently ſpoken) are of two ſorts, *viz.* Simple and Compound; ſo are their Fractions. *Broken* Coſſicks.

Simple Coſſical Fractions are ſome Broken Parts, or a part only of ſome Single Coſſick, and are expreſſed like Vulgar Fractions with the Coſſical Character or Denomination annexed. *Simple how expreſſed.* As $\frac{2}{3}$ ℨ ſignifieth $\frac{2}{3}$ of a Root, let the Root be what it will. *Examples.* So $\frac{3}{5}$ ʒ importeth $\frac{3}{5}$ of a Square, *&c.* And hereby is underſtood that the Coſſical Quantity is divided into ſo many parts as the Denominator denoteth, and a certain number of thoſe parts to be taken as the Numerator denoteth. And hence it may happen ſometime that the Coſſical Fraction may in value be an Integer, and no Fraction. For if the Coſſical Fraction be $\frac{3}{4}$ʒ, if the Square be 16, then ſhall $\frac{3}{4}$ of that Square be 12 Integers. Alſo $\frac{2}{3}$φ ſhall be 18 Integers, if the Cube be 27. All theſe Simple Coſſical Fractions may be ſet as Compound, by placing the Coſſical Denomination to the Numerator, and N to the Denominator; or elſe leaving the Denominator as an Integer: For $\frac{2}{3}$ʒ is as $\frac{2\text{ʒ}}{3\text{N}}$ or $\frac{2\text{ʒ}}{3}$.

Compound of 2 ſorts.

Dual.

Examples.

Compound Coſſical Fractions are of two ſorts, Dual or Plural.

Dual, when the Fraction conſiſteth only of two Coſſical Denominations, and look like Simple Fractions. As $\frac{3\text{ʒ}}{2\varphi}$ which is as much as 3ʒ to be divided by 2φ. So $\frac{4\text{ß}}{9\text{ʒ}\varphi}$ import that 9ʒφ muſt divide 4ß, &c.

Plural.

Example.

Plural Coſſical Fractions are when the Numerator or Denominator, or both, conſiſt of more than two Coſſical Quantities. As $\frac{4\varphi + 3\text{ℨ} - 10\text{N}}{3\text{ℨ} + 12\text{N}}$ that implyeth that 4 Cubes and 3 Roots lacking 10 Numbers are to be divided by 3 Roots and 12 Numbers added together; and ſo of others.

Proper and Improper how known.

Coſſical Fractions are alſo Proper and Improper, when the Denominator is the Greater Coſſick, it is a Proper Fraction, but Improper when the contrary.

CHAP. VII.

Reduction of Broken Coſſicks.

Broken Coſſicks reduced.

To their leaſt Termes.

THE Reduction of Coſſical Fractions, is either to reduce them to their leaſt Terms, or to like Denominators.

The firſt ſort, as in Vulgar Fractions, may be called Abbreviation; for the Simple Coſſical Fractions may be reduced lower ſometime in their Numbers, and the Compound ſometime both in their Numbers and Quantities. But if the Numbers be incommenſurable in either, or any one Quantity of the Compound be N, then each of them reſpectively muſt be kept as they happen, unaltered, unleſs alterable by the ſubſequent Propoſition.

Examples.

As $\frac{24}{28}$ʒ may be reduced in its Numbers, by the Rules of abbreviating Fractions ſeen before in the *Second Part* of the *Firſt Book*, *Chap.* 2. to $\frac{6}{7}$ʒ, as at *A*.

So $\frac{30\varphi}{36\text{ʒʒ}}$ may be abbreviated in its Numbers to $\frac{5\varphi}{6\text{ʒʒ}}$, and in its Quantities to $\frac{5\text{N}}{6\text{ℨ}}$, there being alike Quantities abated from the Coſſical Numerator and Denominator; ℨ being as far diſtant from ʒʒ, as N from φ. See below at *B*.

Alſo in Plural Fractions, as $\frac{3\text{ʒʒ} + 6\text{ʒ}}{3\text{ß} + 12\varphi}$ the Numbers being all commenſurable by 3, may be reduced firſt in its Numbers to $\frac{1\text{ʒʒ} + 2\text{ʒ}}{1\text{ß} + 4\varphi}$, and in the Quantities till one of them be an Abſolute Number, thus, $\frac{1\text{ʒ} + 2\text{N}}{1\varphi + 4\text{ℨ}}$, as at *C*.

		Abbreviated,			
Simple Coſſical Fraction	$\frac{24}{28}$ʒ	$\frac{12}{14}$ʒ	$\frac{6}{7}$ʒ		*A*.
Compound Coſſical — Dual Fraction	$\frac{30\varphi}{36\text{ʒʒ}}$	$\frac{15\text{ʒ}}{18\varphi}$	$\frac{5\text{ℨ}}{6\text{ʒ}}$	$\frac{5\text{N}}{6\text{ℨ}}$	*B*.
		in Numbers	in Quantities		
Compound Coſſical — Plural Fraction	$\frac{3\text{ʒʒ} + 6\text{ʒ}}{3\text{ß} + 12\varphi}$	$\frac{1\text{ʒʒ} + 2\text{ʒ}}{1\text{ß} + 4\varphi}$	$\frac{1\text{ʒ} + 2\text{N}}{1\varphi + 4\text{ℨ}}$		*C*.

The

The Propoſition above-mentioned.

Coſſicks in their leaſt Terms ſometime ſhortned.

Sometimes, and but ſometimes, it happeneth, when yet the Numbers are incommenſurable, and the Quantities reduced as low as N, that ſome part of the compoſition may be abated, and yet the Remain be in like proportion.

Examples.

As $\frac{4\varphi + 12ʒ}{8ʒ\varphi + 24ß}$ reduced in its Numbers is $\frac{1\varphi + 3ʒ}{2ʒ\varphi + 6ß}$ then in its Quantities is $\frac{1ƶ + 3N}{2ʒʒ + 6\varphi}$. But now if I abate the Quantities that follow the Sign of Compoſition +, yet the remaining Fraction will retain the ſame proportion, and $\frac{1ƶ}{2ʒʒ}$ or $\frac{1N}{2\varphi}$ ſhall be equal.

Alſo Reſidual Numbers, as $\frac{4\varphi - 12ʒ}{8ʒ\varphi - 24ß}$ may be reduced to $\frac{1ƶ - 3N}{2ʒʒ - 6\varphi}$, and conſequently to $\frac{1N}{2\varphi}$ as before.

When this happeneth what muſt be.

For the better underſtanding of this kind of Reduction, obſerve;

1. That the Signs muſt be *Synonima's*, that is both + or both —; for if one be more, and the other leſs, it will not ſuffer this Reduction.

2. The Numbers taken away muſt be in like proportion to them that remain. As in the former Examples, 3 to 6, is as 1 to 2, or 4 to 8, as 12 to 24.

3. As the Numbers, ſo the Coſſical Quantities muſt alſo be in like proportion. For in the former Examples, N to φ is as ƶ to ʒʒ, the difference 3, being between their ſeveral *Indices*.

The Reaſon thereof.

So that if the difference of the abatement unto abatement, be as the whole is in proportion to the whole, then ſhall the reſidue be in like proportion to the reſidue, as the whole is to the whole, by *Euclid.* 5 *Lib.* 19 *Prop.*

To reduce Coſſicks to one Denominator. Simple.

The ſecond ſort of Coſſical Reduction, to bring Coſſicks of different Denominations into one, comprehendeth,

1. To reduce Simple Fractions to one Denomination, which is effected as Abſtract Fractions, without altering the Coſſical Quantities.

Example.

And ſo $\frac{2}{3}ʒ + \frac{3}{4}\varphi$ reduced, ſhall ſtand as at *D*, and be $\frac{8ʒ \quad 9\varphi}{12}$.

$$D.\quad \begin{array}{ccc} 8ʒ & & 9\varphi \\ \frac{2}{3}ʒ & \times & \frac{3}{4}\varphi \\ & 12 & \end{array}$$

Dual.

2. To reduce Dual Fractions to one Denomination. And this differs nothing from the other, but in increaſing the Coſſical Quantities according to the nature of Coſſical Multiplication.

Example.

And ſo $\frac{3ʒ}{4\varphi}$ reduced with $\frac{1ƶ}{3N}$ ſhall ſtand as at *E*, and be $\frac{9ʒ \quad 4ʒʒ}{12\varphi}$.

$$E.\quad \begin{array}{ccc} 9ʒ & & 4ʒʒ \\ \frac{3ʒ}{4\varphi} & \times & \frac{1ƶ}{3N} \\ & 12\varphi & \end{array}$$

Plural.

3. To reduce Plural Fractions to one Denomination, which *mutatis mutandis* is like the laſt.

Example.

And ſo $\frac{1ʒ + 2ƶ}{2\varphi + 3N}$ reduced with $\frac{2\varphi - 4N}{3ʒʒ - 5ƶ}$, ſhall ſtand as at *F.*; and be $\frac{3ʒ\varphi + 6ß - 5\varphi - 10ʒ \qquad 4ʒ\varphi - 2\varphi - 12N}{6Bß - 13ʒʒ - 15ƶ}$, as by the reſpective Multiplications of Denominator by Denominator, and then alternately the Denominator of each into the others Numerator will appear.

$$F.\quad \begin{array}{ccc} 3ʒ\varphi + 6ß - 5\varphi - 10ʒ & & 4ʒ\varphi - 2\varphi - 12N \\ \frac{1ʒ + 2ƶ}{2\varphi + 3N} & \times & \frac{2\varphi - 4N}{3ʒʒ - 5ƶ} \\ & 6Bß - 13ʒʒ - 15ƶ & \end{array}$$

Mixt turned into Improper Fractions.

4. To reduce Mixt Numbers, *viz.* Whole and Broken Coſſicks into an Improper Fraction, or to ſet an Whole Coſſick in form of a Fraction, both like as was ſhewed in Abſtract Fractions.

Examples.

And ſo 2ʒ $\frac{2N}{3ℨ}$ ſhall be reduced into $\frac{6φ + 2N}{3ℨ}$ by multiplying 2ʒ, the Whole Coſſick, into 3ℨ the Denominator of the Fraction, which make 6φ, and then adding thereto 2N, makes the Numerator 6φ + 2N, to which the 3ℨ ſhall be the Denominator.

Abſurd Numbers what.

And if $\frac{2N}{3ℨ} - 2ʒ$, which indeed is an abſurd Number, or leſs than nothing were to be reduced, the Reduction ſhall be $\frac{2N - 6φ}{3ℨ}$.

And if 2ʒ + 3N $\frac{3φ - 2N}{4ʒʒ}$ be reduced, the Improper Fraction will ſtand thus, $\frac{8ʒφ + 12ʒʒ + 3φ - 2N}{4ʒʒ}$.

Whole Coſſick ſet as a Fraction.

But if any Whole Coſſick be ſet Fraction wiſe, there is only 1N to be ſubſcribed.

And ſo 3φ + 4ʒ − 2ℨ ſhall be ſet thus, $\frac{3φ + 4ʒ - 2ℨ}{1N}$.

Improper turned back to Integral.

5. To reduce Improper Coſſical Fractions back into Whole Coſſicks, or an Whole and Broken Coſſick, divide the Numerator by the Denominator, whereby ſometime the Denominator is wholly diſcharged, when nothing remains upon the Diviſion, or the Integer turned into a Mixt Number.

Examples.

And ſo $\frac{63φ}{21ʒ}$ will be reduced into 3ℨ. And $\frac{30ß + 24ʒʒ + 18φ}{20ʒ}$ into $1\frac{1}{2}φ + 1\frac{1}{5}ʒ + \frac{9}{10}ℨ$.

Many Fractions to one Denomination.

6. To reduce ſeveral ſorts of Fractions, or many of one ſort, into one Denomination, by the Reduction proper to each of them.

Examples.

As $\frac{3}{4}φ$ reduced with $\frac{2ʒ}{3φ}$ ſhall be $\frac{9ʒφ \quad 8ʒ}{12φ}$.

And ſo $\frac{1ℨ}{2N}$ and $\frac{2N}{3ℨ}$ reduced with $\frac{3ʒ - 2N}{5φ}$ ſhall be $\frac{15ß \quad 20φ \quad 18φ - 12ℨ}{30ʒʒ}$ and abbreviated, $\frac{15ʒʒ \quad 20ʒ \quad 18ʒ - 12N}{30φ}$.

Proof of Coſſical Reduction.

One part of Coſſical Reduction hath the ſame faculty with other Reductions, to prove the other part reciprocal thereto, as may eaſily be diſcerned without Example. And moreover, by reſolving the Coſſical Fractions into abſtract Numbers, as before in the works of Whole Coſſicks, every part of Reduction may be fully proved.

As in the laſt Example, ſuppoſing 2 be a Root, then ſhall each of the Coſſical Fractions be as at *G*, and the reduced Fractions found to agree in value without and with abbreviation.

$$\begin{array}{lccc}
 & 15ß = 480 & 20φ = 160 & 18φ - 12ℨ = 144 - 24 \\
G. & \frac{1ℨ}{2N} = \frac{2}{2} & \frac{2N}{3ℨ} = \frac{2}{6} & \frac{3ʒ - 2N}{5φ} = \frac{12 - 2}{40} \\
 & & 30ʒʒ = 480 &
\end{array}$$

Abbreviated.

$$\begin{array}{ccc}
15ʒʒ = 240 & 20ʒ = 80 & 18ʒ - 12N = 72 - 12 \\
 & 30φ = 240 &
\end{array}$$

CHAP. VIII.

Addition of Broken Coſſicks.

Broken Coſſicks added. Simple and Homogeneal.

IN adding Coſſical Fractions; firſt if they be Simple, and the Numbers and Coſſicks be of one Denomination, then add the Numerators, and ſubſcribe the Common Denominator with the Coſſical Character: But if the Numbers be not of one Denomination,

mination, reduce them as Common Fractions, and then add their Numerators as above.

Examples.

As to add $\frac{3}{5}\text{ʒ}$ to $\frac{1}{5}\text{ʒ}$, make the Total $\frac{4}{5}\text{ʒ}$, as at *A*.

But $\frac{3}{4}\varphi$ added to $\frac{1}{3}\varphi$ muſt firſt be reduced to $\frac{9\quad 4}{12}\varphi$, and then added, make together $\frac{13}{12}\varphi$, or $1\frac{1}{12}\varphi$, as at *B*.

$$A.\quad \overset{4}{\underset{5}{\frac{3}{5}\text{ʒ}+\frac{1}{5}\text{ʒ}}} = \frac{4}{5}\text{ʒ} \qquad B.\quad \overset{\overset{13}{9\ +\ 4}}{\underset{12}{\frac{3}{4}\varphi \times \frac{1}{3}\varphi}} = \frac{13}{12}\varphi \text{ or } \left(1\frac{1}{12}\varphi\right.$$

2. If the Coſſical Quantities of Simple Fractions to be added, be unlike or Heterogeneal, then connex them by the Sign of Addition, or reduce them.

Simple and Heterogeneal.

Example.

As to add $\frac{3}{5}\text{ʒ}$ to $\frac{1}{2}\varphi$, I ſet them as at *C*. or *D*.

$$C.\quad \frac{3}{5}\text{ʒ}+\frac{1}{2}\varphi \qquad D.\quad \overset{6\text{ʒ}\ +\ 5\varphi}{\underset{10}{\frac{3}{5}\text{ʒ} \times \frac{1}{2}\varphi}} = \frac{5\varphi+6\text{ʒ}}{10}$$

3. If the Fractions be Compound, and of like Denominations in Numbers and Coſſicks, then add the Numerators as Coſſicks are to be added, and ſubſcribe the Common Denominator.

Compound and Homogeneal.

Examples.

As $\frac{3\text{N}}{10\text{ȥ}}$ added to $\frac{6\text{N}}{10\text{ȥ}}$ ſhall make the Total $\frac{9\text{N}}{10\text{ȥ}}$, as at *E*.

And $\frac{3\text{ȥ}+2\text{N}}{20\text{ʒ}}$ added to $\frac{2\text{ȥ}+1\text{N}}{20\text{ʒ}}$ ſhall make together $\frac{5\text{ȥ}+3\text{N}}{20\text{ʒ}}$, as at *F*.

And $\frac{3\text{ȥ}+2\text{N}}{20\text{ʒ}}$ added to $\frac{2\text{ȥ}-5\text{N}}{20\text{ʒ}}$ ſhall make the Total $\frac{5\text{ȥ}-3\text{N}}{20\text{ʒ}}$, as at *G*.

$$E.\quad \overset{9\text{N}}{\underset{10\text{ȥ}}{\frac{3\text{N}}{10\text{ȥ}}+\frac{6\text{N}}{10\text{ȥ}}}} = \frac{9\text{N}}{10\text{ȥ}} \qquad F.\quad \overset{5\text{ȥ}+3\text{N}}{\underset{20\text{ʒ}}{\frac{3\text{ȥ}+2\text{N}}{20\text{ʒ}}+\frac{2\text{ȥ}+1\text{N}}{20\text{ʒ}}}} = \frac{5\text{ȥ}+3\text{N}}{20\text{ʒ}}$$

$$G.\quad \overset{5\text{ȥ}-3\text{N}}{\underset{20\text{ʒ}}{\frac{3\text{ȥ}+2\text{N}}{20\text{ʒ}}+\frac{2\text{ȥ}-5\text{N}}{20\text{ʒ}}}} = \frac{5\text{ȥ}-3\text{N}}{20\text{ʒ}}.$$

4. If the Numbers or Compound Coſſicks be not alike, firſt reduce them, and then add their Numerators, as laſt above-mentioned.

Compound and Heterogeneal.

Examples.

As to add $\frac{8\varphi+9\text{ʒ}}{10\text{ʒ}\text{ʒ}}$ to $\frac{6\varphi-3\text{ʒ}}{10\varphi}$, they are firſt reduced to the Denomination of $100\,\text{Bſδ}$, and then added and abbreviated, as at *H*. and *I*.

$$H.\quad \overset{\overset{60\text{Bſδ}+50\text{ʒ}\varphi+90\text{ſδ}}{80\text{ʒ}\varphi+90\text{ſδ}\quad +\quad 60\text{Bſδ}-30\text{ʒ}\varphi}}{\underset{100\text{Bſδ}}{\frac{8\varphi+9\text{ʒ}}{10\text{ʒ}\text{ʒ}} \times \frac{6\varphi-3\text{ʒ}}{10\varphi}}} = \frac{60\text{Bſδ}+50\text{ʒ}\varphi+90\text{ſδ}}{100\text{Bſδ}}$$

Abbreviated in Numbers and Quantities.

$$I.\quad \frac{60\text{Bſδ}+50\text{ʒ}\varphi+90\text{ſδ}}{100\text{Bſδ}} = \frac{6\text{Bſδ}+5\text{ʒ}\varphi+9\text{ſδ}}{10\text{Bſδ}} = \frac{6\text{ʒ}+5\text{ȥ}+9\text{N}}{10\text{ʒ}}$$

Alſo $\frac{2ʒ+3ℨ}{3φ+4N}$ is added to $\frac{4ℨ-2N}{2ʒ+2ℨ}$ in like manner at *K*. The Total whereof may be abbreviated to $\frac{8ʒʒ+2φ+3ʒ+8ℨ-4N}{3ß+3ʒʒ+4ʒ+4ℨ}$.

$$\begin{array}{lccc}
 & \multicolumn{3}{c}{16ʒʒ+4φ+6ʒ+16ℨ-8N} \\
\hline
 & 4ʒʒ+10φ+6ʒ & & 12ʒʒ+16ℨ-6φ-8N \\
\hline
K. & \frac{2ʒ+3ℨ}{3φ+4N} & \overset{+}{\mathbf{X}} & \frac{4ℨ-2N}{2ʒ+2ℨ} \\
\hline
 & \multicolumn{3}{c}{6ß+6ʒʒ+8ʒ+8ℨ}
\end{array}$$

Proof of Coſſical Addition of Fractions.

Addition of Coſſical Fractions is proved both by Subſtraction, as in the next *Chapter*, and by taking ſome fit Number for a Root, and accordingly turning the Coſſical Fractions into Abſtract, and after the Addition to parallel the Totals.

As in the laſt Example, ſuppoſing 2 be a Root, the Fractions given to be added will be in their leaſt Terms, $\frac{1}{2}$ and $\frac{1}{2}$, which added make 1 Integer; and ſo much is the added Coſſick.

$$\frac{2ʒ+3ℨ}{3φ+4N}=\frac{8+6}{24+4}=\frac{14}{28}=\frac{1}{2}$$

$$\frac{4ℨ-2N}{2ʒ+2ℨ}=\frac{8-2}{8+4}=\frac{6}{12}=\frac{1}{2}$$

Numerator.	Denominator.
16ʒʒ = 256	6ß = 192
+ 4φ = 32	+ 6ʒʒ = 96
+ 6ʒ = 24	+ 8ʒ = 32
+ 16ℨ = 32	+ 8ℨ = 16
344	336
− 8N 8	
336	

$$\text{And as }\begin{array}{c}2\\ \hline \frac{1}{2}+\frac{1}{2}\\ \hline 2\end{array}=\frac{1}{1},\text{ or 1, ſo is }\frac{336}{336}(1$$

CHAP. IX.

Subſtraction of Broken Coſſicks.

Broken Coſſicks ſubſtracted. Simple and Homogeneal.

IN ſubſtracting Coſſical Fractions; firſt if they be Simple, and the Numbers and Coſſicks be of like Denominations, then take the Leſſer Numerator from the Greater, and to the Remain ſubſcribe the Common Denominator with the Coſſical Character. But when the Subtrahend is the Greater change the Sign to the difference.

Examples.

As to take $\frac{2}{5}φ$ from $\frac{3}{5}φ$, the Remain ſhall be $\frac{1}{5}φ$, as at *A*.

But to take $\frac{3}{5}φ$ from $\frac{2}{5}φ$, the Remain ſhall be $-\frac{1}{5}φ$, as at *B*.

$$A.\quad \begin{array}{c}1\\ \hline \frac{3}{5}φ-\frac{2}{5}φ\\ \hline 5\end{array}=\frac{1}{5}φ \qquad B.\quad \begin{array}{c}-1\\ \hline \frac{2}{5}φ-\frac{3}{5}φ\\ \hline 5\end{array}=-\frac{1}{5}φ$$

Simple and Heterogeneal.

2. If the Numbers be not of one Denomination, or the Simple Coſſicks Heterogeneal, then reduce the Numbers as Common Fractions, and then ſubſtract the Numerators of the Homogeneal, as above: But let the unlike Coſſicks be connexed with the Sign of Subſtraction.

Example.

As to take $\frac{1}{2}ʒ$ from $\frac{2}{3}ʒ$, the Remain after Reduction of the Fractions will be $\frac{1}{6}ʒ$ as at *C*.

But if $\frac{1}{2}ʒ$ were to be taken from $\frac{2}{3}φ$, I ſet them as at *D*, or *E*.

C.

$$C.\ \underset{6}{\overset{1}{\overset{-}{\frac{4}{\frac{2}{3}ʒ}\ \mathbf{X}\ \frac{3}{\frac{1}{2}ʒ}}}} = \frac{1}{6}ʒ \qquad D.\ \frac{2}{3}φ - \frac{1}{2}ʒ \qquad E.\ \underset{6}{\overset{-}{\frac{4φ}{\frac{2}{3}φ}\ \mathbf{X}\ \frac{3ʒ}{\frac{1}{2}ʒ}}} = \frac{4φ - 3ʒ}{6}$$

3. If the Fractions be Compound, and of like Denominations both in Numbers and Coſſicks, then ſubſtract the Leſſer Numerator out of the Greater, as Coſſicks are to be ſubſtracted, and ſubſcribe the Common Denominator: But when the Subtrahend is the Greater, change the Sign to the Difference, as before. *Compound and Homogeneal.*

Examples. As to take $\frac{1ʑ}{3ʒ}$ from $\frac{2ʑ}{3ʒ}$, the Remain will be $\frac{1ʑ}{3ʒ}$, as at *F.*

But if $\frac{2ʑ}{3ʒ}$ be taken from $\frac{1ʑ}{3ʒ}$, the Remain will be $-\frac{1ʑ}{3ʒ}$; as at *G.*

So if $\frac{3ʑ+2N}{9ʒ}$ be taken from $\frac{6ʑ+7N}{9ʒ}$, the Remain will be $\frac{3ʑ+5N}{9ʒ}$, as at *H.*

And if $\frac{3ʒ+5ʑ}{4φ}$ be abated out of $\frac{5ʒ-6ʑ}{4φ}$, the Remain will be $\frac{2ʒ-11ʑ}{4φ}$; as at *I.*

Alſo $\frac{4ʑ-3N}{10ʒ-2ʑ}$ deducted out of $\frac{7ʑ-2N}{10ʒ-2ʑ}$, the Remain will be $\frac{3ʑ+1N}{10ʒ-2ʑ}$, as at *K.*

$$F.\ \underset{3ʒ}{\overset{1ʑ}{\frac{2ʑ}{3ʒ} - \frac{1ʑ}{3ʒ}}} = \frac{1ʑ}{3ʒ} \qquad G.\ \underset{3ʒ}{\overset{-1ʑ}{\frac{1ʑ}{3ʒ} - \frac{2ʑ}{3ʒ}}} = -\frac{1ʑ}{3ʒ}$$

$$\underset{9ʒ}{\overset{3ʑ+5N}{\frac{6ʑ+7N}{9ʒ} - \frac{3ʑ+2N}{9ʒ}}} = \frac{3ʑ+5N}{9ʒ}\ H. \qquad \underset{4φ}{\overset{2ʒ-11ʑ}{\frac{5ʒ-2ʑ}{4φ} - \frac{3ʒ+5ʑ}{4φ}}} = \frac{2ʒ-11ʑ}{4φ}\ I.$$

$$K.\ \underset{10ʒ-2ʑ}{\overset{3ʑ+1N}{\frac{7ʑ-2N}{10ʒ-2ʑ} - \frac{4ʑ-3N}{10ʒ-2ʑ}}} = \frac{3ʑ+1N}{10ʒ-2ʑ}$$

4. If the Numbers or Compound Coſſicks be of unlike Denominations; firſt reduce them, and then ſubſtract their Numerators, as laſt above-mentioned. *Compound and Heterogeneal.*

As to abate $\frac{2ʑ}{3ʒ}$ from $\frac{9N}{10ʑ}$, they are firſt reduced to the Denomination of 30φ, and the Remain then is $\frac{7ʒ}{30φ}$, and abbreviated $\frac{7N}{30ʑ}$, as at *L.* *Examples.*

And ſo to take $\frac{2}{3}ʒ + \frac{9}{10}ʑ$ from $\frac{3}{4}ʒ + \frac{2}{3}ʑ$, they are firſt to be reduced ſeverally, then jointly; and then ſubſtracted the Remain is $\frac{5ʒ-14ʑ}{60N}$, as at *M.*

Alſo $\frac{2ʒ+3ʑ}{3φ+4N}$ ſubſtracted, as at *N*, from $\frac{4ʑ-2N}{2ʒ+2ʑ}$, will leave remaining, $\frac{8ʒʒ-16φ-6ʒ+16ʑ-8N}{6ß+6ʒʒ+8ʒ+8ʑ}$ which will be abbreviated to $\frac{4ʒʒ-8φ-3ʒ+8ʑ-4N}{3ß+3ʒʒ+4ʒ+4ʑ}$.

$$L.\ \underset{30φ}{\overset{7ʒ}{\overset{-}{\frac{27ʒ}{\frac{9N}{10ʑ}}\ \mathbf{X}\ \frac{20ʒ}{\frac{2ʑ}{3ʒ}}}}} = \frac{7ʒ}{30φ} \text{ or } \frac{7N}{30φ}$$

M.

$$\underset{12}{\underline{\overset{9\text{ʒ}+8\text{ℨ}}{\overline{\tfrac{3}{4}\text{ʒ}+\tfrac{2}{3}\text{ℨ}}}}} - \underset{30}{\underline{\overset{20\text{ʒ}+27\text{ℨ}}{\overline{\tfrac{2}{3}\text{ʒ}+\tfrac{9}{10}\text{ℨ}}}}} \qquad \overset{5\text{ʒ}-14\text{ℨ}}{\overline{\underset{\underbrace{\underset{5}{\overset{\cdots\cdots}{12}}\qquad\qquad\underset{2}{\overset{\cdots\cdots}{30}}}_{60}}{\frac{45\text{ʒ}+40\text{ℨ}}{9\text{ʒ}+8\text{ℨ}}-\frac{40\text{ʒ}+54\text{ℨ}}{20\text{ʒ}+27\text{ℨ}}}}} = \frac{5\text{ʒ}-14\text{ℨ}}{60N}$$

N.

$$\underset{6\text{ß}+6\text{ʒʒ}+8\text{ʒ}+8\text{ℨ}}{\underline{\overset{8\text{ʒʒ}-16\text{φ}-6\text{ʒ}+16\text{ℨ}-8N}{\overline{\overset{12\text{ʒʒ}+16\text{ℨ}-6\text{φ}-8N}{\overline{\frac{4\text{ℨ}-2N}{2\text{ʒ}+2\text{ℨ}}}}\ \mathbf{X}\ \overset{4\text{ʒʒ}+10\text{φ}+6\text{ʒ}}{\overline{\frac{2\text{ʒ}+3\text{ℨ}}{3\text{φ}+4N}}}}}}}$$

Proof of Coſſical Subſtraction of Fractions.

Subſtraction of Coſſical Fractions is proved, as Addition before, by converting the Coſſicks into Abſtract Fractions, by ſome apt Root taken at pleaſure, and after Subſtraction made therewith, the Remains will be alike valuable.

As in the laſt Example, taking 2 for the Root, the Fractions given in their leaſt Terms will be $\frac{1}{2}$ and $\frac{1}{2}$, which ſubſtracted the one from the other, leave 0 for the Remain. And ſo is the Coſſical Remain 8ʒʒ — 16φ — 6ʒ + 16ℨ — 8N, becauſe the negative Quantities equally counterballance the affirmative.

$$\frac{4\text{ℨ}-2N}{2\text{ʒ}+2\text{ℨ}}=\frac{8-2}{8+4}=\frac{6}{12}=\frac{1}{2}$$

$$\frac{2\text{ʒ}+3\text{ℨ}}{3\text{φ}+4N}=\frac{8+6}{24+4}=\frac{14}{28}=\frac{1}{2}$$

Numerator.

$$\begin{array}{rcrcrcr} 8\text{ʒʒ} & = & 128 & \qquad & -16\text{φ} & = & 128 \\ +16\text{ℨ} & = & 32 & & -6\text{ʒ} & = & 24 \\ & & \overline{160} & & -8N & = & 8 \\ & & & & & & \overline{160} \end{array}$$

And as $\overset{0}{\overline{\underset{2}{\underline{\frac{1}{2}-\frac{1}{2}}}}}=\frac{0}{2}$, or 0; ſo 160 — 160 = 0.

Beſides, if the Remain 8ʒʒ — 16φ — 6ʒ + 16ℨ — 8N be Coſſically added to 4ʒʒ + 10φ + 6ʒ, the Number ſubſtracted, the total will be 12ʒʒ + 16ℨ — 6φ — 8N, as above; whereby Addition and Subſtraction of Coſſical Fractions are ſeen to be alternate Proofs of each other.

CHAP. X.

Multiplication of Broken Coſſicks.

Broken Coſſicks multiplyed. Simple.

IN multiplying Coſſical Fractions; firſt if they be Simple, increaſe Numerator by Numerator, and Denominator by Denominator, as Common Fractions are multiplyed, and annex to the Product the Coſſical Character due to the Total of both their *Indices*.

Example.

As $\frac{1}{2}$ʒ multiplyed by $\frac{5}{6}$φ, ſhall make the Product $\frac{5}{12}$ß, as at *A*.

And $\frac{1}{3}$φ with $\frac{2}{3}$ʒ, and $\frac{4}{5}$ℨ multiplyed together ſhall make the Product $\frac{8}{45}$ʒφ, as at *B*.

$$A.\quad \overset{5}{\overline{\underset{12}{\underline{\tfrac{1}{2}\text{ʒ}\times\tfrac{5}{6}\text{φ}}}}}=\frac{5}{12}\text{ß}\qquad \overset{\textit{Indices.}}{\begin{array}{c}2\\ \underline{3}\\ 5\end{array}}$$

$$B.\quad \overset{8}{\overline{\underset{45}{\underline{\tfrac{1}{3}\text{φ}\times\tfrac{2}{3}\text{ʒ}\times\tfrac{4}{5}\text{ℨ}}}}}=\frac{8}{45}\text{ʒφ}\qquad \overset{\textit{Indices.}}{\begin{array}{c}3\\ 2\\ \underline{1}\\ 6\end{array}}$$

Compound.

2. If the Fractions be Compound, then multiply after the manner of Compound

Coſſicks, Numerator by Numerator, and Denominator by Denominator; and both Coſſicks and Signs + and — of the produced Numbers are known, as if they were Whole Coſſicks.

Examples.

As to multiply $\frac{1N}{3\mathfrak{z}}$ by $\frac{1\mathfrak{z}}{2\text{ʒ}}$, the Product will be $\frac{1\mathfrak{z}}{6\varphi}$, and abbreviated $\frac{1N}{6\text{ʒ}}$, as at *C.*

And to multiply $\frac{2\text{ʒ}+3\mathfrak{z}-2N}{4\varphi}$ by $\frac{4\mathfrak{z}+2N}{3\varphi-5\text{ʒ}}$, the Numerators and Denominators being multiplyed as at *D*, the Product of the new Fraction is $\frac{8\varphi+16\text{ʒ}-2\mathfrak{z}-8N}{12\text{ʒ}\varphi-20\text{ſs}}$ and abbreviated $\frac{4\varphi+8\text{ʒ}-1\mathfrak{z}-2N}{6\text{ʒ}\varphi-10\text{ſs}}$.

$$C.\quad \begin{array}{c} 1 \\ \hline \dfrac{1N}{3\mathfrak{z}} \times \dfrac{1\mathfrak{z}}{2\text{ʒ}} \\ \hline 6 \end{array} = \frac{1\mathfrak{z}}{6\varphi}$$

$$D.\quad \begin{array}{c} 8\varphi+16\text{ʒ}-2\mathfrak{z}-4N \\ \hline \dfrac{2\text{ʒ}+3\mathfrak{z}-2N}{4\varphi} \times \dfrac{4\mathfrak{z}+2N}{3\varphi-5\text{ʒ}} \\ \hline 12\text{ʒ}\varphi-20\text{ſs} \end{array} = \frac{8\varphi+16\text{ʒ}-2\mathfrak{z}-4N}{12\text{ʒ}\varphi-20\text{ſs}}$$

$$\begin{array}{l} 2\text{ʒ}+3\mathfrak{z}-2N \\ 4\mathfrak{z}+2N \\ \hline 8\varphi+12\text{ʒ}-8\mathfrak{z} \\ \quad\quad 4\text{ʒ}+6\mathfrak{z}-4N \\ \hline 8\varphi+16\text{ʒ}-2\mathfrak{z}-4N \text{ Numerator.} \\ \hline \end{array} \qquad \begin{array}{l} 3\varphi-5\text{ʒ} \\ 4\varphi \\ \hline 12\text{ʒ}\varphi-20\text{ſs} \text{ Denominator.} \\ \hline \end{array}$$

Proof of Coſſical Multiplication *of Fractions.*

Multiplication of Coſſical Fractions, is proved as well by Diviſion noted in the next *Chapter*, as by converting the Coſſical Fractions into Abſtract; and after Multiplication of them, comparing the equality in their Products.

As in the laſt Example, the given Coſſicks converted into Abſtract Fractions ſuppoſing the Root 2, are $\frac{12}{32}$ and $\frac{10}{4}$, which multiplyed make the Product without abbreviation $\frac{120}{128}$ and with it $\frac{15}{16}$, which agree with the new Coſſical Fraction in equal value.

$$\frac{2\text{ʒ}+3\mathfrak{z}-2N}{4\varphi} = \frac{8+6-2}{32} = \frac{12}{32} = \frac{3}{8}$$

$$\frac{4\mathfrak{z}+2N}{3\varphi-5\text{ʒ}} = \frac{8+2}{24-20} = \frac{10}{4} = \frac{5}{2}$$

$$\begin{array}{c} 120 = 15 \\ \hline \end{array}$$

$$\text{And as } \frac{12}{32}\times\frac{10}{4} \text{ or } \frac{3}{8}\times\frac{5}{2}, \text{ ſo is } \frac{120}{128}\Big|\frac{60}{64}\Big|\frac{30}{32}\Big|\frac{15}{16}$$

$$\begin{array}{c} \hline 128 = 16 \end{array}$$

$$\begin{array}{r} \text{Numerator.} \\ 8\varphi=64 \\ +16\text{ʒ}=64 \\ \hline 128 \\ -2\mathfrak{z}=4 \\ -4N=4 \\ \hline 8 \\ \hline 120 \\ \hline \end{array} \qquad \begin{array}{r} \text{Denominotor.} \\ 12\text{ʒ}\varphi=768 \\ \hline -20\text{ſs}=640 \\ \hline 128 \\ \hline \end{array}$$

CHAP. XI.

Diviſion of Broken Coſſicks.

Broken Coſſicks *divided.*

Simple.

IN dividing Coſſical Fractions; firſt if they be Simple, multiply croſs wiſe, as in Common Fractions, the Numerator of the Dividend by the Denominator of the Diviſor for the Numerator of the Quotient, and the Denominator of the Dividend by the Numerator of the Diviſor for the Denominator of the Quotient: And thereto annex the Coſſical Character due to the Remain of the *Index* of the Diviſor deducted from the *Index* of the Dividend.

As

Examples.

As to divide $\frac{1}{2}$ſß by $\frac{8}{9}$ʒ, the Quotient ſhall be $\frac{9}{16}$φ, ſet as at *A.* or *B.*

And $\frac{3}{4}$φ divided by $\frac{3}{4}$ʒ, ſhall make the Quotient 1 ȥ, as at *C.* or *D.*

$$A.\ \frac{1}{2}\text{ſß}\overset{9}{\underset{16}{\mathbf{X}}}\frac{8}{9}\text{ʒ}=\frac{9}{16}\text{φ}\qquad B.\ \frac{8}{9}\text{ʒ}\Big)\frac{1}{2}\text{ſß}\Big(\frac{9}{16}\text{φ}\qquad C.\ \frac{\overset{1}{3}}{\underset{1}{4}}\text{φ}\overset{1}{\underset{1}{\mathbf{X}}}\frac{\overset{1}{3}}{\underset{1}{4}}\text{ʒ}=\frac{1}{1}\text{ȥ}\qquad D.\ \frac{\overset{1}{3}}{\underset{1}{4}}\text{ʒ}\Big)\frac{\overset{1}{3}}{\underset{1}{4}}\text{φ}\Big(1\,\text{ȥ}$$

Compound.

2. If the Fractions be Compound, then after the manner of Compound Coſſicks, multiply the Numerator of the Dividend into the Denominator of the Diviſor, and contrariwiſe, the Denominator of the Dividend into the Numerator of the Diviſor; and the Coſſicks and Signs + and —, of the produced Quotienary Numbers, ſhall be as if they were Whole Coſſicks.

Examples.

As to divide $\frac{4\,\text{N}}{5\,\text{ȥ}}$ by $\frac{5\,\text{N}}{6\,\text{ȥ}}$ giveth in the Quotient $\frac{24\,\text{ȥ}}{25\,\text{ȥ}}$ or $\frac{24\,\text{N}}{25\,\text{N}}$, as at *E.* or *F.*

And $\frac{12\text{ʒʒ}-16\,\text{ȥ}}{2\text{ʒ}-5\,\text{N}}$ divided by $\frac{9\text{ʒ}-3\,\text{N}}{4\text{ʒ}+5\,\text{N}}$, maketh the Quotient to be $\frac{48\text{ʒφ}+60\text{ʒʒ}-64\text{φ}-80\,\text{ȥ}}{18\text{ʒʒ}-51\text{ʒ}+15\,\text{N}}$, as at *G.* or *H.*

$$E.\ \frac{4\,\text{N}}{5\,\text{ȥ}}\overset{24\,\text{ȥ}}{\underset{25\,\text{ȥ}}{\mathbf{X}}}\frac{5\,\text{N}}{6\,\text{ȥ}}=\frac{24\,\text{ȥ}}{25\,\text{ȥ}}\qquad F.\ \frac{5\,\text{N}}{6\,\text{ȥ}}\Big)\frac{4\,\text{N}}{5\,\text{ȥ}}\Big(\frac{24\,\text{ȥ}}{25\,\text{ȥ}}\text{ or }\frac{24\,\text{N}}{25\,\text{N}}$$

$$G.\ \frac{12\text{ʒʒ}-16\,\text{ȥ}}{2\text{ʒ}-5\,\text{N}}\overset{48\text{ʒφ}+60\text{ʒʒ}-64\text{φ}-80\,\text{ȥ}}{\underset{18\text{ʒʒ}-51\text{ʒ}+5\,\text{N}}{\mathbf{X}}}\frac{9\text{ʒ}-3\,\text{N}}{4\text{ʒ}+5\,\text{N}}=\frac{48\text{ʒφ}+60\text{ʒʒ}-64\text{φ}-80\,\text{ȥ}}{18\text{ʒʒ}-51\text{ʒ}+15\,\text{N}}$$

$$H.\ \frac{9\text{ʒ}-3\,\text{N}}{4\text{ʒ}+5\,\text{N}}\Big)\frac{12\text{ʒʒ}-16\,\text{ȥ}}{2\text{ʒ}-5\,\text{N}}\Big(\frac{48\text{ʒφ}+60\text{ʒʒ}-64\text{φ}-80\,\text{ȥ}}{18\text{ʒʒ}-51\text{ʒ}+15\,\text{N}}$$

$$\begin{array}{lrrrrlrrr}
 & 12\text{ʒʒ} & -16\,\text{ȥ} & & & & 2\text{ʒ} & -5\,\text{N} & \\
 & 4\text{ʒ} & +5\,\text{N} & & & & 9\text{ʒ} & -3\,\text{N} & \\
\hline
 & 48\text{ʒφ} & -64\text{φ} & & & & 18\text{ʒʒ} & -45\text{ʒ} & \\
 & & 60\text{ʒʒ} & -80\,\text{ȥ} & & & & -6\text{ʒ} & +15\,\text{N} \\
\hline
\text{Numerator} & 48\text{ʒφ} & +60\text{ʒʒ} & -64\text{φ} & -80\,\text{ȥ} & & 18\text{ʒʒ} & -51\text{ʒ} & +15\,\text{N}\ \ \text{Denominator} \\
\hline
\end{array}$$

Proof of Coſſical Diviſion of Fractions.

Diviſion of Coſſical Fractions is proved, as other Elementary Operations thereof, by reſolving the Coſſical Fractions into Abſtract, taking ſome fit Number for a Root at pleaſure, and after Diviſion made, as in Common Fractions, the Quotients will be found equal.

As in the laſt Example, ſuppoſing 2 for the Root, the given Fractions will be $\frac{160}{3}$ and $\frac{33}{21}$, and the Quotient thereof $\frac{3360}{99}$, and ſo is the Coſſical Quotient $\frac{48\text{ʒφ}+60\text{ʒʒ}-64\text{φ}-80\,\text{ȥ}}{18\text{ʒʒ}-51\text{ʒ}+15\,\text{N}}$.

$$\frac{12\text{ʒʒ}-16\,\text{ȥ}}{2\text{ʒ}-5\,\text{N}}=\frac{192-32}{8-5}=\frac{160}{3}$$

$$\frac{9\text{ʒ}-3\,\text{N}}{4\text{ʒ}+5\,\text{N}}=\frac{36-3}{16+5}=\frac{33}{21}$$

$$\begin{array}{rrcrr}
\multicolumn{2}{c}{\text{Numerator.}} & & \multicolumn{2}{c}{\text{Denominator.}} \\
48\text{ʒφ}= & 3072 & & 18\text{ʒʒ}= & 288 \\
+60\text{ʒʒ}= & 960 & & +15\,\text{N}= & 15 \\
\cline{2-2}\cline{5-5}
 & 4032 & & & 303 \\
\cline{2-2}\cline{5-5}
-64\text{φ}= & 512 & & & \\
-80\,\text{ȥ}= & 160 & & -51\text{ʒ}= & 204 \\
\cline{2-2}\cline{5-5}
 & 672 & & & 99 \\
\cline{2-2}\cline{5-5}
 & 3360 & & & \\
\cline{2-2}
\end{array}$$

And ſo $\frac{\overset{}{33}}{\underset{7}{21}}\Big)\frac{160}{\underset{1}{3}}\Big(\frac{3360}{99}$ or $\frac{1120}{33}$ being abbreviated.

Beſides if the Quotient be multiplyed by the Diviſor, the Dividend of the Simple Fractions will be returned or reduced thereto by abbreviation. But in Compound Fractions other Coſſical Fractions in ſhew will be produced, yet the ſame in value; which ſerveth to evidence the alternate Proof of Multiplication and Diviſion of Coſſical Fractions one by the other.

For if $\frac{4\varphi+9\text{ʒ}}{12\,N}$ be divided by $\frac{3\text{ʒ}+2\text{ℨ}}{3\text{ℨ}}$, the Quotient will be $\frac{12\text{ʒʒ}+27\varphi}{36\text{ʒ}+24\text{ℨ}}$ and by abbreviation $\frac{4\varphi+9\text{ʒ}}{12\text{ℨ}+8N}$; wherefore if this Quotient be multiplyed by $\frac{3\text{ʒ}+2\text{ℨ}}{3\text{ℨ}}$ the Diviſor, the Product abbreviated will be $\frac{12\text{ʒʒ}+35\varphi+18\text{ʒ}}{36\text{ℨ}+24N}$, which nevertheleſs is but equal in value to the Dividend $\frac{4\varphi+9\text{ʒ}}{12N}$, as by the Root 2 appeareth.

$$\frac{4\varphi+9\text{ʒ}}{12N} = \frac{32+36}{12N} = \frac{68}{12} = 5\frac{2}{3} \qquad \begin{array}{r} 12\text{ʒʒ}=192 \\ 35\varphi=280 \\ 18\text{ʒ}=72 \\ \hline 544 \end{array} \qquad \begin{array}{r} 36\text{ℨ}=72 \\ 24N=24 \\ \hline 96 \end{array} \qquad \begin{array}{l} 16 \\ \cancel{54}4 \\ \cancel{96} \end{array}\Big(5\frac{2}{3}$$

CHAP. XII.

Figuration of Coſſicks.

Figurate Coſſicks *produced.*

TO Figurate any Coſſick is Coſſically to multiply the ſame, be it Simple or Compound by it ſelf to produce the Square, and that again by the Root to produce the Cube, *&c.* as other Figural Numbers are produced.

Examples.

As to produce the Cube of 2ℨ, or the Square of $2\text{ℨ}+3N$, their Figurate Multiplications bring forth 8φ for the one, and $4\text{ʒ}+12\text{ℨ}+9N$ for the other.

$$\begin{array}{lllll} & \text{Simple} & & & \text{Compound} \\ \text{Root} & 2\text{ℨ} & & \text{Root} & 2\text{ℨ}+3N \\ & 2\text{ℨ} & & & 2\text{ℨ}+3N \\ \hline \text{Square} & 4\text{ʒ} & & & 4\text{ʒ}+6\text{ℨ} \\ & 2\text{ℨ} & & & \qquad 6\text{ℨ}+9N \\ \hline \text{Cube} & 8\varphi & & & 4\text{ʒ}+12\text{ℨ}+9N \end{array}$$

What to be noted in extracting of their Roots.

1. Which Coſſicks *are Rooted.*

In extracting the Root of a Coſſick ſeveral things are to be noted. For firſt, every Coſſick is not Coſſically Rooted, no more than every Abſolute Number is a Rooted Number. But thoſe Coſſicks are Rooted, which have a Root agreeable to the Figure or Character of his Quantity; and therefore no Coſſick may be properly called Square, Cubick, or otherwiſe Rooted, except the Root of the Coſſick agreeth with his Character. So 8φ is a Cubick Coſſick, and his Root 2ℨ; becauſe 8 is a Cubick Number agreeable to his Character φ. But 8ʒ hath no Coſſical Root, becauſe 8 hath no Square Root agreeable to the Character ʒ, neither is it a Cubick Coſſick, although the Number have a Cubick Root, becauſe the Cubick Root is diſagreeable to the Character ʒ. Likewiſe 16φ, is no Rooted Coſſick, becauſe 16 hath no exact Cubick Root; but 16ʒ is Coſſically Rooted, and hath 4ℨ for the Root thereof.

2. Which Simple Coſſicks *Compound in their Characters are Rooted.*

2. Simple Coſſicks Compound in their Characters are not Rooted, unleſs the Number annexed will yield a Root according to the compoſition of the Character. So 16ʒʒ becauſe compound of the Square twice, may either have a $\text{ʒ}\text{ℨ}$, or a $\text{ʒʒ}\text{ℨ}$, the one 4ʒ, the other 2ℨ. And 4ʒʒ hath a Square Root 2ʒ, but no $\text{ʒʒ}\text{ℨ}$ agreeable to the whole Character. And $9\text{ʒ}\varphi$ hath a $\text{ʒ}\text{ℨ}$, which is 3φ agreeable to part of his Character, but no $\text{ʒ}\varphi$ Root anſwerable to the compounded Character.

How many Roots ſuch may have.

And therefore if any Coſſick Compound in his Character have a Root agreeable to his whole Character, then may he have alſo as many Roots as their be parts in that compoſition: For ſo $4096\text{ʒʒ}\varphi$ hath not only a $\text{ʒʒ}\varphi$ Root, which is 2ℨ, but alſo a $\text{ʒ}\varphi$

Root which is 4ʒ, and thirdly a ʒʒ Root which is 8φ, and fourthly a φ Root which is 16ʒʒ, and laſtly a ʒ Root which is 64ʒφ.

3.
What proper to Compound Rooted Coſſicks.

3. Compound Coſſicks, if they be Coſſically Rooted, muſt firſt have their greateſt Coſſick a Rooted Coſſick, and the Number annexed to the leaſt Denomination of that Compound Coſſick, muſt be a Rooted Number of the ſame kind or denomination with the greateſt Coſſick. As in the Example above of 4ʒ + 12ƶ + 9N, the greateſt Coſſick 4ʒ is Rooted, and the leaſt Denomination 9N is a ʒ Number.

4.
What in the Compounds the part not Rooted is.

4. In a Compound Rooted Coſſick, every part that is not a Rooted Coſſick is a mean between the greateſt Coſſick and the leaſt Denomination in that Coſſick. And if ƶ be one Denomination, then N ſhall be another: As appeareth in the ſaid Example, where alſo is 12ƶ to be ſeen a mean proportional between 9N and 4ʒ.

Others not with theſe properties what they are.

Other Coſſicks that ſuit not with theſe particulars, are either Surdes, or ſuch affected Coſſicks, whoſe Roots are not properly Coſſical, but being equal to ſome other Number, the inveſtigation of their Roots are to be ſought hereafter among *Æquations*.

Root of a Simple Coſſick extracted. Examples.

Theſe things premiſed, to extract the Root of a Simple Coſſick, extract the Root of the Number according to the Character of the Quantity, and let the Character of the Root be ſet for the Denomination.

As 9ʒ hath for his Root 3ƶ; and 64φ hath 4ƶ for his Root.

And if the Simple Coſſick be Compound in the Character, then as in the former *Book* in *Figural Numbers*, ſearch out the Root from the Number, and affix thereto the Coſſical Character.

As if from 256, a Number of the 8th Quantity from the Root 2, I would ſearch out the ʒʒʒ Root, I find it 2ƶ, as at *A*. If out thereof I would ſeek the ʒʒ Root, I find it 4ʒ, as at *B*. And if the ʒ Root, I find it 16ʒʒ, as at *C*.

So alſo 729ʒφ gives the ʒφ Root 3ƶ, as at *D*. And the φ Root 9ʒ, as at *E*. And the ʒ Root 27φ, as at *F*.

ʒʒʒ ʒʒ ʒ
$\cancel{1}$
A. $\cancel{2}\cancel{5}6$ | $\cancel{1}6$ | 4 | 2ƶ
· ·
1 16 4
156

ʒʒʒ ʒʒ
$\cancel{1}$ ·
B. $\cancel{2}\cancel{5}6$ | $\cancel{1}6$ | 4ʒ
· ·
1 16
156

ʒʒʒ
$\cancel{1}$
C. $\cancel{2}\cancel{5}6$ | 16ʒʒ
· ·
1
156

ʒφ φ
$\cancel{3}$
D. $\cancel{7}\cancel{2}\cancel{9}$ | $\cancel{2}7$ | 3ƶ
· · 27
4
329

ʒφ
E. 729 | 9ʒ
·

ʒφ
$\cancel{3}$
F. $\cancel{7}\cancel{2}\cancel{9}$ | 27φ
· ·
4
329

Root of the Compounds extracted.

But if the Coſſick be Compound, then prick your Coſſick according to the Quantity whoſe Root you would extract, as was before taught in *Figural Numbers* in the *Second Part* of the *Second Book*; and out of the Left Hand pricked Coſſick, take the Coſſical Root, and place in the Quotient, with half his Coſſical Denomination, for the Square; the third part for the Cube, *&c.* as was noted before in *Extraction of the Roots of Aſtronomicals*, and proceed with the reſt of the work as before in *Figural Numbers*; Coſſical Extraction differing therefrom no otherwiſe than as Coſſical Addition and Subſtraction, Multiplication and Diviſion, differs from that of Integers.

Example in the Square.

As to extract the Coſſical Square Root of 16ʒʒʒ — 16Bſδ — 20ʒφ + 20ſδ + 5ʒʒ — 6φ + 1ʒ. The pricked Numbers will be 1ʒ, 5ʒʒ, 20ʒφ, and 16ʒʒʒ, in which laſt the greateſt Coſſical Square Root is 4ʒʒ; for 16 being a Square Number hath 4 for the Root, and half the *Index* of ʒʒʒ which is 8, is 4, whoſe Character is ʒʒ. This Root doubled is 8ʒʒ for the firſt Diviſor, which dividing — 16Bſδ gets — 2φ for the Quotient, and the Square of — 2φ is + 4ʒφ to be ſubſtracted from the next Right Hand Prick: And then doubling the Root, and proceeding to the end of the work, the whole Root obtained is 4ʒʒ — 2φ — 3ʒ + 1ƶ, as is obvious at *G*.

Example in the Cube.

Alſo to extract the Coſſical Cube Root of 27φφ + 54ʒʒʒ — 72Bſδ — 136ʒφ + 96ſδ + 96ʒʒ — 64φ. The pricked Numbers will be 64φ, 136ʒφ, and 27φφ, in which the greateſt Cube Root is 3φ; for 3 is the Cube Root of 27, and the *Index* of φφ is 9, whoſe third part is 3, the Character of which *Index* is φ: This tripled is 9φ, and multiplyed by the Root 3φ is 27ʒφ for the firſt Diviſor; which dividing + 54ʒʒʒ gets + 2ʒ for the Quotient. The other parts of the Gnomon and proceedings in the work are plain at *H*.

G.

G. Square Root

− ~~24~~ʒφ + 8ß − ~~4~~ʒʒ

~~16~~ʒʒʒ − ~~16~~Bß − ~~20~~ʒφ + ~~20~~ß + ~~5~~ʒʒ − ~~6~~φ + ~~1~~ʒ (4ʒʒ − 2φ − 3ʒ + 1ℨ

16ʒʒʒ

Gnom. − 16Bß + 4ʒφ
. . . . . . − 24ʒφ + 12ß + 9ʒʒ
. . . . + 8ß − 4ʒʒ − 6φ + 1ʒ

H. Cube Root

− ~~108~~Bß − ~~144~~ʒφ

~~27~~φφ + ~~54~~ʒʒʒ − ~~72~~Bß − ~~136~~ʒφ + ~~96~~ß + ~~96~~ʒʒ − ~~64~~φ (3φ + 2ʒ − 4ℨ

27φφ

Gnomons 54ʒʒʒ + 36Bß + 8ʒφ
. . . . − 108Bß − 144ʒφ + 96ß + 96ʒʒ − 64φ

The one part of Coſſical Figuration ſerves for Proof of the other; that is, Production by Extraction, and Extraction by Production, as other Figural Numbers before exemplified clearly ſhew; and here may further be ſeen at *I.* and *K.*; where the Productions of the Simple Coſſicks, whoſe Roots were before extracted at *A. B. C.* and *D. E. F.*; and at *L.* and *M.* are the Productions of the Compound Coſſicks, whoſe Roots are extracted at *G.* and *H.*

Proof of Coſſical Figuration.

But beſides, if the Coſſicks be reſolved into Abſtract Numbers, and the Figural Roots thereof be taken, the ſame will not differ with the Coſſical Root, unleſs ſome miſtake occaſion it.

Proof of the Simple Coſſical Extraction.

I.		Root ſuppoſed 2.	*K.*		Root ſuppoſed 2.
2ℨ	1	4	3ℨ	1	6
2ℨ	1	4	3ℨ	1	6
4ʒ	2	16	9ʒ	2	36
4ʒ	2	16	3ℨ	1	6
16ʒʒ	4	96	27φ	3	216
16ʒʒ	4	16	27φ	3	216
96		256	189		1296
16		256	54		216
	8			6	432
256ʒʒʒ		1536	729ʒφ		
		1280			46656
		512			
		65536			

65536 = 256ʒʒʒ by the Root 2. 46656 = 729ʒφ by the Root 2.

Proof of the Compound Coſſical Extraction.

L.

Root 4ʒʒ − 2φ − 3ʒ + 1ℨ
4ʒʒ − 2φ − 3ʒ + 1ℨ

16ʒʒʒ − 8Bß − 12ʒφ + 4ß
− 8Bß + 4ʒφ + 6ß − 2ʒʒ
− 12ʒφ + 6ß + 9ʒʒ − 3φ
4ß − 2ʒʒ − 3φ + 1ʒ

Square 16ʒʒʒ − 16Bß − 20ʒφ + 20ß + 5ʒʒ − 6φ + 1ʒ

Root

M.

Root 3φ + 2ʒ − 4ℨ
3φ + 2ʒ − 4ℨ

9ʒφ + 6ß − 12ʒʒ
6ß + 4ʒʒ − 8φ
−12ʒʒ − 8φ + 16ʒ

Square 9ʒφ + 12ß − 20ʒʒ − 16φ + 16ʒ
3φ + 2ʒ − 4ℨ

27φφ + 36ʒʒʒ − 60Bß − 48ʒφ + 48ß
18ʒʒʒ + 24Bß − 40ʒφ − 32ß + 32ʒʒ
−36Bß − 48ʒφ + 80ß + 64ʒʒ − 64φ

Cube 27φφ + 54ʒʒʒ − 72Bß − 136ʒφ + 96ß + 96ʒʒ − 64φ

So in the tryal by Numbers, ſuppoſing the Root 2, then,

L.

4ʒʒ = 64
+1ℨ = 2
66
−2φ = 16
−3ʒ = 12
28
True value of the Root 38
38
304
114
Square 1444

16ʒʒʒ = 4096
+20ß = 640
+5ʒʒ = 80
+1ʒ = 4
4820
−16Bß = 2048
−20ʒφ = 1280
−6φ = 48
3376
═══ 1444

M.

3φ = 24
+2ʒ = 8
32
−4ℨ = 8
ℨ 24
24
96
48
ʒ 576
24
2304
1152
φ 13824

27φφ = 13824
+54ʒʒʒ = 13824
+96ß = 3072
+96ʒʒ = 1536
32256
−72Bß = 9216
−136ʒφ = 8704
−64φ = 512
18432
═══ 13824

Partis Quartæ Libri Tertii

FINIS.

THE

THE FIFTH PART OF THE THIRD BOOK.

CHAP. I.

Of SURDES.

NEXT after *Coſſicks* follow in order *Surdes*, as the fiſth ſort of Numbers eſpecially Contract, and the ſecond of thoſe whoſe Denominations are expreſſed, becauſe of the variety and uncertainty thereof.

Surdes *placed as the fifth of ſpecial Contracts*, &c.

Surdes are Irrational Numbers, as before in *Book* 1. *Par.* 1. *Chap.* 2. and *Book* 2. *Par.* 2. *Chap.* 3. was noted; that is, Numbers ſet for Roots that cannot be expreſſed by any Abſolute Number: Or Numbers whoſe Roots cannot certainly be expreſſed by Integers, but beſides the Integers contain ſome broken part or parts thereof. As the Square Root of 2, 3, 5, 6, or of any other Number that is not a Square Number. So the Cube Root of 2, 3, 4, 5, or of any other Number that is not Cubick. And in like manner any other Root of any Number that hath no ſuch Root exactly to be meaſured by Whole Numbers, cauſeth that Number to be called a *Surde Number*. And perhaps the Reaſon why ſo called, it being abſurd or irrational to attribute to any thing, or ſeek out thereof a Root or other thing not there to be had. *Latus inexplicabile* (ſaith *Alſted*) *dicitur*, *aſymmetrum*, *incommenſurabile*, *incommunicabile*, *irrationale*, *&* *ſurdum*, *quia ejus explicatio à nobis*, *quaſi exaudiri nequeat*, *ut ſurdam buccinam*, *ſurdos ictus*, *dicimus qui difficulter & obſcurè audiantur.* Encyclop. lib. 14. pag. 830.

Surdes *what they are, and*

why ſo called.

The Denominations of theſe Roots being different according to the Powers of Numbers before-mentioned in Figural Numbers, maketh it neceſſary to expreſs them. And for the Reaſon before rendred in the *Firſt Chapter* of *Coſſicks*, by ſome known Characters to diſtinguiſh them the one from the other. Which Characters, as in *Coſſicks*, are arbitrary and mutable at the pleaſure of the Operator.

Denominators why expreſſed, and how.

Characters arbitrary.

In the enſuing ſurvey of *Surdes*, the following Characters are uſed with their ſignifications, thus;

Characters uſed in this Book.

Characters.	*Significations.*
√	Root. Divers ſet this for the Square Root.
√: or V or V√	Univerſal Root.
w or √ʒ	Square Root.
vw or √φ	Cube Root.
ww or √ʒʒ	Squared Square Root.
vww or √ß	Surſolide Root.

And ſo increaſing the Minnoms according to the *Index* of the Figural Number, or adjoyning the Coſſical Character of the Power to the Character for the Root, √, you have a Character for any Surde Denomination. Wherefore if I ſee √ʒφ 8, I read it the Zenzicube Root of 8. And ſo I underſtand by this Character vwvw 8. And the like is to be done with others; only becauſe the Minnoms increaſing in the Higher Powers take up more room, and are not ſo ſoon made as the Coſſical Characters, it is better to uſe them altogether for the Powers beyond the Zenzicube.

Coſſical *Characters beſt for the Higher Powers.*

Surdes have ſociety with Rational Numbers,

Beſides the taking Denominations from Figural Numbers, and borrowing the Coſſical Characters for them, Surdes admit into their ſociety Rational Numbers; that is, ſuch Numbers whoſe Roots may be expreſſed by Integers. As w 4, the Square Root of 4; ſo ww 8, the Cube Root of 8; and ſuch others: For the Square Root of 4 is 2; and ſo is the Cube Root of 8; and ſo conſequently no Surdes, but often ſet thus for the more apt Operation.

and Abſolute Numbers.

And ſo alſo Abſolute Numbers not Rational are uſed with them as well as Coſſicks. And Coſſicks themſelves I have ſeen wrought together with Surdes.

Nature of Surdes.

Surdes are Simple or Compound, Integral or Fracted. Of Fractionary Surdes, ſee *Chap.* 11. following.

Integral and Simple.

The Integral Simple Surdes conſiſt of one Species or Denomination: As w 5, which is to be read, the Square Root of 5; ſo ww 4, the Cube Root of 4, and the like of others.

Integral and Compound. Theſe of 2 ſorts.

Compound Surdes conſiſt of different Species, or divers Simple Surdes, or ſome Simple Surde with another Number ſet for a Surde, and are of two ſorts, to wit, Particular or Univerſal.

Particular.

Examples.

Particular, when the different Denominations compounded by the ſigns + or —, or both, are to be conſidered diſtinct as to their Roots. As w 5 + w 6, which ſignifieth, the Square Root of 5, and the Square Root of 6. So w 6 — w 5 denoteth the Square Root of 6, lacking the Square Root of 5. In both which, and ſuch others, their Roots are conſidered as two diſtinct Numbers.

Univerſal.

Examples.

Univerſal, when though the Quantities conſiſt of different Species, yet the Root of the whole compounded Number is to be underſtood thereby. As √: 5 + √36; here it ſignifieth, that the Square Root of 6 is to be added to 5, and then the Square Root of that ſumme is to be taken for the Root Univerſal and ſumme of that compounded Surde. So √: 6 — √35; here the Square Root of 5 is to be taken from 6, and the Square Root of the Remain is the Root Univerſal. See further of theſe *Chap.* 7. *Addition of Compound Surdes.*

Compounds otherwiſe conſidered.

Theſe Compound Surdes fall again under a threefold Conſideration: 1. In their Signs. 2. In their Characters. 3. In their Numbers.

1.

In Signs, as Binomials *or* Bimedials. Reſiduals *or* Apotomes. Polynomials *or* Multinomials.

1. As to their Signs, two Surdes, or a Simple Surde with a Rational or other Number conjoyned with the ſign +, are called *Binomials*, and ſometime *Bimedials*; but conjoyned with the ſign —, they are called *Reſiduals* or *Apotomes*. If three Quantities be conjoyned, and but three, they are ſometime called *Trinomials*. But generally where the compoſition hath more than two parts, the Compound is called a *Polynomial* or a *Multinomial*, that is a many named Number, as was before noted in *Coſſicks*.

Examples of Binomials.

Examples of Binomials.

3 + w 8	That is, 3 more the Square Root of 8.
w 24 + 4	Is, the Square Root of 24 more 4.
w 6 + w 2	Is, the Square Root of 6, and the Square Root of 2.
ww 9 + www 8	Signifieth, the Cubick Root of 9, and the Squared Square Root of 8.

Reſiduals.

Examples of Reſiduals.

25 — w 80	That is, 25 lacking the Square Root of 80.
w 160 — 9	Is, the Square Root of 160 wanting 9.
w 180 — w 6	Is, the Square Root of 180 wanting the Square Root of 6.
ww 100 — w 30	Signifieth, the Cubick Root of 100 abating the Square Root of 30.

Polynomials.

Examples of Polynomials.

3 + w 10 + ww 9	That is, 3 more the Square Root of 10, and the Cube Root of 9.
100 — w 20 — ww 5	Is, 100 lacking the Square Root of 20, and the Cube Root of 5.
4 + ww 30 — w 6	Is, 4 and the Cube Root of 30, wanting the Square Root of 6.
w 8 + 100 — www 7 — ww 40	Signifieth, the Square Root of 8, added to 100, lacking the Squared Square Root of 7, and the Cube Root of 40.

2.

In Characters, as Homogeneal, Heterogeneal.

2. Compound Surdes are conſidered in their Characters, and ſo they are divided into *Homogeneal* and *Heterogeneal*.

Homogeneal, when their Characters or Denominations are one and the ſame: *Heterogeneal* when contrary.

Examples of Homogeneals.

Examples of Homogeneals.

Binomials	√5 + √6	√√9 + √√10	√√√19 + √√√18
Reſiduals	√6 — √5	√√10 — √√9	√√√19 — √√√18
Polynomials	√√5 + √√6 + √√4		√√√6 + √√√15 — √√√20

Examples of Heterogeneals.

Heterogeneals.

Binomials	√5 + √√6	√√5 + √√√6	√√√14 + √√√25
Reſiduals	√5 — √√6	√√9 — √√√12	√√√14 — √√15
Polynomials	√√√5 + √√6 + √3		√√√6 + √3 — √√5

3. Their conſideration in their Numbers divides them into commenſurable and incommenſurable. 3.

Commenſurable, called alſo *Symmetral*, is when the given Numbers have a Common Diviſor, that will reduce them into leſs Terms of like nature.

In Numbers as Commenſurable or Symmetral.

Examples of Symmetrals.

Examples.

Binomials	√8 + √32 √√81 + √√24	Common Diviſor 2 3
Reſiduals	√8 — √32 √√81 — √√24	Common Diviſor 2 3
Polynomials	√48 + √75 — √27 √√24 + √√81 — √√192	Common Diviſor 3

Becauſe by theſe Common Diviſors, the Square Surdes will be brought into Square Numbers, and the Cubical Surdes into Cube Numbers, they are therefore called Commenſurable. For by the Diviſor 2, will 8 and 32 in the upper Binomial and Reſidual be brought into 4 and 16, which are both Square Numbers. And by the Diviſor 3, both 81 and 24 in the lower Binomial and Reſidual will be brought into 27 and 8, which are both Cube Numbers. And by the ſame Common Diviſor 3, will the upper Polynomial be brought into 16 + 25 — 9, which are all Squares or Rational Numbers, and the lower into 8 + 27 — 64, which are all Cubical Numbers.

Incommenſurable, or Aſymmetral Surdes, are thoſe which have no ſuch Common Diviſors.

Incommenſurable or Aſymmetral. Examples.

Examples of Aſymmetrals.

Binomials	√5 + √6	√√9 + √8	√√12 + √√√19
Reſiduals	√6 — √5	√8 — √√9	√√√12 — √√19
Polynomials	√6 + √√5 + √√√3		√6 + √√5 — √√√3

Numbers thus Commenſurable or Incommenſurable are ſaid to be Commenſurable or Incommenſurable in Power; to difference this meaſure of Numbers from plain Commenſuration, ſpoken of in Fractions before. For Numbers may be Commenſurable, as 2 and 12, yet Incommenſurable in Power: But 3 and 12 are as well Commenſurable in Power as otherwiſe, ſeeing 12 divided by 3 gives in the Quotient 4, a Square Number.

Commenſurable in Power how different from other Commenſuration.

Symmetral Surdes are diſcovered from Aſymmetral thus: Divide the greater given Number by the leſſer, and if 0 remain, then ſhall the Quotient be a Number of the ſame nature with the given Surdes, that is Square, Cube, or other like Quantity accordingly, if the Surdes are Commenſurable. But if any thing remain upon the Diviſion, reduce the Fraction into its leaſt Terms, and then reduce all into an Improper Fraction, and this ſhall repreſent two Figural Numbers of like Quantity with the given Surdes, if they are Commenſurable. And to find the Common Diviſor do thus: If 0 remain upon the Diviſion as aforeſaid, then by the Quotient of this Diviſion, divide the leaſt of the two given Surdes, and this laſt Quotient ſhall be the Common Diviſor. But if the Diviſion left a Remain, which is to be brought, as aforeſaid, into an Improper Fraction, then by the Numerator thereof divide the greateſt given Surde, or by the Denominator the leaſt, and this Quotient ſhall be the Common Diviſor.

How Symmetral Surdes *are diſcovered.*

As √8 + √32 was before counted Commenſurable, and the Common Diviſor 2; becauſe if 8 divide 32, the Quotient will be 4, a Square Number agreeable to the Surde and 0 remain; and by this 4, if 8 the leaſt of the two Surdes be divided, 2 the Common Diviſor appears in the Quotient, as at *A*. But if √12 + √147, be given, then

Examp'es.

then after Diviſion, becauſe 3 remains, I reduce 3 with 12 the Diviſor to $\frac{1}{4}$, and then 12 in the Quotient and this $\frac{1}{4}$ into an Improper Fraction, which is $\frac{49}{4}$, and being both Square Numbers of the nature with the given Surdes, ſhew them to be Symmetral; and by dividing 147 by 49, or 12 by 4, the Common Diviſor is found in the Quotient to be 3, as at *B*. Examples of Cubes ſee at *C*. and *D*.

A. $\begin{matrix}\cancel{3}\cancel{2}\\8\end{matrix}\Big(4$ A Square and 0 left : *Ergo*, √8 + √32 are Commenſurable.

$\begin{matrix}8\\\cancel{4}\end{matrix}\Big(2$ Common Diviſor.

B. $\begin{matrix}2(3\\\cancel{1}47\\\cancel{1}2\\\cancel{1}2\end{matrix}\Big(12\frac{1}{4}$ or $\frac{49}{4}$ Squares: *Ergo* √12 + √147 are Commenſurable.

$\begin{matrix}\cancel{1}47\\4\cancel{9}\end{matrix}\Big(3$ $\begin{matrix}\cancel{1}2\\\cancel{4}\end{matrix}\Big(3$ Common Diviſor.

C. $\begin{matrix}\cancel{1}28\\\cancel{1}6\end{matrix}\Big(8$ A Cube and 0 remaining : *Ergo*, √√16 + √√128 are Commenſurable.

$\begin{matrix}\cancel{1}6\\8\end{matrix}\Big(2$ Common Diviſor.

D. $\begin{matrix}(9\\8\cancel{1}\\2\cancel{4}\end{matrix}\Big(3\frac{3}{8}$ or $\frac{27}{8}$ Cubes : *Ergo*, √√24 + √√81 are Commenſurable.

$\begin{matrix}8\cancel{1}\\2\cancel{7}\end{matrix}\Big(3$ $\begin{matrix}2\cancel{4}\\8\end{matrix}\Big(3$ Common Diviſor.

Symmetral Surdes may have more Common Diviſors than one.

Some Symmetral Surdes may have more Common Diviſors than one, which is thus known : Divide one of the given Surdes, according to his Quantity, by any Number of like nature that will part it exactly without leaving a Remain; and by this Quotient divide the other given Surde, and if this ſecond Quotient be a Number of like nature, then thoſe given Surdes have more Common Diviſors than one. And ſo proving with all, leſs than the given Surdes; ſo many Quotients as will hold this tryal, ſo many Common Diviſors have thoſe Commenſurable Surdes.

Examples.

As in the Square Surdes above at *A*, if 32 be divided by 4, it giveth 8 in the Quotient; and if this 8, divide 8 the other given Surde, the Quotient is 1 another Square Number, therefore ſhall 8 be another Common Diviſor to √8 + √32 beſides 2 found out as above.

So in the Cube Surdes above at *C*, if 128 be divided by 8, it giveth 16 in the Quotient : And if this 16 divide 16, the other given Surde, the Quotient is 1, another Cube Number; therefore ſhall 16 be another Common Diviſor to √√16 + √√128, beſides 2, found as above.

$\sqrt{}\begin{matrix}\cancel{3}\cancel{2}\\3\ \cancel{4}\end{matrix}\Big(8$ $\begin{matrix}\cancel{8}\\\cancel{8}\end{matrix}\Big(1\ 3$ $\sqrt{}\sqrt{}\begin{matrix}\cancel{1}\cancel{2}8\\0\ \cancel{8}\end{matrix}\Big(16$ $\begin{matrix}\cancel{1}\cancel{6}\\\cancel{1}\cancel{6}\end{matrix}\Big(1\ 0$

Surdes not ſo ſtrictly uſed to place the greateſt foremoſt. Uſe of Signs as in Coſſicks.

All further needful to this *Chapter* is, That Authors do not ſo ſtrictly obſerve to place the Greateſt Surdes foremoſt, as they do the Coſſicks; but ſometime the Leſſer Surdes are ſet to the Left Hand of the Greater. But agreeable to Coſſicks in the uſe of the ſigns with or without Aſteriſques. And where — is not ſet + is underſtood.

CHAP. II.

Reduction of Surdes.

Surdes *reduced.*

To their leaſt Termes.

UNder Reduction of Surdes is comprehended, To leſſen their Terms, and alter their different Denominations into one: Both ſometime called, *Alteration of Surdes.*

1. To leſſen the Terms of a Surde is but abbreviation. And as all Common Fra-

ctions will not be abbreviated; ſo neither will all Surdes have their Terms leſſened. But when the Denomination or Character is a Compound Coſſick, and the annexed Number hath a Root that may be expreſſed by part of that Coſſical Character, then reduce the Number and Character thereto accordingly, by extracting the Root of the Number, and clearing the Character of that part of the Compoſition.

Examples.

As √ʒʒ25 and √ʒφ81, the Characters being compounded of ʒ with ʒ in the firſt, and ʒ with φ in the ſecond, and the Numbers 25 and 81 being both Square Numbers, the Square Roots thereof are to be taken, and Square in the Characters abated from both their Quantities, and both are to be expreſſed in leſs Terms by √ʒ5 and √φ9, or W5 and ww9.

So alſo √ʒφ27 may be reduced to the √ʒ3, being diſcharged of φ. And √ʒſß32 may be abbreviated to the √ʒ2, and diſcharged of ſß.

To reduce Surdes *to one Denominator.*

2. To bring the different Denominations of Surdes into one, belongs to Heterogeneal Surdes, or one Surde with an Abſolute Number.

Surdes *and Abſolute Numbers.*

(1.) If an Abſolute Number be given to be reduced into the Denomination of a Surde, then multiply the Abſolute Number according to the Denomination of the Surde, and ſet before it the like Character.

Examples.

As if W8 and 2 be reduced into one Denomination; 2 muſt be multiplyed Squarely, becauſe the Denomination of the Surde 8 is ſuch. And ſo will the Numbers ſtand thus, W8 and W4.

And if ww9 and 4 be reduced into one Denomination, 4 muſt be multiplyed Cubically: And thus reduced the Surde ſhall be ww9 and ww64.

Surdes *with uncompound* Indices.

(2.) If two different Surdes be given to be reduced into one Denomination, and their *Indices* be uncompounded; then alternately multiply the Number of the one Surde according to the Denomination of the other, and to both the Products adjoyn both the Characters for the Common Denominator.

Example.

As if W8 and ww10 were to be reduced to one Denominator; then muſt 10 be multiplyed Squarely and 8 Cubically, and each production ſhall be Square Cube, ſet after the manner of Surdes thus, √ʒφ100 and √ʒφ512; or thus, www100 and www512.

$$\left.\begin{array}{l}\sqrt{}ʒ\,8 \times 8 \times 8 = 512 \\ \sqrt{}φ\,10 \times 10 = 100\end{array}\right\} ʒφ \qquad \frac{\sqrt{}ʒφ\,512}{\sqrt{}ʒ\,8} \; \underset{\sqrt{}ʒφ}{\mathrm{X}} \; \frac{\sqrt{}ʒφ\,100}{\sqrt{}φ\,10}$$

Surdes *with Compound* Indices.

But (3.) If the *Indices* of the given Quantities be Numbers Compound, then by the greateſt Common Diviſor divide them, and then by the leaſt Terms of the *Indices* multiply alternately as well the *Indices* of the one by the other for a new *Index*, as the Numbers given by the Powers of theſe alternate *Indices* for the reduced Surdes.

Example.

As √ʒʒ10 and √ʒφ7, thus reduced ſhall be √ʒʒφ1000 and √ʒʒφ49. For the *Index* of ʒʒ is 4, and the *Index* of ʒφ is 6, the Common Diviſor of 4 and 6 is 2, which reduceth the one to 2 the *Index* of ʒ, and the other to 3 the *Index* of φ; therefore alternately Squaring 7 it is 49, and Cubing 10 it is 1000, and multiplying 6 by 2, or 4 by 3, the Product 12 is the *Index* of the reduced Powers, which is ʒʒφ.

$$\begin{array}{lllllll} & \sqrt{} & [12] & 1000 & \sqrt{} & [12] & 49 \\ 2) & \sqrt{} & [4] & 10 & \sqrt{} & [6] & 7 \\ & & [2] & & & [3] & \end{array}$$

Proof of Reduction *of* Surdes.

Beſides the Proof of that ſort of Reduction which leſſeneth the Terms, by exalting the leſſened Surdes into the Powers from whence they were abated; and reciprocally that ſort of Reduction which increaſeth their Terms and Denominations, by extracting the Roots, and abating the Characters of the Surdes accordingly; all ſorts of Reduction of Surdes may be proved, by ſuppoſing Rational Numbers inſtead of the Surdes, and working with them as if they were Surdes.

As in the former Example, where √ʒʒ25 was reduced to W5; therefore if W5 be multiplyed Squarely, it ſhall be ww25: And in Rational Numbers it is evident that W4 and the ww16 are equal, being in each but 2.

Again in the ſecond ſort of Reduction, and firſt Example, there 2 reduced to a Square Denomination ſhall be 4, the Rational Number, and W8 and W4, is all one as W8 and 2; for they are both equal: And ſo is the ww64 or 4 Abſolute Numbers in the next Example.

Likewiſe if W 4 and the ww 27, both Rational Numbers, be reduced to one Denomination, they ſhall be √Ʒφ 64 and √Ʒφ 729, agreeable to the next Example of Reduction. And the √Ʒφ of 64 being but 2, equal to the W 4, and the √Ʒφ of 729 being 3, as the ww 27 ſhew the Reduction right.

Moreover, agreeable to the laſt Example of Reduction, if I take √ƷƷ 16 and the √Ʒφ 64, that are both Rational Numbers, and have 2 for their Roots, and reduce them to one Denomination, they ſhall be √ƷƷφ 4096 and √ƷƷφ 4096, and be equal to the other. For the √ƷƷφ of 4096 is 2; whence may be alſo obſerved, that if the Roots of the given Surdes be equal, the reduced Surdes will be equal.

CHAP. III.

Addition of Simple Surdes.

Addition of Simple Surdes.

TO underſtand Addition of Surdes the better, it is meet to Analyſe them.

Into
- Simple
  - Homogeneal and Commenſurable.
  - Homogeneal and Incommenſurable.
  - Heterogeneal.
- Compound
  - Particular.
  - Univerſal.

1. *Homogeneal and Commenſurable.*

1. To add two Simple Surdes that are both Homogeneal and Commenſurable, divide them by the Common Diviſor, and extract the Roots of the Quotients, then multiply the Total of the Roots Figurately according to their Quantities, and this Product multiplyed by the Common Diviſor ſhall be the Total of the added Surdes.

Examples.

As to add W 12 and W 147; both Numbers divided by 3, the Common Diviſor, giveth 4 and 49, which are Square Numbers, and their Roots 2 and 7, the Total whereof 9 multiplyed Squarely produceth 81; this multiplyed by 3 is 243. So is W 243 the Total of W 12 + W 147, as at *A*.

Alſo to add ww 81 and ww 24, Commenſurable alſo by 3; after Diviſion thereby the Cubes are 27 and 8, and their Roots 3 and 2, which together make 5, the Cube whereof is 125; this multiplyed by 3 produceth 375. So is ww 375 the ſumme of ww 81 + ww 24, as at *B*.

An Example of Squared Square Surdes is ſet at *C*.

	A.	*B.*	*C.*
Addends	W 12 + W 147	ww 81 + ww 24	www 648 + www 5000
	3)	3)	8)
	4Ʒ 49Ʒ	27φ 8φ	81ƷƷ 625ƷƷ
	2√ + 7√	3√ + 2√	3√ + 5√
	9	5	8
	9ℨ	25Ʒ	512φ
	81Ʒ	125φ	4096ƷƷ
	3	3	8
Totals	W 243	ww 375	www 32768

Where the Data have many Common Diviſors.

If the given Surdes have many Common Diviſors, any one of them may be uſed. As in W 1152 + W 288, there are 5 Common Diviſors, *viz.* 2, 8, 18, 32 and 72, the Addition by all which agree to be W 2592, as in the Operations following.

Examples.

W 1152 + W 288	W 1152 + W 288	W 1152 + W 288
2)	8)	18)
576Ʒ 144Ʒ	144Ʒ 36Ʒ	64Ʒ 16Ʒ
24√ + 12√	12√ + 6√	8√ + 4√
36	18	12
36ℨ	18ℨ	12ℨ
1296Ʒ	324Ʒ	144Ʒ
2	8	18
W 2592	W 2592	W 2592

W 1152 + W 288	W 1152 + W 288
32)	72)
36ʒ 9ʒ	16ʒ 4ʒ
6√ + 3√	4√ + 2√
9	6
9z	6z
81ʒ	36ʒ
32	72
162	72
243	252
W 2592	W 2592

2. *Square Surdes Incommenſurable. Reducible what, and how added.*

2. Simple Square Surdes Homogeneal and Incommenſurable, if the Product of both multiplyed together bring forth a Figural Number of the ſame kind (as many times happeneth when the Numbers are Commenſurable, or Rational Numbers uſed as Surdes) then are they called reducible, and are added thus: To the Total of the Surdes given prefix their proper Character, afterward either extract the Root of their Product and multiply this Root by the *Index*, and add this Product to the former Total; or elſe multiply the Product of both Surdes by the double *Index*, and extract the Root of this laſt Product, and add this Root to the Total firſt reſerved; and this Number with his Character ſhall be the ſumme of the added Surdes.

Example.

As to add W 3 and W 12; becauſe 3 and 12 multiplyed is 36, a Square Number agreeable to the given Surdes, they are reducible; then 3 and 12 added are 15 to be ſet apart with their Character thus, W 15; afterward the Root of 36, which is 6, is to be multiplyed by 2, the *Index* of Squares, and the Product 12 is to be added to the 15 before reſerved; or elſe 36 is to be multiplyed by 4 the double *Index*, and the Root of 144 which is 12, added to 15, as before, makes the Total of this Addition W 27, as at *D.* and *E.*

	D.		*E.*
Addends	W 3 + W 12		W 3 + W 12
	3		3
	12		12
W 15	W 36 is 6	W 15	36
	2 *Index*		4
	W 15 + W 12		W 144
Totals	W 27		W 15 + 12 or W 15 + W 144
			W 27

When not Reducible.

If Square Surdes are not thus reducible, they with others of higher Denominations Incommenſurable, although Homogeneal, are to be joyned together with the ſign of Addition +.

Examples.

As to add W 6 and W 7, they are ſet thus, W 6 + W 7.

And to add ww 6 and ww 7, they are ſet thus, ww 6 + ww 7.

How ſet by ſome.

Yet ſome ſet the Square, although Incommenſurable, after the form of Addition abovementioned; whereby W 6 + W 7 is brought to W 13 + W 168; for that by this form they come to be Commenſurable with other Numbers with whom they occaſionally may be uſed.

Example.

	W 6 + W 7
	6
	7
W 13	42
	4
	W 13 + W 168

3. *Heterogeneal.*

3. Simple Heterogeneal Surdes are firſt to be reduced, and then if by their Reduction they prove Commenſurable add them as ſuch; if otherwiſe, conjoyn them by the ſign +.

Examples.

As to add W 3 and 9, Abſolute Numbers together, being reduced they are W 3 and W 81, and becauſe Incommenſurable abide ſo; unleſs ſet after the form of Addition laſt mentioned, and then they ſtand thus, W 84 + W 972.

And if W 3 be given to be added to ww 2, after Reduction to √3ϕ 27 + √3ϕ 4, becauſe Incommenſurable they are left ſo without further work.

But if the Rational Number W 16 be given to be added to ww 8, another Rational Number, by Reduction they are brought to √ʒφ 4096 + √ʒφ 64, and being Commenſurable, and added as before, make √ʒφ 46656, which is alſo a Rational Number, and hath 6 for Zenzicube Root thereof.

√ʒφ 4096 + √ʒφ 64   4 + 2 = 6

64) ———

64ʒφ   1ʒφ
2√ + 1√

3

243 ß

729ʒφ

64

2916

4374

√ʒφ 46656 is 6

√ʒφ 4096 / W 16 X √ʒφ 64 / ww 8

√ʒφ

4.
Different Signs

4. If two Simple Surdes of different ſigns be given to be added together, then conjoyn them by the ſign of Subſtraction —.

As to add W 3 with — W 5, they ſhall be ſet thus, W 3 — W 5.

To add any Surde to it ſelf.

By what hath been ſaid of Addition of Simple Surdes, appeareth this Conſectary; That to add any Surde to it ſelf is but to multiply the Square Surdes by 4, Cube Surdes by 8, Squared Squares by 16, &c. See the Examples following.

Examples.

Addends W8 + W8   ww 9 + ww 9   www 10 + www 10

8) ———   9) ———   10) ———

1ʒ 1ʒ   1ʒ 1ʒ   1ʒʒ 1ʒʒ
1√ + 1√   1√ + 1√   1√ + 1√

2   2   2

2ℨ   4ʒ   8φ

4ʒ   W8   8φ   ww 9   16ʒʒ   www 10
8   4   9   8   10   16

Totals W 32   W 32   ww 72   ww 72   www 160   www 160

Addition of Compounds why deferred.

Addition of Compound Surdes is deferred to the *Seventh Chapter*, till Subſtraction of Simple Surdes be learned; becauſe the knowledge thereof is neceſſary to the Addition of ſome Compound Surdes.

Proof of Addition of Simple Surdes.

Subſtraction of Simple Surdes is the Proof of Addition of Simple Surdes, and there to be ſeen. But the truth of Addition will be clear, if inſtead of the Surdes, Rational Numbers be taken, and operation made therewith.

For in the laſt Example of reduced Surdes, the Zenzicube Root of 4096 is 4, and of 64 is 2, which 4 and 2 make 6, and the Total of the Addition bringeth forth 46656 a Zenzicube Number that hath 6 for the Root, as was before noted.

ʒφ   φ ℨ
zß
46656 |216|6
· · ·
4 : :
Gnomon{4: : / 1 :}
Gnomon{252: / 36}

6ℨ
6
36ʒ
6
216φ
216
1296
216
432
46656ʒφ

Indices. 3 / 3 / 6

Value 4 / 2 / 6 Total

C H A P. IV.

Subſtraction of Simple Surdes.

Subſtraction of Simple Surdes.
1.
Homogeneal and Commenſurable.

In the ſame method with Addition doth Subſtraction proceed.

1. Then to ſubſtract one Homogeneal and Commenſurable Surde from another, after dividing them by the Common Diviſor, and extracting the Roots of the Quotients,

tients, take the Leſſer Root from the Greater, then multiply the Remainer Figurately according to their Quantities, and this Product multiplyed by the Common Diviſor ſhall be the remaining Surde deſired with the ſame ſign. But if the Subtrahend be the Greater, then the ſign + ſhall be changed into —.

Examples.

As to ſubſtract w 12 from w 243 by the Common Diviſor 3, they are reduced into the Squares 81 and 4, whoſe Roots are 9 and 2, the difference 7, this multiplyed Squarely is 49, which increaſed by the Common Diviſor is w 147, the Remainer deſired, as at *A.*

Likewiſe if w 147 were taken from w 243, there would remain w 12, as at *B.*

But if w 243 were to be taken from w 147 or w 12, in this there would want w 147, and in that w 12, and then to be marked with —, as at *C.* and *D.*

Greater Surde. Subtrahend.
w 243 — w 12
3)
81ʒ 4ʒ
9√ — 2√
A. 7
7ℨ
49ʒ
3
Remain w 147

Greater Surde. Subtrahend.
w 243 — w 147
3)
81ʒ 49ʒ
9√ — 7√
B. 2
2ℨ
4ʒ
3
Remain w 12

Leſſer Surde. Subtrah. Remain.
C. w 147 — w 243 = — w 12

Leſſer Surde. Subtrah. Remain.
D. w 12 — w 243 = — w 147

Commenſurable Surdes of Higher Powers, are likewiſe thus to be ſubſtracted
Examples to take ww 24 from ww 375, the Remain will be ww 81, but will want ww 81, if ww 375 be taken from ww 24

So www 648 taken from www 32768, will leave www 5000; but the Greater ſubſtracted from the Leſſer, the Remain will be ſo much too ſhort.

Greater Surde. Subtrahend.
ww 375 — ww 24
3)
125φ 8φ
5√ — 2√
3
9ʒ
27φ
3
Remain ww 81

Greater Surde. Subtrahend.
www 32768 — www 648
8)
4096ʒʒ — 81ʒʒ
8√ — 3√
5
125φ
625ʒʒ
8
Remain www 5000

Leſſer Surde. Subtrah. Remain
ww 24 — ww 375 = — ww 81

Leſſer Surde. Subtrah. Remain.
www 648 — www 32768 = — www 5000

If the given Surdes have many Common Diviſors; any one of them may be uſed in Subſtraction, as before in Addition.

If the Data *have many Common Diviſors.*

2. One Simple Square Surde Homogeneal and Incommenſurable to another, may be ſubſtracted therefrom, if the Product of them both multiplyed together produce a Figural Number of the ſame kind: For then to the Total of the Surdes prefix their proper Character; afterward either extract the Root of their Product, and multiply this Root by the *Index*, and deduct this Product from the former Total: Or elſe multiply the Product of both the given Surdes by the double *Index*, and extract the Root of this laſt Product, and ſubſtract this Root from the Total firſt reſerved. And this Number with his Character ſhall be the Remain deſired, with the ſign changed, as before noted, if the Subtrahend be the Greater.

2.
Square Surdes *Incommenſurable. Reducible how added.*

Example.

As to extract w 12 from w 27, being multiplyed they produce 324, the Square Number of 18; therefore 18 multiplyed by the *Index* 2, or 324 by 4 the double *Index*, and the Root of this Product, or the Product 36 ſubſtracted from w 39, the Total of 12 and 27 added, leaves remaining w 3, as at *E.* and *F.* But if w 27 were to be taken from w 12, becauſe the Subtrahend is greateſt, the ſign or places of the Surdes ſhall be changed, as at *G.*

Greater Surde W 27 — W 12

27
12

54
27

E. W 39

W 324 is 18
2 *Ind.*

W 39 —— 36

Remain W 3

W 27 — W 12 Subtrahend

27
12

54
27

F. W 39

324
4 Double *Index.*

W 1296 is 36

W 39 — 36 or W 39 — W 1296

W 3

G.
W 12 — W 27 = { W 1296 — W 1521 or 36 — 39 } = — W 3

Not Reducible. If Square Surdes be not thus reducible to Square Numbers by Multiplication, then they with other Surdes Incommenſurable of Higher Powers, although Homogeneal, are to be joyned together with the ſign of Subſtraction —.

Examples. As to ſubſtract W 6 from W 7, or W 7 from W 6; they are ſet thus;

W 7 — W 6 W 6 — W 7

And to ſubſtract ww 6 from ww 7, or ww 7 from ww 6; they are ſet thus;

ww 7 — ww 6 ww 6 — ww 7

How ſet by ſome. Nevertheleſs for the Reaſon before rendred in Addition, ſome ſet the Square Surdes, though Incommenſurable, after the form of Subſtraction above-mentioned; whereby W 7 — W 6 is brought to W 13 — W 168, and W 6 — W 7 to W 168 — 13.

Examples.

W 7 — W 6

7
6

W 13

42
4

W 168

W 13 — W 168

W 6 — W 7

6
7

W 13

42
4

W 168

W 168 — 13

3. *Heterogeneal.* 3. Simple Heterogeneal Surdes are firſt to be reduced, and then, if by their Reduction they happen to be Commenſurable, ſubſtract them as ſuch: But if otherwiſe, conjoyn them accordingly with the ſign —.

Examples. As to take W 3 from 9, Abſolute Numbers after Reduction, they ſtand, becauſe Incommenſurable, W 81 — W 3, unleſs after the form of Subſtraction laſt abovementioned, and then they ſtand thus, W 84 — W 972.

And if ww 2 be ſubſtracted from W 3, they are reduced, and becauſe Incommenſurable left thus, √3φ 27 — √3φ 4.

But if ww 8 be taken from W 16, they being reduced, and both Rational Numbers and Commenſurable, will be √3φ 4096 — √3φ 64, that is in concluſion √3φ 64, which is alſo a Rational Number, and hath 2 for the Zenzicube Root thereof.

√3φ 4096 — √3φ 64 4 — 2 = 2

64)

64 3φ 1 3φ
2 √ — 1 √

1
1 ß

1 3φ
64

√3φ 64

√3φ 4096 √3φ 64

W 16 — ww 8

√3φ

And if W 16 had been given to have been ſubſtracted from ww 8, then had there wanted √3φ 64, and the Remain ſhould have been — √3φ 64.

4. *Different Signs* 4. If the given Surdes be of different ſigns, the Surdes are to be added, and the ſign of the Remain, as in Coſſicks, ſhall be contrary to the ſign of the Subtrahend, or Number ſubſtracted, that is as the upper Number, if the Surdes be Commenſurable; but if Incommenſurable, then conjoyn them by the ſign of Addition +.

Examples. As to ſubſtract — W 13 from W 52, they are added becauſe the ſigns are unlike, the one + and the other —, the Total, which is W 117, is the Remain, as at *H.*

And W 5 — W 3 and W 5 — W 3 being Incommenſurable, have their Remains, as at *I.* and *K*, W 5 + W 3 and W 5 + W 3.

w 52 — w 13
13)————
4ʒ 13
2√ + 1√
3
H. 3ℨ
93
13
Total w 117 Remain.

I.
w 5 — w 3
w 5 + w 3

K.
w 5 — w 3
w 5 + w 3

From what hath been ſaid of Subſtraction of Simple Surdes, theſe two Conſectaries are apparent.

2 Conſectaries hence.

1. That to ſubſtract any Surde from it ſelf leaves 0 remaining, as in other Numbers. As to take w 10 from w 10, the Remain is w 0.

A Surde taken from it ſelf leaves 0.

w 10 — w 10
10)————
1ʒ 1ʒ
1√ — 1√
0
0ℨ
0ʒ
10
w 0

w 10
w 10
w 0

2. That to take half any Surde is but to divide Square Surdes by 4, Cube Surdes by 8, Squared Squares by 16, *&c.* as is plain by the Examples following.

To take half a Surde.

Greater Surde. Subtrah.
w 32 — w 8
8)————
4ʒ 1ʒ
2√ — 1√
1
1ℨ
1ʒ
8
Remain w 8
w 32/4 (w 8

Greater Surde. Subtrah.
ww 72 — ww 9
9)————
8φ 1φ
2√ — 1√
1
1ʒ
1φ
9
Remain ww 9
ww 72/8 (ww 9

Greater Surde. Subtrah.
www 160 — www 10
10)————
16ʒʒ 1ʒʒ
2√ — 1√
1
1φ
1ʒʒ
10
Remain www 10
www 160/16 (www 10

Subſtraction of Compound Surdes is referred to the *Eighth Chapter*, after their Addition hath been inſpected.

Subſtraction of Compounds why deferred.

Foraſmuch as ſeveral of the Examples in this *Chapter* from which Subſtraction is made, are the Totals of the Additions in the foregoing *Chapter*, and the Subtrahends here are one of the Surde Numbers added, and the Remains the other, it will apparently manifeſt the Proof of Subſtraction of Surdes by Addition, and Addition by Subſtraction.

Proof of Subſtraction of Simple Surdes.

But for a full demonſtration of all ſubſtractionary Operations, let Rational Numbers be ſet inſtead of the Surdes, and after Subſtraction made therewith, the truth will appear by the equal value of the Remain.

For in the Rational Numbers above-mentioned √3φ 4096 is 4, from which if √3φ 64 which is 2, be ſubſtracted, the Remain is √3φ 64, which is 2, and anſwers to the Remain of 4 — 2, which is but 2.

3φ φ ℨ
64 | 8 | 2

2ℨ
2
4ʒ *Indices.*
2
8φ
8 3
64ʒφ 3
6

Value
4
2
2 Remain

CHAP.

CHAP. V.

Multiplication of Simple Surdes.

Multiplication *of Simple* Surdes. *Homogeneal.*

TO multiply Simple Surdes obſerve their Homogenity or Heterogeniety. For,

1. If the Surdes given to be multiplyed be Homogeneal, then multiply Number by Number, Integers as Integers, and Fractions as Fractions, and to the Product prefix the Character common to the given Surdes.

Examples.

As to multiply W 15 by W 5, the Product is W 75, ſet as at *A.* or *B.*
And to multiply W $12\frac{1}{2}$ by W $4\frac{1}{2}$, the Product is W $56\frac{1}{4}$, as at *C.*

A.
Multiplicand W 15
Multiplyer W 5
Product W 75

Md. Mr. Prod.
B. W 15 × W 5 = W 75

C.
Multiplicand W $12\frac{1}{2}$
Multiplyer W $4\frac{1}{2}$
Product W $56\frac{1}{4}$ or $7\frac{1}{2}$

$$\frac{\frac{25}{2}\times\frac{9}{2}=\frac{225}{4}\left(56\tfrac{1}{4}\right.}{}\qquad 225 \qquad 4$$

1 Root
225 | 15

1
{10
25
4 | 2
($7\frac{1}{2}$

Other Examples.

Multiplicands	W 48	ww 12	www 30
Multiplyers	W 5	ww 6	www 3
Products	W 240	ww 72	www 90

Heterogeneal.

2. If the Surdes given to be multiplyed be Heterogeneal, or one Surde with an Abſolute Number; then firſt reduce them to one Denomination, and then multiply Number by Number, as before.

Examples.

As to multiply W 10 by 3; firſt 3 is ſquared, and then by 9 is 10 multiplyed; ſo is the Product W 90, as at *D.*

And to multiply W 8 by ww 10, being reduced they are √3φ 512, and √3φ 100, and 512 multiplyed by 100 produce 51200; ſo is the Product √3φ 51200, as at *E.*

D.
W 10 X W 9
W 10 X 3
W

W 10
W 9
W 90

E.
√3φ 512 X √3φ 100
W 8 X ww 10
√3φ

√3φ 512
√3φ 100
√3φ 51200

Conſectaries hence.

Out of Multiplication of Simple Surdes iſſue theſe enſuing Conſectaries.

1. *To double, triple,* &c. *a* Surde, *what.*

1. That to multiply any Surde, is to increaſe him by the Power of a Root Homogeneal: And ſo to double any Square Surde is to multiply him by 4, which is the Square Power of the Root 2, as before noted in Addition. Likewiſe to triple any Square Surde is to multiply him by 9, *&c.* And to double any Cube Surde is to multiply him by 8, the Cube of 2. As alſo to triple him is to multiply him by 27, *&c.*

Examples.

	Doubled.	Tripled.	Quadrupled.
Square Surdes	W 3 4 W 12	W 3 9 W 27	W 3 16 W 48
Cube Surdes	ww 3 8 ww 24	ww 3 27 ww 81	ww 3 81 ww 243
Squared Squares	www 3 16 www 48	www 3 81 www 243	www 3 243 www 729

2. *Product is often Rational.*

2. That Multiplication of Surdes oftentimes produceth Rational Numbers, whoſe Roots are Abſolute Numbers, and may be ſo expreſſed.

Examples.

Examples in Surdes	Square.	Cubed	Squared Square.
	W 48 W 3 W 144 √ 12	ww 9 ww 3 ww 27 √ 3	www 8 www 2 www 16 √ 2

3. That to multiply the ſide of any Power according to the exigency of his own kind; the Character or Note of the ſide may be cancelled, and the Number left Abſolute. For W 3 multiplyed by the W 3, produceth W 9, which is the Abſolute Number 3. So the Square of the Square Root of 64, and the Cube of the Cube Root of 64, is 64.

3. When the Character may be cancelled.

Examples.

Examples W 3
W 3
W 9 √ 3

Alſo W 64 × W 64 = 64
And WW 64 × WW 64 × WW 64 = 64

4. That the ſide of a Power whoſe *Index* is a Compounded Number, multiplyed into one of the Compounding Powers, produceth a Surde anſwerable to the Quotient of the Greater *Index* divided by the Leſſer, and may be ſet alone accordingly. As ʒʒ whoſe *Index* is 4, compounded of ʒ whoſe *Index* is 2; it ſhall be therefore, that if WW 10 were ſo to be multiplyed, the Product WW 100 ſhall be equal to W 10, becauſe W anſwers to the *Index* 2, brought out by the Diviſion of 4 by 2. So the Square of √ʒφ 64 is √φ 64; and the Cube of √ʒφ 64 is √ʒ 64; for the √ʒφ is √ of 2 multiplyed by 3.

4. What produced by the Side of a Power multiplyed, &c. Examples.

WW 10
WW 10
WW 100 = W 10

Ergo WW 10 × WW 10 = W 10 becauſe
ʒ ʒʒ ʒ
2) 4 (2

√ʒφ 64
√ʒφ 64
256
384
√ʒφ 4096 = √φ 64 which is a Rational Number, and the Root 4.

A Rational Number, and hath the Root 2.

Ergo √ʒφ 64 × √ʒφ 64 = √φ 64 becauſe
ʒ ʒφ φ
2) 6 (3

√ʒφ 64
16384
24576
√ʒφ 262144 = √ʒ 64, alſo a Rational Number, and the Root 8.

Ergo √ʒφ 64 × √ʒφ 64 × √ʒφ 64 = √ʒ 64 becauſe
φ ʒφ ʒ
3) 6 (2

5. That if a Figural Number be multiplyed by an Homogeneal Figural Number, the Product ſhall be a Figural Number of the ſame kind, whoſe Side or Root ſhall be equal to the Product of the ſides of the Numbers multiplyed. As 4 and 9, both Squares, produce 36, whoſe Root is 6, equal to 2 × 3, the ſides of 4 and 9. So 343 the Cube of 7, if multiplyed into 27, the Cube of 3, ſhall produce 9261, the Cube of 21, equal to the Product of 7 × 3.

5. Homogeneal Figurals multiplyed what produced. Examples.

Squares { 4 √ 2
{ 9 3
Square 36 √ 6

Cubes { 343 √ 7
{ 27 3
2401
686
Cube 9261 √ 21

6. That the ſides of Homogeneal Surdes multiplyed procreateth ſides of Homogeneal Surdes.

6. Sides of Homogeneals what they produce. Examples.

Ergo W 2 × W 3 begetteth W 6. And WW 7 × WW 3 begetteth WW 21.

Multiplication of Compound Surdes is remitted to the *Ninth Chapter* of this *Fifth Part* of the *Third Book*, that it may follow in order Compound Addition and Subſtraction.

Multiplication *of Compounds why deferred.*

Diviſion of Simple Surdes is the Proof of Multiplication of Simple Surdes, and there ſet forth. Yet beſides, the truth of this Simple Multiplication will appear by taking Rational Numbers and multiplying them, and extracting the Roots of the Product, which will equalize the Product of the Roots of the Factors multiplyed in Abſolute Numbers.

Proof of Multiplication *of Simple* Surdes.

As if W 9 be multiplyed by W 16, the Product will be W 144, whoſe Root is 12, and ſo will be the Product of 3, the Root of 9, multiplyed into 4, the Root of 16, as at *F*.

And ſo if W 9, which is 3, be multiplyed by WW 8, which is 2, the Product will be 6, agreeable to the √ʒφ 46656, as at *G*.

F.

W 9 √3
W 16 4
W 144 12
3 2
144(12
1
4
4

√3φ 729 X √3φ 64
W 9 ww 8
√3φ

G.

√3φ 729 √3
√3φ 64 2
2916
4374
√3φ 46656 √6

3φ φ 2
25
46656(216(6
4
41
2556

C H A P. VI.

Diviſion of Simple Surdes.

Diviſion of Simple Surdes. Homogeneal.

After Multiplication of Simple Surdes followeth their Diviſion, and with like eaſe and order, as they are Homogeneal or Heterogeneal.

1. If they be Homogeneal, then divide the Number of the Dividend by the Number of the Diviſor, Integers as Integers, and Fractions as Fractions, and to the Quotient annex the Common Character of the Surde.

Examples.

As to divide W 75 by W 5, the Quotient will be W 15, as at A. or B.

And to divide W $56\frac{1}{4}$ by W $4\frac{1}{2}$, the Quotient will be W $12\frac{1}{2}$, as at C. or D.

A.

Dividend W 75 (W 15 Quotient
Diviſor W 5

B.

Diviſor. Dividend. Quotient.
W 5) W 75 (W 15

C.

Dividend. Diviſor. Quotient.

450

$\frac{225}{W\,56\frac{1}{4}}$ X $\frac{9}{W\,4\frac{1}{2}}$ = W $\frac{450}{36}$ (W $12\frac{1}{2}$

36

D.

Diviſor. Dividend. Quotient.

$\frac{9}{W\,4\frac{1}{2}}$) $\frac{225}{W\,56\frac{1}{4}}$ (W $\frac{450}{36}$ (W $12\frac{1}{2}$

Other Examples.

Dividend ww 240 (ww 48 Quotient
Diviſor ww 5

Dividend ww 72 (ww 12 Quotient
Diviſor ww 6

Heterogeneal.

2. If the Surdes be Heterogeneal, or one Surde be given to be divided with, or to divide an Abſolute Number; then firſt reduce them to one Denomination, and then divide the Dividend by the Diviſor, as above.

Examples.

As to divide W 90 by 3; firſt 3 is ſquared, and then by 9 is 90 divided, ſo will the Quotient be W 10, as at E.

And to divide √3φ 51200 by W 8, being reduced they are √3φ 51200 and √3φ 512, and then divided, the Quotient will be √3φ 100, as at F, and may be depreſſed to √φ 10.

W 90 X W 9
W 90 3
W

E.
W 90 (W 10
W 9

2) √[2] 8 [1]
√[6] 512

√[6] 51200
√[6] 51200 [3]

F.
√3φ 51200 (√3φ 100
√3φ 512

Conſectaries hence.

The following Conſectaries flow from Diviſion of Simple Surdes.

1. To take the half, third part, &c. of a Surde.

1. That to divide any Surde is to diminiſh him by the Power of a Root Homogeneal. So as to take the half of any Square Surde is to divide him by 4, the Power of the Root 2, as was before noted in Subſtraction Alſo to take the third part of any Square Surde is to divide him by 9, &c. And to take the half of any Cube Surde is to divide him by 8, the Cube of 2. Likewiſe to take the third part thereof is to divide him by 27, &c.

Examples.

Examples of	The Half.	The Third Part.	The Fourth part.
Square Surdes	W 12/4 (W 3	W 27/9 (W 3	W 48/16 (W 3
Cube Surdes	ww 24/8 (ww 3	ww 81/27 (ww 3	ww 192/64 (ww 3

Squared Squares www $\frac{48}{16}$(www 3 www $\frac{243}{81}$(www 3 www $\frac{729}{243}$(www 3

2. *Quotient is often a Rational. Examples.*

2. That Diviſion of Surdes ſometimes bringeth forth Rational Numbers in the Quotient: The Roots whereof being Abſolute Numbers, may be ſo expreſſed.

Examples in Surdes Square. Cubed Squared Square.

$\frac{\text{W }27}{\text{W }3}$(W 9 $\frac{\text{ww }24}{\text{ww }3}$(ww 8 $\frac{\text{www }48}{\text{www }3}$\ www 16

√ 3 √ 2 √ 2

3. Surde *dividing himſelf gives* 1. *Examples.*

3. That Diviſion of any Surde by himſelf, giveth in the Quotient a Surde Unit.

Examples in Square Surdes. Cube Surdes. Squared Square Surdes.

$\frac{\text{W }5}{\text{W }5}$(W 1 $\frac{\text{ww }9}{\text{ww }9}$(ww 1 $\frac{\text{www }15}{\text{www }15}$(www 1

4. *Quotient of a Power, &c. divided by the Side. Examples.*

4. That a Power whoſe *Index* is compounded, divided by one ſide of the Compounding Powers, ſhall give the Quotient higher or lower according to the diviſing Power. For Diviſion made by the Root, or loweſt Quantity of the Dividend, the Root of the Quotient ſhall be equal to the Root of the higher compounding Power of the Diviſor; and if by the Higher Power, the contrary.

As if √3φ 262144 which is 8, be divided by √3φ 64 which is 2, the Quotient will be 4, the √3φ of 4096. And becauſe 64 is the Root, the √φ 64 which is 4, may be taken, (Cube being the higher compounding Power in 3φ). But if √3φ 262144 be divided by √3φ 4096, the Quotient will be √3φ 64 which is 2, and may be taken in ſtead thereof.

√ 2) √ 8 (√ 4
√3φ 64) √3φ 262144 (√3φ 4096 = √φ 64, that is 4.

√ 4) √ 8 (√ 2
√3φ 4096) √3φ 262144 (√3φ 64 = √ 2

5. *Homogeneal Figurals divided, what the Quotient. Examples.*

5. That if a Figural Number be divided by a Figural Number Homogeneal, the Quotient ſhall be a Figural Number of the ſame kind, whoſe ſide is equal to the Quotient of the ſide of the Dividend applyed to the ſide of the Diviſor, or the Greater divided by the Leſſer, and the contrary.

As if 36 be divided by 4 (both Squares) the Quotient will be 9, whoſe Root 3 is equal to 6 divided by 2, the ſides of 36 and 4. And if 36 be divided by 9, the Quotient 4, whoſe Root 2, is equal to 6 divided by 3, the ſides of 36 and 9.

So 9261, the Cube of 21, divided by 27 and 243, the Cubes of 3 and 7, gives alternately in the Quotient the Powers whoſe Roots are equal to the Diviſion of 21 by 3 or 7 accordingly.

Square. Cube.

√2) √6 (√3 √3) √6 (√2 √3) √21 (√7 √7) √21 (√3
4) 36 (9 9) 36 (4 27) 9261 (343 343) 9261 (27

6. *Sides of ſuch divided what begotten. Examples.*

6. That the ſides of Homogeneal Surdes divided procreateth ſides of Homogeneal Surdes.

Ergo W 2 dividing W 6 begetteth W 3. And ww 3 dividing ww 21 begetteth ww 7.

Diviſion of Compounds where.

Diviſion of Compound Surdes is to be found in its proper place, in the *Tenth Chapter* following of this *Book*.

Proof of Diviſion *of Simple* Surdes.

Becauſe moſt of the Diviſions of this *Chapter* are the Products of the Multiplications in the foregoing *Chapter* divided by one of the Factors, it will ſerve ſufficiently to prove the truth of Surde Multiplication by Diviſion, and Diviſion by Multiplication.

Yet to make all clear, take Rational Numbers for Surdes, and proceed in their Diviſion as if they were Surdes, and the Quotients of ſuch Diviſions will be equal to the Diviſion of their Roots in Abſolute Numbers.

For if W 144 be divided by W 16, the Quotient will be W 9, whoſe Root is 3, agreeable to the Quotient of 12 the √ of 144 divided by 4 the √ of 16, as at *G*.

And ſo if √3φ 46656 which is 6, be divided by W 9 which is 3, the Quotient will be 2, as at *H*.

G. √ 4) √ 12 (√ 3
W 16) W 144 (W 9

√ [6] 46656 √ [6] 729
2) √ [6] 46656 √ [2] 9 $\frac{46656}{729}$(64
H. [3] [1]
√ 3) √ 6 (√ 2
√3φ 729) √3φ 46656 (√3φ 64 = 2

CHAP. VII.

Addition of Compound Surdes.

Compound Surdes *added.*

IN the Addition of Compound Surdes, let them be conſidered as they are Particular or Univerſal.

As the Compound Surdes are made of the Simple, or elſe of Rational Numbers with Surdes; ſo the work of the Compound dependeth on the work of the Simple, and to be wrought alike. And the ſigns + and — to be ordered, as in Addition of Compound Coſſicks.

Particular.

In particular Compound Surdes, as the parts given to be added be, ſo ſhall the Addition be. For like Surdes and Symmetral are to be added with like as Simple, and unlike and Aſymmetral with the ſign of Addition +.

Becauſe Examples are very demonſtrative, the varieties of Examples following with their explanations, are to be born with.

Examples of Binomials.

Examples of Binomials.

Addends $\left\{\begin{array}{l} 9+\sqrt{40} \\ 30+\sqrt{10} \end{array}\right.$

Total $39+\sqrt{90}$

After Addition of the Abſolute Numbers 9 and 30, the Surdes $\sqrt{40}$ and $\sqrt{10}$, are added as before in Simple Surdes.

$\sqrt{40}+\sqrt{10}$
10)————
4ʒ 1ʒ
2√ + 1√
3
3ȥ
————
9ʒ
10
————
$\sqrt{90}$

Addends $\left\{\begin{array}{l} 7+\sqrt{8} \\ 5+\sqrt{3} \end{array}\right.$

Total $12+\sqrt{8}+\sqrt{3}$

The Abſolute Numbers 7 and 5 make 12. The Surdes being Incommenſurable are conjoyned by +. And after the ſecond form of Addition of Simple Surdes, may be ſet thus, $12+\sqrt{11}+\sqrt{96}$.

Addends $\left\{\begin{array}{l} \sqrt{1264}+8 \\ 28+316 \end{array}\right.$

Total $\sqrt{2844}+36$

In this Example the Abſolute Numbers make up 36, and the Surdes added as Simple are $\sqrt{2844}$.

$\sqrt{1264}+\sqrt{316}$
79)————
16ʒ 4ʒ
4√ + 2√
6
6ȥ
————
36ʒ
79
————
$\sqrt{2844}$

Addends $\left\{\begin{array}{l} \sqrt[3]{32}+\sqrt{10} \\ \sqrt[3]{4}+\sqrt{19} \end{array}\right.$

Total $\sqrt[3]{108}+\sqrt{29}+\sqrt{760}$

The Cube Surdes added as Simple make $\sqrt[3]{108}$, but the Square Surdes ſet after the ſecond form of Addition being Incommenſurable might have ſtood thus, $\sqrt{10}+\sqrt{19}$; and the whole Total thus, $\sqrt[3]{108}+\sqrt{10}+\sqrt{19}$.

Addends $\left\{\begin{array}{l} \sqrt[4]{48}+\sqrt{5} \\ \sqrt[4]{243}+\sqrt{45} \end{array}\right.$

Total $\sqrt[4]{1875}+\sqrt{80}$

$\sqrt[4]{}$ added.
$\sqrt[4]{48}+\sqrt[4]{243}$
3)————
16ʒʒ 81ʒʒ
2√ + 3√
5
————
625ʒʒ
3
————
$\sqrt[4]{1875}$

$\sqrt{}$ added.
$\sqrt{5}+\sqrt{45}$
5)————
1ʒ 9ʒ
1√ + 3√
4
————
16ʒ
5
————
$\sqrt{80}$

Examples of Reſiduals.

Examples of Reſiduals.

Addends $\left\{\begin{array}{l} 14-\sqrt{3} \\ 12-\sqrt{12} \end{array}\right.$

Total $26-\sqrt{27}$

After Addition of the Abſolute Numbers which make 26, the Surdes added make $\sqrt{27}$, which is to be adjoyned to 26 with —, becauſe the Numbers to be added had the ſame ſign.

$\sqrt{3}+\sqrt{12}$
3)————
1ʒ 4ʒ
1√ + 2√
3
————
9ʒ
3
————
$\sqrt{27}$

Addends $\left\{\begin{array}{l} 8-\sqrt{5} \\ 6-\sqrt{3} \end{array}\right.$

Total $14-\sqrt{5}-\sqrt{3}$

The Abſolute Numbers added make 14. The Surdes are Incommenſurable, and ſo annexed with their proper ſigns; or elſe after the ſecond form of Simple Addition ſet thus, $14-\sqrt{8}-\sqrt{60}$.

Addends { W50 − 4
W18 − 3

Total W128 − 7

In this Example the work is like the firſt Example of Reſiduals above; for the Surdes added make W128, and 4 and 3 the Abſolute Numbers make 7 to be ſet with —, becauſe they were of the ſame Nature.

W50 + W18
2)
25ʒ 9ʒ
5√ + 3√
8
64ʒ
2
W128

Addends { W128 − 6
8 − W72

Total 2 + W8

Addends { W128 − 8
6 − W72

Total W8 − 2

Becauſe one of the Abſolute Numbers is + and the other —, 6 taken from 8 leaves 2, but in the one Example +, in the other —, according to the nature of the greater Number. Then the W72 being —, and the other Surde +, the Leſſer is taken from the Greater, and the Remain is + W8, according to the ſign of W128.

W128 − W72
2)
64ʒ 36ʒ
8√ − 6√
2
4ʒ
2
W8

Addends { 250 − W99
W44 − 76

Total 174 − W11

The work in this Example is like the laſt; for — 76 taken from + 250 leaves + 174, and W44 which is +, taken from W99 which is —, leaves W11.

W99 − W44
11)
9ʒ 4ʒ
3√ − 2√
1
1ʒ
11
W11

Addends { W44 − W27
W99 − W3

Total W275 − W48

Here W44 + W99 and W27 + W3, are added ſeverally, and their Totals conjoyned by their proper ſign.

W44 + W99
11)
4ʒ 9ʒ
2√ + 3√
5
25ʒ
11
+ W275

W27 + W3
3)
9ʒ 1ʒ
3√ + 1√
4
16ʒ
3
− W48

Addends { ww72 − W96
ww9 − W6

Total ww243 − W150

The Cube Surdes added by themſelves, and the Squares by themſelves, make the Total ww243 − W150.

ww72 + ww9
9)
8ϕ 1ϕ
2√ + 1√
3
27ϕ
9
+ ww243

W96 + W6
6)
16ʒ 1ʒ
4√ + 1√
5
25ʒ
6
− W150

Addends { ww32 − W5
ww32 − W24

Total ww512 − W24 − W5
or ww512 − W29 − W480

This Example is like the ſecond of the Reſidual Examples above in the Square Surdes, and ſo being Aſymmetral may be ſet differently. The ww being added to himſelf, is as 32 × 16.

Examples of Binomials *with* Reſiduals.

Examples of Mixt.

Addends { 8 − W27
11 + W3

Total 19 − W12

Addends { 8 + W27
11 − W3

Total 19 + W12

In both theſe Examples adding 8 and 11, they make 19 abſolute Numbers; and W3 taken from W27, becauſe their Signs are contrary, there reſts W12, which in the one Total is +, in the other —, according to the nature of the Greater Surde.

W27 − W3
3)
9ʒ 1ʒ
3√ − 1√
2
4ʒ
3
W12

```
Addends { W12 — 8
        { W3  + 6
Total     W27 — 2

Addends { W12 + 8
        { W3  — 6
Total     W27 + 2
```

Both theſe Examples adding the Surdes make their Total W27, and taking 6, one of the Abſolute Numbers, from 8, the other leave 2, which in the one is —, and in the other +, according to the ſign of the greater Number.

```
   W12 + W3
3)---------
   4ʒ    1ʒ
   2√ + 1√
      3
   ------
      9ʒ
      3
   ------
    W27
```

```
Addends { W50 + W7
        { W52 — W7
Total     W242
```

Here W7 being found with + and —, both are cancelled, and the Total of the other Surdes only ſet down.

```
   W50 + W72
2)----------
   25ʒ   36ʒ
   5√ + 6√
      11
   ------
     121ʒ
      2
   ------
    W242
```

```
Addends { W63 + W20
        { W7  — W5
Total     W112 + W5
```

In this Addition W5 is taken from W20 becauſe the Signs are contrary, and there reſts W5, which is +, becauſe the Leſſer Surde was ſubſtracted.

```
   W63 + W7              W20 — W5
7)---------           5)---------
   9ʒ    1ʒ              4ʒ    1ʒ
   3√ + 1√               2√ — 1√
      4                      1
   ------                 ------
     16ʒ                     1ʒ
      7                      5
   ------                 ------
    W112                    W5
```

```
Addends { W80 — wwW160
        { W5  + wwW20
Total     W125 — ww20
```

After Addition of the Square Surdes, the leſſer Cube Surde is taken from the greater, their Signs being contrary.

```
   W80 + W5              ww160 — ww20
5)---------          20)-------------
   16ʒ   1ʒ               8φ     1φ
   4√ + 1√                2√ — 1√
      5                       1
   ------                  ------
     25ʒ                      1φ
      5                      20
   ------                  ------
    W125                    ww20
```

```
Addends { W63 + W20 + ww10
        { W7  — W5
Total     W112 + W5 + ww10
```

This Example is like the laſt above ſave one, only the odd Surde ww10, is adjoyned to the Total of the others added.

```
Addends { ww40  + W24
        { ww320 — W56
Total     ww1080 — W80 — W5376

Addends { ww40  — W24
        { ww320 + W56
Total     ww1080 + W80 — W5376
```

Theſe two laſt Examples make the Cube Surdes in both ww1080, in the Square Surdes the Total is — W80 in one, and + W80, becauſe in this the Greater Surde to be added was +, but in the other —. The Totals may be thus ſet,

ww1080 + W24 — W56

ww1080 — W24 + W56

```
    ww40 + ww320              W24 ∓ W56
40)-------------             -----------
    1φ      8φ                    | 24
    1√  +  2√                     | 56
        3                         | ----
    ---------               W80   | 144
       27φ                        | 120
       40                         | ----
    ---------                     | 1344
    ww1080                        |    4
                             -----------
                              W80 ∓ 5376
```

Universal. In Addition of Univerſal Surdes reſpect is to be had to the Mark prefixed; for ſo properly is the Addition to be: So as if there be given to be added thus differently marked;

√ʒ12 to √ʒ12
or
W12 + W12

Then are the Surdes looked on as Compounds, only Particular and not Univerſal, and the Total of both their Roots added as before is W48, which is almoſt 7 abſolute Numbers.

How differently marked and taken.

√:12 to √ʒ12
or
√:12 + W12

Then is to be underſtood that the Square Root of the Dexter 12 is to be added to the Siniſter 12, and the Square Root of that ſumme to be taken for the Univerſal Root.

√:ʒ12 to √ʒ12
or
√:W12 + W12

This form is underſtood by ſome as the laſt above; but others more ſtrictly, after the √ʒ of the Dexter 12 is added to the √ʒ of the Siniſter 12, take the √ʒ of that Total for the Univerſal Root.

Explained by Rationals.

To make all plain let Rational Numbers be taken; as ſuppoſe W81 which is 9, and W361 which is 19, theſe added together make 28; and ſo they are conſidered as Compound only, and their Roots particular and diſtinct; yet as both Roots are added the Total may ſeem to be the Univerſal Root of both Surdes. But uſually conſidered as Univerſal, then muſt 19 be added to 81, which make 100, and the Square Root thereof taken which is but 10. And this *Record* in his *Whetſtone of Wit*, *p.* 11. counts moſt aptly the Univerſal Root. And according to him, and other good Authors, let Univerſal Roots be ſo underſtood here. Otherwiſe more ſtrictly, if the Root of the Dexter Surde be added to the Root of his Siniſter Surde alike denominate, and the Root thereof taken for the Univerſal Root; then after 19 is added to 9, the Root of 28, the Total taken for the Univerſal Root will be 5, and ſomewhat more.

Addends √ʒ81 + √ʒ361
9√ 19√
Total 28 Particular.

Addends √:81 + √ʒ361
19 19√
Total W100 (10√ Universal.

Addends √:W81 + W361
Total ſtrictly 5 and + Univerſal.

√ʒ81 + √ʒ361
361
81
W442
361
2888
29241 (171
2
√:W442 + W342
Total W784 = 28
√9 + √19
(3
√28 (5

ʒ (3 √
784 (28 (5
4
{32
{64
√

Hence it is apparent, that the Totals of Simple or Particular Compound Surdes, as before added, if conſidered as Rooted after a ſort may be taken for the Univerſal Root of the Surdes given to be added. But if from the Total the Character of the Univerſal Surdes or √: be removed or cancelled, and the Siniſter Number left abſolute, the Surdes are commonly conſidered as diſtinct, and their Roots particular.

Steps to the Addition of Univerſals.

To the right underſtanding of the Addition of Univerſal Roots, it is meet to proceed in the following ſteps.

1. If the Data be Incommenſurable or Heterogeneal Example.

1. If the Numbers or Surdes given be Incommenſurable or Heterogeneal, then conjoyn them with +, and before them prefix √:, the Mark that ſignifies the Univerſal Root.

As to add the Univerſal Root of 39, abſolute Numbers, the WW9 and the W8 together, the Total ſhall be √:39 + WW9 + W8; or thus, √:WW9 + W8 + 39, of which this latter form is the better, to ſet the abſolute Numbers to the Right Hand of the Surdes; leſt ſtanding next the Character √: they be taken for a Surde: For there ſometime the Denomination is omitted, and the Number valued as his next Dexter Surde.

2. One Univerſal, and the other Particular, two varieties.

2. If the one be an Univerſal Surde, and the other a Particular, then add the Particular Surde to the Siniſter part of the Univerſal, as Particulars are added; and to the Dexter part of the Univerſal, add double the Root of the Particular (or the Square multiplyed by 4, which is all as one) if the Root of that Dexter part of the Univerſal added to the Siniſter part, make up the next Square Number thereto.

Examples of the firſt variety.

As to add W16 to √:36 + W169, the Square Roots of 16 and 36 added, which are 4 and 6, make 10, the Square whereof is 100; for the Siniſter work to the Dexter W169, whoſe Root is 13, muſt be added 8, the double of 4, which is 21; or 16 multiplyed by 4 makes W64, which added as Surdes to W169, makes the Dexter part W441, whoſe Root is 21. Another Example is at *B.*

Roots

	Roots.	Rationals.		*B.*
Addends	7	√: 36 + W 169	4	√: 9 + W 49
	4	√3 16	2	√3 4
Total	11	√: 100 + W 441	6	√: 25 + W 121

Other variety.

But if the Root of the Dexter part of the Univerſal added to the Siniſter part, make not up the next Square Number, then if one be omitted, quadruple the Root of the Particular which before you doubled, if 2 be omitted then ſexcuple the Root, *&c.*

Examples.]

As to add W 25 to √: 4 + W 144; becauſe 12 the Root of 144, added to 4, make 16, which is not the next Square to 4, but 9 is omitted; therefore 5 the Root of 25 muſt be quadrupled which is 20, and the Square therefore added to 144.

So to add W 9 to √: 4 + W 441; becauſe 21, the Root of 441, added to 4 make 25, which is not the next Square to 4, but 9 and 16, two Squares are omitted, therefore 3, the Root of 9, muſt be ſexcupled, which is 18, and the Square thereof added to 441.

Addends	4	√: 4 + W 144	5	√: 4 + W 441
	5	√3 25	3	√3 9
Total	9	√: 49 + W 1024	8	√: 25 + W 1521

3. Data *both Univerſals, and* 0 *in the Siniſter part of one.*

3. If both Numbers given to be added be Univerſals, and in the Siniſter part of one be 0, then ſubſcribe the Siniſter part as it is, without any alteration, and double the Root Univerſal of that Number wherein the Cypher is not, multiply the other Root thereby, and the Square thereof add with both the Dexter Squares.

But if the Number that hath the Cypher be a Rational, the Square Root thereof may be taken and placed under the Siniſter part, and added as laſt before.

Example.

As to add √: 4 + W 25 to √: 0 + W 256; becauſe 256 is a Rational, and hath 16 for its Root, this may be added with √: 4 + W 25, as before.

But otherwiſe 4 is ſubſcribed in the ſiniſter part only, and 3 the Root Univerſal doubled is 6, which multiplying 4, the other Root makes 24, whoſe Square is to be added with both the other Dexter Squares. The Totals of both Additions are here ſet down.

Addends	3	√: 4 + W 25	3	√: 4 + W 25
	4	√: 0 + W 256	4	√3 16
Total	7	√: 4 + W 2025	7	√: 36 + W 169

4. Data *both Univerſals, without* 0.

4. If both Univerſals be without Cyphers, then as before, add the Siniſter Surdes as Particular: And the Dexter muſt be increaſed to ſuch a Number, that the Root thereof added to the Siniſter may be equal to the Roots of the Numbers given to be added. But herein great circumſpection is to be uſed; for though the Roots be Univerſal, no Univerſal Rule can be given that I know of to work by, but ſometime the double of the Dexters, ſometime the √33 of the Dexters ſeverally taken, added, and then figurate accordingly and ſet as Squares, ſometime the method uſed in the Pathway, but ſometime neither will ſerve.

Examples.

As in the two Examples following, the Dexters in both have their 33 Roots, yet the Dexters at *D.* require 169, a greater Number than their Roots 33 will give, which will be but 81.

		C.		*D.*
Addends	8	√: 48 + W 256	7	√: 48 + W 1
	4	√: 12 + W 16	4	√: 12 + W 16
Total	12	√: 108 + W 1296	11	√: 108 + W 169

Other Examples.

Addends	7	√: 48 + W 1	4	√: 12 + W 16
	2	√: 3 + W 1	2	√: 3 + W 1
Total	9	√: 75 + W 36	6	√: 27 + W 81

5. *Reſidual Univerſals. Examples.*

5. If the Univerſals be Reſiduals, then add them as above, aud keep the ſign Reſidual to the Total.

As to add √: 48 — W 144 which is 6, to √: 12 — W 9 which is 3, the Total will be √: 108 — W 729, whoſe Root is 9. Another Example is ſet at *F.*

E.

	E.			F.
Addends	6	√:48 — W 144	6	√:48 — W 144
	3	√:12 — W 9	5	√:27 — W 4
Total	9	√:108 — W 729	11	√:147 — W 676

6. If the Univerſals be mixt then after Addition of the Siniſters, one of the Dexters is to be taken from the other, as in Addition of Particulars, yet the Remain muſt be left valuable to make the Root of the Siniſters ſufficient for the ſumme of the Addition. See the Examples following.

6. *Mixt Univerſals.*

Examples.

Addends	8	√:48 + W 256	6	√:28 + W 64
	5	√:27 — W 4	2	√:7 — W 9
Total	13	√:147 + W 484	8	√:63 + W 1
Addends	6	√:48 — W 144	7	√:50 — W 1
	4	√:12 + W 16	2	√:2 + W 4
Total	10	√:108 — W 64	9	√:72 + W 81

7. If any Univerſal Square Surde be added to himſelf, the Siniſter may be multiplyed by 4, the next Dexter by 16, and the next by 256, *&c.*

7. *Square Univerſals added to themſelves.*

But if any Abſolute Number be intermixed amongſt the Surdes, they are to be multiplyed by the next Siniſter Multiplyers before them.

Example.

As to add √:3 + W 30 + W 36 to it ſelf, that is 3 to 3, the Total will be √:12 + W 480 + W 9216, whoſe Root Univerſal is 6: For 96, the Root of 9216, added to 480, and the Root of 576, the ſumme which is 24, brought and added to 12, makes 36, whoſe Root is equal to the other Roots of the Addends.

Multiplyers		4	16	256
Addends	3	√:3	+ W 30	+ W 36
	3	√:3	+ W 30	+ W 36
Total	6	√:12	+ W 480	+ W 9216

8. If any Square Univerſal be added to himſelf, the Multiplications may be ſhortned thus: Let the Siniſter Number be multiplyed by 2, the next Dexter by 4, and the next Dexter by 16, *&c.* and the Abſolute Numbers accordingly: For if the outmoſt Dexter be Abſolute and not Figurate, then multiply that by 4; if the middlemoſt be Abſolute, then multiply that by 2; and the outmoſt Dexter by 4, though Figurate. Alſo in getting the abſolute value of the Total ſo added, when the Root is brought to be added to or ſubſtracted from the Siniſter, the ſumme or difference is accordingly to be multiplyed again by 2.

8. *To ſhorten the Multiplications in ſuch Additions.*

Example.

As in the laſt Example the Total will be √:6 + W 120 + W 576; but after 24, the Root of 576, is added to 120, and the Root of 144, the ſumme which is 12, brought and added to 6, then 18, the ſumme there is to be multiplyed by 2, the Root of which Product 36, is 6, the Total as before.

Multiplyers		2	4	16
Addends	3	√:3	+ W 30	+ 36
	3	√:3	+ W 30	+ 36
Total	6	√:6	+ W 120	+ W 576

576 | 24 √
+ 120
144 | 12 √
+ 6
18
× 2
36 | 6 √ value

Other Examples follow with their Multiplyers at top, and the ſumme of their Roots under the Totals.

Other Examples.

	2	4	16	2	4	16	2	4	16
Addends	√:5	+ W 12	+ W 16	√:5	+ W 20	— W 16	√:5	— W 20	— W 16
	√:5	+ W 12	+ W 16	√:5	+ W 20	— W 16	√:5	— W 20	— W 16
Total	√:10	+ W 48	+ W 256	√:10	+ W 80	— W 256	√:10	— W 80	— W 256
	8	16		8	—16		—8	—16	
	18	64 (8 √		18	64 (8 √		2	64 (8 √	
	36 (6 √ value			36 (6 √ value			4 (2 √ value		

Addends	$\overset{2}{\surd}{:}5 + \overset{4}{W}20 - \overset{4}{16}$	$\overset{2}{\surd}{:}2 + \overset{4}{W}20 - \overset{4}{16}$	$\overset{2}{\surd}{:}25 - \overset{2}{12} + \overset{4}{W}16$
	$\surd{:}5 + W20 - 16$	$\surd{:}2 + W20 - 16$	$\surd{:}25 - 12 + W16$
Total	$\surd{:}10 + W80 - 64$	$\surd{:}4 + W80 - 64$	$\surd{:}50 - 24 + W64$
	$4 \quad -64$	$4 \quad -64$	$-32 \quad 8$
	$14 \quad 16\,(4\surd$	$8 \quad 16\,(4\surd$	$18 \quad 32$
	$W28$ value	$16\,(4\surd$ value	$36\,(6\surd$ value

Whence this work laſt mentioned proceeds.

This and the precedent work take original from the *Firſt Conſectary* in *Chap.* 5. *Multiplication of Simple Surdes*, and according thereto may alſo Univerſal Surdes of Higher Powers than Squares be doubled, tripled, *&c.* but little uſe being of any higher Univerſals than Squares, the foregoing Operations are fitted for them.

Proof of Addition *of Compound* Surdes.

The Proof of Addition of Compound Surdes is like the Proof of Simple Addition, either by Subſtraction, Particular by Particular, and Univerſal by Univerſal; or by taking Rational Numbers, and working therewith inſtead of the Surdes. And ſeeing ſeveral of the Examples are of Rational Numbers, they may ſerve without further inſtance.

CHAP. VIII.

Subſtraction of Compound Surdes.

Compound Surdes *ſubſtracted. Particular.*

IN the Subſtraction of Compound Surdes, as in Addition, let them be conſidered as they are Particular or Univerſal.

In Particular Compound Surdes, to Heterogeneal and Incommenſurable ſet the ſign of Subſtraction —; and for Commenſurable, as the parts given to be ſubſtracted, ſo ſhall the Subſtraction be. Like is to be ſubſtracted from like; and the uſe of the ſigns + and — is here as in Subſtraction of Compound Coſſicks.

Examples of Binomials.

Examples of Binomials.

Greater Surde	$10 + W48$
Subtrahend	$7 + W27$
Remain	$3 + W3$
Leſſer Surde	$10 + W27$
Subtrahend	$7 + W48$
Remain	$3 - W3$

After Subſtraction of the Abſolute Number 7 from 10, and the Remain 3 ſet down, in both theſe Examples the $W27$ and $W48$ are ſubſtracted as before in Simple Surdes, and the Greater of them being taken from the Leſſer, makes the Remain to be —.

$$\begin{array}{r} W48 - W27 \\ 3)\overline{} \\ 16\,3 \quad 9\,3 \\ 4\surd - 3\surd \\ 1 \\ \hline 13 \\ 3 \\ \hline W3 \end{array}$$

Greater Surde	$12 + W8 + W3$	Greater Surde	$12 + W11 + W96$
Subtrahend	$5 + W3$	Subtrahend	$5 + W3$
Remain	$7 + W8$	Remain	$7 + W8$

The Abſolute Numbers being ſubſtracted in both theſe Examples, there remains 7; then in the one finding $W3$ in the Greater Surde and Subtrahend both, they are both cancelled, for taking one from the other, 0 remains; ſo $W8$ is only ſet down. But in the latter Example, where the Greater Surde is ſet as the Total of the ſecond form of Simple Addition, 96 is to be divided by 4, and the Quotient by $W3$, and ſo $W8$ is gotten for the Remain.

$$W\frac{\cancel{96}}{\cancel{4}}\Big(\,\frac{\cancel{24}}{W3}\Big(\,W8$$

Greater Surde	$W2844 + 36$
Subtrahend	$28 + W316$
Remain	$W1264 + 8$

In this Example taking $W316$ from $W2844$, the Remain is $W1264$. And 28 Abſolute Numbers taken from 36, there Remains + 8.

$$\begin{array}{r} W2844 - W316 \\ 316)\overline{} \\ 9\,3 \quad 1\,3 \\ 3\surd - 1\surd \\ 2 \\ \hline 4\,3 \\ 316 \\ \hline W1264 \end{array}$$

Greater Surde	ww108 + w29 + w760
Subtrahend	ww 4 + w19
Remain	ww 32 + w10

Here taking ww4 from ww108 remains ww32, then taking w19 from w29, reſts w10, rejecting w760, or dividing as before by 4, and the Quotient by 19, the w10 is obtained, rejecting w29.

$$\frac{\text{w}760}{4}\Big(\frac{190}{\text{w}19}\Big(\text{w}10$$

$$\begin{array}{r} \text{ww}108 - \text{ww}4 \\ 4)\overline{\qquad\qquad\qquad} \\ 27\varphi \qquad 1\varphi \\ 3\surd - 1\surd \\ 2 \\ \hline 8\varphi \\ 4 \\ \hline \text{ww}32 \end{array}$$

Greater Surde	ww1875 + w5
Subtrahend	ww 48 + w80
Remain	ww 243 − w45

ſubſtracted.

$$\begin{array}{r} \text{ww}1875 - \text{ww}48 \\ 3)\overline{\qquad\qquad\qquad} \\ 625ʒʒ \qquad 16ʒʒ \\ 5\surd - 2\surd \\ 3 \\ \hline 81ʒʒ \\ 3 \\ \hline \text{ww}243 \end{array}$$

ſubſtracted.

$$\begin{array}{r} \text{w}5 - \text{w}80 \\ 5)\overline{\qquad\qquad} \\ 1ʒ \qquad 16ʒ \\ 1\surd - 4\surd \\ -3 \\ \hline -9ʒ \\ 5 \\ \hline -\text{w}45 \end{array}$$

Examples of Reſiduals.

Examples of Reſiduals.

Greater Surde	26 − w27
Subtrahend	12 − w12
Remain	14 − w3
Leſſer Surde	26 − w12
Subtrahend	12 − w27
Remain	14 + w3

After Subſtraction of 12 from 26, the Abſolute Number 14 is left in both theſe Examples. And w12 taken from w27 leaves w3, but in the one —, in the other +, according as the Subtrahend was the Greater or Leſſer Surde.

$$\begin{array}{r} \text{w}27 - \text{w}12 \\ + \\ 3)\overline{\qquad\qquad} \\ 9ʒ \qquad 4ʒ \\ 3\surd - 2\surd \\ + \\ 1 \\ \hline 1ʒ \\ 3 \\ \hline \text{w}3 \end{array}$$

Greater Surde	14 − w5 − w3
Subtrahend	8 − w5
Remain	6 − w3

The Abſolute Number 6, is left when 8 is taken from 14. And w5 taken from w5 leaves 0; ſo both cancelled, w3 is brought down to the Remain: But if the Surde had been ſet thus, 14 — w8 — w60, after the ſecond form of Simple Subſtraction, then 60 divided by 4, and the Quotient 15 divided by 5, gives w3 for the Remain, as before.

Greater Surde	w275 − w48
Subtrahend	w99 − w3
Remain	w44 − w27

Here taking w99 from w275, and w3 from w48, the Remain appears thus,

$$\begin{array}{r} \text{w}275 - \text{w}99 \\ 11)\overline{\qquad\qquad} \\ 25ʒ \qquad 9ʒ \\ 5\surd - 3\surd \\ 2 \\ \hline 4ʒ \\ 11 \\ \hline \text{w}44 \end{array} \qquad \begin{array}{r} \text{w}48 - \text{w}3 \\ 3)\overline{\qquad\qquad} \\ 16ʒ \qquad 1ʒ \\ 4\surd - 1\surd \\ 3 \\ \hline 9ʒ \\ 3 \\ \hline \text{w}27 \end{array}$$

Greater Surde	ww243 − w150
Subtrahend	ww72 − w96
Remain	ww9 − w6

Both Cube Surdes and Square Surdes ſeverally ſubſtracted as Simple their Remains are thus;

$$\begin{array}{r} \text{ww}243 - \text{ww}72 \\ 9)\overline{\qquad\qquad\qquad} \\ 27\varphi \qquad 8\varphi \\ 3\surd - 2\surd \\ 1 \\ \hline 1\varphi \\ 9 \\ \hline \text{ww}9 \end{array} \qquad \begin{array}{r} \text{w}150 - \text{w}96 \\ 6)\overline{\qquad\qquad} \\ 25ʒ \qquad 16ʒ \\ 5\surd - 4\surd \\ 1 \\ \hline 1ʒ \\ 6 \\ \hline \text{w}6 \end{array}$$

Greater Surde	www512 − w24 − w5
Subtrahend	www32 − w24
Remain	www32 − w5
Greater Surde	www512 − w24
Subtrahend	www32 − w24 − w5
	www32 + w5

In both theſe Examples the Remain of the Squared Square Surdes is www32. The Remain of the Square Surdes in the one is +, in the other —, as the odd w5 in the one was in the Subtrahend, in the other in the Number from which Subſtraction is made. And if the Square Surdes had been ſet after the ſecond form of Addition, that is w29 — w480, then dividing 480 by 4, and the Quotient 120 by 24, the Remain will be w5, as before.

$$\begin{array}{r} \text{www}512 - \text{www}32 \\ 2)\overline{\qquad\qquad\qquad} \\ 256ʒʒ \qquad 16ʒʒ \\ 4\surd - 2\surd \\ 2 \\ \hline 16ʒʒ \\ 2 \\ \hline \text{www}32 \end{array}$$

Examples of Mixt.

Examples of Binomials *with* Reſiduals.

Greater Surde	19 + √12
Subtrahend	11 − √3
Remain	8 + √27
Greater Surde	19 − √27
Subtrahend	11 + √3
Remain	8 − √27

In both theſe Examples the Abſolute Numbers 11 taken from 19, leaves 8. And √3 added to √12 becauſe their Signs are contrary, makes the Remain √27, in the one +, in the other −, as the ſign of the Number from which Subſtraction is made.

3) √12 + √3
4ȝ 1ȝ
2√ + 1√
3
9ȝ
3
√27

Greater Surde	√72 − 2
Subtrahend	6 − √8
Remain	√128 − 8

Both Surdes and Abſolute Numbers being of contrary Signs, the reſpective ſums of both are taken.

8) √72 + √8
9ȝ 1ȝ
3√ + 1√
4
16ȝ
8
√128

Greater Surde	174 − √11
Subtrahend	√44 − 76
Remain	250 − √99

The ſum of the Abſolute Numbers, and the Total of the Surdes, is the Remain of this Subſtraction, the ſigns of both being contrary.

11) √44 + √11
4ȝ 1ȝ
2√ + 1√
3
9ȝ
11
√99

Greater Surde	√242
Subtrahend	√72 − √7
Remain	√50 + √7
Greater Surde	√242
Subtrahend	√50 + √7
Remain	√72 − √7

In both theſe Examples the work is alike; for √72 taken from √242 leaves √50, and √50 taken out of √242 leaves √72. Then √7 in both having none to be taken from doth remain; but the Signs are changed; for where + is ſubſtracted − ſhall remain, and where leſs, more.

2) √242 − √72
121ȝ 36ȝ
11√ − 6√
5
25ȝ
2
√50

Greater Surde	√112 + √5
Subtrahend	√7 − √5
Remain	√63 + √20

Here √7 taken from √112 leaves √63, and − √5 added to + √5 makes √20, whoſe ſign is as the upper Surde.

7) √112 − √7
16ȝ 1ȝ
4√ − 1√
3
9ȝ
7
√63

5) √5 + √5
1ȝ 1ȝ
1√ + 1√
2
4ȝ
5
√20

Greater Surde	√125 − ∛20
Subtrahend	√5 + ∛20
Remain	√80 − ∛160

The work in this is like that in the laſt Example above.

5) √125 − √5
25ȝ 1ȝ
5√ − 1√
4
16ȝ
5
√80

20) ∛20 + ∛20
1φ 1φ
1√ + 1√
2
8φ
20
∛160

Greater Surde	∛1080 − √80 − √5376
Subtrahend	∛320 − √56
Remain	∛40 + √24
Greater Surde	∛1080 − √80 − √5376
Subtrahend	∛40 + √24
Remain	∛320 − √56

The Operations in both theſe laſt Examples are much alike; for ∛320 ſubſtracted from ∛1080 leaves ∛40, and therefore ∛40 ſubſtracted muſt leave ∛320. In the one √56 may be ſubſtracted from √80, and the Remain ſet down with the ſign of 80; in the other it may be added to √80. Or elſe 5376 divided by 4, and the Quotient divided by 56 in the one, or 24 in the other, gives in the Quotient the Remain accordingly.

40) ∛1080 − ∛320
27φ 8φ
3√ − 2√
1
1φ
40
∛40.

40) ∛1080 − ∛40
27φ 1φ
3√ − 1√
2
8φ
40
∛320

√~~5376~~ / 4 (~~1344~~ / 24 (√56

√~~5376~~ / 4 (~~1344~~ / 56 (√24

In

Univerſal.

In *Subſtraction of Univerſal Surds*, proceed with reſpect to the Mark prefixed, as in *Addition of Univerſals* was before noted. For ſo accordingly muſt the *Subſtraction* be.

Explained by Rationals.

As in Rational Numbers it may be clearly demonſtrated, that if √ʒ25+√ʒ121, which are 5 and 11, be ſubſtracted from √ʒ81+√ʒ361, which are 9 and 19, they being marked only as *Compounds*, ſhall make the particular Remain as at A, 12 Abſolute Numbers. But marked as Univerſal, the Remain ſhall be but 4 as at B. And if ſtrictly taken, as before ſhewed in the foregoing Chapter, it ſhall not amount to 4; but be W12, which is a ſingle *Surd*, and hath 3 and ſomewhat more for his Root, as at C.

Greater Surd.	√ʒ81+√ʒ361	A	W81+W361=9+19=28
Subtrahend.	√ʒ25+√ʒ121		W25+W121=5+11=16
Remain.	12	Particular.	W16+W 64=4+ 8=12
			W81
Greater Surd.	√:81+√ʒ361	B	W361= 19
Subtrahend.	√:25+√ʒ121		W100=10
Remain.	4	Univerſal.	W 25
			W121= 11
			W 36=6
			4
Greater Surd.	√:W81+W361	C.	
Subtrahend.	√:W25+W121		
Remain.	W12	Strictly Univerſal.	√9+19=28
			√5+11=16
			W12

Hereby it appeareth that the Remains of the Subſtraction of Simple or Particular *Compound Surds* as before ſubſtracted, if conſidered as rooted, are after a ſort to be taken for the Univerſal Root. But if the Character of the *Univerſal Surds* be removed or cancelled, or the *Remain* but a ſingle Number, then they ceaſe to be *Univerſal*, but are underſtood as Simple or Particular *Compound Surds*.

Steps to the Subſtraction of Univerſals.

Further to underſtand the Subſtraction of Univerſal Roots, it may be ſafe to tread in the like Steps as before in their *Addition*.

1. If the Data *be Incommenſurable or Heterogeneal.*

1. If the Numbers or *Surds* given be Incommenſurable or Heterogeneal; then conjoin them with —, and prefix before them √: to ſignify the Root Univerſal.

Example.

As to take W8 from √:ww9+W8+39, the Remain ſhall be √:ww9+39.

2. One Particular, and the other Univerſal, 2 Varieties.

2. If a Particular *Surd* be to be ſubſtracted from ſome Univerſal; then take the Particular *Surd* from the ſiniſter Part of the Univerſal, as Particulars are ſubſtracted: And from the dexter Part of the Univerſal, take the Square of double the Root of the Particular, when the Root of that dexter Part of the Univerſal added to the ſiniſter Part make up the next ſquare Number thereto; but if one be omitted, then quadruple the Root; if two be omitted, ſexcuple the Root, &c.

Example of the firſt Variety.

As to take W16 from √:36+W169, the Root of 16, which is 4 doubled, is 8; ſo muſt 64 the Square thereof be taken from 169, after the manner of *Surds*.

Other Variety.

But if W25 be taken from √:36+W784, there 5 the Root of 25 muſt be multiplied by 4; becauſe 28, the Root of 784, added to 36, makes 64; which is not the next ſquare Number to 36, but one is omitted, to wit 49.

Example.

		Roots.		*Roots.*
Greater Surds.	√:36+W169	7	√:36+W784	8
Subtrahends.	√ʒ16	4	√ʒ25	5
Remains.	√:4+W25	3	√:1+W64	3

3. Data both Univerſals, and 0 in the Siniſter Place of one.

3. If both Numbers given be Univerſals, and the ſiniſter Part of the *Subtrahend* be a Cipher; then if the Dexter thereof be Rational, take the Root and place in the ſiniſter Place of the Cipher, and proceed as in the laſt Direction.

Example.

As to take √:0+W81 from √:49+W225, the Root of 81 being 9, if taken from √:49+W225 as above, leaves the Remain √:16+W81.

Greater Surd.	√:49+W225		8
Subtrahend.	√ʒ9	for √:0+W81	3
Remain.	√:16+W81		5

4. Data both *Univerſals without* 0.

4. If both Univerſals be without Ciphers, then take the Siniſter of the *Subtrahend* from the Siniſter of the other, as particular *Surds* are ſubſtracted. And let the Dexter be diminiſhed, that the *Remain* may be juſt. But herein lies the Skill, as before in *Addition*, wherein the Operator muſt have under conſideration ſeveral Directions at once.

Examples.

As to take √:12+W16 from √:48+W256, after 12 is taken from 48, and the Remain 12 ſubſcribed, the Dexter can be but 16, to make 12 half the other *Surd* as it is.

But if √:12+W16 be taken from √:48+W1, the Siniſter Part of the Remain will be as before; but the Dexter will be —W9, becauſe W1 was not ſufficient to ſubſtract W16 from, and therefore the Sign is changed.

Greater Surds.	√:48+W256	8	√:48+W1	7
Subtrahends.	√:12+W16	4	√:12+W16	4
Remains.	√:12+W16	4	√:12—W9	3

5. *Reſidual Univerſals.*

5. If the Univerſals be Reſiduals, they are to be ſubſtracted as above, and the Reſidual Sign kept to the Remain, or changed as the Caſe requires.

Examples.

As to take √:12—W9 from √:48—W144, the Remain ſhall be √:12—W9. But √:2—W4 taken from √:50—W1, ſhall leave the Remain √:32+W289. where the Sign is changed, becauſe W1 was too little to ſubſtract W4 from.

Greater Surds.	√:48—W144	6	√:50—W1	7
Subtrahends.	√:12—W9	3	√:2 —W4	0
Remains.	√:12—W9	3	√:32+W289	7

6. *Mixt Univerſals.*

6. If the Univerſals be mixt, then after ſubſtraction of the Siniſters, the Dexters are to be added; yet ſo as the Dexter Remain muſt have reſpect to the Siniſter, and not exceed its due Proportion. See the Examples following.

Examples.

Greater Surds.	√:4+W144	4	√:48+W256	8
Subtrahends.	√:4—W16	0	√:27—W4	5
Remains.	√:0+W256	4	√:3 +W36	3
Greater Surds.	√:48—W144	6	√:50—W1	7
Subtrahends.	√:12+W16	4	√:2 +W4	2
Remains.	√:12—W64	2	√:32—W49	5

7. *Sq. Univerſals halfed.*

7. If any Square Univerſal be to be halved, divide the Siniſter by 4, the next Dexter by 16, and the next by 256, *&c.* And if Abſolute Numbers be intermixed, they are to be divided by the next Siniſter Diviſors before them.

Example.

As to half √:12+W480+W9216, dividing accordingly by 4.16.256. there will be brought forth the √:3+W30+W36, for the half of the former.

8. *To ſhorten the Diviſion in ſuch Subſtractions.*

8. If any Square Univerſal be divided by 2.4.16, *&c.* orderly, and the Abſolute Numbers, if any, as the Siniſters next before them, the Quotient ſhall be an Univerſal, like thoſe in the laſt Direction mentioned in *Addition*. And if the Root be gotten, the Sum or Difference at laſt muſt be divided by 2, and the Root of the Quotient taken for the Remain.

Example.

As if √:12+W480+W9216 be thus divided, the Quotient will be thus, √:6+W120+W576: then 24 the Root of 576 brought and added to 120, make 144, whoſe Root 12 added to 6 make 18; but this 18 muſt be divided by 2, and the Root of 9 the Quotient taken.

Whence this laſt Work proceeds.

This and the precedent Work take their Original from the firſt Conſectary in Chap. 6. *Diviſion of Simple Surds*; and according thereto may higher Univerſal *Surds* be halfed, *&c.* But theſe are fitted only for Squares, others being ſeldom uſed, as before noted in *Addition*.

Proof of Subſtraction of Compound Surds.

The Proof of *Subſtraction of Compound Surds*, is like the Proof of *Simple Subſtraction*, either by *Addition*, or by taking Rational Numbers inſtead of the *Surds*, and working therewith. For making the Total of any *Addition*, the Number from which *Subſtraction* is made, and one of the Numbers added the Subtrahend: then ſhall the Remain be the other Addend, and ſo *vice verſa*; remembring Particular to try Particular, and Univerſal, Univerſal; of which Inſtances are ſpared here, foraſmuch as many of the Examples in this Chapter are ſo ordered, as they prove the Additions of the former: And divers of the Examples being of Rational Numbers, may be Inſtances ſufficient without farther explanation.

CHAP. IX.

Multiplication of Compound Surds.

Compound Surds multiplied.

TO multiply *Compound Surds*, let them be conſidered as they are, Particular or Univerſal.

Particular.

Multiply the Numbers of Particular *Compound Surds*, as Simple *Surds*, like with like, or reduced thereunto; and let the Signs + and — be ordered as in *Multiplication of Compound Coſſicks*, for like Signs give +, and unlike —.

Example of Binomials.

Examples of Binomials.

Multiplicand. $\surd 26+\surd 3$
Multiplier. $\surd 5$
Product. $\surd 130+\surd 15$

The Multiplier being a Simple *Surd*, tho the Multiplicand a Compound, there is no Difficulty or Difference from Simple Multiplication.

Multiplicand. $5+\surd 10$
Multiplier. $5+\surd 10$
$25+\surd 250$
$\surd 250+10$
Product. $35+\surd 1000$

The Abſolute Numbers multiplied make 25, and $\surd 10$ by $\surd 10$ make 10 Abſolute Nnmbers, the 5 ſquared is 25; which multiplied by 10 is 250, and adding $\surd 250$ to $\surd 250$, the Total is $\surd 1000$, that is 250 multiplied by 4.

Multiplicand. $23+\surd 15$
Multiplier. $6+\surd 8$
$138+\surd 540$
$\surd 4232+\surd 120$
Product. $138+\surd 540+\surd 4232+\surd 120$

After Multiplication of the Abſolute Numbers 23 and 6, and the two *Surds* which make $\surd 120$, the Abſolute Numbers are ſquared; and ſo multiplied alternately into the other *Surd*, all which collected make the Total Product.

Multiplicand. $\surd 120+\surd 12$
Multiplier. $\surd 12+\surd 7$
$\surd 1440+12$
$\surd 840+\surd 84$
Product. $12+\surd 1440+\surd 840+\surd 84$

Here $\surd 120$ multiplied by $\surd 12$, the Product is $\surd 1440$; and $\surd 12$ by $\surd 12$, gives 12 Abſolute Numbers by cancelling $\surd$, as was taught in Simple *Surds*; the other Products are plain.

Example of Reſiduals.

Examples of Reſiduals.

Multiplicand. $\surd 26-\surd 3$
Multiplier. $\surd 5$
Product. $\surd 130-\surd 15$

The Multiplicand being a Compound, but the Multiplier Simple, there is no Difficulty nor Difference from Simple Multiplication.

Multiplicand. $10-\surd 5$
Multiplier. $10-\surd 5$
$100-\surd 500$
$-\surd 500+5$
Product. $105-\surd 2000$

The Abſolute Numbers multiplied, produce 100, and $\surd 5$ by $\surd 5$, make 5. Abſolute Numbers $\surd$ being cancelled, the reſt of the Work is like the ſecond Example of *Binomials* above.

Multiplicand. $23-\surd 15$
Multiplier. $6-\surd 8$
$138-\surd 540$
$-\surd 4232+\surd 120$
Product. $138-\surd 540-\surd 4232+\surd 120$

The Work in this Example is like that in the third Example of *Binomials* above, only altered in the Signs. And $\surd 120$ which is + may be ſet next to 138; which is of the ſame Nature, and ſo the Product will ſtand thus;

$138+\surd 120-\surd 540-\surd 4232.$

$$\begin{array}{ll}
\textit{Multiplicand.} & \surd 24 - \surd 20 \\
\textit{Multiplier.} & \surd 30 - \surd 24 \\
\hline
 & \surd 720 - \surd 600 \\
 & \quad -24 + \surd 480 \\
\hline
\textit{Product.} & \surd 720 + \surd 480 - 24 - \surd 600 \\
\hline
\end{array}$$

In this Example $\surd 24 \times \surd 30$ produceth $\surd 720$, and $\surd 30 \times \surd 20$, makes $\surd 600$; but $\surd 24 \times \surd 24$, giveth 24 Abſolute Numbers, cancelling the Character; and $\surd 24 \times \surd 20$, makes $\surd 480$.

Examples of mixt.

Examples of Binomials with Reſiduals.

$$\begin{array}{ll}
\textit{Multiplic.} & 6 + \surd 3 \\
\textit{Multiplier.} & 6 - \surd 3 \\
\hline
 & 36 + \surd 108 \\
 & \quad - \surd 108 - 3 \\
\hline
\textit{Product.} & 33 \\
\hline
\end{array}$$

After multiplication of the Abſolute Numbers, whoſe Product is 36, the *Surds* are multipli'd into the ſquare of 6, which alternately produce $+\surd 108$ & $-\surd 108$, and in the addition both cancelled being of contrary Signs, and 3 which is – coming of $\surd 3 \times \surd 3$, is taken from 36.

$$\begin{array}{ll}
\textit{Multiplic.} & \surd 52 + 17 \\
\textit{Multiplier.} & 17 - \surd 52 \\
\hline
 & \surd 15028 + 289 \\
 & \quad - 15028 - 52 \\
\hline
\textit{Product.} & 237 \\
\hline
\end{array}$$

Here the Square of 17 multiplied into $\surd 52$, makes in both places $\surd 15028$, the one + and the other —, according to the Signs. And 17×17 gives 289; and $\surd 52 \times \surd 52$ produceth 52, cancelling the Character.

$$\begin{array}{ll}
\textit{Multiplic.} & \surd 124 - 6 \\
\textit{Multiplier.} & 32 + \surd 14 \\
\hline
 & \surd 126976 - 192 \\
 & \qquad \surd 1736 - \surd 504 \\
\hline
\textit{Product.} & \surd 126976 + \surd 1736 - \surd 504 - 192 \\
\hline
\end{array}$$

The Work in this Example is like the laſt before, and the total Product is ſet at large.

$$\begin{array}{ll}
\textit{Multiplic.} & \surd 32 - 3 \\
\textit{Multiplier.} & \surd 8 + 2 \\
\hline
 & \surd 256 - \surd 72 \\
 & \qquad \surd 128 - 6 \\
\hline
\textit{Product.} & \surd 256 + \surd 8 - 6 \\
\hline
\end{array}$$

The Multiplication ended in collecting the Total $\surd 72$ is taken from $\surd 128$, and the Remain $\surd 8$ ſet down.

$$\begin{array}{rc}
8\,\big) & \surd 128 - \surd 72 \\
\hline
 & 16ȝ \quad 9ȝ \\
 & 4\surd - 3\surd \\
 & 1 \\
\hline
 & 1ȝ \\
 & 8 \\
\hline
 & \surd 8 \\
\hline
\end{array}$$

Polynomial multiplied ſquarely, how changeth a Name.

From hence it is obvious, that if a *Multinomial* be multiplied into it ſelf, with one of his Signs changed, the Product ſhall be purged of one Name.

Example.

As $3 + \surd 5 + \surd 2$ multiplied by $3 + \surd 5 - \surd 2$, the Product ſhall be $12 + \surd 180$.

$$\begin{array}{ll}
\textit{Multiplic.} & 3 + \surd 5 + \surd 2 \\
\textit{Multiplier.} & 3 + \surd 5 - \surd 2 \\
\hline
 & 9 + \surd 45 + \surd 18 \\
 & \quad \surd 45 + 5 + \surd 10 \\
 & \qquad - \surd 18 - \surd 10 - 2 \\
\hline
 & 9 + \surd 180 + 5 - 2 \\
\hline
\textit{Product.} & 12 + \surd 180 \\
\hline
\end{array}$$

$$\begin{array}{rc}
45\,\big) & \surd 45 + \surd 45 \\
\hline
 & 1ȝ \quad 1ȝ \\
 & 1\surd + 1\surd \\
 & 2 \\
\hline
 & 4ȝ \\
 & 45 \\
\hline
 & \surd 180 \\
\hline
\end{array}
\qquad
\begin{array}{r}
\surd 45 \\
4 \\
\hline
\surd 180 \\
\hline
\end{array}$$

Univerſals Homogeneal how multiplied.

Multiply the Numbers of Univerſal *Surds* that are Homogeneal, as Compound *Surds*, alternately one into the other, with this difference, that the Siniſter Numbers be figurate, according to the Denomination of the Dexter into which they are multiplied; And the particular Roots of all the Dexter Multiplees added to the Product of the two Siniſter Numbers, and the Root of the Total ſhall be the Product in Abſolute Numbers; which if thereby cannot be expreſſed, is to be marked accordingly with its proper Character.

Example.

Example, to multiply the Root Univerſal of $\surd 2 + \surd 49$, by the Univerſal Root of $\surd 3 + \surd 36$; after multiplication of 3 by 2, both 3 and 2 are ſquared before they are reſpectively multiplied into 49 and 36, and then 49 and 36 are multiplied together; the Roots of all which Dexter Multiplees are added to 6, and the Total makes 81, which is a Square, and the Root 9 Abſolute Numbers is the Product deſired.

Multiplicand.

Multiplicand.	√:2+W49	ʒ √
Multiplier.	√:3+W36	√ 2 × 3 ===== 6
	6+W441	3 · 9 × 49 = · 441 |21
	W144+W1764	
Product.	√:6+W441+W144+W1764	2 · 4 × 36 = 144 |12
Roots.	6+ 21 + 12 + 42	49 × 36 = 1764 |42
Total Product.	W81 = 9 Abſolute Numbers.	81

Univerſals Heterogeneal, how multiplied.

If the given Numbers be *Heterogeneal*, then beſides the Reduction of the one to the other, as in Compounds before, figurate the Siniſter *Surd* according to the Reduction of the Dexter, and then proceed as in other Univerſals.

Example.

Example, to multiply the Univerſal Root of W23+ww8, by the Univerſal Root of W2+ww343. Firſt, 23 is multiplied by 2, for they are *Homogeneal*, that is, both Squares. Then W2 is reduced with ww8 to the √ʒφ8, and √ʒφ64: And moreover, becauſe 64 the Dexter Number is now of the Zenzicube Denomination, 2 the Siniſter Number ſhall be made a Zenzicube alſo, which is 64. So ſhall 64 be multiplied by 64, and the Product with his Character ſet down, which is √ʒφ4096. Likewiſe W23 and ww343 reduced, make the Dexter Number √ʒφ117649; which is to be multiplied by 148035889 the Zenzicube of 23, and the product which is the √ʒφ17416274304961 ſet down. Afterward ww8 and ww343, both Homogeneal Cubes, are multiplied, and the Roots ſeverally taken and added to the Siniſter Product 46; the Total is W225, which in Abſolute Numbers is 15.

Multiplicand.	√:23+ww8
Multiplier.	√: 2+ww343
	46+√ʒφ4096
	√ʒφ17416274304961+ww2744
Product.	√:46+√ʒφ4096+√ʒφ17416274304961+ww2744
Roots.	46 + 4 + 161 + 14
Total Product.	W 225 = 15 Abſolute Numbers.

If the Univerſal Root be ſtrictly taken, then the Root of theſe laſt Numbers muſt be taken for the Product; which in the former Example would be 3, the Square Root of 9, in this Example W15.

Proof of Multiplication of Compound Surds.

As the Multiplication of Simple *Surds* takes for the Proof thereof, either Simple Diviſion or Rational Numbers; ſo doth Multiplication of Compound *Surds*, Particular and Univerſal, prove it ſelf either by the reſpective Diviſions, *viz.* Particular by Particular, and Univerſal by Univerſal, as in the next Chapter is exemplified; or by working with Rational Numbers, as in the Inſtance following.

Suppoſe the Roots, Particular or Univerſal of W4+ww125 were to be multiplied by themſelves, the W4 being 2, and ww125 being 5, added, make the Sum of their Particular Roots 7; which multiplied by it ſelf is 49.

But if ww125 which is 5, be added to 4, the Sum is 9, of which the Square Root is 3 for the Root Univerſal, and 3 multiplied into it ſelf is 9, for the Product of the Univerſal Roots; accordingly by the Work thereof are thoſe Numbers produced.

And both if ſtrictly taken muſt be the Roots of theſe Roots, to wit, 7 and 3 the Roots of 49 and 9.

W 4+ww125	2+5= 7	√:4+ww125	4+5=9√3
W 4+ww125	2+5= 7	√:4+ww125	4+5=9√3
W16+√ʒφ1000000	49	16+√ʒφ64000000	9
√ʒφ1000000+ww15625		√ʒφ64000000+ww15625	
4 + 10 + 10 + 25		16 + 20 + 20 + 25	
49		W81 = 9	

CHAP. X.

Diviſion of Compound Surds.

Compound Surds divided.

TO divide *Compound Surds*, let them alſo as in *Multiplication*, be conſidered as they are Particular or Univerſal.

Particular.

Diviſion of Particular *Compound Surds*, hath ſometime the given Diviſor Simple, ſometime the Dividend Simple, and ſometime both Compound: The Varieties of whoſe Diviſions are compriſed in the following Caſes.

1. Diviſor Simple, or an Abſolute Number.

Caſe 1. When the Diviſor given is a Simple *Surd*, or Abſolute Number, and the Dividend Compound; divide by the Diviſor the ſeveral Numbers of the Dividend that are of like Denomination; and if any be unlike, reduce the Diviſor to the ſame Denomination, and then divide thereby. And order the Signs + and — as in *Diviſion of Compound Coſſicks*; for like Signs give + and unlike —.

Examples of Binomials.

Examples of Binomials.

Dividend. / *Diviſor.* $\dfrac{\surd 130+\surd 15}{\surd 5}\Big(\ \surd 26+\surd 3$ *Quotient.*

Dividend. / *Diviſor.* $\dfrac{\surd 56+\surd 24}{\surd 6}\Big(\ \surd 9\tfrac{1}{3}+\surd 4$ *Quotient.*

Reſiduals.

Examples of Reſiduals.

Dividend. / *Diviſor.* $\dfrac{\surd 130-\surd 15}{\surd 5}\Big(\ \surd 26-\surd 3$ *Quotient.*

Dividend. / *Diviſor.* $\dfrac{\surd 56-\surd 24}{\surd 6}\Big(\ \surd 9\tfrac{1}{3}-\surd 4$ *Quotient.*

In all theſe firſt 4 Examples there appeareth no Difficulty, every Number being divided as Simple *Surds*, and the Quotients + or — according to the likeneſs or unlikeneſs of the Diviſor's Sign with the Sign of the Numbers of the Dividend.

Mixt.

Examples of Binomials with Reſiduals.

Dividend. / *Diviſor.* $\dfrac{6+\surd 10-\surd\surd 16}{2}\Big(\ 3+\surd 2\tfrac{1}{2}-\surd\surd 2$ *Quotient.*

Dividend. / *Diviſor.* $\dfrac{6-\surd 10+\surd\surd 16}{2}\Big(\ 3-\surd 2\tfrac{1}{2}+\surd\surd 2$ *Quotient.*

In both theſe Examples, after the Abſolute Numbers are divided; to divide the Squares, 2 is ſquared, and to divide the Cubes, 2 is cubed.

2. Dividend Simple, or an Abſolute Number.

Caſe 2. When the Dividend given is a Simple *Surd*, or Abſolute Number, and the Diviſor a Compound Binomial or Reſidual, ſo as the one *Surd* be not equal to the other; then multiply the Diviſor, if Binomial, by his Reſidual; if Reſidual, by his Binomial. And by the ſame Number the Diviſor is multiplied, multiply the Dividend: this Product divide by the Number remaining of the Multiplication of the Diviſor, figurate or not, according to the Denominations of the Dividend ſo multiplied, ordering the Signs as before.

Examples.

Example, to divide the Abſolute Number 49 by $4+\surd 9$. Firſt, the Diviſor multiplied by his Reſidual $4-\surd 9$, produceth 7; which figurated or ſet as a *Surd*, is $\surd 49$; then the Dividend multiplied by $4-\surd 9$, doth produce $196-\surd 21609$; whereof 196 being Abſolute Numbers, divided by 7, the Abſolute Number remaining by multiplication of the Diviſor, and $\surd 21609$ divided by $\surd 49$, give in the Quotient $28-\surd 441$, as at A.

And if 49 be divided by $4-\surd 9$, the Quotient will be alike in Numbers, only the Sign contrary, *viz.* $28+\surd 441$, as at B.

But if 49 be divided by $\surd 9+4$, the Quotient will be $\overline{\surd} 441+28$, as at C.

And if 49 be divided by $\surd 9-4$, the Quotient will be $\overline{\surd} 441-28$, as at D.

A	*Diviſor.*	B	*Diviſor.*	C	*Diviſor.*	D	*Diviſor.*
Binomial.	$4+\sqrt{9}$	*Reſidual.*	$4-\sqrt{9}$	*Binomial.*	$\sqrt{9}+4$	*Reſidual.*	$\sqrt{9}-4$
Reſidual.	$4-\sqrt{9}$	*Binomial.*	$4+\sqrt{9}$	*Reſidual.*	$\sqrt{9}-4$	*Binomial.*	$\sqrt{9}+4$
	$16+\sqrt{144}$		$16-\sqrt{144}$		$9+\sqrt{144}$		$9-\sqrt{144}$
	$-\sqrt{144}-9$		$\sqrt{144}-9$		$-\sqrt{144}-16$		$\sqrt{144}-16$
Product.	$16-9$		$16-9$		$9-16$		$9-16$
Remain.	7		7		-7		-7
Figurated.	$\sqrt{49}$		$\sqrt{49}$		$-\sqrt{49}$		$-\sqrt{49}$
	Dividend.		*Dividend.*		*Dividend.*		*Dividend.*
	49		49		49		49
	$4-\sqrt{9}$		$4+\sqrt{9}$		$\sqrt{9}-4$		$\sqrt{9}+4$
Product.	$196-\sqrt{21609}$		$196+\sqrt{21609}$		$\sqrt{21609}-196$		$\sqrt{21609}+196$

Quotients.

$$\frac{196}{7} \mp \frac{\sqrt{21609}}{\sqrt{49}} \Bigg(28 \mp \sqrt{441} \left\{ \begin{matrix} A \\ B \end{matrix} \right. \qquad \frac{\sqrt{21609}}{-\sqrt{49}} \mp \frac{196}{-7} \Bigg(\underset{-}{\overset{-}{\sqrt{441}}} \pm 28 \left\{ \begin{matrix} C \\ D \end{matrix} \right.$$

Other Examples.

Another Example, with the Varieties in dividing a Rational Number; the like of which is to be done where the Dividend is a *Surd.* As ſuppoſe $\sqrt{81}$ be divided by $3+\sqrt{4}$, or $3-\sqrt{4}$, or $\sqrt{4}+3$, or $\sqrt{4}-3$, the ſeveral Quotients appear at E. F. G. H.

E	*Diviſor.*	F	*Diviſor.*	G	*Diviſor.*	H	*Diviſor.*
Binomial.	$3+\sqrt{4}$	*Reſidual.*	$3-\sqrt{4}$	*Binomial.*	$\sqrt{4}+3$	*Reſidual.*	$\sqrt{4}-3$
Reſidual.	$3-\sqrt{4}$	*Binomial.*	$3+\sqrt{4}$	*Reſidual.*	$\sqrt{4}-3$	*Binomial.*	$\sqrt{4}+3$
	$9+\sqrt{36}$		$9-\sqrt{36}$		$4+\sqrt{36}$		$4-\sqrt{36}$
	$-\sqrt{36}-4$		$\sqrt{36}-4$		$-\sqrt{36}-9$		$\sqrt{36}-9$
Product.	$9-4$		$9-4$		$4-9$		$4-9$
Remain.	5		5		-5		-5
Figurated.	$\sqrt{25}$		$\sqrt{25}$		$-\sqrt{25}$		$-\sqrt{25}$
	Dividend.		*Dividend.*		*Dividend.*		*Dividend.*
	$\sqrt{81}$		$\sqrt{81}$		$\sqrt{81}$		$\sqrt{81}$
	$3-\sqrt{4}$		$3+\sqrt{4}$		$\sqrt{4}-3$		$\sqrt{4}+3$
Product.	$\sqrt{729}-\sqrt{324}$		$\sqrt{729}+\sqrt{324}$		$\sqrt{324}-\sqrt{729}$		$\sqrt{324}+\sqrt{729}$

Quotients.

$$\frac{\sqrt{729} \mp \sqrt{324}}{\sqrt{25}} \Bigg(\sqrt{29\tfrac{4}{25}} \mp \sqrt{12\tfrac{24}{25}} \left\{ \begin{matrix} E \\ F \end{matrix} \right. \qquad \frac{\sqrt{324} \mp \sqrt{729}}{-\sqrt{25}} \Bigg(\underset{-}{\overset{-}{\sqrt{12\tfrac{24}{25}}}} \pm \sqrt{29\tfrac{4}{25}} \left\{ \begin{matrix} G \\ H \end{matrix} \right.$$

Example,

Examples in Cubes.

Example, to divide ww27, by 4 + ww8, or 4 — ww8, or ww8 + 4, or ww8 — 4, the ſeveral Quotients are as at I. K. L. M.

	I	K	L	M
	Diviſor.	*Diviſor.*	*Diviſor.*	*Diviſor.*
	Binom. 4+ww8	*Reſid.* 4−ww8	*Binom.* ww8+4	*Reſid.* ww8−4
	Reſid. 4−ww8	*Binom.* 4+ww8	*Reſid.* ww8−4	*Binom.* ww8+4
	16+ww512 −ww512−ww64	16−ww512 ww512−ww64	ww64+ww512 −ww512−16	ww64−ww512 ww512−16
Product.	16—4	16—4	4—16	4—16
Remain.	12	12	—12	—12
Figurat.	ww1728	ww1728	ww1728	ww1728
	Dividend. ww27 4—ww8	*Dividend.* ww27 4+ww8	*Dividend.* ww27 ww8—4	*Dividend.* ww27 ww8+4
Prod.	ww1728−ww216	ww1728+ww216	ww216−ww1728	ww216+ww1728

Quotients.

$$\frac{\text{ww}1728 \mp \text{ww}216}{\text{ww}1728}\Big(\ \text{ww}1 \mp \text{ww}\tfrac{27}{216}\begin{cases}\text{I}\\ \text{K}\end{cases}$$

Quotients.

$$\frac{\text{ww}216 \mp \text{ww}1728}{-\text{ww}1728}\Big(\ \overline{\underline{-}}\,\text{ww}\tfrac{27}{216} \pm \text{ww}1\begin{cases}\text{L}\\ \text{M}\end{cases}$$

3. Diviſor Compound Binomial, but one part equal to the other.

Caſe 3. When the Dividend is Simple or Compound, and the Diviſor a Compound Binomial, and one *Surd* is equal to the other; then add them together, and by the Total divide the Dividend, as in the firſt Caſe, if the Dividend be Compound, or otherwiſe as a Simple *Surd*. And if ſuch Reſidual be given for a Diviſor whoſe Parts are equal, becauſe of the contrary Signs, the Value thereof is 0, and ſo no Diviſion can be made thereby.

Example. Example, to divide w60 by w5 + w5, both added make w20, by which w60 divided maketh the Quotient w3. But if the Diviſor had been propoſed w5 — w5, it being clear the Diviſor is 0, no Diviſion can be made thereby but nugatory.

Diviſor. w5
4
w20

Dividend. w60
Diviſor. w20 (w3 *Quotient.*

4. Data both Compound.

Caſe 4. When both Dividend and Diviſor are Compound, and the Diviſor a Binomial or Reſidual; ſo as the Parts of the Diviſor be not equal, as aforeſaid: Then proceed as in the ſecond Caſe before.

Examples. *Examples of Binomials, Reſiduals, and mixt Surds.*

To divide w68 + w54 by w6 + w3 Binomial, as at N.
To divide w68 — w54 by w6 — w3 Reſidual, as at O.
To divide w456 — w72 by w18 + w6 Mixt, as at P.
To divide w456 + w72 by w18 — w6 Mixt, as at Q.

	Diviſors.			*Dividends.*	
N	w6 ± w3	O	N	w68 ± w54	O
	w6 ∓ w3			w6 ∓ w3	
	6 ± w18 ∓ w18 — 3			w408 ± w324 ∓ w204 — w162	
	6—3			w408 ± w324 ∓ w204 — w162	
	3				
	w 9				

Quotients.

Dividends. $\dfrac{\text{w}408 \pm \text{w}324 \mp \text{w}204 - \text{w}162}{\text{w}9}\Big(\ \text{w}45\tfrac{1}{3} \pm \text{w}36 \mp \text{w}22\tfrac{2}{3} - \text{w}18\begin{cases}\text{N}\\ \text{O}\end{cases}$
Diviſor.

Diviſors.

P √18 ± √6 Q
√18 × √6
18 ± √108
∓ √108 − 6
18 − 6
12
√ 144

P √456 ∓ √72 Q
√ 18 ∓ √6
√8208 ∓ √1296
∓ √2736 + √432
√8208 ∓ √1296 ∓ √2736 + √432

Dividends. √8208 ∓ √1296 ∓ √2736 + √432
Diviſor. √144
(√57 ∓ √9 ∓ √19 + √3 { P / Q *Quotients.*

Caſe 5. When the Diviſor is a Multinomial, in multiplying him by his Reſidual, one Part, or all the Parts thereof except one, may be made Reſidual at pleaſure, always provided the Parts made Reſidual be not equal in Value to the other as aforeſaid. And after addition of the Multiplees, rejecting what may be rejected, by reaſon of contrary Signs, add the Remains of the Total into one Number, if it may be, and proceed as in the ſecond and fourth Caſes before in this Chapter. **5. Diviſor a Polynomial.**

Example, to divide 54 Integers by √4 + √9 + √16 Rational Numbers, the Diviſor may be multiplied either by √4 + √9 − √16, or √4 − √9 + √16, or √4 − √9 − √16, or √9 − √4 − √16, or by any other Reſidual to be made, by exchanging the Places of the Numbers in the Diviſor. And accordingly the Diviſor multiplied by any of them, and the Remains brought into one Number, by this Number I divide the Product of 54, multiplied by the ſame the Diviſor was multiplied. See two of the Varieties at R and S. **Example.**

Diviſor.
R √4 + √9 + √16
√4 + √9 − √16
4 + √36 + √64
√36 + 9 + √144
− √64 − √144 − 16
4 + √144 + 9 − 16
√144 − 3
√81

Diviſor.
S √4 + √9 + √16
√4 − √9 − √16
4 + √36 + √64
− √36 − 9 − √144
− √64 − √144 − 16
4 − √576 − 25
− √576 − 21
− √2025

Dividend.
54
√4 + √9 − √16
√11664 + √26244 − √46656

Dividend.
54
√4 − √9 − √16
√11664 − √26244 − √46656

Quotient.

$\frac{\surd 11664 + \surd 26244 - \surd 46656}{\surd 81}$ (√144 + √324 − √576. R

Quotient.

$\frac{\surd 11664 - \surd 26244 - \surd 46656}{-\surd 2025}$ ($-\surd 5\frac{1539}{2025} + \surd 12\frac{1944}{2025} + \surd 23\frac{81}{2025}$ S

Caſe 6. When both Dividend and Diviſor are Multinomials, and the Remains of the Total after Multiplication of the Diviſor will not be added into one Number as above-mentioned: then divide your given Numbers after the manner of Compound Coſſicks, placing the *Surds* orderly; and by every Number gotten for the Quotient, multiply the Diviſor, and ſubſtract the Product from the Dividend. **6. Data both Polynomials.**

Example, to divide 9 + √180 + 5 − 2 by 3 + √5 + √2; at the firſt Application of the Diviſor 3 being gotten for the Quotient, I multiply the Diviſor thereby, and the Product is 9 + √45 + √18; which ſubſtracted from the Dividend, leaves √45 − √18. Then by Application of the Diviſor is gotten for the Quotient + √5, by which the Diviſor multiplied, produceth √45 + √25 + √10; **Example.**

this ſubſtracted from the Dividend, leaves —W18—W10. And laſtly, applying the Diviſor, —W2 is gotten for the Quotient, and the Product of the Diviſor multiplied thereby is —W18—W10—W4; which ſubſtracted leaves 0 remaining, and the Work ſtands as at W; the other Paragraphs thereof as at T and V.

Dividend.
W 45—W18 T
Diviſor. 3+W 5+W 2) 9+W180+5—2 (3 *Quotient.*
3) 9+W 45+W18 (
9+W45+W18

Dividend.
W45-W18-W10 V
Diviſor. 3+W 5+W 2) 9+W180+5—2 (3+W5 *Quotient.*
3+W 5) 9+W 45+W18 (
9+W45+W18 W45+5+W10
W45+5+W10

Dividend. W
W45-W18-W10
Diviſor. 3+W5 +W2) 9+W180+5—2 (3+W5—W2 *Quotient.*
3+W5 —W2) 9+W45 +W18 (
9+W45+W18 W45 +5+W10
W45+5+W10 —W18—W10—2
-W18-W10-2

When Names are increaſed and changed in Diviſion.

Thus Diviſion of particular *Surds* give evidence, that every Compound Diviſor; by the Work of the 2*d*, 4*th* and 5*th* Caſes, increaſeth the Names in the Quotient; the one a Binomial, makes the other a Reſidual, and the Reſidual begets a Binomial, although the Dividend be ſingle. But if the Dividend be Compound as well as the Diviſor, the Quotient ſhall be a Multinomial. And great heed is to be taken in right placing the Signs, for that according thereto the Quotient is to be valued; all which is plainly to be obſerved in the foregoing Examples.

Heed to be took in placing the Signs.

Univerſals Homogeneal, how divided.

Diviſion of Univerſal *Surds* Homogeneal, imitate the Diviſion of Particular *Surds*, only before the Quotient is to be prefixed the Univerſal Sign. And if Operation be made according to the ſixth Caſe; then let the Siniſter Number of the Diviſor upon every Removal be figurate, as the next Dexter Number he is applied to. And if you proceed according to the fourth Caſe, then in multiplying the Numbers, the Multiplication proper to Univerſal *Surds* is to be obſerved; and beſides, if the Diviſor be Negative, the Order of the Quotient is inverted; for then the Siniſter Number of the Quotient ſhall be ſubſtracted from the Dexter, and the Root of the Remainder taken for the Number deſired.

Examples.

Example, to divide √:6+W144+W441+W1764 by √:3+W36, upon application of 3 in the Diviſor to 6 in the Dividend, 2 is gotten for the Quotient, by which multiplying the Diviſor, the Product is 6+W144: Then 3 is ſquared, and by applying the Product or Square which is 9, to 441, there is W49 gotten for the Quotient; which multiplying the Diviſor, produceth W441+W1764, and ſubſtracted, leaves 0 remaining. And the Diviſion according to the ſixth Caſe ſtands as at X. And if Diviſion were made by √:2+W49, the Numbers ſhould be placed thus, √:6+W441+W144+W1764.

But if nine Abſolute Numbers (which is the Root Univerſal of √:6+W144 +W441+W1764) be divided by √:3+W36, according to the 4*th* Caſe; then multiplying √:3+W36 by his Reſidual √:3—W36, the Numbers to be uſed for Diviſors will be —27 and —W729. And then √:3—W36 multiplying 9, makes the Dividend 243—W236196, and the Quotient will be —9+W324 as at Y. Then becauſe the Diviſor is Negative, 9 ſhall be taken from the Root of 324, and the ſquare Root of the Remainder, which here will be 3, ſhall be the Univerſal Root deſired, equivalent, though in other Terms, to the Root of the Quotient at X.

Diviſor. *Dividend.* *Quotient.*

√:3+W36) √:6+W144+W441+W1764 (√:2+W49 **X**

√:2+W49 6+W144

6+W144 W441+W1764

W441+W1764

Diviſor. *Dividend.* *Quotient.*

√:3+W36 9

√:3—W36 √:3 — W36

9+W324 243—W236196

—W324—36

9—36

—27

—W729

$\frac{243}{-27}$——$\frac{\mathrm{W}236196}{-\mathrm{W}729}$ (√:—9+W324 **Y**

Nevertheleſs where the Diviſor by this laſt Sort is Affirmative, the Quotient ſhall be in the former order.

Other Examples.

Example, to divide 30 by √:30+W36, whoſe Root Univerſal is 6, or by √:30 —W25, whoſe Root Univerſal is 5, the Quotients will accordingly be 5 or 6, and therewith agree the Quotients at Z. z. if their Roots be extracted.

Diviſor. *Dividend.* *Quotient.*

√:30 +W36 30

30 —W36 30 —— W36

900+W32400 27000—W29160000

—W32400—36

900—36 **Z**

864 $\frac{27000}{864}$—$\frac{\mathrm{W}29160000}{\mathrm{W}746496}$ (√:31$\frac{1}{4}$—W39$\frac{1}{16}$.

W746496

Diviſor. *Dividend.*

√:30 —W25 30

30 +W25 30 + W25

900—W22500 27000+W20250000

W22500—25

900—25 **z**

875 $\frac{27000}{875}$+$\frac{\mathrm{W}20250000}{\mathrm{W}765625}$ (√:30$\frac{6}{7}$+W26$\frac{22}{49}$.

W765625

Univerſals Heterogeneal, how divided.

If the given Numbers be Heterogeneal, then proceeding by the *6th* Caſe, beſides upon Removal of the Diviſor to figurate the Siniſter Number thereof, according to the Dexter Number he is applied to; if the Quotient of this Diviſion be of higher Denomination than the next Dexter Number of the Diviſor, then let this Quotient be depreſſed thereto by extracting the Root, and this Root with his proper Character ſhall be ſet in the Quotient of the firſt Diviſion.

Examples.

Example. If √:4+√ʒΦ4096+W16+√ʒΦ4096, the Product of √:2+wW8 by √:2+W4, be divided by the firſt of the ſaid Factors, then upon the Application of the Diviſor 2 to W16 he is to be ſquared; and the Quotient being a Square, and ſo not above the Dexter Number of the Diviſor, which is of the Cube Denomination, this Quotient ſhall ſtand for the Quotient deſired. But if Diviſion be made by √:2+W4 the other Factor, then upon application of 2 to √ʒΦ4096, after 2 is exalted to the Zenzicube Power, and 4096 divided thereby, the Quotient 64 being a Zenzicube, (and ſo above the Denomination of the next

Dexter Number of the Diviſor which is a Square) is to be depreſſed to a Cube by extracting the Square Root thereof.

Diviſor. *Dividend.* *Quotient.*
√:2+ww8) √:4+√ʒφ4096+w16+√ʒφ4096 (√:2+w4

New Diviſor. 2×2=w4) 16 (4

Diviſor. *Dividend.* *Quotient.*
√:2+w4) √:4+w16+√ʒφ4096+√ʒφ4096 (√:2+ww8

2×ʒφ=64) 4096 (64√ʒ whereof is 8.

If the Univerſal Root were ſtrictly to be taken in all theſe Inſtances, then at X and Y it would be w1. at Z $\frac{w5}{w6}$. at z w1$\frac{1}{5}$, and in the two laſt Examples w1.

X Y Z z
$\frac{w3}{w3}$(w1 $\frac{w5}{w6}$($\frac{w5}{w6}$ $\frac{w6}{w5}$(w1$\frac{1}{5}$ $\frac{√:2+w\,4=2}{√:2+ww8=2}$(w1

Proof of Diviſion of Compound Surds.

As the Proof of Diviſion of Simple *Surds* is by their Simple Multiplication, or by Rational Numbers; ſo will Diviſion of Compound *Surds* be proved by Compound Multiplication, Particular by Particular, and Univerſal by Univerſal reſpectively; multiplying the Quotient by the Diviſor to return the Dividend: And alſo by working with Rational Numbers.

Among the Examples of Particular Surds in this Chapter at A, 49 was divided by 4+w9, which is 7; ſo ſhall the Quotient of the Diviſion be 7; the Quotient being 28—w441, multiplied by the Diviſor, ſhall return 49, and the Square Root of 441, which is 21 taken from 28, ſhall leave 7.

Among the Examples of Univerſal *Surds* at X, the Root Univerſal of √:6+w144+w441+w1764 is 9, and the Root Univerſal of the Diviſor √:3+w36 is 3; ſo ſhall the Quotient of that Diviſion be 3, the Quotient at X being √:2+w49 agrees: for if the Square Root of 49 which is 7 be added to 2, it makes 9, whoſe Square Root is 3. As likewiſe doth the Quotient at Y, for from the Square Root of 324 which is 18, let 9 be taken, and the Square Root of the 9 remaining is alſo 3. And if thoſe Quotients be multiplied by the Diviſor, the Dividends will reſpectively be returned, as that at X is to be ſeen in the foregoing Chapter, the other here follow.

Diviſor.
4+w9)
4—w9
16—9
7
w49

Dividend.
49
4—w9
196—w21609
7 w49

Quotient.
(28—w441 (√ 21
4+w9
112—w7056
w7056—w3969 (√ 63
112—63
49

4+3=7) 49 (7
28—21=7

Univerſal Root.

Diviſor. *Dividend.* *Quotient.*
$\frac{√:3+w36}{\text{or } w9}$) $\frac{√:6+w144+w441+w1764}{\text{or } w81}$ ($\frac{√:2+w49}{\text{or } w9}$ $\frac{w81 \text{ or } 9}{w9 \text{ or } 3}$ (w9 or 3.

$$\begin{array}{llll}
\textit{Divisor.} & \textit{Dividend.} & \textit{Quotient.} & \\
\sqrt{:}3+W36) & 9 & & W9=3)9(3 \\
3-W36 & 3-W36 & & W18-9=W9=3 \\
\hline
9-36 & 243-W236196 & \Big(\dfrac{\sqrt{:}-9+W324}{3+W36}\Big(\overset{\sqrt{}}{18} & \\
-27 & -27-W729 & & \\
-W729 & & -27+W2916 & \\
 & & -W2916+W11664\Big(\overset{\sqrt{}}{108} & \\
 & & -27+108 & \\
 & & W81=9 &
\end{array}$$

CHAP. XI.

Of Fractionary Surds.

Fractionary Surds, how to be ordered.

ALthough little mention of Fractions hath been made before in this Part of *Surds*, yet they oft-times ariſe upon Diviſion of *Surds*, and ſometimes with and without Integers are uſeful among *Surds*; Wherefore it is neceſſary to remember, that the Operations proper to common Fractions, mixed with the Operations proper to *Surds*, ſhall add and ſubſtract, multiply and divide them, whether Simple or Compound; ſo as the particular Rules of their ſeveral Elements, being nothing but what hath been before ſhewed, need no further be ſpoken to here than to give a few Examples.

Fractionary Surds added and ſubſtracted.

Examples of Addition and Substraction.

$$\text{Examples.}\ \left.\begin{array}{l} W\frac{8}{10} \\ W\frac{72}{4} \\ WW\frac{7}{12}\end{array}\right\}\text{is added to, and ſubſtracted from}\left\{\begin{array}{l} W\frac{50}{10}\text{ at A and B.}\\ 32\text{ Integers at C and D.}\\ WW\frac{56}{12}\text{ at E and F.}\end{array}\right.$$

$$\begin{array}{lll}
\text{A . B} & \text{C . D} & \text{E . F}\\
W\frac{50}{10}\ \begin{smallmatrix}+\\-\end{smallmatrix}\ W\frac{8}{10} & 32\ \begin{smallmatrix}+\\-\end{smallmatrix}\ W\frac{72}{4} & WW\frac{56}{12}\ \begin{smallmatrix}+\\-\end{smallmatrix}\ WW\frac{7}{12}\\
\hline
2)\,W50\ (W25\ .\ 5 & 8)\,W128\ (W16\ .\ 4 & 7)\,WW56\ (WW8\ .\ 2\\
W\,8\ (W\,4\ .\ 2 & W72\ (W9\ .\ 3 & WW7\ (WW1\ .\ 1\\
\hline
\text{A. } W\frac{98}{10}\quad W49\ .\ 7 & \text{C. } W\frac{392}{4}\ .\ W49\ .\ 7 & \text{E. } WW\frac{189}{12}\ .\ 27\ .\ 3\ \textit{Sum.}\\
\text{B. } W\frac{18}{10}\ .\ W9\ .\ 3 & \text{D. } W\frac{8}{4}\ .\ W1\ .\ 1 & \text{F. } WW\frac{7}{12}\ .\ 1\ .\ 1\ \textit{Difference.}
\end{array}$$

Fractionary Surds *multiplied and divided.*

Examples of Multiplication and Division.

$$\text{Examples,}\left.\begin{array}{r} W\frac{5}{9}\ldots\ldots\\ 16\text{ Integers}\\ WW\frac{2}{9}\ldots\ldots\end{array}\right\}\text{is multiplied and divided by}\left\{\begin{array}{l} W\frac{5}{7}\text{ at G and H.}\\ W2\frac{1}{4}\text{ at I and K.}\\ WW1\frac{1}{3}\text{ at L and M.}\end{array}\right.$$

$$\text{G. } W\frac{5}{9}\times\frac{5}{7}=W\frac{25}{63}\qquad \text{I. } \overset{\substack{64\\ \vdots\\ W256\\ \vdots}}{\frac{16}{1}}\times \overset{\substack{9\\ \vdots}}{\underset{\substack{\vdots\\ 1}}{W2\frac{1}{4}}}=W\frac{576}{1}\qquad \text{L. } WW\frac{2}{9}\times WW\overset{\substack{4\\ \vdots}}{1\frac{1}{3}}=WW\frac{8}{27}\ \text{Product.}$$

$$\text{H. } \overset{\substack{1\\ \vdots}}{W\frac{5}{7}}\Big)\overset{\substack{1\\ \vdots}}{W\frac{5}{9}}\Big(W\frac{7}{9}\qquad \text{K. } \overset{\substack{9\\ \vdots}}{W2\frac{1}{4}}\Big)\overset{W256}{\frac{16}{1}}\Big(W\frac{1024}{9}\qquad \text{M. } WW\overset{\substack{2\\ \vdots\\ 4\\ \vdots}}{\underset{\substack{\vdots\\ 1}}{1\frac{1}{3}}}\Big)WW\overset{\substack{1\\ \vdots}}{\underset{\substack{\vdots\\ 3}}{\frac{2}{9}}}\Big(WW\frac{1}{6}\ \text{Quotient.}$$

Proof of the Fractionary Operations.

Common with other Fractions are theſe Operations of *Fractionary Surds* proved, *Addition* by *Subſtraction*, *Multiplication* by *Diviſion*, and the contrary. Proper to *Surds*, the truth of their Fractionary Operations may be proved, by taking Rational Numbers, and working therewith as in other *Surds*, as may be eaſily examined.

CHAP. XII.

Figuration of Surds.

Figurate Surds produced.

To figurate *Surds*, is to multiply them after the manner of *Surds*; for any *Surd* multiplied by himſelf, produceth the Square and Product again, by the Root bringeth forth the Cube, &c. as other Figural Numbers. But this is proper to *Compound Surds*, for any Simple *Surd* multiplied figurately, produceth a Rational Number, and ſo ceaſeth to be a *Surd*.

Examples.

As to multiply √3 by √3, produceth √9, which is a Rational Number, and hath 3 for the Root.

To multiply ſquarely the particular Surds √3+√5, the Square produced is 3+√60+5: but the Univerſal Root ſquared, as here appeareth, is √:9+√180+√25.

	Simple.		*Particular.*		*Univerſal.*
Root.	√3	*Root.*	√3+√5	*Root.*	√:3+√5
	√3		√3+√5		√:3+√5
Square.	√9		3+√15		9+√45
			√15+5		√45+√25
		Square.	3+√60+5	*Square.*	√:9+√180+√25

In extraction of Roots to be noted.

Touching the Extraction of the Root of *Surds* is to be minded,

1. Simple Surds can have no Roots. Rationals have their Roots as before extracted. Examples.

1. That Simple *Surds* having no Roots to be expreſſed by Integers or Fractions exactly, can have no Root extracted: but the Rational Numbers ſet as *Surds*, have their Roots extracted as figural Numbers before ſpoken to in the ſecond Part of the ſecond Book.

And ſo √2025 ſhall be 45, and ∛729 ſhall be 9 Integers; becauſe both 2025 and 729 are Rational Numbers, and not proper *Surds*.

Rational. √2025 |45 Root. (4 above, 16 below; √ over 45)

Gnomon. { 40
25

Rational. ∛729. |9 Root. (√ over 9)

2. Particular Compounds, ſome in a ſort Irradical, what then to be done. When the Siniſter Number is Abſolute. Examples.

2. That among particular Compound *Surds*, ſome are in a ſort Irradical, and have their Roots extracted only by altering their Characters. This ſort have their Siniſter Number, either Abſolute or a *Surd*.

If Abſolute, then place before the given Number the Character belonging to the Root to be extracted.

As to extract the Square Root of 10+√5, or the Cube Root thereof, they are ſet as at A and B.

A	B
Square 10+√5 (√10+√5 Root.	Cube 10+√5 (∛10+√5 Root.

When the Siniſter is a Surd.

If the Siniſter Number be a *Surd*, then multiply the Index of the Root to be extracted, by the Index of the Siniſter *Surd*, and the Index amounting ſhall be the Index of the Root, whoſe Character is to be prefixed before the given *Surd*.

Examples.

As to extract the Square Root of √10+√5, becauſe 2 and 2 make 4, the Index of ſquared Squares, the Root ſhall be ∜10+√5; ſometime ſet thus, √.√10+√5, and is as much as to ſay, the Square Root of the Square Root of 10, and the Square Root of 5.

So to take the Square Root of ∛18—2, is √.∛18—2. And the like is to be done for the Cube and Higher Powers.

Square w10+w5 (ww10+w5 Root, Square ww18—2 (√ʒφ18—2 Root.
2×2=4 Index ʒʒ 2×3=6 Index ʒφ.

3. Thoſe Extractions of the Roots of particular Compound *Surds*, that are properly Radical, alter the Numbers in Homogeneals, both Numbers and Characters in Heterogeneals.

3. Particular Compounds Radical, what to be done.

When the Siniſter is Abſolute.

If the Siniſter Number be Abſolute, and the Dexter a Square *Surd*, then ſquare the Siniſter Number, and ſubſtract the Dexter Number from it; take the Square Root of the Difference, which add to the Siniſter Number, and alſo ſubſtract it therefrom: half the Sum, and half the Difference, joined with the Sign +, ſhall be the Binomial Root, and with the Sign — ſhall be the Reſidual Root.

Example.

As to extract the Square Root of 7+w40, the Square of 7 is 49; from which 40 taken, leaves 9, whoſe Square Root is 3, which added to 7, makes 10; the half is 5, and taken from 7, leaves 4; the half is 2: therefore 2+5 ſhall be the Binomial Root, and 2—5 the Reſidual Root, or the contrary.

7	Siniſter.	7		7		w2+w5	Binomial Root.
7		3		3		w2—w5	Reſidual Root.
49	ʒ.	10	Sum.	4	Difference.		of 7+w40
40	Dexter.	5	Half.	2	Half.		or 7—w40
9	Difference.					w5+w2	Binomial Root.
3	Root.					w5—w2	Reſidual Root.

When the Siniſter is a Surd.

If the given Numbers be Homogeneal ſquare *Surds*, take the one Square from the other, and the Root of the Difference add to the Root of the greater Square, and alſo ſubſtract from it: half the Sum and half the Difference of theſe Roots joined with the Sign +, ſhall be the Binomial Root, and with the Sign — ſhall be the Reſidual Root as before.

Example.

For ſuppoſe the Numbers given were w49+w40, or w40+w49: then 40 taken from 49, leaves 9; whoſe Root 3 added to and ſubſtracted from 7, the Root of 49, the Greater, makes the Sum and Difference, and conſequently the Halfs, and the Binomial and Reſidual Roots as before, becauſe the Product of w2+w5, or w5+w2 multiplied ſquarely, is alike.

49	Greater } Square.	7	Root.	7	Root.	w5±w2 } Roots {	Binom.
40	Leſſer }	3		3		w2±w5 }	Reſidual
9	Difference.	10	Sum.	4	Difference.		of w49±w40
3	Root.	5	Half.	2	Half.		or w40±w49

When the Data are Cubes or Higher Powers.

If the given Numbers be Cubes or Higher Powers, or an Abſolute Number, and a Cube, or Higher Power, take the Roots of the Numbers, and work as if they were Square Roots till you get the Halfs as before; then advance the Roots of thoſe Halfs into the Powers of the Denominations given, or the Dexter Denomination, if but one.

Examples.

As if the Root be deſired of 13+ww1728, or ww2197+ww1728, the Cube Root of 2197 is 13, and of 1728 is 12; both Roots ſquared are 169 and 144, the Difference 25, whoſe Square Root 5 added to 13, makes 18, the Half thereof is 9, whoſe Root is 3; the ſaid 5 taken from 13, leaves 8, the half whereof is 4 whoſe Root is 2: theſe Roots 2 and 3 advanced to be Cubes, according to the Denominations given, are the deſired Numbers.

√ 3
ww 2197 |13×13 = 169
1
Gnomon { 9, 27, ...27

13
5
18 Sum.
9 Half.
3 Root.

13
5
8 Differ.
4 Half.
2 Root.

ww8 + ww27 Binom. Root.
ww8 — ww27 Reſid. Root.
of 13 + ww1728
or 13 — ww1728

√ 3
ww 1728 |12×12 = 144
1
Gnomon { 6, 12, ...8

27φ
25 Difference.
5 Root.

8φ

ww27 + ww8 Binom. Root.
ww27 — ww8 Reſid. Root.

When the Siniſter is Abſolute, and the Dexter an Higher Power.

If the Siniſter Number be Abſolute, and the Dexter an Higher Power than a Square, different Roots may be had according to the Powers into which the Roots of the Halfs be exalted; for the ſame Value in the Roots produce Products of like Value, though different in Terms.

Example.

As in the laſt Example, where the Roots of the Halfs were 3 and 2, let them be both ſquared, as w9 + w4, or the one ſquared and the other cubed, as w9 + ww8, or ww27 + w4; any of theſe ſhall produce Products alike valuable with the Product of ww27 + ww8, but all different in Terms.

When the Dexter higher Power may have the Index halfed.

Yet if the Higher Power on the Dexter Part of the given Number may have the Index evenly halfed, it is moſt uſual to exalt the Root of one half into the Power anſwering to half the Index of the Dexter Number, and take the other half for the next Inferior Power thereto.

Example.

As ſuppoſe the Binomial and Reſidual Roots be deſired of 29 + √3φ64000000, the Zenzicube Root of 64000000 is 20; which ſquared is 400, and 29 ſquared is 841, the Difference is 441, whoſe ſquare Root is 21; this added to and ſubſtracted from 29, makes the Sum 50, the Difference 8, whoſe Halfs are 25 and 4, their Roots 5 and 2: Of which may be made ſeveral Roots, but moſt uſual thus; becauſe the Index of 3φ which is 6, may be halfed, 3 the Index of Cube ſhall be taken for one Root; then ſhall the other half be a Square, which is next inferior to the Cube.

3φ φ √
29 + √3φ64000000 |8000|20

3
29 × 29 = 841
20 × 20 = 400
441 |21 √

29
21
50
25 Half.
5 √

29
21
8
4 Half.
2 √

w4 + ww125 Binomial } Root.
w4 — ww125 Reſidual
Or,
w25 + ww8 Binomial } Root.
w25 — ww8 Reſidual

4. Univerſal Surds, how to extract their Roots. When the Siniſter is Abſolute.

4. The Roots of Univerſal *Surds* are extracted in all things like thoſe particular Compound *Surds*, properly Radical, laſt ſpoken to, till you come to the Halfs: Then for thoſe that are Homogeneal, the Root of half the Sum ſhall be the one Root deſired, and the half Difference the other, this Dexter, and that Siniſter. But if the Root of the half Difference be made the Siniſter, then the half Sum ſhall be the Dexter.

Example.

As to extract the Root Univerſal of 25 + w576, the Square of 576 taken from 625 the Square of 25, leaves 49 whoſe Root is 7; which added to 25 makes 32, whoſe half is 16, and the Root thereof 4 for the one Root ſought: Then 7 taken from 25 leaves 18, whoſe half is 9 for the other Root ſought. Or if 3 the Root of 9 be the Siniſter, then 16 ſhall be the Dexter.

$$\begin{array}{lll}
25+\mathrm{W}576 & 25 & 25 \\
\underline{25} & \underline{7} & \underline{7} \\
125 & 32\ \text{Sum.} & 18\ \text{Difference.} \\
\underline{50} & 16\ \text{Half.} & 9\ \text{Half.} \\
625 & 4\ \text{Root.} & 3\ \text{Root.} \\
\underline{576} & & \\
49\ \text{Difference.} & & \\
7\ \text{Root.} & &
\end{array}$$

$$\left.\begin{array}{l}\surd{:}4+\mathrm{W}9\\ \surd{:}4-\mathrm{W}9\end{array}\right\}\text{Root}\left\{\begin{array}{l}\text{Binomial.}\\ \text{Reſidual.}\end{array}\right.$$

$$\left.\begin{array}{l}\surd{:}3+\mathrm{W}16\\ \surd{:}3-\mathrm{W}16\end{array}\right\}\text{Root}\left\{\begin{array}{l}\text{Binomial.}\\ \text{Reſidual.}\end{array}\right.$$

When the Dexter is an higher Power.

If the Root of Univerſals be ſought, that have the Siniſter Number given Abſolute, and the Dexter an higher Power than a Square: Then proceed as above till the Roots of the Halfs be obtained, and thereby different Roots may be had according to the Powers into which the Roots of the Halfs are advanced. But moſt uſual it is to exalt the Root of one half into the Power anſwering to half the Index of the Dexter Denomination given, if the ſame Index may be equally halfed: And take the Root of the other half, with the Character of the next inferior Power thereto, for the other part of the Root deſired.

Example.

As if the Roots Binomial and Reſidual be deſired of 41+√ʒφ4096000000, the Zenzicube Root of 4096000000 is 40, which ſquared is 1600, taken from 1681 the Square of 41 leaves 81, whoſe Root is 9; which added to and ſubſtracted from 41, makes the Sum 50, the Difference 32, the Halfs whereof are 25 and 16, whoſe ſquare Roots are 5 and 4. Then becauſe the Index of Zenzicube, which is 6, may be equally halfed into 3, the Index of the Cube, either 5 or 4, may be cubed, and the other that is not ſhall be ſet with the Character belonging to the Square.

$$\begin{array}{llll}
 & \text{ʒ}\varphi & \varphi & \surd \\
41+\surd\text{ʒ}\varphi & 4\dot{0}9\dot{6}0\dot{0}\dot{0}0\dot{0}0 & \underline{|6\dot{4}00\dot{0}|} & \underline{40} \\
 & \text{ʒ} & & \\
41\times41=1681 & & 41 & 41 \\
40\times40=\underline{1600} & \surd & \underline{9} & \underline{9} \\
81|\underline{9} & & 50 & 32 \\
 & & 25\ \text{Half.} & 16\ \text{Half.} \\
 & & 5\surd & 4\surd
\end{array}$$

$$\left.\begin{array}{l}\surd{:}4+\mathrm{ww}125\\ \surd{:}4-\mathrm{ww}125\end{array}\right\}\text{Root}\left\{\begin{array}{l}\text{Binomial.}\\ \text{Reſidual.}\end{array}\right.$$

Or,

$$\left.\begin{array}{l}\surd{:}5+\mathrm{ww}64\\ \surd{:}5-\mathrm{ww}64\end{array}\right\}\text{Root}\left\{\begin{array}{l}\text{Binomial.}\\ \text{Reſidual.}\end{array}\right.$$

Proof of Figuration of Surds.

Beſides the Proof of Extraction of theſe Roots by the Production of the *Surds*, and their Production by Extraction, Simple by Simple, Particular by Particular, and Univerſal by Univerſal reciprocally: The Truth of all may be tried, by taking Rational Numbers, and working with them inſtead of the *Surds*; nevertheleſs for brevity ſake Examples thereof are omitted here.

Partis quintæ Libri tertii

FINIS.

The Sixth PART of the Third BOOK.

CHAP. I. *Of Species.*

Species the laſt ſort of ſpecial Contracts.

SPECIES, as the ſixth Sort of Contract Numbers, and Third of thoſe whoſe Denominations are uncertain, come now to be inſpected in the Cloſe of this third Book.

Whence the Name.
What Species are.
Characters uſed, and why.

According to the Name, (which with the Latins ſerveth for the Figure, Form or Shape of any thing) *Species* are Quantities or Magnitudes, denoted by Letters, ſignifying Numbers, Lines, Lineats, Figures Geometrical, *&c.* And for avoiding the prolix and often reſcription of Words, ſeveral Marks or Characters are uſed therewith for Terms Artificial: Some whereof, moſt cuſtomary, are already ſpecified, *Book* 1. *Part* 1. *Chap.* 3. which being remembred, may ſave their tranſcribing here.

The ſame Letter denotes different Species.

Further let it be remembred here, that A while the Queſtion is in, may ſignify any Number of Pounds, Yards, Ells, Length, Breadth, *&c.* But when the Work is ended, and another Queſtion begun, A may denote another Quantity different from the former. And the like is to be underſtood of B. C. D. or any other *Species*.

Powers how differenced from plain Numbers.

It is alſo neceſſary to obſerve, That ſometime to difference Powers (with whom *Species* alſo converſe) from plain Numbers, the Letters for theſe are commonly Capital, but for thoſe Small. Likewiſe given Quantities or Numbers, ſome will have noted with Conſonants, and thoſe ſought with Vowels; but this is not eſſentially neceſſary, ſo as by any Diſtinction the *Data* and *Quæſita* be diſcerned apart.

Form of the Species to be kept while the Queſtion is working.

Moreover, while a Queſtion is working, the Form of the Letters is ſtrictly to be kept: For if A and E be two given Numbers, then AE, or Æ, ſhall be the Product, Z the Sum, *&c.* But if *a* and *e* be the given Numbers, then *ae*, or *æ*, ſhall be the Product, *z* the Sum, *&c.*

The ſame Species differently called.

One and the ſame Letter in the beginning and end of a Queſtion may be differently called: As in Extraction of Roots and Equations, hereafter ſpoken to in the 4*th* Book, A in working the Queſtion is called the Suppoſititious or Queſitious Root; but when the Root is found, or the Queſtion brought to an Equation, A ſhall be called the Eductitious or Reſolved Root.

Figures prefixed make a Number of Quantities, &c.

As every ſingle Letter ſignifies a certain Quantity or Magnitude, ſo by prefixing of Figures to their left Hand, there ſhall be accordingly made a number of Quantities or Magnitudes after the manner of Coſſicks. Wherefore if A ſignify one Yard, B one Houſe, *&c.* then 10A ſhall ſignify 10 Yards, and 19B 19 Houſes, *&c.* But if A ſignify 10 Yards, B 10 Houſes; then ſhall 10A ſignify 100 Yards, and 19B 190 Houſes; and ſo of others.

Nature of Species.
1. Whole, and they, Homogeneal, or Heterogeneal.

Touching the Nature of *Species*, they are diverſly conſidered.
Firſt, As they are Whole or Integral, Broken or Fracted.
  Integral, as A. B. C. *&c.* 4A. 5B. 3C. *&c.* Theſe are alſo
    Homogeneal, as A and A, that is 2A. Or,
    Heterogeneal, as A and B, that is A+B.

Broken, & they Commenſurable, Or Incommenſurable.

  Fracted, as $\frac{2}{3}$A. $\frac{A}{B}$. $\frac{3B}{4A}$ or the like. Theſe are alſo
    Commenſurable, as $\frac{BA}{BC}$. $\frac{15BD}{27BD}$ *&c.* Or,
    Incommenſurable, as $\frac{BD}{C}$. $\frac{3AB}{4E}$ *&c.*

Fractions alſo divided as others.

All fracted *Species* may alſo be divided as Common Fractions, into Proper, Improper, and Equal Fractions; and the Proper into Conjunct and Divided, or Fractions of Fractions, as *Book* 1. *Part* 2. *Chap.* 1.

Examples of Fractions.

Examples of
- Equal, as $\frac{A}{A}$ or $\frac{B}{B}$. *&c.* which is always an Unit.
- Improper, as $\frac{BD+C}{D}$, *&c.* when the Numerator is the greater *Species.*
- Proper
  - Conjunct, as $\frac{C}{CA}$ and $\frac{B}{CA}$, *&c.*
  - Divided, as $\frac{C}{CA}$ of $\frac{B}{CA}$, *&c.*
  - when the Denominator is the greater.

2. Simple, and they

2. *Species* both Integral and Fracted, are conſidered as they are Simple or Compound.

Simple, are ſuch as have ſome one ſingle *Species* and no more,

Whether

Whole, or Broken.

Integral, as 6A. 8B. 9C. *&c.* Or,

Fracted, as $\frac{1}{2}$A. $\frac{2}{3}$B. $\frac{3}{4}$C. *&c.* Theſe, as was noted in Coſſicks before, may be repreſented as Compound, by placing the *Species* to the Numerator, and leaving the Denominator ſolitary, for $\frac{3}{4}$A is all one with $\frac{3A}{4}$.

Compound, and they

Compound, conſiſts of ſeveral *Species.* And theſe are alſo

Whole, as Binomials. Reſiduals. Polynomials.

Integral, conjoined with the Signs + or —; or both, As

Binomials, A+E . B+C . B+D. *&c.*

Reſiduals, A—E . B—C . B—D. *&c.*

Polynomials, A+E—B . A—E+B. *&c.* Or,

Broken, as Dual.

Fracted, compound in *Species*, or Signs, or both; as

Dual, compound in *Species* $\frac{A}{B}$. $\frac{2B}{3D}$. *&c.*

Plural.

Plural, compound in Signs $\frac{A+B+E}{D+C}$. $\frac{A-B-E}{D-C}$. *&c.*

Mixt.

Mixt, compound in both $\frac{A+B-E}{D-C}$. $\frac{A-B+E}{D+C}$. *&c.*

3. Plain or

3. *Species*, both Simple and Compound, are conſidered as they are Plain or Figurate.

Plain, as A. B. C. or any other *Species* Integral or Fracted, below the Power of any Number.

Figurate.

Figurate, as Aq. Bq. or Ac. Bc. or any other Figural *Species.*

Theſe are Rational or

Theſe are alſo divided into Rational or Irrational.

Rational, are ſuch as denote ſome figural rooted Number, or Quantity of figural rooted Numbers: The former reſembling the figural rooted Numbers handled in *Book* 2. *Part* 2. The latter Coſſicks in the 4*th* Part of this third Book: Thoſe without Numbers annexed; theſe with Numbers prefixed to their left Hand.

As Aq. Ac. Aqq. *&c.* Bq. Bc. Bqq. *&c.* Figural
and } *Species.*
2Aq. 4Ac. 5Aqq. *&c.* 3Bq. 4Bc. 6Bqq. *&c.* Coſſical

Irrational.

Irrational *Species*, as Surds, treated of before, have no Roots to be expreſſed by Abſolute Numbers.

As √q5 . √c7 . *&c.* Surd
and } *Species.*
√qB . √cBA. *&c.* Irrational

Both admit like Diviſions as Coſſicks and Surds reſpectively.

Both Rational and Irrational figurate *Species* are capable of like Diviſions with Coſſicks and Surds into Whole and Broken, and either ſort into Simple and Compound. Compound again into Binomial, Reſidual and Polynomial, Homogeneal and Heterogeneal, Symmetral and Aſymmetral; and their Fractions into Single, Dual and Plural: And all are ſometime intermixt, and aſſume Society one with another, both Plain and Figural; of all which Examples will but take up room.

Notes of Figurate Species.

Touching Figurate *Species*, let be further noted:

1. Species for all Powers compoſed of the two Prime.

Examples.

Firſt, That the *Species* for all the Powers, are compoſed of the *Species* for the two Prime Powers, *viz.* the Square and Cube. The *Species* for the Square or Quadrat being q, and for the Cube c: Of theſe are all the others compounded according to the Addition of their Indices. So ſhall the 4*th* Power be noted by qq. the 5*th* by qc. the 6*th* by cc. *&c.* The proper *Species* for Root is l, ſignifying *Latus*, latin for *a Side*; yet the Character √ is uſed as before for Root, and √: for Root Univerſal, and other Characters for Surds occaſionally.

Indices.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	*&c.*
Species.	l.	q.	c.	qq.	qc.	cc.	qqc.	qcc.	ccc.	qqcc.	qccc.	cccc.	*&c.*
Powers in Numbers.	2.	4.	8.	16.	32.	64.	128.	256.	512.	1024.	2048.	4096.	*&c.*
Coſſical Characters.	ȥ.	ʒ.	φ.	ʒʒ.	ſs.	ʒφ.	Bſs.	ʒʒʒ.	φφ.	ʒſs.	Cſs.	ʒʒφ.	*&c.*
Example in the Root A.	A.	Aq.	Ac.	Aqq.	Aqc.	Acc.	Aqqc.	Aqcc.	Accc.	Aqqcc.	Aqccc.	Acccc.	*&c.*

For A multiplied into A, or A×A, or AA, is Aq. So A×A×A, or AAA, or AqA, is Ac; and the like is to be underſtood of others.

2. Magnitudes Parodical.

Secondly, All Magnitudes under the Power propoſed, are called *Parodical to the Power*; as A. Aq. Ac. Aqq. are Parodical to Aqc.

3. Coefficient, what.

Thirdly, If the Parodical Degree have a known Magnitude joined therewith, it is called the *Coefficient*, as in Aq—AB, where B is the Coefficient, and A joined to it, is the Parodical Degree under Aq.

Signs uſed as before.

The Signs + and —, with or without Aſterisk, are uſed in all things, as before in *Coſſicks* and *Surds*.

Examples of ſome Species, and what underſtood thereby.

By theſe previous Directions it will not be difficult to underſtand both the Name and Nature of any given *Species*. And ſuppoſing ſome Abſolute Number for the Prime *Species* in any Operation, the Value of the reſt will be ſeen as in a *Speculum*. For if A and E ſignify two Numbers, and you appoint A the Greater, and E the Leſſer; as let A be 3, and E 2, then ſhall

Species.			*Value.*
Z. or A+E,	ſignify the Sum which is	(by Addition)	3+2= 5
X. or A—E,	Difference	(by Subſtraction)	3−2= 1
Æ. AE. or A×E	Product.	(by Multiplication)	3×2= 6
$\frac{R}{S}$.	Ratio, or Subratio	(by Diviſion)	2) 3 (= $\frac{3}{2}$
Aq.	Greater Square		3×3= 9
Eq.	Leſſer Square		2×2= 4
Ac.	Greater Cube		3×3×3=27
Ec.	Leſſer Cube		2×2×2= 8
Z..	Sum of the Squares		9+4=13
X..	Difference of the Squares		9−4= 5
˙Z.	Sum of the Cubes		27+8=35
˙X.	Difference of the Cubes		27−8=19
AqE.	Greater Square multiplied by the leſſer Number		9×2=18
EqA.	Leſſer Square multiplied by the greater Number		4×3=12
AqEq.	Greater Square multiplied by the Leſſer		9×4=36
AcEc.	Greater Cube multiplied by the Leſſer		27×8=216
$\frac{Aq}{Eq}$.	Greater Square divided by the Leſſer		4) 9 (=$2\frac{1}{4}$
$\frac{Ac}{Ec}$.	Greater Cube divided by the Leſſer		8) 27 (=$3\frac{3}{8}$

Species.		*Value.*
$ZA+Aq$.	Sum of the two Numbers multiplied by the Greater, and added to the greater Square	$5\times3+9=24$
$ZA-Aq$.	Sum of the two Numbers multiplied by the Greater, and made leſs by the Square of the Greater	$5\times3-9=6$
$ZA+Eq$.	Sum of the two Numbers multiplied by the Greater, and added to the leſſer Square	$5\times3+4=19$
$ZA-Eq$.	Sum of the two Numbers multiplied by the Greater, and made leſs by the Square of the Leſſer	$5\times3-4=11$
$ZE+Aq$.	Sum of the two Numbers multiplied by the Leſſer, and added to the greater Square	$5\times2+9=19$
$ZE-Aq$.	Sum of the two Numbers multiplied by the Leſſer, and made leſs by the greater Square	$5\times2-9=1$
$ZE+Eq$.	Sum of the two Numbers multiplied by the Leſſer, and added to the leſſer Square	$5\times2+4=14$
$ZE-Eq$.	Sum of the two Numbers multiplied by the Leſſer, and made leſs by the leſſer Square	$5\times2-4=6$
$\frac{SqAq}{Rq}$.	Square of the leſſer Proportional multiplied by the Square of the greater Number, and the Product divided by the Square of the greater Proportional	$\frac{4\times9}{9}=\frac{36}{9}=4$
$\frac{RqEq}{Sq}$.	Square of the greater Proportional multiplied by the Square of the leſſer Number, and the Product divided by the Square of the leſſer Proportional	$\frac{9\times4}{4}=\frac{36}{4}=9$
$Zq+Eq-2ZE$ or $Zq-2ZE+Eq$	Sum of the 2 Numbers multiplied by the Leſſer, and the Product doubled, is to be ſubſtracted from the Sum of the 2 Numbers ſquared and added to the leſſer Square	$25+4-20$ or $25-20+4$ $=9$
$Zq\overset{*}{-}2ZE+Eq$.	Double Sum of the 2 Numbers multiplied by the Leſſer, and added to the leſſer Square, ſubſtracted from the Sum of the 2 Numbers ſquared	$25\overset{*}{-}20+4=1$
$WZ.-Aq$.	Square of the greater Number ſubſtracted from the Sum of the Squares of the 2 Numbers, and the ſquare Root of the Remain is to be taken	$W13-9=W4-2$
$\frac{WZ.Rq-RqAq}{Aq}$.	Sum of the Squares multiplied by the Square of the greater Proportional, made leſs by the ſame Square, multiplied into the Square of the greater Number. The Remain is to be divided by the Square of the greater Number, & the ſquare Root of the Quot. to be taken.	$\frac{W13\times9-9\times9}{9}=\frac{36}{9}(W4=2$
$\frac{Z}{2}+\frac{WZq-4Æ}{4}$ or $\frac{Z}{2}+\frac{WZq-4P}{4}$	Half the Sum of the 2 Numbers added to the ſquare Root of the Sum ſquared, made leſs by 4 times, the Rectangle or Product divided by 4 (for P is ſometime the Species for Product as well as Periphery).	$\frac{5}{2}+\frac{W25-24}{4}$ that is $2\frac{1}{2}+W\frac{1}{4}=3$

Species.		*Value.*
$\surd{:}\frac{Z}{2}-\frac{\mathrm{w}Zq-4Pq}{4}$	The Univerſal ſquare Root of half the Sum of the Squares, made leſs by the ſquare Root of the Sum of the Squares ſquared and leſſened by 4 Rectangles ſquared and divided by 4.	$\surd{:}\frac{13}{2}-\frac{\mathrm{w}169-144}{4}$ or $\surd{:}\frac{13}{2}-\mathrm{w}\frac{25}{4}$ or $\surd{:}\frac{13}{2}-\frac{5}{2}=\frac{8}{2}=\mathrm{w}4=2$
$E=\frac{Zq-X}{2Z}$	The leſſer Number is equal to the Quotient of the Sum of both Numbers ſquared, made leſs by the Difference of the Squares, and the Remain divided by the doubled Sum —	$2=\frac{25-5}{10}$ or $2=\frac{20}{10}=2$
$A=\frac{X+Xq}{2X}$.	The greater Number is equal to the Difference of the Squares, and the Difference of the Numbers ſquared & divided by the ſame Difference doubled.	$3=\frac{5+1}{2}$ or $3=\frac{6}{2}=3$
$2Aq-2XA=Z-Xq$	The Square of the greater Number doubled, made leſs by the greater, multiplied by the Difference doubled, is equal to the Sum of the Squares made leſs by the Difference ſquared.	$9+9-6=13-1$ or $18-6=12$

Thus with wonderful Variety, and unimaginable Celerity, may *Species* be written and beheld, of both which this is but a Drop; yet may ſerve as an Introduction, as well to their Knowledg and true Poſition, as for a Baſis to the Reſolution of ſeveral Propoſitions deducible therefrom; which is eaſily diſcernable by the different *Species*, whereby one and the ſame Number in value may be expreſſed. For if the Sum of two Numbers be Z, and the two Numbers A+E, then ſhall A+E be equal to Z and Z to them. So in like manner ſhall other *Species* of different Forms be equal in value, of which more in the next Book among *Equations*.

The ſame Number in value expreſſed by different Species.

CHAP. II. *Addition of Integral and Rational Species.*

Addition of Integral and Rational Species.

THE *Addition of Integral and Rational Species*, whether Sole or Mixt, with Integers, Simple or Compound, may be compriſed under the four Caſes following.

1. Homogeneal, and of like Signs.

Caſe 1. If the *Species* be Homogeneal, and of like Signs, then add the Number of the *Species* as if they were Integers, and annex to the Total the ſame Signs.

Examples.

As A added to A, ſhall be 2A; and —A added to —A, ſhall be —2A. The like is to be obſerved in the reſt of the Examples following.

Addends	A A	5A A	—A—B -2A	A+B A+B	3C—E 2C—E
Total.	2A	6A	-3A—B	2A+2B	5C—2E

2. Homogeneal, and of unlike Signs.

Caſe 2. If the *Species* be Homogeneal and of unlike Signs, then ſubſtract the leſſer Number from the Greater, and to the Difference ſubſcribe the Sign of the *Species* wherein the Exceſs lieth.

Examples.

As to add 2A to —3A, the 2 taken from 3, leaves 1; which is here to be Negative, becauſe 3 which exceeded 2 was Negative. See further in the following Examples.

Addends.	A —A	5A -3A	3B—C -5B	2D—E -D+E	A+3C -2A—2C	
Total.	0A	2A	-2B—C	D	—A+C	Remaining.

3. Heterogeneal.

Caſe 3. If the *Species* be Heterogeneal, then conjoin the *Species* to be added with their proper Signs.

Examples.

As A added to B, ſhall be A+B; but -A added to B, ſhall be B—A. This is further clear in the Examples enſuing.

Addends.	A E	4B 5D	—A -4B	A+E B	A—B —E	Ac+A Aq
Total.	A+E	4B+5D	-A-4B	A+B+E	A-B-E	Ac+Aq+A

4. Mixt.

Caſe 4. If the *Species* be mixt Homogeneal and Heterogeneal, with Signs like and unlike; then as the Caſe is, ſo ſhall the Addition be.

Examples.

As A+B added to A—B, the *Species* B being of contrary Signs by the ſecond Caſe, are to be ſubſtracted one from the other, and ſo 0 will remain. But A added to A, makes the Total 2A by the firſt *Caſe*.

Alſo A+B added to A—C, the *Species* B and C being Heterogeneal, are by the third *Caſe* to be conjoined with their proper Signs, and A added to A by the firſt *Caſe*, makes the Total 2A+B—C.

More Examples of this kind follow.

Addends.	—B+C C—D	A+ D A—5D	3B—A 5C+B	A—B+C —A—B	A+B—C A—B+2C
Total.	2C-B-D	2A-4D	4B+5C-A	C—2B	2A+C

Proof of Addition of Int. and Rat. Species.

As in other Numbers, ſo in *Species*, *Subſtraction* will prove the Truth of *Addition*, as in the next Chapter appeareth.

Moreover, ſuppoſing the *Species* to be Abſolute Numbers, compare the Addition of the one with the other, and the Truth of the Work will appear thereby. As in the laſt Example where the Total is 2A+C, ſuppoſing A 10, and C 6, then ſhall the Total be 26, and ſo ſhall anſwer the Value of the *Species* to be added when the Quantities — are taken from them with +.

Suppoſing A=10. B=5. C=6. Then ſhall

$$\textit{Species}\left\{\begin{array}{l} A+B-C=10+5-6=9 \\ A-B+2C=10-5+12=17 \end{array}\right\}\text{Value.}$$

$$\begin{array}{ccc} 2A+C & 20+6 & 26 \end{array}$$

CHAP. III. *Subſtraction of Integral and Rational Species.*

Subſtraction of Int. and Rat. Species.

1. Homogeneal, and of like Signs.

I*Ntegral and Rational Species* are ſubſtracted under like Caſes with *Addition*.

Caſe 1. If the *Species* be Homogeneal and of like Signs, then abate the Quantities to be ſubſtracted from the other; and to the Difference prefix the Common Sign, except the Greater be the *Subtrahend*; then prefix the contrary Sign to the Difference.

Examples.

As to take 2A from 3A, there reſts 1A. But to take 3A from 2A, there will remain —1A, becauſe the greater Quantity is the Subtrahend. More of like ſort the following Examples ſhew.

From	A	3A	A	E	3A+5B	3C—E
Subſtract	A	A	4A	2E	4A+ B	2C—E
Remaineth	0A	2A	-3A	-E	4B— A	C

2. Homogeneal, and of unlike Signs.

Caſe 2. If the *Species* be Homogeneal and of unlike Signs, then add the Quantities together, and to the Total prefix the Sign of the *Species* from which Subſtraction is made. And if any odd *Species* have none to fellow him, annex him to the Remain with his own Sign if in the upper Number; but with the contrary Sign if in the *Subtrahend*.

Examples.

As to take 2A from —3A, the Remain ſhall be —5A. But —2A taken from 3A, ſhall leave remaining 5A. The like may be obſerved in the Examples following.

From

From	A	−3A	3B—C	−5B	2D—E
Subſtract	−A	5A	−5B	3B—C	−D+E
Remaineth	2A	−8A	8B—C	−8B+C	3D—2E

3. *Heterogeneal.* *Caſe* 3. If the *Species* be Heterogeneal, then conjoin the *Species* to be ſubſtracted with the contrary Signs.

Examples. As to take E from A, makes the Remain A—E; but —E taken from A, makes the Remain A+E. See further in the following Examples.

From	4B	—A	A+E	A	A	Aq
Subſtract	5D	—4B	B	B—C	B+C	A
Remaineth	4B—5D	—A+4B	A+E—B	A—B+C	A—B—C	Aq—A

4. *Mixt.* *Caſe* 4. If the *Species* be mixt Homogeneal and Heterogeneal, with Signs like and unlike; then as the Caſe is, ſo ſhall the Subſtraction be.

Examples. As to take C—D from —B+C, the *Species* C being Homogeneal, and of like Signs, the one taken from the other, by the firſt *Caſe*, leaves the remaining Quantity cleared of both. And —D ſubſtracted from —B being Heterogenal, makes the Remain —B+D by the third *Caſe*.

Alſo A+D ſubſtracted from A—5D, the *Species* A in both Homogeneal, and of like Signs, leaves o remaining of that Quantity by the firſt *Caſe*. But +D taken from —5D, leaves —6D by the ſecond *Caſe*. The like is to be obſerved in the Examples enſuing.

From	2A—4D	4B+5C—A	C—2B	2A+C	3Aq—3BA+CD
Subſtract	A—5D	5C+B	−A−B	A—B+2C	CD—Dq—Aq
Remaineth	A+D	3B—A	A−B+C	A+B—C	4Aq—3BA+Dq

Proof of Subſtraction of Int. and Rat. Species. The Quantities from which Subſtraction is made in four of theſe laſt Examples, being the Total of the Additions in the fourth *Caſe* of the foregoing Chapter: And the Remains here being one of the Addends there, and the Subtrahends here the other; ſufficiently ſhew the Proof of Addition by Subſtraction, and Subſtraction by Addition, without farther Example.

Alſo ſuppoſing the *Species* to be Abſolute Numbers, compare the Subſtraction of the one with the other; and by their exact Agreement in Value of the Remains, will the Truth of the Work be made manifeſt.

As in the laſt Example ſave one, where the Remain is A+B—C, ſuppoſing A10, B5, and C6, then ſhall the Remain be 9, that is 15 lacking 6, and ſo accordingly will remain, when the Value of the *Species* to be ſubſtracted is taken from the other.

Suppoſing A = 10. B = 5. C = 6. Then ſhall
2A + C = 20 + 6 = 26.
A−B+2C = 10—5+12 = 17
A+B—C 10+5—6 9

CHAP. IV. *Multiplication of Integral and Rational Species.*

Multiplication of Int. and Rat. Species. FOUR *Caſes* comprehend all needful to the *Multiplication of Integral and Rational Species*, whether Homogeneal or Heterogeneal, Simple or Compound, with or without Integers; in all which, as before in *Coſsicks* and *Surds*, like Signs produce +, and unlike —.

1. *Simple and Homogeneal.* *Caſe* 1. If the *Species* be Simple and Homogeneal, then to the right Hand of one of the given Quantities adjoin q, which ſignifieth a Quadrate or Square; becauſe any Number multiplied by himſelf, produceth the Square thereof.

As to multiply A by A. B by B, *&c.* they are ſet thus. **Examples.**

Multiplicands.	A	B	C	*&c.*	Roots.
Multipliers.	A	B	C		
Products.	Aq	Bq	Cq		Squares.

Caſe 2. If the *Species* be Simple and Heterogeneal; then ſet one of the given *Species* beſides the other, in a Right Line, from the Right Hand to the Left. *2. Simple and Heterogeneal.*

As to multiply A by B, the Product ſhall be AB or BA; and ſometimes ſet with the Sign of Multiplication between them thus, A×B or B×A. Other Examples of this ſort follow. **Examples.**

Multiplicands.	B	A	D	—B	*&c.*
Multipliers.	C	E	A	A	
Products.	BC	AE	AD	—AB	

Caſe 3. If the *Species* have any Numbers joined with them, or annexed to them by the Signs + or —: then multiply the Numbers as Integers, and the *Species* as *Species*. *3. Species with Numbers.*

Examples.

Multiplicands.	3E	4A	2B+1
Multipliers.	2E	3E	A
Products.	6Eq	12Æ	2BA+A

Caſe 4. If the *Species* or one of them be Compound, or one of the Factors be ſome multiplied *Species*; then multiply every Quantity in the Multiplicand, by every Quantity in the Multiplier, as before in Compound Coſſicks, Integers as Integers, and Species as Species. And place the Multiplees orderly from the Left Hand forwards to the Right, whether the Signs be more or leſs. And when the *Species* are multiplied by themſelves, or any other Power by the Root, or by ſome higher Power, whereby figural Numbers ariſe; let the Character of the Magnitude or Power ariſing by the Addition of their Indices, be added to the Product. *4. Compound Species.*

Examples.

Multiplicands.	AE	A+E	A—E	A+E—I
Multipliers.	AE	B	B	Z
Products.	AqEq	BA+BE	BA—BE	ZA+ZE—ZI

Multiplicands.	A+BE	A—B	A+E	A+E
Multipliers.	B	BA	A+E	A—E
Products.	BA+BqE	BAq—BqA	Aq+AE	Aq+AE
			AE+Eq	—AE—Eq
			Aq+2AE+Eq	Aq— Eq

Multiplicands.	3A—2E	5A+CD
Multipliers.	4B—C	3BA—2CD
	12BA—8BE	15BAq+3BACD
	—3CA+2CE	—10DCA—2CqDq
Products.	12BA—8BE—3CA+2CE	15BAq+3BACD—10DCA—2CqDq

Species, as other Numbers, prove the Truth of their Multiplications by Diviſion, as will appear in the next Chapter. And by ſuppoſing the *Species* to be Abſolute Numbers, the Product of their Multiplication will agree with the Product of their *Species* multiplied, if the Work be right. *Proof of Multiplication of Int. & Rat. Species.*

As in the laſt Example, if A be 3, B 4, C 5, D 6, then ſhall 5A be 15, BA 12, and 3BA 36, CD 30, and 2CD 60. So will the Multiplicand being 5A+CD be 15+30 or 45. And the Multiplier being 3BA—2CD, will be 36—60, that is —24; by which 45 multiplied, makes the Product —1080; and ſo is the Value of the other Product at the ſuppoſed Rate aforeſaid, when the *Species* with + are taken from thoſe with —.

Suppoſing A=3. B=4. C=5. D=6. Then ſhall

$$\begin{array}{ll}
5A+CD = & 5\times3+5\times6=15+30=45\\
3BA-2CD = & 3\times4\times3-2\times5\times6=36-60=-24\\
\hline
15BAq+3BACD = & 15\times4\times9+3\times4\times3\times5\times6=540+1080\\
\quad -10DCA-2CqDq = & -10\times6\times5\times3-2\times25\times36=-900-1800\\
\hline
15BAq+3BACD-10DCA-2CqDq & 540+1080-900-1800\\
\hline
\end{array}$$

$$\begin{array}{lrlrr}
15BAq = & 540 & -10DCA = & 900 & 45\\
3BACD = & 1080 & -2CqDq = & 1800 & -24\\
\hline
 & 1620 & & & 180\\
 & & & -2700 = & 90\\
 & & & & -1080
\end{array}$$

CHAP. V. *Diviſion of Integral and Rational Species.*

Diviſion of Int. & Rat. Species. TO divide *Integral and Rational Species*, conſider the ſix *Caſes* following; in all which, as in *Coſſicks*, *Surds*, and other Contract Numbers, like Signs ſhall give + and unlike —. And where Integers are adjoined, divide them as Integers are divided.

1. *Simple.* *Caſe* 1. If any Simple *Species* be to divide the ſame Simple *Species*, then ſet in the Quotient an Unit; but if Integers be annexed to the given *Species*, divide Integers by Integers, as if there were no *Species*.

Examples. As to divide A by A, or B by B, the Quotients ſhall be 1. But 6A divided by 2A, ſhall give 3 in the Quotient. And 8B by —2B, ſhall give —4B.

Diviſor $A\,\Big)\overset{\textit{Dividend}}{A}\Big(\,1$ *Quotient*, or thus $\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{A}{A}\Big(\,1A$ *Quotient.*

Diviſor $B\,\Big)\overset{\textit{Dividend}}{B}\Big(\,1$ *Quotient*, or thus $\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{B}{B}\Big(\,1B$ *Quotient.*

$\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{6A}{2A}\Big(\,3A$ *Quotient.* $\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{8B}{-2B}\Big(\,-4B$ *Quotient.*

2. *Simple Species of the Diviſor figurate in the Dividend.* *Caſe* 2. If the Simple *Species* of the Diviſor be figurate in the Dividend; then place the Diviſor in the Quotient with ſuch a Note of Abatement in the Power of the *Species*, as the Index of the Diviſor being ſubſtracted from the Index of the Dividend will leave.

Examples. As if Aq be divided by A, the Quotient ſhall be A only; becauſe A being the Root, whoſe Index 1 taken from 2, the Index of Aq, the Square leaves 1 the Index of the Root.

$\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{Aq}{A}\Big(\,A$ *Quotient.* $\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{4Ac}{2A}\Big(\,2Aq$ *Quotient.*

Indices $2-1=1$ *Indices* $3-1=2$

$\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{AqEq}{AE}\Big(\,AE$ *Quotient.* $\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\dfrac{Aqqc}{Aqq}\Big(\,Ac$ *Quotient.*

Indices $2-1=1$ *Indices* $7-4=3$

3. *Simple Species of the Diviſor ſpecified in the Dividend.* *Caſe* 3. If the Simple *Species* of the Diviſor be ſpecified in the Dividend, then after the *Species* of like Form are cancelled or ſubſtracted, the one from the other, let the Reſidue be ſet in the Quotient.

Examples. As if BA be divided by A, the *Species* A being in both, only B is placed in the Quotient. So 10AE divided by 5A, makes the Quotient 2E. Other Examples of like ſort are ſet with them as followeth.

$\begin{matrix}\textit{Dividends}\\ \textit{Diviſors}\end{matrix}\dfrac{BA}{A}\Big(\,B \qquad \dfrac{10AE}{5A}\Big(\,2E \qquad \dfrac{BA+A}{A}\Big(\,B+1$ *Quotients.*

$\begin{matrix}\textit{Dividends}\\ \textit{Diviſors}\end{matrix}\dfrac{BA-BE}{B}\Big(\,A-E \qquad \dfrac{BA+BE}{B}\Big(\,A+E$ *Quotients.*

Caſe 4. If the *Species* of both Dividend and Diviſor be Compound, then as before in the third *Caſe*, ſubſtract like *Species*; and for every two of the remaining *Species* in the Dividend, from which Subſtraction is made, let one be ſet in the Quotient. 4. *Compound Species.*

As to divide BA—BE by A—E, ſubſtracting A—E from the Dividend, there is left only B—B, for which B only is put in the Quotient. See further in the following Examples. Examples.

$$\begin{array}{l}\textit{Dividends}\\ \textit{Diviſors}\end{array}\ \frac{BA-BE}{A-E}\Big(\,B \qquad \frac{BA+CA}{B+C}\Big(\,A\ \ \textit{Quotients.}$$

$$\begin{array}{l}\textit{Dividend}\\ \textit{Diviſor}\end{array}\ \frac{BA-BE-CA+CE}{B-C}\Big(\,A-E \qquad \textit{Quotient.}$$

Caſe 5. If the *Species* be Heterogeneal, then place the Diviſor under the Dividend in form of a Fraction. 5. *Heterogeneal.*

As to divide B by A, being ſet thus $\frac{B}{A}$ they are left as not otherwiſe dividable. The Examples following are of like ſort. Examples.

$$\begin{array}{l}\textit{Dividends}\\ \textit{Diviſors}\end{array}\ \frac{-BC}{E}\Big(-\frac{BC}{E} \qquad \frac{-BC}{-A}\Big(\frac{BC}{A} \qquad \frac{B+C}{A-E}\Big(\frac{B+C}{A-E}\ \ \textit{Quotients.}$$

Caſe 6. If the *Species* given to be divided be mixt, then accordingly let their Diviſion be by mixture of the Caſes under which they fall. 6. *Mixt.*

As to divide BAc by Aq, by the ſecond *Caſe*, Aq dividing Ac, ſhall give A in the Quotient; and becauſe B hath nothing ſubſtracted from him, he ſhall be ſet in the Quotient by the third *Caſe*. Examples.

So BA—BE divided by A, maketh the Quotient $B-\frac{BE}{A}$; that is, by the third *Caſe* is B left for the Quotient, and by the fifth *Caſe* $\frac{BE}{A}$ becauſe they are Heterogeneals.

$$\begin{array}{l}\textit{Dividends}\\ \textit{Diviſors}\end{array}\ \frac{BAc}{Aq}\Big(\,BA \qquad \frac{BA-BE}{A}\Big(\,B-\frac{BE}{A}\ \ \textit{Quotients.}$$

The Quotient of every Diviſion in *Species* multiplied by the Diviſor, returning the Dividend; and the Product of every Multiplication in *Species* divided by one of the Factors, giving the other in the Quotient, is a ſufficient Teſtimony, that the proof of either is reciprocally by each other. *Proof of Diviſion of Int. and Rat. Species.*

Likewiſe by ſuppoſing the *Species* to be divided Abſolute Numbers, and dividing them as ſuch, the Quotient of this Diviſion will be the Value of the *Species* in the Quotient of their Diviſion, if the Work be right.

As in the laſt Example; Suppoſe A 6, B 2, E 4, then ſhall BA be 12, and BE be 8, and the Dividend 12—8 to be divided by 6; ſo ſhall the Quotient be $2-\frac{8}{6}$, agreeing with $B-\frac{BE}{A}$.

Suppoſing A=6. B=2. E=4. Then ſhall

$$\frac{BA-BE}{A}=\frac{2\times6-2\times4}{6}=\frac{12-8}{6}=2-\tfrac{8}{6}=B-\frac{BE}{A}.$$

CHAP. VI. Reduction of Fracted and Rational Species.

INtegral and Rational *Species* diſcuſſed, the next that ſhew themſelves are their Fractions. Theſe in the firſt Chapter of *Species*, were divided and ſubdivided into ſeveral ſorts, and ſo need no farther Deſcription here; wherefore I proceed to *Reduction*. *Reduction of Fract. and Rat. Species.*

Fracted and Rational *Species* are to be reduced either into their leaſt Terms, or into like Denominators when they will admit thereof.

The firſt ſort is called *Abbreviation*, as in Common Fractions; and if the Numbers be commenſurable in Single Fractions, or the *Species* in Dual, or both Numbers *To their leaſt Terms.*

bers and *Species* in Plural Fractions, or either of them: then dividing them by the Common Diviſor, they will be brought to their leaſt Terms.

Examples. As $\frac{6}{8}$ A will be reduced to $\frac{3}{4}$ A, by the Common Diviſor 2; and $\frac{6}{9}$ B, by the Common Diviſor 3, will be reduced to $\frac{2}{3}$ B.

And $\frac{BA}{BC}$ will be reduced to $\frac{A}{C}$ by the Common Diviſor B. And $\frac{BCD}{BDA}$ by the Common Diviſor BD will be reduced to $\frac{C}{A}$.

And $\frac{6BA}{8BC}+\frac{DE}{DC}$ will be reduced in Numbers to $\frac{3BA}{4BC}+\frac{DE}{DC}$ and in *Species* to $\frac{3A}{4C}+\frac{E}{C}$ by the Common Diviſors B and D.

Abbreviated.

Plain { Simple Fracted *Species* 2) $\frac{6}{8}$A ($\frac{3}{4}$A 3) $\frac{6}{9}$B ($\frac{2}{3}$B.

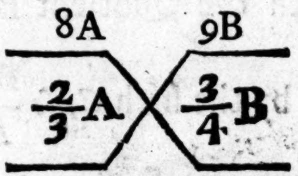
Compound { Dual Fracted *Species* B) $\frac{BA}{BC}$ ($\frac{A}{C}$ BD) $\frac{BCD}{BDA}$ ($\frac{C}{A}$

Plural Fracted Species 2 B + D) $\frac{6BA}{8BC}+\frac{DE}{DC}$ ($\frac{3A}{4C}+\frac{E}{C}$

To one Denominator. The ſecond ſort of Reduction of Fracted and Rational *Species*, to bring different Denominators into one comprehendeth,

1. *Simple.* *Firſt*, To reduce ſimple Fracted *Species*, in which proceed with the Numbers, as in Common Fractions, without altering the *Species*.

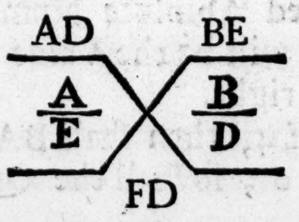
Example. And ſo $\frac{2}{3}$ A, and $\frac{3}{4}$ B, ſhall be reduced to $\frac{8A \quad 9B}{12}$

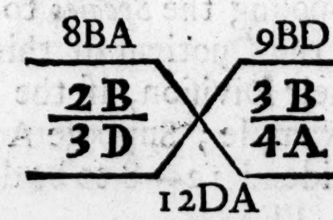


2. *Dual.* 2*ly*, To reduce Dual Fracted Species into like Denominators, multiply the Numbers as Common Fractions, and the *Species* as *Species*.

Examples. And ſo $\frac{A}{E}$ and $\frac{B}{D}$ ſhall be reduced to $\frac{AD \quad BE}{ED}$

And $\frac{2B}{3D}$ and $\frac{3B}{4A}$ ſhall be reduced to $\frac{8BA \quad 9BD}{12DA}$

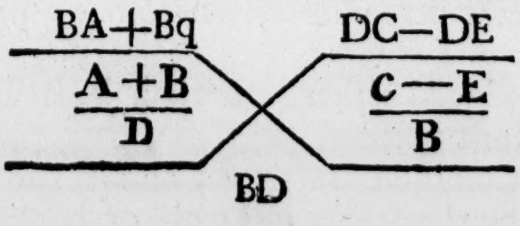


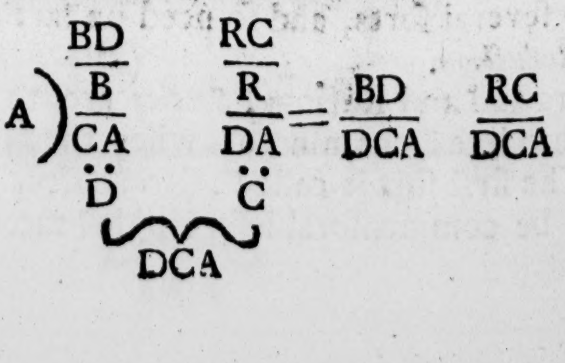


3. *Plural.* 3*ly*, To reduce Plural Fracted *Species* into one Denomination, is alike to the Reduction of Dual, *mutatis mutandis*. And if in either of theſe the Denominations given are Commenſurable, reduce them to their leaſt Terms.

Examples. As to reduce $\frac{A+B}{D}$ and $\frac{C-E}{B}$ into one Denomination, multiplying alternately D into C—E, and B into A+B, the new Numerators are gotten, and D into B is the common Denominator. So are the new Fractions $\frac{BA+Bq}{BD}$ and $\frac{DC-DE}{BD}$.

But if $\frac{B}{CA}$ and $\frac{R}{DA}$ be given to be reduced to one Denomination; then becauſe their Denominators are commenſurable by A, they are firſt reduced to their leaſt Terms C and D, and then multiplied alternately as before.





4*ly*,

4. Many proper Fractions.

4*ly*, To reduce many proper Fractions of *Species* into one Denomination, is like the Method uſed in vulgar Fractions, by multiplying the Denominators one into another for the common Denominator; and every fraction's Numerator into the other Denominators except his own.

Example.

And ſo $\frac{B}{D}$ and $\frac{C}{A}$ and $\frac{E}{H}$ reduced to one Denomination, are $\frac{BAH}{DAH}$ & $\frac{CDH}{DAH}$ & $\frac{EDA}{DAH}$

5. Fractions of Fractions.

5*ly*, To reduce Fractions of Fractions, is to multiply after the manner of *Species*, Numerator by Numerator, and Denominator by Denominator.

Examples.

As $\frac{B}{D}$ of $\frac{E}{A}$ reduced, ſhall be $\frac{BE}{DA}$ by multiplying B into E, and D into A.

And $\frac{A}{C}$ of $\frac{3Aq}{2B}$ reduced, ſhall be $\frac{3Ac}{2BC}$ by multiplying A into 3 Aq, and C into 2B.

6. Mixt into Improper.

6*ly*, To reduce Integral *Species* and Fracted, into improper Fractions, multiply the Integral *Species* by the Denominator of the Fraction, and to the Product add the Numerator.

Examples.

As $B\frac{C}{D}$ and $D\frac{C}{A}$ reduced into improper Fractions, ſhall be $\frac{BD+C}{D}$ and $\frac{DA+C}{A}$ by multiplying B into D in the one, and D into A in the other, and adjoining to the Products C, with the Sign of Addition $+$.

So $B-C\frac{R+D}{S}$ reduced, ſhall be $\frac{B3-CS+R+D}{S}$ the improper Fraction.

7. Improper into Mixt.

7*ly*, To reduce improper Fracted *Species* back into Integers, or an Integral and Fracted Species, divide the Numerator by the Denominator after the manner of *Species*.

Example.

As $\frac{BD+C}{D}$ divided by D, makes the Quotient B an Integral *Species*, and the remaining C is ſet over D as a Fraction thus, $B\frac{C}{D}$.

8. Species turned to a Denomination desired.

8*ly*, To reduce an Integral *Species* into ſome deſired Denomination, multiply the whole *Species* by the given Denominator: And any whole *Species* may be ſet as a Fraction, by placing 1 under the *Species*.

Examples.

As if B be deſired as a Fraction, whoſe Denominator ſhall be A, then is B to be multiplied into A, and ſet thus $\frac{BA}{A}$.

So B+C into D, ſhall be $\frac{BD+CD}{D}$.

Integer ſet as a Fraction.

And B, or B+C ſet as Fractions, ſhall be $\frac{B}{1}$ and $\frac{B+C}{1}$.

Some Fractions by Abbreviation brought to Integers.

Hence it appeareth, that as by this laſt-mentioned Reduction, whole *Species* may be ſet as Fractions: So by the firſt ſort of Reduction may ſome fracted *Species* be turned into Integral, being abbreviated into their leaſt Terms.

As $\frac{BA}{B}$ may be abbreviated into A the Integer.

And $\frac{4Aq}{2A}$ into 2A. And $\frac{3Aq}{6A}$ into $\frac{A}{2}$.

Proof of Reduction of Fract. and Rat. Species.

Reduction of Fracted and Rational Species, as well as other Reductions, may clearly be diſcerned to prove one part thereof, by the other part reciprocal thereto. And beſides by ſuppoſing the Fracted *Species* abſolute Numbers or common Fractions, every Part of Reduction may be proved with ſufficient Demonſtration.

As in the laſt Example, if A be ſuppoſed 3, then ſhall 6A be 18, and Aq ſhall be 9, and 3 Aq 27; which abbreviated or divided by 18, ſhall be $1\frac{1}{2}$ equal to $\frac{A}{2}$.

Suppoſing A = 3. Then ſhall

$$\frac{3Aq = 3\times 3\times 3 = 27}{6A = 6\times 3 = 18}\left(\tfrac{3}{2} = \frac{A}{2}.\right.$$

CHAP. VII. *Addition of Fracted and Rational Species.*

Addition of Fract. and Rat. Species.

THE ſeveral Caſes and Varieties in Addition of Common Fractions, *Book* 1. *Part* 2. *Chap.* 3. might here be run over again: But three Caſes contain ſufficient for the *Addition of Fracted and Rational Species*, all the reſt differing materially only in Examples.

1. Like Denominators without Numbers.

Caſe 1. If the Fractions to be added be without Numbers annexed, and of the ſame Denomination; then add the Numerators as *Species* are added, that is, by conjoining them with the Signs proper thereto, if the Numerators be unlike; or if alike, by adding the Number of them together, their Signs alſo being alike; but when unlike, take their Difference, and ſubſcribe the common Denominator.

Examples.

As to add $\frac{B}{D}$ to $\frac{A}{D}$ the Total ſhall be $\frac{B+A}{D}$ by joining B to A, becauſe they are Heterogeneal by the Sign of Addition.

But $\frac{B}{D}$ added to $\frac{B}{D}$ ſhall make the Sum $\frac{2B}{D}$; becauſe both Numerators are Homogeneal, they are added as Integral *Species*.

So Z the Integer added to the Fraction $\frac{E}{B}$, makes the Total $Z\frac{E}{B}$ or $\frac{ZB+E}{B}$.

And $\frac{3D}{B}$ added to $\frac{-D}{B}$, makes the Sum $\frac{2D}{B}$, becauſe the Signs are contrary; when $-D$ is taken from $+3D$, the Remain is $+2D$.

Other Examples.

Addends.	*Total.*	*Addends.*	*Total.*
$\frac{BA}{DE}$ and $\frac{CD}{DE}$ added, are	$\frac{BA+CD}{DE}$.	$\frac{A+E}{B}+\frac{D-C}{B}=$	$\frac{A+E+D-C}{B}$.

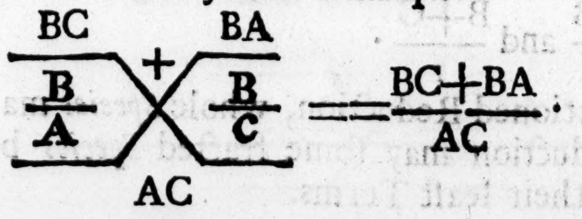
2. Unlike Denominators without Numbers.

Caſe 2. If the Fractions to be added be without Numbers, and of different Denominations, then firſt reduce them as fracted *Species* are reduced, and afterwards add their Numerators as before.

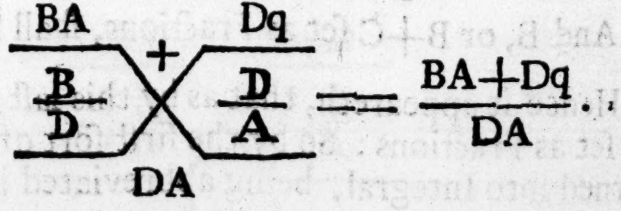
Examples.

As $\frac{B}{A}$ added to $\frac{B}{C}$ makes, firſt by Reduction $\frac{BC}{AC}$ and $\frac{BA}{AC}$, and then joined by the Sign of Addition $\frac{BC+BA}{AC}$. So $\frac{B}{D}$ added to $\frac{D}{A}$ make the Total $\frac{BA+Dq}{DA}$.

The former Example.



The latter Example.

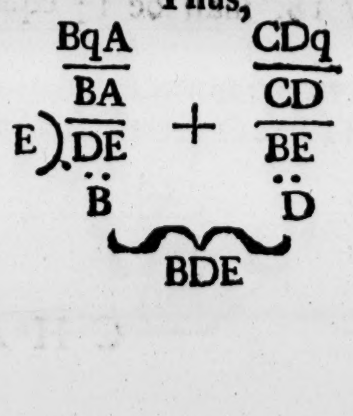


Likewiſe $\frac{B}{D}$ the proper Fraction added to $\frac{ZB+E}{B}$ the improper Fraction, makes the Total $\frac{ZBD+ED+Bq}{BD}$.

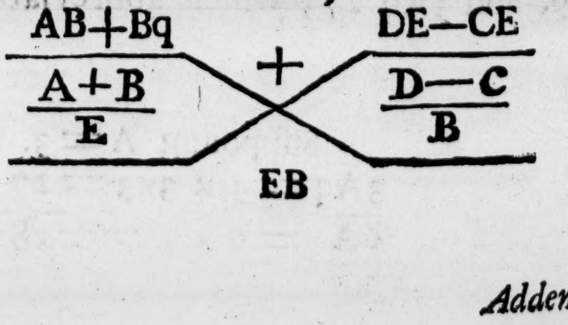
Other Examples.

Addends.	*Total.*	*Addends.*	*Total.*
$\frac{BA}{DE}+\frac{CD}{BE}=$	$\frac{BqA+CDq}{BDE}$.	$\frac{A+B}{E}+\frac{D-C}{B}=$	$\frac{AB+Bq+DE-CE}{EB}$

Thus,



Thus,



Addends. *Total.*

$$\frac{B+C}{Dq+A}+\frac{DE+1}{A-C}=\frac{BA+CA-BC-Cq+DcE+Dq+ADE+A}{DAq+Aq-DqC-AC}$$

Caſe 3. If the Fractions have Numbers annexed to them, then order the Numbers as common Fractions, and the *Species* as *Species*. **3. With Numbers.**

As to add $\frac{2}{5}A$ to $\frac{1}{5}A$, the Sum ſhall be $\frac{3}{5}A$.

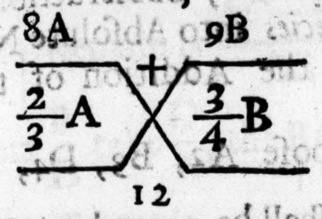
So 2B and $\frac{1}{2}A$ added, make the Total $2B+\frac{1}{2}A$, or $\frac{4B+A}{2}$. **Examples.**

Other Examples.

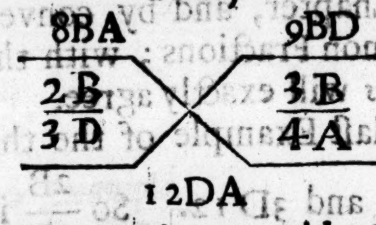
Addends. *Total.*

$$\frac{2}{3}A+\frac{3}{4}B=\frac{8A+9B}{12}. \qquad \frac{2B}{3D}+\frac{3B}{4A}=\frac{8BA+9BD}{12DA}$$

Thus,



Thus,



Here are fitly to be inſerted ſuch Propoſitions as require to add a Part or Parts of a given Number or Magnitude to the ſame Number or Magnitude, or to any other of his Parts. In both which the Deſire is thus obtained; Firſt by the Reduction of Fractions of Fractions get the Part or Parts to be added, and then by Addition add them as the Caſe requires.

Examples of the firſt Sort. **Part of a Number added thereto.**

As $\frac{1}{2}$ of $\frac{3A}{5}$ added to $\frac{3A}{5}$, maketh the Total $\frac{9A}{10}$.

And $\frac{2}{5}$ of $\frac{3A+D}{E}$ added to $\frac{3A+D}{E}$, maketh the Total $\frac{21A+7D}{5E}$.

For in the firſt of theſe $\frac{1}{2}$ of $\frac{3A}{5}$ is found by Reduction to be $\frac{3A}{10}$, which added to $\frac{3A}{5}$ as Common Fractions are added, make the Total as above.

And in the latter Example $\frac{2}{5}$ of $\frac{3A+D}{E}$ is found by Reduction to be $\frac{6A+2D}{5E}$, which by Addition to $\frac{3A+D}{E}$, makes the Total as before.

$$\begin{array}{c} 3A \\ \hline \frac{1}{2}\text{ of }\dfrac{3A}{5} \\ \hline 10 \end{array} \qquad \begin{array}{c} 9A \\ \hline 3A \quad 6A \\ \hline 5)\,\dfrac{3A}{10}+\dfrac{3A}{5} \\ \underbrace{\ddot{1}\qquad\ddot{2}}_{10} \end{array} \qquad \begin{array}{c} 6A+2D \\ \hline \frac{2}{5}\text{ of }\dfrac{3A+D}{E} \\ \hline 5E \end{array} \qquad \begin{array}{c} 21A+7D \\ \hline \qquad\quad 15A+5D \\ \dfrac{6A+2D}{5E}+\dfrac{3A+D}{E} \\ \hline 5E \end{array}$$

Examples of the ſecond Sort. **Part of a Number added to other Parts thereof.**

As $\frac{1}{3}$ of 5B added to $\frac{2}{5}$ of 5B, makes the Total $\frac{11B}{3}$.

And $\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{15A-12}{8}$ added, make the Total $\frac{25A-20}{16}$.

For in the firſt of theſe $\frac{1}{3}$ of 5B is $\frac{5B}{3}$, and $\frac{2}{5}$ of 5B is $\frac{2B}{1}$; both which added make $\frac{11B}{3}$ as before.

And in the latter Example $\frac{1}{2}$ of the *Data* is $\frac{15A-12}{16}$, and $\frac{1}{3}$ is $\frac{5A-4}{8}$, which added together make the Total as above.

$$\begin{array}{c} 5B \\ \hline \tfrac{1}{3}\text{ of }\frac{5B}{1} \\ \hline 3 \\ 2B \\ \hline \underset{\because}{1} \\ \tfrac{2}{5}\text{ of }\frac{5B}{1} \\ \hline \ddot{1} \\ \hline 1 \end{array} \qquad \begin{array}{c} 11B \\ \hline 6B \\ \hline \frac{5B}{3}+\frac{2B}{1} \\ \hline 3 \end{array} \qquad \begin{array}{c} 15A-12 \\ \hline \tfrac{1}{2}\text{ of }\frac{15A-12}{8} \\ \hline 16 \\ 5A-4 \\ \hline \ddot{5}A-\ddot{4} \\ \tfrac{1}{3}\text{ of }\frac{15A-12}{8} \\ \hline 8 \end{array} \qquad \begin{array}{c} 25A-20 \\ \hline 10A-8 \\ \hline 8)\underbrace{\frac{15A-12}{\underset{1}{16}}+\frac{5A-4}{\underset{2}{8}}}_{16} \end{array}$$

Proof of Addition of Fract. & Rat. Species.

Addition of Fracted and Rational Species is to be proved both by Subſtraction, as in the next Chapter, and by converting the Fracted *Species* into Abſolute Numbers or Common Fractions; with the Addition whereof the Addition of theſe Fracted *Species* will exactly agree.

As in the laſt Example of the third *Caſe* above, ſuppoſe A2, B3, D4, then ſhall 2B be 6, and 3D 12. So $\frac{2B}{3D}$ is $\frac{6}{12}$ or $\frac{1}{2}$. And 3B ſhall be 9, and 4A 8. So $\frac{3B}{4A}$ is $\frac{9}{8}$. And $\frac{1}{2}$ and $\frac{9}{8}$ added together, make $\frac{13}{8}$ or $1\frac{5}{8}$ equal to the Total $\frac{8BA+9BD}{12DA}$

Suppoſing A=2. B=3. D=4. Then ſhall

$$\frac{2B=6}{3D=12}+\frac{3B=9}{4A=8} \qquad \begin{array}{l} 8BA=8\times3\times2=48 \\ 9BD=9\times3\times4=108 \\ \hline 12DA=12\times4\times2=96 \end{array}\!\!\left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \frac{=156}{=96}\Big(1\tfrac{60}{96}\text{ or }1\tfrac{5}{8}$$

$$2)\underbrace{\overset{4}{\frac{1}{\underset{4}{2}}}+\overset{9}{\frac{9}{\underset{1}{8}}}}_{8}=\frac{13}{8}\Big(1\tfrac{5}{8}$$

CHAP. VIII. *Subſtraction of Fracted and Rational Species.*

Subſtraction of Fract. and Rat. Species.

As in *Addition*, ſo here, ſufficient for the *Subſtraction of Fracted and Rational Species*, is contained under the three *Caſes* following. The other *Caſes* mentioned in Subſtracting of Common Fractions, *Book* 1. *Part* 2. *Chap.* 4. being more occaſional than eſſentially neceſſary.

1. Like Denominators without Numbers.

Caſe 1. If the Fractions to be ſubſtracted be without Numbers annexed, and of like Denomination, then after the manner of *Species* ſubſtract the Numerator of the Subtrahend from the other Numerator ſet to the left Hand of the Subtrahend; that is, by connexing them with the Sign of Subſtraction, if the Numerators be unlike and their Signs alike: But if the Numerators be alike, by taking the one from the other, or adding with contrary Signs both Numerators as before in Coſſicks, and under ſuch Sum or Difference ſubſcribe the Common Denominator.

Examples.

As to take $\frac{D}{C}$ from $\frac{B}{C}$, the Remain is $\frac{B-D}{C}$ conjoining B to D, becauſe they are Heterogeneal by the Sign of Subſtraction.

But $\frac{B}{D}$ taken from $\frac{3B}{D}$, ſhall leave remaining $\frac{2B}{D}$, becauſe both Numerators are Homogeneal, the one is ſubſtracted from the other as Integral *Species*.

So $\frac{E}{B}$ the proper Fraction taken from $\frac{ZB+E}{B}$ the improper Fraction, the Remain will be Z.

And $\frac{-D}{B}$ taken from $\frac{2D}{B}$, leaves remaining $\frac{3D}{B}$; for the Signs being contrary, −D ſhall be added to 2D, which makes 3D as before in Coſſicks.

Other Examples.

Subtrahend. *Remain.*

$\frac{AB}{D}$ ſubſtracted from $\frac{BC}{D}$, ſhall leave $\frac{BC-AB}{D}$.

Subtrahend. *Remain.*

$\frac{A-E}{B}$ ſubſtracted from $\frac{D-C}{B}$, ſhall leave $\frac{D-C-A+E}{B}$.

Caſe 2. If the Fractions to be ſubſtracted be without Numbers, and of different Denominations; then firſt reduce them as Fracted *Species* are reduced, and then ſubſtract the Numerator of the Subtrahend from the other as before.

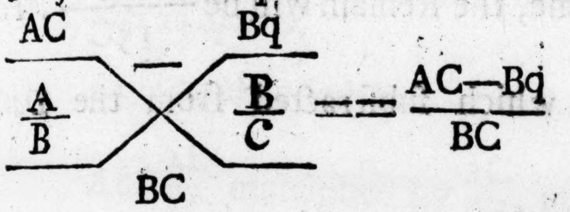
2. *Unlike Denominators without Numbers.*

Examples.

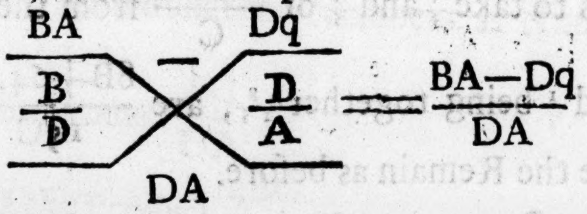
As from $\frac{A}{B}$ ſubſtract $\frac{B}{C}$, the Rednction makes them $\frac{AC}{BC}$ and $\frac{Bq}{BC}$; then by Subſtraction the Remain is $\frac{AC-Bq}{BC}$.

And from $\frac{B}{D}$ take $\frac{D}{A}$, the Remain ſhall be $\frac{BA-Dq}{DA}$.

The former Example.



The latter Example.



Other Examples.

Subtrahend. *Remain.* *Subtrahend.* *Remain.*

$\frac{BA}{DE} - \frac{CD}{BE} = \frac{BqA-CDq}{BDE}$. $\frac{BqA+CDq}{BDE} - \frac{CD}{BE} = \frac{BA}{DE}$.

Thus, Thus,

$$E\Big)\begin{array}{c} BqA \\ \hline BA \\ \hline DE \\ \ddot{B} \end{array} - \begin{array}{c} CDq \\ \hline CD \\ \hline BE \\ \ddot{D} \end{array} \qquad BE\Big)\frac{BqA+CDq}{BDE} - \begin{array}{c} CDq \\ \hline CD \\ \hline BE \\ \ddot{D} \end{array} = \frac{BqA}{BDE} = \frac{BA}{DE}$$

$$\underbrace{B \qquad D}_{BDE} \qquad\qquad \underbrace{\ddot{1} \qquad D}_{BDE}$$

☞ In which laſt Example is to be noted, that in the Reduction the Common Diviſor BE takes away all the Denominator of the Subtrahend; and ſo nothing being left to be brought to the Denominator of the other Number, an Unit is there placed and not a Cipher, leſt it ſhould be taken for a *Species*. After the Reduction the Numbers ſtand thus.

$\frac{BqA+CDq-CDq}{BDE}$; where becauſe CDq is both Affirmative and Negative, in the Numerators they are both to be cancelled, and the Remain $\frac{BqA}{BDE}$ abbreviated to $\frac{BA}{DE}$, becauſe Bq in the Numerator is figurate; which divided by B, leaves B only to A for the Numerator.

Caſe 3. If the Fractions have Numbers annexed to them, then order the Numbers as Common Fractions, and the *Species* as *Species*.

3. *With Numbers.*

Examples.

As from $\frac{2}{7}$ A take $\frac{1}{7}$A, and the Remain ſhall be $\frac{1}{7}$A.

But to take $\frac{3}{4}$A from $\frac{13}{12}$A, the Fractions muſt firſt be reduced to one Denomination; and then from $\frac{13}{12}$A let $\frac{9}{12}$A be taken, and the Remain will be $\frac{4}{12}$A or $\frac{1}{3}$ A.

So 2B taken from $\frac{4B+A}{2}$ leaves $\frac{A}{2}$ or $\frac{1}{2}$A.

Other Examples.

Subtrahend.	*Remain.*		*Subtrahend.*	*Remain.*
$\frac{2}{3}A - \frac{3}{4}B$	$= \frac{8A - 9B}{12}$.		$\frac{8A + 9B}{12} - \frac{3}{4}B$	$= \frac{2}{3}A$.

Thus,

$$\begin{array}{ccc} \dfrac{8A}{\frac{2}{3}A} & - & \dfrac{9B}{\frac{3}{4}B} \\ & 12 & \end{array}$$

Thus,

$$4)\frac{8A + 9B}{\underset{1}{12}} - \frac{9B}{\underset{3}{\frac{3}{4}B}} = \frac{8A}{12} = \frac{2}{3}A.$$

$$\underbrace{1 \qquad 3}_{12}$$

In this place may be fitly inſerted ſuch Propoſitions as require to take a Part or Parts of a Number or Magnitude given, from the ſame Number or Magnitude, or from any other of his Parts. In both which the Queſited is gotten thus: Firſt by Reduction of Fractions of Fractions, get the Part or Parts to be ſubſtracted, and then by Subſtraction, as the Caſe may require, you will have the Remain.

Part of a Number taken therefrom.

Example of the firſt Sort.

As to take $\frac{1}{3}$ and $\frac{1}{5}$ of $\frac{B + 8A}{C}$ from the ſame, the Remain will be $\frac{7B + 56A}{15C}$. For $\frac{1}{3}$ and $\frac{1}{5}$ being together $\frac{8}{15}$, are $\frac{8B + 64A}{15C}$; which ſubſtracted from the *Data*, leave the Remain as before.

$$\frac{8}{\dfrac{\overset{5}{\frac{1}{3}} + \overset{3}{\frac{1}{5}}}{15}} \qquad \frac{8B + 64A}{\dfrac{\frac{8}{15} \text{ of } \frac{B + 8A}{C}}{15C}} \qquad \frac{7B - 56A}{\dfrac{\overset{15B + 120A}{\frac{B + 8A}{C} - \frac{8B + 64A}{15C}}}{15C}}$$

Parts of a Number taken from other Parts thereof.

Example of the ſecond Sort.

As to take $\frac{3}{4}$ of $3B + C$ from $\frac{4}{5}$ thereof, the Remain is $\frac{3B + C}{20}$.

For $\frac{3}{4}$ of $3B + C$ is $\frac{9B + 3C}{4}$, and $\frac{4}{5}$ is $\frac{12B + 4C}{5}$, and the former taken from this latter, leaves the Remain as aforeſaid.

$$\frac{9B + 3C}{\dfrac{\frac{3}{4} \text{ of } \frac{3B + C}{1}}{4}} \qquad \frac{12B + 4C}{\dfrac{\frac{4}{5} \text{ of } \frac{3B + C}{1}}{5}} \qquad \frac{3B + C}{\dfrac{\dfrac{48B + 16C}{\frac{12B + 4C}{5}} - \dfrac{45B + 15C}{\frac{9B + 3C}{4}}}{20}}$$

Proof of Subſtraction of Fract. and Rat. Species.

Foraſmuch as in the laſt Example here in the third *Caſe*, and alſo in the laſt Example of the ſecond *Caſe* above, the Numbers from which the Subſtraction is made, are the Totals of their Additions in the former Chapter; and the Subtrahends in both, are one of the Addends there: It is enough to ſhew the Reciprocal Proof of Addition by Subſtraction, and Subſtraction by Addition in theſe Fracted *Species*.

Subſtraction alſo, as Addition before, may be proved by ſuppoſing the *Species* to be Numbers Abſolute; becauſe after Subſtraction made with them, the Remain will be equally valuable with the Fracted *Species* remaining.

As in the laſt mentioned Example, if A be ſuppoſed 2, and B 3, then ſhall 8A be 16, and B 27. And $8A + 9B$, that is $16 + 27$, ſhall be 43; which divided by 12, ſhall be $3\frac{7}{12}$: From which $\frac{3}{4}B$ taken which is $\frac{9}{4}$, the Remain is $\frac{16}{12}$, that is 8A to be divided by 12, or $\frac{2}{3}A$, as the *Species* above ſhewed.

Suppoſing $A = 2$. $B = 3$. Then ſhall

	Subtrahend.	*Remain.*
$\left.\begin{array}{l} 8A = 16 \\ 9B = 27 \end{array}\right\} \frac{43}{12}$	$- \frac{3}{4}B = \frac{9}{4}$	$\frac{2}{3}A = \frac{4}{3} = 1\frac{1}{3}$

16

$$\frac{16}{} \qquad \text{Or thus, } 16+27-27=16$$

$$\frac{27}{} \qquad\qquad \frac{9B}{\overset{\cdot}{3}}$$

$$4)\underbrace{\underset{\overset{\cdot\cdot}{1}}{\frac{43}{12}}-\underset{\overset{\cdot\cdot}{3}}{\frac{9}{4}}}_{12}=\frac{16}{12}=1\frac{1}{3} \qquad 4)\underbrace{\underset{\overset{\cdot\cdot}{1}}{\frac{8A+9B}{12}}-\underset{\overset{\cdot\cdot}{3}}{\frac{3}{4}}}_{12}B=8A \qquad 12 = \frac{}{12}$$

CHAP. IX. *Multiplication of Fracted and Rational Species.*

Multiplication of Fract. and Rat. Species.

SPECIES Fracted and Rational, mixed with Integral *Species*, or purely Fracted, are multiplied as Common Fractions, *Book* 1. *Part* 2. *Chap.* 5. by comparing the Heterologal Terms, and multiplying the Homologal. And therefore but two *Caſes* are neceſſary, in both which the Signs are to be ordered as in Coſſical Fractions.

1. Heterologal Terms not reducible.

Caſe 1. If the Heterologal Terms need no Reduction, then multiply Numerator by Numerator, and Denominator by Denominator, Numbers as Numbers, and *Species* as *Species*.

Examples.

As $\frac{B}{D}$ multiplied into $\frac{C}{A}$, makes the Product $\frac{BC}{DA}$.

And $\frac{2B}{3D}$ multiplied by $\frac{2C}{3A}$, makes the Product $\frac{4BC}{9DA}$.

So Z the Integral *Species* multiplied by the Fractio n$\frac{A}{B}$ makes the Product $\frac{ZA}{B}$; where I ſet or ſuppoſe an Unit for the Denominator to Z, as if it were a Fraction.

Other Examples.

Mds. *Mrs.* *Products.* *Mds.* *Mrs.* *Products.*

$$\frac{A}{B}\times\frac{ZA}{C}=\frac{ZAq}{BC}. \qquad B\times\frac{DA}{C}=\frac{BDA}{C}.$$

$$\frac{3B}{4C}\times\frac{1}{5}A=\frac{3BA}{20C}. \qquad \frac{2B+C}{D-E}\times\frac{3A+O}{P}=\frac{6BA+3CA+2BO+CO}{DP-EP}.$$

2. Heterologal Terms reducible.

Caſe 2. If the Heterologal Terms need Reduction, then reduce them according to the Nature of the Fraction, Numbers as Numbers, and *Species* as *Species*; And after Reduction into their leaſt Terms, multiply the Homologal Terms as before.

Examples.

As $\frac{B}{A}$ multiplied into $\frac{A}{C}$ produceth $\frac{B}{C}$, becauſe A in the Numerator of the one, and Denominator of the other, being alike, are both ſet aſide.

And $\frac{2B}{3D}$ multiplied by $\frac{2C}{5B}$ produceth $\frac{4C}{15D}$, the Heterologal B in both Fractions being uſeleſs in the Product.

So B the Integral *Species* multiplied into the Fraction $\frac{A}{B}$ makes the Product A; for A is equal to $\frac{BA}{B}$.

Other Examples,

Mds. *Mrs.* *Reductions.* *Products.*

$$B\frac{C}{A}\times\frac{C}{D}=\frac{BA+C}{A}\times\frac{C}{D}=\frac{BAC+Cq}{AD}.$$

$$\frac{BA}{RE}\times\frac{DE}{CA}=\begin{matrix}A)\\E)\end{matrix}\underset{\overset{\cdot\cdot}{R}}{\overset{\overset{B}{\cdot\cdot}}{\frac{BA}{RE}}}\times\underset{\overset{\cdot\cdot}{C}}{\overset{\overset{D}{\cdot\cdot}}{\frac{DE}{CA}}}=\frac{BD}{RC}.$$

$$\frac{2B}{3C}\times\frac{3}{4}A=\begin{matrix}2)\\3)\end{matrix}\underset{\overset{\cdot\cdot}{1}}{\overset{\overset{1}{\cdot\cdot}}{\frac{2B}{3C}}}\times\underset{\overset{\cdot\cdot}{1}}{\overset{\overset{1}{\cdot\cdot}}{\frac{3}{4}A}}=\frac{1BA}{2C}.$$

Hence

Heterolog. Terms equal.

Hence as in Multiplication of Common Fractions, if the Heterologal Terms either way are equal, the other Terms ſhall ſtand for the Product. And if they are alike both ways, then ſhall the Product be an Unit or equal Fraction. For

Examples.

$\frac{2B}{3C} \times \frac{3C}{5B} = \frac{2BC}{5CB}$, and omitting the *Species* to $\frac{2}{5}$ the Number 3 in both being ſet aſide. And $\frac{2B}{3C} \times \frac{3C}{2B} = \frac{6BC}{6CB}$, that is 1.

To find a Part or Parts of a Number.

Alſo here properly may be inſerted a Propoſition to find a Part or Parts of a given Number or Magnitude, which is no more than to multiply the ſame by the Part or Parts, after the manner of Fractions.

Examples of both Sorts.

Examples.

As to know what $\frac{3}{4}$ of 3B is, the Product ſhews it $\frac{9B}{4}$.

And if $\frac{1}{3}$ and $\frac{1}{4}$ of $\frac{B+8A}{C}$ be demanded, the Multiplication produceth $\frac{7B+56A}{12C}$ for Anſwer. For $\frac{1}{3}$ and $\frac{1}{4}$ added, make $\frac{7}{12}$, which multiplying the *Data*, makes the Product as laſt above-mentioned.

Proof of Multiplication of Fract. and Rat. Species.

Multiplication of Fracted and Rational Species is to be proved, both by Diviſion as in the next Chapter; and by turning the Fracted *Species* into Common Fractions or Abſolute Numbers, with the Multiplication whereof will agree the Multiplication of theſe Fracted *Species*.

For ſuppoſe in the Example above-mentioned (where $\frac{2B}{3C} \times \frac{3C}{5B}$ produce $\frac{2BC}{5CB}$) B be 2, and C 3, then ſhall 2B be 4, and 3C 9. So will $\frac{2B}{3C}$ be $\frac{4}{9}$ and $\frac{3C}{5B}$ will be $\frac{9}{10}$, and the Product by Abbreviation $\frac{2}{5}$, as is the Value of the Product $\frac{2BC}{5CB}$ by that Suppoſition.

Suppoſing B = 2. C = 3. Then ſhall

$$\frac{2B = 4}{3C = 9} \times \frac{3C = 9}{5B = 10} = \frac{2BC = 2\times2\times3}{5CB = 5\times3\times2} = \tfrac{12}{30} \text{ or } \tfrac{2}{5}.$$

C HAP. X. *Diviſion of Fracted and Rational Species.*

Diviſion of Fract. and Rat. Species.

Contrary to Multiplication *Species* Fracted and Rational, Pure or Mixt with Integral *Species*, are divided by comparing the Homologal Terms, and multiplying the Heterologal as in Common Fractions, *Book* 1. *Part* 2. *Chap.* 6. And ſo but two Caſes are here neceſſary, and in both of them the Signs are to be ordered as in Coſſical Fractions.

1. Homologal Terms not reducible.

Caſe 1. When the Homologal Terms cannot be reduced lower, then multiply the Numerator of the Dividend by the Denominator of the Diviſor, to produce the Numerator of the Quotient: And the Denominator of the Dividend by the Numerator of the Diviſor, to produce the Denominator of the Quotient; Numbers as Numbers, and *Species* as *Species*.

Examples.

As $\frac{B}{D}$ divided by $\frac{C}{A}$ gives in the Quotient $\frac{BA}{DC}$.

And $\frac{2B}{3D}$ divided by $\frac{3C}{4A}$, gives in the Quotient $\frac{8BA}{9DC}$.

So $\frac{Aq}{D}$ divided by the Integral *Species* B, makes the Quotient $\frac{Aq}{BD}$: Suppoſing or ſetting an Unit for Denominator to B.

Other Examples.

Diviſors.	*Dividends.*	*Quotients.*	*Diviſors.*	*Dividends.*	*Quotients.*
$\frac{Bc}{C}$)	$\frac{Ec}{A}$	($\frac{EcC}{BcA}$	$\frac{A}{D}$)	$\frac{BC}{1}$	($\frac{BCD}{A}$
$\frac{DA}{C}$)	$\frac{B}{1}$	($\frac{BC}{DA}$	$\frac{3B}{4C}$)	$\frac{1}{5}A$	($\frac{4AC}{15B}$.

Caſe 2. When the Homologal Terms may be reduced, then, according to the Nature of the Fractions, reduce them, Numbers as Numbers, and *Species* as *Species*: And after Reduction into their leaſt Terms, multiply the Heterologal Terms as before.

2. Homologal Terms reducible.

As $\frac{A}{C}$ dividing $\frac{B}{C}$ giveth in the Quotient $\frac{B}{A}$, becauſe C the Denominator of both the given Fractions is ſet aſide as uſeleſs.

Examples.

And $\frac{4B}{15C}$ divided by $\frac{7B}{11D}$ brings in the Quotient $\frac{44D}{105C}$, the Homologal B in both being underſtood as cancelled.

So BC the Integral *Species* divided by $\frac{B}{A}$, makes the Quotient CA.

Other Examples.

Diviſors. Dividends. Reductions. Quotients.

$$\frac{BA}{RT}\Big)\ \frac{CA}{DT} == \frac{A)}{T)}\underset{\underset{R}{\cdot\cdot}}{\overset{\overset{B}{\cdot\cdot}}{\frac{BA}{RT}}}\Big)\underset{\underset{D}{\cdot\cdot}}{\overset{\overset{C}{\cdot\cdot}}{\frac{CA}{DT}}}\Big(\frac{CR}{BD}.$$

$$\frac{BA}{CR}\Big)\ \frac{A}{R} == \frac{A)}{R)}\underset{\underset{C}{\cdot\cdot}}{\overset{\overset{B}{\cdot\cdot}}{\frac{BA}{CR}}}\Big)\underset{\underset{1}{\cdot\cdot}}{\overset{\overset{1}{\cdot\cdot}}{\frac{A}{R}}}\Big(\frac{C}{B}.$$

$$\tfrac{3}{4}A\Big)\ \tfrac{1}{2}\frac{BA}{C} == \frac{A)}{2)}\underset{\underset{2}{\cdot\cdot}}{\overset{\overset{1}{\cdot\cdot}}{\tfrac{3}{4}A}}\Big)\underset{\underset{1}{\cdot\cdot}}{\tfrac{1}{2}}\overset{\overset{B}{\cdot\cdot}}{\frac{BA}{C}}\Big(\frac{2B}{3C}.$$

Hence, as in Diviſion of Common Fractions, if the Homologal Terms be either way equal, the other Terms ſhall be taken for the Quotient: And if they are alike both ways, the Quotient ſhall be an Unit or equal Fraction.

Homologal Terms equal.

For $\frac{2B}{3C}\Big)\frac{2B}{5D}\Big(\frac{3C}{5D}$ the Number 2 being ſet aſide with the *Species* B, becauſe alike in both Diviſor and Dividend. And $\frac{2B}{3D}\Big)\frac{1C}{3D}\Big(\frac{1C}{2B}$, becauſe 3D in both Denominators are equal, the other *Species* make up the Quotient.

Example.

And $\frac{2B}{3D}\Big)\frac{2B}{3D}\Big($ 1 is the Quotient, the Terms and *Species* in both Numerators and Denominators being alike.

Sometimes it is the eaſieſt way to reduce the Numbers given, part before Diviſion and part after; which happens if one or both be figurate.

Beſt ſometime to reduce the Data, part before and part after Diviſion.

As in dividing $\frac{ZAq}{BC}$ by $\frac{ZA}{C}$ and $\frac{BAC+Cq}{AD}$ by $\frac{C}{D}$ in the firſt Quotient $\frac{Aq}{BA}$ is reduced to $\frac{A}{B}$, and the other $\frac{BA+C}{A}$ to $B\frac{C}{A}$.

Examples.

Diviſors. Dividends. Reductions. Quotients Reduced.

$$\frac{ZA}{C}\Big)\ \frac{ZAq}{BC} == \frac{Z)}{C)}\underset{\underset{1}{\cdot\cdot}}{\frac{ZA}{C}}\Big)\underset{\underset{B}{\cdot\cdot}}{\frac{ZAq}{BC}}\Big(\frac{Aq}{BA}\Big(\frac{A}{B}.$$

$$\frac{C}{D}\Big)\frac{BAC+Cq}{AD} = \frac{C)}{D)}\frac{C}{D}\Big)\frac{BAC+Cq}{AD}\Big(\frac{BA+C}{A}\Big(B\frac{C}{A}.$$

Here may be inſerted a Propoſition, to find the principal Number or Magnitude, if any of his Parts be given; which is done by dividing the ſame Number or Magnitude by the Part or Parts, after the manner of Fractions.

To find a Number by any of its Parts.

Examples of both Sorts.

Examples.

As to know what is the principal Number whereof B is the $\frac{4}{5}$?

Anſwer, $\frac{5B}{4}$.

And if $\frac{B+8A}{C}$ be $\frac{2}{3}$ of another Number which is deſired, it will be found to be $\frac{3B+24A}{2C}$. For by dividing B by $\frac{4}{5}$, and $\frac{B+8A}{C}$ by $\frac{2}{3}$, their Quotients are as before.

Proof of Diviſion of Fract. & Rat. Species. Seeing the Quotients of theſe Diviſions multiplied by their Diviſors will return the Dividends, and the Dividends in both the Examples above are the Products of their Multiplications in the former Chapter, it is ſufficient to ſhew the Alternate Proof of Multiplication by Diviſion, and Diviſion by Multiplication in theſe Fracted *Species.*

Diviſion alſo, as Multiplication before, may be proved by ſuppoſing the *Species* to be Numbers Abſolute: For after Diviſion made thereby, the Quotient will be equally valuable with the Quotient of the divided Fractions.

As in the Example above, ſuppoſing A 2. B 3. C 4. D 5. then will the Diviſor $\frac{C}{D}$ be $\frac{4}{5}$; and the Dividend $\frac{BAC+Cq}{AD}$ be $\frac{40}{10}$ or $\frac{4}{1}$, that is $\frac{3\times2\times4+4\times4}{2\times5}$; and the Quotient of that Diviſion 5 Integers equal to $B\frac{C}{A}$ that is $3\frac{4}{2}$.

Suppoſing A=2. B=3. C=4. D=5. Then ſhall

$$\frac{C}{D} = \frac{4}{5}\Bigg)\overset{24+16}{\frac{BAC+Cq}{AD}} = \frac{40}{10}\Bigg(B\frac{C}{A} = 3\frac{4}{2} \qquad \begin{matrix}4)\\5)\end{matrix}\frac{\overset{\overset{1}{:}}{4}}{\underset{\underset{1}{:}}{5}}\Bigg)\frac{\overset{\overset{10}{:}}{40}}{\underset{\underset{2}{:}}{10}}\Bigg(\frac{10}{2}\Bigg(5$$

CHAP. XI. *Figuration of Rational Species.*

Figurate Rational Species produced. HOW to add, ſubſtract, multiply and divide whole and broken *Species*, as well Plain as Figurate, or Rational, intermixt one with another, is already diſpatcht in the foregoing Chapters; wherein becauſe the Coſſical or Rational *Species* are ſubject to like Orders and Directions, and receive like Reſolutions with plain *Species*, little need be added here ſave ſomething of their Figuration.

Common Way. Touching the Production of Rational *Species*, as others before them, ſo they are produced by Multiplication: For any *Species* multiplied into it ſelf, produceth the Square; and that multiplied by the Root, produceth the Cube, *&c.* And whether the Root be Simple or Compound, or if one Power be multiplied by another; by Addition of their Indices, the due Index of the Product is had, as was touched in their Multiplication before, and fully ſeen in Coſſicks in the *4th* Part of this Book, and Figural Numbers in the ſecond Part of the ſecond Book.

Examples.

Examples

Of Simple Rational Species *produced.*

Species	A	Aq	Ac	2Ac	Aqc	Aqc	multiplied.
	A	A	A	3A	Aq	Aqq	
	Aq	Ac	Aqq	6Aqq	Aqqc	Accc	
Indices	1	2	3	3	5	5	added.
	1	1	1	1	2	4	
	2	3	4	4	7	9	

Examples

Examples
Of Compound Rational Species *produced.*

√ A +E
A +E
Aq+AE
AE+Eq
q Aq+2AE+Eq
A+E
Ac+2AqE+AEq
AqE+2AEq+Ec
c Ac+3AqE+3AEq+Ec
&c.

√ 2A +3E
2A +3E
4Aq+6AE
6AE+9Eq
q 4Aq+12AE+9Eq
2A+3E
8Ac+24AqE+18AEq
12AqE+36AEq+27Ec
c 8Ac+36AqE+54AEq+27Ec.
&c.

√ A—E
A—E
Aq-AE
-AE+Eq
q Aq—2AE+Eq
A—E
Ac—2AqE+AEq
— AqE+2AEq—Ec
c Ac—3AqE+3AEq—Ec

√ A +B+C
A +B+C
Aq+BA+CA
BA+Bq+BC
CA+BC+Cq.
q Aq+2BA+Bq+2CA+2BC+Cq.

Power of a Binomial otherwise produced.

Besides this ordinary way of Production, the Power of any Binomial *Species* may ſpeedily be had, thus: Set down all the Parodical Degrees of both *Species* under the Power to the Root, and then place the higheſt Powers of each in oppoſition, and let the reſt be coupled contrary, the higheſt of one ſort to the loweſt of the other, and to them prefix the Numbers proper to the Power to be produced, mentioned in the Table for Extraction of Roots, *Book* 2. *Part* 2. *Chap.* 3. which ſaid Numbers are ſometime called *Unciæ*.

Unciæ what.

As if the 7*th* Power of A+E (that is the ſecond Surſolid) be ſought.

Example.

Parodical Degrees.		*Rightly placed*		*with the Unciæ.*
		Aqqc		
A	E			Aqqc
Aq	Eq	Acc	E	7AccE
Ac	Ec	Aqc	Eq	21 AqcEq
Aqq	Eqq	Aqq	Ec	35 AqqEc
Aqc	Eqc	Ac	Eqq	35 AcEqq
Acc	Ecc	Aq	Eqc	21 AqEqc
Aqqc	Eqqc	A	Ecc	7 AEcc
			Eqqc	Eqqc

The Parodical Degrees with the *Unciæ* to the 10*th* Power, ſtand as in the Table following, which may be enlarged at pleaſure.

A Table of the Parodical Degrees, with the **Unciæ** *to the* **10th** *Power.*

1	2	3	4	5	6	7	8	9	10
A E	Aq 2AE Eq	Ac 3AqE 3AEq Ec	Aqq 4AcE 6AqEq 4AEc Eqq	Aqc 5AqqE 10AcEq 10AqEc 5AEqq Eqc	Acc 6AqcE 15AqqEq 20AcEc 15AqEqq 6AEqc Ecc	Aqqc 7AccE 21AqcEq 35AqqEc 35AcEqq 21AqEqc 7AEcc Eqqc	Aqcc 8AqqcE 28AccEq 56AqcEc 70AqqEqq 56AcEqc 28AqEcc 8AEqqc Eqcc	Accc 9AqccE 36AqqcEq 84AccEc 126AqcEqq 126AqqEqc 84AcEcc 36AqEqqc 9AEqcc Eccc	Aqqcc 10AcccE 45AqccEq 120AqqcEc 210AccEqq 252AqcEqc 210AqqEcc 120AcEqqc 45AqEqcc 10AEccc Eqqcc

Theorem of Euclid *demonſtrated.*

By this Table we have a farther Demonſtration of the Truth of that Theorem of *Euclid* touching the Square; and the other of *Ramus* touching the Cube, mentioned before, *Book* 2. *Part* 2. *Chap.* 2. *Sect.* 6, 7.

As in producing the Square and Cube of 845; firſt A ſhall be ſet for 8, and E for 4; then 84 ſhall be A, and 5 ſhall be E; the reſt of the Work, with the *Species* annexed, follow.

Square.

8	4	5		
64			Aq	
6	4		2AE	*Gnomon.*
	16		Eq	
70	56		Aq	
	84	0	2AE	*Gnomon.*
		25	Eq	
71	40	25		

Cube.

8	4	5		
512			Ac	
76	8		3AqE	
3	84		3AEq	*Gnomon.*
	64		Ec	
592	704		Ac	
10	584	0	3AqE	
	63	00	3AEq	*Gnomon.*
		125	Ec	
603	351	125		

What the Diagonals, Complements Unciæ and Gnomon.

The two extream Powers of every kind in this Table, are ſometime called *Diagonals*, and the intermediate Species *Complements*. The Numbers or *Unciæ* affixed, ſhew the number of Complements to be taken in the Conſtitution of the Power; and all the Complements with the leſſer Power make up the *Gnomon*.

Mr. Oughtred'*s Obſervations on the Table.*

Mr. *Oughtred* in the 17*th* Chapter of his *Clavis*, makes farther inſpection into the Table, and obſerves:

1. What Species Affirmative, and what Negative, &c.

1. That all the *Species* of the Powers of a Binomial Root are Affirmative, and thoſe Parts ſimply taken without Unity, are in continual Proportion: But all the *Species* of the Powers of a Reſidual Root, are alternately Negative.

Example in Binomials.

As Powers.	*Binomial.*		*Parts.*		*Ratio.*
Q:			Aq+AE+Eq 9 . 6 . 4	∷	$\frac{9}{6}$($1\frac{1}{2}$ $\frac{6}{4}$($1\frac{1}{2}$
C:	A + E 3 + 2	is	Ac+AqE+AEq+Ec 27 . 18 . 12 . 8	∷	$\frac{27}{18}$($1\frac{1}{2}$ $\frac{18}{12}$($1\frac{1}{2}$ $\frac{12}{8}$($1\frac{1}{2}$
QQ:			Aqq+AcE+AqEq+AEc+Eqq 81 . 54 . 36 . 24 . 16	∷	$\frac{81}{54}$($1\frac{1}{2}$ $\frac{54}{36}$($1\frac{1}{2}$, &c.

Examples in Reſiduals.

Powers.	*Reſidual.*		*Parts.*
Q:			Aq —2AE +Eq.
C:	A—E	is	Ac —3AqE+3AEq—Ec.
QQ:			Aqq—4AcE+6AqEq—4AEc+Eqq. &c.

2. Difference of Names, what.

2. That the difference of the Names of every Binomial or Reſidual, is the Homogeneal Power of the Difference of the Names of the Root.

Example.

Scilicet, Ac+3AEq$\overset{*}{—}$3AqE+Ec, or Ac+3AEq—3AqE—Ec, is the Cube of A—E.

3. Difference of Squares, &c. what.

3. That the Difference of the Squares of the Names of every Binomial or Reſidual, is the Homogeneal Power of the difference of the Squares of the Names of the Root.

Example.

Scilicet, Q: Ac+3AEq$\overset{*}{—}$Q: 3AqE+Ec, is C: Aq—Eq.

4. Species aggregate in Names, &c. what.

4. That if the *Species* aggregate in Names alternately be Affirmative and Negative, the Sum of the Squares of the Names is the Homogeneal Power of the Sum of the Squares of the Names of the Root.

Example.

Scilicet, Q: Ac—3AEq$\overset{*}{+}$ Q: 3AqE—Ec, is C: Aq+Eq.

5. Intermediate Species, Powers of the Means.

5. That all the intermediate *Species* of every Order, are alſo Powers of the mean Proportionals between A and E.

Example.

Scilicet, between Ac and Ec, there are two mean Proportionals AqE and AEq, which are alſo Cubes of M and N, (the four continual Proportionals being A.M.N.E): Wherefore A.√cAqE.√cAEq.E. are continual Proportionals. For AqE=AMN=Mc. And AEq=MNE=Nc.

Invention of Means ariſe hence.

And hence ariſeth the Invention of any mean Proportionals between A and E; of which further is to be ſeen in the next Book. As if five mean Proportionals be ſought, then ſhall the Power be the ſixth Quantity or cc; the Index whereof exceeds by an Unit the Number of the Means ſought, and ſhall be A. √ccAqcE. √ccAqqEq. √ccAcEc. √ccAqEqq. √ccAEqc. E. ÷

6. Mean Species made of what.

6. That every mean *Species* in every kind is made of the two Powers of the Names of the Root, the Indices whereof together, are equal to the Index of the ſame kind; but the mean Diſtances of the ſame *Species* from their Extreams, ſhall be equal to the Indices of the Alternate Factors, and anſwer to them in the Common Angle.

Example.

Scilicet. AqEc in the 5*th* Power, is made of Aq multiplied into Ec, which anſwer in the common Angle; and is from Aqc the Third, and Eqc the Second.

Hence came the production of Figurals in the Pages before.

And hence came the former ſpeedy and eaſy Way of producing the Power of any Binomial by the Parodical Degrees. The Example given by Mr. *Oughtred* there, is in the 5*th* Power of the Binomial Root A+√qÆ, of which the Parodical Degrees may be thus placed with their Indices.

Example.

4 3 2 1
Aqq . Ac . Aq . A
5
Aqc √qÆqc
5
√qÆ . Æ . √qÆc . Æq
1 2 3 4

Aqc
5 √qÆAqcc
10 ÆAc
10 √qÆcAqq
5 ÆqA
√qÆqc

Explained in Numbers.

This explained in Numbers, taking for the Root A+E, 82 of which the Segments being 80+2 or 8. 2. the Rectangle or Æ is 16, the ſquare Root or √qÆ is 4. the Numbers that anſwer to the Parodical Degrees, and make up the whole Power of 84, *viz.* 4182119424, are as followeth.

A .8	√qÆ	.4 .		Aqc	32768
Aq .64	Æ	.16 .	5	√qÆAqcc	81920
Ac .512	√qÆc	.64 .	10	ÆAc	81920
Aqq.4096	Æq	.256 .	10	√qÆcAqq	40960
Aqc.32768	√qÆqc	.1024.	5	ÆqA	10240
				√qÆqc	1024
					4182119424

7. What produced by a Species, × in Æ.

7. That if any *Species* be multiplied in Æ, the Magnitude produced ſhall be the mean *Species* collateral in the following alternate Order, and the ſame in Number from their Extreams.

Examples.

As Ac×Æ, is AqqE, which is the firſt from Aqc, and the fourth from Eqc.
So AcE×Æ, is AqqEq, which is the ſecond from Acc, and the fourth from Ecc.

8. What the Product of A—E× *into a Species.* Examples.

8. That if any *Species* be multiplied by A—E or X, the Magnitude produced ſhall be the Difference between the two *Species* next each of the following Order.

As AcX =Aqq —AcE. AqEX=AcE—AqEq.
AEqX=AqEq—AEc. EcX =AEc— Eqq.

Wherefore if all the *Species* of every Order be multiplied by X, the Difference of the two extream Powers of the next upper Order will be produced.

As of Ac+AqE+AEq+Ec multiplied into X, will be made Aqq—Eqq.

9. Sum of unequal difference of the equal Indices × into a Species, what produced.

9. That in the Orders of unequal Indices (l. c. qc. &c.) the Sum of the two extream Powers; but in the Orders of equal Indices, (q. qq. cc. &c.) the Difference of them, made of A+E multiplied into the ſingular *Species* of the leſſer Order precedent, is alternately Affirmative and Negative.

Examples.

As Ac+Ec, made of Aq—AE+Eq, multiplied into A+E.
Alſo Aqq—Eqq, made of Ac—AqE+AEq—Ec, multiplied into A+E.

10. Magnitude in contrary, what they produce.

10. That if the ſame Magnitude be multiplied into two contrary Magnitudes, the Magnitudes made of them ſhall be alſo contrary.

Examples.

As Aq−2AE+Eq multiplied into A−E, ſhall be made Ac−3AqE+3AEq−Ec.
But the ſame multiplied into −A+E, ſhall be made −Ac+3AqE−3AEq+Ec.

11. Unciæ *what they are, and how called.*

11. That the *Unciæ* or Numbers prefixt to the *Species*, are numerary Figures: For all under A and E are Roots; all under Aq and Eq are Triangulars; all under Ac and Ec are Pyramidals; all under Aqq and Eqq are Triangle-Triangulars; all under Aqc and Eqc are Triangle-Pyramidals; all under Acc and Ecc are Pyramidal-Pyramidals, *&c.*

12. Unequal Sides Negative, ſo the Species.

12. That if in any *Species* the number of the negative Sides be unequal, that *Species* ſhall be Negative.

Examples.

As $$Q{:}A+B-C=Aq+2AB+Bq-2BC+Cq-2AC.$$
$$C{:}A+B-C=Ac+3AqB+3ABq+Bc-3BqC+3BCq-Cc-3CAq+3CqA-6ABC.$$

Root.	*Square.*		*Cube.*		
A+B−C	Aq	2AB	Ac	Bc	−Cc
	Bq	−2BC	3AqB		3ABq
	Cq	−2CA	−3BqC		3BCq
			3CqA		−3CAq
				−6ABC	

Roots of Rational Species extracted.

Thus much may ſuffice for the Geneſis. Now for the Analyſis of Rational *Species* or Extraction of their Roots, wherein nothing of difficulty occurs, the Extraction of Roots in other Numbers being learned, and eſpecially Coſſicks, between which and theſe Rational *Species* there is great Concinity: For Figurate *Species* Simple, as Simple Coſſicks, and compound Figurate *Species* Binomial and Reſidual, as compound Coſſicks, have their Roots extracted. Yet *Species* of a Polynomial Root differ in the pointing or pricking of the Number; for whereas in all other Extractions the figural Numbers were pricked according to their Quantities, in *Species* reſpect is to be had both to the Quantities and to the Number of diſtinct *Species* in the Root: For the Square that hath three *Species* in the Root, ſhall leave two Magnitudes unprickt between the right-hand Prick and the next, and but one Magnitude unprickt between the two next left-hand Pricks. But if the Root have four *Species* in it, then between the Dexter and next Siniſter Prick, ſhall three Magnitudes be left unprickt, and 2 between the two next Pricks, and but one between that and the uttermoſt Siniſter Prick. Likewiſe in pricking Cubes, if the Root conſiſt of three *Species*, there ſhall be five Magnitudes left unprickt, between the Dexter and next Siniſter Prick, and between that and the utmoſt Siniſter Prick but two: And if the Root have four *Species* in it, then are nine Magnitudes left unprickt between the firſt two right-hand Pricks, five between the two next, and two between thoſe next the left-hand, &c.

Figurate Species of a Polynomial Root pricked different from others.

Examples in the Square.

As to extract the Root of Aq+2AE+Eq, becauſe the Root is a Binomial, only one *Species* is left unprickt as others: And the Root of Aq which is A, is placed in the Quotient; this doubled, that is 2A, is Diviſor, which gets E in the Quotient: Then multiplying the Diviſor thereby, and adjoining the Square of E, makes the *Gnomon* to be ſubſtracted.

$$\begin{array}{lll}
\text{Square} & \dot{A}q+2AE+\dot{E}q & |\underline{A+E}\quad \text{Root.}\\
 & Aq & \\
\text{Diviſor} & 2A & \\
 & E & \\
 & \overline{\underline{2AE+Eq}}\ \textit{Gnomon.} &
\end{array}$$

But to extract the Square Root of Aq+2BA+Bq+2CA+2BC+Cq, becauſe the Root is a Polynomial of three *Species* Aq. Bq. & Cq. are the pricked *Species*. And after A+B is gotten in the Quotient, as by the Work above, then 2A+2B is Diviſor, which gets C in the Quotient; and the Diviſor multiplied thereby, and the Square of C adjoined, makes the ſecond *Gnomon*.

$$\begin{array}{llll}
\text{Square} & \dot{A}q+2BA+\dot{B}q+2CA+2BC+\dot{C}q & | \underline{A+B}+C. \quad \text{Root.} & \\
 & Aq & & \\
\text{Diviſor} & 2A & & \\
 & B & & \\
 & \overline{\underline{2BA+Bq}}\ \textit{Gnomon.} & & \\
 & \text{Diviſor}\ 2A+2B & & \\
 & C & & \\
 & \overline{\underline{2CA+2BC+Cq}}\ \textit{Gnomon.} & &
\end{array}$$

Examples in the Cube.

So to extract the Cube Root of Ac+3AqE+3AEq+Ec, the Root being a Binomial, the *Species* unprickt are as others: And the Root of Ac which is A, is placed in the Quotient: This ſquared and tripled is Diviſor, which gets E in the Quotient, whereby the Diviſor multiplied, and the triple of A by the Square of E and the Cube of E added together, make up the *Gnomon* to be ſubſtracted.

Cube Ac+3AqE+AEq+Ec | A+E Root.
Ac
Divisor 3Aq 3A
E Eq
3AqE+3AEq+Ec *Gnomon.*

But to extract the Cube Root of Ac+3AqB+3ABq+Bc+3CAq+6ABC+3BqC+3ACq+3BCq+Cc, becauſe the Root is a Polynomial of three *Species*, according to the Inſtructions aforeſaid, the Number is pricked: And after A+B is gotten in the Quotient, as by the Work of the other Cube laſt mentioned; then A+B ſquared and tripled, makes the Diviſor 3Aq+6AB+3Bq, whereby C is gotten for the Quotient; this multiplying the Diviſor, and the Square of C by the triple of A+B, and added to the Cube of C, makes up the ſecond *Gnomon* to be ſubſtracted.

Cube. *Root.*
Ac+3AqB+3ABq+Bc+3CAq+6ABC+3BqC+3ACq+3BCq+Cc | A+B+C
Ac
Diviſor 3Aq 3A
B Bq
Gnomon 3AqB+3ABq+Bc
Diviſor 3Aq+6AB+3Bq 3A+3B
C Cq
Gnomon 3AqC+6ABC+3BqC+3ACq+3BCq+Cc.

Diviſors uſed in Extraction, how different from others.

Beſides this real Difference in pointing the Numbers, there is another ſeeming Difference in extraction of the Roots of Rational *Species* and others in the Diviſor of all Powers above the Square: For whereas in all other Extractions, the Power next inferior to the given Number, whoſe Root is to be extracted, multiplied by the Index of the given Power, is counted the Diviſor; As in the Square the Root multiplied by 2, the Index of the Square; in the Cube the Square multiplied by 3, the Index of the Cube; in the ſquared Square the Cube multiplied by 4, the Index of the ſquared Square, *&c.* In *Species* the Diviſor is uſually reckoned to conſiſt of all the Parodical Degrees under the Power whoſe Root you are extracting, with the *Unciæ* annexed to thoſe Degrees.

Examples.

As in the Square 2A, in the Cube 3Aq and 3A, in the ſquared Square 4Ac and 6Aq and 4A, *&c.* Nevertheleſs, inquiry for the next Quotient, Figure or *Species*, is made only by the left-hand *Species* thereof, which is the ſame with the Diviſor in other Extractions: And the other Parts of the Diviſor are ſet down, the better to direct the Operator to place the Parodical Degrees of the new-gotten Quotient, Figure or *Species* thereby, for Multiplication of them one into another to make up the *Gnomon.*

Square.
Aq
Diviſor | 2A:E
Eq } *Gnomon.*

Cube.
Ac
Diviſor { 3Aq:E
3A :Eq
Ec } *Gnomon.*

Squared Square.
Aqq
Diviſor { 4Ac:E
6Aq:Eq
4A :Ec
Eqq } *Gnomon.*

Practice of Mr. Oughtred.

Thus that moſt curious Analyſt Mr. *Oughtred* practiſes, not only in all his Reſolution of Affected Equations, but even in extracting the Roots of plain Figural Numbers, to which as more demonſtrative he adjoineth the *Species*, after the manner in the two Examples following, of the Square and Cube of 845; in which latter the firſt Diviſor is 1944, which by the common Way is but 192: And the next Diviſor, which ordinarily would be but 21168, is 211932. In the Square there ariſeth no Difference in the Diviſor, becauſe the Quantity is next the Root.

Square

Examples.

Square 714025 (above: 784) Root. (845

64			Aq	
1	6		2A *Diviſor.*	
6	4		2AE	*Gnomon.*
	16		Eq	
6	56			
	16	8	2A *Diviſor.*	
	84	0	2AE	*Gnomon.*
		25	Eq	
	84	25		

Cube 603351125 (above: 91647, 10) Root. (845

512			Ac	
19	2		3Aq	*Diviſor.*
	24		3A	
18	44			
76	8		3AqE	*Gnomon.*
3	84		3AEq	
	64		Ec	
80	704			
2	116	8	3Aq	*Diviſor.*
	2	52	3A	
2	119	32		
10	584	0	3AqE	*Gnomon.*
	63	00	3AEq	
		125	Ec	
10	647	125		

Proof of Figuration of Rational Species.

Production of Rational *Species* and Extraction of their Roots, are mutual Proofs of the Truth of each other's Operations, as in other Figural Numbers. Beſides which, if any Scruple ariſe in either, trial may be made of both, by taking Abſolute Numbers, and working therewith. As inſtead of other Examples, the laſt of the Analyſis of the Square and Cube of 845, with the Examples of their Production before in this Chapter, are ſufficient Evidence.

CHAP. XII. *Reduction of Irrational Species.*

Reduction of Irrational Species.

In purſuit of *Species*, I am now come to *Irrationals*, which in their Operations and Reſolutions follow *Surds*, as the Rational *Species Coſſicks*. And having been deſcribed before, *Chap.* 1. of *Species*, I proceed forthwith to their Reduction.

Irrational Species are either to be reduced to their leaſt Terms, or from different Denominations into one.

To their leaſt Terms.

To abbreviate or leſſen the Terms of an *Irrational Species*, is when the Denomination is Compound, and the Number annexed hath a Root that may be expreſſed by part of that Compound Denomination; then extract the Root of the Number, and alter the *Species* accordingly.

Examples.

As √qq25 and √cc81, the Indices 4 and 6 reduced to their leaſt Terms, are 2 and 3, by the common Diviſor 2 the Index of the Square: If therefore the Square Root of 25 and 81 be taken, the Surd *Species* ſhall be reduced to √q5 and √c9.

	2) 4	6	2	3	
√qq25 and √cc81	qq	cc	q	c	√q5 and √c9
	25	81	5	9	

So √cc27 may be reduced to √q3, and diſcharged of c.
And √qqcc32 diſcharged of qcc, may be reduced to √q2.

To one Denominator.

To reduce *Irrational Species* of different Denominations into one, comprehends,

With an Abſolute Number.

Firſt, To reduce an Abſolute Number into the Denomination of a Surd or *Irrational Species*; and this is done by multiplying the Number according to the Power of the *Species*.

Examples.

As 2 and √qB reduced, is √q4 and √qB.
So 2 and √cB reduced, is √c8 and √cB.

Secondly, To reduce two *Irrational Species* of uncompounded Indices into one Denomination; And this is effected by multiplying each Quantity as his Alternate Power or *Species* doth ſhew. *With uncompound Indices.*

Example.

As √qB & √cC reduced, is √ccBc & √ccCq

√q		√c	√q
B	X	C	√c
Bc		Cq	√cc

Thirdly, To reduce two *Irrational Species* of compounded Indices into one Denomination. And this is performed by dividing the Indices by the common Diviſor, and then by the leaſt Terms thereof multiplying alternately both the given Indices for the new Index, and the Numbers into the Powers of theſe leaſt Terms. *With compound Indices.*

Examples.

	Examples in Numbers.			*Species.*		
Surds to be reduced	√qq10 and √cc7			√qAq and √qqBq		
Common Diviſor	2) 4qq		6cc	2) 2q		4qq
	2 q	X	3c	1√	X	2q
Leaſt Terms.	3		2	2		1
Surds reduced	√cccc1000		√cccc49	√qqAqq		√qqBq

Proof of Reduction of Irrational Species. As before in Reduction of Surds one part of Reduction, which leſſened the Terms, was obſerved to prove the other, which exalted the leſſened Surds into the Powers from whence they were abated; and reciprocally that part of Reduction which increaſeth their Terms and Denominations, by extracting the Roots and abating the Characters: So here.

Alſo all ſorts of Reduction of *Irrational Species* may be proved, by taking in their ſtead Rational Numbers, and working with them after the manner of Surds, as in the Chapter of *Reduction of Surds* is ſo fully exemplified that Example here need not.

CHAP. XIII. *Addition of Simple Irrational Species.*

Addition of Irrational Species Simple. Irrational *Species* agreeing in their Simple Elements with Surds, before explicated in the 5*th* Part of this third Book, the fewer Examples, and a more brief Repetition of the Rules may ſerve turn here.

Addition of the Simple may be included in four *Caſes*.

Caſe 1. If the *Species* and Numbers (if any be annexed) be commenſurable, then order the Numbers, whether Integers or Fractions, as Simple Surds before, and the *Species* in like manner; that is, figurate the Sum of their Roots according to their Quantities, and multiply that Quantity by the Common Diviſor. *1. Commenſurable.*

Examples. As to add WRPq to WRSq. The Common Diviſor is R. The Squares Pq & Sq. The Roots P & S; which ſquared and multiplied into R, makes the Total WRPq +2PRS+RSq, which is all one in equality with WRPq+WRSq.

Common Diviſor	WR)	*Addends.* WRPq WRSq	Squares	(Pq (Sq	Roots	P. S.
Total		WRPq+2PRS+RSq				

P+S Root.

P+S

Pq+PS

PS+Sq

Pq+2PS+Sq Square.

WR Com.Diviſor.

WRPq+2PRS+RSq Total.

Another Example with Numbers.

	Addends.		
W3)	W27Aq	(9Aq .	3A.
	W 3Bq	(1Bq .	1B.
Total	W27Aq+18BA+3Bq		

3A +B

3A +B

9Aq+3BA

3BA+Bq

9Aq+6BA+Bq

W3

W27Aq+18BA+3Bq=W27Aq+W3Bq.

2. Incommenſurable. *Caſe* 2. If the *Species* or Numbers annexed, or both be incommenſurable, then join them together with +.

Examples. As √qB and √qC added, are √qB + √qC.

So √c5B added to √c10C, make √c5B + √c10C.

3. Heterogeneal. *Caſe* 3. If the Denominations of the Quantities be Heterogeneal, they may be reduced, and then added or conjoined as before.

Example. As to add the √qB & √cC, they are reduced as in the precedent Chapter to the Denomination of cc, and then conjoined with the Sign of Addition thus; √ccBc + √ccCq.

4. Different Signs. *Caſe* 4. If the Signs be different, conjoin the given Irrationals with the Sign —.

Example. As to add √qB with —√qA, the Total ſhall be √qB—√qA.

Irrational added to it ſelf. Here alſo, as in Simple Surds, may be obſerved, to add any *Irrational Species* to it ſelf, is to multiply the Squares by 4, the Cubes by 8, *&c.*

Examples. As √qB + √qB, is √q4B. And √cD + √cD, is √c8D.

Proof of Addition of Simp. Irrat. Species. *Addition of Simple Irrational Species*, is to be proved by their Subſtraction in the next Chapter, and by taking Rational *Species* or Numbers inſtead of Irrational. As in the laſt Example, ſuppoſing the √cD be 2, then ſhall D be 8, and 8D 64, whoſe Cube Root will be 4 equal to 2 added to it ſelf.

CHAP. XIV. *Subſtraction of Simple Irrational Species.*

Subſtraction of Irrational Species Simple. AS Addition, ſo the Subſtraction of theſe Irrationals may be included in four *Caſes*.

1. Commenſurable. *Caſe* 1. If the *Species* and Numbers (if any be annexed) be commenſurable, then (whether Integers or Fractions) order the Numbers as in Subſtraction of Simple Surds before, and the *Species* likewiſe; that is, figurate the Difference of their Roots according to the given Quantities, and multiply that Quantity by the Common Diviſor.

Examples. As to take WRSq from WRPq, the Common Diviſor is R, the Squares Pq and Sq, the Roots P and S, the Difference P—S; which ſquared and multiplied into R, makes the Remain WRPq—2PRS + RSq, which is equal to WRPq—WRSq.

Subtrahend.
WRPq—WRSq Squares (Pq Roots P
Common Diviſor WR (Sq S
Remain WRPq—2PRS + RSq

P—S
P—S
Pq-PS
-PS + Sq
Pq-2PS+Sq
WR
WRPq-2PRS + RSq

Another Example with Numbers.

Subtrahend.
W27Aq—W3Bq (9Aq . 3A.
Common Diviſor W3 (1Bq . 1B.
Remain W27Aq-18BA+3Bq

3A—B
3A—B
9Aq-3BA
-3BA + Bq
9Aq-6BA + Bq
W3
W27Aq—W3Bq = W27Aq—18BA + 3Bq

2. Incommenſurable. *Caſe* 2. If the *Species* or Numbers annexed, or both, be Incommenſurable, then join them together with —.

Examples. As to take √qC from √qB, the Remain ſhall be √qB—√qC.

So √c9B taken from √c10C, ſhall make the Remain √c10C—√c9B.

3. Heterogeneal. *Caſe* 3. If the given Denominations of the Quantities be Heterogeneal, they may be reduced, and then ſubſtracted or conjoined as before.

Example. As to take √qB from √cC, being reduced as before in *Chap.* 12. *Of Reduction of Irrational Species*, to the Denomination of cc, they are then conjoined with the Sign of Subſtraction thus, √ccCq—√ccBc.

Case 4. If the Signs be different, the Irrationals given are to be added; And the Sign of the Remain ſhall be contrary to the Sign of the Subtrahend, as before in Simple Surds, if they be commenſurable; but if incommenſurable, conjoin them by the Sign of Addition +.

4. Different Signs.

As to take —√q3BD from √q12A, becauſe the Signs are unlike, the one + and the other — they are added, and the Remain ſhall be √q12A+√q3BD.

Examples.

And —√qC taken from √qB, and √qC taken from —√qB, leave their Remains √qB+√qC, and —√qB+√qC.

Here alſo, as in Subſtraction of Simple Surds, may be obſerved, that to take any Irrational from it ſelf, leaves nothing remaining; but to take half any Irrational *Species*, is to divide the Squares by 4, the Cubes by 8, &c.

Irrational taken from it ſelf, or the Half taken.

As to take √qB from √qB, the Remain is Nought.

Examples.

But to take half the √qB is $\frac{WB}{4}$. And half the √cB is $\frac{wWB}{8}$

Subſtraction of Simple Irrational *Species* is proved by their Addition: And by taking Rational *Species* or Numbers, inſtead of thoſe Irrational, and working therewith: As in the laſt Example, if half the √cB be $\frac{\surd cB}{8}$, then by the Addition thereof to it ſelf, it ſhall be $\frac{\surd c8B}{8}$, and by clearing the Number of 8, it ſhall be √cB as before.

Proof of Subſtraction of Sim. Irrat. Species.

Alſo ſuppoſe B were 64, the √c thereof ſhall be 4; which halfed, ſhall be 2 equal to the √c8.

CHAP. XV. *Multiplication of Simple Irrational Species.*

TO the Multiplication of theſe Irrationals, two Caſes will ſerve.

Caſe 1. If the *Species* be Homogeneal in their figurate Denominations, then multiply Numbers as Numbers, and *Species* as *Species*, Integers as Integers, and Fractions as Fractions.

Multiplication of Irrational Species Simple.
1. Homogeneal.

As √c12B multiplied by √c5C, produceth √c60BC.

Examples.

Factors.		*Product.*	*Factors.*		*Product.*
√qA×√qB	═══	√qBA	√qq12B×√qqD	═══	√qq12BD.

Caſe 2. If the *Species* be Heterogeneal in their figurate Denominations; then after Reduction to one Denomination, multiply them as before.

2. Heterogeneal.

As √qA to be multiplied into √cB, is reduced to √ccAc, and √ccBq; and then multiplied to √ccAcBq.

Example.

Reduction. *Multiplication.*

$$\begin{array}{c}A\\ \surd q\\ \hline \underline{Bq}\end{array} \;\times\; \begin{array}{c}B\\ \surd c\\ \hline \underline{Ac}\end{array} \quad \begin{array}{c}\surd q\\ \surd c\\ \hline \underline{\surd cc}\end{array} \qquad\qquad \begin{array}{c}\surd cc\;\; Ac\\ Bq\\ \hline \underline{\surd ccAcBq}\end{array}$$

Here, as in Multiplication of Simple Surds, theſe Conſectaries take place.

Conſectaries.

1. To multiply any *Irrational Species*, is to increaſe him by the Power of a Root Homogeneal.

1. How to double, triple, &c.

Examples.

Irrational.	*Doubled.*	*Tripled.*	*Quadrupled.*
	√qB	√qB	√qB
Squares.	4	9	16
	√q4B	√q9B	√q16B
	√cD	√cD	√cD
Cubes.	8	27	64
	√c8D	√c27D	√c64D

2. To multiply ſome Irrationals, produce Rationals, and the Product, as occaſion is, may be cleared of the Surd Character.

2. Rationals produced.

Examples.

Irrationals.	*Square.*	*Cube.*
	√q18B	√c250C
	√q 2E	√c 4D
Rationals.	√q36BE=6BE.	√c1000CD=10CD.

3. To

3. When to cancel the Character.

3. To multiply the Side of any Power according to the Exigency of his Kind, is but to blot out or cancel the Note of the Side, and leave the *Species* abſolute.

Examples. As $\surd qB\times\surd qB = B$. And $\surd c12D\times\surd c12D\times\surd c12D = 12D$.

4. What produced by the Side of a Power, &c.

4. To multiply the Side of a Power, whoſe Index is a Compound Number, according to the Exigency of one of the Kinds compounding; the Side of either Kind may be prefixed to the ſpecial Number alone.

Examples. As the Square of the $\surd ccB$, is $\surd cB$; and the Cube of the $\surd ccB$, is $\surd qB$: For $\surd cc$ is $\surd$ of 2×3.

5. Homogeneal Figurals multiplied, what produced.

5. To multiply a figural Number by an Homogeneal figural Number, the Product ſhall be a figural Number of the ſame Kind, whoſe Side or Root ſhall be equal to the Product of the Sides of the Number multiplied.

Examples.

As $Aq\times Eq = AqEq$. And $Ac\times Ec = AcEc$.

q. $25\times 16 = 400$ c $27\times 8 = 216$

√. $5\times 4 = 20$ √ $3\times 2 = 6$

6. Sides of ſuch multiplied, what produced.

6. To multiply the Sides of Homogeneal *Species* Irrational, begetteth Sides of Irrational *Species* Homogeneal.

Examples. As $\surd qA\times\surd qE = \surd qAE$. And $\surd cB\times\surd cC = \surd cBC$.

Proof of Multiplication of Sim. Irrat. Species.

Multiplication of Simple Irrational *Species*, is to be proved by their Diviſion in the next Chapter; and by taking Rational *Species* or Numbers, and working therewith inſtead of Irrational. As in the laſt Example, if B be 27, and C 8, the Product will be 216; the Cube Root whereof is 6, equal to 3 the Root of 27, multiplied into 2 the Root of 8.

CHAP. XVI. *Diviſion of Simple Irrational Species.*

Diviſion of Irrational Species Simple.

As Multiplication, ſo the Diviſion of theſe Irrationals may be included in two Caſes.

1. Homogeneal.

Caſe 1. If the *Species* be Homogeneal in their figurate Denominations, then divide the Dividend by the Diviſor, Numbers as Numbers, and *Species* as *Species*, Integers as Integers, and Fractions as Fractions.

Examples. As $\surd qBA$ divided by $\surd qB$, giveth in the Quotient $\surd qA$.

And $\surd c60BC$ divided by $\surd c12B$, giveth in the Quotient $\surd c5C$.

Dividend. *Dividend.*

Diviſor $\surd qB)\ \surd qBA\ (\surd qA$ Quotient. $\surd c12B)\ \surd c60BC\ (\surd c5C$.

2. Heterogeneal.

Caſe 2. If the *Species* be Heterogeneal in their figurate Denominations, then firſt reduce them into an Homogeniety, and afterward divide them as before.

Example. As $\surd ccAcBq$ divided by the $\surd qA$, the Diviſor is firſt reduced to $\surd ccAc$; and then Diviſion being made, the Quotient is $\surd ccBq$, which may be depreſſed to $\surd cB$.

Reduction. *Diviſion.*

$$2\left)\begin{array}{c}\surd ccAc\\ \surd qA\\ \surd\\ \hline \surd ccAc\\ \hline\end{array}\right. \times \begin{array}{c}\surd ccAcBq\\ \surd ccAcBq\\ c\\ \hline \surd ccAcBq\\ \hline\end{array} \qquad \surd ccAc)\ \surd ccAcBq\ (\surd ccBq$$

Conſectaries.

Here, as in Diviſion of Simple Surds, theſe Conſectaries are fitly inſerted.

1. To take half, &c.

1. To divide any Irrational *Species*, is to diminiſh him by the Power of a Root Homogeneal.

Examples.

Irational.	*The Half.*	*The third Part.*	*The fourth Part.*
Squares.	$\frac{\surd q12B}{4}(\ \surd q3B$	$\frac{\surd q27B}{9}(\ \surd q3B$	$\frac{\surd q48B}{16}(\ \surd q3B$
Cubes.	$\frac{\surd c24B}{8}(\ \surd c3B$	$\frac{\surd c81B}{27}(\ \surd c3B$	$\frac{\surd c192B}{64}(\ \surd c3B$

2. Quotient ſome time Rational.

2. To divide ſome Irrationals, brings forth Rationals in the Quotient. And the Quotients, as occaſion is, may be cleared of the Surd Character.

Examples.

$$\text{Irrational.}\quad \overset{\text{Square.}}{\frac{\sqrt{q}27BE}{\sqrt{q}3B}}\Big(\sqrt{q}9E=3E\quad \text{Rational.}$$

$$\text{Irrational.}\quad \overset{\text{Cube.}}{\frac{\sqrt{c}24DC}{\sqrt{c}\,3\,C}}\Big(\sqrt{c}8D=2D\quad \text{Rational.}$$

3. To divide any *Irrational Species* by himſelf, giveth in the Quotient one *Irrational Species* or Unit.

3. Irrational dividing himſelf gives 1.

Examples.

$$\text{Examples in}\quad \overset{\text{Squares.}}{\frac{\sqrt{q}B}{\sqrt{q}B}}\Big(\,1,\ \text{or } B.\qquad \overset{\text{Cubes.}}{\frac{\sqrt{c}9B}{\sqrt{c}9B}}\Big(\,1,\ \text{or } B.$$

4. To divide a Power whoſe Index is compounded, by one Side of the Compounding Powers, ſhall give the Quotient higher or lower according to the Dividing Power.

4. Quotient of a Power, &c. *divided by the Side.*

Examples.

As $\sqrt{q}B)\,\sqrt{cc}B\,(\sqrt{c}B$. And $\sqrt{c}B)\,\sqrt{cc}B\,(\sqrt{q}B$.

5. To divide a Figural Number by another Homogeneal figural Number, the Quotient ſhall be a figural Number of the ſame Kind, whoſe Side is equal to the Quotient of the greater Side divided by the Leſſer.

5. Homogeneal Figurals divided, what the Quotient.

Example.

$$\text{As } \sqrt{q}\frac{AqEq}{Bq}\Big(\frac{AE}{B}.$$

6. To divide the Sides of Homogeneal Irrational *Species*, begetteth Sides of Irrational *Species* Homogeneal.

6. Sides of ſuch divided, what begotten.

Example.

Ergo, $\sqrt{q}A)\,\sqrt{q}AE\,(\sqrt{q}E$.

Diviſion of Simple Irrational Species, is proved by their Multiplication; and by taking Rational *Species* or Numbers inſtead of Irrational, and working therewith. As in the laſt Example, and ſeveral others of this Chapter made exemplary in Multiplication, bring here in the Quotient one of the Factors there.

Proof of Diviſion of Simple Irrat. Species.

Alſo ſuppoſe A 25 and E 9, then $\sqrt{q}A$ which is 5 multiplied into $\sqrt{q}E$ that is 3, will be 15 for $\sqrt{q}AE$: And this divided by $\sqrt{q}A$ that is 5, giveth 3 in the Quotient equal to $\sqrt{q}E$.

CHAP. XVII. *Addition of Compound Irrational Species.*

ADdition of Irrational *Species* that are Compound, includes the Addition of Particulars and Univerſals.

Addition of Comp. Irrat. Species. Particulars.

Particulars, like or unlike, are added as the Simple, and joined together into one Total with the reſpective Signs.

Examples.

Examples.	*Binomials.*	*Reſiduals.*	*Mixt.*
Addends.	$Z+\sqrt{q}BC$ $R+\sqrt{q}DE$	$\sqrt{q}B-3D$ $\sqrt{q}B-2C$	$\sqrt{c}Rq+6B$ $\sqrt{c}Rq-6B$
Totals.	$Z+R+\sqrt{q}BC+\sqrt{q}DE$	$\sqrt{q}4B-3D-2C$	$\sqrt{c}8Rq$

Another Example.

$$\text{Addends}\begin{cases}\frac{Z}{2}\overset{*}{+}\sqrt{q}\frac{Zq-4P}{4}\\ \frac{Z}{2}\overset{*}{-}\sqrt{q}\frac{Zq-4P}{4}\end{cases}$$

$$\text{Total.}\quad \underline{\overline{Z}}$$

Here the plain and fracted Siniſter *Species* being Z and Z, that is 2Z to be divided by 2, makes Z only to be left for the Total. All the dexter Part of the Addends being of contrary Signs, *viz.* one + and the other −, and ſo Equal, are taken away.

Univerſals are added as other Compound Irrationals, with reſpect had to their Signs, and the Sign Univerſal $\sqrt{:}$ prefixed, as was largely diſcourſed in the Addition of Univerſal Surds before.

Univerſals.

Examples.

Examples.	*Binomials.*	*Reſiduals.*	*Mixt.*
Addends.	$\surd{:}\ Zq+4Pq$ $\surd{:}\ Zq+4Pq$	$\surd{:}\ Zq-4Pq$ $\surd{:}\ Zq-4Pq$	$\surd{:}\ Zq+4Pq$ $\surd{:}\ Zq-4Pq$
Totals.	$\surd{:}4Zq+16Pq$	$\surd{:}4Zq-16Pq$	$\surd{:}2Zq+\surd q4Zqq-64Pqq$

The two firſt of theſe Examples are wrought according to the 7*th* Direction in Addition of Univerſal Surds, *Chap.* 7. *Book* 3. *Part* 5. but the laſt according to the Second Form of Addition of Square Surds, *Chap.* 3. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

Other Examples in Species *and Numbers correſponding.*

$$\text{Suppoſing}\left\{\begin{array}{l}A\ 3.\ E\ 2.\ Z\ 5.\ P\ 6.\\ Aq\ 9.\ Eq\ 4.\ Z\ 13.\ X\ 5.\end{array}\right.$$

$$\begin{array}{l}\surd{:}\ Z+\surd q\ Zq-4Pq\\ \surd{:}\ Z+\surd q\ Zq-4Pq\\ \hline \surd{:}4Z+\surd q16Zq-64Pq\end{array}\qquad \left.\begin{array}{l}\surd{:}13+\surd q\ 169-144\\ \surd{:}13+\surd q\ 169-144\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd{:}13+\surd q25\\ \surd{:}13+\surd q25\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd q18\\ \surd q18\\ \hline \surd q72\end{array}\right.$$

$$\begin{array}{l}\surd{:}52+\surd q2704-2304\\ \hline 20\quad -\underline{2304}\ \ \surd\\ \underline{}\quad\ 400\,(\,20\\ \surd q72\ \textit{Value}\ \dot{}\ \dot{}\end{array}$$

$$\begin{array}{l}\surd{:}\ \frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4Pq}{4}\\ \surd{:}\ \frac{Z}{2}+\surd q\frac{Zq-4Pq}{4}\\ \hline \surd{:}\frac{4Z}{2}+\surd q\frac{16Zq-64Pq}{4}\\ \hline \text{or}\ \surd{:}2Z+\surd q4Zq-16Pq\end{array}\qquad \left.\begin{array}{l}\surd{:}\tfrac{13}{2}\overset{*}{+}\surd q\frac{169-144}{4}\\ \surd{:}\tfrac{13}{2}+\surd q\frac{169-144}{4}\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd{:}\tfrac{13}{2}+\surd q\tfrac{25}{4}\\ \surd{:}\tfrac{13}{2}+\surd q\tfrac{25}{4}\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd q\tfrac{18}{2}=3\\ \surd q\tfrac{18}{2}=3\\ \hline 6\end{array}\right.$$

$$\begin{array}{l}\surd{:}\tfrac{52}{2}+\surd q\frac{2704-2304}{4}\\ \hline \surd{:}26+\surd q\ 676-576\\ \hline 10\quad -\underline{576}\ \ \surd\\ \underline{}\quad\ 100\,(\,10\\ \surd q36\\ \quad (6\ \textit{Value.}\end{array}$$

$$\begin{array}{l}\surd{:}\ Z-\surd q\ Zq-4Pq\\ \surd{:}\ Z-\surd q\ Zq-4Pq\\ \hline \surd{:}4Z-\surd q16Zq-64Pq\end{array}\qquad \left.\begin{array}{l}\surd{:}\ 13-\surd q\ 169-144\\ \surd{:}\ 13-\surd q\ 169-144\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd{:}13-\surd q25\\ \surd{:}13-\surd q25\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd q8\\ \surd q8\\ \hline \surd q32\end{array}\right.$$

$$\begin{array}{l}\surd{:}\ 52-\surd q2704-2304\\ \hline -20\quad -\underline{2304}\ \ \surd\\ \underline{}\quad\ 400\,(\,20\\ \surd q\ 32\\ \quad (\textit{Value.}\end{array}$$

$$\begin{array}{l}\surd{:}\ \frac{Z}{2}\overset{*}{-}\surd q\frac{Zq-4Pq}{4}\\ \surd{:}\ \frac{Z}{2}-\surd q\frac{Zq-4Pq}{4}\\ \hline \surd{:}\frac{4Z}{2}-\surd q\frac{16Zq-64Pq}{4}\\ \hline \text{Or}\ \surd{:}2Z-\surd q4Zq-16Pq\end{array}\qquad \left.\begin{array}{l}\surd{:}\tfrac{13}{2}\overset{*}{-}\surd q\frac{169-144}{4}\\ \surd{:}\tfrac{13}{2}-\surd q\frac{169-144}{4}\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd{:}\tfrac{13}{2}-\surd q\tfrac{25}{4}\\ \surd{:}\tfrac{13}{2}-\surd q\tfrac{25}{4}\end{array}\right\}\text{or}\left\{\begin{array}{l}\surd q\tfrac{8}{2}=2\\ \surd q\tfrac{8}{2}=2\\ \hline 4\end{array}\right.$$

$$\begin{array}{l}\surd{:}\tfrac{52}{2}-\surd q\frac{2704-2304}{4}\\ \hline \surd{:}26-\surd q676-576\\ \hline -10\quad -\underline{576}\ \ \surd\\ \underline{}\quad\ 100\,(\,10\\ \surd q16\\ \quad (4\ \textit{Value.}\end{array}$$

All theſe four laſt Examples are wrought according to the 7*th* Direction in Addition of Univerſal Surds, *Chap.* 7. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

$$\begin{array}{l}
\surd{:}Z+\surd q\,Zq-4Pq\\
\surd{:}Z-\surd q\,Zq-4Pq\\
\hline
\surd{:}2Z+4P
\end{array}
\quad
\left.\begin{array}{l}
\surd{:}13+\surd q\,169-144\\
\surd{:}13-\surd q\,169-144\\
\hline
\surd{:}\ 26+24
\end{array}\right\}\text{or}\left\{\begin{array}{l}
\surd{:}13+\surd q25\\
\surd{:}13-\surd q25
\end{array}\right\}\text{or}\left\{\begin{array}{l}
\surd q18\\
\surd q\ 8\\
\hline
\surd q50
\end{array}\right.$$

$$\begin{array}{l}
\surd{:}\frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4Pq}{4}\\
\surd{:}\frac{Z}{2}-\surd q\frac{Zq-4Pq}{4}\\
\hline
\surd{:}\ Z+2P
\end{array}
\quad
\left.\begin{array}{l}
\surd{:}\frac{13}{2}\overset{*}{+}\surd q\frac{169-144}{4}\\
\surd{:}\frac{13}{2}-\surd q\frac{169-144}{4}\\
\hline
\surd{:}\ 13+12
\end{array}\right\}\text{or}\left\{\begin{array}{l}
\surd{:}\frac{13}{2}+\surd q\frac{25}{4}\\
\surd{:}\frac{13}{2}-\surd q\frac{25}{4}
\end{array}\right\}\text{or}\left\{\begin{array}{l}
\surd q\frac{18}{2}=3\\
\surd q\frac{8}{2}=2\\
\hline
\qquad 5
\end{array}\right.$$

The Work of theſe two laſt Examples is like the ſecond Form of Addition of ſquare Surds, *Chap.* 3. *Book* 3. *Part* 5. For firſt the Surds are added, which in the upper Example make 2Z, in the lower $\frac{2Z}{2}$, that is Z being divided; then they are multiplied, the upper Product is Zq the Siniſter part, and —Zq+4Pq the Dexter part, the lower Product is $\frac{Zq}{4}-\frac{Zq+4Pq}{4}$: for according to the third Conſectary in Multiplication of Simple Surds, and alſo of Simple Irrational *Species*, the Surds multiplied by themſelves have their Characters cancelled: So as √qZq—4Pq multiplied by it ſelf, ſhall be Zq—4Pq; but in the Examples above it is Zq+4Pq, becauſe the Signs of the given Numbers were + and —. The Products thus gotten, the Parts found with + and — of like Denomination are ſet aſide; ſo as of the upper Product is left only 4Pq, of the lower $\frac{4Pq}{4}$, which divided is Pq. The Roots of theſe are 2P and P; which multiplied by 2, the Index of the Square, make 4P to be added to the upper 2Z, and 2P to the lower Z.

Another Example explained with Numbers.

$$\begin{array}{l}
\surd{:}\surd\frac{Xq+4Pq}{4}\overset{*}{+}\frac{X}{2}\\
\surd{:}\surd\frac{Xq+4Pq}{4}-\frac{X}{2}\\
\hline
\surd{:}\surd Xq+4Pq+2P
\end{array}
\quad
\left.\begin{array}{l}
\surd{:}\frac{25+144}{4}\overset{*}{+}\frac{5}{2}\\
\surd{:}\frac{25+144}{4}-\frac{5}{2}\\
\hline
\surd{:}25+144+12\\
\hline
\frac{144}{169}\left(\begin{array}{l}\surd\\13+12\end{array}\right.
\end{array}\right\}\text{or}\left\{\begin{array}{l}
\surd q\frac{169}{4}+\frac{5}{2}\\
\surd q\frac{169}{4}-\frac{5}{2}
\end{array}\right\}\text{or}\left\{\begin{array}{l}
\surd q\ \frac{18}{2}\ (3\\
\surd q\ \frac{8}{2}\ (2\\
\hline
\qquad 5
\end{array}\right.$$

The Total of this Example is miſprinted in *Moore*, being there √Xqq+Pq+2P; which cannot be, for the Addition is $\frac{4Xq+16Pq}{4}$, and by Diviſion to clear the Fraction, is Xq+4Pq; the Multiplication is thus, $\frac{Xq+4Pq}{4}-\frac{Xq}{4}$, and by cancelling the contrary *Species*, is left only $\frac{4Pq}{4}$; which by Diviſion is Pq, whoſe Root is P; this multiplied by 2, is 2P to be added to the firſt Addition, as I have ſet it.

Proof of Addition of Compound Irrat. Species.

Addition of Compound Irrational Species is proved by Subſtraction, or by Rational Numbers, like Addition of Compound Surds before; the Work of Particulars by Particulars, and Univerſals by Univerſals: And many of theſe Examples have their Rational Numbers beſides them to ſave farther Inſtance.

CHAP. XVIII. *Subſtraction of Compound Irrational Species.*

Subſtraction of Comp. Irrat. Species. Particulars.

As Addition before, ſo *Subſtraction of Compound Irrational Species*, includes the Subſtraction both of Particulars and Univerſals.

Particulars, as the Simple Irrationals take like *Species* from like, and to the Remain their proper Sign is ſubſcribed unleſs the greater be taken from the leſſer, for then the Sign is to be changed: And if the Signs be contrary, though the

Species be alike, then the Sign of the Sum is to be contrary to the Sign of the Subtrahend. Alſo *Species* unlike and Aſymmetral as before, are ſubſtracted, by conjoining them with —.

Examples.

Examples.	*Binomials.*	*Reſiduals.*	*Mixt.*
Greater Numbers.	$Z+\surd qSR$	$\surd qB-8$	$\surd cRq+4D$
Subtrahends.	$R+\surd qAE$	$\surd qB-4$	$\surd cRq-4D$
Remains.	$Z-R+\surd qSR-\surd qAE$	4	$8D$

Another Example.

Greater Number. $\frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4P}{4}$

Subtrahend. $\frac{Z}{2}\overset{*}{-}\surd q\frac{Zq-4P}{4}$

Remain. $\surd q\frac{4Zq-16P}{4}$

Or $\surd qZq-4P$

In this Example the ſiniſter Parts of the *Data* being both alike and equal, *viz.* $\frac{Z}{2}$, are ſet aſide; the Reſidue being of contrary Signs, are added together; and afterward Diviſion by 4 being made to diſcharge the Fraction, the Remain is $\surd qZq-4P$.

Univerſals. Univerſals, like other Compound Irrationals, are ſubſtracted with reſpect to their Signs, and the Sign Univerſal prefixed, as before diſcourſed in Surds.

Examples.

Examples.	*Binomials.*	*Reſiduals.*	*Mixt.*
Greater Numbers.	$\surd{:}4\text{Ƶ}q+16Pq$	$\surd{:}4\text{Ƶ}q-16Pq$	$\surd{:}2\text{Ƶ}q+\surd q4\text{Ƶ}qq-64Pqq$
Subtrahends.	$\surd{:}\ \text{Ƶ}q+4Pq$	$\surd{:}\ \text{Ƶ}q-4Pq$	$\surd{:}\ \text{Ƶ}q-4Pq$
Remains.	$\surd{:}\ \text{Ƶ}q+4Pq$	$\surd{:}\ \text{Ƶ}q-4Pq$	$\surd{:}\ \text{Ƶ}q+4Pq$

The firſt two of theſe Examples are wrought according to the 7*th* Direction in Subſtraction of Univerſal Surds, *Chap.* 8. *Book* 3. *Part* 5. but the laſt according to the ſecond Form of Subſtraction of Square Surds, *Chap.* 4. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

Other Examples in Species and Numbers correſponding.

Suppoſing $\left\{\begin{array}{llll} A\ 3. & E\ 2. & Z\ 5. & P\ 6. \\ Aq\ 9. & Eq\ 4. & \text{Ƶ}\ 13. & X\ 5. \end{array}\right.$

$$\begin{array}{ll} \surd{:}\frac{\text{Ƶ}}{2}\overset{*}{+}\surd q\frac{\text{Ƶ}q-4Pq}{4} & \surd{:}\frac{13}{2}\overset{*}{+}\surd q\frac{169-144}{4} \\ \surd{:}\frac{\text{Ƶ}}{2}-\surd q\frac{\text{Ƶ}q-4Pq}{4} & \surd{:}\frac{13}{2}-\surd q\frac{169-144}{4} \\ \hline \surd{:}\text{Ƶ}-2P & \surd{:}13-12 \end{array}\left.\right\}\text{or}\left\{\begin{array}{l} \surd{:}\frac{13}{2}+\surd q\frac{25}{4} \\ \surd{:}\frac{13}{2}-\surd q\frac{25}{4} \end{array}\right\}\text{or}\left\{\begin{array}{l} \surd q\frac{18}{2}=3 \\ \surd q\frac{8}{2}=2 \\ \hline 1 \end{array}\right.$$

$$\begin{array}{ll} \surd{:}\surd\frac{Xq+4Pq}{4}\overset{*}{+}\frac{X}{2} & \surd{:}\frac{25+144}{4}\overset{*}{+}\frac{5}{2} \\ \surd{:}\surd\frac{Xq+4Pq}{4}-\frac{X}{2} & \surd{:}\frac{25+144}{4}-\frac{5}{2} \\ \hline \surd{:}\surd Xq+4Pq-2P & \surd{:}25+144-12 \\ & \frac{144}{169}\ \surd\ (13-12 \end{array}\left.\right\}\text{or}\left\{\begin{array}{l} \surd{:}\frac{169}{4}+\frac{5}{2} \\ \surd{:}\frac{169}{4}-\frac{5}{2} \end{array}\right\}\text{or}\left\{\begin{array}{l} \surd q\frac{18}{2}=3 \\ \surd q\frac{8}{2}=2 \\ \hline 1 \end{array}\right.$$

The Work of theſe two laſt Examples may be ſeen in the precedent Chapter of their Addition; between which and their Subſtraction, all the Difference is to connex the multiplied Root 2P, to the Sum of the Addition of the Siniſter part of the *Data*, with the Sign of Subſtraction —.

Another Example explained with Numbers.

$$\begin{array}{ll} \surd{:}\frac{2\text{Ƶ}q-4Pq}{4}\overset{*}{+}\surd q\frac{4\text{Ƶ}qq-16Pq\text{Ƶ}q}{16} & \surd{:}\frac{338-144}{4}\overset{*}{+}\surd q\frac{114244-97344}{16} \\ \surd{:}\frac{2\text{Ƶ}q-4Pq}{4}-\surd q\frac{4\text{Ƶ}qq-16Pq\text{Ƶ}q}{16} & \surd{:}\frac{338-144}{4}-\surd q\frac{114244-97344}{16} \\ \hline \surd{:}\ \text{Ƶ}q-4Pq & \surd{:}169-144 \\ \hline & \surd q25=5 \end{array}$$

The Work of this laſt Example in *Moore* is thus to be underſtood: The Addition at firſt being $\frac{4Zq-8Pq}{4}$, by Diviſion is brought to $Zq-2Pq$. The Multiplication is $\frac{4Zqq-16ZqPq+16Pqq}{16}-\frac{4Zqq+16PqZq}{16}$; and abating 16 the Denominator, becauſe alike in both Parts, they are reduced to Integers, and by cancelling the contrary *Species*, reſteth only $\frac{16Pqq}{16}$, and by Diviſion Pqq; whoſe Root Pq multiplied by 2, the ſquare Index is 2Pq, which is to be adjoined to the firſt Addition: So will the Remain be $\surd{:}Zq-2Pq-2Pq$, that is $\surd{:}Zq-4Pq$. But if the Total of thoſe *Data's* had been required, it would have been $\surd{:}Zq-2Pq+2Pq$, that is $\surd qZq$; and by Reduction only Z, and in Numbers thus:

$$\begin{array}{r}338\\-144\\\hline 194\end{array}\qquad \begin{array}{r}114244\\-97344\\\hline 16900\end{array}\overset{\surd}{\Big(}\;\frac{130}{16\big(\;4}\qquad \begin{array}{l}\frac{194}{4}+\frac{130}{4}=\frac{324}{4}\Big(\overset{\surd}{\frac{18}{2}}\Big(9\\[2ex]\frac{194}{4}-\frac{130}{4}=\frac{64}{4}\Big(\frac{8}{2}\Big(4\end{array}$$

$$\begin{array}{rl}\hline 13 & \text{Total.}\\\hline 5 & \text{Difference.}\\\hline\end{array}$$

Proof of Subſtraction of Comp. Irrat. Species.

Subſtraction of Compound Irrational Species, Particulars and Univerſals reſpectively, is proved as Subſtraction of Compound Surds, both by Addition and by Rational Numbers. Enough of which may be ſeen by the Examples of this Chapter compared with thoſe in the former of Addition.

CHAP. XIX. *Multiplication of Compound Irrational Species.*

Multiplication of Comp. Irrat. Species.

IN the Multiplication of theſe Compound Irrationals, Particulars agree with the Multiplication of particular Compound Surds; and Univerſals here with Univerſals there.

Particulars.

Particulars therefore are like to be multiplied with like, or elſe reduced thereto, and the Signs + and — ordered as in the Multiplication of Compound Coſſicks.

Examples.

Examples.	*Binomials.*	*Reſiduals.*	*Mixt.*
Multiplicands.	$\surd qB+\surd qC$	$\surd qD-\surd qB$	$\surd cK+\surd cP$
Multipliers.	$\surd qB+\surd qD$	$\surd qB-\surd qD$	$\surd cK-\surd cP$
	$B+\surd qBC$	$\surd qBD-B$	$\surd cKq+\surd cKP$
	$\surd qBD+\surd qDC$	$-D+\surd qBD$	$-\surd cKP-\surd cPq$
Products.	$B+\surd qBC+\surd qBD+\surd qDC$	$\surd q4BD-B-D$	$\surd cKq-\surd cPq$

Another Example out of Moore *explained with Numbers.*

Another Example explained with Numbers.

Suppoſing A 3. E 2. Z 5. P 6.

$$\begin{array}{ll}\text{Multiplicand.} & \frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4P}{4}\\[1ex] \text{Multiplier.} & \frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4P}{4}\\\hline & \frac{Zq}{4}\overset{*}{+}\surd q\frac{Zqq-4ZqP}{16}\\[1ex] & \qquad\surd q\frac{Zqq-4ZqP}{16}\overset{*}{+}\frac{Zq-4P}{4}\\\hline \text{Product.} & \frac{2Zq-4P}{4}\overset{*}{+}\surd q\frac{4Zqq-16ZqP}{16}\\\hline\end{array}$$

By breaking the Line into two Segments, the two Squares thereof are $\frac{Zq}{4}+\frac{Zq-4P}{4}$, that is, $\frac{2Zq-4P}{4}$ each of the two Rectangles is $\surd q\frac{Zqq-4ZqP}{16}$, which added, after the manner of Surds, by their multiplication with 4, make the Product as in the ordinary way of Multiplication here ſet down.

The Explanation by Numbers.

$$\begin{array}{ll}
\text{Multiplic.} & \tfrac{5}{2} \overset{*}{+} \surd q\,\frac{25-24}{4} \\
\text{Multiplier.} & \tfrac{5}{2} \overset{*}{+} \surd q\,\frac{25-24}{4} \\ \hline
 & \tfrac{25}{4} \overset{*}{+} \surd q\,\frac{625-600}{16} \\
 & \qquad \surd q\,\frac{625-600}{16} \overset{*}{+} \frac{25-24}{4}
\end{array}
\quad \text{or} \left\{\begin{array}{l} \tfrac{5}{2}+\surd q\tfrac{1}{4} \\ \tfrac{5}{2}+\surd q\tfrac{1}{4} \end{array}\right\} \text{or} \left\{\begin{array}{l} \tfrac{6}{2}\,(3 \\ \tfrac{6}{2}\,(3 \\ \hline \quad 9 \ \text{Product.} \end{array}\right.$$

$$\text{Product.}\quad \frac{50-24}{4} \overset{*}{+} \surd q\,\frac{2500-2400}{16} \ \text{or} \left\{\frac{26}{4}+\surd q\,\frac{100}{16}\right\} \text{or} \left\{\frac{26}{4}+\frac{10}{4}\right\} \text{or} \left\{\frac{36}{4}\,(9\right.$$

Universals. Universals Homogeneal, as Surds before, are to be multiplied one into the other alternately, the Siniſter Numbers and *Species* being Figurate, according to the Denomination of the Dexter into which they are multiplied ; and Heterogeneals according to the Reduction.

Examples in Homogeneals.

$$\begin{array}{lll}
\textit{Examples.} & \textit{Binomials.} & \textit{Residuals.} \\
\text{Multiplicands.} & \surd{:}B+\surd qH & \surd{:}D-\surd qI \\
\text{Multipliers.} & \surd{:}D+\surd qK & \surd{:}B-\surd qI \\ \hline
 & BD+\surd qDqH & DB-\surd qBqI \\
 & \qquad \surd qBqK+\surd qHK & \qquad -\surd qDqI+\surd qIq \\ \hline
\text{Products.} & \surd{:}BD+\surd qDqH+\surd qBqK+\surd qHK & \surd{:}DB-\surd qBqI-\surd qDqI+\surd qIq
\end{array}$$

$$\begin{array}{ll}
 & \textit{Mixt.} \\
\text{Multiplicand.} & \surd{:}D+\surd qP \\
\text{Multiplier.} & \surd{:}\,B-\surd qR \\ \hline
 & BD+\surd qBqP \\
 & \qquad -\surd qDqR-\surd qPR \\ \hline
\text{Product.} & \surd{:}BD+\surd qBqP-\surd qDqR-\surd qPR.
\end{array}$$

Examples in Heterogeneals, Binomial and Residual.

Examples in Heterogeneals.

$$\begin{array}{lll}
\text{Multiplicands.} & \surd{:}B+\surd cD & \surd{:}B-\surd cD \\
\text{Multipliers.} & \surd{:}B+\surd cD & \surd{:}B-\surd cD \\ \hline
 & Bq+\surd ccDqBcc & Bq-\surd ccDqBcc \\
 & \qquad \surd ccDqBcc+\surd cDq & \qquad -\surd ccDqBcc+\surd cDq \\ \hline
\text{Products.} & \surd{:}Bq+\surd cc64DqBcc+\surd cDq & \surd{:}Bq-\surd cc64DqBcc+\surd cDq
\end{array}$$

The following Examples by the former Suppoſition in this Chapter, are explained by Numbers.

Examples explained by Numbers.

$$\surd\!:\frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4Pq}{4} \qquad\qquad \surd\!:\frac{13}{2}\overset{*}{+}\surd q\frac{169-144}{4}\;.$$

$$\surd\!:\frac{Z}{2}-\surd q\frac{Zq-4Pq}{4} \qquad\qquad \surd\!:\frac{13}{2}-\surd q\frac{169-144}{4}$$

$$\frac{Zq}{4}+\surd q\frac{Zqq-4ZqPq}{16} \qquad\qquad \frac{169}{4}+\surd q\frac{28561-24336}{16}$$

$$-\surd q\frac{Zqq-4ZqPq}{16}-\frac{Zq-4Pq}{4} \qquad\qquad -\surd q\frac{28561-24336}{16}-\frac{169-144}{4}$$

$$\surd q\frac{4Pq}{4}\text{ or }\surd qPq\text{, or }P \qquad\qquad \surd q\frac{144}{4}\Big(\frac{12}{2}^{\surd}\Big(6$$

$$\surd\!:\frac{Z}{2}\overset{*}{+}\surd q\frac{Zq-4Pq}{4} \qquad\qquad \surd\!:\frac{13}{2}\overset{*}{+}\surd q\frac{169-144}{4}$$

$$\surd\!:\frac{Z}{2}+\surd q\frac{Zq-4Pq}{4} \qquad\qquad \surd\!:\frac{13}{2}+\surd q\frac{169-144}{4}$$

$$\frac{Zq}{4}+\surd q\frac{Zqq-4ZqPq}{16} \qquad\qquad \frac{169}{4}+\surd q\frac{28561-24336}{16}$$

$$\surd q\frac{Zqq-4ZqPq}{16}+\frac{Zq-4Pq}{4} \qquad\qquad \surd q\frac{28561-24336}{16}+\frac{169-144}{4}$$

$$\surd\!:\frac{2Zq-4Pq}{4}\overset{*}{+}\surd q\frac{4Zqq-16ZqPq}{16} \qquad\qquad \surd\!:\frac{338-144}{4}\overset{*}{+}\surd q\frac{114244-97344}{16}$$

$$\begin{array}{r} 338 \\ -144 \\ \hline 194 \\ \hline 4 \end{array} \qquad \begin{array}{r} 114244 \\ -97344 \\ \hline 16900 \\ \hline 16 \end{array}\Big(\frac{130}{4}^{\surd} \qquad \frac{194}{4}+\frac{130}{4}=\frac{324}{4}\Big(\frac{18}{2}^{\surd}\Big(9\Big(3\surd$$

$$\frac{194}{4}+\frac{130}{4}=\frac{324}{4}\Big(\frac{18}{2}\Big(9\Big(\underline{3\surd}$$

$$\underline{9}\ \text{Product.}$$

Multiplication of Compound Irrational Species, both Particulars and Universals, like Multiplication of Compound Surds before, will be proved by Division and by Rational Numbers, as already hath been sufficiently seen.

Proof of Multiplication of Com. Irrat. Species.

CHAP. XX. *Division of Compound Irrational Species.*

Division of Com. Irrat. Species. Particulars.

THE *Division of Compound Irrationals*, takes in Particulars and Universals.

Particulars are to be divided by a Mixture of Division of *Species* and Compound Surds, variegated as the *Case* requires.

Case 1. When the Divisor is Simple, and the Dividend Compound.

1. Divisor Simple.

Examples.

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\surd qB)$	$\surd qBS+\surd qBD$	$(\surd qS+\surd qD$
Residual.	$\surd qC)$	$\surd qBC-\surd qBD$	$(\surd qB-\surd q\frac{BD}{C}$
Mixt.	$3B)$	$6B+\surd q54BC-\surd c81BAD$	$(2B+\surd q6C-\surd c3AD$

These Examples are like the first in Division of Compound Surds.

Case 2. When the Divisor is Compound, and the Dividend Simple.

2. Dividend Simple.

Examples.

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\surd qB+\surd qC)$	$\surd qD$	$\Big(\frac{\surd qD}{\surd qB+\surd qC}$
Residual.	$\surd qB-\surd qC)$	$\surd qB$	$\Big(\frac{\surd qB}{\surd qB-\surd qC}$
Mixt.	$\surd qB+\surd qC-\surd qD)$	$\surd qC$	$\Big(\frac{\surd qC}{\surd qB+\surd qC-\surd qD}$

The Quotients of theſe Examples are ſet as Fractions, like thoſe in the *5th Caſe* of Diviſion of Integral and Rational *Species*; for to divide *Compound Irrational Species*, it is requiſite the *Species* be Homogeneal, as well as the Figurate Quantities to be divided.

3 Diviſor reduced to a Simple. *Caſe* 3. When the Diviſor is Compound, but both Parts being equal, may be reduced to a Simple as the Dividend is.

Examples.	*Diviſor.*	*Reduced.*	*Dividend.*	*Quotient.*
Binomial.	$\sqrt{q}B+\sqrt{q}B$)	$\sqrt{q}4B$)	$\sqrt{q}8Bq$	($\sqrt{q}2B$
Reſidual.	$\sqrt{q}4B-\sqrt{q}B$)	$\sqrt{q}B$)	$\sqrt{q}Bc$	($\sqrt{q}Bq$
Mixt.	$\sqrt{q}4B-\sqrt{q}4B+D$)	D)	$\sqrt{q}BD$	($\sqrt{q}B$

The Work of theſe Examples is according to the third *Caſe* of Diviſion of Compound Surds.

4 Data Compound. *Caſe* 4. When both Diviſor and Dividend are Compound.

Examples.

$$\begin{array}{llll}
 & \textit{Diviſor.} & \textit{Dividend.} & \textit{Quotient.} \\
\text{Binomial.} & \sqrt{q}B+\sqrt{q}D) & \sqrt{q}Bq+\sqrt{q}BC+\sqrt{q}BD+\sqrt{q}DC & (\sqrt{q}B+\sqrt{q}C \\
 & \underline{\sqrt{q}B+\sqrt{q}C} & \sqrt{q}Bq + \sqrt{q}BD & \\
 & \sqrt{q}Bq+\sqrt{q}BD & \quad\sqrt{q}BC+ + \sqrt{q}DC & \\
 & \quad\sqrt{q}BC+\sqrt{q}DC & & \\
\hline
 & & \sqrt{q}BD & \\
\text{Reſidual.} & \sqrt{q}B-\sqrt{q}D) & \sqrt{q}4BD-B-D & (\sqrt{q}D-\sqrt{q}B \\
 & \underline{\sqrt{q}D-\sqrt{q}B} & \sqrt{q}BD - D & \\
 & \sqrt{q}BD-D & \sqrt{q}BD-B & \\
 & \quad -B+\sqrt{q}BD & & \\
\hline
 & & \quad -\sqrt{c}KP & \\
\text{Mixt.} & \sqrt{c}K+\sqrt{c}P) & \sqrt{c}Kq-\sqrt{c}Pq & (\sqrt{c}K-\sqrt{c}P \\
 & \underline{\sqrt{c}K-\sqrt{c}P} & \sqrt{c}Kq+\sqrt{c}KP & \\
 & \sqrt{c}Kq+\sqrt{c}KP & \quad -\sqrt{c}KP-\sqrt{c}Pq & \\
 & \quad -\sqrt{c}KP-\sqrt{c}Pq & & \\
\hline
\end{array}$$

Theſe Examples are wrought according to the *6th Caſe* of Diviſion of Compound Surds.

5. Heterogeneal. *Caſe* 5. When the Quantities are Heterogeneal, reduce them to like Denominations before Diviſion.

Examples.

$$\begin{array}{lcccc}
 & \textit{Diviſor.} & & \textit{Dividend.} & \textit{Quotient.} \\
 & & & B+\sqrt{q}4BD+D & \\
\text{Binomial.} & \sqrt{q}B+\sqrt{q}D & \Bigg) & \text{Reduced.} & \Bigg(\sqrt{q}B+\sqrt{q}D \\
 & & & \sqrt{q}Bq+\sqrt{q}4BD+\sqrt{q}Dq & \\
 & \textit{Diviſor.} & & \textit{Dividend.} & \textit{Quotient.} \\
 & \sqrt{q}B-\sqrt{q}D & & \sqrt{cc}Bc-\sqrt{cc}Dc & \\
\text{Reſidual.} & \text{Reduced.} & \Bigg) & \text{Reduced.} & \Bigg(\sqrt{cc}Bc-\sqrt{cc}Dc \\
 & \sqrt{cc}Bc-\sqrt{cc}Dc & & \sqrt{cc}Bcc-\sqrt{cc}64BcDc+\sqrt{cc}Dcc &
\end{array}$$

In theſe two Examples the Reduction agrees with that of Surds and Irrational *Species*, and the Diviſion according to the ſecond *Caſe* of Diviſion of the Simple Irrationals in both, is performed after the Reduction, as the next precedent *Caſe* of this Chapter.

6. Fractionary. *Caſe 6.* When the *Species* given are Fractionary, the Work is mixed after the manner of Fractions and Surds both.

Example.

$$\begin{array}{ccc}
\textit{Diviſor.} & \textit{Dividend.} & \textit{Quotient.} \\
\frac{Z}{2}\overset{*}{+}\sqrt{q}\frac{Zq-4P}{4}\Bigg) & \frac{2Zq-4P}{4}\overset{*}{+}\sqrt{q}\frac{4Zqq-16ZqP}{16}\Bigg(& \frac{4Zq-8P}{4Z}\overset{*}{+}\sqrt{q}\frac{16Zqq-64ZqP}{16Zq-64P}
\end{array}$$

Explained in Numbers.

$$\frac{5}{2}\overset{*}{+}\sqrt{q}\frac{25-24}{4}\Bigg)\frac{50-24}{4}\overset{*}{+}\sqrt{q}\frac{2500-2400}{16}\Bigg(\frac{100-48}{20}\overset{*}{+}\sqrt{q}\frac{10000-9600}{400-384}$$

The Quotients being depreſſed by Reduction in *Species*, may be brought to $\frac{Z}{2}\overset{*}{+}\sqrt{q}\frac{Zq-4P}{4}$, and in Numbers to $\frac{100-48}{20}\overset{*}{+}\sqrt{q}\frac{400}{16}$, or $\frac{52}{20}+\frac{20}{4}$. But as in Multiplication of this Dividend in the foregoing Chapter, the Denominator 2 of the Siniſter part, was led into the Numerator of the Dexter part, contrary to the manner of other Fractionary Multiplication. So here this Denominator 4 is to be doubled and made Numerator to 20; which $\frac{8}{20}$ added to $\frac{52}{20}$, make $\frac{60}{20}$, that is 3 Integers for the Value of the Root of the Quotient. And this is worthy to be noted in other Examples of like Nature in Particular Irrationals, when as noted by the Aſteriſque any Quantity Compound is affirmed or denied of a Simple, or a Simple of a Compound. But in others the Diviſion of Fracted Irrationals agreeth with the Diviſion of Fractionary Surds and *Species* mixt: And will appear plainer in this Example, if there be made two Diviſions thereof, as in effect there is, *viz.* the Siniſter part of Dividend and Diviſor for one, and the Dexter part for the other.

Univerſals. Homogeneal

Diviſion of Univerſal *Species* Homogeneal, is like the Diviſion of Univerſal Homogeneal Surds; and according to the *4th Caſe* of Diviſion of *Species* in this Chapter, only before the Quotient prefix the Sign Univerſal, and upon every Removal of the Diviſor figurate the Siniſter Number thereof according to the Dexter Number to which he is applied; and in multiplying the Diviſor by the Quotienary *Species*, the Multiplication muſt be alſo proper to Univerſals.

Example.

Diviſor. *Dividend.* *Quotient.*

√:B+√qH) √:BD+√qDqH+√qBqK+√qHK (√:D+√qK

D+√qK BD+√qDqH BqK+√qHK

BD+√qDqH

BqK+√qHK

Heterogeneals.

In Heterogeneals, beſides Figuration of the Siniſter part of the Diviſor as aforeſaid, if the Quotienary *Species* gotten on the ſecond Removal of the Diviſor be of higher Denomination than the next Dexter Number of the Diviſor, this Quotienary *Species* ought to be depreſſed in Quantity by extracting the Root and abating the Index, and this ſhall be the true Quotienary Number.

Example.

Diviſor. *Dividend.* *Quotient.*

DqBcc

√:B+√cD) √:Bq+√cc64DqBcc+√cDq (√:B+√cD

B+√cD Bq+√ccDqBcc

Bq+√ccDqBcc DqBcc+√cDq

√ccDqBcc+√cDq

Here beſides figurating the diviſing B to Bcc, when applied to DqBcc, becauſe Dq will be gotten thereby for the Quotient, which is a Power above D the next Number of the Diviſor, therefore the Root D ſhall be ſet in the Quotient with the Cube Index.

Example explained by Numbers.

Another Example.

√:B+√qD) √:Bq+√qBqD+√ccBccCq+√cCqDc (√:B+√cC

Explained in Numbers.

√:2+W4) √:4+W16+√ʒφ4096+√ʒφ4096 (√:2+ww8.

Proof of Diviſion of Comp. Irrat. Species.

The Proof of Diviſion of theſe Compound Irrationals, is by their Multiplications, or by Rational Numbers, that is to ſay, Particular by Particular, and Univerſal by Univerſal, as is to be ſeen before in Diviſion of Surds, and may further be beheld in theſe two laſt Examples, where the Dividends are returned, by multiplying the Diviſors into the Quotients reſpectively, and ſo may others be examined.

Species.

√:B+√cD

√:B+√cD

Bq+√ccBccDq

√ccBccDq+√cDq

√:Bq+√cc64BccDq+√cDq

Numbers ſuppoſed.

√:2+ww8 2+2=4

√:2+ww8 2+2=4

4+√ʒφ4096 W16

√ʒφ4096+ww64

√:4+√ʒφ262144+ww64

4 + 8 + 4

W16 = 4

Species.	*Numbers ſuppoſed.*	
√:B+√qD	√:2+w4	2+2=4
√:B+√cC	√:2+ww8	2+2=4
Bq+√qBqD	4+w16	w16
√ccBccCq+√ccDcCq	√ʒφ4096+√ʒφ4096	
√:Bq+√qBqD+√ccBccCq+√ccDcCq.	4+w16+√ʒφ4096+√ʒφ4096	
	4+4+4+4	
	w16=4	

CHAP. XXI. *Figuration of Irrational Species.*

Figurate Irrational Species produced.

As was obſerved before in Figuration of Surds; ſo here it may be remembred that any *Species* ſimply Irrational, multiplied figurately, produceth a Rational: But Compound Irrational *Species* may be ſquared, cubed, *&c.* by multiplying them figurately as other Figural Numbers are multiplied; but the *Unciæ* to the Parodical Degrees here, will be the Squares of them in Rational *Species*. See the Examples that follow.

	Simple.		*Particular.*	*Univerſal.*
Root	√qB	Root	√qB+√qD	√:B+√qD
	√qB		√qB+√qD	√:B+√qD
Square	√qBq		B+√qBD	Bq+√qBqD
	√qB		√qBD+D	√qBqD+√qDq
Cube	√qBc	Square	B+√q4BD+D	√:Bq+√q4BqD+√qDq
			√qB+√qD	√:B+√qD
			√qBc+√q4BqD+√qBDq	Bc+√q4BqqD+√qBqDq
			√qBqD+√q4BDq+√qDc	√qBqqD+√q4BqDq+√qDc
		Cube	√qBc+√q9BqD+√q9BDq+√qDc.	√:Bc+√q9BqqD+√q9BqDq+√qDc

Explained in Numbers, ſuppoſing B 4. *and* D 9.

	Particular.		*Univerſal.*	
Root	w4+w9	2+3=5 ℨ	√: 4+w9	4+3=7ℨ
	w4+w9	2+3=5	√: 4+w9	4+3=7
	4+w36	25ʒ	16+w144	49√7
	w36+9	5	w144+w81	
Square	4+w144+9	125φ	√:16+w576+w81	
	4+12+9=25ʒ		16+24+9=49√7	
	w4+w9		√:4+w9	
	w64+w576+w324		64+w9216+w1296	
	w144+w1296+w729		w2304+w5184+w729	
Cube	w64+w1296+w2916+w729		√:64+w20736+w11664+w729	
	8+36+54+27=125φ		√:64+144+108+27=343√7	

Roots of Irrational Species extracted.

Extraction of the Roots of Irrational *Species* is performed like the Extraction of Surd Roots, at large diſcuſſed *Chap.* 12. of the precedent part of this third Book, with a mixture of the Extraction of Roots in Coſſicks: So as a Retroſpection thither may ſave a large Repetition of the Rules here in the different *Caſes* occurrent.

1. *Simple.*

Caſe 1. If the Surd be Simple, he hath no Root to be otherwiſe expreſſed, than by prefixing the proper Character before the *Species*.

Example.

As to extract the Square Root of B, or the Cube Root of B+D, they are ſet thus:

√qB   √cB+D

But any Rational *Species* ſet as an Irrational, may have the Root extracted.

As the Square Root of $\surd qBq$ is B.
And the Cube Root of $\surd cBc$ is B.

2. *Particular Irradical.*

Caſe 2. If any particular Compound *Species* Irradical in the *Species*, have the Siniſter *Species* abſolute, then prefix before the ſame the proper Character for the Root to be extracted.

Example.

As to extract the Square Root of $B + \surd qD$, then it is to be ſet thus: $\surd qB + \surd qD$.

But if the Siniſter *Species* be figurate, then multiply the Index thereof by the Index of the Root to be extracted.

As to extract the Square Root of $\surd qB + \surd qD$, it ſhall be $\surd qqB + \surd qD$.

3. *Particular Radical if formally Rational.*

Caſe 3. If the Compound *Species* be not Irradical, then are they formally Rational or Irrational: And if they be formally Rational, the Root may be extracted after the manner of Coſſicks, keeping the Addition and Subſtraction of the Multiplees as in Surds. But becauſe Operations in *Species* keep the Prime and Original *Species* for the moſt part throughout the Work, and in all formal Figuration only increaſe their Quantities; therefore it is eaſy to ſee the Root in any Figural Quantity in *Species* of never ſo high a Power.

Example.

As in the former Examples in this Chapter of
$\surd qBc + \surd q9BqD + \surd q9BDq + \surd qDc$.
$\surd{:}\ Bc + \surd q9BqqD + \surd q9BqDq + \surd qDc$.
there being no other *Species* uſed ſave the Notes of Quantities B and D, it is eaſy to diſcern, the Root did conſiſt of B and D.

4. *Particular or Univerſal, if not formally Rational.*

Caſe 4. If the Particular or Univerſal Compound *Species* be not formally Rational, then figurate the Siniſter Part as the Dexter, and from thence take the Dexter, add the Square Root of the Difference to the Siniſter, and ſubſtract it therefrom, half the Sum and half the Difference joined with + ſhall be the Binomial Root, and with — ſhall be the Reſidual Root.

Examples.

As to extract the ſquare Root of $\frac{2Zq - 4P}{4} \overset{*}{+} \surd q \frac{4Zqq - 16ZqP}{16}$

Siniſter Part ſquared is $\frac{4Zqq - 16ZqP + 16P}{16}$. From whence if the

Dexter Part $\frac{4Zq4 - 16ZqP}{16}$ be ſubſtracted, then will the

Remain be 16P, whoſe Root is 4P; and this Root when

Added to the Siniſter part, makes the Sum $\frac{2Zq - 8P}{4}$

Subſtracted therefrom, makes the Difference $\frac{2Zq}{4}$

Half the Sum.	*Half the Difference.*
$\frac{Zq - 4P}{4}$	$\frac{Zq}{4}$ or $\frac{Z}{2}$

Binomial Root $\frac{Z}{2} + \surd q \frac{Zq - 4P}{4}$

Reſidual Root $\frac{Z}{2} - \surd q \frac{Zq - 4P}{4}$

So to extract the Root of $\surd{:}\,\frac{2Z,q-4Pq}{4} \overset{*}{+} \surd q\,\frac{4Z,qq-16Z,qPq}{16}$

$\frac{4Z,qq-16Z,qPq+16Pqq}{16}$ Siniſter part ſquared.

$\frac{4Z,qq-16Z,qPq}{16}$ Dexter part ſubſtracted.

$16Pqq$ Remain. $4Pq$ Root.

$\frac{2Z,q-8Pq}{4}$ Sum. $\frac{2Z,q}{4}$ Difference.

$\frac{Z,q-4Pq}{4}$ Half Sum. $\frac{Z,q}{4}$ or $\frac{Z,}{2}$ Half Difference.

Root $\frac{Z,}{2}+\surd q\,\frac{Z,q-4Pq}{4}$ Binomial. $\frac{Z,}{2}-\surd q\,\frac{Z,q-4Pq}{4}$ Reſidual.

Proof of Extraction of Roots of Irrat. Species.

Theſe Extractions are to be proved by Production of their Figurals, as thoſe Productions by theſe Extractions. And alſo by Rational Numbers, working with them inſtead of the Irrationals; as by the laſt Example explained in Numbers will appear.

Species $\surd{:}\,\frac{2Z,q-4Pq}{4} \overset{*}{+} \surd q\,\frac{4Z,qq-16Z,qPq}{16}$

Numbers $\surd{:}\,\frac{338-144}{4} \overset{*}{+} \surd q\,\frac{114244-97344}{16}$

Siniſter Part ſquared $\frac{114244-97344+20736}{16}$

Dexter Part ſubſtracted $\frac{114244-97344}{16}$

Remain 20736 | Root 144

Sum $\frac{338-288}{4}$ Half $\frac{169-144}{4}$

Difference $\frac{338}{4}$ Half $\frac{169}{4}$ or $\frac{13}{2}$

Binomial Root $\surd{:}\,\frac{13}{2}+\surd q\,\frac{169-144}{4}$

Reſidual Root $\surd{:}\,\frac{13}{2}-\surd q\,\frac{169-144}{4}$

$$\begin{array}{llll}
 & 338-144 & & \\
 & 338-144 & & \\
\hline
 & 2704-1152+576 & & \\
1014 & 432 & 576 & \\
1014 & 432 & 144 & \\
\hline
114244 & -48672+ & & \\
 & -48672 & +20736 & \\
\hline
114244 & -97344 & +20736 & \\
\hline
\end{array}$$

And thus much may ſuffice in this place to be ſpoken of *Species*, and in them of the Simple Elements of both Abſtract and Contract Numbers.

Partis ſextæ & Libri tertii

FINIS.

ARITHMETICK.

The Fourth BOOK.

CONCERNING
Numbers Proportional, Abſtract and Contract.

In Four PARTS.

WHEREIN

Ratio's *Proportions diſjunct* *Proportions continued* *Aequations*	are	Deciphered. Diſſected. Computed. Enodated.

And their *Comparative* ELEMENTS.

CHAP. I.

Of RATIO'S.

Numbers in their common Nature as Relational, and their Comparative Elements, treated of in this Fourth Book.

HAVING thus in the three former Books run through the *Simple Elements* of *Numbers*, in their proper Nature, *viz. Abſtract*, and generally and ſpecially *Contract*: I now come to review them in their common Nature, as they are Relational, and uncover their *Comparative Elements*; which ſhew the Compariſon of Numbers among themſelves, the Deſcription whereof takes up this Chapter, and the Computation the reſt of this Book.

Deſcription, *what.*

Deſcription gives us an Account of the ſeveral Species or Kinds of Compariſon.

Computation, *what.*

Computation is an Account of the ſeveral Operations belonging to theſe Species.

Compariſon *twofold.*

Compariſon of Numbers, ſhews what Likeneſs of Relation there is between the Numbers compared; and is twofold, *Ratio*, or *Proportion*.

Ratio, *what. How ſometime called.*

Ratio is a Compariſon of Terms, and is, when the Relation or Conference is extended, but to two Numbers or Magnitudes only, as 21 to 12, or 3 to 6, or any ſuch-like: This is ſometimes promiſcuouſly called *Proportion.*

Proportion, *what, and how called.*

Proportion is properly a Compariſon of *Ratio's*, and is when the Conference reacheth unto many Numbers: This is ſometimes called *Proportionality*, and often *Analogy.*

Ratio twofold. Every Relation between two Numbers, is either *Ratio* of Equality or Inequality.

Ratio of Equality. *Ratio* of Equality, is when one Number is compared to himſelf, as 5 to 5, always agreeing in Unity.

Ratio of Inequality. *This twofold.* *Ratio* of Inequality, is when one Number is compared to another different from him: Wherein alſo there is a double Conference, *viz.* *Ratio* of the Greater, and *Ratio* of the Leſſer Inequality.

Greater Inequality alſo twofold. *Ratio* of the Greater Inequality, is when the greater Number is compared to the Leſſer; as 6 to 3, or 5 to 2, *&c.* And this is of two Sorts, either Simple and Prime, or Compound and Conjunct.

Prime, and this, Prime, or Simple *Ratio* of the greater Inequality, is of two ſorts.

1. *Containeth the Leſſer once and a Part.* The firſt is, when the greater Number containeth the Leſſer more than once, and not twice, but once and a Part more, called generally by the Latin Name, *Superparticularis*, or a Part more: For that one Part of the leſſer Number, is the juſt Difference or Exceſs betwixt it and the Greater. Specially, and for diſtinction-ſake, each receiveth his Name from that Part it containeth: As if it contain the Half more, then it is called *Seſquialtera*, as 3 to 2.

A farther view of ſuch *Ratio's*.

Examples.

Seſquialtera ———	as 3 to 2.	6 to 4, &c.	1 $\frac{1}{2}$	an Half more.
Seſquitertia ———	as 4 to 3.	8 to 6, &c.	1 $\frac{1}{3}$	a Third more.
Seſquiquarta ———	as 5 to 4.	10 to 8, &c.	1 $\frac{1}{4}$	a Fourth more.
Seſquiquinta ———	as 6 to 5.	12 to 10, &c.	1 $\frac{1}{5}$	a Fifth more.
Seſquiſexta ———	as 7 to 6.	14 to 12, &c.	1 $\frac{1}{6}$	a Sixth more.
Seſquiſeptima ———	as 8 to 7.	16 to 14, &c.	1 $\frac{1}{7}$	a Seventh more.
Seſquioctava ———	as 9 to 8.	18 to 16, &c.	1 $\frac{1}{8}$	an Eighth more.
Seſquinona ———	as 10 to 9.	20 to 18, &c.	1 $\frac{1}{9}$	a Ninth more.
Seſquidecima ———	as 11 to 10.	22 to 20, &c.	1 $\frac{1}{10}$	a Tenth more.
Seſquiundecima ———	as 12 to 11.	24 to 22, &c.	1 $\frac{1}{11}$	an Eleventh more.
&c.				

2. *Containeth the leſſer once, and ſome Parts.* The ſecond Simple ſort is, when the Difference is 2, 3, or more Parts of the whole; this is generally called *Superpartiens*, or Parts more, intimating above one Part; but ſpecially every one hath his proper Name, according to his Content, as the former had. For if 5 be compared to 3, then is it called *Superbipartiens-tertias*, becauſe it containeth the Whole, and $\frac{2}{3}$ of the Whole. And ſo may other Names be given infinitely. Some Examples whereof follow.

Examples.

Superbipartiens.	*Tertias* ———	as 5 to 3.	10 to 6, &c.	1 $\frac{2}{3}$	Two Parts more.
	Quintas- ———	as 7 to 5.	14 to 10, &c.	1 $\frac{2}{5}$	
	Septimas ———	as 9 to 7.	18 to 14, &c.	1 $\frac{2}{7}$	
	Nonas ———	as 11 to 9.	22 to 18, &c.	1 $\frac{2}{9}$	
	Undecimas ———	as 13 to 11.	26 to 22, &c.	1 $\frac{2}{11}$	
	Decimas tertias--	as 15 to 13.	30 to 26, &c.	1 $\frac{2}{13}$	
	&c.				
Supertripartiens.	*Quartas*-- ———	as 7 to 4.	14 to 8, &c.	1 $\frac{3}{4}$	Three Parts more.
	Quintas ———	as8 to 5.	16 to 10, &c.	1 $\frac{3}{5}$	
	Septimas ———	as 10 to 7.	20 to 14, &c.	1 $\frac{3}{7}$	
	Octavas- ———	as 11 to 8.	22 to 16, &c.	1 $\frac{3}{8}$	
	Decimas - ———	as 13 to 10.	26 to 20, &c.	1 $\frac{3}{10}$	
	Undecimas ———	as 14 to 11.	28 to 22, &c.	1 $\frac{3}{11}$	
	Decimas tertias--	as 16 to 13.	32 to 26, &c.	1 $\frac{3}{13}$	
	&c.				
Superquadrupartiens.	*Quintas* ———	as 9 to 5.	18 to 10, &c.	1 $\frac{4}{5}$	Four Parts more.
	Septimas ———	as 11 to 7.	22 to 14, &c.	1 $\frac{4}{7}$	
	Nonas- ———	as 13 to 9.	26 to 18, &c.	1 $\frac{4}{9}$	
	Undecimas ———	as 15 to 11.	30 to 22, &c.	1 $\frac{4}{11}$	
	Decimas tertias--	as 17 to 13.	34 to 26, &c.	1 $\frac{4}{13}$	
	Decimas quintas-	as 19 to 15.	38 to 30, &c.	1 $\frac{4}{15}$	
	&c.				

Superquintupartiens.	*Sextas*	as 11 to 6.	22 to 12, &c.	$1\frac{5}{6}$	Five Parts more.
	Septimas	as 12 to 7.	24 to 14, &c.	$1\frac{5}{7}$	
	Octavas	as 13 to 8.	26 to 16, &c.	$1\frac{5}{8}$	
	Nonas	as 14 to 9.	28 to 18, &c.	$1\frac{5}{9}$	
	Undecimas	as 16 to 11.	32 to 22, &c.	$1\frac{5}{11}$	
	Duodecimas	as 17 to 12.	34 to 24, &c.	$1\frac{5}{12}$	
	&c.				

Here is to noted, that thoſe *Ratio's* which ſome have ſet to fill up the Complement of the other, as *Superbipartiens-Secundas*, *Quartas*, *Sextas*, *Octavas*, &c. and *Supertripartiens-Secundas*, *Tertias*, *Sextas*, *Nonas*, &c. are all Irregular; for properly there are no ſuch *Ratio's*; but thoſe are *Ratio's* that fall under ſome other Name agreeable thereto: As 6 to 4, called by ſome *Superbipartiens-Quartas*, is indeed *Seſquialtera*. So 10 to 6, called by ſome *Superquadrupartiens-Sextas*, is properly no other than *Superbipartiens-Tertias*. The like is to be underſtood of many others: For as in Common Fractions, it is moſt regular to ſet them in their leaſt Terms; ſo in *Ratio's*: wherefore 10 to 6 ſhall be accounted $1\frac{2}{3}$, not as $1\frac{4}{6}$.

Compound, and this

Compound, otherwiſe called *Conjunct Ratio's* of the greater Inequality, are of three Kinds.

1. *Containeth the Leſſer many times.*

The firſt is, when the greater Number containeth the Leſſer, divers times generally called *Multiplex*, or *Manifold*; and particularly named, according to the Times that the leſſer Number is contain'd in the Greater: So that if it contain it twice, then it is called *Dupla*, or *Double*, as 2 to 1, *&c.* A farther Account of their Names follows.

Examples.

Dupla	as 2 to 1.	4 to 2, &c.	$\frac{2}{1}$	*Duple*, or Double.
Tripla	as 3 to 1.	6 to 2, &c.	$\frac{3}{1}$	*Triple*, or Threefold.
Quadrupla	as 4 to 1.	8 to 2, &c.	$\frac{4}{1}$	*Quadruple*, or Fourfold.
Quintupla	as 5 to 1.	10 to 2, &c.	$\frac{5}{1}$	*Quintuple*, or Fivefold.
&c.				

2. *Containeth the Leſſer many times, and a Part.*

The ſecond Sort of *Ratio's* of the greater Inequality compound, is named *Multiplex-Superparticularis*; which importeth, that the greater Number containeth the leſſer many times, and a Part more, as 5 to 2, which contains 2 twice and a half more, and therefore is called *Dupla-Seſquialtera*. Theſe Kinds of *Ratio's* may be diverſly divided, as into *Double*, *Triple*, *Quadruple*, &c. and every of them into their ſeveral Subdiviſions.

As for Inſtance.

Examples.

Duplex	*Seſquialtera*	as 5 to 2.	10 to 4, &c.	$2\frac{1}{2}$
	Seſquitertia	as 7 to 3.	14 to 6, &c.	$2\frac{1}{3}$
	Seſquiquarta	as 9 to 4.	18 to 8, &c.	$2\frac{1}{4}$
	&c.			
Triplex	*Seſquialtera*	as 7 to 2.	14 to 4, &c.	$3\frac{1}{2}$
	Seſquitertia	as 10 to 3.	20 to 6, &c.	$3\frac{1}{3}$
	Seſquiquarta	as 13 to 4.	26 to 8, &c.	$3\frac{1}{4}$
	&c.			
Quadruplex	*Seſquialtera*	as 9 to 2.	18 to 4, &c.	$4\frac{1}{2}$
	Seſquitertia	as 13 to 3.	26 to 6, &c.	$4\frac{1}{3}$
	Seſquiquarta	as 17 to 4.	34 to 8, &c.	$4\frac{1}{4}$
&c.	&c.			

3. *Containeth the Leſſer many times, and ſome Parts.*

The third Sort is called *Multiplex-Superpartiens*, and implieth, that the greater Number containeth the Leſſer divers times, and ſome Parts thereof beſides: And are likewiſe diſtinguiſhed into *Double*, *Triple*, &c. Some of theſe, with their Subdiviſions, appear in the following Examples.

Dupla

Examples.

Dupla	*Superbipartiens*	*Tertias*	as 8 to 3, &c.	$2\frac{2}{3}$
		Quintas	as 12 to 5, &c.	$2\frac{2}{5}$
		Septimas	as 16 to 7, &c.	$2\frac{2}{7}$
		&c.		
	Supertripartiens	*Quartas*	as 11 to 4, &c.	$2\frac{3}{4}$
		Quintas	as 13 to 5, &c.	$2\frac{3}{5}$
		Septimas	as 17 to 7, &c.	$2\frac{3}{7}$
		&c.		
	Superquadripartiens	*Quintas*	as 14 to 5, &c.	$2\frac{4}{5}$
		Septimas	as 18 to 7, &c.	$2\frac{4}{7}$
		Nonas	as 22 to 9, &c.	$2\frac{4}{9}$
		&c.		
Tripla	*Superbipartiens*	*Tertias*	as 11 to 3, &c.	$3\frac{2}{3}$
		Quintas	as 17 to 5, &c.	$3\frac{2}{5}$
		&c.		
	Supertripartiens	*Quartas*	as 15 to 4, &c.	$3\frac{3}{4}$
		Quintas	as 18 to 5, &c.	$3\frac{3}{5}$
		&c.		
	Superquadripartiens	*Quintas*	as 19 to 5, &c.	$3\frac{4}{5}$
		Septimas	as 25 to 7, &c.	$3\frac{4}{7}$
		&c.		
Quadrupla	*Superbipartiens*	*Tertias*	as 14 to 3, &c.	$4\frac{2}{3}$
		Quintas	as 22 to 5, &c.	$4\frac{2}{5}$
		&c.		
	Supertripartiens	*Quartas*	as 19 to 4, &c.	$4\frac{3}{4}$
		Quintas	as 23 to 5, &c.	$4\frac{3}{5}$
		&c.		
	Superquadripartiens	*Quintas*	as 24 to 5, &c.	$4\frac{4}{5}$
		Septimas	as 32 to 7, &c.	$4\frac{4}{7}$
	&c.	&c.		

Leſſer Inequality twofold.

Ratio of the leſſer Inequality, is when the leſſer Number is conferred to the Greater, as 3 to 5, or 2 to 6, *&c.*

Containing a Part.

Theſe *Ratio's* are alſo divided into two Sorts, *viz.* either ſuch as contain a Part of the Number only, as one Third, one Fourth, *&c.* as 1 to 3, and 4 to 16, *&c.*

Containing ſome Parts.

Or ſuch as contain many Parts of the greater Number, as Three Quarters, Four Fifths, *&c.* as 3 to 4, and 4 to 5, *&c.*

How differenced in the Names from the other.

Both theſe kinds of *Ratio's* of the leſſer Inequality, have the ſame Names which the *Ratio's* of the greater Inequality had; ſave that to the beginning of every Name *Sub* is to be adjoined, as *Subdupla*, *Subtripla*, *Subſeſquialtera*, &c. Examples of both Sorts follow.

Examples of both.

Examples of the firſt Sort.

Subdupla	as 1 to 2.	2 to 4, &c.	$\frac{1}{2}$	one Part.
Subtripla	as 1 to 3.	2 to 6, &c.	$\frac{1}{3}$	
Subquadrupla	as 1 to 4.	2 to 8, &c.	$\frac{1}{4}$	
&c.				

Examples of the ſecond Sort.

Subſeſquialtera	as 2 to 3.	4 to 6, &c.	$\frac{2}{3}$	many Parts.
Subſeſquitertia	as 3 to 4.	6 to 8, &c.	$\frac{3}{4}$	
Subſeſquiquarta	as 4 to 5.	8 to 10, &c.	$\frac{4}{5}$	

Ratio's properly converſing with Abſtract Numbers, appear in the Front of Relational Numbers, and ſo are the Subject of the firſt Part of this 4*th* Book.

Proportion diſſected into Simple and Compound.

Proportion, as was ſaid before, is referred to more than two Numbers, and therein may be a Conference of the former ſeveral *Ratio's* in their ſeveral Terms: But as converſing both with Abſtract and Contract Numbers, and ſo beſt befitting Arithmetick, the uſual Diſſection of all Analogy, is into Simple and Compound Proportion.

Simple into Diſcontinual and Continual.

Simple, becauſe the Numbers ſo compared, need make no uſe of the Signs + or —. This is alſo divided into Diſcontinual and Continual Proportion.

Diſcontinual twofold.

Diſcontinual, called alſo *Disjunct Proportion*, is when there is an Equality of the Difference or *Ratio* between ſome of the Terms given; but not current through all, either ſort ſeldom exceeding four Terms.

This kind of Proportion is double, either Arithmetical or Geometrical.

Arithmetical, what.

Arithmetical Diſcontinual Proportion is, when the Equality of the Difference is not continued alike throughout all the Terms, but the Difference of the firſt and ſecond Numbers is ſomewhere diſtracted. As 4 . 7 : 5 . 8 . where 3, the Difference between 4 and 7, is diſcontinued between 7 and 5.

Hence ariſe Muſical Proportions.

What they are.

How to make them.

Hence aroſe thoſe Proportions called *Muſical*, which Mr. *Blundevil*, and ſome others, make a third Species: And as Mr. *Oughtred*, in Chap. 6. of his *Clavis* tells us is, when in 4 Numbers; As the Firſt is to the Fourth, ſo is the Difference of the Firſt and Second, to the Difference of the Third and Fourth. As 5. 8. 12. 30. are Muſical Proportions, becauſe 5. 30 : : 8 — 5. 30 — 12 : : 3. 18. Alſo in Species, A, M, N, E; let A. E : : M — A. E — N. Wherefore A E — A N = M E — A E in theſe Terms duly ordered, the Rule ſhall be $\frac{AN}{2A-M} = E$ and $\frac{EM}{2E-N} = A$. That is, If the Product of the Firſt and Third be divided by the Exceſs of the Firſt doubled above the Second, the Quotient ſhall be a Fourth in Muſical Proportion.

Geometrical Proportion, what.

Geometrical Diſcontinual Proportion is, when the *Ratio* is diſtracted: As 5. 15 : : 6. 18. where the *Subtriple Ratio* between 5 and 15, agreeth not with the *Ratio Duplaſeſquialtera* between 15 and 6.

This twofold.

This ſort of Proportion is either Plain or Figurate.

Plain.

Plain or Simple, becauſe the Number found thereby, agreeth in the ſimple Nature of the *Data*.

How ſorted by Euclid.

How here.

Such as theſe, by the 12, 13, 14, 15, and 16 Definitions of the 5*th* Book of *Euclid*, may be divided into *Alternate*, *Inverſe*, *Compounded*, *Divided*, and *Converſe*; but as moſt ſutable to *Arithmetick*, they may be divided into *Direct* or *Indirect*.

Direct what, and what founded thereon.

Direct is, when the Term by which the Queſtion is made, (which is the Third) by how much it is greater than the Firſt, by ſo much it requireth a Fourth Number greater than the Second: And by how much it is leſſer, by ſo much it requireth a Leſſer. On this is founded the *Golden Rule Direct*: For of 3, Numbers given, if the Second multiply the Third, and that Product be divided by the Firſt, the Quotient ſhall be a Fourth Proportional to the three given Numbers.

Examples.

Example when the Greater requireth a Greater. If 7 give 28, (being *quadrupled*) What ſhall 9 give? *facit* 36; which is in proportion to 28, as 9 is to 7.

Example when the Leſſer requireth a Leſſer. If 6 give 4, (being diminiſhed by $\frac{1}{3}$) what ſhall 3 give? *facit* 2; for 2 is in proportion to 4, as 3 is to 6.

Indirect what, how called, and what founded thereon.

Indirect, called alſo *Reciprocal* or *Reverſed Proportion*, is, when the Term by which the Queſtion is made, by how much it is greater than the Firſt, by ſo much it requireth a Fourth Number leſſer than the Second: And by how much it is leſſer, by ſo much it requireth a Greater. And on this is bottomed the *Backward Rule of Three*, or *Golden Rule Reverſed*: For of three Numbers given, if the Firſt multiply the Second, and that Product be divided by the Third, the Quotient ſhall be a Fourth, proportional to the three given Numbers.

Examples.

Example when the Greater requireth a Leſſer. If 4 give 10, then ſhall 8 give 5; becauſe as 8 is double to 4, ſo the Double of 5 is 10.

Example when the Leſſer requireth a Greater. If 4 give 10, then ſhall 2 give 20; for 20 is double to 10, as 4 is to 2.

Figurate Proportions *how to be underſtood here.*

Figurate Proportions here, are not to be taken ſo much for any Simple *Ratio* or Proportion that is between Figural Numbers *Homogeneal* or *Heterogeneal*, or any of their Complements or Parodical Degrees mentioned before *in Species*; nor yet only for the Proportional Figural Numbers, found out by the *Direct* or *Indirect* Analogy aforeſaid; but the Proportions uſed about Geometrical Figures; and together with theſe, ſuch as are diſcovered by the Operations proper to the Figural Numbers themſelves, or depending thereupon. And theſe receive Names according to the *Index* of the Figural Numbers they deal with; as if Squares, then are they called *Doubled Proportions*; if with Cubes, *Tripled Proportions*; if with Squared-Squares, *Quadrupled Proportions*, &c.

All theſe Plain and Figurate-Geometrical-Diſcontinual Proportions, with their farther Subdiviſions, and the Iſſues and Operations to them properly belonging, fill up the Second Part of this 4*th* Book, in the Computation thereof.

The other Sort of Proportions before-mentioned, are *Continual.*

Continual Pro*portion twofold.*

Continual, otherwiſe called *Conjunct Proportion*, is, when three or more Numbers bear like Proportion in their Progreſſion: So as well the Second Number may be referred to the Third, as the Firſt to the Second. And the ſame Difference, or *Ratio*, ſhall be between the two laſt Terms, as between the two Firſt.

This kind of Proportion is alſo both *Arithmetical* and *Geometrical.*

Arithmetial, *what.*

Arithmetical Continual Proportion is, when between every two Numbers, or Terms, the Difference or Exceſs is equal, as 2, 4, 6, 8, &c. where the Exceſs 2 is continued throughout all the Terms, *&c.*

Geometrical, *what.*

Geometrical Continual Proportion is, when between every two Numbers, or Terms, the *Ratio* is equal, as 4, 8, 16, 32, &c. where the *Ratio* 2 is continued throughout all the Terms.

Both called Progreſſion.

Theſe two Sorts deal with plain Numbers eſpecially, yet the latter in a manner figurates the firſt Terms; both are called *Progreſſion*, and have their uſeful Operations computed in the Third Part of this 4*th* Book.

Compound *Proportion how otherwiſe called.*

Compound Proportions hold Community eſpecially with Contract Numbers; make uſe of the Signs + and —, and are thoſe called *Æquations*; that is, Numbers equal to others. This may be called *Proportion of Equality.* And of theſe there are two principal Sorts.

Æquations of *two ſorts.*

Firſt, *Pure*, when one Number is compared as equal to another.

Secondly, *Mixt*, (commonly called *Affected*) when one or divers Numbers are compared in Equality to divers others.

Examples of Pure.

Pure { Example in *Geodædicals* – 1 *s.* = 12 *d.*
Example in *Coſſicks* — 2♉ = 6ℨ Root 3.

Examples in *Abſtract* Numbers.

Examples of Affected.

Mixt {
4 = 2 + 2   4 = 6 — 2
4 + 2 = 3 + 3   4 — 2 = 3 — 1

Examples in Contract Numbers.

1 Φ = 2♉ + 3ℨ   1 Φ = 4♉ — 3ℨ
1Φ + 1♉ = 9ℨ + 9 N   1 Φ — 1♉ = 7ℨ — 3 N   Root 3.
1Φ + 1♉ = 13ℨ – 3N   1 Φ — 1♉ = 5ℨ + 8 N

Theſe, as the moſt profound Part of *Arithmetick*, occupy the laſt Part of this Fourth Book, and cloſe up the whole Survey of this Numbering Art.

Notes of Ratio's.

Touching *Ratio's* is here further to be noted.

1. Antecedent *&* Conſequent.

1*ſt*, As in *Fractions* there is *Numerator* and *Denominator*: So in *Ratio's* there are two Terms; the firſt whereof is called the *Antecedent*, and the ſecond the *Conſequent.*

2. Ratio's, *how expreſſed.*

2*dly*, Though *Ratio's* are ſet commonly one before another, as in the Inſtances before-mentioned; yet for better conveniency in working, they are alſo ſet one over another like Fractions. And by ſome, to diſtinguiſh them from Fractions, inſtead of the intervening Line, two Pricks are ſet; and ſo the *Ratio Seſquialtera* is thus expreſſed $\overset{3}{\underset{2}{:}}$

3. Ratio's *are Commenſurable, or Incommenſurable.*

3*dly*, *Ratio's*, as common Fractions before, are either Commenſurable, or Incommenſurable. For if the two Terms compared have any common Part, that will equally divide them both: then they are Commenſurable, as 12 to 21; becauſe 3, a Part of them, is a Common Diviſor to both. But on the contrary, if the Numbers have no ſuch Part for a Common Diviſor; they are Incommenſurable, as 18 to 25: for 25 can evenly be divided by no Number but 5; and 18 cannot be divided equally thereby.

4. Ratio *and Proportion promiſcuouſly uſed.*

4*thly*, The words *Ratio* and *Proportion* may be found promiſcuouſly uſed one for the other in good Authors; which the curious cannot ſtumble at, ſince they agree in the Genus, for *Ratio* is a Single Proportion, and Proportion but Plural *Ratio's.*

5. *Conjunct* Ratio's *have their Names ſo.*

5*thly*, In Conjunct *Ratio's*, the Names are Conjunct; as $3\,\overset{1}{\underset{4}{:}}$ *Triple-Seſquiquarta.*

6thly, The Difference between two Numbers, is the Diſtance of a Number from a Number, and is found by *Subſtraction*: But the *Ratio* is the containing of a Number in a Number, and is found by *Diviſion*, and therefore to be underſtood differently.

6. *Difference and Ratio how they differ.*

7thly, The ſame Difference between Terms may fall out, and the *Ratio* divers. As 6 to 3, the *Ratio* is double, and the Difference 3: But 12 to 9, the *Ratio* is *Seſquitertia*, yet the Difference 3 as before.

7. *Terms alike in Difference, not in* Ratio.

8thly, If one Number ſhall multiply two Numbers, the Products ſhall be proportional. And if one Number ſhall divide two Numbers, the Quotients alſo ſhall be proportional. As if 4 multiply 7 and 9, the Products 28 and 36 will be alike proportional: And if 28 and 36 be divided by 4, their Quotients 7 and 9 will be alike proportional.

8. *A Number multiply or divide two others, the Products and Quotients equal.*

$$\text{Multiplier } 4\left\{\begin{matrix}7.\ 28\\ 9.\ 36\end{matrix}\right. \qquad \text{Diviſor } 4\Big)\begin{matrix}28\,(7\\ 36\,(9\end{matrix} \quad \frac{9}{7}\Big(1\tfrac{2}{7} \quad \frac{36}{28}\Big(1\tfrac{2}{7}$$

Example in Species.

$$A\times\left\{\begin{matrix}B.\ BA\\ C.\ CA\end{matrix}\right. \qquad A\Big)\begin{matrix}BA.\ (B\\ CA.\ (C\end{matrix}$$

9thly, Mr. *Oughtred* in his *Clavis*, Chap. 6. adds; If the Conſequents of two *Ratio's* are equal, they are as the Antecedents: but if the Antecedents are equal, they are reciprocal as the Conſequents.

9. *What follows if the Terms are equal.*

$$\text{As } \frac{7}{1}\,.\,\frac{9}{1} :: 7\,.\,9 \qquad \text{And } \frac{1}{7}\,.\,\frac{1}{9} :: 7\,.\,9$$

10thly, The *Ratio* of the Antecedent to the Conſequent, is compounded either of the *Ratio* of the Antecedent to the Third, and of the Third to the Conſequent; or of the *Ratio* of the Third to the Conſequent, and of the Antecedent to the Third.

10. Ratio *of the Terms, of what compounded.*

$$\text{As } 7\,.\,9 :: \times\left\{\begin{matrix}7A\\ A9\end{matrix}\right. \qquad \text{Alſo } 7\,.\,9 :: \times\left\{\begin{matrix}A9\\ 7A\end{matrix}\right.$$

CHAP. II.

Reduction of RATIO'S.

THE Deſcription of *Relational Numbers* paſſed in the former Chapter, *Computation* comes next on the Stage.

Computation of Relational Numbers.

Ratio's as they are ſingle Proportions, ſo they challenge the Simple Elements of Numbers to their Accompt; and as Contract Numbers have their Ortive Numeration before their Original.

Ratio's *have their Simple Elements.*

The Ortive Numeration of *Ratio's* conſiſts in *Reduction*.

Reduction of *Ratio's*, is to reduce them to their leaſt Terms, or to like Antecedents or Conſequents. The firſt is to be performed as they are of the greater or leſſer Inequality.

Reduction *of* Ratio's, *of two ſorts.*

Ratio's of the greater Inequality are to be divided as Integers; and if any thing remain abbreviated as common Fractions with the Diviſor to the leaſt Terms: And then if Occaſion be, this Quotient, and the leaſt Terms of the Remain and Diviſor ſo abbreviated, may be reduced into the Form of an improper Fraction.

To reduce Ratio's *of the greater Inequality to their leaſt Terms.*

As 80 to 32, after Diviſion 16 remaineth; which abbreviated with 32, makes $\frac{1}{2}$ this, and the Quotient 2 is $2\frac{1}{2}$; or reduced like an improper Fraction is $\frac{5}{2}$ and ſheweth the *Ratio* of 80 to 32 in its leaſt Terms, is *Dupla-Seſquialtera*.

Example.

Ratio's of the leſſer Inequality, are to be abbreviated as common Fractions to their leaſt Terms. As 32 to 80, when abbreviated, is $\frac{2}{5}$ *Subdupla-Seſquialtera*.

To reduce Ratio's *of the Leſſer Inequality,* &c.

Example.

Of

To reduce Ratio's *to like Conſequents.*

Of the other ſort of *Reduction*, that to reduce *Ratio's* to like Conſequents, is like Reduction of Common Fractions to like Denominators.

Example.

As $\begin{smallmatrix}1\\ \cdot\cdot\\ 20\end{smallmatrix}$ and $\begin{smallmatrix}1\\ \cdot\cdot\\ 30\end{smallmatrix}$ reduced to like Conſequents, is $\begin{smallmatrix}30\\ \cdot\cdot\\ 600\end{smallmatrix}$ and $\begin{smallmatrix}20\\ \cdot\cdot\\ 600\end{smallmatrix}$, and abbreviated is $\begin{smallmatrix}3\\ \cdot\cdot\\ 60\end{smallmatrix}$ and $\begin{smallmatrix}2\\ \cdot\cdot\\ 60\end{smallmatrix}$.

To reduce Ratio's *to like Antecedents.*

If *Ratio's* are to be reduced to like Antecedents; then, contrary to the other, their Antecedents are to be multiplied one into another for the common Antecedent; and then croſswiſe every Antecedent is to be multiplied into the other Conſequent, except his own.

Example.

As to reduce $\begin{smallmatrix}20\\ \cdot\cdot\\ 1\end{smallmatrix}$ and $\begin{smallmatrix}30\\ \cdot\cdot\\ 1\end{smallmatrix}$ to one Antecedent, 20 is to be multiplied into 30 for the common Antecedent; and then croſswiſe, 20 into 1, and 30 into 1. So is this Reduction $\begin{smallmatrix}600\\ \cdot\cdot\\ 30\end{smallmatrix}$ and $\begin{smallmatrix}600\\ \cdot\cdot\\ 20\end{smallmatrix}$, and by Abbreviation $\begin{smallmatrix}60\\ \cdot\cdot\\ 3\end{smallmatrix}$ and $\begin{smallmatrix}60\\ \cdot\cdot\\ 2\end{smallmatrix}$.

Proof of Reduction of Ratio's.

Reduction of *Ratio's* is proved one part by another, after the manner of other *Reductions*.

CHAP. III.

Addition of RATIO'S.

Geneſis *and* **Analyſis** *of* **Ratio's. Continuation** *and* **Diminution,** *what.*

RATIO'S have their Original Numeration in their *Geneſis* and *Analyſis*; and their *Geneſis* Prime in *Addition*, (called ſometime *Continuation*) Compound in *Multiplication*: their *Analyſis* Prime in *Subſtraction*, (ſometime called *Diminution*) Compound in *Diviſion* as others before them; but being Comparative herein they differ, for the Operations of the prime Parts of their Numeration are as the Compound Parts of others, and their Compound Parts as Figurals: Wherefore *Addition* of *Ratio's*, in all reſpects, is performed as Multiplication of Fractions. And ſo two *Caſes* are ſufficient.

Addition *of* **Ratio's** *in two Caſes.*

1. **Heterologal** *Terms not reducible.*

Caſe 1. Where the *Heterologal* Terms need no Reduction, multiply Antecedent by Antecedent, for a new Antecedent: And in like manner Conſequent by Conſequent, for a new Conſequent of the Total.

Example.

As if I would add $\begin{smallmatrix}2\\ \cdot\cdot\\ 3\end{smallmatrix}$ to $\begin{smallmatrix}2\\ \cdot\cdot\\ 1\end{smallmatrix}$, that is the *Ratio* of 2 to 3, which is *Sub-Seſquialter* to the *Ratio* 2 to 1, which is double: I multiply 2 by 2, and 4 is the new Antecedent of the Total, and 3 by 1. So is 3 the new Conſequent, and the Total *Ratio* is *Seſquitertia*.

Antecedents.

$$\textit{Addends}\ \begin{smallmatrix}2\\ \cdot\cdot\\ 3\end{smallmatrix} + \begin{smallmatrix}2\\ \cdot\cdot\\ 1\end{smallmatrix} = \begin{smallmatrix}4\\ \cdot\cdot\\ 3\end{smallmatrix}\ \text{Total.}$$

Conſequents.

2. **Heterologal** *Terms reducible.*

Caſe 2. Where the *Heterologal* Terms, or either of them may be reduced, reduce them like Fractions as low as you can; and then multiply the reduced Terms as above.

Example.

As to add $\begin{smallmatrix}4\\ \cdot\cdot\\ 3\end{smallmatrix}$ to $\begin{smallmatrix}3\\ \cdot\cdot\\ 2\end{smallmatrix}$, they may be reduced to $\begin{smallmatrix}2\\ \cdot\cdot\\ 1\end{smallmatrix}$ and $\begin{smallmatrix}1\\ \cdot\cdot\\ 1\end{smallmatrix}$, and then their *Addition* will make the Total $\begin{smallmatrix}2\\ \cdot\cdot\\ 1\end{smallmatrix}$.

$$\textit{Addends}\ \begin{smallmatrix}2\\ \cdot\cdot\\ 4\\ \cdot\cdot\\ 3\\ \cdot\cdot\\ 1\end{smallmatrix} + \begin{smallmatrix}1\\ \cdot\cdot\\ 3\\ \cdot\cdot\\ 2\\ \cdot\cdot\\ 1\end{smallmatrix} = \begin{smallmatrix}2\\ \cdot\cdot\\ 1\end{smallmatrix}\ \text{Total}\qquad \begin{smallmatrix}12\\ \cdot\cdot\\ 6\end{smallmatrix}\ \Big|\ \begin{smallmatrix}2\\ \cdot\cdot\\ 1\end{smallmatrix}$$

Muſical Proportions, how gotten.

Hereby it appeareth that the *Ratio's* commonly called *Harmonica*, or *Harmoniaca Ratio*; and ſometime Muſical Proportions, (which are ſuch as are to be found in Muſical Conſorts) though accompted by ſome a diſtinct Species from the *Ratio's* before deſcribed; yet are nothing elſe but ſeveral *Ratio's* of the former Sorts, only they have other Names, as appeareth in the ſpeculative Part of Muſick. For the *Diapente* is *Seſquialter*. *Diateſſeron*, *Seſquitertia*, *Diapaſon*, *Dupla*. *Dia-*

paſon with the *Diapente*, *Tripla*; and the *Tone*, *Seſquioctava.* So that the *Diapaſon* is made of the *Diateſſeron* added to the *Diapente*: And the *Diapente* is made of the *Diateſſeron* added to the *Tone*.

Examples.

$$\begin{array}{lcccccccc} & \textit{Diapaſon.} & \textit{Diateſſeron.} & & \textit{Diapente.} & & \textit{Diapaſon,} & \text{or} & \textit{Dupla.}\\ \textit{Ant.} & 2 & 4 & & 3 & & 12 & & 2\\ & : & : & + & : & \text{added make} & : & = & :\\ \textit{Conſ.} & 1 & 3 & & 2 & & 6 & & 1\end{array}$$

$$\begin{array}{lcccccccc} & \textit{Diapente.} & \textit{Diateſſeron.} & & \textit{Tone.} & & \textit{Diapente,} & \text{or} & \textit{Seſquialter.}\\ \textit{Ant.} & 3 & 4 & & 9 & & 36 & & 3\\ & : & : & + & : & \text{added make} & : & = & :\\ \textit{Conſ.} & 2 & 3 & & 8 & & 24 & & 2\end{array}$$

Many Ratio's *added.*

If many *Ratio's* are to be added together, proceed as in Reduction of Fractions of Fractions; that is, multiply all their Antecedents one into another, and likewiſe all their Conſequents. And if the Terms be reduced before Multiplication, the Total will be in its leaſt Terms.

Example.

Antecedents. *Reduced.*

$$\textit{Addends}\ \overset{3}{\underset{2}{:}} + \overset{4}{\underset{3}{:}} + \overset{2}{\underset{1}{:}} = \overset{24}{\underset{6}{:}}\ \text{Total.} \qquad \overset{\overset{1}{\cancel{3}}}{\underset{\underset{1}{\cancel{2}}}{:}} + \overset{\overset{2}{\cancel{4}}}{\underset{\underset{1}{\cancel{3}}}{:}} + \overset{2}{\underset{1}{:}} = \overset{4}{\underset{1}{:}}$$

Conſequents.

In *Addition* of *Ratio's* may be obſerved;

Obſervations. 1. *What* Addition *of* Ratio's *ſhews.*

1*ſt*, That *Addition* of *Ratio's*, ſhews how far the *Ratio's* added are diſtant from the *Ratio* of *Equality*. For *Dupla* and *Subdupla* added, ſhall make the *Ratio* of *Equality*.

Examples.

As $\overset{2}{\underset{1}{:}} + \overset{1}{\underset{2}{:}} = \overset{2}{\underset{2}{:}}$ or $\overset{1}{\underset{1}{:}}$ So $\overset{3}{\underset{5}{:}} + \overset{4}{\underset{1}{:}} = \overset{12}{\underset{5}{:}}$ Therefore the Complements of $\overset{3}{\underset{5}{:}} + \overset{4}{\underset{1}{:}}$ to Unity, which are $\overset{5}{\underset{3}{:}} + \overset{1}{\underset{4}{:}} = \overset{5}{\underset{12}{:}}$ & $\overset{5}{\underset{12}{:}} + \overset{12}{\underset{5}{:}}$ make the *Ratio* equal.

Otherwiſe if out of each particular *Ratio* I add or ſubſtract the Remains of the Complement, and make compariſon between the Remains, it will likewiſe ſo appear. For 3 is leſs than 5 by 2, and more than 1 by 2; after the Subſtraction there reſts 1. Likewiſe in the *Ratio* of the Complement ſo working there will alſo remain 1, which is in the *Ratio* of *Equality* with the other 1.

$$\begin{array}{c}12\\ \hline \overset{3}{\underset{5}{:}} + \overset{4}{\underset{1}{:}}\\ \hline 5\end{array} \quad \begin{array}{c}5\\ \hline \overset{5}{\underset{3}{:}} + \overset{1}{\underset{4}{:}}\\ \hline 12\end{array} \quad \begin{array}{c}3\\4\\5\\1\\ \hline 13\\ \hline\end{array} \quad \begin{array}{c}5\\1\\3\\4\\ \hline 13\\ \hline\end{array} \quad \begin{array}{c}12\\5\\ \hline 17\\ \hline\end{array} \quad \begin{array}{c}5\\12\\ \hline 17\\ \hline\end{array} \quad \begin{array}{c}60\\ \hline \overset{12}{\underset{5}{:}} + \overset{5}{\underset{12}{:}}\\ \hline 60\end{array}$$

Alſo $-1\overset{2}{\underset{5}{:}} + 3 = +1\overset{2}{\underset{5}{:}} - 3.$ And $\begin{array}{l}-2+3=+2-3\\ \quad +1=-1\end{array}$

2. *When the Total will be Equal.* *Example.*

2*dly*, That Equal *Ratio's* added to Equal *Ratio's*, make the Total Equal.

As $\overset{2}{\underset{2}{:}} + \overset{3}{\underset{3}{:}} = \overset{6}{\underset{6}{:}}$ or $\overset{1}{\underset{1}{:}}$.

3. *When the Total is Inequal.*

3*dly*, That Equal *Ratio's* added to Inequal *Ratio's*, render the Total in the ſame *Ratio* of Inequality the Inequal *Ratio* was before *Addition*.

Examples.

$$\text{As}\ \begin{array}{c}\overset{3}{\underset{3}{:}} + \overset{3}{\underset{2}{:}} = \overset{9}{\underset{6}{:}}\\ \hline \textit{Ratio}\ 0 + 1\tfrac{1}{2} = 1\tfrac{1}{2}\\ \hline\end{array} \qquad \text{And}\ \begin{array}{c}\overset{3}{\underset{3}{:}} + \overset{2}{\underset{3}{:}} = \overset{6}{\underset{9}{:}}\\ \hline \textit{Ratio}\ 0 + \overset{2}{\underset{3}{:}} = \overset{2}{\underset{3}{:}}\\ \hline\end{array}$$

Queſtions Reſolved by Addition *of* Ratio's.

Queſtions.

1. *Ratio of a Shilling to a Farthing.*

1. The *Ratio* of a Penny to a Farthing is as 4 to 1. The *Ratio* of a Shilling to a Penny is as 12 to 1. What is the *Ratio* of a Shilling to a Farthing?

Anſw. As 48 to 1, for ſo is the Total of the added *Ratio's*.

$$\overset{4}{\underset{1}{:}} + \overset{12}{\underset{1}{:}} = \overset{48}{\underset{1}{:}}$$

2. *Of two Horſes Draught.*

2. An Horſe draws a Weight; but can draw three times as much; and if another Horſe be joined to him, How much will both Horſes draw betwixt them, when the firſt draws as much as he can, and the other but three quarters as much as he can?

Anſw. Twice as much and a quarter more, as the *Addition* of the given *Ratio's* ſhews.

$$\substack{3\\ \cdot\cdot\\ 1}+\substack{3\\ \cdot\cdot\\ 4}=\substack{9\\ \cdot\cdot\\ 4}\ (2\tfrac{1}{4}$$

Proof of Addition of Ratio's.

Addition of *Ratio's* is to be proved by *Subſtraction*, as in the next Chapter is made plain.

CHAP. IV.

Subſtraction of RATIO'S.

Subſtraction of Ratio's in two Caſes.

RATIO'S are ſubſtracted as Fractions are divided. So two Caſes are ſufficient for their *Subſtraction*.

1. *Homologal Terms not reducible.*

Caſe 1. When the *Homologal* Terms need no Reduction, multiply croſswiſe the Antecedent of the *Ratio*, out of which *Subſtraction* is to be made by the Conſequent of the *Ratio* to be ſubſtracted, and the Product ſhall be the remaining Antecedent. And likewiſe the Conſequent of the *Ratio*, from which *Subſtraction* is to be made by the Antecedent of the *Ratio* to be ſubſtracted, and you ſhall have the Conſequent of the Remain.

Examples.

As the *Ratio* of $\substack{4\\ \cdot\cdot\\ 3}$ ſubſtracted from the *Ratio* of $\substack{3\\ \cdot\cdot\\ 2}$, the Remain will be $\substack{9\\ \cdot\cdot\\ 8}$ And if the *Ratio* of $\substack{3\\ \cdot\cdot\\ 2}$ be taken from the *Ratio* of $\substack{4\\ \cdot\cdot\\ 3}$, there will remain $\substack{8\\ \cdot\cdot\\ 9}$

$\substack{3\\ \cdot\cdot\\ 2}-\substack{4\\ \cdot\cdot\\ 3}=\substack{9\\ \cdot\cdot\\ 8}$ Remain. $\substack{4\\ \cdot\cdot\\ 3}-\substack{3\\ \cdot\cdot\\ 2}=\substack{8\\ \cdot\cdot\\ 9}$ Remain.

2. *Homologal Terms reducible.*

Caſe 2. Where the *Homologal* Terms, or either of them, may be reduced, reduce them like Fractions as low as you can; and then multiply the reduced Terms as above.

Example.

As to take the *Ratio* of $\substack{4\\ \cdot\cdot\\ 3}$ from $\substack{2\\ \cdot\cdot\\ 1}$, becauſe 4 and 2 will abbreviate, they are reduced to 2 and 1: And being multiplied, the Remain ſhall be in its leaſt Terms $\substack{3\\ \cdot\cdot\\ 2}$, which otherwiſe will be $\substack{6\\ \cdot\cdot\\ 4}$.

$$\substack{1\\ \cdot\cdot\\ 2\\ \cdot\cdot\\ 1}-\substack{2\\ \cdot\cdot\\ 4\\ \cdot\cdot\\ 3}=\substack{3\\ \cdot\cdot\\ 2}$$

Subſtraction of many from one, or one from many.

If many *Ratio's* were to be ſubſtracted from one, or one *Ratio* from many, then firſt add together the *Ratio's* exceeding one, and afterward make *Subſtraction* as above.

Example.

As to ſubſtract $\substack{2\\ \cdot\cdot\\ 1}$ from $\substack{4\\ \cdot\cdot\\ 3}$ and $\substack{3\\ \cdot\cdot\\ 2}$, I firſt add $\substack{4\\ \cdot\cdot\\ 3}$ and $\substack{3\\ \cdot\cdot\\ 2}$ and the Total is $\substack{12\\ \cdot\cdot\\ 6}$, or by Reduction $\substack{2\\ \cdot\cdot\\ 1}$; From which if $\substack{2\\ \cdot\cdot\\ 1}$ be taken, there remaineth the *Ratio* of Equality $\substack{2\\ \cdot\cdot\\ 2}$ or $\substack{1\\ \cdot\cdot\\ 1}$.

Addition. *Subſtraction.* Without *Reduction.*

$$\substack{2\\ \cdot\cdot\\ 4\\ \cdot\cdot\\ 3\\ \cdot\cdot\\ 1}+\substack{1\\ \cdot\cdot\\ 3\\ \cdot\cdot\\ 2\\ \cdot\cdot\\ 1}=\substack{2\\ \cdot\cdot\\ 1}\qquad \substack{1\\ \cdot\cdot\\ 2\\ \cdot\cdot\\ 1}-\substack{1\\ \cdot\cdot\\ 2\\ \cdot\cdot\\ 1}=\substack{1\\ \cdot\cdot\\ 1}\qquad \substack{4\\ \cdot\cdot\\ 3}+\substack{3\\ \cdot\cdot\\ 2}=\substack{12\\ \cdot\cdot\\ 6}-\substack{2\\ \cdot\cdot\\ 1}=\substack{12\\ \cdot\cdot\\ 12}\text{ or }\substack{1\\ \cdot\cdot\\ 1}$$

Subſtraction of many from many.

If many *Ratio's* are to be ſubſtracted from many, proceed in like manner, adding together the *Ratio's* from which *Subſtraction* is to be made, and alſo the *Ratio's* to be ſubſtracted: And then make *Subſtraction* as above.

Example.

As $\substack{4\\ \cdot\cdot\\ 3}$ and $\substack{3\\ \cdot\cdot\\ 2}$ to be ſubſtracted from $\substack{2\\ \cdot\cdot\\ 1}$ and $\substack{3\\ \cdot\cdot\\ 4}$, the Remain will be $\substack{3\\ \cdot\cdot\\ 4}$

For $\substack{4\\ \cdot\cdot\\ 3}+\substack{3\\ \cdot\cdot\\ 2}=\substack{2\\ \cdot\cdot\\ 1}$ And $\substack{2\\ \cdot\cdot\\ 1}+\substack{3\\ \cdot\cdot\\ 4}=\substack{3\\ \cdot\cdot\\ 2}$ And $\substack{3\\ \cdot\cdot\\ 2}-\substack{2\\ \cdot\cdot\\ 1}=\substack{3\\ \cdot\cdot\\ 4}$

If Signs be uſed, how to proceed.

Sometime more to puzzle a young Practitioner than otherwiſe, many *Ratio's* are given to be ſubſtracted from many, annexed with the Signs + and —, the which though improper, becauſe, as was noted before, thoſe Signs properly belong to *Æquations*; yet there is more Difficulty apparent than real therein. For it is but to add what is to be added, and ſubſtract what is to be ſubſtracted of the *Data*. And for a final Reſolution, work with the New *Ratio's*.

Example.

As if $\begin{smallmatrix}2\\:\\1\end{smallmatrix} + \begin{smallmatrix}3\\:\\2\end{smallmatrix} - \begin{smallmatrix}1\\:\\3\end{smallmatrix}$ were to be ſubſtracted from $\begin{smallmatrix}4\\:\\3\end{smallmatrix} + \begin{smallmatrix}5\\:\\6\end{smallmatrix} - \begin{smallmatrix}4\\:\\5\end{smallmatrix}$; then firſt I add $\begin{smallmatrix}2\\:\\1\end{smallmatrix} + \begin{smallmatrix}3\\:\\2\end{smallmatrix}$, and from the Total $\begin{smallmatrix}3\\:\\1\end{smallmatrix}$ I ſubſtract $\begin{smallmatrix}1\\:\\3\end{smallmatrix}$ and there reſteth $\begin{smallmatrix}9\\:\\1\end{smallmatrix}$; Alſo $\begin{smallmatrix}4\\:\\3\end{smallmatrix} + \begin{smallmatrix}5\\:\\6\end{smallmatrix}$ added, make $\begin{smallmatrix}10\\:\\9\end{smallmatrix}$, from which $\begin{smallmatrix}4\\:\\5\end{smallmatrix}$ taken, leaves $\begin{smallmatrix}25\\:\\18\end{smallmatrix}$. Laſtly, From thence I withdraw the *Ratio* $\begin{smallmatrix}9\\:\\1\end{smallmatrix}$, and the ultimate Remain is $\begin{smallmatrix}25\\:\\162\end{smallmatrix}$.

$$\begin{smallmatrix}2\\:\\4\\:\\3\end{smallmatrix} + \begin{smallmatrix}5\\:\\6\\:\\3\end{smallmatrix} = \begin{smallmatrix}5\\:\\10\\:\\9\end{smallmatrix} - \begin{smallmatrix}3\\:\\4\\:\\5\end{smallmatrix} = \begin{smallmatrix}25\\:\\18\end{smallmatrix} \qquad \begin{smallmatrix}1\\:\\2\\:\\1\end{smallmatrix} + \begin{smallmatrix}3\\:\\2\\:\\1\end{smallmatrix} = \begin{smallmatrix}3\\:\\1\end{smallmatrix} - \begin{smallmatrix}1\\:\\3\end{smallmatrix} = \begin{smallmatrix}9\\:\\1\end{smallmatrix}$$

$$\text{And } \begin{smallmatrix}25\\:\\18\end{smallmatrix} - \begin{smallmatrix}9\\:\\1\end{smallmatrix} = \begin{smallmatrix}25\\:\\162\end{smallmatrix} \text{ Ultimate Remain.}$$

Obſervations.

In *Subſtraction* of *Ratio's* may be obſerved:

1. *When the Remain will be equal.*

1*ſt*, That Equal *Ratio's*, ſubſtracted from Equal *Ratio's*, leave the Remains Equal.

Example.

$$\text{For } \begin{smallmatrix}2\\:\\2\end{smallmatrix} - \begin{smallmatrix}3\\:\\3\end{smallmatrix} = \begin{smallmatrix}6\\:\\6\end{smallmatrix}$$

2. *When the Remain will be inequal.*

2*dly*, That if Equal *Ratio's* be ſubſtracted from Inequal *Ratio's*, the Remains will be left in the ſame *Ratio* of Inequality the Inequal *Ratio* was before *Subſtraction* made: And is all one as if the *Ratio's* were added.

Examples.

$$\text{As } \begin{smallmatrix}1\\:\\3\\:\\2\end{smallmatrix} - \begin{smallmatrix}1\\:\\3\\:\\3\end{smallmatrix} = \begin{smallmatrix}3\\:\\2\end{smallmatrix} \qquad \text{And } \begin{smallmatrix}2\\:\\3\\:\\1\end{smallmatrix} - \begin{smallmatrix}3\\:\\3\\:\\1\end{smallmatrix} = \begin{smallmatrix}2\\:\\3\end{smallmatrix}$$

$$\text{Ratio } 1\tfrac{1}{2} - 0 = 1\tfrac{1}{2} \qquad \text{Ratio } \begin{smallmatrix}2\\:\\3\end{smallmatrix} - 0 = \begin{smallmatrix}2\\:\\3\end{smallmatrix}$$

3. *Inequal from Equal, what the Remain.*

3*dly*, Inequal *Ratio's* ſubſtracted from Equal, leave the Remain of the ſame Inequal Terms the ſubſtracted *Ratio* was, but alters the ſame ſo, that if before *Subſtraction* the *Ratio* were of the Greater Inequality, now it ſhall be of the Leſſer; and if before of the Leſſer, now of the contrary.

Examples.

$$\text{As } \begin{smallmatrix}1\\:\\3\\:\\3\end{smallmatrix} - \begin{smallmatrix}1\\:\\3\\:\\2\end{smallmatrix} = \begin{smallmatrix}2\\:\\3\end{smallmatrix} \qquad \text{And } \begin{smallmatrix}3\\:\\3\\:\\1\end{smallmatrix} - \begin{smallmatrix}2\\:\\3\\:\\1\end{smallmatrix} = \begin{smallmatrix}3\\:\\2\end{smallmatrix}$$

$$\text{Ratio } 0 - 1\tfrac{1}{2} = \begin{smallmatrix}2\\:\\3\end{smallmatrix} \qquad \text{Ratio } 0 - \begin{smallmatrix}2\\:\\3\end{smallmatrix} = 1\tfrac{1}{2}$$

4. *What* Subſtraction *of* Ratio's *ſhews.*

4*thly*, *Subſtraction* ſhews which is the greateſt *Ratio* of two given; for after the *Subſtraction*, if the Remain be of the greater Inequality, then was the *Ratio* from which *Subſtraction* was made greater than the ſubſtracted *Ratio*: But if the Remain be of the leſſer Inequality, then underſtand the contrary. Both theſe are ſo evident in the laſt Examples, that no farther Demonſtration thereof is needful.

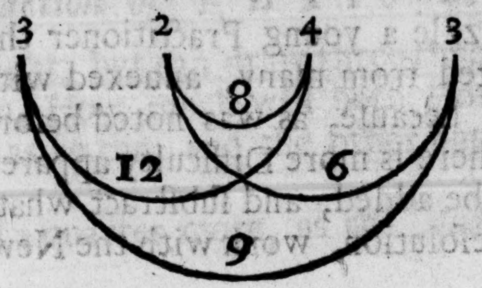
5. *What* Addition *and* Subſtraction *of* Ratio's *ſerves for.*

5*thly*, *Addition* and *Subſtraction* of *Ratio's* ſerves to find out new Proportional *Ratio's*; for the new *Ratio's* found by both, are reciprocally proportional.

Examples.

As if $\begin{smallmatrix}4\\:\\3\end{smallmatrix}$ be added to $\begin{smallmatrix}3\\:\\2\end{smallmatrix}$ without Reduction, the Total will be $\begin{smallmatrix}12\\:\\6\end{smallmatrix}$. And if $\begin{smallmatrix}3\\:\\2\end{smallmatrix}$ be ſubſtracted from $\begin{smallmatrix}4\\:\\3\end{smallmatrix}$, the Remain will be $\begin{smallmatrix}8\\:\\9\end{smallmatrix}$. Then it is evident that 12 to 8 is as 3 to 2; and 12 to 9, is as 4 to 3: Alſo 8 to 6, is as 4 to 3; and 9 to 6, is as 3 to 2: So that 8 and 9 have proportion with the other reciprocally, as by the Scheme following may appear: After which Form the Antient Writers of Muſick were wont to expreſs both the *Addition* and *Subſtraction* of *Ratio's*, as Mr. *Oughtred* teſtifieth, Chap. 10. of his *Clavis*.

The Antient Form of expreſſing Addition *and* Subſtraction *of* Ratio's.



$\overset{3}{\underset{2}{:}} + \overset{4}{\underset{3}{:}} = \overset{12}{\underset{6}{:}}$ *Ergo* $\overset{12}{\underset{8}{:}} + \overset{8}{\underset{6}{:}} = \overset{96}{\underset{48}{:}}$ or $\overset{12}{\underset{6}{:}}$ Alſo $\overset{9}{\underset{6}{:}} + \overset{12}{\underset{9}{:}} = \overset{108}{\underset{54}{:}}$ or $\overset{12}{\underset{6}{:}}$.

$\overset{4}{\underset{3}{:}} - \overset{3}{\underset{2}{:}} = \overset{8}{\underset{9}{:}}$ *Ergo* $\overset{8}{\underset{12}{:}} - \overset{9}{\underset{12}{:}} = \overset{96}{\underset{108}{:}}$ or $\overset{8}{\underset{9}{:}}$ Alſo $\overset{9}{\underset{6}{:}} - \overset{8}{\underset{6}{:}} = \overset{54}{\underset{48}{:}}$ or $\overset{9}{\underset{8}{:}}$

Queſtions.

Queſtions reſolved by Subſtraction *of* Ratio's.

1. Ratio *of a Pound to a Shilling.*

1. The *Ratio* of a Pound to a Penny, is as 240 to 1. The *Ratio* of a Shilling to a Penny, is as 12 to 1. What is the *Ratio* of a Pound to a Shilling?

Anſw. As 20 to 1, for ſo is the Remain left after *Subſtraction*.

$$\begin{smallmatrix}20\\:\\240\\:\\1\end{smallmatrix} - \begin{smallmatrix}1\\:\\12\\:\\1\end{smallmatrix} = \overset{20}{\underset{1}{:}} \qquad \frac{240}{12}\Big(20$$

2. *Of the Draught of one Horſe.*

2. Suppoſe two Horſes together draw twice as much, and a quarter part more at one time as at another when the Ways are bad: One of which Horſes alone can draw a Weight of no greater *Ratio* than 3 to 4. What *Ratio* ſhall the Weight drawn by the other Horſe bear?

Anſw. 3 to 1: for if the *Ratio* $\overset{3}{\underset{4}{:}}$ be ſubſtracted from the *Ratio* $2\frac{1}{4}$, the Remain will be $\overset{3}{\underset{1}{:}}$.

$$\begin{smallmatrix}3\\:\\9\\:\\4\\:\\1\end{smallmatrix} - \begin{smallmatrix}1\\:\\3\\:\\4\\:\\1\end{smallmatrix} = \overset{3}{\underset{1}{:}} \qquad \frac{36}{12}\Big(3$$

Proof of Subſtraction *of* Ratio's.

Subſtraction and *Addition* of *Ratio's* are Reciprocal Proofs of each other, as *Multiplication* and *Diviſion* of Fractions, and need no more than theſe two Queſtions, being of a contrary Nature to thoſe in *Addition*, compared one with another to make it plain: For the Remain of any *Subſtraction* of *Ratio's*, added to the *Subtrahend*, after the manner of *Ratio's*, will return the Number from which *Subſtraction* was made, that is, the Total of their *Addition*.

CHAP. V.

Multiplication of RATIO'S.

Multiplication *of* Ratio's.

THE prime Parts of *Numeration* of *Ratio's*, in their *Addition* and *Subſtraction*, have been now ſeen to be like the Compound Parts of *Numeration* in other Numbers. And as before obſerved, their Compounds Parts of *Numeration*, viz. *Multiplication* and *Diviſion*, are like Figuration of other Numbers.

Rule.

To multiply therefore any *Ratio*, is to multiply the Antecedent and Conſequent ſo often into themſelves reſpectively, as there be Units in the Multiplier.

Examples.

As to double $\overset{3}{\underset{2}{:}}$, the *Ratio Seſquialter*, or multiply it by 2, is to multiply 3 by 3, and 2 by 2: So is the Product $\overset{9}{\underset{4}{:}}$.

And to triple the *Ratio Seſquitertia*, I either ſet down $\overset{4}{\underset{3}{:}}$ three times, and multiply the Antecedents one into another, and ſo likewiſe the Conſequents: Or multiply 4 cubically, and 3 likewiſe: Whereby there is produced $\overset{64}{\underset{27}{:}}$.

Factors. *Product.* *Factors.* *Product.*

$\overset{3}{\underset{2}{:}} \times \overset{3}{\underset{2}{:}} == \overset{9}{\underset{4}{:}} \qquad \overset{4}{\underset{3}{:}} \times \overset{4}{\underset{3}{:}} \times \overset{4}{\underset{3}{:}} == \overset{64}{\underset{27}{:}}$

Multiplication figurates the Terms.

In like manner as to multiply any *Ratio* by 2, is to ſquare the ſame; and to multiply a *Ratio* by 3, is to cube it. So to multiply by any other Number, is to figurate the Terms of the *Ratio* accordingly.

Queſtion of the Bigneſs of the Sun.

Queſt. *Ptolomy* in his *Almageſt*, propoſes the Diameter of the Sun to the Diameter of the Earth, to be as 11 to 2. And by *Prop.* 18. of *Euclid*'s 12th Book, Spheres or Globes have triple Proportion to their Diameters. How much then ſhall the Sun be bigger than the Earth?

Anſwer.

Anſw. 166 times and $\frac{3}{8}$; for the *Ratio* of $\overset{11}{\underset{2}{:}}$ tripled that is cubed, produceth $\overset{1331}{\underset{8}{:}}$, which reduced by *Diviſion*, gives $166\frac{3}{8}$ as before.

$$\overset{11}{\underset{2}{:}} \times \overset{11}{\underset{2}{:}} \times \overset{11}{\underset{2}{:}} = \overset{1331}{\underset{8}{:}} \big(166\tfrac{3}{8}$$

Proof of Multiplication of Ratio's.

Multiplication of *Ratio's* is to be proved by their *Diviſion*, as in the next Chapter is to be ſeen.

CHAP. VI.

Diviſion of RATIO'S.

Diviſion of Ratio's is an Extraction of Roots.

AS *Multiplication* figurates the Terms of the *Ratio*; ſo on the contrary, *Diviſion* of *Ratio's* is to extract from each Term of the *Ratio* given to be divided, a Root of the ſecond, third, or fourth Quantity, *&c.* according to the Units contained in the Diviſor. As to divide a *Ratio* by 2, is to extract the Square Root thereof: And to divide by 3, is to extract the Cube Root, *&c.*

Examples:

Thus $\overset{9}{\underset{4}{:}}$ divided by 2, gives in the Quotient $\overset{3}{\underset{2}{:}}$

And $\overset{27}{\underset{8}{:}}$ divided by 3, gives in the Quotient $\overset{3}{\underset{2}{:}}$

But $\overset{256}{\underset{81}{:}}$ if divided by 4, makes the Quotient $\overset{4}{\underset{3}{:}}$

Dividend.

$$\text{Diviſor } \overset{4}{\underset{1}{:}} \Big) \overset{256}{\underset{81}{:}} \Big(\overset{4}{\underset{3}{:}} \text{ Quotient} \qquad \begin{array}{r|c|l} 256 & 16 & 4 \\ \hline 81 & 9 & 3 \end{array}$$

Queſtion of the Diameters of the Sun and the Earth.

Queſt. If the Body of the Sun, according to *Ptolomy*, be bigger than the Globe of the Earth 166 times and $\frac{3}{8}$; then what *Ratio* is there between their Diameters?

Anſwer.

Anſw. $\overset{11}{\underset{2}{:}}$: for $166\frac{3}{8}$ reduced, and the *Ratio* divided by 3, that is, the Cube Root of each Term taken, the Quotient or Root is $\overset{11}{\underset{2}{:}}$ as aforeſaid.

Reduced. Divided.

$$166\tfrac{3}{8} = \overset{1331}{\underset{8}{:}} \qquad \overset{3}{\underset{1}{:}} \Big) \overset{1331}{\underset{8}{:}} \Big(\overset{11}{\underset{2}{:}} \qquad \begin{array}{r|l} 1331 & 11 \\ \hline 8 & 2 \end{array}$$

Proof of Diviſion of Ratio's.

Diviſion and *Multiplication* of *Ratio's*, being as Production of Figural Numbers, and Extraction of Roots; it muſt needs be that the one ſhall be Proof of the other reciprocally as they were, and hath been largely diſcourſed before in Figural Numbers. And the laſt Queſtion being but the Reverſe of the Queſtion before in *Multiplication*, will clearly evince without farther Teſtimony.

Partis primæ Libri quarti

FINIS.

The Second PART of the Fourth BOOK.

CHAP. I.

Of PROPORTIONS *Disjunct.*

Proportions, *their Computations and Comparative Elements.*

NEXT to the Computation of *Ratio's*, come the Computation of *Proportions*, and their Comparative Elements, to be viewed.

Among *Proportions*, the firſt that preſent themſelves are Simple, and of them thoſe called *Disjunct.*

Disjunct, where deſcribed.

Disjunct *Proportions*, in the firſt Chapter of this 4*th* Book, were generally deſcribed to be *Arithmetical* and *Geometrical.*

Thoſe Arithmetical of little uſe. Geometrical excel.

Thoſe *Arithmetical* (yet of little uſe in *Arithmetick*) were before remembred to give being to ſome Muſical *Proportions*, and have ſomewhat mentioned of them hereafter in this Chapter, with others *Geometrical*; but give place to theſe, as being of much more excellent Uſe, not only in *Arithmetick*, but in ſeveral other Arts and Sciences.

Wherein their Comparative Elements conſiſt.

The Comparative Elements of Geometrical Disjunct *Proportions*, conſiſt in the Invention of new Proportional Numbers, Plain or Figural, according to the *Data* by which they are found.

What they are, vide antea. *Comparative Elements twofold.*

What Plain and Figural *Proportions*, Disjunct and Geometrical, are, the firſt Chapter of this 4*th* Book hath told us. And that theſe plain *Proportions* are Direct and Indirect; both which, with their Comparative Elements, may be ſubdivided into *Primitive* and *Derivative.*

Primitive, *what.*

Primitive give three Numbers to find out a Fourth or new Proportional by a ſingle Operation, and need nothing before Operation ſave a due and orderly Diſpoſition of the *Data.*

Primitives are again ſubdivided

How divided, and where handled.

Into
- Common, in the *Rule of Three*
  - *Direct*, Chap. 2.
  - *Indirect*, Chap. 3.
- Peculiar, in *Practice*, Chap. 4.

Derivative *of four ſorts.*

Derivative before Operation, need ſome or other of the Simple Elements of Numbers to fit them for Reſolution.

Or, 2. give more than three Numbers.
Or, 3. deal with ſome particular Subject.
Or, 4. have ſome peculiar Operation requiſite to their Reſolution.

Where handled.

Derivatives
- of the firſt Sort, are Specificks *Direct* and *Indirect*, ——— Chap. 5.
- of the ſecond Sort, The Rule of 5 Numbers
  - *Direct*, —— Chap. 6.
  - *Indirect*, —— Chap. 7.
- of the third Sort
  - *Fellowſhip*, ——— Chap. 8.
  - *Alligation*, ——— Chap. 9.
  - *Barter* and *Exchange*, ——— Chap. 10.
  - *Loſs* and *Gain*, ——— Chap. 11.
  - *Æquation* of Paiment, ——— Chap. 12.
  - *Factorſhip*, ——— Chap. 13.
- of the fourth Sort, *Falſhood* or *Poſition*, ——— Chap. 14.

Figural Proportions, where handled.

Figural Disjunct Proportions follow them,
- *Doubled*, ——— Chap. 15.
- *Tripled*, &c. ——— Chap. 16.

Concerning Disjunct Proportions in general, take this farther Account.

Notes on Diſjunct Proportions.

1. The *Data*, or Numbers given, are called *Terms*; of which the firſt is ſet to the left Hand, and the Reſidue in order towards the Right.

1. Terms what; and how ſet.

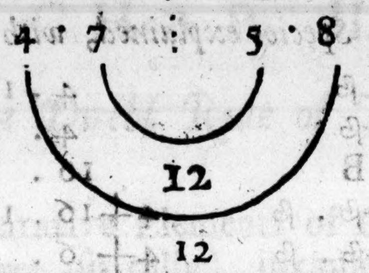
2. The leaſt and greateſt Terms are called *the Extreams*, and the others *the Means*, or *middle Proportionals*. Alſo the Siniſter Term is called the *Antecedent*, and the Second his *Conſequent*. So likewiſe the Third and Fourth.

2. Extreams and Means, what.

3. In Arithmetical *Disjunct Proportions*, the Aggregate of the Extreams ſhall be equal to the Aggregate of the Means.

3. Arithmetical Disjunct Proportions, *the Total of the Means and Extreams equal. Example.*

As in 4 . 7 : 5 . 8, the Sum of 4 and 8, is equal to the Sum of 7 and 5.



4. If of three Terms in Arithmetical *Disjunct Proportion*, a fourth be ſought; from the Second added to the Third, the Firſt ſhall be ſubſtracted.

4. Three Terms thereof given, to find a fourth. Example.

As in the former Example, if 4 . 7 . 5 be given; then from 12, the Sum of 7 and 5, ſhall 4 be abated; ſo will the Remain be 8.

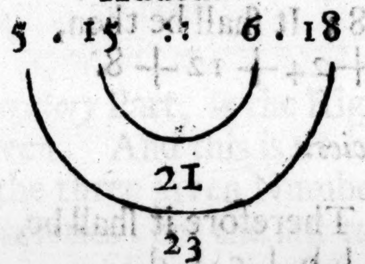
For $7 + 5 - 4 = 8$.

5. Numbers or Magnitudes in Geometrical *Disjunct Proportion*, have the Aggregate of the greateſt and leaſt Terms, greater than the Aggregate of the Reſidue: but the Product of the Extreams, equal to the Product of the Means.

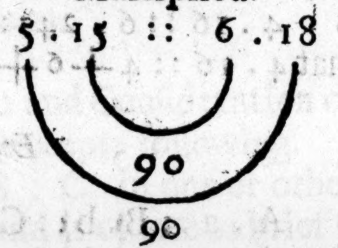
5. Geomet. Diſjunct Proportions, the Product of the Extreams and Means equal. Example.

As 5 . 15 :: 6 . 18, the Total of 5 and 18 is 23, greater than 21, the Total of 15, and 6; but the Product of 5 by 18, which is 90, is equal to the Product of 15 by 6.

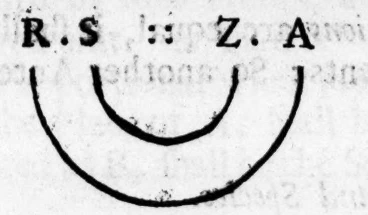
Added.

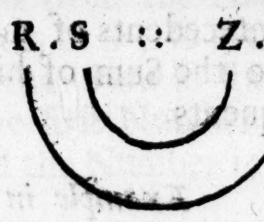


Multiplied.



Example in Species.





Ergo, $R + A$ ſhall be greater than $S + Z$. But $SZ = RA$, and by Conſequence $\frac{ZS}{R} = A$ in Numbers $\frac{90}{5} = 18$.

From hence ſprang the *Rule of Three*, ſpoken to in the next Chapter.

Whence the Rule of Three.

6. Four Numbers or Magnitudes in Disjunct Geometrical *Proportion* Direct, ſhall alſo be proportional; if they be Alternate, Inverſed, Compounded, Parted, Converſed and Mixt.

6. Four in Geom. Disjunct Proportion, how otherwiſe Proportional.

Alternate.

As admit 7 get 28, and accordingly 9, 36, then alternately ſhall 7 . 9 :: 28 . 36; that is, Antecedent to Antecedent, and Conſequent to Conſequent; and there the *Ratio* of 7 to 9, is alike to that of 28 to 36.

Inverſed.

And Inverſed 28 . 7 :: 36 . 9, the *Quadruple Ratio* of 28 to 7, is the ſame with 36 to 9. Here the Conſequent is taken as the Antecedent, and ſo compared to the Antecedent as the Conſequent.

Compounded.

Alſo Compounded; as 7 and 28 which is 35 is to 28, ſo ſhall 9 and 36 that is 45 to 36. This is when the Antecedent and Conſequent, together as one, are compared to the Conſequent.

Parted.

Again, being divided 7 — 28 . 28 :: 9 — 36 . 36. And if withdrawing the firſt Term from the Second, the Reſidue be compared the Second, yet the *Ratio* ſhall be alike to the *Ratio* of the Remainder of the third Term ſubſtracted from the Fourth, compared to the Fourth. For divided *Ratio* is, when the Exceſs wherein the Antecedent exceedeth the Conſequent, is compared to the Conſequent, as 21 . 28 :: 27 . 36 ; where as well 27 to 36, as 21 to 28, are in the *Ratio Subſeſquitertia.*

Converſed. Likewiſe, if converſed, that is, when the Antecedent is compared to the Exceſs, wherein the Antecedent exceeds the Conſequent, 7 to 35, which is the Sum of the firſt and ſecond Terms, are in a like *Ratio* as 9 to 45, the Sum of the third and fourth Terms.

Mixt. Moreover, Mixt, as 7 + 28 . 7 — 28 :: 9 + 36 . 9 — 36 ; which in effect, is as 35 . $\overline{21}$ :: 45 . $\overline{27}$, both agreeing in the *Ratio Superbipartiens-Tertias.*

Examples.

An Example in Species explained, with other Numbers.

Data	A . α :: B . β	4 . 16 :: 6 . 24	
Alternate	A . B :: α . β	4 . 6 :: 16 . 24	*Subſeſquialter.*
Inverſed	α . A :: β . B	16 . 4 :: 24 . 6	*Quadruple.*
Compounded	A + α . α :: B + β . β	4 + 16 . 16 :: 6 + 24 . 24	*Seſquiquarta.*
	A + B . B :: α + β . β	4 + 6 . 6 :: 16 + 24 . 24	*Superbipartiens-Tertias.*
Divided	A — α . α :: B — β . β	4 — 16 . 16 :: 6 — 24 . 24	*Subſeſquitertia.*
	A — B . B :: α — β . β	4 — 6 . 6 :: 16 — 24 . 24	*Subtriple.*
Converſed	A . A ± α :: B . B ± β	4 . 4 ± 16 :: 6 . 6 ± 24	*Subquintuple.* *Subtriple.*
	A . A ± B :: α . α ± β	4 . 4 ± 6 :: 16 . 16 ± 24	*Subduplaſeſquial.* *Subdupla.*
Mixt	A + α . A − α :: B + β . B − β	4 + 16 . 4 — 16 :: 6 + 24 . 6 − 24	*Superbipartiens-Tertias.*
	A + B . A − B :: α + β . α − β	4 + 6 . 4 — 6 :: 16 + 24 . 16 − 24	*Quintuple.*

7. Numbers Proportional, the Conſequents. 7. Certain Numbers or Magnitudes proportional, it ſhall be, That as one Antecedent to his Conſequent ; ſo the Sum of the Antecedents to the Sum of the Conſequents.

Examples.

Example in Numbers.

As 4 . 16 :: 6 . 24 :: 3 . 12 :: 2 . 8 ; It ſhall be then,
That 4 . 16 :: 4 + 6 + 3 + 2 . 16 + 24 + 12 + 8.

Example in Species.

As A . a :: B . b :: C . c :: D . d ; Therefore it ſhall be,
That A . a :: A + B + C + D . a + b + c + d.

8. Antecedents equal the Conſequents. 8. When the Antecedents of many *Proportions* are equal, it ſhall be ; That as one Antecedent, to the Sum of his Conſequents : So another Antecedent to the Sum of his Conſequents.

Example in Numbers and Species.

Examples. As 4 . 16 :: 6 . 24 And 4 . 12 :: 6 . 18 And 4 . 10 :: 6 . 15
A . B :: a . b A . C :: a . c A . D :: a . d

Ergo 4 . 16 + 12 + 10 :: 6 . 24 + 18 + 15
A . B + C + D :: a . b + c + d That is, 4 . 38 :: 6 . 57.

9. Four in Arith. Disjunct *Proportion added to, or taken from others the Conſequents.* 9. If 4 Numbers or Magnitudes in Arithmetical *Proportion Disjunct*, be added to, or ſubſtracted from 4 others alike Proportional, the Totals and Remains will be Proportionals.

Example.

	3 . 5	9 . 11	Exceſs	2
	2 . 4	5 . 7	Exceſs	2
Totals	5 . 9	14 . 18		4
Remains	1 . 1	4 . 4		0

10. Four in Geo. Disjunct *Proportion, multiply or divide others the Conſequents.* 10. If four Numbers, or Magnitudes, in Disjunct Geometrical Proportion, be multiplied or divided by four others reſpectively Proportional, The Products and Quotients ſhall be accordingly Proportional : And the *Ratio* in the Products will likewiſe be multiplied, but in the Quotients divided.

7 . 28

	7 . 28 :: 9 . 36	Ratio	4
	8 . 32 :: 9 . 36	Ratio	4
Products	56 . 896 :: 81 . 1296		16
Quotients	$1\frac{1}{7}$. $1\frac{1}{7}$ :: 1 . 1		1

Example.

CHAP. II.

The Direct Rule of Three.

IN the Front of the Comparative Elements of Disjunct Geometrical Proportions ſtands *The Rule of Three*, ſo called, becauſe three Numbers are given to find out a Fourth Proportional Number; ſometime *The Rule of Proportion*, becauſe the Likeneſs or Agreement of ſome Numbers within themſelves, or each to other, is thereby declared: But moſt frequently it is called *The Golden Rule* for its Excellency, the Concluſions wrought and obtained thereby being ſo many, and ſo profitable and uſeful, as exceed all Credence in the Unskilful. And to difference this from *The Double Rule of Three*, ſpoken to hereafter, in the 6*th* and 7*th* Chapters, this is called *The Simple Golden Rule*.

Direct Rule of Three, *why ſo called.*

Why called the Rule of Proportion.

Why the Golden Rule.

Why the Simple Golden Rule.

This *Rule of Three*, as in the Chapter before noted, is *Direct* and *Indirect*: to the Performance and perfect Underſtanding of *The Direct Rule of Three*,

Rule of Three, Direct *and* Indirect.

Some things are { *Preparatory.*
Operatory.
Probationary.

The *Preparatory* Part, is the Right Diſpoſition and Conſideration of the *Data*, or Numbers given. And this is to be had in the Precepts following.

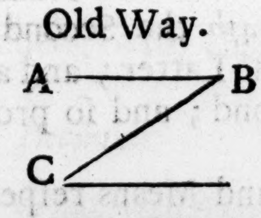
Preparatory *to the Direct.*

1. Place the three given Numbers, as A . B : C, in one or other of the three following Varieties: but the laſt to the right Hand is of late as brief and beſt, moſt in uſe; that is, to divide the firſt Term from the Second by a Point, the Second from the Third by four Points, and the Third from the Fourth when found by another ſingle Point. So ſhall it be read, as A is to B; ſo is C to D, ſignifying the Fourth new Proportional when found. And the *Data* ſo ſtanding, the Number ſtanding in the Place of A, ſhall be called the firſt Number or Term; and the Number placed as B, ſhall be the Second; and the Number in the Place of C, the Third.

1. *How to place the Numbers.*

Common Way.
1 2 3
A — B — C —

Old Way.



Late Way.
A . B :: C.

2. Let the Number upon which the Queſtion propounded depends, be always ſet in the third Place, or C, and called the third Number.

2. *Where to ſet that on which the Queſtion depends.*

3. That Number of the *Data* which is of one Denomination and Nature with the Third, muſt be ſet in the Place of A, for the firſt Number.

3. *Which muſt be firſt.*

4. The right Places found for two of the three given Numbers, of neceſſity the Number of another Nature or Denomination, left unplaced, ſhall be ſet in the ſecond Place inſtead of B; and ſo will all the Three be rightly placed.

4. *Which the ſecond Number.*

5. When the Denominations ſeem to be doubled, as Yards-long, Yards-broad, Pounds-principal, or Pounds-profit, *&c.* the latter Denomination is to be reſpected in placing the Numbers.

5. *Denominations doubled, how to place them.*

6. The fourth Number, when found, ſhall be of like Denomination with the Second.

6. *Fourth of what Denomination.*

The *Operatory* Part conſiſts in the { Invention of the *Diviſor*.
Reſolution of the *Queſtion*.

Operatory *in the Direct.*

To find the Diviſor.

To find the Diviſor after the *Data* are rightly placed, conſider whether the fourth Number, which is the Number queſited, muſt be greater or leſſer than the Second. For if greater, then the leaſt of the two Extreams (which are the firſt and third Numbers) ſhall be Diviſor: But if leſs, then the greateſt of thoſe two Extreams is to be Diviſor.

To reſolve the Queſtion.

To reſolve the Queſtion, when the firſt Number is found to be Diviſor, the Rule is called the *Direct Rule of Three*, and is to multiply the ſecond Number by the Third, and divide the Product by the Firſt. The Quotient of this Diviſion ſhall be the Anſwer to the Queſtion, and the fourth Proportional: For ſuch Proportion as the third Number beareth to the Firſt, and is greater or leſs, ſuch Proportion ſhall this fourth Number bear to the Second.

Example.

Example. What ſhall 18 ℔. of Spice [Yards of Cloth, Buſhels of Wheat, *&c.*] coſt me, when 3 ℔. [Yards, Buſhels, *&c.*] of the ſame Commodity coſteth me 5 *s?*

The Queſtion thus propounded, I ſee, by the ſecond Precept, 18 ℔. muſt be the third Number, becauſe the Queſtion depends thereon; and by the third Precept 3 ℔. ſhall be the firſt Number, becauſe it is alike denominate to 18, the third Number. So of neceſſity 5 *s.* which is the odd denominate Number, ſhall be ſet in the ſecond Place, according to the *4th* Precept; and the Numbers thus rightly placed, ſtand as here ſet.

℔. *s.* ℔.
As 3 . 5 :: 18. Commonly if 3—5—18?

And if wrote at length ſhall be, If 3 ℔. coſt me 5 *s.* what ſhall 18 ℔?

Then to find the Diviſor, conſulting with Reaſon, it is evident, that 18 ℔. ſhall coſt me more than 3 ℔. Wherefore enquiring for a greater Number than 5 *s.* the leaſt Extream, which here is 3, ſhall be Diviſor. And ſo 5 and 18 muſt be multiplied together, and the Product 90 divided by 3, the fourth Number and Reſolution is obtained, which is found to be 30, and they to be Shillings, denominate as 5 *s.* by the *6th* Precept.

℔. *s.* ℔. *s.*
As 3 . 5 :: 18 . 30

$$\frac{\begin{array}{r}18\\5\end{array}}{90} \qquad \frac{90}{3}(30\,s.$$

Proof of the Direct Rule of three, by

The *Probationary* Part is either to { Reverſe the Queſtion, Or, Multiply the Extreams and Means.

Reverſing the Queſtion.

To reverſe the Queſtion: Let the third Number of the one Queſtion, be the Firſt of the other; the former *4th* the Second of the next Work; and the firſt of the former, the Third of the Latter; and after Operation, the Fourth of this Latter will be the former ſecond; and ſo prove both, if there be no Error in the Operations.

Multiplying the Extreams and Means.

To multiply the Extreams and Means reſpectively, according to the Fifth of the foregoing Chapter, the Products being equal, will alſo prove the Truth of any Queſtion reſolved by the *Rule of Three.*

For in the Inſtance before, If the Queſtion be ſtated thus;

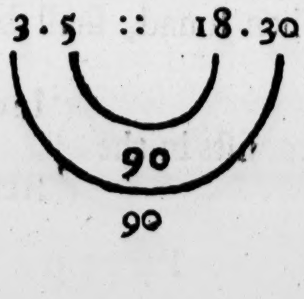
Example.

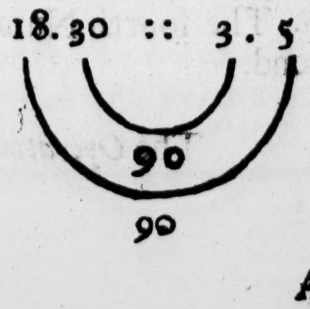
If 18 ℔. coſt 30 *s.* what ſhall 3 ℔. coſt? Seeing a leſſer Number than 30 is looked for, 18 the greater of the two Extreams ſhall be Diviſor; which ſhall divide 90, the Product of 30, into 3. So will 5, the Quotient, be equal to the former ſecond Number, and prove the former Work true.

Alſo if the Extreams and Means be multiplied in either Operation, they will produce in both Operations 90.

℔. *s.* ℔. *s.*
As 18 . 30 :: 3 . 5

$$18)\frac{\begin{array}{c}3\\ \hline 90\end{array}}{}(5$$





And the *Sexcuple Ratio* between 18 and 3, is equal to the *Ratio* of 30 to 5.

Four Numbers in Proportion, how diverſly tranſpoſed.

Nevertheleſs though this above be the common Courſe in reverſing the Queſtion, it may be noted further, That where four Numbers are proportional, their Order may be ſo tranſpoſed, that each of thoſe Terms may be laſt in Proportion. As, firſt, 1 . 2 :: 3 . 6. Second, 3 . 6 :: 1 . 2. Third, 2 . 1 :: 6 . 3. Fourth, 6 . 3 :: 2 . 1. So as every Proportion doth implicitly contain four Orders, two deſcending, and two aſcending; by one of which Orders, if of four Proportionals any three be given, the other Unknown may be found out.

In Reſolution of all Queſtions proper, Multiplication and Diviſion to be uſed.

The Benefit thereof.

That nothing may be wanting to compleat the Neceſſaries of this Chapter, let it be obſerved, that to the Reſolution of every Queſtion propounded, the Multiplication and Diviſion uſed be proper to the Nature of the Numbers given: And this will ſave a burdenſome Charge of the Memory with many Rules; which is always to be avoided, where one Rule and Method of proceeding is ſufficient. As to reſolve a Queſtion propounded in Fractions, or other Contract Numbers, it is but to multiply and divide after the manner of Fractions, or other Contract Numbers proper to the *Data, mutatis mutandis:* Whereas moſt Arithmetical Writers deliver the Rule in Fractions, thus, Multiply the Numerator of the firſt Fraction, by the Denominator of the ſecond Fraction, and that Product again by the Denominator of the third Fraction, and this Product ſhall be Diviſor. Again, multiply the Denominator of the firſt Fraction, by the Numerator of the Second, and that Product again by the Numerator of the Third for the Dividend. Much more long and tedious to the Memory, than to multiply the Second by the Third, and divide by the Firſt, as Fractions are to be multiplied and divided.

Example in Fractions.

Examples in Fractions of the Price of ½ an Ell.

If $\frac{3}{4}$ of one Ell coſt me $\frac{4}{5}$ of a Pound *Sterling*, What ſhall $\frac{1}{2}$ of an Ell coſt? Here $\frac{3}{4}$ being Diviſor, $\frac{4}{5}$ and $\frac{1}{2}$ are multiplied together; and the Product $\frac{2}{5}$ divided by $\frac{3}{4}$, giveth in the Quotient $\frac{8}{15}$ of a Pound, or 10 *s.* 8 *d.*

$$\begin{array}{cccc} \textit{Ell.} & \textit{l.} & \textit{Ell.} & \textit{l.} \\ \text{As } \frac{3}{4} \;. & \frac{4}{5} \;::& \frac{1}{2} \;. & \frac{8}{15} \end{array} \qquad \text{For } \frac{\overset{2}{4}}{5}\times\frac{1}{\underset{1}{2}}=\frac{2}{5} \qquad \text{And } \frac{3}{4})\,\frac{2}{5}\,(\,\frac{8}{15}$$

The other Way uſed by ſeveral, is thus.

$$\begin{array}{cccc} \textit{Ell.} & \textit{l.} & \textit{Ell.} & \textit{l.} \\ \text{As } \frac{3}{4} \;. & \frac{4}{5} \;::& \frac{1}{2} \;. & \frac{8}{15} \end{array} \qquad \begin{array}{c} 3 \\ 5 \\ \hline 15 \\ 2 \\ \hline 30 \end{array} \qquad \begin{array}{c} 4 \\ 4 \\ \hline 16 \\ 1 \\ \hline 16 \end{array} \qquad \frac{16}{30}\Big|\frac{8}{15}$$

$$\text{Diviſor } 30 \qquad \text{Dividend } 16 \qquad \text{For } \begin{cases} 4\times4\times1=16 \\ 3\times5\times2=30 \end{cases}$$

Examples in other Contract Numbers, with their proper Multiplications and Diviſions.

Decimals.

Decimals.

Of the Price of 640 Ells.

If 158¼ Ells coſt 79 *l.* 2 *s.* 6 *d.* What will 640 Ells coſt at that Rate? The firſt and ſecond Numbers turned into Decimals, are 158,25; and 79,125. Operation being made as aforeſaid, the Reſolution is 320 *l.* and the whole Work is as here followeth.

```
                 Ells.        l.        Ells.   l.
              As 158,25 .  79,125  ::  640  .  320,0
     (3)                      640
     (2)               3165 000              3165     l.
Index (1) of the      47475 0             50640000( 320,0
   Quotient.         50640,000              158255
                                             1582
```

Aſtronomicals.

Aſtronomicals.

Of the Digits eclipſed.

If the Diameter of the Moon be ſuppoſed 33′ 28″, and the deficient Scruples at a Lunar Eclipſe are found to be 29′ 5″, What are the Digits eclipſed? Aſtronomers allowing 12 Digits for the whole, 12 ſhall be the ſecond Number, by the proper multiplication whereof into the third Number, after Diviſion made there is found 10 Digits, 25′ 41″, and almoſt 50‴.

Digits. ′ ″ Digits. ′ ″
As 33 , 28 . 12 :: 29 , 5 . 10 , 25 , 41, &c.

$\overset{\prime}{29} \cdot \overset{\prime\prime}{5}$
12°
348 . 60
$\overset{\circ}{5} \cdot \overset{\prime}{49} \cdot \overset{\prime\prime}{00}$

$\cancel{2}\cancel{3}$ | 27
$\cancel{1}\cancel{4} \cdot \cancel{20} \cdot \cancel{20}$ | 52
) $\overset{\circ}{\cancel{5}}$, $\overset{\prime}{\cancel{49}}$, $\overset{\prime\prime}{\cancel{00}}$, $\overset{\prime\prime\prime}{\cancel{00}}$, $\overset{\prime\prime\prime\prime}{\cancel{00}}$, &c. ($\overset{\circ}{10}$, $\overset{\prime}{25}$, $\overset{\prime\prime}{41}$
5 . 34 . 40
13 . 56 . 40
22 . 52 . 08

Logarithms.

Logarithms. *Of Intereſt-Money.*

If at Simple Intereſt, *6 s.* be the Gain of 5 *l.* Principal Money, What in the ſame ſpace of Time, and at the ſame Rate, ſhall 100 *l.* gain? Becauſe Addition of Logarithms is equivalent to Multiplication, and Subſtraction to Diviſion; The Logarithms of the ſecond and third Numbers added, and the Log. of 5 *l.* the firſt Number ſubſtracted, the Log. for Reſolution will be 2,07918,12461, which is the Log. for 120 *s.* or 6 *l.*

l. . *s.* :: *l.* . *s.* . *l.*
As 5 . 6 :: 100 . 120 . or 6
Or, 0,69897,00043. 0,77815,12504 : 2,00000,00000. 2,07918,12461.

Log. of 6. 0,77815,12504
Log. of 100. 2,00000,00000
Sum 2,77815,12504 Product. Log. of 600.
Log. of 5. 0,69897,00043
Difference 2,07918.12461 Quotient. Log. of 120.

Coſſicks.

Coſſicks. *Of the Equality of 30. φ.*

If 16ɣ be equal to 2ſs, what ſhall 30φ (coming from the ſame Root) be equal to? Anſwer, by the following Operation to 3ɣφ and ¾.

As 16ɣ . 2ſs :: 30φ . 3¾ ɣφ.

For 2ſs × 30φ = 60ɣɣɣ. And 16ɣ) 60ɣɣɣ (3¾ɣφ.
Indices 5+3 = 8 Indices 8—2 = 6

The like for Compound *Coſſicks* whole and broken.

Surds.

Surds. *Of the Likeneſs of 5 + √ 4.*

If 3+√16 be alike to 4+√9, What ſhall 5+√4 be like to? By the Operation following, (performed according to the Nature of the *Data*) the Anſwer appears to be √25+√4.

As 3+√16 . 4+√9 :: 5+√4 . √25+√4.

3+√16
3—√16
9+√144
—√144-16
9—16
— 7
—√49

5+√4
20+√225
√.64+√36
20+√ 225 +√ 64 +√36
3—√ 16
60+√2025+√ 576 +√ 324
—√6400—√3600—√1024—√576
—√1225—√ 196

—√49)—√1225—√196(√25+√4

The like for *Simple Surds* as well as *Compound*, both Integers and Fractionary.

Species.

Species.

Of 7AD, *what required.*

If 8 AB require 56 BC, What ſhall 7 AD require? Operation being made according to the Simple Elements of Species, the Reſolution for Anſwer will be 49 CD.

$$\text{As } 8AB \,.\, 56BC :: 7AD \,.\, 49CD.$$

$$\text{For } 56BC \times 7AD = 392BCAD. \text{ And } 8AB)392BCAD(49CD.$$

The like for other *Species*, Simple, Compound, Integral, Fracted, Rational or Irrational, *mutatis mutandis*.

Proof of Contract like others.

And as the Proceſs in the Work of theſe Contract Numbers: So the Proof of the Work, when done, needs no new Inſtitution. For the Reverſe of the Queſtion, or the Multiplication of the Extreams and Means, ſhall prove them all. And over and above, as the Simple Elements of Contract Numbers had their ſingularity of Proof in reducing them to other Numbers: So the Comparative Elements of Contract Numbers may be proved, by taking other Numbers in their ſtead, and comparing the Reſolutions together: As in the laſt Example thus proved is evident.

Example.

$$\begin{array}{lcc} \text{Reverſe of the Queſtion.} & \text{As } 7AD \,.\, 49CD & ::\ 8AB \,.\, 56BC. \\ \text{Extreams in the one,} \Big\} & 56BC & 49CD \\ \text{Means in the other,} & \underline{7AD} & \underline{8AB} \\ & 392BCAD & = 392CDAB \end{array}$$

Reduction into other Numbers, ſuppoſing $A = 1 . B = 2 . C = 3 . D = 4$.

Then $8AB \,.\, 56BC :: 7AD \,.\, 49CD$. ſhall be

$$\text{As } 16 \,.\, 336 :: 28 \,.\, 588 \,.$$

$$\text{For } 336 \times 28 = 9408. \text{ And } 16)9408(588.$$

CHAP. III.

The Indirect Rule of Three.

Indirect Rule of Three. *How called.*

THE next of the Comparative Elements of *Disjunct Proportions Geometrical*, is, *The Indirect Rule of Three*, called ſeverally by Authors, as, *The Reverſed*, *Converſed*, *Inverſed*, *Reciprocal*, and *Backer Rule*; though improperly, except the Firſt and two Laſt.

What herein like to the Direct.

In this *Indirect Rule of Three*, all the Preparatory and Probationary Parts, and the finding of the Diviſor, is exactly the ſame with that in the precedent Chapter of *The Direct Rule of Three*, the only Difference is in the Reſolution.

The third Number always Diviſor.

To reſolve the Queſtion propounded; the third Number being found to be Diviſor, as it always will in Indirect Proportions, the Rule is, Multiply the firſt and ſecond Numbers together, and divide the Product by the Third. The Quotient of this Diviſion ſhall be the 4*th* Proportional Number, and anſwer the Queſtion. For the Proportion between the third and firſt Numbers, is the ſame between the Second and Fourth.

Queſtion of the Weight of Bread.

Example. If at 4*s.* the Price of a Buſhel of Wheat, the Penny white Loaf muſt weigh 9 Ounces *Troy*, What ſhall it weigh at 6 *s.* a Buſhel?

The Numbers right placed, according to the Precepts of the former Chapter, to find the Diviſor, I conſider, the dearer the Corn, the leſſer the Loaf: So the Number queſited being leſs than the Second, the greater of the two Extreams, which is 6, the third Number muſt be Diviſor. Therefore 4 multiplied into 9, and the Product 36 divided by 6, giveth 6 Ounces for the Number deſired.

$$\begin{array}{lcccc} & s. & \textit{Ounces.} & s. & \textit{Ounces.} \\ \text{As} & 4 \,. & 9 & ::\ 6 \,. & 6 \\ & & \underline{4} & & \\ & & \underline{36} & & \end{array} \qquad \begin{array}{l} \qquad\quad \textit{Ounces.} \\ \dfrac{36}{6}(6 \end{array}$$

Proof of the Indirect Rule of Three.

Proof as in the *Rule of Three Direct*, by

Reverse of the Question. Ratio.

$$\text{As } \overset{s.}{6} \cdot \overset{\textit{Ounces.}}{6} :: \overset{s.}{4} \cdot \overset{\textit{Ounc.}}{9} \qquad \begin{array}{ccc}\textit{Extreams.} & & \textit{Means.}\\ \underbrace{4\cdot 9} & :: & \underbrace{6\cdot 6}\\ 36 & = & 36\end{array} \qquad 2)\frac{6}{4}(\frac{3}{2} \quad 3)\frac{9}{6}(\frac{3}{2} \text{ abbreviated.}$$

$$4)\overset{6}{\underline{36}}(9 \text{ Ounces.} \qquad\qquad 4)6(1\tfrac{1}{2}\; 6)9(1\tfrac{1}{2} \text{ divided.}$$

Proper Multiplication and Division here to be used. The Benefit thereof.

Further as neceſſary to this Chapter is to be remembred, for the Reaſon before rendred in the *Direct Rule of Three*, That whenſoever any Queſtion reſolvable by the *Indirect Rule of Three*, is propounded in Fractions, or other Contract Numbers, the Multiplication and Diviſion proper to the *Data* be uſed. It being much ſhorter to remember, to multiply the firſt and ſecond Numbers after the manner of Fractions, and divide the Product by the Third as Fractions are divided, than to multiply the Numerator of the firſt Fraction, by the Numerator of the ſecond Fraction; and that Product again by the Denominator of the Third for the Dividend: And again, to multiply the Denominator of the Firſt by the Denominator of the Second; and that Product again by the Numerator of the Third for the Diviſor, as ſome deliver the Rule.

Example in Fractions.

Examples in Fractions. Of keeping Mony lent.

If I lend $\frac{3}{4}$ of a Pound to a Friend for 7 Months. And when I come to borrow of him, he can ſpare me but $\frac{5}{12}$ of a Pound: How long may I keep from paiment thereof again, to equalize the Intereſt of his Loan to me?

Anſw. 12 Months and $\frac{3}{5}$ of a Month.

$$\text{As } \overset{l.}{\frac{3}{4}} \cdot \overset{M.}{\frac{7}{1}} :: \overset{l.}{\frac{5}{12}} \cdot \overset{M.}{12\tfrac{3}{5}} \qquad \text{For } \frac{3}{4}\times\frac{7}{1} = \frac{21}{4} \qquad \text{And } \underset{3}{\frac{5}{12}})\underset{1}{\frac{21}{4}}(\frac{63}{5}(12\tfrac{3}{5}$$

The other Way uſed by ſeveral.

$$\text{As } \overset{l.}{\frac{3}{4}} \cdot \overset{M.}{\frac{7}{1}} :: \overset{l.}{\frac{5}{12}} \cdot \overset{M.}{12\tfrac{3}{5}} \qquad \begin{array}{rr} & 3\\ & 7\\ \hline & 21\\ & 12\\ \hline & 42\\ & 21\\ \hline \text{Dividend} & 252\\ \hline \end{array} \quad \begin{array}{rr} & 4\\ & 1\\ \hline & 4\\ & 5\\ \hline \text{Diviſor} & 20\\ \hline \end{array} \quad \begin{array}{l} (1\\ 2\cancel{5}(2(12\tfrac{3}{5}\\ \hline 20 \end{array}$$

Example in Decimals.

Decimals. Of the Weight of Bread.

Suppoſe at 5 *s.* 9 *d.* the Buſhel of Wheat, the Penny white Loaf weigh 9 Ounces and 6 Penny Weights; What ſhall the ſame weigh, when Wheat is riſen to 6 *s.* 6 *d.* the Buſhel?

Anſw. 8 Ounces, 4½ Penny Weights, and ſomewhat more.

$$\text{As } \overset{s.}{0{,}2875} \cdot \overset{\textit{Ounces.}}{0{,}775} :: \overset{s.}{0{,}325} \cdot \overset{\textit{Ounces.}}{0{,}6855}$$

$$\begin{array}{c} (4)\\ (3)\\ \hline (7) \end{array} \quad \begin{array}{r} {,}775\\ \hline 14375\\ 20125\\ 20125\\ \hline {,}2228125 \end{array} \qquad \begin{array}{c} (7)\\ (3)\\ \hline (4) \end{array} \quad \begin{array}{l} \phantom{{,}22}\cancel{1}(25\\ \phantom{{,}2}\cancel{7}\cancel{1}8\cancel{7}|0\\ {,}\cancel{2}\cancel{2}\cancel{2}8\cancel{1}\cancel{2}\cancel{5}(,6855\\ {,}\cancel{3}\cancel{2}\cancel{5}\\ {,}\cancel{3}\cancel{2}\cancel{5}\\ {,}\cancel{3}\cancel{2}\cancel{5}\\ {,}\cancel{3}\cancel{2}\cancel{5} \end{array}$$

Example in Logarithms.

Logarithms. Of paying Money lent.

In caſe I lend a Friend 320 *l.* for 20 Months: And when I want, he courteouſly lendeth me 400 *l.* When ſhall I pay him again, not to treſpaſs on his Kindneſs?

Anſw. At 16 Months end.

l. . *Mon.* . *l.* . *Mon.*
As 320 . 20 :: 400 . 16

Or, 2,50514,99783 . 1,30102,99957 : 2,60205,99913 . 1,20411,99827

Log. of 320 . 2,50514,99783
Log. of 20 . 1,30102,99957
Sum 3,80617,99740 Product. Log. of 6400.
Log. of 400 . 2,60205,99913
Difference 1,20411,99827 Quotient. Log. of 16.

Example in Species.

Species. Of Workmen to build a Wall.

If 15 Pioneers, with the help of 3 Boys, can dig a Trench, and build a Wall in 12 Days; and Expedition requires the ſame to be done in 9 Days: How many Men and Boys are there needful?

Anſw. 20 Pioneers and 4 Boys.

As 12 A . 15 B + 3 C :: 9 A . 20 B + 4 C.
For 12 A × 15 B + 3 C = 180 BA + 36 CA.
And 9 A) 180 BA + 36 CA (20 B + 4 C.

Proof of theſe as others.

The Proof of theſe, and ſuch others, are as before, by reverſing the Queſtion, or multiplying the Extreams and Means; or, moreover, by taking other Numbers inſtead of theſe Contracts, and working therewith. All which by the laſt Example is here inſtanced.

Example.

Reverſe of the Queſtion. As 9A . 20 B + 4 C :: 12 A . 15 B + 3 C.

$$\text{Extreams}\left\{\begin{array}{c} 20\,B + 4\,C \\ 9A \\ \hline 180BA + 36CA \end{array}\right. \qquad \text{Means}\left\{\begin{array}{c} 15\,B + 3\,C \\ 12\,A \\ \hline 180BA + 36CA \end{array}\right.$$

Omitting the Species. As *Days.* 12 . *Labourers.* 18 :: *Days.* 9 . *Labourers.* 24.
12 × 18 = 216   9) 216 (24

CHAP. IV.

PRACTICE.

Practice.

THE common Comparative Elements, in the *Direct* and *Indirect Rule of Three viſited*, the other Part of the Primitives which is Peculiar, comes to be patent, and this is *Practice*.

What, and why ſo called.

Practice is ſo called, from the frequent Uſe and general Practice thereof, and is a Compendium or Breviat of the brief Rules and moſt expeditious Method of reſolving the Propoſitions reſolvable by the *Rule of Three*, after the Common Way. So as if any Proceſs therein be ſhorter than other, it falls under conſideration here.

Heads of Practice.

The practical ſhortning the common Work of the *Rule of Three*, may be compriſed under one of theſe three Heads.

1. When the firſt and third Numbers will abbreviate.

1. When the firſt and third Numbers may be abbreviated, then reduce them to their leaſt Terms, and by thoſe Terms proceed, as in the *Rule of Three*, to the Reſolution of the Queſtion.

Example in the Direct Rule of Three.

Examples. Of the Price of Rods of Wall.

If 30 Rods of Wall making coſt 8 *s.* What ſhall 48 Rods coſt? Becauſe 30 and 48 will abbreviate, they may be reduced to 5 and 8 for the firſt and third Numbers, and the Reſolution thereby gotten will be 12 *s.* $\frac{4}{5}$, as by the Common Way.

$6)\frac{30}{48}(\frac{5}{6}$

Rods. *s.* *Rods.* *s.*
As 5 . 8 :: 8 . $12\frac{4}{5}$
8
$5)\overline{\underline{64}}(12\frac{4}{5}$

Common Way.
As 30 . 8 :: 48 . $12\frac{4}{5}$
8
$30)\overline{\underline{384}}(12\,s.\,\frac{4}{5}$

Example in the Indirect Rule of Three.

Of Labourers to finiſh a Piece of Work.

If 18 Labourers in 12 Days can finiſh a Piece of Work; In how many Days ſhall 24 Labourers finiſh the ſame Work? Here 18 and 24, the firſt and third Numbers, will be reduced to their leaſt Terms, 3 and 4; by which the Reſolution will be 9 Days, as by the *Common Way.*

$6)\frac{18}{24}(\frac{3}{4}$

Lab. *Days.* *Lab.* *Days.*
As 3 . 12 :: 4 . 9
3
$4)\overline{\underline{36}}(9$

Common Way.
As 18 . 12 :: 24 . 9
18
96
12
$24)\overline{\underline{216}}(9$

2. When the common Work may be ſhortned.

2. Though the firſt and third Numbers will not abbreviate, yet may the common Work of the *Rule of Three* be ſhortned, if in the *Direct Rule* the third Number be divided by the Firſt; and in the *Indirect Rule*, the firſt Number be divided by the Third, and the Quotient multiplied by the Second.

Example in the Direct Rule of Three.

Examples. Of Gain by Trade.

If 750 *l.* Stock in merchandiſing gain 360 *l.* What ſhall 1250 *l.* Stock gain?

Anſw. 600 *l.* For 1250 divided by 750, gives in the Quotient $1\frac{2}{3}$, which multiplying 360, makes the Product 600.

l. Stock. *l. Gain.* *l. Stock.* *l. Gain.*
As 750 . 360 :: 1250 . 600
$1\frac{2}{3}$
360
120
120
·600*l.*

150
$\cancel{125}$|0 $(1\frac{2}{3}$
$\cancel{75}$|0

Common Way.
As 750 . 360 :: 1250 . 600
360
75000
3750
$750)\underline{450000}(600$

Example in the Indirect Rule of Three.

Of Length, &c. to make an Acre of Land.

If 8 Perches broad require 20 Perches long to make an Acre of Land: What length ſhall a Piece of Land have that is 2 Perches broad to make an Acre?

Anſw. 80 Perches: For 8 divided by 2, gives 4 in the Quotient; which multiplying 20, makes the Product 80.

Perches-broad. *Perches-long.* *Perches-broad.* *Perches-long.*
As 8 . 20 :: 2 . 80
$\frac{8}{2}(4$ $\frac{4}{80}$ Perches long.

Common Way.
As 8 . 20 :: 2 . 80
8
$2)\underline{160}(80$

The firſt and third Numbers in theſe two laſt Examples, abbreviated as abovementioned, the Work will ſhew it ſelf thus.

As 750 . 360 :: 1250 . 600 As 8 . 20 :: 2 . 80

$250)\frac{750}{1250}(\frac{3}{5}$ — $3)\frac{5}{1800}(600$ $2)\frac{8}{2}(\frac{4}{1}$ $\frac{4}{80}$

3. Becauſe one doth neither multiply nor divide. If an Unit among *Geodeticals* be one of the three given Numbers, and the Firſt and Third be of like Denominations, and ſo need no Reduction as ſome do, which will be ſeen among Specificks in the next Chapter; then the Unit is ſet by, and Operation being made with the other two Numbers of the *Data*, the Work may be ſhorter than that commonly by the *Rule of Three*, eſpecially when the Unit happens to ſtand in the firſt Place, as in the enſuing Proceſs of this Chapter will be proved: Which Operations are all that with ſome Authors paſs by the Name of *Practice.*

3. When an Unit is one of the Data.

The enſuing Operations of *Practice* under this third Head, will be comprehended under theſe ten Caſes following.

This ſort of Practice ſeen in ten Caſes.

Caſe 1. If the given Price of one Ell, Yard, Pound-weight, *&c.* be any certain Number of Shillings, then multiply the given Quantity of Ells, Yards, Pounds, *&c.* whoſe Price is deſired, by half the Number of Shillings which one ſhall coſt: And from the Product cut off by a Daſh of the Pen the right-hand Figure for Primes, (every Unit whereof is in value 2 *s.*) the Reſidue is Pounds. If a Cipher be in the Dexter Place cut off, it ſignifies nothing.

1. If the Price be any Number of Shillings.

Example 1. If one Ell coſt 4 *s.* what ſhall 401 Ells coſt at that Price?

Anſw. 80 *l.* 4 *s.* For multiplying 401 by 2, which is the half of 4 the Price given; the Product is 802, of which the Right-hand Figure 2, cut off for Primes, is 4 *s.*

Example in Numbers. Even and Odd.

Common Way.

```
                   Ells.          Ell.  s.   Ells.   l.   s.
At 4 s. per Ell, what coſts 401 ?   As  1 . 4 :: 401 . 80 : 4
                             2                  4
½ of 4 s. is 2.        l. 80|2 Primes.        160|4
                                             l. 80:4 s.
```

Example 2. If one Ell coſt 19 *s.* what will 300 Ells coſt?

Anſw. 285 *l.* For after Multiplication by 9½, half the Price of 1 Ell, the total Product is 2850; from which the Cipher as inſignificant cut off, the Reſidue is 285.

Odd and Even.

Common Way.

```
                    Ells.           Ell.  s.   Ells.   l.
At 19 s. per Ell, what coſts 300 ?   As  1 . 19 :: 300 . 285
                              9½                 19
½ of 19 s. is 9½            2700               2700
                             150                300
                       l. 285|0 Primes.        570|0
                                             l.285:0 s.
```

Example 3. If 1 Yard coſt 15 *s.* what ſhall 343 Yards coſt?

Anſw. 257 *l.* 5 *s.* For multiplying 343 by 7½, the half Price of one Yard, there comes to be cut off 2½ Primes, which is 5 *s.*

Both odd.

Common Way.

```
                      Yards.         Yard.  s.   Yards.   l.    s.
At 15 s. per Yard, what coſts 343 ?   As  1 . 15 :: 343 . 257 : 5.
                               7½                  15
½ of 15 s. is 7½             2401                1715
                             171½                 343
                       l.257|2½ Prim.           514|5
                                              l.257:5 s.
```

2. If the Price be an Aliquot Part of a Pound or Shilling.

Caſe 2. If the given Price of one Yard, Ell, Pound-weight, *&c.* be any *Aliquot*, or even Part of a Pound or Shilling: Then divide the Quantity of Yards, Ells, *&c.* whoſe Price is deſired by that *Aliquot* Part; and the Quotient ſhall be the Reſolution in Pounds or Shillings, according to the Part divided by. And if any thing remain upon the Diviſion, every Unit thereof is in Value ſo much as that Part of a Pound or Shilling which was the Diviſor.

The Collection of the Aliquot *Parts of a Pound and Shilling, commonly called* Practice Tables, *here follow.*

Practice *Tables.*

Aliquot Parts of a Pound.

Parts	*Value* s. d.
1	20.0
2	10.0
3	6.8
4	5.0
5	4.0
6	3.4
8	2.6
10	2.0
12	1.8
15	1.4
16	1.3

Parts	*Value* s. d.
20	1.0
24	0.10
30	0.8
40	0.6
48	0.5
60	0.4
80	0.3
120	0.2
240	0.1
480	0.0$\frac{1}{2}$
960	0.0$\frac{1}{4}$

Aliq. Parts of a Shilling.

Parts	*Value* d.
1	12
2	6
3	4
4	3
6	2
8	1$\frac{1}{2}$
12	1
16	0$\frac{3}{4}$
24	0$\frac{1}{2}$
48	0$\frac{1}{4}$

The Tables explained.

The Tables are eaſy to be underſtood, the Value of every Aliquot Part ſtanding directly againſt the ſame on the ſame Line: As the third Part of a Pound is 6 *s.* 8 *d.* the third Part of a Shilling 4 *d.* the like of all the reſt.

How made.

The Value of every Part is found, by dividing the Shillings in a Pound by the Parts under 20: The Pence in a Pound by the Parts under 240, *&c.* The like for the Parts of a Shilling, or the loweſt Denomination, 960 Farthings in a Pound, and 48 Farthings in a Shilling by the Part deſired; and ſo is the Quotient Farthings, which may be reduced into Pence and Shillings by *Geodetical* Reduction.

Their Uſe.

Operation by the Parts of a Pound.

Examples of the firſt Table.

Example 1. If one Yard coſt 6 *s.* 8 *d.* what ſhall 348 Yards coſt?

Anſw. 116 *l.* For 348 divided by 3, or the third part thereof taken, becauſe 6 *s.* 8 *d.* is $\frac{1}{3}$ of a Pound, the Quotient will be 116.

Common Way.

At 6 *s.* 8 *d.* *per* Yard, what coſts 348? *Yards.*

$\frac{1}{3}$ *l.* 3) 348 (116 *l.*

As 1 *Yard.* . 80 *d.* :: 348 *Yards.* . 116 *l.*

$$\begin{array}{r} 348 \\ 80 \\ \hline 27840 \\ \hline 12 \end{array} \left(\begin{array}{r} 232|0 \\ \hline l.\,116 \\ \hline \end{array} \right.$$

Example 2. If one pound Weight coſt 1 *s.* 4 *d.* what ſhall 8976 Pounds coſt?

Anſw. 598 *l.* 8 *s.* For dividing 8976 by 15, becauſe 1 *s.* 4 *d.* is the fifteenth part of a Pound: the Quotient is 598, and 6 remaining is $\frac{6}{15}$; which, if $\frac{1}{15}$ be 1 *s.* 4 *d.* is 8 *s.* that is, 6 Shillings and 6 Groats.

℔.
At 1 *s.* 4 *d. per* ℔, what coſts 8976?

$\frac{1}{15}$ *l.*
1
142 (*l.* *s.*
8976 (598 . 8
15
15
15

Common Way.

℔. *d.* ℔. *l.* *s.*
As 1 . 16 :: 8976 : 598 : 8.
16
53856
8976
143616 (1196|8
12 (*l.* 598:8 *s.*

Operation by the Parts of a Shilling.

Examples of the Second.

Example 1. If 1 ℔ coſt 3 *q.* what ſhall 4048 ℔. coſt?
Anſw. 12 *l.* 13 *s.* For dividing 4048 by 16, which part of a Shilling 3 *q.* is, the Quotient is 253 *s.* and by Reduction 12 *l.* 13 *s.*

℔.
At 3 *q. per* ℔, what coſts 4048?

$\frac{1}{16}$ *s.*
8
4048 (25|3
16 (*l.* 12:13 *s.*
16
16

Common Way.

℔. *q.* ℔. *l.* *s.*
As 1 . 3 :: 4048 . 12 : 13.
3
12144 (6
4 (3036 (25|3
12 *l.* 12:13 *s.*

Example 2. If one Pound coſt 2 *q.* what ſhall 1227 ℔. coſt?
Anſw. 2 *l.* 11 *s.* 1 $\frac{1}{2}$ *d.* For 2 *q.* being $\frac{1}{24}$ of a Shilling, and dividing 1227, the Quotient ſhall be 51 *s.* and the 3 which remain 3 odd Half-pence.

℔.
At 2 *q. per* ℔. what coſts 1227?

$\frac{1}{24}$ *s.*
(3
1227 (51 *s.*
24 (*l.* *s.* *d.*
24 2:11:1$\frac{1}{2}$

Common Way.

℔. *q.* ℔. *l.* *s.* *d.*
As 1 . 2 :: 1227 : 2 : 11 : 1$\frac{1}{2}$
2
2454
(2*q* (1*d.* (1*s.*
2454 (613 (5(1 (2 *l.*
4 12 20

Price oft-times to be found in both Tables.

The price of 1 ℔. in this laſt Example, 2 *q.* being alſo an Aliquot part of a Pound, (as oftentimes happeneth in the given Price) Operation may be made by the Parts of a Pound.

℔.
As at 2 *q. per* ℔. what coſts 1227?

$4\frac{1}{8}$. *l.*
26
122|7 (2 *l.*
480

2(3
267 (11$\frac{1}{8}$ *s.*
24

3. If the Price be no Aliquot Part of a Pound or Shilling.

Caſe 3. If the given Price of one Yard, Ell, *&c.* be not any even Part of a Pound or Shilling: Then break the given Price into any Aliquot Parts, and work as before with every of thoſe Parts, and add their Quotients together, with the Remains if any be. Or for the Shillings, work as in the firſt Caſe; and for the Pence as in the ſecond Caſe.

Example 1.

Example 1. If one hundred Weight coſt 13 *s.* 7 *d.* what ſhall 245 hundred Weight coſt?
Anſw. 166 *l.* 7 *s.* 11 *d.* as appeareth by both Works ſo plain, that nothing is needful for Explication.

By the firſt and ſecond Caſes.

At 13*s.*7*d.* *per* C. what coſts 245 C.?

$\frac{1}{2}$ of 13 *s.* is 6$\frac{1}{2}$

$\frac{1}{40}$ *l.* 0 : 6 *d.*

$\frac{1}{240}$ *l.* 0 : 1

13 : 7

6$\frac{1}{2}$

1470

122$\frac{1}{2}$

l. 159|2$\frac{1}{2}$ Primes.

159 : 5 *s.* *d.*

$\frac{\cancel{2}4(5}{\cancel{4}0}$ (6 : 2 : 6

$\frac{\cancel{2}4(5}{\cancel{2}4\ 0}$ (1 : 0 : 5

166 : 7 : 11

By the ſecond Caſe.

At 13 *s,* 7 *d.* *per* C. what coſts 245 C.?

s. *d.*

$\frac{1}{2}$ *l.* 10 : 0

$\frac{1}{6}$ *l.* 3 : 4

$\frac{1}{80}$ *l.* 0 : 3

13 : 7

(1

$\frac{24\cancel{5}}{\cancel{2}}$ (*l.* *s.* *d.* 122 : 10 : 0

$\frac{\cancel{2}4(5}{6}$ (40 : 16 : 8

$\frac{\cancel{2}4(5}{8\ 0}$ (3 : 01 : 3

166 : 7 : 11

Example 2.

Example 2. If 1 C. coſt 11 *s.* 8 *d.* what ſhall 146 C. coſt?
Anſw. 85 *l.* 3 *s.* 4 *d.*

By the firſt and ſecond Caſes.

At 11*s.*8*d.* *per* C. what coſts 146 C.?

$\frac{1}{2}$ of 11 *s.* is 5$\frac{1}{2}$

$\frac{1}{30}$ *l.* 0 : 8*d.*

11 : 8

5$\frac{1}{2}$

730

73

80|3 Primes.

l. 80 : 6*s.* *d.*

(2

$\frac{\cancel{2}4(6}{\cancel{3}\ 0}$ (4 : 17 : 4

85 : 3 : 4

By the ſecond Caſe.

At 11 *s.* 8 *d.* *per* C. what coſts 146 C.?

s. *d.*

$\frac{1}{3}$ *l.* 6 : 8

$\frac{1}{4}$ *l.* 5 : 0

11 : 8

2(2

$\frac{\cancel{1}4\cancel{6}}{\cancel{3}}$ (*l.* *s.* *d.* 48 : 13 : 4

2(2

$\frac{\cancel{1}46}{\cancel{4}}$ (36 : 10 : 0

85 : 3 : 0

4. Operation by the Parts of 24.

Caſe 4. If the Aliquot Parts of 24 be taken for the Price given, and Operation made thereby as before by the Aliquot Parts of 20, under the ſecond Caſe: A brief Reſolution of the Queſtion will be had in the Quotient of Diviſion by theſe Parts; which Quotient ſhall be Pounds alſo, becauſe 24 hath but the Cipher cut off from 240, the Pence in a Pound; and therefore the Dexter Figure or Cipher of the Quotient ſhall be cut off for Primes: And if any thing remain upon the Diviſion, it is always leſs than one Prime, or 2 *s.* every Unit being in Value ſo much as the Diviſor contained of 24.

The Aliquot Parts of 24, are known by the ſingle Numbers in this Table ſtanding againſt them, the uneven Parts having ſome 2, and ſome 3 Numbers againſt them.

A Table of the Parts of 24.

Pence.	*Parts.*		*Pence.*	*Parts.*
1	24		13	2.24
2	12		14	3.4
3	8		15	2.8
4	6		16	3.3
5	12.8		17	3.4.8
6	4		18	2.4
7	8.6		19	2.8.6
8	3		20	2.3
9	4.8		21	2.4.8
10	4.6		22	2.4.6
11	3.8		23	2.3.8
12	2		24	1

The Table explained. The Numbers under the Title *Parts* are for Diviſors, when ſuch a Number of Pence as ſtands againſt them happens to be the given Price of one Ell, Yard, Pound, *&c.* As if 4 *d.* be the Price given, then ſhall 6 be Diviſor: But if 5 *d.* be the given Price, which is no even Part of 24, then 12 and 8 ſhall be Diviſors, 12 becauſe it is the Part anſwering to 2 *d.* and 8, becauſe the Part that anſwers to 3 *d.* which 2 and 3 make 5, and both the Quotients muſt be added together, as thoſe under the third Caſe.

Operation by the Parts of 24.

Example 1. *Example* 1. If 1 ℔. coſt 8 *d.* what will 447 ℔. coſt?

Anſw. 14 *l.* 18 *s.* For 447 divided by 3, becauſe 8 is the third Part of 24, the Quotient is 149; from which 9 cut off for the Primes, which is 18 *s.* the reſt is Pounds.

℔.
At 8 *d.* *per* ℔. what coſt 447?

$\frac{1}{3}$ of 24 447/3 (14|9 Primes.
l. 14:18 *s.*

By the ſecond Caſe.

℔.
At 8 *d.* *per* ℔. what coſts 447?

$\frac{1}{30}$ *l.* (2 *l.* *s.*
44(7/30 (14 : 18

Example 2. *Example* 2. If 1 ℔. coſt 9 *d.* what will 4638 ℔. coſt?

Anſw. 173 *l.* 18 *s.* 6 *d.* For dividing 4638 by 4 and 8, the Diviſors for 9, and adding the Quotients together, the Total is 173 *l.* 9¼ Primes, or 18 *s.* 6 *d.*

℔.
At 9 *d.* *per* ℔. what coſts 4638?

d.
$\frac{1}{4}$ of 24 is 6
d.
$\frac{1}{8}$ of 24 is 3
9*d.*

2(2
4638/4 (1159½
7(6
4638/8 (579¾
l. 173|9¼ Pri.
l. 173:18:6*d.*

By the ſecond Caſe.

℔.
At 9 *d.* *per* ℔. what coſts 4638?

$\frac{1}{40}$ *l.* 6 *d.*
$\frac{1}{80}$ *l.* 3 *d.*
9 *d.*

2 *l.* *s.* *d.*
46(38/40 (115 : 19 : 0
(7
463(8/80 (57 : 19 : 6
173 : 18 : 6

5. If the Price be above 2 *s.* *Caſe* 5. If the Price given be above 24 *d.* or 2 *s.* and yet the ſame may be evenly divided by ſome Aliquot Part of 24: then ſee how many ſuch Parts there is in the given Price, and after Diviſion by that Aliquot Part, multiply the Quotient by the other, and cut off the dexter Figure or Cipher as before.

Example 1. *Example* 1. If 1 Ell coſt 3 *s.* 6 *d.* what ſhall 400 Ells coſt?

Anſw. 70 *l.* For 6 *d.* is an Aliquot Part of 24, and ¼ thereof, and in 3 *s.* 6 *d.* there are 7 Six-pences: Therefore 400 divided by 4, gives 100, which muſt be multiplied by 7; and from that Product 700, a Cipher cut off leaves 70.

Ells.
At 3 *s.* 6 *d.* *per* Ell, what coſts 400?

d.
$\frac{1}{4}$ of 24 is 6
7
3:6

400/4 (100
7
l. 70|0 Primes.

By the ſecond Caſe.

Ells.
At 3 *s.* 6 *d.* *per* Ell, what coſts 400?

s. *d.*
$\frac{1}{8}$ *l.* 2 : 6
$\frac{1}{20}$ 1 : 0
3 : 6

400/8 (50 *l.*
400/20 (20
70 *l.*

Example 2. *Example* 2. If 1 Ell coſt 2 *s.* 8 *d.* what ſhall 500 Ells coſt?

Anſw. 66 *l.* 13 *s.* 4 *d.* For 4 *d.* is an even Part of 24, and in 2 *s.* 8 *d.* are 8 Groats: Therefore dividing by 6, becauſe 4 is the ſixth Part of 24, and multiplying the Quotient by 8, there is 66 *l.* 6⅔ Primes, or 13 *s.* 4 *d.*

Ells.
At 2 *s.* 8 *d.* *per* Ell, what coſts 500?

d.
$\frac{1}{6}$ of 24 is 4
8
2:8

2(2
500 / 6 (83$\frac{1}{3}$
8
664
2$\frac{2}{3}$
l. 66|6$\frac{2}{3}$ Primes.
l. 66:13:4 *d.*

By the ſecond Caſe.

Ells.
At 2 *s.* 8 *d.* *per* Ell, what coſts 500?

s. d.
$\frac{1}{8}$ *l.* 2 : 6
$\frac{1}{12}$. 2
2 : 8

2(4
500 / 8 (*l.* *s.* *d.* 62 : 10 : 0
2
50|0 / 12|0 (4 : 03 : 4
66 : 13 : 4

6. If a Fraction be in the Data.

Caſe 6. If beſides the whole Quantity given, there be ſome ſmall Part annexed, as $\frac{3}{4}$ or $\frac{1}{2}$, or ſuch-like: After Operation for the whole Quantity, the Price of that Part is to be added.

Example.

Example. If one Yard coſt 3 *s.* 4 *d.* what ſhall 300$\frac{1}{2}$ Yards coſt?

Anſw. 50 *l.* 1 *s.* 8 *d.* Where after 300 is divided by 6, becauſe 3 *s.* 4 *d.* is $\frac{1}{6}$ of a Pound, 1 *s.* 8 *d.* for the half Yard is added to the Quotient.

Yards.
At 3 *s.* 4 *d.* *per* Yard, what coſts 300$\frac{1}{2}$?

$\frac{1}{6}$ *l.* 300 / 6 (*l.* 50 *s.* *d.*
1 : 8 for the $\frac{1}{2}$ Yard.
50 1 : 8

7. If the Price exceed Pounds.

Caſe 7. If the Price given exceed Pounds, the Pounds are to be multiplied by the Quantity, and the reſt of the Price wrought out by ſome or other of the foregoing Caſes: And ſometime Geodetical Multiplication is uſed as the ſhorter Way.

Example.

Example. If one Hundred Weight coſt 3 *l.* 8 *s.* 4 *d.* what ſhall 7 C?

Anſw. 23 *l.* 18 *s.* 4 *d.* For ſo by Multiplication of 3 *l.* 8 *s.* 4 *d.* into 7, it will appear.

By Geodetical Multiplication.

C.
At 3 *l.* 8 *s.* 4 *d.* *per Cent.* what coſts 7?

7
21:56:28
23:18: 4

Otherwiſe.

C.
At 3 *l.* 8 *s.* 4 *d.* *per* C. what coſts 7?

3 × 7
$\frac{1}{2}$ of 8 is 4.
$\frac{1}{6}$ *l.* 4 *d.*
$\frac{7}{6}$.

7 7
3 4
l. 21 2|8 Primes.
2 : 16
2:4
23 : 18:4

8. If the Price of an hundred Weight, &c.

Caſe 8. If the Price of an hundred Weight be given, and it be required to know what ſome certain Number of Pounds will coſt, which are not the equal Half or Quarters of the hundred Weight, and ſo eaſily reckoned without the Pen: then thoſe Pounds may be broken into Aliquot Parts of 112, (the Weight of one Hundred) and Operation made thereby, like as in the other Caſes before.

A Table of the Aliquot Parts of 112.

The Aliquot Parts of 112.

Pounds.	*Parts.*		*Pounds.*	*Parts.*
1	112		14	8
2	56		16	7
4	28		28	4
7	16		56	2
8	14		112	1

Example of the Uſe of the Table.

Example. If 1 C. coſt 16 *l.* what will 35 ℔. coſt ?

Anſw. 5 *l.* For breaking 35 into Aliquot Parts of 112, that is, 28 and 7; and dividing 16 by 4, which ſtands againſt 28, and by 16 which ſtands againſt 7, among the Aliquot Parts, and adding the Quotients together, the Total will be 5 *l.*

At 16 *l.* *per* C. what coſts 35 ℔. ?

$\left.\begin{matrix}\frac{1}{4}\\ \frac{1}{16}\end{matrix}\right\}$ of 112 $\left\{\begin{matrix}28\\ 7\end{matrix}\right.$ — 35

$\frac{\cancel{16}}{\cancel{4}}\Big($ 4 *l.*

$\frac{\cancel{16}}{\cancel{16}}\Big($ 1

5 *l.*

Common Way.

℔.	*l.*		*l.*	*l.*
As 112	. 16	::	35	. 5

16 × 35: 80, 48 — 560

$\frac{\cancel{560}}{\cancel{112}}\Big($ 5 *l.*

9. The Price of 1 Pound given, to know the Price of the Hundred. **Billingſgate-Rule.**

Caſe 9. If the Price of one Pound be given, and it be required to know what the hundred Weight will coſt; or the contrary by the Price of the Hundred to know the Price of 1 ℔. the Rule called the *Billingſgate Rule*, is made uſe of, (being ſo eaſily remembred, that every common Coſtermonger at *Billingſgate* can preſently tell what it comes to) which is thus: For every Farthing in one Pound, take ſo many Groats, and double the Number of Shillings, and this ſhall be the Price of the hundred Weight. So that every Farthing in the Pound increaſes the hundred Weight ſo many times ſeven Groats.

Example.

Example. If 1 ℔. coſt 2 *d.* $\frac{1}{2}$, what ſhall 1 C. coſt ?

Anſw. 1 *l.* 3 *s.* 4 *d.* For in 2 *d.* $\frac{1}{2}$ there being 10 Farthings, the Hundred ſhall coſt 10 Groats, and twice 10 Shillings, which is 23 *s.* 4 *d.*

10. If 1 be in the third place.

Caſe 10. If an Unit ſtand in the third Place of the *Data*, and the firſt Number be an Article, then, according to the Ciphers in the Article cut off (by a Perpendicular Line from the higheſt Geodetical Denomination in the ſecond Number) ſo many Figures, and reduce the reſt by common Reduction, and the Numbers that exceed that Line to the Left-hand, ſhall be the Reſolution.

Example 1.

Example 1. If 10 Pieces of Cloth coſt 42 *l.* 5 *s.* 5 *d.* what doth one Piece thereof coſt ?

Anſw. 4 *l.* 4 *s.* 6 *d.* $\frac{1}{2}$. Here I cut off 2, of the 42 *l.* becauſe of the Cipher in 10; and that 2 *l.* with the 5 *s.* reduced, makes 45 *s.* of which 4 exceeds the Line; the other 5 *s.* with 5 *d.* is turned into 65 *d.* of which 6 exceeds the Line; and 5 left, the half of 10, is for the Half-penny; or multiplied by 4, is 20, of which the 2 exceeds the Line, and are Farthings.

Pieces.	*l.*	*s.*	*d.*		*Piece.*	*l.*	*s.*	*d.*
As 10	. 4|2	: 5	: 5	::	1	. 4	: 4	: 6 $\frac{1}{2}$

	|	
	|	20
s. 4	|	5
	|	12
d. 6	|	5
	|	4
q. 2	|	0

Common Way.

P.	*l.*	*s.*	*d.*		*P.*	*l.*	*s.*	*d.*
As 10	. 42	: 5	: 5	::	1	. 4	: 4	: 6 $\frac{1}{2}$

20 — 845, 12 — 1690, 845.5

10) 1014|5 ($\frac{\cancel{1014}}{12}$ 5(6 *d.* ($\frac{8_{,}4 \; s.}{l.\ 4:4:6\frac{1}{2}}$

Example 2.

Example 2. If 100 ℔. of Spice coſt 28 *l.* 10 *s.* 6 *d.* what doth one Pound thereof coſt ?

Anſw. 5 *s.* 8 *d.* 1 *q.* $\frac{21}{25}$. Here, becauſe 100 hath 2 Ciphers, 28 is wholly cut off. So the Price of one ℔. ſhall not reach to one *l.* The Work in the reſt is as the laſt above.

℔. *l.* *s.* *d.* ℔. *s.* *d.* *q.*
As 100 . 28 : 10 : 6 :: 1 . 5 : 8 : $1\frac{21}{25}$.
| 20
s. 5 | 70
| 12
d. 8 | 46
| 4
q. 1 | $\frac{84}{100}$, or $\frac{21}{25}$.

Common Way.
℔. *l.* *s.* *d.* ℔. *s.* *d.* *q.*
As 100 . 28 : 10 : 6 :: 1 . 5 : 8 : $1\frac{21}{25}$
20
570
12
1140
5706
6846
4
100) 273|84 (3 (1 q
273 / 4 (6 (8 d / 12 (5 *s.*

Proof of Practice. The Proof of all Proceedings in *Practice*, is by the common Work of the *Rule of Three*, and one Variety by another, as by the Examples before is ſufficiently clear.

Variety of Proof. Some prove the Operations of *Practice* by a contrary Queſtion, thus; Mark the Number given for the Price of one Ell, Yard, Pound, *&c.* in the firſt Queſtion, and how much it wanteth of 20 *s.* and by that which it wanteth, work with the firſt given Quantity ſtated into a ſecond Queſtion. And if both Reſolutions added together be equal to the Quantity, the Work of both is right.

As if 8 *s.* buy 1 Ell, then will 100 Ells coſt 40 *l.* on the contrary, becauſe 8 *s.* wanteth 12 *s.* of 20. if the Price of 100 Ells be enquired at 12 *s.* *per* Ell, the Reſolution will be 60 *l.* Now becauſe 40 and 60 make up 100, both Works are found right.

At 8 *s.* *per* Ell, what coſts 100 *Ells*?
4
l. 40|0

At 12 *s.* *per* Ell, what coſts 100 *Ells*?
6
l. 60|0

Proof.
40 + 60 = 100.

CHAP. V.

SPECIFICKS.

Specificks. THE laſt Chapter ended with thoſe Comparative Elements called *Primitive*; therefore this muſt begin with thoſe called *Derivative*, becauſe they borrow their Being from the former.

Foremoſt, in the order of *Derivatives*, ſtand *Specificks*; ſo called, becauſe ſomething is ſpecifical, in or about the Reſolution of the Queſtions propounded, either by reaſon of the contract Numbers mixed in the *Data*, or the different Denominations of ſome ſorts of Contract Numbers in one or more of the 3 given Numbers; or the right Preparation of the *Data*, or the intermixture of Queſtions or Proportions in the Propoſition, and that both in the *Direct* and *Indirect Rule of Three*. *Why ſo called.* *The Sorts thereof.*

Of theſe Diverſities, the two firſt Sorts are ſpecifical in reſpect of the Nature of the Numbers given, *viz.* the firſt in Fractions and Geodeticals; the ſecond in Geodeticals and Aſtronomicals: But the two laſt Sorts are Specifical in reſpect of the Nature of the Queſtion propounded.

1. If Fractions and Geodeticals be in the Data. 1. When Fractions and Geodeticals are mixed in the *Data*, place an Unit under the Geodeticals; and if any Fraction be mixed with an Integer, reduce the ſame into an Improper Fraction: then for Reſolution of the Queſtion, proceed as in the *Rule of Three*, multiplying and dividing after the manner of Fractions.

Operation by the Direct Rule of Three.

Examples in the Direct Rule of Three. *Example* 1. If 2 Ells coſt $\frac{3}{4}$ *l.* what ſhall $1\frac{3}{4}$ Ell coſt?

Anſw. $\frac{7}{12}$ *l.* or 11 *s.* 8 *d.* Here $1\frac{3}{4}$ is to be reduced to $\frac{7}{4}$, and multiplied into $\frac{2}{3}$, as Fractions whoſe Product is to be divided by $\frac{2}{1}$, or 2 the firſt Number.

Of the Price of $1\frac{3}{4}$ *Ell.*

Ells. *l.* *Ells.* *l.*

As $2 \,.\, \frac{2}{3} :: 1\frac{3}{4} \,.\, \frac{7}{12}$

For $\frac{\overset{1}{\cancel{2}}}{3} \times \frac{7}{\underset{2}{\cancel{4}}} = \frac{7}{6}$ And $\frac{2}{1})\frac{7}{6}(\frac{7}{12}$ *l.*

Example 2. If $1\frac{1}{2}$ Ell coſt $1\frac{1}{4}$ *l.* what ſhall $3\frac{3}{4}$ Ells coſt?

Of the Price of $3\frac{3}{4}$ *Ells.*

Anſw. $3\frac{1}{8}$ *l.* or 3 *l.* 2 *s.* 6 *d.* The *Data* reduced into improper Fractions, make $\frac{3}{2} \,.\, \frac{5}{4} \,.\, \frac{15}{4}$; the reſt of the Work needs nothing to explain it.

$\frac{3}{\textit{Ell.}}$ $\frac{5}{\textit{l.}}$ $\frac{15}{\textit{Ells.}}$ *l.*

As $1\frac{1}{2} \,.\, 1\frac{1}{4} :: 3\frac{3}{4} \,.\, 3\frac{1}{8}$

For $\frac{5}{4} \times \frac{15}{4} = \frac{75}{16}$ And $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}})\frac{\overset{25}{\cancel{75}}}{\underset{8}{\cancel{16}}}(\frac{25}{8}$ or $3\frac{1}{8}$ *l.*

Operation by the Indirect Rule of Three.

Examples in the Indirect Rule of Three.

Example 1. If at 4 *s.* the Buſhel, the Penny White-loaf muſt weigh 9 Ounces *Troy*: what muſt it weigh at 5 *s.* $\frac{1}{2}$ *per* Buſhel?

Anſw. $6\frac{6}{11}$ Ounces. Here $5\frac{1}{2}$ reduced into an Improper Fraction, is $\frac{11}{2}$, under the other Numbers an Unit is placed, and the Operation is as the Indirect Rule of Three in Fractions.

Of the Weight of Bread.

s. *Ounc.* $\frac{11}{\textit{s.}}$ *Ounc.*

As $\frac{4}{1} \,.\, \frac{9}{1} :: 5\frac{1}{2} \,.\, 6\frac{6}{11}$

For $\frac{4}{1} \times \frac{9}{1} = \frac{36}{1}$ And $\frac{11}{2})\frac{36}{1}(\frac{72}{11}$, or $6\frac{6}{11}$. *Ounces.*

Example 2. If $6\frac{6}{11}$ Ounces *Troy* be the Weight of a Loaf, when Wheat is ſold for 5 *s.* 6 *d.* the Buſhel: what ſhall the ſame Loaf weigh when Wheat is ſold for 12 *s.* the Buſhel?

Another of the Weight of Bread.

Anſw. 3 Ounces. Here $6\frac{6}{11}$ is reduced to $\frac{72}{11}$, and $5\frac{1}{2}$ to $\frac{11}{2}$; and under 12 is placed an Unit; and the Multiplication and Diviſion is as before.

$\frac{11}{\textit{s.}}$ $\frac{72}{\textit{Oun.}}$ *s.* *Ounces.*

As $5\frac{1}{2} \,.\, 6\frac{6}{11} :: \frac{12}{1} \,.\, 3$

For $\frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{36}{\cancel{72}}}{\underset{1}{\cancel{11}}} = \frac{36}{1}$ And $\frac{\overset{1}{\cancel{12}}}{1})\frac{\overset{3}{\cancel{36}}}{1}(\frac{3}{1}$, or 3. *Ounces.*

2. When the *Data* are Geodeticals or Aſtronomicals of different Denominations, either the Multiplication and Diviſion uſed for Reſolution of the Queſtions propounded in ſuch Numbers, muſt be proper according to the Nature of the Numbers; or elſe they muſt be either reduced Geodetically, by common Reduction, into the loweſt Denomination, or turned into Decimals.

2. *If the* Data *be of divers Denominations. Theſe reduced or turned into Decimals.*

Touching Reduction of the given Numbers, obſerve:

Their common Reduction is to be noted in four things.

§. 1. If the ſecond Number be reduced, he muſt be reduced into the loweſt Denomination of them given, lower than which he need not be brought. As if the ſecond Number be Pounds, Shillings, and Pence, it muſt be brought all into Pence: And if Pounds, Shillings, Pence, and Farthings, then all muſt be reduced into Farthings, *&c.* 1.

§. 2. If the ſecond Number be reduced, then the fourth Number, when found, ſhall be of like Denomination to the Second when reduced, whether Shillings, Pence, Farthings, *&c.* 2.

3. §. 3. If the ſecond Number multiplied into the Third, in the *Direct Rule of Three*, or multiplied into the Firſt in the *Indirect Rule of Three*, be too little to be divided by the Diviſor; then to avoid the Work in Fractions, the ſecond Number may be reduced from a groſs Denomination into a ſmaller or ſubtiler, as from Pounds to Shillings, *&c.* And likewiſe the fourth Number, when found, often-times admits of Reduction from a ſmaller Denomination into a Groſſer.

4. §. 4. If the firſt and third Numbers be of plural Denominations, or ſingle Numbers but of different Denominations, as they muſt be reduced into the loweſt of the given Denominations; ſo they both muſt be reduced to one and the ſame Denomination. As if the one be Pounds and Shillings, and the other Pounds, Shillings and Pence, or Shillings and Pence; both Numbers muſt be reduced into Pence, *&c.*

Theſe things obſerved, the reſt of the Work differs not from that in the *Rule of Three* Direct or Indirect for *Geodeticals*, or Direct only in *Aſtronomicals*, they being all Direct Proportions.

Operation in Geodeticals, *by the* Direct Rule of Three.

Examples in Geodeticals. *What* 3 *l. will buy.*

Example. If 4 *l.* 14 *s.* 4 *d.* buy 2 C. 12 ℔. 6 ℥. what ſhall 3 *l.* buy?

Anſw. 1 C. 38 ℔. 5 ℥ $\frac{145}{283}$. Here the firſt and third Numbers reduced into Pence, are 1132, and 720. The ſecond Number brought into Ounces, is 3782. And after multiplication of 720 into 3782, and diviſion of the Product by 1132, the Quotient is 2405$\frac{145}{283}$ Ounces, according to the Reduction of the ſecond Number, and may be brought into the groſſer Denominations of Pounds and Hundreds. And if the firſt and third Numbers after Reduction were abbreviated, the Multiplication and Diviſion would be ſhorter.

```
     l.   s.  d.    C.  ℔.  ℥.    l.     C.   ℔.   ℥.
As   4 : 14 : 4 .   2 : 12 : 6 :: 3 .    1 : 38 : 5 145/283.
    20              112           20
    ---            ----           ---
    94 s.          236 ℔          60 s.
    12              16            12
    ---            ----           ---
   188             1416          120
   944             2366           60
   ---             ----           ---
  1132 d.          3782 ℥.       720 d.
                    720                    6|5
                  -----                 4592|8                      ℔.
                  75640                 ----           8  ℥       |38      C.
                 26474               2723040|0 ---(  240|5    (   150   (  1
                -------                 1132           16         112
                2723040                 1132           16
                                        1132           16
                                        1132
```

Operation in Geodeticals *by the* Indirect Rule of Three.

What Stock to raiſe a Profit propoſed.

Example. If Stock to the Value of 10 *l.* 13 *s.* 3 *d.* raiſe a conſiderable Profit in 12 Months: what Stock ſhall raiſe the ſame Profit in 16 Months?

Anſw. 7 *l.* 19 *s.* 11 *d.* $\frac{1}{4}$. Here the ſecond Number only needs Reduction into Pence, or may be turned into a Decimal. And becauſe the firſt and third Numbers will abbreviate, may be wrought as in *Practice.*

```
Mon.  l.  s.  d.   Mon. l.  s.  d.         Mon.  l.  s.  d.     Mon.  l.   s.  d.
As 12 . 10: 13 : 3 :: 16 . 7: 19: 11 1/4   As 12 . 10: 13 : 3 :: 16 . 7 : 19: 11 1/4
    3   20              4                      3 . 10,6625   ::  4 . 7,9968 1/4
       ---                                          3
       213              2 d.                 4)31,9875(7,9968 1/4
        12
       ---                                          |  20
       426   3 3(1    7|11                      ----|----
      2133   7677  (  1919  ( 15|9             s. 19|9375
      ----                                          |  12
      2559    4        22     ----------         ---|----
         3                    l.7:19:11 1/4         1|8750
      ----                                          9|375
      7677                                      ----|----
                                               d. 11|2500
                                                    |   4
                                                ----|----
                                               q.  1|0000
```

Operation in Aſtronomicals.

Example in Aſtronomicals. Of the Æquation of Mars.

Example. Suppoſe the Æquation of ♂ be deſired, and his Anomaly given be o Sign, 18 Deg. 12′. 30″. and the Difference of Æquations found by the Aſtronomical Tables, between the 18*th* and 19*th* Degree, be 9′ : 36″ decreaſing, the Queſtion then will ſtand thus:

If 1 Degree, or 60′, give 9′ 36″, what ſhall 12′ 30″?

And the Proportional Part gained will be 2′, to be ſubſtracted from the given Anomaly, as the following Operations make plain.

By Geodetical Reduction.

```
    ′    ′    ″      ′    ″    ′
As 60 .  9 : 36 :: 12 : 30 .  2
   60   60          60
 ----  ---         ---
 3600  576         750
                   576
                  ----
                  4500
                 5250
                3750
                ------
     3600 ) 432000 ( 120″ ( 2′
                    ----
                     60
```

By the Sexagenary Table.

```
 Deg.  ′    ″      ′    ″    ′
As 1 . 9 : 36 :: 12 : 30 .  2
      12 : 30
      -------------
       1 : 55 . 12‴
           4 : 48 . 00⁗
      ------------------
       2 . 00 . 00 . 00
```

By Decimals.

```
    ′    ′        ′
As 60 . 9,6 :: 12,5 . 2
                 9,6
                ----
                 750
               1125
              ------
      60 ) 120,00 ( 2′,00
```

```
                       s.   d.   ′    ″
Anomaly of ♂        .  0 . 18 . 12 . 30
Proportional }
   Part      }                   2 . 00
                    ---------------------
Æquation of ♂       .  0 : 18 . 10 . 30 deſired.
```

3. When the *Data* are not the ſame three Numbers Reſolution is to be had by, but theſe are included in the Queſtion, and according to the State thereof, by a due preparation of the *Data*, thoſe more covert Numbers are diſcovered, through help of ſome or other of the Simple Elements of Numbers which they call to their Aid. *3. If the Data muſt be prepared.*

In this third Sort of *Specificks*, diligent conſideration muſt be had of the State of the Queſtion, and Nature of the Number queſited thereby to find the three Numbers to work by in the *Rule of Three*, ſince no Rule can be given to reach all Caſes; but ſometime one, ſometime another, and ſometime more than one of the Simple Elements of Numbers are needful to prepare the *Data*. So as much depends on the Ingenuity of the Operator, as *Ptolomy* once ſaid, *Abs te & à Scientia.* *State of the Queſtion to be diligently heeded, and the reaſon.*

After the Numbers fit for Reſolution are obtained, which muſt be firſt done, (ſeeing without the true *Data* no Queſtion can be truly reſolved) let the Numbers, however propoſed in the Queſtion, (ſometime only to try the Skill of the Reſolver) be duly diſpoſed, according to the Precepts before given in the *Rule of Three*, and then operate by the Direct or Indirect Rule as the Caſe requires; for all Operation by a wrong Rule will render the Reſult falſe. *Numbers to be duly diſpoſed, however propounded.*

Addition needful.

Addition uſeful.

Example 1. Two Poſts, (ſuppoſe *A* and *B*) depart one from another, one directly Eaſtward, and the other directly Weſtward: *A* travelleth 18 Miles a Day, and *B* 30: How far are they diſtant the third Day after their Departure? *Q. Of the Diſtance of two Poſts.*

Anſw. 144 Miles. Here 18 and 30, the Travel of both in one Day, are to be added, and the Work with the Total committed to the *Direct Rule of Three*, thus. *Anſwer.*

A . 18 — *Day.* As 1 . *Miles.* 48 :: *Days.* 3 . *Miles.* 144.
B . 30 — 48 × 3
48 — 144 Miles.

Q. Of the lining of two Cloaks.

Example 2. If one Cloak may be lined with $4\frac{1}{2}$ Yards of Stuff that is yard broad: what Pluſh will line two of thoſe Cloaks when the Pluſh is but $\frac{3}{4}$ of a yard broad, and one Cloak will be lined with $\frac{1}{2}$ Yard leſs than the other?

Anſwer.

Anſw. $11\frac{1}{3}$ Yards of Pluſh. Here the breadth of both Cloaks is to be added, and the Work with the Total committed to the *Indirect Rule of Three*, thus.

1. Cloak $4\frac{1}{2}$
2. Cloak 4

$8\frac{1}{2}$

	Yard-broad.		Yards long.		Yards broad.		Yards long.
As	$\frac{1}{1}$	.	$8\frac{1}{2}$	::	$\frac{3}{4}$	.	$11\frac{1}{3}$

$$\overline{17}$$

$$\frac{2}{\overset{\cdot\cdot}{\underset{2}{4}}}\Big)\frac{17}{\overset{\cdot\cdot}{\underset{1}{2}}}\Big(\frac{34}{3}\Big(11\tfrac{1}{3}\ \text{Yards long.}$$

Subſtraction useful.

Subſtraction needful.

Q. Of the filling of a Ciſtern.

Example 1. A Conduit-Pipe rnnning into a Ciſtern, holding 250 Gallons, poureth in every Hour 24 Gallons; but the Ciſtern by another Pipe running out thereof, emptieth of it ſelf in every Hour 16 Gallons: in what time ſhall the Ciſtern be filled, both the Pipes running?

Anſw.

Anſw. $31\frac{1}{4}$ Hours. Here the Gallons emptied by the one Pipe, muſt be ſubſtracted from the Gallons filled by the other, that is, 16 from 24; and the Work with the Remain committed to the *Direct Rule of Three*, thus.

Greater Pipe 24
Leſſer Pipe 16

8

	Gallons.		Hour.		Gallons.		Hours.
As	8	.	1	::	250	.	$31\frac{1}{4}$

$$\frac{\overset{1(2}{250}}{8}\Big(31\tfrac{1}{4}\ \text{Hours.}$$

Q. Of the length, &c. to ½ Acre.

Example 2. If 4 Perches in breadth, when 40 is the length of a Piece of Land, make 1 Acre: what length muſt there be to make one half-Acre, when the breadth is 6 Perches?

Anſw.

Anſw. $13\frac{1}{3}$ Perches in length. Here, becauſe the Enquiry is but for half an Acre, and 4 was a Proportion of breadth given for a whole Acre; therefore half the 4 ſhall be taken away, and the Work with the Remain committed to the *Indirect Rule of Three*: Thus,

1. Acre 4
½. Acre 2

2

	Perches broad.		Perches long.		Perches broad.		Perches long.
As	2	.	40	::	6	.	$13\frac{1}{3}$

$$6\Big)\frac{\overset{2}{\overline{80}}}{}\Big(13\tfrac{1}{3}\ \text{Perches long.}$$

Multiplication useful.

Multiplication needful.

Q. Of the Powder ſpent by ſeven Guns.

Example 1. Suppoſe 5 Guns being often diſcharged, ſpend 60 Barrels of Powder: what will 7 Guns which ſpend three times as much as the other at every ſhot?

Anſw.

Anſw. 252 Barrels. Here 60 is multiplied by 3, and the Work with the Product is committed to the *Direct Rule of Three*: Thus,

Barrels 60
Increaſed 3

180

	Guns.		Barrels.		Guns.		Barrels.
As	5	.	180	::	7	.	252

$$5\Big)\frac{\overset{7}{\overline{1260}}}{}\Big(252\ \text{Barrels.}$$

Example

Q. *Of Paſturage of Sheep.*

Example 2. If one have right to paſture on a Common 100 Sheep 40 Days, and he paſtureth there four times as many 12 Days: hath he tranſgreſſed or not?

Anſwer.

Anſw. Yes, by the ſpace of two Days. Here 4 is to multiply 100, and the Work with the Product to be committed to the *Indirect Rule of Three*, by which it is reſolved he ought to have paſtured 400 Sheep but 10 Days.

Sheep 100
Increaſed 4
400

Sheep. *Days.* *Sheep.* *Days.*
As 100 . 40 :: 400 . 10
100
400) 4000 (10 Days.

Diviſion needful.

Diviſion *uſeful.*

Q. *Of Wine drunk.*

Example 1. Four Gueſts at Table drank 16 *d.* in Wine: how many Gueſts that drink but half ſo much as the former will 18 Pennyworth of Wine ſerve?

Anſwer.

Anſw. 9 Gueſts. Here 16 is to be halfed, or divided by 2, and the Work with the Quotient committed to the *Direct Rule of Three*: Thus,

Wine 16 (8
Halfed 2

d. *Gueſts.* *d.* *Gueſts.*
As 8 . 4 :: 18 . 9
4
8) 72 (9 Gueſts.

Q. *Of Hay to feed Cattel.*

Example 2. If a Parcel of Hay will maintain 90 Head of Cattel 10 Weeks, and ⅓ of the Cattel be put out to keeping: how long will the Hay maintain the Reſt?

Anſwer.

Anſw. 15 Weeks. Here 90 is divided by 3, and the Quotient, or the third Part of 90 taken from thence, the Reſidue is committed to the *Indirect Rule of Three*: Thus,

Cattel 90 (30
Divided 3

90—30=60

Cattel. *Weeks.* *Cattel.* *Weeks.*
As 90 . 10 :: 60 . 15
90
60) 900 (15 Weeks.

Diviſion and Addition needful.

Div. *and* Add. *uſed.*

Q. *Of Reapers.*

Example. Certain Reapers in 5 Days can reap 24 Acres of Corn: in how many Days can they, with each Man his Servant, every of which doth half as much Work as his Maſter, reap 144 Acres?

Anſwer.

Anſw. In 20 Days. Here the half of 24 being added thereto, the Work is committed to the *Direct Rule of Three*: Thus,

Acres 24 (12 24
Halfed 2 12
36

Acres. *Days.* *Acres.* *Days.*
As 36 . 5 :: 144 . 20
5
36) 720 (20 Days.

Subſtraction and Multiplication needful.

Sub. *and* Multi. *uſed.*

Q. *Of purſuing a Thief.*

Example. *A* having ſtolen certain Goods, fleeth 40 Miles a Day: *B* ſetting out 4 Days after him, purſueth 50 Miles a Day: in how many Days may *B* overtake *A*?

Anſwer.

Anſw. In 16 Days. Here the Difference between the Journey of *A* and *B* in one Day firſt taken, and the Day's Journey of *A*, multiplied by the number of Days *B* ſet out after *A*, the Work is committed to the *Direct Rule of Three*: Thus,

				Miles.	*Day.*		*Miles.*	*Days.*
B . 50	*A* . 40		As	10	. 1	::	160	. 16
A . 40	4							
Differ. 10 Miles.	160							

☞ In like manner Queſtions may be compoſed, wherein *Multiplication* and *Addition*, or *Diviſion* and *Subſtraction*, &c. may be needful: but theſe are ſufficient for Example here.

4. If Queſtions be intermixt. 4. When there is an Intermixture of Queſtions or Proportions in the Propoſition, the one will be *expreſs*, and the other *implied*.

Expreſly. §. 1. When the ſeveral Queſtions are expreſſed in the Propoſition, then proceed according to the State of the Queſtion in the Reſolution of either; firſt with one, and then with the other.

Q. Of the Miles travel to overtake a Thief. *Example* 1. Suppoſe in the laſt Propoſition it had been demanded, not only in how many Days *B* ſhould overtake *A*, but alſo after how many Miles travel? Then after the firſt Work, as above, had reſolved *B* to overtake *A* in 16 Days, another Queſtion ſhould be committed to the *Direct Rule of Three*, for the Reſolution of this latter Query: Thus,

If *B* travel in 1 Day 50 Miles; how far ſhall he travel in 16 Days?

Anſwer. *Anſw.* 800 Miles.

	Day.		*Miles.*		*Days.*		*Miles.*
As	1	.	50	::	16	.	800
					50		
					800 Miles.		

Q. Of a Caſtle beſieged. *Example* 2. A Caſtle beſieged hath Victuals enough for 1200 Men 7 Months; but the Captain finding the Siege is like to be long, and that fewer Men will defend it, he would disband ſome of his Men, and lengthen out his Proviſion to 12 Months: how many Men ſhall he retain, and how many disband?

Anſwer. *Anſw.* By the *Indirect Rule of Three*, 700 Men are to be retained; which found, that Number is to be taken from the 1200 propounded, and the Reſidue, that is 500, to be disbanded.

	Months.		*Men.*		*Months.*		*Men.*
As	7	.	1200	::	12	.	700
			7				
	12)		8400 (		700 Men retained. 500 Men disbanded.		
			1200				

Implicitly, with a Mixture of Ratio's, &c. §. 2. When in the Propoſition there is ſuch an Intermixture of *Ratio's* or Proportions, that as neceſſary to the Reſolution of the Demand, either the Elements proper to *Ratio's* muſt be uſed, or more than one Operation of the *Rule of Three*, though perhaps but three Numbers given; and in ſuch Operations the Quotient of the one Work is to be added to, or ſubſtracted from ſome or other of the *Data*, or the contrary, before Reſolution can be had by the other: which differs from thoſe Queſtions falling under the *Rule of five Numbers*, treated of in the two next Chapters, becauſe they may be reſolved at one Operation; yet they always give five Numbers; and if they be reſolved by two Works, the Quotient of the firſt is taken whole without Alteration for the next Work, which is not ſo in theſe *Specificks*. *Wherein theſe differ from thoſe that fall under the Rule of five Numbers.*

Data, 3 Numb. and Add. uſed. Data, 3 *Numbers and Addition needful.*

Q. Of two doing a piece of Work. *Example* 1. *A* and *B* are hired to do a Piece of Work, which *A* can do alone in 30 Days, and *B* in 20 Days: in how many Days can they do it together?

Anſwer. *Anſw.* In 12 Days. If I work by *Ratio's*, always where the *Ratio* is manifold, or the contrary, that is, to an Unit, it matters not which of the Terms be made Antecedent: For if the Unit be made Conſequent, the *Ratio's* are to be reduced to like Antecedents; and if Antecedent, to like Conſequents. Now here being two *Ratio's*, *viz.* that of *A* to the Work, as 30 to 1; and that of *B*, as 20 to 1;

Firſt I reduce them to like Antecedents: So are they $\overset{600}{\underset{20}{:}}$ and $\overset{600}{\underset{30}{:}}$, and in their leaſt Terms $\overset{60}{\underset{2}{:}}$ and $\overset{60}{\underset{3}{:}}$: after Reduction I divide this common Antecedent by 30 added to 20, the Sum of the reduced Conſequents, or 60 by 5, that is, 3+2.

Or otherwiſe, if I work by the *Rule of Three*, my firſt Queſtion is, If 30 Days of *A* can do 1 Work: what ſhall 20 Days of *B*? To which the Anſwer ⅔ muſt be added to the Piece of Work to be done; and that 1⅔ Work ſhall be the firſt Number of the ſecond Queſtion, thus: If 1⅔ Work come of 20 Days when *B* works with *A*, of what ſhall 1 Work come? *Variety of Operation.*

By Ratio's.

$$\overline{A\ \overset{30}{\underset{1}{:}}\quad B\ \overset{20}{\underset{1}{:}}}^{\,600}$$

$$\begin{array}{l} 20\times30=\dfrac{60|0}{5|0}\Big(\,12\text{ Days.} \\ 20+30=5 \end{array}$$

By the Rule of Three.

	Days.		*Work.*		*Days.*		*Work.*
As	30	.	1	::	20	.	⅔

$$\frac{2|0}{3|0}\ \text{Work.}$$

	Work.		*Days.*		*Work.*		*Days.*
As	1⅔	.	20	::	1	.	12

$$\frac{\overset{1}{5}}{3}\Big)\frac{\overset{4}{20}}{1}\Big(\frac{12}{1}\ \text{Days.}$$

Example 2. A Conduit hath three Cocks; if the Ciſtern be full, and Water run out by the greateſt Cock, the Ciſtern will be empty in three Hours; if it run out by the ſecond Cock, the Water will be all run out in four Hours; but if the Water run out by the leaſt Cock, it will be five Hours e're the Ciſtern be emptied: In what Time will all the Water be run out if it run by all the Cocks together? *Q. Of emptying a Ciſtern.*

Anſw. In 1 Hour and $\frac{13}{47}$ of an Hour. Here the three *Ratio's* given being reduced to like Antecedents, the common Antecedent will be 60, to be divided by 47 the Sum of the reduced Conſequents. Anſwer.

$$\overline{\text{Great Cock } \overset{3}{\underset{1}{:}},\ \text{Second Cock } \overset{4}{\underset{1}{:}},\ \text{Leaſt Cock } \overset{5}{\underset{1}{:}}.}^{\,60}$$

$$\underline{\phantom{\text{Great Cock}}}\quad 20 \qquad 15 \qquad 12$$

$$\begin{array}{l} \text{Antecedents } 3\times4\times5 = 60 \\ \text{Conſequents } 20+15+12 = 47 \end{array}\Big(\,1\tfrac{13}{47}\ \text{Hour.}$$

Otherwiſe, if the Work be wrought by the *Rule of Three*, the firſt Queſtion is, If three Hours running of the great Cock empty the Ciſtern: what will four Hours running of that Cock? The Anſwer to which, 1⅓ Ciſtern muſt be added to the Ciſtern: And this 2⅓ ſhall be the firſt Number of the ſecond Queſtion, thus; If 2⅓ Ciſterns be run out in four Hours, when will 1 Ciſtern be run out? Then $1\frac{5}{7}$ Hour, the Anſwer to this Queſtion, ſhall be the firſt Number of the third Queſtion, thus; If $1\frac{5}{7}$ Hour empty 1 Ciſtern, what will 5 Hours do? And $2\frac{11}{12}$ the Anſwer to this, ſhall be added again to the 1 Ciſtern, and the laſt Queſtion ſtands thus; If $3\frac{11}{12}$ Ciſterns will be emptied in 5 Hours, when ſhall 1 Ciſtern be emptied? The Anſwer to which is $1\frac{13}{47}$ Hour as before. *Variety of Operation.*

		Hours.		*Ciſtern.*		*Hours.*		*Ciſtern.*
1.	As	3	.	1	::	4	.	1⅓

$$\frac{4}{3}\Big(\,1\tfrac{1}{3}\ \text{Ciſtern.}$$

		Ciſterns.		*Hours.*		*Ciſtern.*		*Hour.*
2.	As	2⅓	.	4	::	1	.	$1\frac{5}{7}$

$$\frac{7}{3}\Big)\frac{4}{1}\Big(\frac{12}{7}\ \text{Hour.}$$

		Hour.		*Ciſtern.*		*Hours.*		*Ciſterns.*
3.	As	$1\frac{5}{7}$	.	1	::	5	.	$2\frac{11}{12}$

$$\frac{12}{7}\Big)\frac{5}{1}\Big(\frac{35}{12}\ \text{Ciſterns.}$$

		Ciſterns.		*Hours.*		*Ciſtern.*		*Hour.*
4.	As	$3\frac{11}{12}$	.	5	::	1	.	$1\frac{13}{47}$

$$\frac{47}{12}\Big)\frac{5}{1}\Big(\frac{60}{47}\ \text{Hour.}$$

Data, 3 *Numb. and Sub. uſed.*

Data 3, *Numbers and Subſtraction needful.*

Q. *Of* 1 *doing a piece of Work.*

Example 1. *A* and *B* do a Piece of Work together in 12 Days, which *A* alone can do in 30 Days: in how many Days can *B* do the ſame if he work alone?

Anſwer.

Anſw. In 20 Days. Here the two *Ratio's* given are as 12 to 1, and 30 to 1; which being reduced to like Antecedents, this common Antecedent is to be divided by the Difference of their reduced Conſequents.

Variety of Operation.

Otherwiſe to work by the *Rule of Three*, the firſt Queſtion is; If 30 Days of *A* can do 1 Piece of Work, what ſhall 12 Days of *A* joined with *B* do? To which the Anſwer $\frac{2}{5}$ of a Piece of Work ſhall be taken from the whole Piece, and the Reſidue, which is $\frac{3}{5}$, ſhall be the firſt Number of the ſecond Queſtion, thus; If $\frac{3}{5}$ of the Work ſhall come of 12 Days, of what comes the whole Work?

By *Ratio's.*

$$A \text{ and } B\ \overline{\overset{12}{\ddot{1}}\ \ A\ \overset{30}{\ddot{1}}}^{\,360}$$

$$\begin{array}{l} 30\times 12 = 360 \\ 30-12 = 18 \end{array} \Big(\text{20 Days.}$$

By the Rule of Three.

$$\begin{array}{lcccc} & \textit{Days.} & \textit{Work.} & \textit{Days.} & \textit{Work.} \\ \text{As} & 30\ . & 1\ :: & 12\ . & \frac{2}{5} \end{array}$$

$$6\Big)\frac{12}{30}\Big(\frac{2}{5}\text{ Work.}$$

$$\begin{array}{lcccc} & \textit{Work.} & \textit{Days.} & \textit{Work.} & \textit{Days.} \\ \text{As} & \frac{3}{5}\ . & 12\ :: & 1\ . & 20 \end{array}$$

$$\frac{\overset{1}{\ddot{3}}}{5}\Big)\frac{\overset{4}{\ddot{12}}}{1}\Big(\frac{20}{1}\text{ Days.}$$

Q. *Of filling a Ciſtern.*

Example 2. Water runneth into a Ciſtern by a Pipe that can fill it in 8 Hours, and runneth out by another Pipe which can empty the Ciſtern in 22 Hours: in what Time both running will the Ciſtern be full?

Anſwer.

Anſw. In 12 Hours and $\frac{4}{7}$ of an Hour. Here the *Ratio's* of 8 to 1, and 22 to 1, reduced to like Antecedents, make $\overset{176}{\underset{22}{\ddot{}}}$ and $\overset{176}{\underset{8}{\ddot{}}}$, and in their leaſt Terms $\overset{88}{\underset{11}{\ddot{}}}$ and $\overset{88}{\underset{4}{\ddot{}}}$. This common Antecedent 88, divided by 7, the Difference of the Conſequents gives in the Quotient $12\frac{4}{7}$ as before.

$$\overline{\text{Filling Pipe } \overset{8}{\underset{22}{\ddot{1}}}\text{, Emptying Pipe } \overset{22}{\underset{8}{\ddot{1}}}}^{\,176}$$

$$\begin{array}{l} \text{Antecedents } 8\times 22 = 176 \\ \text{Conſequents } 22-8 = 14 \end{array} \Big(\overset{\text{Hours.}}{12\tfrac{4}{7}}, \text{ or } \tfrac{88}{7}\big(12\tfrac{4}{7}$$

Variety of Operation.

Otherwiſe, to work by the *Rule of Three*, the firſt Queſtion is, If 8 Hours fill 1 Ciſtern, what ſhall 22 Hours fill? The Anſwer whereof $2\frac{3}{4}$ ſhall be ſubſtracted from 1; ſo will there want $\frac{7}{4}$, which ſhall be the firſt Number of the ſecond Queſtion to be wrought as the other in the former Example.

$$\begin{array}{lcccc} & \textit{Hours.} & \textit{Ciſtern.} & \textit{Hours.} & \textit{Ciſterns.} \\ \text{As} & 8\ . & 1\ :: & 22\ . & 2\frac{3}{4} \end{array}$$

$$\frac{\overset{(6}{\cancel{22}}}{8}\Big(2\tfrac{3}{4}\text{ Ciſterns.}$$

$$\frac{4}{4} - \frac{11}{4} = \frac{7}{4}$$

$$\begin{array}{lcccc} & \textit{Ciſtern.} & \textit{Hours.} & \textit{Ciſtern.} & \textit{Hours.} \\ \text{As} & \frac{7}{4}\ . & 22\ :: & 1\ . & 12\frac{4}{7} \end{array}$$

$$\frac{7}{4}\Big)\frac{22}{1}\Big(\frac{88}{7}\Big(12\tfrac{4}{7}\text{ Hours.}$$

Data, *above* 3 *Numb. and Add. uſed.*

Data, *more than* 3 *Numbers and Addition needful.*

Q. *Of* 4 *Mills grinding.*

Example 1. Suppoſe four Mills, whereof the firſt grindeth 4 Quarters in 3 Hours; the ſecond 5 Quarters in 4 Hours; the third 6 Quarters in 5 Hours; and the fourth 7 Quarters in 6 Hours: in what Time ſhall they all grind 30 Quarters?

Anſw. In 6 Hours and $\frac{2}{33}$ of an Hour. The *Ratio's* in this Propoſition having both Terms greater than Units, they are to be reduced to like Conſequents, and the Sum of the Antecedents divided by the common Conſequent. And becauſe there is a Limitation in the Demand of 30 Quarters, the Quotient of this Diviſion is to be the firſt Number of a Queſtion to be reſolved by the *Rule of Three*, with an Unit in the ſecond Place, and the 30 inquired for in the Third. Anſwer.

$$\begin{array}{lcccc} & \underline{480} & \underline{450} & \underline{432} & \underline{420} \\ \text{Quarters} & 4 & 5 & 6 & 7 \\ & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \text{Hours} & 3 & 4 & 5 & 6. \\ \hline & & 360 & & \end{array}$$

$$\begin{array}{ll} \text{Antecedents} & 480+450+432+420=\overset{|34}{\cancel{1}78|2} \\ \text{Conſequents} & 3\times 4\times 5\times 6=\cancel{3}\cancel{6}0 \end{array}\Big(4\tfrac{19}{20}\ \text{Quarters.}$$

Then, If $4\frac{19}{20}$ Quarters be ground in 1 Hour, in what Time ſhall 30 Quarters be ground? *Facit* as aforeſaid, $6\frac{2}{33}$ Hours.

$$\text{As } \tfrac{99}{20} \cdot 1 :: 30 \cdot 6\tfrac{2}{33} \qquad \tfrac{\overset{33}{\cancel{99}}}{20}\Big)\tfrac{\overset{10}{\cancel{30}}}{1}\Big(\tfrac{200}{33}\Big(6\tfrac{2}{33}$$

The Work by the *Rule of Three*, where the Antecedents and Conſequents of the *Ratio's* given are both greater than Units, is different from that where one Term is an Unit in two things. Firſt, In working the ſeveral Queſtions, the Conſequents muſt be made Antecedents, and the Antecedents Conſequents. And, 2*ly*, at the laſt the Conſequent is to be divided by the Antecedent, as in the following Operation. *Variety of Operation. The Difference herein from the former.*

1. $\text{As } \frac{3}{4} \cdot 1 :: \frac{4}{5} \cdot 1\frac{1}{15}$

$$\tfrac{3}{4}\Big)\tfrac{4}{5}\Big(\tfrac{16}{15} \qquad \begin{array}{c} 1 \text{ added.} \\ \hline 2\frac{1}{15} \\ \hline \end{array}$$

2. $\text{As } 2\frac{1}{15} \cdot \frac{4}{5} :: 1 \cdot \frac{12}{31}$

$$\tfrac{31}{\underset{3}{\cancel{15}}}\Big)\tfrac{4}{\underset{1}{\cancel{5}}}\Big(\tfrac{12}{31} \qquad 3 :: 1$$

3. $\text{As } \frac{12}{31} \cdot 1 :: \frac{5}{6} \cdot 2\frac{11}{72}$

$$\tfrac{12}{31}\Big)\tfrac{5}{6}\Big(\tfrac{155}{72} \qquad \begin{array}{c} 1 \text{ added.} \\ \hline 3\frac{11}{72} \\ \hline \end{array}$$

4. $\text{As } 3\frac{11}{72} \cdot \frac{5}{6} :: 1 \cdot \frac{60}{227}$

$$\tfrac{227}{\underset{12}{\cancel{72}}}\Big)\tfrac{5}{\underset{1}{\cancel{6}}}\Big(\tfrac{60}{227}$$

5. $\text{As } \frac{60}{227} \cdot 1 :: \frac{6}{7} \cdot 3\frac{17}{70}$

$$\tfrac{\overset{10}{\cancel{60}}}{227}\Big)\tfrac{\overset{1}{\cancel{6}}}{7}\Big(\tfrac{227}{70} \qquad \begin{array}{c} 1 \text{ added.} \\ \hline 4\frac{17}{70} \\ \hline \end{array}$$

6. $\text{As } 4\frac{17}{70} \cdot \frac{6}{7} :: 1 \cdot \frac{20}{99}$

$$\tfrac{\overset{99}{\cancel{297}}}{\underset{10}{\cancel{70}}}\Big)\tfrac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}}\Big(\tfrac{20}{99}$$

This $\frac{20}{99}$ the Quotient of the laſt Work muſt be ſet as $\frac{99}{20}$, and then divided, giveth $4\frac{19}{20}$ as before in the Work by *Ratio's* for the firſt Number of the Queſtion there ſtated and reſolved.

Example 2. Suppoſe a Conduit, whoſe Ciſtern holdeth 1000 Gallons, hath three Cocks; by the firſt Cock will run out 20 Gallons in 3 Hours; by the ſecond, 30 Gallons in 7 Hours; and by the third, 40 Gallons in 9 Hours: in what Time will there be run out by all the Cocks $973\frac{1}{3}$ Gallons, and the firſt Cock be opened half an Hour before the other? Q. *Of emptying a Ciſtern.*

Anſw. In $63\frac{1}{2}$ Hours, or 2 Days 15 Hours and an Half. The Work of the *Ratio's* is as in the laſt Example, by which will be gotten $15\frac{25}{63}$ Gallons to be run out by all the Cocks running together in 1 Hour: but before the Queſtion can be ſet therewith to know in what Time $973\frac{1}{3}$ Gallons will be run out, there muſt be known how much runs out in the half Hour the firſt Cock runs alone, which will be $3\frac{1}{3}$ Gallons: Anſwer.

lons: this taken from 973$\frac{1}{3}$, leaves 970, which ſhall give 63 Hours, to which the half Hour muſt be added.

$$\begin{array}{lccc} & 1260 & 810 & 840 \\ \hline \text{Gallons} & 20 & 30 & 40 \\ \text{Hours} & 3 & 7 & 9 \\ \hline & & 189 & \end{array}$$

$$\begin{array}{l} \frac{|7}{\cancel{102}|5} \\ \text{Antecedents } 1260+810+840 = \cancel{2910} \\ \text{Conſequents } \;\; 3 \times 7 \times 9 = \cancel{189} \end{array} \Big(15\tfrac{25}{63} \text{ Gallons.}$$

Hours. *Gallons.* *Hour.* *Gallons.*
If 3 run out 20: what ſhall $\frac{1}{2}$? *facit* 3$\frac{1}{3}$

$$\frac{3}{1}\Big)\frac{20}{2}\Big(\frac{20}{6}\Big(3\tfrac{1}{3} \qquad 973\tfrac{1}{3} - 3\tfrac{1}{3} = 970.$$

Gallons. *Hour.* *Gallons.* *Hours.*
If 15$\frac{25}{63}$ run out in 1: in what 970? *facit* 63.

$$\frac{\overset{1}{970}}{63}\Big)\frac{\overset{1}{970}}{1}\Big(\frac{63}{1} \qquad \begin{array}{r} \text{Added } \tfrac{1}{2} \\ \hline 63\tfrac{1}{2} \text{ Hours.} \end{array}$$

Variety of Operation.

By the *Rule of Three* the Work is as in the laſt Example till the 15$\frac{25}{63}$ Gallons be gotten, and then the other work as above.

As $\frac{3}{20}$. 1 :: $\frac{7}{30}$. 1$\frac{5}{9}$

$$\frac{3}{\underset{2}{20}}\Big)\frac{7}{\underset{3}{30}}\Big(\frac{14}{9} \qquad \begin{array}{r} 1 \text{ added.} \\ \hline 2\tfrac{5}{9} \end{array}$$

As 2$\frac{5}{9}$. $\frac{7}{30}$ :: 1 . $\frac{21}{230}$

$$\frac{23}{\underset{3}{9}}\Big)\frac{7}{\underset{10}{30}}\Big(\frac{21}{230}$$

As $\frac{21}{230}$. 1 :: $\frac{9}{40}$. 2$\frac{13}{28}$

$$\frac{21}{\underset{23}{230}}\Big)\frac{9}{\underset{4}{40}}\Big(\frac{207}{48} \qquad \begin{array}{r} 1 \text{ added.} \\ \hline 3\tfrac{13}{28} \end{array}$$

As 3$\frac{13}{28}$. $\frac{9}{40}$ :: 1 . $\frac{63}{970}$

$$\frac{97}{\underset{7}{28}}\Big)\frac{9}{\underset{10}{40}}\Big(\frac{63}{970} \qquad \frac{970}{63}\Big(15\tfrac{25}{63}$$

Data above 3 Numb. and Sub. uſed.

Data *more than* 3 *Numbers and Subſtraction needful.*

Q. Of hunting an Hare.

Example 1. A Gentleman in hunting an Hare, by their tracing in the Snow findeth that the Hare had 60 Lengths of the Hound's Paces before the Hound: And as often as the Hare runneth 8 Paces, the Hound runneth but 6 Paces; but 2 Paces of the Hound are as much as 3 of the Hare's Paces: In how many Paces of the Hound ſhall he overtake the Hare?

Anſwer.

Anſw. In 540 Paces of the Hound. Here are three Queſtions included. Firſt, Seeing the 60 Lengths were of the Hound's Paces, how many that made of the Hare's Paces? Secondly, Since 2 Paces of the Hound are equal to 3 of the Hare, that is gain upon the Hare 1 Pace, how many 6 Paces of the Hound ſhall gain or be equal to? Thirdly, The Gain of 6 Paces of the Hound known, the third Queſtion ariſeth, and is reſolved by the Quotient of the firſt Work with the 6 Paces, and the Gain found as hereunder is to be noted.

Hound's *Hare's* *Hound's* *Hare's*
If 2 Paces make 3 Paces: what ſhall 60 Paces? *facit* 90 Paces.

$$2\Big)\frac{60}{180}\Big(90 \text{ Hare's Paces.}$$

Hound's *Hare's* *Hound's* *Hare's*
If 2 Paces make 3 Paces: what ſhall 6 Paces? *facit* 9 Paces.

$2)\frac{6}{18}(9$ Hare's Paces. Subſtract 8 Hare's Paces.
Gain 1

Hare's *Hound's* *Hare's*
If 1 Pace be gotten by 6 Paces: in how many Hound's Paces will 90 Paces be gotten? *facit* 540 Hound's Paces, as before.

As 1 . 6 :: 90 . 540.

Q. Of hunting an Hare. *Example* 2. A Gentleman hunteth an Hare; and as often as the Hound runneth 6 Paces, the Hare runneth 8; and 2 Paces of the Hound make 3 of the Hare, and the Hound overtaketh the Hare in 540 Paces of his own: how many Hound's Paces had the Hare before the Hound?

Anſwer. *Anſw.* 60 Paces of the Hound. Here alſo are 3 Queſtions to be reſolved, yet with this difference from the former Example, that as there Subſtraction was made from the Quotient of the ſecond Work, here the Quotient of the firſt Work ſhall be ſubſtracted from the firſt Number of the ſecond Work; the reſt of the Works are ſimilar to the former, as hereunder is evident.

Hound's *Hare's* *Hound's* *Hare's*
If 6 Paces make 8 Paces: what ſhall 2 Paces? *facit* $2\frac{2}{3}$ Paces.

$6)\frac{2}{16}(2\frac{2}{3}$ Hare's Paces. *Hare's Paces.* $3-2\frac{2}{3}=\frac{1}{3}$ loſs.

Hare's *Hound's* *Hare's* *Hound's*
If 3 Paces make 2 Paces: what ſhall $\frac{1}{3}$ Pace? *facit* $\frac{2}{9}$ Pace.

$\frac{3}{1})\frac{2}{3}(\frac{2}{9}$ Hound's Paces.

Paces *Hound's* *Paces* *Hound's*
If 2 Hound come of $\frac{2}{9}$ Pace: of what comes 540 Hound? *facit* 60 Paces.

For $\frac{2}{\underset{1}{9}} \times \frac{\overset{60}{540}}{1} = \frac{120}{1}$ And $\frac{\overset{1}{2}}{1})\frac{\overset{60}{120}}{1}(\frac{60}{1}$ Hound's Paces.

Thus, (as was before noted, and might be ſeen by many other Inſtances) beſides the Rule, due conſideration muſt be had to the Nature of the Queſtion propounded, duly to prepare or diſpoſe the *Data*, and when to add and ſubſtract, &c. in ſuch kind of Specifical Propoſitions as theſe are; to conclude which, one Example more ſhall be added.

Q. Of a Worm creeping out of a Well. A Worm in a Well 24 Feet deep, creepeth upwards every Day $5\frac{1}{4}$ Feet, and downwards every Night $4\frac{1}{3}$ Feet: in how many Days ſhall he creep out of the Well?

Anſwer. *Anſw.* In $21\frac{19}{21}$ Days. For in 21 Days the Deduction of the downward Motion taken from the upward, yet leaveth $19\frac{1}{4}$ Feet for the Worm to crawl up: And the other $4\frac{3}{4}$ Feet to make up 24, he creepeth up on the 22*d* Day, without any deduction, he being up $\frac{2}{21}$ of a Day before Night.

Here $4\frac{1}{3}$ taken from $5\frac{1}{4}$, leaveth $\frac{11}{12}$ the Gain upward in 1 Day: Then becauſe the laſt Day he gets up to the top of the Well, there is no deduction to be made, but 1 Night's deduction is to be taken from 24; and ſo $4\frac{1}{3}$ taken from 24, there remaineth $19\frac{2}{3}$.

Foot. *Day.* *Feet.* *Days.* *Days.* *Feet.* *Days.* *Feet.*
Then as $\frac{11}{12}$. 1 :: $19\frac{2}{3}$. $21\frac{5}{11}$ And as $21\frac{5}{11}$. $19\frac{2}{3}$ :: 21 . $19\frac{1}{4}$

And becauſe $19\frac{1}{4}$ Feet wants $4\frac{3}{4}$ of 24, this muſt be the Motion of the Worm the laſt Day in which he got up to the top of the Well.

$$\begin{array}{lcccc} & \textit{Feet.} & \textit{Day.} & \textit{Feet.} & \textit{Day.} \\ \text{Therefore as} & 5\frac{1}{4} & .\ 1\ :: & 4\frac{3}{4} & .\ \frac{19}{21} \end{array}$$

And this $\frac{19}{21}$ is to be added to the 21 Days in which he got up to the $19\frac{1}{4}$ Feet, ſo both make up $21\frac{19}{21}$ Days as before.

Proof of Specificks Common and Specifical.

Specificks, beſides their Proof in Common with other Works of the *Rule of Three*, (before ſeen in the foregoing Chapters) have their Specifical Proof.

Of the firſt ſort.

Specifical Proof of the firſt ſort, is by turning the Fractions with Integers all into Integers, and working therewith.

This for Queſtions reſolved by the *Direct Rule of Three*, is to multiply the Numerator of the firſt Number of the three given Numbers into the Denominators of the Second and Third, and this Product ſhall be the firſt Number of the Probationary Work. The Numerator of the ſecond given Number ſhall be the ſecond in this Work: And the third Number here ſhall be the Product of the Numerator of the Third, into the Denominator of the Firſt.

But for Queſtions reſolved by the *Indirect Rule of Three*, multiply the Numerator of the firſt of the three given Numbers into the Denominator of the Third, and the Product ſhall be the firſt Number of this probationary Work. The Numerator of the ſecond given Number, ſhall be the ſecond in this Work: And the third Number here ſhall be the Product of the Numerator of the Third into the Denominators of the Firſt and Second.

Examples.

Example in two of the former Inſtances, where

$$\begin{array}{ccccccc} \textit{Ell} & & \textit{l.} & & \textit{Ells} & & \textit{l.} \\ 1\frac{1}{2} & \text{at} & 1\frac{1}{4}, & \text{is as} & 3\frac{3}{4} & \text{to} & 3\frac{1}{8}, \end{array}$$ by the *Direct Rule.*

$$\begin{array}{ccccccc} \textit{s.} & & \textit{Ounces} & & \textit{s.} & & \textit{Ounces} \\ 5\frac{1}{2} & \text{to} & 6\frac{6}{11}, & \text{ſo is} & 12 & \text{to} & 3, \end{array}$$ by the *Indirect Rule.*

The *Data* being reduced into Improper Fractions, ſtand thus.

$$\begin{array}{lccccc} & \textit{Ell} & & \textit{l.} & & \textit{Ells} \\ 1. & \frac{3}{2} & . & \frac{5}{4} & :: & \frac{15}{4}. \end{array} \qquad \begin{array}{lccccc} & \textit{s.} & & \textit{Ounc.} & & \textit{s.} \\ 2. & \frac{11}{2} & . & \frac{72}{11} & :: & \frac{12}{1}. \end{array}$$

Then $3\times4\times4=48$. Firſt Number. And $11\times1=11$. Firſt Number.
5. Second Number. 72. Second Number.
$15\times2=30$. Third Number. $12\times2\times11=264$. Third Number.

As 48 . 5 :: 30 . $3\frac{1}{8}$. As 11 . 72 :: 264 . 3
$\ddot{8}$ $\ddot{5}$ Firſt and Third abbrev. $\ddot{1}$ $\ddot{24}$

$$8\,\Big)\,\frac{5}{25}\,\Big(\,3\tfrac{1}{8}\ \textit{l.} \qquad \frac{72}{24}\Big(\,3\ \text{Ounces.}$$

Of the ſecond ſort.

Specifical Proof of the ſecond Sort, is to turn the Geodeticals or Aſtronomicals into Decimals, or Decimals into them, and ſo work the Queſtion in both, and compare the Reſolutions together; this may be ſeen in ſome of the Examples before wrought in their proper Place, ſo as farther Inſtance is here purpoſely omitted.

Of the third ſort.

Specifical Proof of the third Sort, is by making two Queſtions, and after Reſolution, adding, ſubſtracting, *&c.* the Quotients that are to be added, ſubſtracted, *&c.* according to the Nature of the Propoſition.

Example.

Example in the firſt of the former Inſtances concerning the two Poſts, where *A* travelling 18 Miles a Day, and *B* on the contrary 30, they were found diſtant the third Day 144 Miles.

$$\begin{array}{lcccl} & \textit{A}\ 18 & & \textit{B}\ 30 & \\ & \underline{\quad 3} & & \underline{\quad 3} & \\ \text{That is} & \underline{54} & + & \underline{90} & \end{array} \qquad \begin{array}{lccccc} \text{For as} & 1 & .\ 18 & :: & 3 & .\ 54 \\ \text{And as} & 1 & .\ 30 & :: & 3 & .\ \underline{90} \\ & & & & & \underline{144} \end{array}$$

Of the fourth ſort.

Specifical Proof of the fourth ſort, is by working the Queſtions by their *Ratio's*, or by the *Rule of Three*; and the Agreement in their Reſolutions wrought both ways, declare their Operations right; and becauſe moſt of the Examples before are ſo wrought, farther Inſtances need not here.

CHAP. VI.

The Rule of five Numbers Direct.

Rule of five Numbers. *Why called* Compound. *Why the* Double Rule of Three. *Different from* Specificks.

AFTER *Specificks* follow the next Sort of derived Proportions to be ſeen in the *Rule of five Numbers*, ſometimes called the *Compound Golden Rule*, and ſometimes the *Double Rule of Three*: which latter Name is put upon it, becauſe the Propoſition may be reſolved by a double Operation of the *Simple Golden Rule*, in oppoſition to which this is called *Compound*. But as the two Queſtions are Compound in one; ſo by the *five Numbers* given, from whence it took the firſt Name, Reſolution may be had, and a ſixth Proportional found by one Operation. And though they are reſolvable by two, yet differ from *Specificks*, as before in the precedent Chapter was, and further in this and the next Chapter may be obſerved.

Rule of five Numbers, Direct *and* Indirect.

The *Rule of five Numbers* is both *Direct* and *Indirect*; the latter is referred to the next Chapter: to the knowledg of the other are conſiderable,

Some things { Preparatory in the right Diſpoſition of the *Data*.
Operatory in the Reſolution.
Probationary in the Proof.

Preparatory to the Direct.

The Precepts Preparatory, may be theſe and ſuch-like.

1. All Superfluous to be diſcharged.

1. Among the *Data* diſcharge all the Numbers that are ſuperfluous, for ſometime the Propoſition giveth five Numbers, when two of them may be diſcharged as more than needful; and the Numbers reduced to 3, the Queſtion may be reſolved by the *Simple Rule of Three*.

Example.

As, If 100 *l.* in 12 Months gain 8 *l.* what ſhall 100 *l.* in 16 Months? Here 100 in both Places may be cancelled, and Reſolution had without.

Months. *l.* *Months.* *l.*
For as 12 . 8 :: 16 . $10\frac{2}{3}$
3 4

$3\overline{)\,\underline{\overset{4}{32}}\,}(10\frac{2}{3}$

2. How many Conditional and Interrogative.

2. Of the 5 given Numbers, let 3 be Conditional and Antecedents, or Suppoſitious; and the other 2 Interrogative and Conſequents, correſponding to ſome of the former Antecedents.

Example.

As in the former Inſtance 100. 12. 8. are Conditional and Antecedent, the other 100. 16. are Interrogative. And becauſe this latter 100 is Conſequential and equal to the former Antecedent, 100 in both Places may be cancelled as before.

3. To place the Numbers aright.

3. Let the three Conditionals be firſt placed towards the left Hand; and then the two Terms on which the Queſtion depends, in the 4*th* and 5*th* Places: So as the firſt Term be the principal Cauſe of Loſs or Gain, Increaſe or Decreaſe, Action or Paſſion. That whoſe Sirname or Denomination betokeneth the Space of Time, Diſtance of Place, *&c.* the ſecond and the third Term the odd Denomination, which is unlike in Nature to all the other Numbers. Then let the fourth Term and firſt be of one Denomination; likewiſe the fifth and ſecond.

Example.

As in the former Inſtance, the firſt 100 being the principal Cauſe of the Gain, is firſt ſet; the 12 Months being the Space of Time, is the ſecond Number; the 8 *l.* Gain is the odd Denomination, and ſo ſet in the third Place: Then the latter 100 *l.* Principal-Money anſwering in the Interrogative Part to the firſt Number, is placed in the 4*th* Place: And 16 Months of like Denomination to the ſecond in the 5*th* Place.

4. Denominations double, which regarded.

4. When any of the Numbers ſeem to be doubly denominate, the latter Denomination is to be regarded in placing the Numbers, as before noted in the *Direct Rule of Three.*

5. Data when need to be reduced.

5. If any of the Numbers be of ſeveral Denominations, as Pounds and Shillings, or Pounds, Shillings and Pence, *&c.* reduce the ſame and his correſpondent Number into the loweſt of thoſe given Denominations; as the Firſt and Fourth, Second and Fifth: but if the third Number want Reduction, he having no Correſpondent but the Sixth not yet found, is only brought into the loweſt Denomination given, without altering any of the other *Data:* And according to the Third, or Reduction thereof, ſhall the Sixth Number when found be denominate.

Sixth when found, how denominate.

6. Terms if abbreviated, ſhorten the Work.

6. Sometime the Conditional Terms given may be reduced to leſſer Proportional Numbers, and working by them will ſhorten the Work. As in the former Inſtance, inſtead of 100 *l.* 12 Months, 8 *l.* may be taken; 50 *l.* 12 Months, 4 *l.* or 25 *l.* 12 Months, 2 *l.* or 5 *l.* 12 Months, 8 *s.* or 5 *l.* 3 Months, 2 *s.* &c.

7. To find if Reſolution be by the Direct Rule, or not.

7. To find whether a Queſtion propounded be reſolvable by the *Rule of five Numbers Direct*; conſider whether the principal Cauſe of Loſs or Gain, Intereſt or Decreaſe, Action or Paſſion, *&c.* be the odd denominate Number in the Propoſition: For if ſo, then is the Reſolution by the *Indirect Rule* as in the next Chapter; but if otherwiſe, by the *Direct.* As in Queſtions of Money, the *Direct Rule* enquireth always for the Intereſt, and gives the Principal Money, the *Indirect,* for Stock or Time of Continuance, and gives the Intereſt.

8. Operation to be as the Nature of the Data is.

8. According to the Nature of the *Data,* whether Integers, Fractions, *&c.* ſo let the Operation be: And if any Propoſition be ſpecifical, mix the Work in order to the Reſolution accordingly.

Reſolution by 1 or 2 Works.

The Premiſſes obſerved, the Reſolution will be had by one or two Operations.

The Rule for one Operation.

How by one.

Multiply the two firſt Numbers together for the Diviſor, and the three laſt Numbers one into another for the Dividend; and dividing the one by the other, the Quotient ſhall be the ſixth Proportional, and anſwer the Queſtion.

The Rule for the Double Work.

How by two.

Take the firſt, third and fourth Numbers, and work as in the *Direct Rule of Three* for another Proportional: Then take this new Proportional for the ſecond Number of the next Work, and the ſecond and fifth Numbers of the five given Numbers for the firſt and third Numbers of this ſecond Work, and the Quotient of this laſt working by the *Rule of Three,* ſhall anſwer the Queſtion firſt propounded.

Q. *Of Intereſt-Money.*

Example 1. If 400 *l.* be put out for 16 Months, at the Rate of 8 *l.* *per Centum per Annum,* (accounting 12 Months to the Year) I demand what will the Intereſt come to?

Anſwer.

Anſw. $42\frac{2}{3}$ *l.* Here the three Conditional Terms are the 100 *l.* that in one Year (or 12 Months) gains 8 *l.* and 100 *l.* the Principal Cauſe of the Profit or Increaſe, that ſhall be the firſt Number; and the 400 *l.* being alſo Principal Money the firſt Conſequent, and of like Nature to the 100, ſhall be ſet in the fourth place; 12 Months the ſecond Conditional, being the Space of Time in which the Increaſe groweth, ſhall be the ſecond Number, and the Conſequent 16 Months of like Nature to be 12, ſhall be in the fifth Place: So as the 8 *l.* Uſe-Money having none of the five to parallel him, falls into the third Place.

By one Work.

Then to reſolve the Queſtion at one Operation, multiply 100 into 12, the Product 1200 is Diviſor: And 8 multiplied into 400, and the Product by 16 makes the Dividend 51200, and after Diviſion $42\frac{2}{3}$ found in the Quotient, is denominate like the third Number, and is the Uſe-Money gained by 400 *l.* in 16 Months, at the Rate aforeſaid.

l. *Months.* *l.* *l.* *Months.* *l.*

As 100 . 12 . 8 :: 400 . 16 . $42\frac{2}{3}$

12 400

12|00 3200

16

19200

3200

512|00

$\cancel{3}$(8

$\cancel{5}\cancel{1}\cancel{2}$ ($42\frac{2}{3}$ *l.*

$\cancel{1}\cancel{2}$

If the Operation be double, then is 100 *l.* 8 *l.* 400 *l.* the three Numbers of the firſt Work, 32 the Number gotten thereby the ſecond Number of the ſecond Work, and 12 and 16 the firſt and third Numbers of the ſame Work: Thus, *By two Works.*

l. *l.* *l.* *l.* *Months.* *l.* *Months.* *l.*

As 100 . 8 :: 400 . 32 As 12 . 32 :: 16 . $42\frac{2}{3}$

3 4

4 4

32 3)128($42\frac{2}{3}$

If the conditional Terms be reduced to ſmaller Proportionals, then the Work at one Operation will ſtand thus. *Example where the Conditionals are reduced.*

l. *Mon.* *s.* *l.* *Mon.* *l.* *s.* *d.*

As 5 . 3 . 2 :: 400 . 16 : 40 : 13 : 4

15

32

800

1200

12800

$\cancel{5}$(5

$\cancel{1}\cancel{2}\cancel{8}\cancel{0}\cancel{0}$ (85|3

$\cancel{1}\cancel{5}$

$\cancel{1}\cancel{5}$ 42:13:4

$\cancel{1}\cancel{5}$

Example 2. If 3 Mowers in 6 Days mow $24\frac{1}{2}$ Acres: how many Acres will 8 Mowers mow in 10 Days? *Q. Of Mowers.*

Anſw. 108 Acres and $\frac{8}{9}$ of an Acre. *Anſwer.*

Mowers. *Days.* *Acres.* *Mowers.* *Days.* *Acres.*

As 3 . 6 . $24\frac{1}{2}$ :: 8 . 10 . $108\frac{8}{9}$

18 245

$\cancel{1}$(16

1960 $\cancel{1}\cancel{9}\cancel{6}\cancel{0}$ ($108\frac{8}{9}$ Acres.

$\cancel{1}8$

Specificks.

Example 1. If 5 Guns in 2 Days ſpend 60 Barrels of Powder: what will 7 Guns ſpend in 5 Days, which ſpend $\frac{1}{3}$ part leſs than the 5 at every ſhot? *A ſpecifical Queſtion of Powder ſpent by 7 Guns.*

Anſw. 140 Barrels of Powder.

Barrel $\cancel{6}\cancel{0}$(20

$\frac{1}{3}$ $\cancel{3}$

60 —— 20 == 40

Guns. *Days.* *Bar.* *Guns.* *Days.* *Bar.*

As 5 . 2 : 40 :: 7 . 5 . 140

10 200

1400 $\cancel{1}400$ (140 Barrels.

$\cancel{1}0$

Example 2. If 12 Penny-worth of Wine ſatisfy 8 Perſons at a Meal, when Wine is at 6 *d.* a Quart: how much Wine at 4 *d.* a Quart will ſuffice 40 Perſons that drink twice as much as the other? *Another of Wine drunk.*

Anſw. 80 Pennyworth of Wine. *Anſwer.*

Wine 12 *d.*
Increaſed 2
24

Perſons. *d.quart.* *d.* *Perſons.* *d.quart.* *s.* *d.*
As 8 . 6 . 24 :: 40 . 4 . 6 : 8.
48 96
3840
$\frac{3840}{48}$ (80 *d.*

Proof of the Rule of five Numbers Direct. The Operations of the *Direct Rule of five Numbers* wrought at once, and by the double Work, ſeeing they agree in the Reſolution, and identify the Anſwer, prove each other, as by the Reſolution of the Gain of 400 *l.* in 16 Months wrought before both ways is made plain.

CHAP. VII.

The Rule of five Numbers Indirect.

Rule of five Numbers Indirect. THE Knowledg of this *Indirect Rule of five Numbers*, conſiſts chiefly in the Preparatory Part rightly to diſpoſe the *Data*, for which theſe Precepts are neceſſary.

1. All ſuperfluous Numbers to be diſcharged. 1. Diſcharge all the Numbers ſuperfluous among the *Data*, as before in the *Direct Rule of five Numbers*.

2. How to place the Data. 2. Of the five given Numbers as before, if three be Conditionals and Antecedents, and the other two Interrogative and Conſequents, then diſpoſe them thus; Let the odd Denomination, which here will be the principal Cauſe of Loſs or Gain, Increaſe or Decreaſe, Action or Paſſion, *&c.* be the firſt Term, and the other two Conditionals the Second and Third, as in the *Direct Rule of five Numbers*; and let the Second and Fourth, and Third and Fifth, be of like Denominations.

3. How to place them if there be 4 Conditionals. 3. But if there be four Conditionals or Terms of Explanation; then place the ſingle or odd denominate Term in the firſt Place as before; the Term on which the Queſtion depends, ſet in the ſecond Place; the Number countervailed by the firſt located Term, let be the Third; the Number denominate like the ſecond located Term, place in the fourth Place; and then the other Term of the *Data* countervailing the Fourth, and denominate like the Third located Term, will ſupply the fifth Place.

4. Sixth when found, how denominate. 4. Let the ſixth Proportional, when found, be always accounted of like denomination to the Firſt, or the Reduction thereof if reduced. *Other Precepts before uſeful here.* The reſt of the 4*th*, 5*th*, 6*th*, 7*th* and 8*th* Precepts of the *Direct Rule of five Numbers*, conſider and make uſe of as occaſion ſhall require.

Reſolution by 1 or 2 Works. The *Data* duly diſpoſed, as aforeſaid, the Reſolution will be had by one or two Operations.

The Rule for one Operation.

How by 1. Multiply the third and fourth Numbers together for the Diviſor, and the other three Numbers one into another for the Dividend; and dividing the one by the other, the Quotient ſhall be the deſired Number, and the ſixth Proportional.

The Rule for the double Work.

How by 2. Take the Fourth for the Firſt, the Fifth for the Second, and the Second for the Third; and the Quotient of this Work by the *Rule of Three*, make the third Number of the ſecond Work: and for the Firſt, take the Third of the *Data*; and for the Second, the Firſt of the *Data*, and the Reſult of this Work by the *Rule of Three* ſhall reſolve the Queſtion firſt propounded.

Q. Of Principal Money. *Example* 1. If 42⅔ *l.* be Intereſt for 16 Months, after the Rate of 8 *l. per Centum per Annum*: what Principal Money was delivered to raiſe that Intereſt?

Anſw. 400 *l.* Here being but three Conditionals, the *Data* is to be diſpoſed according to the ſecond Precept. And becauſe 100 is the principal Cauſe of Gain in the Conditionals, and the ſingle Denomination, it is firſt placed, the Year or

12

12 Months being the Space of Time muſt be Second: And 8 the third Conditional ſhall occupy the Third Place. Then muſt 16, becauſe agreeing with the Second, be ſet in the fourth Place; and $42\frac{2}{3}$ in the fifth Place, being of like denomination with the Third, that is both Uſe-Money: So the Queſtion ſtands thus.

If 100 *l.* in 12 Months gain 8 *l.* what Principal-Money in 16 Months will gain $42\frac{2}{3}$ *l*?

By 1 Work.

Then to reſolve the Queſtion at one Operation, 8 is multiplied into 16, and the Product 128 is Diviſor. The other three of the *Data* being multiplied, make the Dividend 51200; which divided by 128, give 400 as the Principal-Money to raiſe that Intereſt in the Time propoſed.

l. *Months.* *l.* *Months.* *l.* *l.*
As 100 . 12 . 8 :: 16 . $42\frac{2}{3}$. 400.

$$\begin{array}{r} 100 \\ 12 \\ \hline 1200 \\ 42\frac{2}{3} \\ \hline 2400 \\ 4800 \\ 400 \\ 400 \\ \hline 51200 \end{array} \qquad \begin{array}{r} 16 \\ 48 \\ 8 \\ \hline 128 \end{array} \qquad \frac{51200}{128}\Big(\,400\ l.\ \text{Principal.}$$

By 2 Works.

If the Work be double, then is 16 . $42\frac{2}{3}$. 12 . the three Numbers of the firſt Work; 32 the Number gotten thereby, ſhall be the third Number of the ſecond Work, and 8 and 100 the other Numbers of that Work: Thus,

Months. *l.* *Months.* *l.* — *l.* *l.* *l.* *l.*
As 16 . $42\frac{2}{3}$ :: 12 . 32 — As 8 . 100 :: 32 . 400

$$\begin{array}{r} 42\frac{2}{3} \\ 12 \\ \hline 8 \\ 42.8 \\ \hline 512 \end{array} \qquad \frac{512}{16}\Big(\,32\ l. \qquad \begin{array}{r} 100 \\ 32 \\ \hline \end{array} \quad 8)\,3200\,(\,400\ l.$$

One Work by the Indirect, yet both Direct ſometimes.

Here is worthy to be noted, that in this and ſeveral other Inſtances, though the Reſolution at one Operation is by the *Indirect Rule of five Numbers*; yet by the double Work both are ſometimes reſolved by the *Direct Rule of Three*.

Example, where the Conditionals are reduced.

If the Conditional Terms be reduced to leſſer Proportionals, the Work at one Operation will ſtand thus.

l. *Mon.* *l.* *Mon.* *l.* *l.*
As 5 . 3 . $\frac{1}{1}$ :: 16 . $42\frac{2}{3}$. 400.

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 42\frac{2}{3} \\ \hline 30 \\ 60 \\ 5 \\ 5 \\ \hline 640 \end{array} \qquad \frac{1}{\underset{5}{10}} \times \frac{\overset{8}{16}}{1} = \frac{8}{5} \qquad \frac{\overset{1}{8}}{5}\Big)\frac{\overset{80}{640}}{1}\Big(\frac{400}{1}\ l.$$

Q. Of Mowers.

Example 2. If 3 Mowers in 6 Days mow $24\frac{1}{2}$ Acres: how many Mowers in 10 Days can mow $108\frac{8}{9}$ Acres?

Anſwer.

Anſw. 8 Mowers.

```
     Mowers. Days.  Acres.     Days.  Acres.  Mowers.
  As   3   .  6  .  24½   ::   10  .  108⅜  .  8
       6            10
     ----          ----
      18           240
      108⅜           5
     ----          ----
      144          245
     180                     1960
      16                     ----( 8 Mowers.
     ----                     245
     1960
```

Specificks.

A ſpecifical Queſtion of Travel.

Example 1. If in 10 Days of 8 Hours long, a Man may journey 200 Miles: in how many Days twice as long may he travel 500 Miles?

Anſwer.

Anſw. In 12½ Days.

```
                     Days.  Hours.  Miles.    Hours.  Miles.  Days.
Hours      8     As   10  .   8   .  200   ::   16  .  500  .  12½
Increaſed  2           8             16       |1
         ----        ----          -----      ----
          16          80           32,00      8|6
         ----        500           -----      ----
                     ----                     400
                    400,00                    ----( 12½ Days.
                    ------                     32
```

Another of Perſons drinking.

Example 2. If 12 pennyworth of Wine ſatisfy 8 Perſons at a Meal, when Wine is at 6 Pence a Quart: how many Perſons that drink but half ſo much Wine at a Meal, will 20 pennyworth of Wine ſuffice when Wine is at 4 *d.* a Quart?

Anſw. 40 Perſons.

```
                      Perſons. d.quart.  d.     d.quart.  d.   Perſons.
Wine  12          As    8   .   6   .   6   ::   4    .  20  .  40
      --( 6             6               4
½      2              ----            ----
12—6=6                 48              24
                       20             ----
                      ----
                       960                     960
                      ----                     ---( 40 Perſons.
                                                24
```

Where four Conditionals are propoſed.

Examples of 4 Conditionals.

Q. Of Paris Pence.

Example 1. If 4 *d.* of *Paris* be worth 5 *d. Tournois*, and 5 *d. Tournois* 6 *d.* of *Savoy*: how many Pence *Paris* are 15 *d. Savoy*?

Anſwer.

Superfluous Numbers diſcharged.

Anſw. 10 *d. Paris.* Here two of the Terms, that is, both the Pence of *Tournois* are ſuperfluous, and ſo may be omitted, and the Queſtion with three Numbers ſtand thus.

```
      d.Savoy.  d.Paris.    d.Savoy.   d.Paris.
  As    6    .    4    ::     15    .    10
                               4
                         6) ------ (10 d. Paris.
                              60
```

And ſo will the Reſolution be, if the *Data* be diſpoſed according to the third Precept: Thus,

```
     d.Paris  d.Sav.   d.Tourn.   d.Sav.  d.Tourn.  d.Paris.
  As   4   .   15   .    5    ::    6   .    5    .   10
      15                 6
     ----              ----
      60                30
       5                          300
     ----                         ---( 10 d.Paris.
     300                           30
```

Q. Of Angels and Crowns.

Example 2. If 2 Angels countervail 20 *s. Sterling*, and 18 *s.* countervail 3 Crowns *French*: how many Angels will countervail 10 Crowns? or how many Crowns will countervail 12 Angels?

In

Numbers placed.

In theſe two Queſtions the *Data* diſpoſed by the third Precept in the former 2 Angels, being the odd Denomination, will ſtand in the firſt place, but in the latter 3 Crowns. The Term on which the Queſtion depends, to be ſet in the ſecond place, muſt be 10 Crowns in the former, and 12 Angels in the latter. The third Place being filled with the Number countervailing the firſt located Term, ſhall be in the former 20 *s.* and in the latter 18 *s.* The Number denominate like him, placed in the Second, to be ſet in the fourth Place, muſt be in the former 3 Crowns, and in the latter 2 Angels. Then the remaining Number denominate like the Third, and countervailing the fourth located Terms, will be placed in the fifth Place, which in the former is 18 *s.* and in the latter 20 *s.*

Single Operation.

Operation ſingle.

```
            Angels.     Crowns.      s.        Crowns.        s.       Angels.
Q. 1.   If    2     .     10    .    20    ::    3      .    18 ? facit 6.
             10                       3
             --                      ---
             20                      6,0
             18
             --                            36
            160                           ---( 6 Angels.
             20                             6
            ----
            36,0
```

Double Operation.

Double.

```
     Cr.    s.      Cr.     s.           s.    Ang.     s.    Ang.
As    3  .  18  ::  10  .   60      As   20  .  2   ::  60  .  6
            10                                 60
         3)180(60 s.                       20)120(6 Angels.
```

Single Operation.

Single.

```
          Crowns.    Angels.      s.        Angels.      s.       Crowns.
Q. 2. If    3    .    12    .    18    ::     2     .   20 ? facit 20
           12                     2
           --                    --
           36                    36
           20                                720
          ---                                ---( 20 Crowns.
          720                                 36
```

Double Operation.

Double.

```
   Ang.   s.     Ang.    s.              s.    Cr.     s.     Cr.
As  2  .  20  ::  12  .  120        As   18  .  3  ::  120  .  20
          12                                  120
          --                             18)360(20
          40
          20
      2)024(120 s.
```

The Operations of the *Indirect Rule of five Numbers*, are proved as thoſe of the *Direct* for the Reſolution by one Operation, and by two is the ſame, as by the laſt Examples wrought both ways is ſufficiently convincing.

Proof of the Rule of five Numbers Indirect.

CHAP.

CHAP. VIII.

FELLOWSHIP.

Fellowſhip *how otherwiſe called.*

DERIVATIVES of the third Sort, which deal with ſome particular Subject, are next to be exhibited; and among them, firſt, thoſe Operations that converſe about *Fellowſhip*, called alſo the *Rule of Society* and *Partnerſhip*, becauſe Queſtions of *Partnerſhip* are reſolved thereby.

The Sorts thereof.

Fellowſhip may be divided into four Sections,
- Without Time, §. 1.
- With Time, §. 2.
- With diverſity of Time, §. 3.
- With diverſity of Parts, §. 4.

Without Time of two Sorts.

§. 1. *Fellowſhip* without Time, either propounds the ſeveral Disburſments of the particular Partners, and the general Gain or Loſs by their Traffique, and requires to know each Man's Part or Share thereof: Or by the general Stock and particular Shares of the Gain or Loſs, requires to know their particular Disburſments.

To reſolve the Firſt.

To reſolve the Propoſitions of the firſt Sort, add all the Monies disburſed, or Stock in Partnerſhip together, for the firſt Number of the *Rule of Three:* Then ſet the Gain or Loſs in the ſecond Place; and each Man's ſeveral Stock or Disburſments in the Third: And by ſo many Operations of the *Rule of Three* as there be Partners, the Deſire is obtained.

Q. Of each Partner's Gain.

Example. Three Men, or *A*, *B*, and *C*, conſenting to trade together, make a Bank of Money, in which *A* disburſed 100 *l.* *B* 150 *l.* and *C* 250 *l.* And trading therewith, they gained 300 *l.* what is each Man's Part of the Gain?

Anſwer.

Anſw. *A* 60 *l.* *B* 90 *l.* and *C* 150 *l.* Here 100 + 150 + 250 = 500 make the firſt Number, 300 the Gain the Second, and the ſeveral Sums disburſed the Third Numbers, as followeth.

	Stock.
A	100 *l.*
B	150
C	250
	50,0

l. *l.* *l.* *l.*
As 500 . 300 :: 100 . 60
10
3000
3000 / 50 (60 *l.* *A.*

l. *l.* *l.* *l.*
As 500 . 300 :: 150 . 90
15
4500
4500 / 50 (90 *l.* *B.*

l. *l.* *l.* *l.*
As 500 . 300 :: 250 . 150
25
7500
7500 / 50 (150 *l.* *C.*

To reſolve the ſecond Sort.

To reſolve the Propoſitions of the latter Sort, place the Gain or Loſs for the firſt Number; the whole Stock for the Second; and the ſeveral Parts of the Gain or Loſs for the ſeveral third Numbers, and proceed by the *Rule of Three*, as before.

Q. Of each Partner's Stock.

Example. *A*, *B*, and *C*, made a Stock to trade with, and all laid in together 560 *l.* wherewith they gain 150 *l.* of which at the end of their Partnerſhip, *A* took 40 *l.* *B* 50 *l.* and *C* 60 *l.* what had each Man in Stock?

Anſwer.

Anſw. *A* 149⅓ *l.* *B* 186⅔ *l.* and *C* 224 *l.*

$$\begin{array}{lllll} & l. & l. & l. & l. \\ \text{As} & 15\underline{0}\ \cdot & 560\ :: & 4\underline{0}\ \cdot & 149\frac{1}{3} \end{array} \qquad \begin{array}{r} 1 \\ 7(5 \\ 2240 \\ \hline 15 \end{array}\Big(\ 149\tfrac{1}{3}\ l.\quad A.$$

$$\begin{array}{r} 4 \\ \hline 2240 \\ \hline \end{array}$$

$$\begin{array}{lllll} \text{As} & 15\underline{0}\ \cdot & 560\ :: & 5\underline{0}\ \cdot & 186\frac{2}{3} \end{array} \qquad \begin{array}{r} 1 \\ 13(1 \\ 280(0 \\ \hline 15 \end{array}\Big(\ 186\tfrac{2}{3}\ l.\quad B.$$

$$\begin{array}{r} 5 \\ \hline 2800 \\ \hline \end{array}$$

$$\begin{array}{lllll} \text{As} & 15\underline{0}\ \cdot & 560\ :: & 6\underline{0}\ \cdot & 224 \end{array} \qquad \begin{array}{r} 3360 \\ \hline 15 \end{array}\Big(\ 224\ l.\quad C.$$

$$\begin{array}{r} 6 \\ \hline 3360 \\ \hline \end{array}$$

§. 2. ***Fellowſhip with Time***, ſimply propounds the ſeveral Disburſments of the particular Partners at once, and the Time they continue in Stock, with the Gain or Loſs in general, and requires to know each Man's Part thereof: Or by the whole Gain or Loſs, and Particulars of the Stock, and ſome particular Partner's Time of Trade, requireth to know the Time of the others Continuance: Or by the Gain or Loſs of ſome particular Partner's Stock and Time according to the *Data*, ſeeketh the particular Gain or Loſs, Stock or Time of another Partner: Or by Mixtures of theſe, propoſeth Propoſitions mixt.

Fellowſhip with Time, of four Sorts.

To reſolve the Propoſitions of the firſt Sort, multiply ſeverally every Man's Money by the Time it continued in Stock; and the Total of all thoſe Products added together, ſhall be the firſt Number of the *Rule of Three*; the Gain or Loſs the Second; and the ſeveral Products ſhall be the ſeveral third Numbers; and by ſo many Operations of the *Rule of Three* as there be Partners, Reſolution will be had.

To reſolve the firſt.

Example. *A*, *B*, and *C* disburſe as followeth; *viz.* *A* 50 *l.* *B* 60 *l.* and *C* 80 *l.* And *B* drew away his Money from the Stock at 10 Months end: But *A* and *C* continued the Trade till 16 Months; when the Stock was broken up, there was found 400 *l.* Gain: what is each Man's Share thereof?

Q. Of each Partner's Gain.

Anſw. *A* $119\frac{27}{67}$ *l.* *B* $89\frac{37}{67}$ *l.* and *C* $191\frac{3}{67}$ *l.*

Anſwer.

Stock. *Time.*

$$\begin{array}{llcr} A & 50 \times 16 & = & 800 \\ B & 60 \times 10 & = & 600 \\ C & 80 \times 16 & = & 1280 \\ \hline & & & 268{,}0 \\ \hline \end{array}$$

$$\begin{array}{lllll} & & l. & & l. \\ \text{As} & 268\underline{0}\ \cdot & 400\ :: & 80\underline{0}\ \cdot & 119\frac{27}{67} \end{array} \qquad \begin{array}{r} (1 \\ 25(0 \\ 522(8 \\ 32000 \\ \hline 268 \end{array}\Big(\ \overset{l.}{119\tfrac{27}{67}}\quad A.$$

$$\begin{array}{r} 80 \\ \hline 32000 \\ \hline \end{array}$$

$$\begin{array}{lllll} \text{As} & 268\underline{0}\ \cdot & 400\ :: & 60\underline{0}\ \cdot & 89\frac{37}{67} \end{array} \qquad \begin{array}{r} (14 \\ 256(8 \\ 24000 \\ \hline 268 \end{array}\Big(\ \overset{l.}{89\tfrac{37}{67}}\quad B.$$

$$\begin{array}{r} 60 \\ \hline 24000 \\ \hline \end{array}$$

$$\begin{array}{lllll} \text{As} & 268\underline{0}\ \cdot & 400\ :: & 128\underline{0}\ \cdot & 191\frac{3}{67} \end{array} \qquad \begin{array}{r} 2(1 \\ 2448(2 \\ 51200 \\ \hline 268 \end{array}\Big(\ \overset{l.}{191\tfrac{3}{67}}\quad C.$$

$$\begin{array}{r} 128 \\ \hline 51200 \\ \hline \end{array}$$

To reſolve the Propoſitions of the ſecond Sort, multiply that Partner's Stock, whoſe Time is given by the Time for the ſecond Number of the *Rule of Three*: The firſt Number ſhall be that Partner's Gain or Loſs; And the ſeveral third Numbers ſhall be the other Partner's Gain or Loſs reſpectively. Then proceed as before by the *Rule of Three*, and the Quotients of theſe Works will be their Money and Time multiplied together; which if divided by their Money reſpectively, give the Time deſired.

To reſolve the ſecond Sort.

Example. *A*, *B*, and *C* are in Partnerſhip. *A* layeth in 200 *l.* for 10 Months, *B* layeth in 350 *l.* and *C* 100 *l.* and through ill Adventure ſuſtained the Loſs of 160 *l.* Whereupon breaking off their Trade, *A* found himſelf a Loſer 80 *l.* *B* 56 *l.* and *C* 24 *l.* The Queſtion is, how long the Money of *B* and *C* continued in Stock?

Q. *Of the Time of Trade.*

Anſwer. *Anſw.* *B* 4 Months, and *C* 6 Months.

$$\begin{array}{llll} \textit{Stock. Time.} & & \textit{A Loſs} & \textit{B Loſs} \\ A\ 200 \times 10 = 2000. & \text{As} & 80 \ .\ 2000 & ::\ 56\ .\ 1400 \end{array}$$

$$\begin{array}{r} 56 \\ \hline 11200{,}0 \\ \hline \end{array} \qquad \frac{\cancel{1}\cancel{1}200}{8}\Big(\frac{\cancel{1}4\cancel{0}\cancel{0}}{\cancel{3}\cancel{5}\cancel{0}}\Big(\ 4 \text{ Months } B.$$

$$\begin{array}{llll} & \textit{A Loſs} & & \textit{C Loſs} \\ \text{As} & 80\ .\ 2000 & :: & 24\ .\ 600 \end{array}$$

$$\begin{array}{r} 24 \\ \hline 4800{,}0 \\ \hline \end{array} \qquad \frac{4800}{8}\Big(\frac{600}{100}\Big(\ 6 \text{ Months } C.$$

To reſolve the third ſort. To reſolve the Propoſitions of the third Sort, commit the Queſtion to the *Rule of five Numbers*, or the *Double Rule of Three*, *Direct* or *Indirect*, as the Caſe requires.

Q. Of the Stock of 1 Partner. *Example.* *A* and *B* are in Company: *A* putteth in 240 *l.* and gaineth 50 *l.* in 6 Months: what ſhall *B* put in to gain 30 *l.* in 4 Months?

Anſwer. *Anſw.* 216 *l.*

$$\begin{array}{lcccccc} & l. & \textit{Months.} & l. & \textit{Months.} & l. & l. \\ \text{As} & 240 & .\ 6\ . & 50 & ::\ 4\ . & 30\ . & 216 \end{array}$$

$$\begin{array}{r} 6 \\ \hline 1440 \\ 30 \\ \hline 432{,}00 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ \hline 2{,}00 \\ \hline \end{array} \qquad \frac{43\cancel{2}}{\cancel{2}}\Big(\ 216\ l.$$

To reſolve the fourth Sort. *Propoſitions various.* The Propoſitions of the fourth Sort are many and various: For ſometime the Stock is enquired after, ſometime the Time, and ſometime the Gain or Loſs; ſometime in general, and ſometime particularly of one or more of the Copartners: And ſometime more than one of them is included in the Queſtion; and accordingly the *Data* with much Variety mixed, as in the Examples following.

$$1.\ \text{Example, where are} \begin{cases} \textit{Data} \begin{cases} \text{Stock, generally.} \\ \text{Time and Gain, particularly.} \end{cases} \\ \textit{Quæſita}, \text{ Stocks, particularly.} \end{cases}$$

Q. Of each Partner's Stock. *A*, *B*, and *C*, traffique together with a Stock of 638 *l.* wherewith they gain 90 *l.* And *A* having had his Money in Stock 5 Months, *B* 8 Months, and *C* 7 Months; the Gain was parted, to *A* 18 *l.* *B* 12 *l.* and *C* 60 *l.* what Monies had each in Stock?

Anſwer. *Anſw.* *A* 168 *l.* *B* 70 *l.* and *C* 400 *l.*

Here, after the particular Gains are ſeverally divided by the Times of Continuance, the Analogy is,

As the Sum of thoſe Quotients to the whole Stock:
So are thoſe Quotients ſeverally to the ſeveral Stocks.

$$\begin{array}{l} \text{Gains of } A \\ \text{Times} \end{array} \frac{18}{5}\Big(\ 3\tfrac{3}{5} \qquad B\ \frac{12}{8}\Big(\ 1\tfrac{1}{2} \qquad C\ \frac{60}{7}\Big(\ 8\tfrac{4}{7} \qquad 3\tfrac{3}{5} + 1\tfrac{1}{2} + 8\tfrac{4}{7} = 13\tfrac{47}{70}$$

$$\textit{As}\quad 13\tfrac{47}{70}\ .\ 638\ :: \begin{cases} 3\tfrac{3}{5}\ .\ 168\ .\ \text{Stock of } A. \\ 1\tfrac{1}{2}\ .\ \ 70\ .\ \text{Stock of } B. \\ 8\tfrac{4}{7}\ .\ 400\ .\ \text{Stock of } C. \end{cases}$$

$$2.\ \text{Example, where are} \begin{cases} \textit{Data} \begin{cases} \text{Stock of One.} \\ \text{Time and Gain of Another.} \end{cases} \\ \textit{Quæſita}, \text{ Stocks ſeverally.} \end{cases}$$

Q. Of each Partner's Stock. In a joint Trade *B* putteth in 200 *l.* more than *A*; *B* continueth his Stock but 5 Months, *A* 7½ Months; they gain alike: what Money had each of them in Stock?

Anſw. *A* 400 *l.* *B* 600 *l.* Anſwer.

Here ſubſtracting one Time from the other, the Analogy is ;
As the Difference of Times, to the Stock propounded :
So are the Times themſelves, to the ſeveral Stocks of each other.

$$\text{Times of}\begin{cases} A & 7\tfrac{1}{2} - 5 = 2\tfrac{1}{2} \\ B & 5 \end{cases}$$

$$\text{As } 2\tfrac{1}{2} \,.\, 200 :: \begin{cases} 5 \,.\, 400 \,.\, \text{Stock of } A. \\ 7\tfrac{1}{2} \,.\, 600 \,.\, \text{Stock of } B. \end{cases}$$

$$\text{3. Example, where are}\begin{cases} \textit{Data}\begin{cases} \text{Stocks and Gains, particularly.} \\ \text{Time, generally.} \end{cases} \\ \textit{Quæſita}, \text{ Times particularly.} \end{cases}$$

Three are in Company, the Stock of *A* in their Trade is 168 *l.* of *B* 70 *l.* of *C* 400 *l.* they gain 90 *l.* whereof *A* hath 18 *l.* *B* 12 *l.* and *C* 60 *l.* they withdrew their Stock ſeverally ; but all the ſeveral Times their Monies were in Stock, added together, make 20 Months : how long did each Man continue his Money in Stock? Q. *Of the Time of Trade.*

Anſw. *A* 5 Months, *B* 8 Months, and *C* 7 Months.

Here abrreviate every Man's Gains with his Stock to the leaſt Terms, and take theſe Parts of any Number that will equally be divided by all the Denominators : Or if no ſuch Number come readily to mind, multiply all the Denominators one into the other ; and divide the Product by the reſpective Denominators, and multiply the Quotients by the ſeveral Numerators : And theſe laſt Products abbreviate with the Sum of them. Then the Analogy is ; Anſwer.

As the Sum of theſe Parts, is to the whole Time given :
So is every Man's Part ſeverally, to his reſpective Time.

$$\begin{array}{lccc} & A. & B. & C. \\ \begin{array}{l}\text{Gains}\\ \text{Stock}\end{array} & \frac{18}{168}\Big|\frac{3}{28} & \frac{12}{70}\Big|\frac{6}{35} & \frac{60}{400}\Big|\frac{3}{20} \end{array} \qquad 28 \times 35 \times 20 = 19600$$

$$\frac{19600}{28}\Big(\begin{array}{r}700\\ 3\\ \hline 2100\end{array} \qquad \frac{19600}{35}\Big(\begin{array}{r}560\\ 6\\ \hline 3360\end{array} \qquad \frac{19600}{20}\Big(\begin{array}{r}980\\ 3\\ \hline 2940\end{array} \qquad \begin{array}{r|ll} 2100 & 5 & A. \\ 3360 & 8 & B. \\ 2940 & 7 & C. \\ \hline 8400 & 20 & \end{array}$$

$$\begin{array}{lcccc} & \textit{Parts.} & \textit{Months.} & & \begin{array}{ccc}\textit{Parts.} & \textit{Months.} & \end{array} \\ \text{As} & 20 & . \; 20 & :: & \begin{cases} 5 \;.\; 5 \;.\; \text{Time of } A. \\ 8 \;.\; 8 \;.\; \text{Time of } B. \\ 7 \;.\; 7 \;.\; \text{Time of } C. \end{cases} \end{array}$$

$$\text{4. Example, where are}\begin{cases} \textit{Data}\begin{cases} \text{Stock and Time of one.} \\ \text{Time of another.} \\ \text{Stock of a Third.} \\ \text{Gains of all particularly.} \end{cases} \\ \textit{Quæſita}\begin{cases} \text{Stock of One.} \\ \text{Time of Another.} \end{cases} \end{cases}$$

A in Company with *B* and *C*, putteth in 168 *l.* for 5 Months, *B* putteth in a Sum of Money for 8 Months, and *C* 400 *l.* for a certain Time ; they gain 90 *l.* whereof *A* muſt have 18 *l.* *B* 12 *l.* and *C* 60 *l.* how much was the Stock of *B*? and what Time did the Stock of *C* continue in the Company? Q. *Of the Stock and Time of Trade.*

Anſw. The Stock of *B* was 70 *l.* And the Time the 400 *l.* of *C* was in Stock was 7 Months. Anſwer.

Here the Analogies are ; As the Gain of one Partner is to his Stock multiplied by his Time : So is the Gain of the other Partners ſeverally to theirs ; Which when found, is to be divided by their Time or Stock reſpectively.

$$\begin{array}{lr} \text{Stock of } A. & 168 \\ \text{Time} & 5 \\ \hline & 840 \end{array}$$

$$\text{As } 18 \,.\, 840 :: 12 \,.\, \frac{560}{\text{Time } 8}\Big(70 \,.\, \text{Stock of } B.$$

$$\text{As } 18 \,.\, 840 :: 60 \,.\, \frac{2800}{400}\Big(7 \text{ Time of } C.$$

5. Example, where are
- *Data*
  - Stock and Gain of One.
  - Time and Gain of Another.
  - Rate on the Hundred of Both.
- *Quæſita*
  - Stock of One.
  - Time of Another.

Q. Of the Stock of one, and Time of another.

A joint Stock of *A* and *B* gained them, after the Rate of 20 *l. per Centum per Annum*, 50 *l.* apiece; *A* had 400 *l.* in Stock, and the Stock of *B* continued but 5 Months: what Money had *B* in Stock, and how long did *A* continue in the Company?

Anſw. *A* continued $7\frac{1}{2}$ Months, *B* put into Stock 600 *l.*

Here firſt finding the Gain of the Stock given by the Rate propounded, the Analogies are,

As the Gain according to the Rate, to 1 Year:
So is the Gain received, to the Time required.
And then, As the Time of one Partner is, to the Stock of the other:
So is the Time of the Partner found, to the Stock of the other ſought.

As 100 . 20 :: 400 . 80 for *A*. Then
l. *Months.* *l.* *Months.*
As 80 . 12 :: 50 . $7\frac{1}{2}$. Time of *A*.
Time B. *Stock A.* *Time A.* *Stock B.*
As 5 . 400 :: $7\frac{1}{2}$. 600. Stock of *B*.

6. Example, where are
- *Data*
  - Stock, Time, and part of the Gain of One.
  - Time and part of the Gain of Another.
  - Time of a third Partner.
- *Quæſita*, Stocks of two Partners.

Q. Of the Stock of two Partners.

Three Partners trade together. *A* continuing his Stock in the Trade 5 Months, muſt have $\frac{1}{5}$ of the Gain. *B* keepeth his Money in Stock 8 Months. *C* layeth in 400 *l.* for 7 Months, and taketh $\frac{2}{3}$ of the Gain: how much Money had *A* and *B* in the Stock?

Anſwer.

Anſw. *A* 168 *l.* and *B* 70 *l.*

Here, after adding the Parts of Gains given together, and taking the Total out of 1, the Analogies are;

As the Part of Gain of one Partner given, is to his Stock multiplied by his Time: So is the Part of Gain of another: And alſo ſo is the Remainder of the Parts taken from 1, to his Stock multiplied by his Time. Which divided by the Time, giveth the Stock of that Partner ſought reſpectively.

$$\overset{A}{\tfrac{1}{5}} + \overset{C}{\tfrac{2}{3}} = \tfrac{13}{15} \text{ Total.} \qquad 1 - \tfrac{13}{15} = \tfrac{2}{15} \text{ Remainder.}$$

$$\text{As } \tfrac{2}{3} \;.\; \overset{(400\times 7)}{2800} \;::\; \tfrac{1}{5} \;.\; \frac{840}{\text{Time of } A\ 5}\Big(\ 168 \text{ Stock of } A.$$

$$\text{As } \tfrac{2}{3} \;.\; 2800 \;::\; \tfrac{2}{15} \;.\; \frac{560}{\text{Time of } B\ 8}\Big(\ 70 \text{ Stock of } B.$$

Fellowſhip with diverſity of Time of three ſorts.

§. 3. *Fellowſhip*, with diverſity of Time, altereth the Stock and Time of ſome, or all of the particular Partners continuance; and thereby with the general Gain or Loſs, requireth their due Proportions thereof: Or by their particular Gains or Loſſes, and Times of Occupation with the general Stock, ſeeketh their particular Disburſments: Or by the Gain or Loſs of ſome particular Partners, Stock and Time, enquireth the particular Gain or Loſs, Stock or Time of another Partner, according to the *Data*.

To reſolve the firſt.

To reſolve the Propoſitions of the firſt Sort, multiply the ſeveral Disburſments of the particular Partners, by the reſpective Times thoſe Disburſements continue in Stock, adding or ſubſtracting for every Partner according to the Propoſition: Theſe Products added together, ſhall be the firſt Number of the *Rule of Three:* And the Sum of the Products of each Partner's Stock, multiplied by his reſpective

Times ſhall be the third Numbers. The ſecond Number ſhall be the Gain or Loſs as before.

Q. Of each Partner's Gain.

Example. *A*, *B*, and *C*, trade together for 12 Months: *A* putteth in preſently 40 *l.* and 4 Months after 30 *l.* more, and 3 Months after 20 *l.* more. *B* at firſt putteth in 100 *l.* but 3 Months after taketh away 20 *l.* and 5 Months after 20 *l.* more. *C* layeth down firſt 60 *l.* and five Months after taketh away 10 *l.* but 3 Months after putteth in 20 *l.* They gain 500 *l.* what is each Man's Part thereof?

Anſwer.

Anſw. *A* 164$\frac{164}{249}$ *l.* *B* 188$\frac{188}{249}$ *l.* and *C* 146$\frac{146}{249}$ *l.*

$$A\left\{\begin{array}{r} \textit{Stock. Time.} \\ 40 \times 4 = 160 \\ +\underline{30} \qquad\qquad \\ 70 \times 3 = 210 \\ +\underline{20} \qquad\qquad \\ 90 \times 5 = \underline{450} \\ \underline{820} \end{array}\right. \qquad B\left\{\begin{array}{r} \textit{Stock. Time.} \\ 100 \times 3 = 300 \\ -\underline{20} \qquad\qquad \\ 80 \times 5 = 400 \\ -\underline{20} \qquad\qquad \\ 60 \times 4 = \underline{240} \\ \underline{940} \end{array}\right. \qquad C\left\{\begin{array}{r} \textit{Stock. Time.} \\ 60 \times 5 = 300 \\ -\underline{10} \qquad\qquad \\ 50 \times 3 = 150 \\ +\underline{20} \qquad\qquad \\ 70 \times 4 = \underline{280} \\ \underline{730} \end{array}\right.$$

$$\begin{array}{lr} A & 820 \\ B & 940 \\ C & \underline{730} \\ & \underline{249{,}0} \end{array}$$

$$\text{As } 249\underset{\llcorner}{0} \;.\; \overset{l.}{500} :: 82\underset{\llcorner}{0} \;.\; \overset{l.}{164}\tfrac{164}{249} \qquad \begin{array}{r} 500 \\ \underline{82} \\ \underline{41000} \end{array} \qquad \begin{array}{l} \cancel{1} \\ \cancel{1}\cancel{6}|\underline{164} \\ \underline{4\cancel{1}\cancel{0}\cancel{0}\cancel{0}} \Big(\overset{l.}{164}\tfrac{164}{249}\ A. \\ 249 \end{array}$$

$$\text{As } 249\underset{\llcorner}{0} \;.\; 500 :: 94\underset{\llcorner}{0} \;.\; 188\tfrac{188}{249} \qquad \begin{array}{r} \underline{94} \\ \underline{47000} \end{array} \qquad \begin{array}{l} \cancel{2}\cancel{2}|\underline{188} \\ \underline{47\cancel{0}\cancel{0}\cancel{0}} \Big(\overset{l.}{188}\tfrac{188}{249}\ B. \\ 249 \end{array}$$

$$\text{As } 249\underset{\llcorner}{0} \;.\; 500 :: 73\underset{\llcorner}{0} \;.\; 146\tfrac{146}{249} \qquad \begin{array}{r} \underline{73} \\ \underline{36500} \end{array} \qquad \begin{array}{l} |\underline{1} \\ \cancel{1}\cancel{1}\cancel{6}|\underline{46} \\ \underline{\cancel{3}65\cancel{0}\cancel{0}} \Big(\overset{l.}{146}\tfrac{146}{249}\ C. \\ 249 \end{array}$$

To reſolve the ſecond Sort.

To reſolve the Propoſitions of the next Sort; multiply the ſeveral Gains or Loſs by the reſpective Times: the Total of theſe Products place for the firſt Number; the general Stock for the ſecond Number; and the particular Products for the third Numbers: And proceed as before by the *Rule of Three.*

Q. Of the Paſturage of Cattel, what Rent each ſhall pay.

Example. *A*, *B*, and *C*, together hire certain Paſture-Land for 30 *l.* Rent: And beſides their joint Stock of Sheep, equal in Number, *A* feedeth there at firſt 20 Oxen 80 Days; and taking them away, putteth in 100 other Oxen, which he keepeth there 14 Days. *B* at firſt putteth in 40 Oxen 50 Days; and removing them, afterward putteth in 16 Oxen for 100 Days. *C* putteth to Paſture only 70 Oxen, and keepeth them there, without alteration, 60 Days: what part of the Rent ſhall each Man pay?

Anſwer.

Anſw. *A* 8 *l.* 6 *s.* 8 *d.* *B* 10 *l.* and *C* 11 *l.* 13 *s.* 4 *d.*

$$A\left\{\begin{array}{r} \textit{Oxen. Days.} \\ 20 \times 80 = 1600 \\ 100 \times 14 = \underline{1400} \\ \underline{3000} \end{array}\right. \qquad B\left\{\begin{array}{r} \textit{Oxen. Days.} \\ 40 \times 50 = 2000 \\ 16 \times 100 = \underline{1600} \\ \underline{3600} \end{array}\right. \qquad \begin{array}{r} \textit{Oxen. Days.} \\ C\ 70 \times 60 = 4200 \end{array}$$

$$\begin{array}{lr} A & 3000 \\ B & 3600 \\ C & \underline{4200} \\ & \underline{108{,}00} \end{array}$$

$$\text{As } 108 \;.\; \overset{l.}{30} :: 30 \;.\; \overset{l.}{8}\tfrac{1}{3} \qquad \begin{array}{r} 30 \\ \underline{30} \\ \underline{900} \end{array} \qquad \begin{array}{l} |\underline{36} \\ \underline{\cancel{9}\cancel{0}\cancel{0}} \Big(\overset{l.}{8}\tfrac{1}{3}\ A. \\ 108 \end{array}$$

$$\text{As } 108 \;.\; 30 :: 36 \;.\; 10 \qquad \begin{array}{r} 30 \\ \underline{36} \\ \underline{1080} \end{array} \qquad \begin{array}{l} \underline{\cancel{1}\cancel{0}8\cancel{0}} \Big(\overset{l.}{10}\ B. \\ 108 \end{array}$$

```
             l.         l.                   |7
As 108 . 30 :: 42 . 11 2/3                   18|2
         42                                       l.
        ----                                 1260
        1260                                 ----( 11 2/3 C.
        ----                                 108
```

To reſolve the third Sort.

To reſolve the Propoſitions of the latter ſort, where the Time or Stock is diverſly given, as before in 2 §. of *Fellowſhip*, where the Time is ſimply given: Let the Numbers be orderly diſpoſed, and the Queſtion committed to the *Rule of five Numbers*, *Direct* or *Indirect*, as the Nature of the Queſtion requireth.

Q. Of the Paſturage of Cattel, what one ſhall pay.

Example. *A* hired Paſturage for 56 Bullocks 150 Days, and was to pay therefore 5 *l.* but meeting with a Market at 90 Days end, ſelleth off 40, and keeping the Remainder, taketh in 10 Bullocks of *B* at one time for 10 Days, and another time 5 Bullocks for 12 Days; and hireth out Paſturage to *C* for 36 Bullocks for 60 Days: what ſhall *C* pay for the ſame?

Anſwer.

Anſw. $1\frac{2}{7}$ *l.* For if 5 *l.* buy 150 Days Paſturage for 56 Bullocks, then will $1\frac{2}{7}$ *l.* buy 60 Days Paſturage for 36 Bullocks, by the *Direct Rule of five Numbers.*

```
    Bull.  Days.    l.    Bull.  Days.   l.
As   56  .  150  .  5  ::  36  .  60  .  1 2/7

          84,00           300             |24
          -----          ------           ----   l.
                         108,00           108
                         ------           ---( 1 2/7
                                          84
```

If the Queſtion had been, what Cattel to keep.

But if the Paiment of *C* $1\frac{2}{7}$ *l.* had been given, and the Queſtion had been, How many Bullocks he ſhould have depaſtured there, then the Queſtion had been reſolved by the *Indirect Rule of five Numbers.* If 5 *l.* buy 150 Days Paſturage for 56 Bullocks: what will $1\frac{2}{7}$ *l.* buy for 60 Days?

Anſwer.

Anſw. Paſturage for 36 Bullocks.

```
    Bull.  Days.   l.    Days.   l.    Bull.
As   56  :  150  :  5  ::  60  :  1 2/7 :  36
            56             5
           ----           ----
            900           3,00
            750
           ----
           8400
           2400 1 2/7                  1
           ----                       108
          108,00                      ---( 36 Bullocks.
           ----                        3
```

Queſtions may be various.

Here may be a Mixture of the Propoſitions, as in the ſecond Section before.

Fellowſhip, *with diverſity of Parts, of 2 ſorts.*

§. 4. *Fellowſhip* with diverſity of Parts, is either when a certain Sum is to be divided among ſeveral ſorts of Partners, ſo as Parcel of the Partners have their Dividend in proportion to other Parcel of them: Or elſe when a certain Sum is to be divided among ſingle Partners, according to the *Ratio* of their ſingle parts:

Latter, how called.

This latter Sort is called *Fractionary Fellowſhip*, and ſtored with variety of Examples.

To reſolve the firſt.

To reſolve the Propoſitions of the firſt Sort, multiply the Number of each Parcel of Partners by the Proportional Part each Partner is to receive, and the Total of theſe Products ſhall be the firſt Number of the *Rule of Three*; the Sum to be divided the Second; and the ſeveral Products the ſeveral third Numbers: And proceed as before by the *Rule of Three.*

Q. Of the Expences of Canons and Vicars.

Example. Suppoſe in a Cathedral were 30 Canons and 40 Vicars, which have allowed to ſpend 360 *l.* *per Annum:* But every Canon is to have 5 *l.* to the Vicar's 3 *l.* How much then are their Annual Expences ſeverally?

Anſwer.

Anſw. The Canons 200 *l.* and the Vicars 160 *l.*

Canons 30 × 5 = 150
Vicars 40 × 3 = 120
270

As 270 . 360 :: 150 . 200 *l.*

15
1800
360
5400

$\frac{5400}{27}$(200 *l.* Canons.

As 270 . 360 :: 120 . 160

12
720
360
4320

16
$\frac{4320}{27}$(160 *l.* Vicars.

To reſolve the ſecond Sort.

To reſolve the Propoſitions of the latter Sort, take a Number that will equally divide by all the Conſequents or Denominators; or to find one, multiply all the Denominators or Conſequents one into another, or ſo many of them as be not equal Halfs of ſome other of them, and divide this ultimate Product, or the Number firſt taken, by the Conſequents ſeverally, and multiply theſe Quotients by the Antecedents or Numerators: Add all theſe ſeveral Products (or the Quotients where one is Antecedent) into one Total, which with the ſeveral Addends abbreviate if it may be: Then with the leaſt Terms, or with the Total, and the ſeveral Products or Quotients, if 1 be Antecedent, where they will not be abbreviated, proceed as in the other ſort of *Fellowſhip* to the Work of the *Rule of Three.* What other Neceſſaries occur are inſerted, with the ſeveral Examples following.

Q. *Of a Legacy.*

Example 1. A certain Man bequeathed to charitable Uſes 100 *l.* to be divided in ſuch Proportion, that *A* ſhould have $\frac{1}{2}$, *B* $\frac{1}{3}$, and *C* $\frac{1}{4}$: how ſhould the 100 *l.* be divided according to the Intent of the Teſtator?

Anſwer.

Anſw. To *A* 46$\frac{2}{13}$ *l.* *B* 30$\frac{10}{13}$ *l.* and *C* 23$\frac{1}{13}$ *l.*

The Ratio *of the* Data *to be noted, and the Intent of the Queſtion.*

Here, and in all Queſtions of like ſort, the *Ratio* of the *Data* is to be minded, for otherwiſe the Queſtion were impoſſible, ſeeing $\frac{1}{2}$ and $\frac{1}{3}$ added to $\frac{1}{4}$, after the manner of Fractions, make more than the Whole, the $\frac{1}{2}$ of 100 *l.* being 50 *l.* the $\frac{1}{3}$ more 33$\frac{1}{3}$ *l.* with $\frac{1}{4}$ which is 25 *l.* added together, make in all 108 *l.* 6 *s.* 8 *d.* But the Intent of the Queſtion being, that every time *A* had $\frac{1}{2}$ Pound, or Shilling, *B* ſhould have $\frac{1}{3}$ of a Pound or Shilling, and *C* $\frac{1}{4}$; So as to every 6 Pence *A* had, *B* ſhould have 4 Pence, and *C* 3 Pence: Wherefore taking $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of 12, a Number which will be equally parted by 2, 3, 4. or multiplying thoſe Conſequents together, they make 24; which divided by 2, 3, and 4 ſeverally, giveth 12, 8, and 6: theſe added together (their Antecedents being Units) make 26; which abbreviated with them, make 6, 4, 3, and together 13, like to the Parts of 12. This 13 ſhall be the firſt Number, and the other the third Numbers of the *Rule of Three*, as followeth.

How to proceed.

1 : 2   1 : 3   1 : 4
8
24

$\frac{24}{2}$(12 $\frac{24}{3}$(8 $\frac{24}{4}$(6

12	6	*A*
8	4	*B*
6	3	*C*
26	13	

$\frac{12}{2}$(6
$\frac{12}{3}$(4
$\frac{12}{4}$(3
13

As 13 . 100 *l.* :: 6 . 46$\frac{2}{13}$ *l.*

6
600

8(2
$\frac{600}{13}$(46$\frac{2}{13}$ *l.* *A.*

As 13 . 100 :: 4 . 30$\frac{10}{13}$

4
400

(1
40(0
$\frac{400}{13}$(30$\frac{10}{13}$ *l.* *B.*

As 13 . 100 :: 3 . 23$\frac{1}{13}$

3
300

4(1
$\frac{300}{13}$(23$\frac{1}{13}$ *l.* *C.*

Q. Of a Ship, what each Partner paid.

Example 2. *A*, *B*, *C*, and *D*, bought a Ship for 890 *l.* and were to have their Parts according to theſe Proportions, *viz.* *A* $\frac{1}{2}$ and $\frac{1}{10}$, *B* $\frac{1}{4}$ and $\frac{1}{10}$, *C* $\frac{1}{6}$, and *D* $\frac{1}{20}$: what muſt each Man pay?

Anſwer.

Anſw. *A* $457\frac{5}{7}$ *l.* *B* 267 *l.* *C* $127\frac{1}{7}$ *l.* and *D* $38\frac{1}{7}$ *l.*

What to be done when one hath more Parts than one.

Here, and in ſuch Queſtions where more Parts or Proportions than one belong to one Partner, I may, after the manner of Fractions, add them into one, and work as before, or otherwiſe: After Diviſion of the Number taken, or the ultimate Product of the Conſequents, add the ſeveral Quotients belonging to one Partner together for his third Number of the *Rule of Three*; and in this Example, becauſe 60 is a Number that will be equally divided by all the Conſequents, as into Halfs, Fourths, Sixths, Tenths and Twentieth Parts, I may take 60 to divide by the Conſequents, or in multiplying the Conſequents to find a Number, I may omit 2, becauſe the half of 4, and 10 the half of 20, and take only 4, 6, and 20.

$$\overset{A}{\frac{1}{2}+\frac{1}{10}=\frac{3}{5}} \;:\; \overset{B}{\frac{1}{4}+\frac{1}{10}=\frac{7}{20}} \;:\; \overset{C}{\frac{1}{6}} \;:\; \overset{D.}{\frac{1}{20}}$$

Thus, or thus,

$$\frac{60}{\cancel{5}}\Big(\begin{array}{r}12\\3\\\hline 36\ A.\end{array}\quad \frac{60}{20}\Big(\begin{array}{r}3\\7\\\hline 21\ B.\end{array}\quad \frac{60}{\cancel{6}}\Big(10\ C.\quad \frac{60}{20}\Big(3\ D.\qquad \begin{array}{rl}36&A.\\21&B.\\10&C.\\3&D.\\\hline 70&\end{array}$$

$$A\begin{cases}\frac{60}{\cancel{2}}\Big(30\\[4pt]\frac{60}{\cancel{1}0}\Big(6\end{cases}\!\!\Big\}36\qquad \text{As } 70 : \overset{l.}{890} :: 36 : \overset{l.}{457\tfrac{5}{7}}.\qquad \begin{array}{r}890\\36\\\hline 5340\\2670\\\hline 3204{,}0\end{array}\qquad \begin{array}{l}\cancel{4}\cancel{5}(5\\\cancel{3}\cancel{2}\cancel{0}4\Big(\overset{l.}{457\frac{5}{7}}\ A.\\\hline \cancel{7}\end{array}$$

$$B\begin{cases}\frac{60}{\cancel{4}}\Big(15\\[4pt]\frac{60}{\cancel{1}0}\Big(6\end{cases}\!\!\Big\}21\qquad \text{As } 70 : 890 :: 21 : 267.\qquad \begin{array}{r}890\\21\\\hline 890\\1780\\\hline 1869{,}0\end{array}\qquad \begin{array}{l}\cancel{4}\cancel{4}\\\cancel{1}\cancel{8}\cancel{6}\cancel{9}\Big(\overset{l.}{267}\ B.\\\hline \cancel{7}\end{array}$$

$$C\ \frac{60}{\cancel{6}}\Big(10\qquad \text{As } 70 : 890 :: 10 : 127\tfrac{1}{7}.\qquad \begin{array}{r}890\\10\\\hline 890{,}0\end{array}\qquad \begin{array}{l}\cancel{1}\cancel{5}(1\\8\,\cancel{9}\,0\Big(\overset{l.}{127\frac{1}{7}}\ C.\\\hline 7\end{array}$$

$$D\ \frac{60}{20}\Big(3\qquad \text{As } 70 : 890 :: 3 : 38\tfrac{1}{7}.\qquad \begin{array}{r}890\\3\\\hline 267{,}0\end{array}\qquad \begin{array}{l}\cancel{5}(1\\\cancel{2}\cancel{6}\cancel{7}\Big(\overset{l.}{38\frac{1}{7}}\ D.\\\hline \cancel{7}\end{array}$$

$$\begin{array}{c}\\\hline 70\\\hline\end{array}$$

Q. Of a Sum to be paid.

Example 3. The Sum of 300 *l.* was to be paid by *A*, *B*, and *C*, in ſuch Proportions, that *A* was to pay 6 *l.* more than $\frac{1}{2}$, *B* $\frac{1}{3}$ and 12 *l.* over, and *C* 8 *l.* leſs than $\frac{2}{3}$: what muſt each Man pay?

Anſwer.

Anſw. *A* $102\frac{2}{3}$ *l.* *B* $76\frac{4}{9}$ *l.* and *C* $120\frac{8}{9}$ *l.*

When there is Money overplus, or to be abated.

Here, and in thoſe Queſtions where there is Overplus-Money, or Money to be abated; From the Sum to be divided, the Overplus-Money is to be ſubſtracted, and to that Sum the Abatement muſt be added, and then proceed as before; and to the reſpective Portions obtained by the *Rule of Three*, add the Overplus-Money to be added, and ſubſtract the other to be ſubſtracted.

$\frac{1}{2}\ \ \frac{1}{3}\ \ \frac{2}{3}$ 6

$\frac{6}{2}(3$ $\frac{6}{3}(2$ $\frac{6}{3}(2 \times 2 = 4$

3 *A.*
2 *B.*
4 *C.*
9

A 6 *l.* } +
B 12
18
C 8 −
10
300—10=290

As 9 . $\overset{l.}{290}$ :: 3 . $\overset{l.}{96\frac{2}{3}}$
$290 \times 3 = 870$
$\frac{\cancel{8}\cancel{7}\cancel{0}}{\cancel{9}}(\overset{l.}{96\frac{2}{3}}$ 6(6
6 +
$102\frac{2}{3}$ A.

As 9 . 290 :: 2 . $64\frac{4}{9}$
$290 \times 2 = 580$
$\frac{\cancel{5}80}{\cancel{9}}(\overset{l.}{64\frac{4}{9}}$ 4(4
12 +
$76\frac{4}{9}$ B.

As 9 . 290 :: 4 . $128\frac{8}{9}$
$290 \times 4 = 1160$
$\frac{\cancel{1}\cancel{1}\cancel{6}\cancel{0}}{9}(\overset{l.}{128\frac{8}{9}}$ 28(8
8 —
$120\frac{8}{9}$ C.

Q. of the Charge to ſet out a Ship.

Example 4. Three Owners of a Ship ſet her forth; the Charge whereof *A* payeth $\frac{1}{4}$ Part, *B* 300 *l.* and *C* $\frac{9}{16}$ Parts of the Whole: how much Money paid *A* and *C* for their Parts? and what Part of the Ship had *B*? and how much was the whole Charge?

Anſwer.

Anſw. *A* paid 400 *l.* *C* 900 *l.* *B* had $\frac{3}{16}$ Parts of the Ship, and the whole Charge was 1600 *l.*

How to proceed when Queſtions are interwoven.

Here, and in others alike, where ſeveral Queſtions are interwoven, ſeveral things are to be done in order to their Reſolution: For firſt the Parts of *A* and *C* being given, added together, and the Total ſubſtracted from an Unit, or the Whole, diſcovers the Part of *B*; that known, ſeeing the Charge thereof is given, the Charge of the other Parts will be had by the *Rule of Three:* which when found, added to the Charge of *B*, anſwers the whole Charge.

Parts of $\overset{A.}{\frac{1}{4}} + \overset{C.}{\frac{9}{16}} = \overset{Sum.}{\frac{13}{16}}$ Unit, or $\frac{16}{16} - \frac{13}{16} = \overset{Remain.}{\frac{3}{16}}$ Part of *B*.

As $\frac{3}{16}$. $\overset{l.}{300}$ :: $\frac{1}{4}$. $\overset{l.}{400}$

$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{16}}}) \frac{\overset{100}{\cancel{300}}}{\underset{1}{\cancel{4}}} (\frac{\overset{l.}{400}}{1}$ Charge of *A* for $\frac{1}{4}$.

Particular Charges.
A for $\frac{1}{4}$. 400 *l.*
B for $\frac{3}{16}$. 300 *l.*
C for $\frac{9}{16}$. 900 *l.*
Total 1600 *l.* Charge.

As $\frac{3}{16}$. 300 :: $\frac{9}{16}$. 900.

$\frac{\overset{1}{\cancel{3}}}{\cancel{16}}) \frac{\overset{900}{\cancel{2700}}}{\cancel{16}} (\frac{\overset{l.}{900}}{1}$ Charge of *C* for $\frac{9}{16}$.

Q. Of Gain parted.

Example 5. Three Merchants have gained 100 *l.* which they divide in ſuch manner, that the $\frac{1}{2}$ Part of *A* was as much as $\frac{5}{8}$ of what *B* had; and $_{8}$ Part of *B* was equal to $\frac{1}{2}$ Part of *C*: how much had each Man for his Part?

Anſwer.

Anſw. *A* 50 *l.* *B* 40 *l.* *C* 10 *l.*

The firſt Number how gotten.

Here, and in ſuch others as before, any Number may be taken that will equally be divided by the Denominators or Conſequents; and if none come to hand, by their Multiplication as before ſuch a Number may be had: then accounting that for *A*, take the Parts thereof for *B* and *C*; and the other Partners, if more, according to the Propoſition: add all theſe Numbers together for the firſt Number of the *Rule of Three*, the Number firſt taken, and the Parts thereof ſhall be the ſeveral third Numbers, and the Sum to be divided the Second, as before.

Denominators $2 \times 8 = 16$.

A 16 *l.* then $\frac{1}{2}$ is $8 = \frac{5}{8}$ *B*. And as $\frac{5}{8}$ *B* . 8 *l.* :: 1 *B* . $12\frac{4}{5}$ *l.* $\frac{5}{8}) \frac{8}{1} (\frac{64}{5}$ *B*.

B $12\frac{4}{5}$ then $\frac{1}{8}$ is $1\frac{3}{5} = \frac{1}{2}$ *C*. And as $\frac{1}{2}$ *C* . $1\frac{3}{5}$ *l.* :: 1 *C* . $3\frac{1}{5}$ *l.* $\frac{1}{2}) \frac{8}{5} (\frac{16}{5}$ *C*.

C $3\frac{1}{5}$

32

As 32 . 100 *l.* :: 16 . 50 *l.*

$$\begin{array}{r} 100 \\ 16 \\ \hline 1600 \end{array} \qquad \frac{1600}{32}\Big(\overset{l.}{50}\ A.$$

As 32 . 100 :: $12\frac{4}{5}$. 40

$$\begin{array}{r} 100 \\ 12\frac{4}{5} \\ \hline 1200 \\ 80 \\ \hline 1280 \end{array} \qquad \frac{1280}{32}\Big(\overset{l.}{40}\ B.$$

As 32 . 100 :: $3\frac{1}{5}$. 10

$$\begin{array}{r} 100 \\ 3\frac{1}{5} \\ \hline 300 \\ 20 \\ \hline 320 \end{array} \qquad \frac{320}{32}\Big(\overset{l.}{10}\ C.$$

Q. Of a Legacy. *Example* 6. A certain Man on his Death-bed, by his laſt Will and Teſtament, bequeathed 360 *l.* to be thus diſpoſed, *viz.* If his Wife, being then with Child, brought forth a Son, then to his Son the $\frac{1}{2}$, and his Wife $\frac{1}{3}$: But if ſhe brought forth a Daughter, then his Wife to have $\frac{1}{2}$, and his Daughter $\frac{1}{3}$. And it happened that the Woman brought forth both a Son and a Daughter. Now the Queſtion is, how the 360 *l.* ſhould be divided, that the Will of the Teſtator ſhould be fulfilled?

Anſwer. *Anſw.* To the Son $170\frac{10}{19}$ *l.* to the Mother $113\frac{13}{19}$ *l.* and to the Daughter $75\frac{15}{19}$ *l.*

How to get the Proportional Numbers. For here, and in ſuch-like Queſtions, Numbers are to be ſought out that bear ſuch Proportion one to another, as the ſeveral Parts propounded, the Intention of the Teſtator being, that the Mother ſhould not have $\frac{1}{3}$ to the Son's Part, and $\frac{1}{2}$ to the Daughter's Part both, ſeeing theſe Parts added together would be $\frac{5}{6}$, and ſo more than the Son's Part: But as the Son was to have $\frac{1}{2}$, ſo the Mother $\frac{1}{3}$, that is proportionally as 3 to 2; whereby the Son is intended to have as much as the Mother, and half as much more; and the like muſt the Mother have to the Daughter. Therefore to find the Proportional Numbers of this Sort, multiply the firſt and laſt Terms or Parts together, as here 2 by 3, which is 6, and this ſhall be the middle Number for the Mother's Portion. Then multiply this middle Number 6 by the leſſer Term 2, and divide the Product 12 by the greater Term 3, and the Quotient ſhall ~~be the~~ leſſer Term, and being here 4, ſhall be for the Daughter's Portion. Again, multiply that middle Number by the greater Term, and divide the Product by the Leſſer, for the greateſt Portion that is here for the Son; ſo 6 multiplied into 3 produceth 18, and this divided by 2 gives 9 in the Quotient. And theſe Numbers found, are the ſeveral third Numbers of the *Rule of Three*, the Total of them the Firſt, and the ſecond Number the Sum to be divided as before.

2x3= 6 Mother.
6x2=$\frac{12}{3}$(4 Daughter.
6x3=$\frac{18}{2}$(9 Son.
19

l. *l.*
As 19 . 360 :: 9 . $170\frac{10}{19}$
9
3240

$\cancel{1}\cancel{3}|1$
$\cancel{3}\cancel{2}\cancel{4}|0$ (*l.* $170\frac{10}{19}$ Son.
$\cancel{1}\cancel{9}$

As 19 . 360 :: 6 . $113\frac{13}{19}$
6
2160

$|1$
$\cancel{2}\cancel{7}|3$
$\cancel{2}\cancel{1}\cancel{6}\,\cancel{0}$ (*l.* $113\frac{13}{19}$ Mother.
$\cancel{1}\cancel{9}$

As 19 . 360 :: 4 . $75\frac{15}{19}$
4
1440

$\cancel{1}|\cancel{1}5$
$\cancel{1}\cancel{4}\,\cancel{4}\cancel{0}$ (*l.* $75\frac{15}{19}$ Daughter
$\cancel{1}\cancel{9}$

Proof 360

The proper Proof of *Fellowſhip* (other than ſuch Propoſitions as are determined by ſome ſingle Operation of the *Rule of five Numbers*, which have their Proof in common therewith) is to add into one Total the ſeveral Quotients that anſwer the Propoſition; which if the Work be right, will return the ſecond Number. As in the laſt Example $170\frac{10}{19}$ *l.* $113\frac{13}{19}$ *l.* and $75\frac{15}{19}$ *l.* added together, make 360 *l.* the Sum to be divided. *Proof of Fellowſhip.*

CHAP. IX.

ALLIGATION.

LEAVING *Fellowſhip*, wherein was a Mixture of Partners and their Stock; the next Subject Derivatives deal with, is the Mixture of Merchandiſes, as Corn, Wine, Wool, Metals, Medicines, *&c.* under the Title of *Alligation.* *Alligation mixeth divers Merchandiſes.*

Alligation is of two kinds, Medial and Alternate: Medial properly ſeeketh a Mean in the Price, Quantity or Quality between the Extreams. And Alternate altereth the placing of the Differences falling out between the mean Price and the Extreams: And both propoſeth the Numbers and Quantities Homogeneal, or reduceth them into ſuch, and properly intendeth ſimple Mixtures, or thoſe done but once; thoſe often repeated being either Figurals, or continued Proportions; of which more in the laſt Chapter of this Part, and the fifth Chapter of the next, in Section 6. *Two ſorts of Alligation.* *Numbers to be Homogeneal.*

Medial Alligation contains ſix Propoſitions. *Medial the Propoſitions.*

Prop. 1. By the Quantities to be mixed, and the particular Prices, to find the Price or Value of ſome part of the Mixture. *1. To find the Price of part.*

Multiply the Quantities to be mixed ſeverally by their own Prices, and divide the Sum of theſe Products by the Sum of the Quantities mixed. Rule.

Example 1. A Merchant would mix 100 Buſhels of Rye at 4 *s.* the Buſhel, with 40 Buſhels of Barley at 3 *s.* 6 *d.* the Buſhel, and 60 Buſhels of Wheat at 6 *s.* the Buſhel: and know what one Buſhel of that Miſcellane would be worth. *Q. Of Miſcellanes.*

Anſw. 4 *s.* 6 *d.* For the Quantities 100, 40, and 60, added, make 200, and theſe Quantities multiplied ſeverally by the particular Prices, make 400, 140, and 360; which Products added, make 900; this divided by 200, gives $4\frac{1}{2}$ *s.* as before. Anſwer.

Quantities.	*Buſhels.*	*Prices.* *s.*		
Rye	100	× 4	=	400
Barley	40	× $3\frac{1}{2}$	=	140
Wheat	60	× 6	=	360
	2,00			9,00

Analogy.

As 200 . 900 :: 1 . $4\frac{1}{2}$

$\frac{9}{2}$(*s.* $4\frac{1}{2}$ Buſhel.

Q. Of the Worth of a Cask of Wine.

Example 2. Five Casks that hold 60 Gallons apiece, are to be filled with Wine, *viz.* 75 Gallons of Sack at 4 *s. per* Gallon, and 225 Gallons of White-Wine at 2 *s. per* Gallon: what ſhall one Cask of this mixed Wine be worth?

Anſwer.

Anſw. 7 *l.* 10 *s.* For after the Value of 1 Gallon of the Mixture is found, as before, to be 2 *s.* 6 *d.* the Value of 60 Gallons, the Content of 1 Cask, is had by Multiplication.

	Quantit. *Gallons.*	*Prices.* *s.*				*Cask.*
						60 Gallons.
Sack	75 ×	4 =	300	(1		2½
White	225 ×	2 =	450	7(5	(*s.* 2½ Gallon.	120
	30,0		75,0	30		30
						15,0
						l. 7:10 *s.*

2. To find the Quantity.

Prop. 2. By the particular Prices of the Quantities, and Sum paid or received for a Mixture bought or ſold; to find what Quantity of each kind was bought or ſold.

Rule.

Divide the Sum paid or received for the Mixture bought or ſold, by the Sum of the particular Prices.

Q. Of Pieces of Money ſent for to the Mint.

Example 1. A certain Noble-man ſent his Servant with the King's Majeſties Warrant to the Mint-maſter for 4000 *l.* and he muſt bring it in Pieces of 12 *d.* 6 *d.* 4 *d.* 2 *d.* 1 *d.* and he muſt bring of each ſort of Pieces a like Number: how many of each ſort muſt he bring?

Anſwer.

Anſw. 38400 Pieces of each ſort: For 4000 *l.* brought into Pence, becauſe the other Pieces are in Pence, the Reſult 960000 *d.* divided by 25, that is 12 + 6 + 4 + 2 + 1, giveth the Anſwer aforeſaid.

Prices.	*Sum paid.*		
12 *d.*	4000 *l.*		
6	240	21	
4	160000	960000 (	38400 Pieces.
2	8000	25	
1		25	
	960000 *d.*	25	
25			

Q. Of Spice ſold, how much of a Sort.

Example 2. A Grocer ſold four ſorts of Spice, of each a like Quantity, but at ſeveral Rates; *viz.* Large Mace at 8 *s.* 4 *d. per* ℔. Cinamon at 6 *s.* 8 *d. per* ℔. Nutmegs at 5 *s. per* ℔. and Ginger at 2 *s. per* ℔. For what he ſold, he received 23 *l.* 2 *s.* The Queſtion is, how many Pounds of each ſort he ſold to make up the ſaid Sum of 23 *l.* 2 *s.*

Anſwer.

Anſw. 21 ℔. of each ſort: For the Prices given added make 22 *s.* and 23 *l.* 2 *s.* reduced into Shillings that they may be Homogeneal, make 462 *s.* which divided by 22 *s.* give 21 ℔ as before.

	Prices.		*Sum received.*		
Large Mace	8 *s.*	: 4 *d.*	23 *l.* 2 *s.*	2	℔.
Cinamon	6	: 8	20	462 (	21 of each Sort.
Nutmegs	5	: 0	460	22	
Ginger	2	: 0	2	22	
	22	: 0	462		

3. To increaſe or leſſen a Mixture.

Prop. 3. By the Quantities of a Mixture, to augment or diminiſh the Mixture proportionally.

Rule.

Sum up the Quantities, and then by the *Rule of Three*, as that Sum is to the Augmentation or Diminution: So is the Quantity of each Parcel of the Mixture, to the Quantity of the Mixture deſired.

Q. Of increaſing an Ointment.

Example 1. In the Ointment called *Unguentum album Camphoratum*, there is put to Oil of Roſes ℥ 12, white Wax ℥ 3, Ceruſe ℥ 6, and Camphire beaten with Oil of Roſes ʒ 2; which reduced all into Drams, make 96 . 24 . 48 . 2. and in the Total 170. And if I would make the Quantity to contain 210

Drams: how much of each Ingredient muſt be taken?

Anſw. Oil $118\frac{10}{17}$ ʒ, Wax $29\frac{11}{17}$ ʒ, Ceruſe $59\frac{5}{17}$ ʒ, Camphire $2\frac{8}{17}$ ʒ.

Anſwer.

As 170 . 210 :: 96 . $118\frac{10}{17}$ Oil.

96

1260
1890

20160

$\cancel{2}(1$
$\cancel{3}\cancel{4}(0$
$\cancel{2}\,\cancel{0}\cancel{1}\cancel{6}(118\frac{10}{17}$
$\cancel{1}7$

As 170 . 210 :: 24 . $29\frac{11}{17}$ Wax.

24

840
420

5040

$(1$
$\cancel{1}\cancel{6}(1$
$\cancel{5}\cancel{0}4(29\frac{11}{17}$
$\cancel{1}7$

As 170 . 210 :: 48 . $59\frac{5}{17}$ Ceruſe.

48

1680
840

10080

$1\cancel{5}(5$
$\cancel{1}\cancel{0}\cancel{0}8(59\frac{5}{17}$
$\cancel{1}7$

As 170 . 210 :: 2 . $2\frac{8}{17}$ Camphire.

2

420

$(8$
$\cancel{4}2(2\frac{8}{17}$
$\cancel{1}7$

Example 2. A Pectoral Pouder of 10 ℔. is made up with Sugar ℔ 6, Licoriſh ℔ 2, Anni-ſeeds ℔ $1\frac{1}{2}$, and Fennel-ſeeds ℔ $\frac{1}{2}$: But I would make no more than 4 ℔ of the Pouder; how much of the ſeveral Ingredients muſt be taken?

Q. *Of leſſening a Pouder.*

Anſw. Sugar $2\frac{2}{5}$ ℔, Licoriſh $\frac{4}{5}$ ℔. Anni-ſeeds $\frac{3}{5}$ ℔, and Fennel-ſeeds $\frac{1}{5}$ ℔.

Anſwer.

As 10 . 4 :: 6 . $2\frac{2}{5}$ Sugar.

6

$10)\underline{24}(2\frac{2}{5}$

As 10 . 4 :: 2 . $\frac{4}{5}$ Licoriſh.

2

$10)\underline{8}(\frac{4}{5}$

As 10 . 4 :: $1\frac{1}{2}$. $\frac{3}{5}$ Anniſeeds.

$1\frac{1}{2}$

4

2

$10)\underline{6}(\frac{3}{5}$

As 10 . 4 :: $\frac{1}{2}$. $\frac{1}{5}$ Fennel-ſeeds.

$\frac{1}{2}$

$10)\underline{2}(\frac{1}{5}$

Prop. 4. By the Qualities or Nature of the ſeveral Ingredients in a Mixture, to find the Temperament or Emergent Quality of the Mixture.

4. *To find the Temperament of a Mixture.*
Rule.

Diſpoſe the Quantities of the Mixture ſeverally in Rows, ſetting orderly them their ſeveral Qualities; multiply each Quantity by its own Quality. And if the Qualities of the Ingredients be contrary, ſubſtract the contrary Qualities ſo multiplied one from the other, as Hot from Cold, Moiſt from Dry, &c. and ſet down the Difference of the Products. Then as the Sum of all the Quantities is, to the Difference of the Products, or the Products where no contrary Qualities are: So is an Unit to the Quality emergent, and always of the ſame Kind with the greater Product where the Qualities are contrary.

Example 1. A Goldſmith mixeth 20 Portions of Silver of 6 ʒ fine, with 5 of 8 ʒ fine, 5 of 10 ʒ fine, and 10 Portions of Copper; what Fineneſs ſhall the melted Maſs be of?

Q. *Of the fineneſs of Metal.*

Anſw. $5\frac{1}{4}$ ʒ.

Anſwer:

	Quantities. *Portions.*		*Fineneſs.* ʒ.		
Silver	20	×	6	=	120
Silver	5	×	8	=	40
Silver	5	×	10	=	50
Copper	10	×	0	=	00
	40				210

As 40 . 210 :: 1 . $5\frac{1}{4}$

$\frac{21}{4}(5\frac{1}{4}$ ʒ. Fine.

Example 2. If the Species called *Dianthus*, be made according to the *London Diſpenſatory*, and the Qualities of the Simples taken according to *Sennertus* in his *Inſtitutes*, *Lib.* 5. *Par.* 1. *Chap.* 3. *Parkinſon* in his *Herbal*, and other approved Authors: And it be enquired to know the Quality, Emergent or Temperament of the Compoſition; then diſpoſing orderly the Quantities and Qualities as below,

Q. *Of the Quality of* Dianthus.

below, and multiplying reſpectively the one by the other, and ſubſtracting the contrary Qualities one from the other, the Remains left to be divided by the Total of the Quantities, declare the Medicine of a fine Temperament, *viz.* Hot in the firſt Degree and ſomewhat above, and Dry not full a Degree.

Anſwer.

	Ingredients.	*Quantities.*		*Qualities.*				*Products.*			
				hot.	*cold.*	*moiſt.*	*dry.*				
Dianthus	Roſemary Flowers	ʒ 8, or ℈ 24	×	2	0	0	2	48	0	0	48
	Red Roſes	ʒ 6, or . 18	×	0	1	0	1	0	18	0	18
	Violets	ʒ 6, or . 18	×	0	1	2	0	0	18	36	0
	Licoriſh	ʒ 6, or . 18	×	1	0	1	0	18	0	18	0
	Cloves	4	×	3	0	0	3	12	0	0	12
	Indian Spiknard	4	×	1	0	0	2	4	0	0	8
	Nutmegs	4	×	2	0	0	2	8	0	0	8
	Galanga	4	×	3	0	0	3	12	0	0	12
	Cinamon	4	×	2	0	0	2	8	0	0	8
	Ginger	4	×	3	0	0	3	12	0	0	12
	Zedoary	4	×	2	0	0	2	8	0	0	8
	Mace	4	×	2	0	0	2	8	0	0	8
	Wood of Aloes	4	×	2	0	0	2	8	0	0	8
	Cardamoms the Leſs	4	×	3	0	0	3	12	0	0	12
	Anniſeeds	4	×	2	0	0	1	8	0	0	4
	Dillſeeds	4	×	2	0	0	3	8	0	0	12
		126						174 .	36 .	54 .	178

Hot. *Cold.* $|12$

$174 - 36 = \frac{\cancel{1}\cancel{3}8}{\cancel{1}2\cancel{6}}\Big(1\frac{2}{21}$ Hot.

Temperament.

Dry. *Moiſt.*

$178 - 54 = \frac{124}{126}\Big(\frac{62}{63}$ Dry.

As 126 . 138 :: 1 . $1\frac{2}{21}$.

As 126 . 124 :: 1 . $\frac{62}{63}$.

When among the Simples there is ſome Compound.

When in any Compoſition among the Simples, ſome compound Ingredient is mixed, then the Temperament of that Compound being firſt gotten, the other is to be found in like manner. As in Maſtick Pills, becauſe among the Simples *Hiera picra* is uſed, which is a Compound, the Temperament thereof is firſt found to be Hot in the ſecond Degree, and Dry in the ſecond Degree, and nigh ¾ of a Degree more: And then the Temperament of the Pills is found to be almoſt two Degrees Hot, and above two Degrees and an half Dry.

	Ingredients.	*Quantities.* ʒ		*Qualities.* *hot.*	*dry.*	*Products.*		
Hiera picra ſimple	Cinamon	6	×	2	2	12	12	
	Xylobalſamum	6	×	2	2	12	12	$\frac{\cancel{2}7\cancel{2}}{\cancel{1}\cancel{3}6}\Big($ 2 Hot.
	Roots of *Aſarabacca*	6	×	3	3	18	18	
	Spiknard	6	×	1	2	6	12	
	Maſtich	6	×	2	2	12	12	$\lvert 100$
	Saffron	6	×	2	1	12	6	
	Aloes 12½ ʒ, or	100	×	2	3	200	300	$\frac{\cancel{3}7\cancel{2}}{\cancel{1}\cancel{3}6}\Big(2\frac{25}{34}$ Dry.
		136				272 .	372	

	Ingredients.	*Quantities.* ʒ		*Qualities.* *hot.*	*dry.*	*Products.*		
Maſtich Pills.	Maſtich	2	×	2	2	4	4	$\overset{3}{\vdots}$ $\overset{11}{\vdots}$
	Aloes	4	×	2	3	8	12	$\frac{9}{1}\Big)\frac{33}{2}\Big(\frac{11}{6}\Big(1\frac{5}{6}$ Hot.
	Agarick	1½	×	1	2	1½	3	
	Hiera Simple	1½	×	2	2¾	3	4⅛	$\frac{9}{1}\Big)\frac{185}{8}\Big(\frac{185}{72}\Big(2\frac{41}{72}$ Dry.
		9				16½ .	23⅛	

5. To find the Quantity, &c. in a ſimple Mixture.

Prop. 5. By the Quantities of a Mixture, to find the particular Quantity of any Ingredient in any Part of the Mixture.

If the Mixture be Simple, or but once, then by the *Rule of Three*,

Rule.

As the Total of the Ingredients in the Compoſition is, to the Quantity of the Doſe, (or part of the Mixture propoſed): So is the Quantity of the Ingredient propoſed in the whole Compoſition, to the Quantity of the Ingredient in the Doſe.

Q. Of Miſcellane.

Example. If 700 Buſhels of Wheat be mixed with 100 Buſhels of Rye: how much Rye is there in one Buſhel of that Miſcellane?

Anſwer.

Anſw. $\frac{1}{8}$ of a Buſhel, that is a Gallon.

		Buſhels.
Wheat	:	700
Rye	.	100
Miſcellane	:	800

As 800 : 100 :: 1 . $\frac{1}{8}$

$\frac{1|00}{8|00}$ ($\frac{1}{8}$ Rye.

In a compound Mixture, how beſt to proceed.

But if the Mixture be compound, that is, often repeated, then the beſt way is to proceed by figural Proportions, as afterward in the 16*th* Chapter. Otherwiſe thus, by the *Rule of Three*, proceed to find the Quantity deſired after the firſt Mixture as before. Then proceed accordingly to repeat the like Work upon every Mixture till your Deſire be obtained.

Common Way.

Q. Of Wine and Water mixed.

Example. A Merchant hath a Piece of Wine of 128 Gallons, out of which he draweth 16 Gallons, and filleth it up again with Water. Again, he draweth out 16 Gallons, and filleth it up again with Water; and the third time doth the like: how much Wine and Water was at laſt in the Caſk?

Anſwer.

Anſw. 85$\frac{3}{4}$ Gallons of Wine, and 42$\frac{1}{4}$ Gallons of Water: For by the firſt Draught there was left but 112 Gallons of Wine in the Caſk; which filled up with Water, and 16 Gallons of that Mixture drawn, there was 14 of Wine and 2 of Water drawn out: So upon the ſecond Draught there were but 98 Gallons of Wine in the Caſk: Then the Caſk filled, there muſt be 30 Gallons of Water to make up 128. And upon the third Draught there were 12$\frac{1}{4}$ Gallons of Wine more drawn out, and 3$\frac{3}{4}$ of Water; which 12$\frac{1}{4}$ taken from 98, leaves 85$\frac{3}{4}$ Gallons of Wine as before, the Reſidue was Water to fill up the Caſk.

Wine.
128
Wine drawn out 16 Water put in.
Wine remaining 112 at the firſt Draught.
14
Wine remaining 98 at the ſecond Draught.
12$\frac{1}{4}$
Wine remaining 85$\frac{3}{4}$ at the third Draught.

Wine. *Water.*
85$\frac{3}{4}$ + 42$\frac{1}{4}$ = 128

As 128 . 16 :: 16 . 2
16
96
16
128) 256 (2 Water run out.
14 Wine run out.
16

As 128 . 30 :: 16 . 3$\frac{3}{4}$
16
180
30
128) 480 (3$\frac{3}{4}$ Water run out.
12$\frac{1}{4}$ Wine run out.
16

Quantity of each in part, how diſcovered.

Now if the Queſtion had deſired to know the Quantity of Wine or Water in any ſmaller Quantity of the Mixture than the Whole; as ſuppoſe in 12 Gallons, then the Analogy is thus: For

```
Wine ;   As 128 . 85 3/4  ::  12 . 8 5/128
                  12
                 ----
                  170              (5
                  859             1029 (
                 ----             ---- ( 8 5/128 Gallons in 12.
                 1029              128 (

Water ;  As 128 . 42 1/4  ::  12 . 3 123/128
                  12
                 ----
                  84              |123
                  423              507 (
                 ----             ---- ( 3 123/128 Gallons in 12.
                  507              128 (
```

6. To find the Quantities mixed, tho unequally. *Prop.* 6. By the Total of a Mixture, with the Total Value, and the Values of the ſeveral Ingredients mixed, to find the ſeveral Quantities mixed, though unequally.

Rule. Multiply the Total of the Mixture by the leaſt Value, ſubſtract the Product from the Total Value, and the Remainer is the firſt Dividend: Then take the ſaid leaſt Value from the greateſt valued Ingredient, and the Remainer is the firſt Diviſor. The Quotient of this Diviſion ſhews the Quantity of the higheſt prized Ingredient, the other is the Complement to the Whole. And when more Ingredients than two are in the Compoſition, the Diviſors are the ſeveral Remains of the leaſt Value taken from the other. The Dividends are the Remains left upon the Diviſions till 0 remain there; which will be one ſhort of the Number of Ingredients, and this defective Ingredient is to be ſupplied as a Complement. And in *Diviſion* no more muſt be taken in every Quotient, than that there may be left enough for the other Diviſors, and the laſt to leave 0 remaining.

Example in a Mixture of two Ingredients.

Q. Of Sack mixed. A Merchant mixeth to the Quantity of 128 Gallons of Sack, which he ſelleth for 37 *l.* 3 *s.* in which Mixture was Malaga at 6 *s.* the Gallon, and Sherry at 4 *s.* the Gallon: how many Gallons of each ſort were in the Mixture?

Anſwer. *Anſw.* 115 1/2 Malaga, 12 1/2 Sherry.

```
Total Mixture.                      Total Value.
                   { 6 s. Malaga. }                          (1
128 Gallons        { 4 s. Sherry. }  37 l.  3 s.            231 (
                   ---------------                          --- ( 115 1/2 Malaga.
   4  Divisor 2                         20                    2 (
 ---          ---                      ---
 512                                   743        Complement  12 1/2 Sherry.
                                       512                   -------
                                       ---                    128
                          Dividend     231                   -------
                                       ---
```

Example, in a Company of three Sexes.

Q. Of Expences of ſeveral Sexes. Twenty four Perſons, Men, Women, and Children, ſpent 37 *s.* 8 *d.* whereof every Man was to pay 2 *s.* every Woman 18 *d.* and every Child 8 *d.* how many of each Sex were there?

Anſwer. *Anſw.* 15 Men, 2 Women, 7 Children.

```
Company. {24 d. a Man.   } Total Expence.
  24     {18 d. a Woman. } 37 s. 8 d.       24—8=16 First Divisor.
   8     { 8 d. a Child. } 12               18—8=10 Second Divisor.
 ----                      ----
 192                        74
                           378      (2
                           ----     20
                           452      26(0 ( 15 Men.
                           192      ----
                           ----     16
         First Dividend    260
                           ----

              Second Dividend 20 (  2 Women.
                              --
                              10
                   Complement  7 Children.
                             ----
                              24
                             ----
```

Example in a mixt Sale at four Rates.

A Baker ſold 12 Loaves of Bread of four Sorts, for 12 Pence, *viz.* Twopenny-Loaves, Penny-Loaves, Halfpenny-Loaves, and Farthing-Loaves: how many of each Sort were there?

Q. Of Loaves of Bread.

Anſw. 4 Twopenny-Loaves, 2 Penny-Loaves, 2 Halfpenny-Loaves, and 4 Farthing-Loaves.

Anſwer.

```
Loaves.   q.           Total Sum.
        {8 Twopenny.  } 12 d.        8 | 7 First Diviſor.
  12    {4 Penny.     }  4           4 | 3 Second Diviſor.
   1    {2 Halfpenny. } ----         2 | 1 Third Diviſor.
 ----   {1 Farthing.  } 48 q.        1 |
  12                    12
 ----                   ----
        First Dividend  36             (8
                        ----           36 ( 4 Twopenny.
                                       --
                                        7
                                       (2
              Second Dividend           8 ( 2 Penny.
                                       --
                                        3

               Third Dividend           2 ( 2 Halfpenny.
                                       --
                                        1
                       Complement       4   Farthing.
                                      ----
                                       12
                                      ----
```

In the Queſtions falling under this Propoſition, two things are to be noted.

What to be noted here.

Firſt, When there is no definite Number of the Species allotted to be had, (as was in the laſt Example, where the Demand was limited to four Sorts of Loaves) there is no certainty as to the particular Numbers deſired: But the Queſtion may oftentimes be reſolved by other Numbers than thoſe found out as above. For in the ſecond Example above, if 10 Men, 10 Women, and 4 Children, ſpend at the Rates aforeſaid, the Sum of 37 *s.* 8 *d.* may be paid by them as exactly, as if there be 15 Men, 2 Women, and 7 Children.

1. If the Number allotted be certain or not.

Secondly, Sometimes a Queſtion is ſo propounded, that before a final Reſolution thereof, ſome Operations of the *Rule of Three* muſt precede.

2. If other Work muſt precede Reſolution.

As a Merchant ſold 24 ℔ of Pepper, Ginger and Sugar for 48 *s.* *viz.* 4 ℔ of Pepper for 9 *s.* and ſo much was 6 ℔ of Ginger valued at; and 12 ℔ of Sugar was rated as 9 ℔ of Ginger: how much of each Sort was ſold?

Anſw. 18 ℔ of Pepper, 2 ℔ of Ginger, and 4 ℔ of Sugar.

℔ s. ℔ s.
As 4 . 9 :: 1 . $2\frac{1}{4}$ Pepper.
℔ s. ℔ s.
As 6 . 9 :: 1 . $1\frac{1}{2}$ Ginger.

℔ s. ℔ s.
As 6 . 9 :: 9 . $13\frac{1}{2}$ Ginger.
℔ s. ℔ s.
As 12 . $13\frac{1}{2}$ :: 1 . $1\frac{1}{8}$ Sugar.

24
27
———
168
48
———
648

ob.
54 a Pound of Pepper.
36 a Pound of Ginger.
27 a Pound of Sugar.

s.
48
24
———
192
96
———
1152 *ob.*
648
———
First Dividend 504

54—27=27 First Diviſor.
36—27= 9 Second Diviſor.

1
$2\cancel{3}$|8
$\cancel{5}\cancel{0}\cancel{4}$ (℔ 18 Pepper.
27

Second Dividend 18 (2 Ginger.
9

Complement 4 Sugar.
———
24

Alligation Alternate.

Alternate Alligation, to declare the due Proportion of every Ingredient entring the Mixture, doth alter or change the Places of ſuch Exceſſes or Differences as fall out between the mean Price and the Extreams, aſcribing that to the greater Extream which proceeds from the Leſſer, and the contrary. And for better direction, Lines (called *Lines of Combination*) are commonly drawn to link or tie together a Number greater than the common Price to one Leſſer; from which the Name of *Alligation* was firſt borrowed, and afterwards became common alſo to Queſtions of Mixture reſolved by *Medial Alligation*, though there be no ſuch tying or alligating the Numbers together, as in this called *Alternate*; which from the interchanging of the Differences, was added to that of *Alligation*, to diſtinguiſh the Species from that of *Medial*.

Lines of Combination, *what.*

Whence the Name of Alligation.

Theorems.

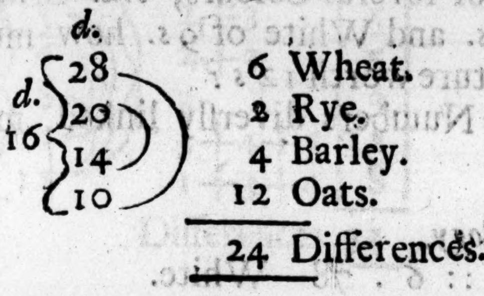
Neceſſary Theorems *to the Reſolution.*

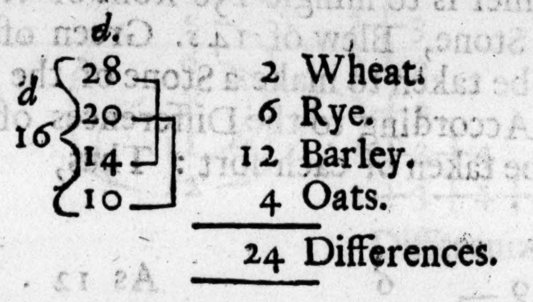
1. Let every greater Extream be linked with one leſſer.
2. When either of the Extreams be Single, and the other Extreams be Plural, the ſingle Extream muſt be linked to all the reſt.
3. If both greater and leſſer Extreams are not ſingle, then they may be linked ſo diverſly, that ſundry Differences may be taken, and diverſities of Anſwers to the Queſtion, yet all true. But if one of the Extreams be ſingle, there can be but one Anſwer.
4. The Numbers being linked, take the Difference of each Number from the mean or common Price, and place this Difference againſt the Number he is linked to alternately.
5. Every Number linked with more than one, muſt have all the Differences of the Numbers he is linked to ſet againſt him.
6. Thoſe Differences reſolve the Queſtion, when the Price of every of the Ingredients is given, without their Quantities, and the Demand be to mix them ſo as to ſell a certain Quantity at a mean Rate.
7. But when a Quantity of one, with the Prices of all the Ingredients is given, and the Demand is to know the Quantities of the other Ingredients, then the *Rule of Three* is to be uſed.
8. And when the Price of every Ingredient is given, without any of their Quantities, and the Demand be to make up a certain Quantity to be ſold at a mean Rate, Then all the Differences added together ſhall be the firſt Number in the *Rule of Three*; the whole Quantity to be mixed ſhall be the ſecond Number; and each Difference apart the ſeveral third Numbers: And ſo many Sorts mixed, ſo many Operations of the *Rule of Three*.
9. A Queſtion may be ſo propounded, as both ſorts of *Alligation* are needful to the Reſolution.

Examples, where the mean Rate is required according to the 6th Theorem. *Mean Rate required.*

A Merchant hath Wheat at 28 *d.* the Buſhel, Rye at 20 *d.* *Barley* at 14 *d.* and Oats at 10 *d.* and would mix the ſame ſo as a Buſhel of the Miſcellane may be ſold for 16 *d.* how much of each ſort muſt be taken? *Q. Of Miſcellane.*

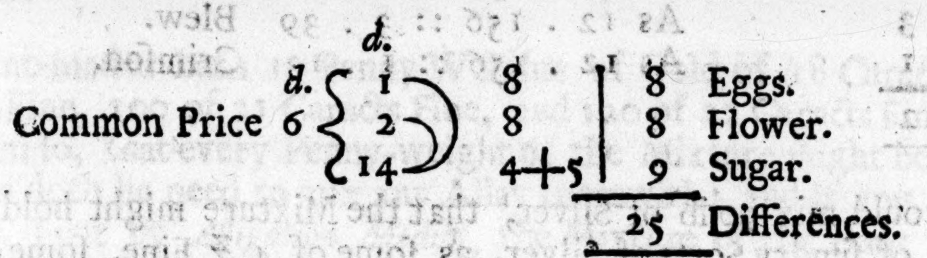
Anſw. Becauſe two of the Extreams are greater than 16, the common Price, and two are leſſer, the Numbers may be linked two ways, and the Mixture accordingly different: For either to 6 Buſhels of Wheat may be taken 2 of Rye, 4 of Barley, and 12 of Oats: or to 2 of Wheat may be taken 6 of Rye, 12 of Barley, and 4 of Oats. *Anſwer.*


d.
d.
16
28
20
14
10
6 Wheat.
2 Rye.
4 Barley.
12 Oats.
24 Differences.


d.
d
16
28
20
14
10
2 Wheat.
6 Rye.
12 Barley.
4 Oats.
24 Differences.

Sugar-Cakes are made with Sugar of 14 *d.* the Pound, Flower at 2 *d.* the Pound, and Eggs at 1 *d.* the Pound: what Quantities of each may be taken to make the Paſte worth 6 *d.* the Pound? *Q. Of Sugar-Cakes.*

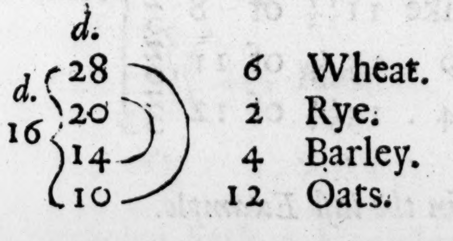
Anſw. Eggs and Flower of each 8 ℔, of Sugar 9 ℔; for the Differences of 1 and 2 from 6, are 4 and 5, which added make 9, belonging to the greater Extream, being ſingle, and ſo linked to both the leſſer Extreams. *Anſwer.*


d.
Common Price
d.
6
1
2
14
8
8
4+5
8
8
9
Eggs.
Flower.
Sugar.
25 Differences.

Examples, where the Quantities of ſome Ingredients are required, as in the 7th Theorem. *Quantities of ſome Ingredients required.*

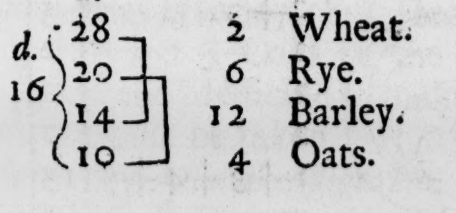
Ten Buſhels of Wheat at 28 *d.* the Buſhel, is to be mixed with Rye at 20 *d.* Barley at 14 *d.* and Oats at 10 *d.* how many Buſhels of thoſe other Sorts may be taken to afford a Buſhel of the Miſcellane at 16 *d*? *Q. Of Miſcellane.*

Anſw. The Extreams being Plural, and Numbers the ſame in the firſt Example above, according to the Differences ſituate by the different linking the Numbers, ſo ſhall the Anſwer be by help of the *Rule of Three*, in the manner following, according to both the above-mentioned Varieties. *Anſwer.*


d.
d.
16
28
20
14
10
6 Wheat.
2 Rye.
4 Barley.
12 Oats.

Analogy.

As 6 . 2 :: 10 . $3\frac{1}{3}$ Rye.
As 6 . 4 :: 10 . $6\frac{2}{3}$ Barley.
As 6 . 12 :: 10 . 20 Oats.


d.
d.
16
28
20
14
10
2 Wheat.
6 Rye.
12 Barley.
4 Oats.

As 2 . 6 :: 10 . 30 Rye.
As 2 . 12 :: 10 . 60 Barley.
As 2 . 4 :: 10 . 20 Oats.

One hundred Quarts of Canary at 12 *d.* the Quart, are to be mixed with Malaga at 9 *d.* and White-wine at 6 *d.* how many Quarts of the two latter muſt be taken to ſell a Quart of the Mixture at 10 *d*? *Q. Of Sack mixed.*

Anſw. Of each 40 Quarts: For one of the Extreams being ſingle, there can be no Variety in linking the Numbers. *Anſwer.*

	d.			
d. 10	12	1+4	5	Canary.
	9	2	2	Malaga.
	6	2	2	White.

Analogy.

As 5 . 2 :: 100 . 40 Malaga.

$$5\,)\,\frac{100}{200}\,(\,40$$

As 5 . 2 :: 100 . 40 White.

Quantities at a mean Rate required.

Examples, where the Quantities are required at a mean Rate, according to the 8th Theorem.

Q. Of Wool mixed.

A Clothier is to mingle 156 ſtone of Wool of ſeveral Colours, *viz.* Crimſon of 18 *s.* the Stone, Blew of 14 *s.* Green of 11 *s.* and White of 9 *s.* how much of each may be taken to make a Stone of the Mixture worth 12 *s.*?

Anſwer.

Anſw. According to the Differences of the Nuumbers diverſly linked, more or leſs may be taken of each ſort: Thus,

	s.	
s. 12	9	6
	11	2
	14	1
	18	3
		12

Analogy.

As 12 . 156 :: 6 . 78 White.
As 12 . 156 :: 2 . 26 Green.
As 12 . 156 :: 1 . 13 Blew.
As 12 . 156 :: 3 . 39 Crimſon.

	s.	
s. 12	9	2
	11	6
	14	3
	18	1
		12

As 12 . 156 :: 2 . 26 White.
As 12 . 156 :: 6 . 78 Green.
As 12 . 156 :: 3 . 39 Blew.
As 12 . 156 :: 1 . 13 Crimſon.

Q. Of Silver mixed.

A Goldſmith would mix 90 ℔ of Silver, that the Mixture might hold out 9 ℥ Fine; and taketh of ſundry Sorts of Silver, as ſome of 4 ℥ Fine, ſome of 5, of 6, of 8, with others of 11 and 12 ℥ Fine; how much of each ſort may be taken?

Anſwer.

Anſw. If the 11 ℥ be alligated to 4 and 5, and the 12 ℥ to 6 and 8, then muſt be taken of each of 4 and 5, the quantity of $7\frac{19}{23}$ ℔, and of each of 6 and 8 the Quantity of $11\frac{17}{23}$ ℔, and of 11 ℥ Fine $35\frac{5}{23}$ ℔, and of 12 ℥ Fine $15\frac{15}{23}$ ℔.

	℥		
s. 9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	4+5	9
	12	1+3	4
			23

Analogy.

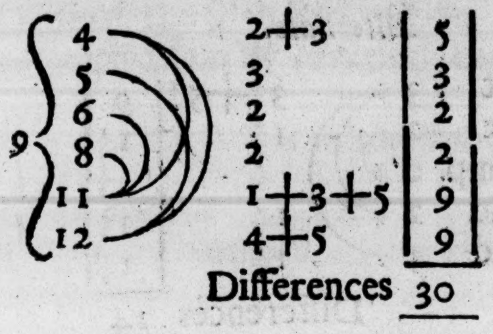
As 23 . 90 :: 2 . $7\frac{19}{23}$ of 4 ℥
The like $7\frac{19}{23}$ of 5 ℥
As 23 . 90 :: 3 . $11\frac{17}{23}$ of 6 ℥
The like $11\frac{17}{23}$ of 8 ℥ Fine.
As 23 . 90 :: 9 . $35\frac{5}{23}$ of 11 ℥
As 23 . 90 :: 4 . $15\frac{15}{23}$ of 12 ℥

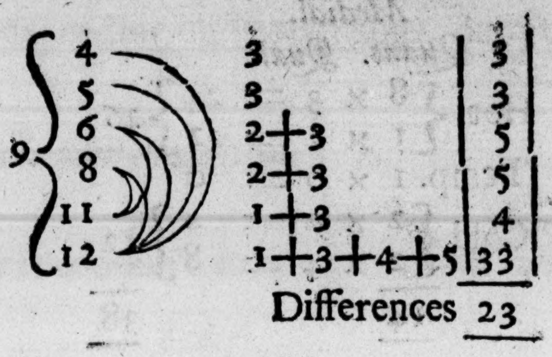
Data diverſly alligated.

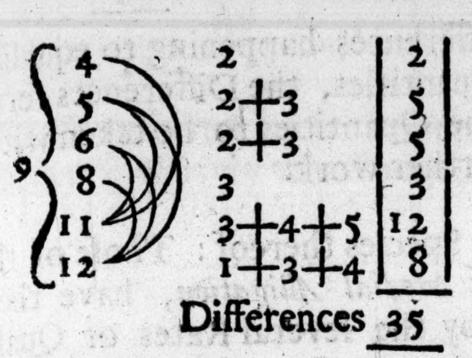
Other Varieties of alligating the Data *in the laſt Example.*

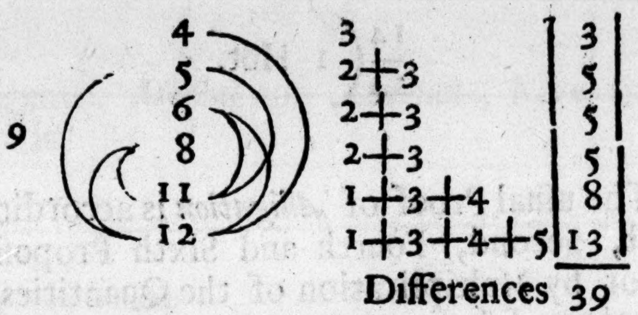
9	4	3	3
	5	3	3
	6	3	3
	8	2	2
	11	1	1
	12	3+4+5	12
		Differences	24

9	4	2+3	5
	5	2	2
	6	2	2
	8	2	2
	11	1+3+4+5	13
	12	5	5
		Differences	29









Beſides theſe, the Lines of Combination may be otherwiſe drawn, according as more or leſs of any one Ingredient is intended to be uſed: For by *Alternate Alligation* the Quality of a Mixture may be augmented or diminiſhed, or made finer or coarſer at pleaſure.

Examples, where both Sorts of Alligation *are needful, according to the ninth* Theorem.

Alligation of both Sorts uſed.

Q. Of Gold allayed.

A Mint-maſter hath 10 Penny Weights of Gold of 18 Caracts Fine, 20 of 19 Caracts Fine, 100 of 21 Caracts Fine, and 120 of 22 Caracts Fine; and he would mix them ſo, that every Penny-weight of the Mixture might be 20 Caracts Fine: whether doth he need to mix any Allay therewith; and if any, how much?

Anſwer.

Anſw. Firſt by *Alligation Medial*, the Mixture of the *Data* will be found $21\frac{1}{5}$ Caracts Fine, ſo as there muſt be ſome Allay. Then by *Alligation Alternate* is found, that for every 20 Penny-weights of the Gold muſt be taken $1\frac{1}{5}$ Penny-weight of Allay: So as for the whole 250 Penny-weights of Gold, muſt be 15 Penny-weights of Copper, or other Allay.

Medial.

10 × 18 = 180
20 × 19 = 380
100 × 21 = 2100
120 × 22 = 2640
25,0 — 530,0

(5
530 ($21\frac{1}{5}$ Fine.
25
25

Alternate.

$20\begin{cases} 21\frac{1}{5} & 20 \text{ Gold.} \\ 0 & 1\frac{1}{5} \text{ Allay.} \end{cases}$

As 20 . $1\frac{1}{5}$ :: 250 . 15

$1\frac{1}{5}$
250
50
20)300(15 Allay.

Q. Of compoſing a Medicine.

A Phyſician hath a Medicine compounded of Simples, Hot, Cold, and Temperate; that is to ſay, 8 ʒ Hot in the third Degree, 1 ʒ Hot in the Second, 1 ʒ Temperate; 2 ʒ Cold in the Second, and 2 ʒ Cold in the Fourth; and would compound the Medicine to make it Hot in $1\frac{1}{2}$ Degree: what Quantities of the Ingredients muſt be taken?

Anſwer.

Anſw. By *Medial Alligation*, the Temperament of the Medicine is found to be Hot iu the firſt Degree; and then as the Numbers be linked together by *Alternate Alligation*, the particular Quantities may be taken to make the Medicine of the deſired Temperament.

Medial.

$$\begin{array}{lccccc} & \textit{Quant.} & & \textit{Qual.} & & \\ \text{Hot} & \left\{\begin{array}{c} 8 \\ 1 \end{array}\right. & \begin{array}{c} \times \\ \times \end{array} & \begin{array}{c} 3 \\ 2 \end{array} & \begin{array}{c} = \\ = \end{array} & \left.\begin{array}{r} 24 \\ 2 \end{array}\right\} 26 \\ \text{Temp.} & 1 & \times & 0 & = & 0 \\ \text{Cold} & \left\{\begin{array}{c} 2 \\ 2 \end{array}\right. & \begin{array}{c} \times \\ \times \end{array} & \begin{array}{c} 2 \\ 4 \end{array} & \begin{array}{c} = \\ = \end{array} & \left.\begin{array}{r} 4 \\ 8 \end{array}\right\} 12 \\ \hline & 14 & & & & 38 \end{array}$$

$$\frac{14}{14}\Big(1 \text{ Hot.}$$

Alternate.

$$\begin{array}{rlc|c|} & \text{Hot } 3 & 3\tfrac{1}{2}+5\tfrac{1}{2} & 9 \\ & \text{Hot } 2 & 1\tfrac{1}{2} & 1\tfrac{1}{2} \\ 1\tfrac{1}{2} & \text{Temp. } 0 & \tfrac{1}{2} & \tfrac{1}{2} \\ & \text{Cold } 2 & 1\tfrac{1}{2} & 1\tfrac{1}{2} \\ & \text{Cold } 4 & 1\tfrac{1}{2} & 1\tfrac{1}{2} \\ \hline & & \text{Differences} & 14 \end{array}$$

The Differences happening to equalize the Quantities, the Differences ſerve for the Quantities to be taken without farther work.

Proof of Alligation.
Of the 1, 2, 4 and 6 Propoſitions Medial.

The uſual Proof of *Alligation* is according to the Species thereof: Thoſe of the Firſt, Second, Fourth and Sixth Propoſitions of *Medial Alligation*, have their Proof by Multiplication of the Quantities mixed by the ſeveral Rates or Qualities thereof before Mixture; and the whole Quantity or Quantities ſo mixed by the new Rate or Quality; which when the Work is right, will be both equal.

As in the firſt Example of the firſt Propoſition.

		s.					
						Buſhels 200	whole Quantity.
Rye	100 ×	4	= 400			4½ s.	new Rate.
Barley	40 ×	3½	= 140			800	
Wheat	60 ×	6	= 360			100	
	200		900 s.	=		900 s.	or 45 *l.*

And in the laſt Example of the ſecond Propoſition.

Mace.	*Cinamon.*	*Nutmegs.*	*Ginger.*		
21 ℔.	21 ℔.	21 ℔.	21 ℔.	21 ℔.	Quantity of each.
8⅓ s.	6⅔ s.	5 s.	2 s.	22 s.	Total Price.
168	126	105	42	42	
7	14			42	
175	140			462	

175 + 140 + 105 + 42 = 462 or 23 *l.* 2 *s.*

Of the third Medial, and of Alternate.

Thoſe of the third Propoſition in *Medial Alligation*, and thoſe alſo of *Alligation Alternate*, generally are proved as Operations in *Fellowſhip* before, by adding all the Quotients together, to return the ſecond Number or Total of the Quantities mixed. Neverthelesſ thoſe of *Alligation Alternate* may be proved, and it ſeems the better way, as others of the firſt Propoſition in *Alligation Medial*, by multiplying each Quotient by the Rate or Quality thereof before Mixture, to agree with the whole Quantity mixed, multiplied by the new Rate or Quality.

As in the firſt Example of the third Propoſition.

	Oil.		*Wax.*		*Ceruſe.*		*Camphire.*		*Total Quantity.*
Quotients	$118\frac{10}{17}$	+	$29\frac{11}{17}$	+	$59\frac{5}{17}$	+	$2\frac{8}{17}$	=	210 Second Number.

And in the firſt Example of the Work by the 8th Theorem.

	White.		*Green.*		*Blew.*		*Crimſon.*		*Total Quantity.*
Quotients	78	+	26	+	13	+	39	=	156 Second Number.

And by the Rates thus.

White	78 × 9 s.	= 702		156	Total Quantity.
Green	26 × 11	= 286		12 s.	New Rate.
Blew	13 × 14	= 182		312	
Crimſon	39 × 18	= 702		156	
	156	1872	=	1872	or 93 *l.* 12 *s.*

Thoſe

Of the fifth Medial.

Thoſe of the fifth Propoſition, and others depending on Proportions, admit of Proof with them by reverſing the Queſtion.

As in the firſt Example of the fifth Propoſition.

	Buſhel of Miſcellane		*Rye.*		*Buſhels of Miſcellane.*		*Rye.*
If	1	contain	$\frac{1}{8}$:	How much Rye is contain'd in	800 ?	*Anſw.*	100.

As 1 . $\frac{1}{8}$ :: 800 . 100 $\frac{800}{8}$(100

Of Both.

Thoſe reſolved by both Sorts of *Alligation*, *Medial* and *Alternate*, have their Proofs reſpectively where any Difference is.

CHAP. X.

Barter and Exchange.

Barter and Exchange, how they agree.

BOTH *Barter* and *Exchange* agreeing eſſentially, are placed together in this Chapter: For *Barter* is but an Exchange of Wares or Merchandiſes one for another: And Exchange a *Barter* of one ſort of Money or Merchandiſe for another; or the ſame Merchandiſe by the Accompt, Weight or Meaſure of another Countrey.

Barter how called.

Barter, (vulgarly called *Truck* and *Scoſing*) and the Concerns thereof relating to the Exchange of one Commodity for another, ſo as the Merchant may ſave his own, have Part in Money, or get ſome Overplus by the Bargain, may be compriſed under the 10 following *Caſes*.

10 Caſes thereof.

1. One Quantity for another.

Caſe 1. If the Price of both Commodities be given, to know how much of one Commodity may be given for any Quantity of the other.

Rule.

By the *Rule of Three*, get the Total Value of the Quantity to be exchanged, and afterward by another Operation of the ſame Rule, get the Quantity deſired.

Q. Of Wheat for Cloth.

Example. *A* and *B* barter; *A* hath 24 Broadcloths, at 10 *l.* the Piece; *B* hath Wheat at 5 *s.* the Buſhel: how much Wheat will pay for the Cloth?

Anſwer.

Anſw. Firſt the Value of the Broadcloths is found to be 240 *l.* then at 5 *s.* the Buſhel, 240 *l.* will buy 960 Buſhels: And ſo much Wheat ought *A* to have, or elſe will loſe by the Bargain.

	Cloth.		*l.*		*Cloths.*		*l.*		*s.*		*Buſhel.*		*l.*		*Buſhels.*
As	1	.	10	::	24	.	240	As	5	.	1	::	240	.	960
			24										20		
			240									5)	4800	(	960

2. Barter-price of one for the other.

Caſe 2. If the Price in ready-Money, and *Barter*-Price of one be given, to know the *Barter*-Price of the other, and how much at that Price of the one Commodity may be given for any Quantity of the other.

Rule.

See what is gained on the Shilling, Pound, Hundred, *&c.* by the one Party: then by the *Rule of Three* the Gain or Overplus of the other is found, as alſo the queſited Quantity.

Q. Of Sugar for Raiſins.

Example. *A* and *B* barter, *A* hath Raiſins at 30 *s. per* C. ready Money; but in *Barter* will ſell for 40 *s. per* C. *B* hath Sugar at 12 *d. per* ℔. ready Money, but would gain proportionally to the other: how therefore muſt *B* rate his Sugar in *Barter*? and how many ℔. of Sugar may he give for 4 C. of Raiſins?

Anſwer.

Anſw. The Gain found by the Queſtion to be 10 *s.* in 40 *s.* the firſt Queſtion ſtands thus: If 30 *s.* gain 10 *s.* what ſhall 1 *s.* which is the ready-Money Price of the Sugar? and by the Work 4 *d.* is gained: So muſt the *Barter*-price of the Sugar be 16 *d.* whereby the firſt Queſtion is anſwered. Then if 16 *d.* buy 1 ℔. of Sugar, 8 *l.* (the *Barter*-Price of 4 C. of Raiſins) ſhall buy 120 ℔. of Sugar; which anſwereth the ſecond Queſtion.

s. *s.* *s.* *s.* *s.* *s.* *s.* *s.*
As 30 . 10 :: 1 . 0$\frac{1}{3}$ or As 30 . 40 :: 1 . 1$\frac{1}{3}$

	s.		*s.* *d.*
A Ready-Money Price	30	*B* Ready-Money Price	1 . 0
Barter Price	40	Barter Price	1 . 4
Difference	10	Difference	0 . 4

C. *s.* C. *s.* *s.* ℔. *s.* ℔.
As 1 . 40 :: 4 . 160 As 1$\frac{1}{3}$. 1 :: 160 . 120 . Sugar.

3. Ready-Money Price of one for another. *Caſe* 3. If the Price in Ready-Money, and *Barter* of one Party with the *Barter*-price of the other be given, to know the Ready-Money Price of the other, and how much at that Price of the one will countervail a Quantity of the other Commodity.

Rule. Find by the *Rule of Three* the Ready-Money Price deſired; and by another Work of the ſame Rule the Quantity ſought.

Q. Of Salt for Wine. *Example. A* hath Salt at 4 *s.* the Buſhel Ready-Money; but in *Barter* will have 4 *s.* 6 *d.* and will exchange with *B* for Wine at 18 *l.* the Tun: how is the Wine rated in Ready-Money? and how much Salt ſhall *B* have for 3 Tuns of Wine?

Anſwer. *Anſw.* By reſolving the firſt Queſtion, 16 *l.* is found to be the Ready-Money Price of the Wine: And by reſolving the ſecond Queſtion, for 3 Tuns of Wine *B* ſhall receive 240 Buſhels of Salt.

s. *s.* *l.* *l.* *l.* *l.* *s.* *Buſhel.* *l.* *Buſhels.*
As 4$\frac{1}{2}$. 4 :: 18 . 16 Tuns 3 × 18 = 54 As 4$\frac{1}{2}$. 1 :: 54 . 240

20
360
4

$\frac{9}{2}$) $\frac{1440}{1}$ ($\frac{2880}{9}$ (32,0 / 16:0 *s.*

20
1080

$\frac{9}{2}$) $\frac{1080}{1}$ ($\frac{2160}{9}$ (240 Buſhels.

4. Gains on the 100, which the moſt. *Caſe* 4. If the Rates, both in ready Money and *Barter*, of both Parties be given; to know the Gains of each upon the Hundred, and which is the greateſt Gainer.

Rule. After the Gains of each Party on the 100 is found by the *Rule of Three*; ſubſtract the one from the other, and the Difference ſhall be the Gain of one Party above the other.

Q. Of Figs for Ginger. *Example. A* and *B* will *barter*: *A* hath Figs at 24 *s.* *per* C. ready Money; but in *Barter* will have 30 *s.* *B* hath Ginger at 4 *l.* 5 *s.* *per* C. but in *Barter* will have 4 *l.* 15 *s.* *per* C. how much did each gain on the 100 by the *Barter*? and which muſt have Money of the other to ballance the *Barter*, and how much?

Anſwer. *Anſw. A* will be found to gain 25 *l.* on the 100, and *B* but 11 *l.* 15 *s.* $\frac{5}{17}$; it follows therefore *A* is the greateſt Gainer. And to ballance the *Barter*, *B* muſt have of *A* one half of the Difference, 13 *l.* 4 *s.* 8 *d.* $\frac{8}{17}$.

s. *s.gain.* *l.* *l.gain.*
As 24 . 6 :: 100 . 25
20
2000
6
24) 12000 (50,0
25:0 *A* gains *per* 100 *l.*

	l.	*s.*	*d.*	
A .	25	0	0	gains.
B .	11	15	3$\frac{9}{17}$	gains.
	13	4	8$\frac{8}{17}$	Difference.
	6	12	4$\frac{4}{17}$	half.

l. *l.gain.* *l.* *l.gain.*
As 4$\frac{1}{4}$. $\frac{1}{2}$ :: 100 . 11$\frac{13}{17}$

$\frac{17}{4}$) $\frac{100}{2}$ ($\frac{200}{17}$ (11$\frac{13}{17}$ *B* gains *per* 100 *l.*
2 1

Caſe

5. Barter-Price and ready Money of one for the other.

Caſe 5. If the Rates, both in ready Money and *Barter*, of one Party be given, and he will have a part of his *Barter*-Price in ready Money; to know how the other Party may rate his Goods to be equal in the *Barter*.

Rule.

Subſtract the demanded Part from the *Barter*-Price, and the other Price, and with theſe two Remains, and the other Party's ready-Money Price, commit the Work to the *Rule of Three*.

Q. Of Kerſey for Linen.

Example. A hath Kerſeys at 15 *l.* ready Money, but in *Barter* will have 18 *l.* and beſides will have $\frac{1}{3}$ of his *Barter*-price in ready Money: *B* hath Linen at 3 *s.* *per* Ell ready Money: how ſhall *B* rate his Linen to be equal with *A*?

Anſwer.

Anſw. Taking 6, which is $\frac{1}{3}$ of 18, from 15 and 18, the Remains 9 and 12 are the firſt and ſecond Numbers of the *Rule of Three*, and 3 *s.* the Third; by the Work whereof it appears *B* muſt rate his Linen at 4 *s.* *per* Ell.

$\frac{1}{3}$ of 18 = 6 15 − 6 = 9
18 − 6 = 12

	l.		*l.*		*s.*		*s.*
As	9	.	12	::	3	.	4
	20		20				
	18,0 *s.*		240 *s.*				
			3				
			72,0				

$\frac{72}{18}\big($ 4 *s.* for 1 Ell of Linen.

6. Both Prices, and part Money, to gain a Sum on the 100.

Caſe 6. If both the Price in ready Money and *Barter* of one Party be given, and a Part of his *Barter*-Price in ready Money deſired; to know how the other may ballance the *Barter*, and gain a Sum on the 100.

Rule.

Take as in the laſt *Caſe* the Part deſired from both the Prices given, and find, by the *Rule of Three*, the Advance of the other Party's ready-Money Price, according to the Sum to be gained on the 100; this Number found, with the other Remains, commit to another Work of the *Rule of Three*.

Q. Of Stuffs for Stockins.

Example. A hath Stuffs which he rateth at 25 *s.* the Piece ready Money; but in *Barter* will have 30 *s.* and will have $\frac{1}{4}$ of his *Barter*-Price in ready Money. *B* hath Stockins at 40 *s.* the Dozen ready Money, and would ballance the *Barter*, and gain 10 *l.* *per Cent.* how ſhall *B* rate the Stockins in *Barter*?

Anſwer.

Anſw. Subſtracting 7 *s.* 6 *d.* which is $\frac{1}{4}$ Part of 30 *s.* from 25 and 30, the Remains are 17 *s.* 6 *d.* and 22 *s.* 6 *d.* And finding the Gain of 40 *s.* at the rate of 10 *l.* *per Cent.* to be 4 *s.* it appears by the *Rule of Three*, *B* muſt rate his Stockins at $2\frac{29}{35}$ *l.* the Dozen, or reduced lower at 2 *l.* 16 *s.* $6\frac{6}{7}$ *d.*

	s.		*s.*	*d.*
$\frac{1}{4}$ of	30	=	7 :	6

s.		*s.*	*d.*		*s.*	*d.*
25	−	7 :	6	=	17 :	6
30	−	7 :	6	=	22 :	6

	l.		*l.*		*l.*		*l.*
As	100	.	110	::	2	.	$2\frac{1}{5}$

$$100\,\big)\,\overset{2}{\overline{220}}\,\big(\,2\tfrac{1}{5}$$

	l.		*l.*		*l.*		*l.*	
As	$\frac{7}{8}$	.	$1\frac{1}{8}$	::	$2\frac{1}{5}$	.	$2\frac{29}{35}$	Stockins rated *per* Dozen.

$$\frac{7}{\underset{1}{8}}\Big)\frac{99}{\underset{5}{40}}\Big(\frac{99}{35}\Big(\,2\tfrac{29}{35}$$

7. Rates and ready Money of one, Gain on the 100 for the other.

Caſe 7. If beſides the different Rates given of one Party, he would gain a Sum on the 100, and have a Part ready Money; to know how the other Party ſhall ballance the *Barter*.

Rule.

Enquire, by the *Rule of Three*, the Gains upon the 100, after the *Barter*-Price; from which take the Part demanded in ready Money, as alſo from the *Barter*-Price; and with theſe two Remains, and the other's ready-Money Price, commit the Work to the *Rule of Three*.

Q. Of Peaſe for Flax.

Example. A and *B* will *barter*: *A* hath Peaſe at 2 *s.* *per* Buſhel ready Money; but in *Barter* will have 2 *s.* 6 *d.* and will gain 10 *l.* *per Cent.* and have $\frac{1}{3}$ of his over-price in ready Money. *B* hath Flax at 7 *d.* *per* ℔. how ſhall he rate his Flax to ballance the *Barter*?

Anſwer.

Anſw. At 10 *l.* *per* C. 2 *s.* 6 *d.* will be 2 *s.* 9 *d.* of which $\frac{1}{3}$ is 11 *d.* which taken from both, the other Queſtion will ſtand thus: If 19 *d.* ready Money make

22 *d.* in *Barter*; what will 7 *d.* in ready Money? The Anſwer to which is, $8\frac{2}{19}$ *d.* and ſo muſt *B* rate his Flax in *Barter*.

$$\begin{array}{rrrrrrr} & l. & l. & s. & d. & s. & d. \\ \text{As} & 100\;. & 110\;:: & 2 & :6\;. & 2 & :9 \\ & \underline{240} & & \underline{12} & & & \\ & \underline{240{,}00} & & 30 & & & \\ & & & \underline{110} & & & \\ & & & \underline{33{,}00} & & & \end{array} \qquad \frac{33}{240}\Big(\overset{l.}{\tfrac{11}{80}} \text{ or } \overset{s.}{2}:\overset{d.}{9}$$

$$\tfrac{1}{3} \text{ of } \overset{s.}{2} : \overset{d.}{9} \text{ or } \overset{d.}{33} = \overset{d.}{11}$$

$$\overset{s.}{2} : \overset{d.}{6} - 11 = \overset{d.}{19}$$
$$2 : 9 - 11 = 22$$

$$\text{As } \overset{d.}{19}\;.\;\overset{d.}{22}\;::\;\overset{d.}{7}\;.\;\overset{d.}{8\tfrac{2}{19}} \text{ The Rate in } \textit{Barter} \text{ of 1 ℔. of Flax.}$$

$$19\big)\overset{7}{\overline{154}}\big(8\tfrac{2}{19}$$

8. *Rates of one, Mony and Goods of the other.* *Caſe* 8. If the different Rates of one Party be given, and the other will have ſome Money and ſome Goods, to know the Quantity of the latter.

Rule. To the Total Value of the Price in ready Money and *Barter* ſeverally, add the Sum to be paid ready down; by theſe Totals, with the ready-Money Price of the other, will be found the *Barter*-Price, with which the Quantity of Goods by another Work of the *Rule of Three* will be had accordingly.

Q. *Of Aſhes for Allum.* *Example.* *A* hath 20 Tuns of Aſhes, at 54 *l.* the Tun ready Money; but in *Barter* rateth them at $55\frac{1}{2}$ *l.* *B* hath Allum at $12\frac{1}{4}$ *l.* the C. ready Money, and he will have 360 *l.* ready Money of *A*: how muſt *B* rate the Hundred of Allum in *Barter*? and how much Allum muſt he deliver for the ſaid 20 Tuns of Aſhes, and 360 *l.* in Money?

Anſwer. *Anſw.* The ready-Money Price of the Aſhes found to be 1080 *l.* and the *Barter*-price 1110 *l.* to both which 360 *l.* added, makes 1440 *l.* and 1470 *l.* And if 1440 *l.* give 1470 *l.* then ſhall $12\frac{1}{4}$ *l.* give $12\frac{97}{192}$ *l.* the *Barter*-Price of the Allum. Then if $12\frac{97}{192}$ *l.* be for 1 C. of Allum, 1470 *l.* ſhall be for $117\frac{27}{49}$ C. which is the Quantity deſired, and to be delivered for the 20 Tuns of Aſhes, with 360 *l.* in Money.

$$\left.\begin{array}{l} \quad\;\; \textit{Tun.}\;\; l. \quad\; \textit{Tun.} \quad l. \\ \text{As } 1\;.\;54\;\;::\;20\;.\;1080 \text{ Ready Money} \\ \text{As } 1\;.\;55\tfrac{1}{2}::\;20\;.\;1110\; \textit{Barter}\text{-Price} \end{array}\right\}\text{Aſhes.} \qquad \begin{array}{l} \\ 1080 + 360 = 1440 \\ 1110 + 360 = 1470 \end{array}$$

$$\text{As } \overset{l.}{1440}\,.\,\overset{l.}{1470} :: \overset{l.}{12\tfrac{1}{4}}\,.\,\overset{l.}{12\tfrac{97}{192}} \;\overset{\textit{Allum.}}{\textit{Barter}\text{-Price.}} \qquad \text{As } \overset{l.}{12\tfrac{97}{192}}\,.\,\overset{C.}{1} :: \overset{l.}{1470}\,.\,\overset{C.}{117\tfrac{27}{49}} \text{ Allum.}$$

9. *Time for delivery: how to rate the other.* *Caſe* 9. If the Propoſition include ſome Time for delivery of the Goods, and propounds the Rates of the one Party, to know how to rate the Goods of the other Party:

Rule. State the Queſtion for Reſolution by the *Rule of five Numbers*, and what is gotten thereby add to the given Price.

Q. *Of Wine for Sugar.* *Example.* *A* hath Wine at 24 *l. per* Tun in Money, but in *Barter* 27 *l.* to be delivered at 3 Months. *B* hath Sugar at 5 *l. per* Hundred in Money, to be delivered at 6 Months: how ſhall *B* rate his Sugar in *Barter*?

Anſwer. *Anſw.* At $6\frac{1}{4}$ *l. per* Hundred, for ſo it will be when $1\frac{1}{4}$ gotten by the Work is dded to the Price given.

$$\text{As } \underbrace{\overset{l.}{24}\;.\;\overset{\textit{Months.}}{3}}_{72}\;.\;\underbrace{\overset{l.}{3}\;::\;\overset{l.}{5}\;.\;\overset{\textit{Months.}}{6}}_{18}\;.\;\overset{l.}{1\tfrac{1}{4}}$$

$$\underline{72} \qquad \begin{array}{c} \underline{18} \\ \underline{90} \end{array} \qquad \begin{array}{c} (18 \\ \hline 90 \\ \hline 72 \end{array}\Big(\overset{l.}{1\tfrac{1}{4}} + \overset{l.}{5} = \overset{l.}{6\tfrac{1}{4}}$$

$$\left.\begin{array}{l} \;\;l. \\ 24 \text{ Ready-Money Price} \\ \underline{27 \; \textit{Barter}\text{-Price}} \\ \;\;\underline{3} \text{ Difference.} \end{array}\right\}\text{of the Wine.} \qquad \left.\begin{array}{l} \;\;l. \\ 5 \;\; \text{Ready-Money Price} \\ \underline{6\tfrac{1}{4} \; \textit{Barter}\text{-Price}} \\ \underline{1\tfrac{1}{4}} \text{ Difference.} \end{array}\right\}\text{of the Sugar.}$$

Caſe 10. If the Propoſition propound a Time for delivery with the *Barter*-Price of both, and the ready-Money Price of one Party, with part of his over-Price; to know the ready-Money Price of the other.

10. Time and Price of one, for the Price of the other.

proceed according to the State of the Queſtions by the ſeveral Caſes needful to the Reſolution, as in other-like mixture of Queſtions, *mutatis mutandis.*

Rule.

Example. A hath Hemp at 16 *l.* the C. in Money, in *Barter* at 20 *l.* the C. to be delivered at 4 Months, and will have $\frac{1}{4}$ of his *Barter*-Price in ready Money: *B* hath Saffron in *Barter* 10 *l.* $\frac{9}{11}$ *per* ℔. at 6 Months to be delivered: what is the Saffron worth in ready Money?

Q. *Of Hemp for Saffron.*

Anſw. 7 *l. per* ℔. For by the 5*th Caſe* the $\frac{1}{4}$ of the *Barter*-Price, which is 5 *l.* is taken from 16 and 20: Then the Difference of the two Remains with the Times gets a Number to be added to the ready-Money Price, after the $\frac{1}{4}$ Part is taken from it; which with the *Barter*-Price of the Saffron, the ready-Money Price of the Saffron is gotten by the third *Caſe.*

Anſwer.

l. *l.* *l.* *Mon.* *l.* *Mon.* *l.*
Ready Money 16 — 5 = 11 As 4 . 4 :: 6 . 6 11 + 6 = 17
Barter-Price 20 — 5 = 15
Difference 4 As 17 . 11 :: 10$\frac{9}{11}$. 7
$\frac{1}{4}$ of 20 = 5

10$\frac{9}{11}$
110
9
17)119(7*l.* Ready-Money Price of the } Saffron

Exchange of one ſort of Money or Merchandiſe for another, or the ſame by different Accompts, and the Neceſſaries thereof, proper for this Place, though placed by ſome under *Reduction*, as in *Reduction of Geodeticals* was before noted, may be compriſed under the two following *Caſes.*

Exchange. *Where placed by ſome. Two Caſes thereof.*

Caſe 1. If the Propoſition be ſingle, as to the Exchange of one ſort of Money or Merchandiſe for another; or the ſame Merchandiſe by different Weights, Meaſures, *&c.*

1. *Propoſal ſingle.*

When the Proportion of one to the other is not readily known, or cannot be reduced to Digits or ſmall Numbers, whereby the Parts may be eaſily taken, commit the Work to the *Rule of Three.*

Rule, *without Reduction.*

Example. A Merchant tranſporteth from *London* to *Bourdeaux* 13 C. Weight of Copperas, and the ſame is exchanged at *Bourdeaux* for 13 C. of Prunes their Weight: The Queſtion is, how much the Prunes weighed *London*-weight.

Q. *Of Copperas for Prunes.*

Anſw. Becauſe the Proportion between the Weights of *London* and *Bourdeaux* being as 104 to 94$\frac{3}{4}$, is not eaſy to be brought to ſmall Parts, therefore the Queſtion is committed to the *Rule of Three*, by which is gotten 1426$\frac{346}{379}$, which divided by 112 the Pounds in 1 C. *Averdupois*, there is found but 12 C. 82$\frac{346}{379}$ ℔ at *London.*

Anſwer.

Bourdeaux. *London.* *Bourdeaux.*
As 94$\frac{3}{4}$ ℔. . 104 ℔. :: 1300 ℔.
1300
31200
104
135200

$\frac{379}{4}$)$\frac{135200}{1}$($\frac{540800}{379}$(℔. 1426$\frac{346}{379}$ *London.*

When the Proportions are eaſy to be reduced to ſmall Numbers, the Reſolution may be had by *Practice*, or the *Rule of Three.*

Rule *with Reduction.*

Examples in Meaſure.

Examples in Meaſure.

A Merchant buyeth at *Antwerp* 1440 Ells of Silks, and ſelleth the ſame at *London*: how many Ells ſhall he make out by *London*-Meaſure?

Q. *Of Silks* Antwerp *Meaſure ſold at* London.

Anſw. Becauſe by the firſt Chapter of *Geodeticals* 100 Ells *Antwerp* make but 60 Ells *London*, the Proportion is ſoon eſpied to be as 5 to 3. If I multiply 1440 by 3, and divide the Product by 5; or elſe add $\frac{3}{5}$ of the Number together; or otherwiſe ſubſtract $\frac{2}{5}$ therefrom; either Way make 864, the Number of Ells at *London.*

Anſwer.

As

Antwerp. *London.* *Antwerp.* *London.* *Otherwiſe by* Practice.
As 5 Ells . 3 Ells :: 1440 Ells . 864 Ells. $\frac{1}{5}$ of 1440 = 288
3 3
5)4320(864 Ells 864 *London.*

As 100 . 60 :: 1440 . 864
60 $\frac{2}{5}$ = 576
100)864,00(864 864 Ells *London.*

London *Ells turned into* Antwerp.

On the contrary, to turn Ells *Engliſh* into Ells *Antwerp*, add $\frac{2}{3}$ of the Number thereto.

As if it were deſired to know how many Ells *Antwerp* 864 Ells *London* would make; $\frac{2}{3}$ thereof added thereto, make 1440 Ells *Antwerp*, as before.

864 Ells *London.*
$\frac{2}{3}$ { 288
288
1440 Ells *Antwerp.*

Q. Of Cloth, whether any loſt.
Anſwer.

A Merchant buyeth at *London* 738 Yards of Cloth, and at *Antwerp* maketh out but 980 Ells? whether did he loſe any by the Way?

Anſw. The Proportions being as 75 to 100, or 3 to 4 by the *Rule of Three*, or *Practice*, by adding $\frac{1}{3}$ of 738 thereto, there ſhould be 984 Ells made out at *Antwerp*; ſo as 4 Ells *Antwerp* were loſt.

London. *Antwerp.* *London.* *Antwerp.*
As 3 Yards . 4 Ells :: 738 Yards . 984 Ells. *Otherwiſe by* Practice.
4 738 Yards *London.*
3)2952(984 $\frac{1}{3}$ 246
984 Ells *Antwerp.*

As 75 . 100 :: 738 . 984
100
75)73800(984

Antwerp *Ells turned into* London *Yards.*

On the contrary, to turn Ells *Antwerp* into Yards *London*, the Proportion being as 4 to 3, take $\frac{3}{4}$ of the *Antwerp*-Meaſure, either by ſubſtracting $\frac{1}{4}$ thereof, or taking half the given Number, and adding thereto half that half.

As if it were deſired to know how many Yards *London* 984 Ells *Antwerp* would make: either 246, which is $\frac{1}{4}$ thereof, taken away, leaves the Remain 738; or $\frac{3}{4}$ of the given Number is taken, which is all alike.

984 Ells *Antwerp.*
$\frac{1}{4}$ 246
738 Yards *London.*

984 Ells *Antwerp.*
$\frac{1}{2}$ 492 }
$\frac{1}{4}$ 246 }
738 Yards *London.*

Examples in Money.

Examples in Money.

Q. Of Sterling *and* French *Money.*

A Merchant delivereth to the Exchangers at *London* 300 l. *Sterling*, after 54 *d.* the French Crown, (that is 3 *Liures*) and taketh a Bill to receive at *Paris* 60 *Sols Tournois* for every Crown: how much *Tournois* or French Money will pay the ſaid Bill?

Anſwer.

Anſw. By the *Rule of Three* 4000 *Liures*: and ſo by *Practice*, if 300 *l.* be brought into Shillings, and $\frac{1}{3}$ taken away, becauſe 54 *d.* to 3 *Liures* in leſs Terms, is as 18 *d.* to 1; that is, $1\frac{1}{2}$ *s.* for 1 *Liure.*

Sterl. *Liure.* *Sterl.* *Liures.*
As $1\frac{1}{2}$ *s.* . 1 :: 300 *l.* . 4000
20
6000

1 2000
$\frac{3}{2}$) $\frac{6000}{1}$ ($\frac{4000}{1}$

d. *Liures.* *l.* *Liures.*
As 54 . 3 :: 300 . 4000
240
72000
3
54) 216000 (4000

Otherwise by Practice.
l. *s.* *s.*
300 × 20 = 6000 *Sterling.*
$\frac{1}{3}$ 2000
4000 *Liures.*

If in *Sols* 80000
But in Crowns $1333\frac{1}{3}$

On the contrary, to turn French Money into English: So much as the *Liure* is valued above an English Shilling, convert into Parts one or more of a Shilling, and add thereto, and the Total is Shillings *Sterling*.

French Money turned into English.

As if 4000 *Liures* at 18 *d.* a piece were to be turned into *Sterling* Money, because 18 *d.* is $1\frac{1}{2}$ *s.* that is $\frac{1}{2}$ above a Shilling, $\frac{1}{2}$ of 4000 is added thereto: So is the Total 6000 *s.* or 300 *l.* as before.

But if the *Liure* be rated at 20 *d.* then 8 *d.* above a Shilling being $\frac{2}{3}$ of a Shilling, $\frac{2}{3}$ of 4000 is to be added thereto; and the Total will be $6666\frac{2}{3}$ *s.* or 333 *l.* 6 *s.* 8 *d.*

4000 *Liures.*
$\frac{1}{2}$ 2000 at 18 *d.* per *Liure.*
600,0 *s.*
l. 300:0 *Sterling.*

4000 *Liures.*
$\frac{2}{3}$ $\left\{\begin{matrix}1333\frac{1}{3}\\1333\frac{1}{3}\end{matrix}\right\}$ at 20 *d.* per *Liure.*
$666,6\frac{2}{3}$ *s.*
l. 333:6:8 *d.* *Sterling.*

A Merchant in *Spain* taketh up 300 Pieces of Eight, at 4 *s.* 4 *d.* a Piece, and payeth for the same in *London* 64 *l.* whether hath he paid his due?

Q. Of Spanish Money paid in London.

Answ. 300 Pieces of Eight, at 4 *s.* 4 *d.* amount to in *Sterling* Money 65 *l.* by the *Rule of Three*, or *Practice*, multiplying by the Shillings, and adding $\frac{1}{3}$ for the 4 *d.* So it appeareth he paid 20 *s.* too little.

Answer.

Piece. *Sterl.* *Pieces.* *Sterl.*
As 1 . $4\frac{1}{3}$ *s.* :: 300 . 65 *l.*
$4\frac{1}{3}$
1200
100
130,0 *s.*
l. 65:0 *s.* *Sterl.*

Piece. *d.* *Pieces.* *l.*
As 1 . 52 :: 300 . 65
300
15600
12
15600 (65 *l.* *Sterl.*
240

On the contrary, to turn English Money into Spanish, commit the Question to the *Rule of Three*, because the Proportions are as 60 to 13, uneasy Parts to take of some Numbers that may be given.

English Money turned into Spanish.

As if 65 *l.* *Sterling* were to be turned into Pieces of Eight, at 4 *s.* 4 *d.* the Piece, the Work would be thus.

s. *Piece.* *l.* *Pieces of 8.*
As $4\frac{1}{3}$. 1 :: 65 . 300
20
$\frac{13}{3}$ 1300

1 100
$\frac{13}{3}$) $\frac{1300}{1}$ ($\frac{300}{1}$ Pieces of Eight.

Case 2. If the Proposition be double, as to express or include two Questions therein:

2. Proposal double.

Rule.

Commit the Work of either to the *Rule of Three*, or *Practice*, as before, or both to the *Rule of five Numbers*, as the State of the Queſtion will admit.

Q. *Of Braces and Ells.*

Example. Seeing by *Geodeticals* 100 Ells of *Antwerp* are equal to 108 Silk Braces at *Venice*, and 100 Ells of *Antwerp* make 60 Ells at *London*: how many Braces *Venice* are in 3648 Ells *London*?

Anſwer.

Anſw. $6566\frac{2}{5}$ Braces, as by the following Works appear.

By the Rule of Three.

London. *Antwerp.* *London.* *Antwerp.*
As 3 Ells . 5 Ells :: 3648 Ells . 6080

$3648 \times 5 = \frac{18240}{3}(6080$

Antwerp. *Venice.* *Antwerp.* *Venice.*
As 100 Ells. 108 Braces :: 6080 Ells. $6566\frac{2}{5}$ Braces.

$6080 \times 108 = \frac{656640}{100}(6566\frac{2}{5}$

By the Rule of five Numbers.

Venice. *London.* *Antw.* *Lond.* *Antw.* *Venice.*
As 108 . 3648 . 100 :: 60 . 100 . $6566\frac{2}{5}$

108
29184
3648
393984,00

60,00

$\frac{393984}{6}(6566\frac{2}{5}$

Or omitting the ſuperfluous Numbers.

Lond. *Venice.* *Lond.* *Venice.*
As 60 . 108 :: 3648 . $6566\frac{2}{5}$

$\frac{393984}{6}(6566\frac{2}{5}$

Of Millan *Braces.*

But if the Demand had been of *Millan* Braces for Linen, and not for Silk; then becauſe 60 Ells *London*, or 100 Ells *Antwerp* are equal to 120 Braces uſed at *Millan* for Linen Cloth, there needed nothing but to double the 3648 Ells given, and 7296 had been the Braces deſired.

And on the contrary, to take ½ the *Millan* Braces for Linen, you have forthwith the *London* Ells.

Nevertheleſs in the Silk Braces at *Millan*, becauſe 141 of them anſwer to 60 Ells of *London*, which will not be reduced to ſmall Numbers, and ſo in others of like ſort, it is beſt to work by the *Rule of Three*, or *Five Numbers*, as aforeſaid.

Proof of Barter *and* Exchange.

All Queſtions of *Barter and Exchange*, receiving their Reſolution by the *Rule of Three*, *Practice*, *Specificks*, or *Rule of five Numbers*, will be accordingly proved: or the Queſtion may be reverſed, which will add to the Demonſtration.

CHAP. XI.

Loſs and Gain.

Loſs and Gain, *the Subject thereof.* *Four Caſes thereof.*

THE Title of this Chapter ſhews the Subject thereof, to converſe with Queſtions reſolving what *Loſs* or *Gain* may accrue by Traffick; and this in one of theſe four Caſes following.

1. Bare enquiry of either.

Caſe 1. When there is a bare enquiry of *Loſs* or *Gain*.

Either the Rates of buying and ſelling are given, to find the *Gain* or *Loſs* in general, or upon the Hundred.

Or on the contrary, to find the Rates of a Quantity, the *Gain* or *Loſs* of the Whole, or upon the Hundred is given.

How reſolved.

In all which, Reſolution is to be had by the *Rule of Three*, the *Data* being duly diſpoſed or prepared, according to the State of the Queſtion propounded.

Q. *Of* Loſs.

Example 1. I have 10 Yards of Cloth that coſt me 8 *l.* 5 *s.* which I ſell again for 15 *s.* the Yard: what do I loſe thereby?

Anſwer.

Anſw. 15 *s.* For by the *Rule of Three*, every Yard bought in, is found to coſt 16 *s.* 6 *d.* which is 1 *s.* 6 *d.* a Yard more than the ſelling Price, and this in 10 Yards amounts to 15 *s.*

Yards. *l.* *s.* *Yard.* *s.*
As 10 . 8 : 5 :: 1 . $16\frac{1}{2}$
20
10)165($16\frac{1}{2}$ *s.*

$16\frac{1}{2}$ *s.* Buying Price.
15 Selling Price.
$1\frac{1}{2}$ *s.* Loſs in 1 Yard.

10 Yards.
$1\frac{1}{2}$ *s.*
10
5
15 *s.* Total Loſs.

Example 2. If 1 Yard coſt 5 *s.* 4 *d.* and be ſold again for 6 *s.* 8 *d.* how much is gained on the 100?

Q. *Of* Gain *on the* 100.

Anſw. 25 *l.* For taking the buying Price from the ſelling Price, the Gain of 1 Yard is had, the reſt appeareth by the Work of the *Rule of Three.*

Anſwer.

s. *d.*
6 : 8
5 : 4
1 : 4 Gain.

s. *Gain.* *l.* *Gain.*
As $5\frac{1}{3}$. $1\frac{1}{3}$ *s.* :: 100 . 25 *l.*
$\frac{16}{3}$

100
20
2000
$1\frac{1}{3}$
2000
$666\frac{2}{3}$
$2666\frac{2}{3}$

$\frac{16}{3}$)$\frac{8000}{3}$($\frac{500}{1}$ = 50,0 / *l.* 25 : 0 *s.* Gain on the 100 *l.*

Example 3. If I pay 34 *l.* for 560 Yards of Cloth, and would gain 17 *l.* 6 *s.* 8 *d.* thereby: how muſt I ſell 1 Yard?

Q. *Of the Price of a Yard.*

Anſw. For 22 *d.* Here adding the Gain to the buying Price, the Total 51 *l.* 6 *s.* 8 *d.* is the ſecond Number of the *Rule of Three*, the other Numbers are the Yards given.

Anſwer.

l.
34
17 : 6 : 8
51 : 6 : 8

Yards. *l.* *Yard.*
As 560 . $51\frac{1}{3}$ :: 1 . $\frac{11}{120}$ *l.* or 22 *d.*

$\frac{560}{1}$)$\frac{154}{3}$($\frac{11}{120}$ *l.* Price of 1 Yard.
(40, 11)

Example 4. If I buy Cloth at 7 *s.* 6 *d.* the Ell, and it proving worſe than expected, I am reſolved to loſe 5 in the 100: how muſt I rate the Cloth an Ell?

Q. *Of the Price of an Ell.*

Anſw. 7 *s.* $1\frac{1}{2}$ *d.* For if 100 *l.* loſe 5 *l.* then 7 *s.* 6 *d.* ſhall loſe $4\frac{1}{2}$ *d.* which taken from 7 *s.* 6 *d.* leaves 7 *s.* $1\frac{1}{2}$ *d.*

Anſwer.

l. *Loſs.* *l.* *Loſs.*
As 100 . 5 *l.* :: $\frac{3}{8}$. $\frac{3}{160}$ *l.* in 1 Yard.

$\frac{100}{1}$)$\frac{15}{8}$($\frac{3}{160}$ × 240 = $\frac{720}{160}$($4\frac{1}{2}$ *d.*
(20, 3) (8)

s. *d.*
7 : 6 Buying Price.
$4\frac{1}{2}$ Loſs.
7 : $1\frac{1}{2}$ Selling Price.

Caſe 2. When the Enquiry is of *Loſs* or *Gain* with Time.

The *Data* duly diſpoſed, Reſolution is to be had by the *Rule of five Numbers.*

2. Loſs *or* Gain *with Time, how reſolved.*

Example 1. If 1 Ell of Holland coſt 2 *s.* 6 *d.* and it be ſold for 2 *s.* 8 *d.* to be paid at the end of four Months: what after that rate is gained upon the 100 in 12 Months?

Q. *Of* Gain *in* 12 *Months.*

Anſw. 20 *l.*

Anſwer.

s. *d.*
2 : 8 Selling Price.
2 : 6. Buying Price.
0 : 2 Gain.

l. *Mon.* *d.* *l.* *Mon.* *l.*
As $\frac{1}{8}$. 4 . 2 :: 100 . 12 . 20
$\frac{1}{2}$
24
$\frac{1}{2}$)$\frac{2400}{1}$($\frac{4800}{240}$ *d.*(20 *l.*

Example 2. If a parcel of Goods that coſt 15 *l.* be ſold again for 14 *l.* 16 *s.* $10\frac{1}{2}$ *d.* to be paid at the end of 3 Months: whether at that rate is there loſt 6 in the 100 for 12 Months?

Q. *Of* Loſs *in* 3 *Months.*

Anſw.

Anſwer. *Anſw.* No, but 4 *l.* 3 *s.* 4 *d.*

l. : *s.* : *d.*
15 : 00 : 00 Buying Price.
14 : 16 : $10\frac{1}{2}$ Selling Price.
00 : 3 : $1\frac{1}{2}$ Loſs.

$$\begin{array}{lllllll} & l. & Mon. & s. & l. & Mon. & l. \\ \text{As} & \underbrace{15\ .\ 3}_{45} & . & 3\frac{1}{8} & :: & 100\ .\ 12 & .\ 4\frac{1}{6} \end{array}$$

$37\frac{1}{2}$

$45)\overline{3750}(\ 8{,}3\frac{1}{3}$ *s.*

l. 4 : 3 : 4 *d.*

3. Loſs or Gain, with Allowance or Rebatement, how reſolved. *Caſe* 3. When the Enquiry is of *Loſs* or *Gain* with Allowance, or Rebatement in the Weight; both paſſing commonly by the Names of *Tare*, *Tret* and *Cloff.*

Addition for the Allowance, and *Subſtraction* for the Rebatement being made, the reſidue of the Work is by the *Rule of Three*, or *Practice.*

Q. Of the Hundred with Tret. *Example* 1. At 14 *l.* the 100 Suttle, what will 890 ℔. Suttle be worth, giving 6 ℔. weight upon every 100 for Tret?

Anſwer. *Anſw.* $117\frac{29}{53}$ *l.* For ſo is the Reſolution, adding the Allowance, and working by the *Rule of Three.*

℔.
100 Suttle.
6 Tret allowed.
106 Sum.

℔. *l.* ℔. *l.*
As 106 . 14 :: 890 . $117\frac{29}{53}$
890
1260
112
12460

(5
280(8
$\frac{12460}{106}$(*l.* $117\frac{29}{53}$

Q. Of the Hundred with Tret. *Example* 2. At 2 *s.* 6 *d.* the Pound, what ſhall 780 ℔. be worth, allowing 4 ℔. Tret on the groſs Hundred?

Anſwer. *Anſw.* $94\frac{4}{29}$ *l.* For by the firſt Work of the *Rule of Three*, or *Practice*, the Value of 1 C. at the rate of 2 *s.* 6 *d.* the ℔. is found to be 14 *l.* And then by another Work of the ſame Rule is gotten $94\frac{4}{29}$ *l.*

℔.
112 Great Hundred.
4 Tret allowed.
116 Sum.

At 2 *s.* 6 *d.* *per* ℔. what coſts ℔. $\frac{112}{8}$(14 *l.*
$\frac{1}{8}$

℔. *l.* ℔. *l.*
As 116 . 14 :: 780 . $94\frac{4}{29}$
14
3120
780
$116)\overline{10920}($ *l.* $94\frac{4}{29}$

Q. Of the Hundred rebating for Tare and Cloff. *Example* 3. If 100 ℔ be worth 38 *s.* what will 860 ℔ be worth, rebating 5 ℔ upon every 100 for Tare and Cloff?

Anſwer. *Anſw.* 15 *l.* $10\frac{23}{50}$ *s.* For according to that Abatement there is but 817 ℔ to be accompted for; which at the rate of 38 *s.* for 100, gives the ſaid Sum.

℔.
100 Suttle.
5 Tare rebated.
95 Remain.

℔. ℔. ℔. ℔.
As 100 . 95 :: 860 . 817
95
4300
7740
$100)\overline{81700}($817 ℔.

℔. *s.* ℔. *l.* *s.*
As 100 . 38 :: 817 . 15 : $10\frac{23}{50}$
38
6536
2451
$100)\overline{31046}($ $31{,}0\frac{23}{50}$
15 : $10\frac{23}{50}$

Example 4. If a Merchant ſell two Baskets of Raiſins weighing 183 ℔, and 159 ℔, for 26 *s.* the Hundred, allowing 3 ℔ Tare on each Basket: what do the Raiſins amount to?

Q. Of Baskets of Raiſins with Tare.

Anſw. 3 *l.* 18 *s.* For ſo it appears by the *Rule of Three*, the Tare being firſt deducted.

Anſwer.

℔.
183
159
342 Weight in Groſs.
6 Tare rebated.
336 Weight *Netto.*

℔. *s.* ℔. *l.* *s.*
As 112 . 26 :: 336 . 3 : 18
26
2016
672
112)8736(7,8 *s.*
l. 3:18 *s.*

Caſe 4. When the Enquiry is of *Loſs* or *Gain*, with different Allowance or Abatement.

The *Gain* or *Loſs* of one Side being gotten, by ſome or other of the Ways foregoing, compare it with the other.

4. Loſs or Gain with different Allowance or Rebatement, how reſolved.

Example 1. Whether doth he loſe more that giveth 5 ℔ upon the 100, or he that rebateth 5 ℔ upon the 100?

Q. Of the moſt Loſs, by giving or abating 5 on 100.

Anſw. Becauſe he that giveth 5 ℔ upon the 100, giveth 105 ℔ for 100 ℔; and he that rebateth 5 ℔ upon the 100, giveth 100 ℔ for 95 ℔: The Queſtion therefore may be ſet thus; If 105 ℔ be given for 100 ℔, what ſhall 100 ℔ be delivered for? The Anſwer to which being $95\frac{5}{21}$ ℔, it appeareth that he who rebateth 5 ℔ on the Hundred is the Loſer, by ſo much as $95\frac{5}{21}$ ℔ is greater than 95 ℔, that is $\frac{5}{21}$ for the other makes $95\frac{5}{21}$ ℔ of the Hundred.

Anſwer.

100 ℔
5 Allowance.
105 Sum.
100 ℔
5 Rebatement.
95 Remain.

℔. ℔. ℔. ℔.
As 105 . 100 :: 100 . $95\frac{5}{21}$
100
105)10000($95\frac{5}{21}$
95 Hundred with Rebate.
$\frac{5}{21}$ Difference.

Example 2. *A* oweth *B* 600 *l.* to be paid in 3 Months; and *B* oweth *A* 500 *l.* to be paid in 4 Months: But if *A* will clear the Score preſently, *B* offereth to take 98 *l.* Which of them, conſidering the Intereſt, loſeth, and how much?

Q. Of the Sum loſt in paying two Debts.

Anſw. *B* loſeth 3 *l.* For accounting the Intereſt of 600 *l.* for 3 Months, it is 9 *l.* And the Intereſt of 500 *l.* for 4 Months is 10 *l.* accounting the Intereſt at 6 *per Cent.* which abated from the reſpective Sums, 591 *l.* is left for *A* to pay, and 490 *l.* for *B* to pay; the Difference is 101 *l.* due from *A* to *B*: For which if *B* take 98 *l.* he loſeth 2 *l.* of the Principal, and 1 *l.* of the Intereſt.

Anſwer.

l. *Mon.* *l.* :: *l.* *Mon.* *l.*
As 100 . 12 . 6 :: 600 . 3 . 9 *A* Intereſt.
:: *l.* *Mon.* *l.*
:: 500 . 4 . 10 *B* Intereſt.

l. *l.* *l.*
A oweth 600 — 9 = 591
B oweth 500 — 10 = 490 *Paid.* *Loſt.*
Due to *B* 101 — 98 = 3

According to the Reſolution of the Propoſitions in *Loſs* or *Gain*, either by the *Rule of Three*, *Practice*, *Specificks*, or *Rule of five Numbers*, ſo ſhall be the Proof: And if any Doubt ariſe, work the Queſtion reverſed in any of the Caſes.

Proof of Loſs and Gain.

CHAP. XII. *Equation of Paiment.*

Equation of Paiment in 3 Caſes.

SOmetime, as well on the Sale of Goods as Loan of Monies, divers Days or Times of *Paiment* are appointed; and upon a new Agreement, the Debtor willing to be diſcharged, deſireth to know when the whole Sum to be paid may be paid all at once, the Intereſt duly accounted: How therefore the ſeveral Days of paiment may be equated and brought all into one, is the Work of this Chapter, and the Contents thereof may be reduced to three *Caſes*.

1. Time ſingly propounded. **Rule.**

Caſe 1. When the Time of Paiment is ſingly propounded.

Multiply the Sums to be paid by the ſeveral Times of Paiment, and the Total of the Products divide by the whole Sum to be paid.

Q. Of 90 l. to be paid at once, tho due at ſeveral Times.

Example 1. A Debtor owing 90 *l.* agreed with his Creditor to pay it at three ſeveral Days, *viz.* 45 *l.* at 4 Months end, 30 *l.* at 8 Months end, and the remaining 15 *l.* at 10 Months end; but receiving Money unexpectedly, was willing to pay all at one Paiment: what Time muſt then be given him?

Anſwer.

Anſw. 6⅓ Months.

$$\begin{array}{lccccccc} & l. & & l. & & l. & & \\ \text{Sums to be paid} & 45 & + & 30 & + & 15 & & \\ \text{Times of Paiment} & 4 & & 8 & & 10 & & \\ \hline \text{Products} & 180 & + & 240 & + & 150 & = & \overset{(3}{\cancel{5}7(0} \\ & & & & & \text{Whole Debt} & & \cancel{9}\ \cancel{0} \end{array} \Big(\; 6\tfrac{1}{3} \text{ Months.}$$

Reſolved by Fractions.

Or if the ſeveral Sums to be paid be propounded in Fractions, or elſe be reduced into Fractionary Parts of the whole Debt, then multiply after the manner of Fractions, and add the Products into one Total.

As becauſe 45 *l.* is ½ of 90, and 30 *l.* is ⅓, and 15 *l.* is ⅙; therefore ½ multiplied by 4, and ⅓ by 8, and ⅙ by 10, ſhall be together 6⅓ as before.

$$\frac{45}{90} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2}\ (2$$
$$\frac{30}{90} = \frac{1}{3} \times \frac{8}{1} = \frac{8}{3}\ (2\tfrac{2}{3}$$
$$\frac{15}{90} = \frac{1}{6} \times \frac{10}{1} = \frac{10}{6}\ (1\tfrac{2}{3}$$
$$\underline{\overline{6\tfrac{1}{3}}}\ \text{Months.}$$

Q. Of 500 l. ſome due preſent, and ſome at other Times, when to be paid at once.

Example 2. *A* is indebted to *B* 500 *l.* to be paid as followeth, *viz.* 100 *l.* preſent, 260 *l.* at 6 Months, and 140 *l.* at 9 Months: when are theſe Paiments on Time to be paid at once? or when ſhall the whole Debt of 500 *l.* be paid together?

Anſwer.

Anſw. At 7$\frac{1}{20}$ Months, if 100 *l.* be paid preſent; but otherwiſe at the end of 5$\frac{16}{25}$ Months the whole 500 *l.* ſhall be paid.

By Integers.

$$\begin{array}{lccccccc} & l. & & l. & & l. & & \\ \text{Sums to be paid} & 100 & + & 260 & + & 140 & & \\ \text{Times of Paiment.} & \text{Preſent.} & & 6 & & 9 & & \\ \hline & & & 1560 & + & 1260 & = & 282|0 \\ & & & & \text{Remaining Debt} & & & \cancel{4}0|0 \end{array} \Big(\; 7\tfrac{1}{20} \text{ Months.}$$

$$\text{Whole Debt} \quad \frac{\overset{(3}{28(2|0}}{\cancel{5}\ 0|0} \Big(\; 5\tfrac{16}{25} \text{ Months.}$$

By Fractions.

Preſent 100 *l.*

$$\frac{260}{400} = \frac{13}{20} \times \frac{6}{1} = \frac{78}{20}\ (3\tfrac{18}{20} \qquad \frac{260}{500} = \frac{13}{25} \times \frac{6}{1} = \frac{78}{25}\ (3\tfrac{3}{25}$$
$$\frac{140}{400} = \frac{7}{20} \times \frac{9}{1} = \frac{63}{20}\ (3\tfrac{3}{20} \qquad \frac{140}{500} = \frac{7}{25} \times \frac{9}{1} = \frac{63}{25}\ (2\tfrac{13}{25}$$
$$\text{Months } 7\tfrac{1}{20} \qquad\qquad \text{Months } 5\tfrac{16}{25}$$

Example 3. A Debt of 600 *l.* was to be paid thus; 200 *l.* preſent, $\frac{1}{3}$ at 8 Months, and $\frac{1}{5}$ at 10 Months, and the Reſidue at the Year's End: when may this Money be all paid together?

Anſw. At $6\frac{4}{15}$ Months.

Q. Of 600 l. due, ſome preſent, ſome afterward, when to be paid at once.

Anſwer.

By Integers.

$$\begin{array}{lcl}
\text{Whole Debt} & 600\ l. & \\
\frac{1}{3} & 200 & 200 \times 8 = 1600 \\
\frac{1}{5} & 120 & 120 \times 10 = 1200 \\
\text{Reſt } \frac{2}{15} & 80 & 80 \times 12 = 960
\end{array}$$

$$\frac{3760}{200 + 200 + 120 + 80 = 600}\Bigg(6\frac{4}{15}\ \text{Months.}$$

By Fractions.

Preſent 200 *l.*

$$\frac{200}{600} = \frac{1}{3} \times \frac{8}{1} = \frac{8}{3}\ (2\frac{2}{3}$$
$$\frac{120}{600} = \frac{1}{5} \times \frac{10}{1} = \frac{10}{5}\ (2$$
$$\frac{80}{600} = \frac{2}{15} \times \frac{12}{1} = \frac{24}{15}\ (1\frac{3}{5}$$
$$6\frac{4}{15}\ \text{Months.}$$

Caſe 2. When the Time of Paiment is propounded with *Loſs* or *Gain*.

Equate the Times of Paiment as before, and then by the *Rule of five Numbers* enquire for the *Gains* or *Loſs*.

2. *Time with* Loſs *or* Gain. Rule.

Example 1. A Merchant buyeth Silks at 10 *s.* the Yard, and ſelleth the ſame at 12 *s.* the Yard, giving 2 Days of Paiment; *viz.* 4 Months for the one Half, and 8 Months for the other: what doth he gain on the 100 in 12 Months?

Q. Of Silks ſold on Time, what Gain.

Anſw. 40 *l.* For the Times of Paiment equated make 6 Months: And then if 10 *s.* in 6 Months gain 2 *s.* an hundred Pounds in 12 Months ſhall gain 40 *l.*

Anſwer.

$$\frac{1}{2} \times \frac{4}{1} = \frac{4}{2}\ (2$$
$$\frac{1}{2} \times \frac{8}{1} = \frac{8}{2}\ (4$$
$$6\ \text{Months.}$$

$$\begin{array}{l}
\qquad\quad l.\ \ Mon.\ \ s.\qquad\quad l.\quad Mon.\quad l. \\
\text{As } \underbrace{\tfrac{1}{2}\ .\ 6}_{3}\ .\ \underbrace{2\ ::\ 100\ .\ 12}_{24}\ .\ 40
\end{array}$$

$$3\Big)\frac{24}{2400}\Bigg(\frac{80|0\ s.}{40{:}0\ s.}$$

Example 2. A Merchant buyeth 20 Cloths, at 6 *l.* the Cloth ready Money; and afterwards ſelleth 5 of them for 7 *l.* the Cloth to be paid at 4 Months; and the other 15 he ſelleth for 8 *l.* the Cloth, and giveth 6 Months Time for the Paiment: how much is gained thereby on the 100 in 12 Months?

Q. Of Cloth, ſome for ready Money; and ſome on Time, what Gains

Anſw. $63\frac{7}{86}$ *l.* For the Money paid for the Cloths bought being 120 *l.* and to be paid for them ſold being 155 *l.* there is 35 *l.* difference; and the Times equated being $5\frac{17}{31}$ Months; by the *Rule of five Numbers* is obtained $63\frac{7}{86}$ *l.*

Anſwer.

$$\begin{array}{llllll}
\text{Cloths bought} & 20 & \text{Cloths ſold} & 5 & + & 15 \\
\text{Price of one} & 6\ l. & \text{Price of one} & 7\ l. & & 8\ l. \\
 & 120 & & 35 & + & 120 = 155
\end{array}$$

$$\begin{array}{llll}
\text{Sums to be paid} & 35 & + & 120 \\
\text{Times of Paiment} & 4 & & 6 \\
 & 140 & + & 720 = \dfrac{8\cancel{60}}{\cancel{155}}\overset{(85}{}\Bigg(5\frac{17}{31}\ \text{Months.}
\end{array}$$

$$\begin{array}{l}
\qquad\ \ l.\qquad\ Mon.\qquad\ l.\qquad\quad l.\qquad Mon.\qquad l. \\
\text{As } \underbrace{120\ .\ 5\tfrac{17}{31}}\ .\ \underbrace{35\ ::\ 100\ .\ 12}_{420}\ .\ 63\tfrac{7}{86}
\end{array}$$

$$\frac{20640}{31}\Bigg)\frac{42000}{1}\Bigg(\frac{130200}{2064}\Bigg(63\frac{7}{86}\ l.$$

Q. Of Cloves ſold on Time, Gain on the 100.

Example 3. A Merchant buyeth Cloves at 8 *s.* the Pound ready Money: how ſhall he ſell the hundred Weight thereof to gain, after the Rate of 10 *l.* on the 100 for a Year, and be paid $\frac{1}{2}$ at 3 Months, and the reſt at 6 Months end?

Anſwer.

Anſw. For 46 *l.* 9 *s.* 7$\frac{1}{5}$ *d.* where the Times being equated for one Paiment, make 4$\frac{1}{2}$ Months; And the Rate of 1 C. at 8 *s.* the ℔, being found to be 44 *l.* 16 *s.* the Ready-money Price, Then by the *Rule of five Numbers* is found 1 *l.* 13 *s.* 7$\frac{1}{5}$ *d.* to be added to the Ready-money Price.

Paiments $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}\ (1\frac{1}{2}$

$\frac{1}{2} \times \frac{6}{1} = \frac{6}{2}\ (3$

$4\frac{1}{2}$ Months

℔. *s.* ℔. *l.* *s.*

As 1 . 8 :: 112 . 44 : 16

4

l. 44|8 Primes.

l. *Mon.* *l.gain.* *l.* *Mon.* *l.* *s.* *d.*

$$\text{As } \underbrace{100 \ .\ 12}\ .\ 10 :: \underbrace{44\tfrac{4}{5}\ .\ 4\tfrac{1}{2}}\ .\ 1 : 13 : 7\tfrac{1}{5}$$

1200

$$1200\,\Big)\,\frac{45}{2016}\Big(\overset{l.}{1\tfrac{17}{25}} \times 20 = \frac{\overset{(1}{\overset{9(5}{\cancel{3}\cancel{4}\cancel{0}}}}{\cancel{2}\cancel{5}}\Big(\overset{s.}{13\tfrac{3}{5}} \times 12 = \frac{\overset{(1}{\cancel{3}\cancel{6}}}{\cancel{5}}\Big(7\tfrac{1}{5}\,d.$$

	l.	*s.*	*d.*	
Ready-money Price	44	16	00	
Gain to be added	1	13	07$\frac{1}{5}$	
	46	09	07$\frac{1}{5}$	Selling Price of the Hundred.

3. *Time ſooner for Part, and later for other Part.* Rule.

Caſe 3. When the Time of Paiment is anticipated for Part of the Debt, and procraſtinated for the Reſidue.

Subſtract the Paiment or Paiments from the whole Debt, and if the Times of Paiment be more than one, equate them; and this equated Time take from the Time of Paiment, and commit the Work to the *Rule of Three:* But if one Time be propounded, ſubſtract the Time of Paiment from the Time when to be paid, and the Sum paid from the whole Debt, and work with the Remainers. Or inſtead of the Difference of the Time equated or not, if the Times of Paiment be ſubſtracted from the Time when the Whole was to be paid, and the Remains be equated, this may be taken to work with.

Q. Of 480 *l. if* 280 *paid before the Day, when the reſt.*

Example 1. *A* is indebted to *B* 480 *l.* to pay it in 5$\frac{1}{2}$ Months; and at 4 Months end he will pay 280 *l.* upon Condition that *B* will ſtay (accounting by the Intereſt) for the other 200 *l.* as long after the Time as this 280 *l.* is paid before the Time: when ſhall the 200 *l.* be paid?

Anſwer.

Anſw. 2$\frac{1}{10}$ Months after the Expiration of 5$\frac{1}{2}$ Months.

	l.		*Months.*
Whole Debt	480	to be paid in	5$\frac{1}{2}$
Paiment	280	made in	4
Reſidue	200	Difference	1$\frac{1}{2}$

l. *Months.* *l.* *Months.*

$$\text{As } \underbrace{200}\ .\ 1\tfrac{1}{2} :: 280\ .\ 2\tfrac{1}{10}$$

$$\begin{array}{r} 1\tfrac{1}{2} \\ \hline 280 \\ 140 \end{array}$$

$$20\,\Big)\,\underline{42,0}\Big(2\tfrac{1}{10} \text{ Months.}$$

Q. Of 240 *l. ſome paid before due, when the reſt.*

Example 2. A Merchant oweth 240 *l.* to be paid in 6 Months; but 1$\frac{1}{2}$ Month paſt he payeth 60 *l.* and 3 Months after that he payeth 80 *l.* more: in what Time ſhall he pay the Reſt, conſidering his Time was 6 Months?

Anſwer.

Anſw. 3$\frac{9}{10}$ Months after the end of 6 Months.

l. *Months.* *Months.*

Whole Debt 240 to be paid in 6 6

$$\text{Paiments} \left\{\begin{array}{c} 60 \\ 80 \end{array}\right\} \text{ in } \left\{\begin{array}{l} 1\tfrac{1}{2}\times 60 = 90 \\ 4\tfrac{1}{2}\times 80 = 360 \end{array}\right\}\begin{array}{l}(3 \\ \frac{4\cancel{5}(\cancel{0}}{\cancel{1}\cancel{4}\cancel{0}}\Big(\ 3\tfrac{3}{14} \text{ Time equated.}\end{array}$$

Reſidue 100 Paiments

$2\frac{11}{14}$ Difference.

$$\begin{array}{llcll} & l. & \textit{Months.} & l. & \textit{Months.}\\ \text{As} & 100 & .\ 2\tfrac{11}{14}\ :: & 140 & .\ 3\tfrac{9}{10}\\ & & & 2\tfrac{11}{14} & \\ \hline & & & 280 & \\ & & & 110 & \\ \hline & & & 10)39{,}0(3\tfrac{9}{10} & \text{Months.} \end{array}$$

Q. *Of* 300 *l. paid, part before due; when the reſt.*

Example 3. A Merchant is indebted 300 *l.* to be paid the 24*th* of *May*; whereof he payeth the 29*th* of *April* 80 *l.* and the 9*th* of *May* after 120 *l.* upon what Day ſhall he pay the remaining 100 *l?*

Anſwer.

Anſw. 38 Days after the 24*th* of *May*, which will be the firſt Day of *July*.

$$\begin{array}{lll} \text{Whole Debt } 300\,l. \text{ to be paid in } 25 \text{ Days.} & & 25 \text{ Days.}\\ \text{Paiments} \left\{\begin{array}{ll} 80 & \text{Preſent}\\ 120 & \text{in} \end{array}\right. & \begin{array}{r} 10\times120 = 1200\\ \text{Paiments } 200 \end{array} & \Big(\begin{array}{l} 6 \text{ Time equated.}\\ \hline 19 \text{ Difference.} \end{array}\\ \text{Reſidue } \ \ 100 & & \end{array}$$

$$\begin{array}{llcll} & l. & \textit{Days.} & l. & \textit{Days.}\\ \text{As} & 100 & .\ 19\ :: & 200 & .\ 38\\ & & & 19 & \\ & & & 100)38{,}00(38 & \text{Days.} \end{array}$$

How otherwiſe wrought.

Or becauſe 80 *l.* was paid *April* 29, which was 25 Days before *May* 24, when the Whole was to be paid; and 120 *l.* being paid *May* the 9*th*, which was 15 Days before *May* 24; if 25 and 15 be equated with the Paiments, the Time equated will be 19 Days, and the Reſolution as above.

$$\begin{array}{l} \text{Whole Debt } 300\,l. \text{ to be paid in } 25 \text{ Days, that is from } \textit{April } 29, \text{ to } \textit{May } 24.\\ \text{Paiments} \left\{\begin{array}{l} 80 \text{ before } \overset{\textit{Days.}}{25}\times 80 = 2000\\ 120 \text{ before } 15\times120 = 1800 \end{array}\right\} \begin{array}{r} 3800\\ \text{Paiments } 200 \end{array} \Big(\begin{array}{l} \textit{Days}\\ 19 \text{ before the Time.} \end{array}\\ \qquad \text{As } 100 \ .\ 19 \ ::\ 200\ .\ 38. \end{array}$$

Proof of Equation of Paiment.

The Operations of the firſt *Caſe* are to be proved thus: Rate the Intereſt of the ſeveral Sums to be paid, according to the Times of Paiment, and the Total thereof ſhall be equal to the Intereſt of the Total Sum accounted to the Time equated.

Of the firſt Caſe.

As in the firſt *Example* of the firſt *Caſe*, the Intereſt reckoned at 6 *l. per Cent. per Annum*, in both is found 57 *s.*

$$\begin{array}{l} \begin{array}{lllll} & l. & \textit{Mon.} & l. & \\ \text{As} & 100 & .\ 12 & .\ 6 & :: \end{array} \begin{array}{ccc} l. & \textit{Mon.} & s.\\ 45 & .\ 4 & .\ 18\\ 30 & .\ 8 & .\ 24\\ 15 & .\ 10 & .\ 15 \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right\} \text{Intereſt of the ſeveral Paiments in their Times.}\\ \text{As } 100\ .\ 12\ .\ 6\ ::\ 90\ .\ 6\tfrac{1}{3}\ .\ \underline{57} \ \ \text{Intereſt of the Total Paiment at the Time equated.} \end{array}$$

Of the ſecond Caſe.

In the Operations of the ſecond *Caſe*, ſo far as concerns the Equations of the Paiments, is to be proved as the firſt. And what further refers to the *Rule of Three*, or *Rule of five Numbers*, admits of their Proof.

Of the third Caſe.

The Operations of the third *Caſe* are to be proved much like thoſe of the Firſt: For account the Intereſt of the whole Sum till the Time of Paiment thereof to be made, and this ſhall be equal with the Intereſt of the ſeveral Sums paid, reckoned till the Times of their Paiments.

As in the firſt *Example* of the third *Caſe*, rating the Intereſt at 6 *l.* in the Hundred, the Sum found for both is 13 *l.* 4 *s.*

$$\begin{array}{l} \quad\ \ l.\ \ Mon.\ \ l. \\ \text{As } 100 \,.\, 12 \,.\, 6 :: \end{array} \left.\begin{array}{l} \quad l.\ \ Mon.\ \ l.\ \ s. \\ 280 \,.\, 4 \,.\, 5 : 12 \\ \\ 200 \,.\, 7\tfrac{3}{5} \,.\, 7 : \underline{12} \end{array}\right\} \begin{array}{l} \text{Intereſt of } 280\,l. \text{ for 4 Months, and} \\ \quad 200\,l. \text{ for } 7\tfrac{3}{5} \text{ Months (that is } 5\tfrac{1}{2} \\ \quad \text{and } 2\tfrac{1}{10}\text{) when paid.} \end{array}$$

$$\text{As } 100 \,.\, 12 \,.\, 6 :: 480 \,.\, 5\tfrac{1}{2} \,.\, 13 : \underline{\ 4\ } \quad \begin{array}{l} \text{Intereſt of } 480\,l. \text{ the whole Debt} \\ \quad \text{to } 5\tfrac{1}{2} \text{ Months.} \end{array}$$

CHAP. XIII. *Factorſhip.*

Factorſhip.

Queſtions relating to Merchants and Factors, cloſe up the third Sort of Derivatives, and are treated of in this Chapter under the Title of *Factorſhip*, but with ſome placed under *Fellowſhip*.

How placed by ſome.
What to be noted therein.

In *Factorſhip* are to be minded two things.

Firſt, The Eſtimation of the Factor.

Secondly, The Partition of the Gains between the Merchant and the Factor, according to their Agreement or Bargain.

Eſtimation of the Factor, what.
How to be accounted, with or without Stock.

Eſtimation of the Factor, is an Allowance to the Factor for his Pains or Imploiment, to countervail ſome part of the Merchant's Stock. And when the Factor layeth in no Stock, his Eſtimation is in ſuch *Ratio*, or Proportion to the Merchant's Stock, as the Gains of the ſaid Factor are to the Gains of the Merchant. But when the Factor layeth in Stock with the Merchant, then, after the whole Eſtimation of the Perſon of the Factor with his Stock is valued according to the Merchant's Stock, the Factor's Stock is to be deducted, and the Remain is the Eſtimation of his Perſon.

Examples of both.

Q. Of the Factor's Eſtimation without Stock.
Anſwer.

1. A Merchant delivereth unto his Factor 900 *l.* to govern in the Trade of Merchandiſe, upon Condition that he ſhould have $\frac{1}{3}$ of the Gain: what is his Eſtimation eſteemed at?

Anſw. 450 *l.* For that Sum beareth the ſame *Ratio* to 900 *l.* which $\frac{1}{3}$ doth to $\frac{2}{3}$, the Reſidue of the Gains belonging to the Merchant.

$$\begin{array}{lllllll} & & l. & & & & l. \\ \text{As} & \frac{2}{3} & .\ 900 & :: & \frac{1}{3} & . & 450\,. \end{array}$$

$$\frac{2}{3}\Bigg)\,\dfrac{\frac{1}{3}}{\dfrac{300}{1}}\Bigg(\dfrac{900}{2}\Bigg(\overset{l.}{450} \text{ Eſtimation.}$$

Q. Of the Factor's Eſtimation with Stock.
Anſwer.

2. A Merchant delivered unto his Factor 600 *l.* and the Factor layeth in Stock therewith 250 *l.* beſides his Perſonal Eſtimation; therefore it is agreed he ſhall have $\frac{2}{5}$ of the Gain: what is the Eſtimation of his Perſon?

Anſw. 150 *l.* For the whole Eſtimation of his Stock and Perſon is 400 *l.* from which 250 *l.* the Factor's Stock deducted, leaves 150 *l.* for the Eſtimation of his Perſon.

$$\begin{array}{lllllll} & & l. & & & & l. \\ \text{As} & \frac{3}{5} & .\ 600 & :: & \frac{2}{5} & . & 400 \end{array}$$

$$\frac{3}{5}\Bigg)\,\dfrac{\frac{2}{5}}{\dfrac{240}{1}}\Bigg(\dfrac{1200}{3}\Bigg(\begin{array}{ll} l. & \\ 400 & \text{Whole Eſtimation.} \\ \underline{250} & \text{Factor's Stock.} \\ \underline{150} & \text{Factor's Perſonal Eſtimation.} \end{array}$$

Partition of the Gains how accounted at, or out of the ſending of the Merchant.

Partition of the Gains enquired, propounds the Eſtimation or Agreement between the Merchant and his Factor: And ſometimes accounts the Eſtimation according to the Merchant's Stock, called *Eſtimation* upon, or at the ſending of the Merchant. And ſometimes the Gain is to be parted proportionally, according to the Merchant's Stock and the Factor's Eſtimation added together, called *Eſtimation* out of the ſending of the Merchant.

Examples

Examples of both.

Q. *Of Gain parted to the Merchant and Factor.*

1. A Merchant putteth into his Factor's Hands to improve in the way of Merchandiſe 800 *l.* upon condition that the ſaid Factor ſhall have $\frac{1}{4}$: And after certain Time they found in Profit 135 *l.* 6 *s.* 8 *d.* how much ſhall the Factor have thereof?

Anſwer.

Anſw. Becauſe $\frac{1}{4}$ is underſtood to be $\frac{1}{4}$ of the Gains; the Gains ſhall be divided, $\frac{3}{4}$ to the Merchant, and $\frac{1}{4}$ to the Factor: That

		l.	*s.*	*d.*
$\frac{3}{4}$	To the Merchant	101	10	00
$\frac{1}{4}$	To the Factor	33	16	08
	Total Gains	135	06	08

But then his Eſtimation is but $\frac{1}{3}$, becauſe $\frac{1}{4}$ of the whole Gains is but $\frac{1}{3}$ of $\frac{3}{4}$ the Merchant's Part.

And if the Factor's Eſtimation were $\frac{1}{4}$, then muſt he have but $\frac{1}{5}$ of the Gains, and the Merchant $\frac{4}{5}$; and ſo muſt the Gains be parted thus,

	l.	*s.*	*d.*
To the Merchant	108	05	04
To the Factor	27	01	04
	135	06	08

The Reaſon whereof is, becauſe if his Eſtimation had been but $\frac{1}{4}$, then the $\frac{1}{4}$ of 800 *l.* being 200 *l.* had made the 800 *l.* to be 1000 *l.* of which $\frac{1}{5}$ is equal to $\frac{1}{4}$ of 800 *l.* the Merchant's Stock.

Q. *Of Gains how parted to the Merchant and Factor.*

2. A Merchant delivereth to his Factor 800 *l.* upon Condition that his Factor ſhall have the Profit of 160 *l.* as though he laid in ſo much ready Money: what Portion of the Gains ſhall the ſaid Factor take up for himſelf at the Reckoning?

Anſwer.

Anſw. If it be intended that the ſaid 160 *l.* ſhall be reckoned as Stock to the other 800 *l.* then the whole Stock maketh 960, of which 160 is $\frac{1}{6}$, and ſuch part of the Gains ſhall the Factor have.

But if the Intent of their Covenants between them were, that the Factor ſhould have the Gains of 160 *l.* of the 800 *l.* then ſhall the Factor take $\frac{1}{5}$ of the Gains; for 160 to 800, is as 5 to 1.

Examples mixt.

Mixt Examples.

Q. *Of Stock laid in by the Factor.*

1. A Merchant puts into his Factor's Hands 390 *l.* to trade with, and the Factor's Eſtimation is accounted 60 *l.* how much Money muſt the Factor put in Stock, that he may have $\frac{1}{4}$ of the Gain?

Anſwer.

Anſw. 70 *l.* For $\frac{1}{4}$ of the Gain is underſtood of the whole Gain, Eſtimation and all accounted thereto: So as $\frac{3}{4}$ Gain for the Merchant coming of 390 *l.* the $\frac{1}{4}$ Gain for the Factor muſt ariſe of 130 *l.* that is 60 *l.* Eſtimation, and 70 *l.* Stock.

		l.			*l.*	
As	$\frac{3}{4}$.	390	::	$\frac{1}{4}$.	130	
					60	Factor's Eſtimation.
					70	Factor's Stock.

Q. *Of Gains how to be parted, and of the Factor's Eſtimation.*

2. A Merchant hath 400 *l.* to trade with in Merchandiſe; and agreeth with a Factor, that if he put in 90 *l.* to the Stock, then he ſhall have $\frac{1}{5}$ for his Pains. Afterward another Merchant deſireth to be a Copartner, and putteth in Stock 350 *l.* and promiſeth to obſerve the ſame Agreement made with the Factor: when the Trade is over they find 250 *l.* gained; how muſt the Gains be divided? and what is the Factor's Pains eſteemed at?

Anſwer.

Anſw. The Factor's Eſtimation being valued according to the firſt Merchant's Stock, is 110 *l.* his Stock 90 *l.* which together with the Stock of both Merchants, make 950 *l.* by which the Gains are to be divided as in *Fellowſhip.*

$\text{As } \frac{2}{3} \;.\; 400\,l. \;::\; \frac{1}{3} \;.\; 200\,l.$

90 Factor's Stock.

110 Factor's Eſtimation.

	Stock.					
A .	400 *l.*					
B .	350				*l.*	
Factor	200	*l.gain.*		400 .	$105\frac{5}{19}$ *A*	Gain divided.
As	950	. 250	::	350 .	$92\frac{2}{19}$ *B*	
				200 .	$52\frac{12}{19}$ Factor	

But if the Factor's Eſtimation had been valued according to both the Merchants Stocks at $\frac{1}{3}$, then his 90 *l.* Stock and Eſtimation had been together 375 *l.* and his Stock ſubſtracted, his Eſtimation would have been 285 *l.* and the Gain divided by this Account, the Factor will have almoſt as much as the Firſt, and more than the ſecond Merchant.

$\text{As } \frac{2}{3} \;.\; 750\,l. \;::\; \frac{1}{3} \;.\; 375\,l.$

90 Factor's Stock.

285 Factor's Eſtimation.

	Stock.					
A .	400 *l.*					
B .	350				*l.*	
Factor	375	*l.gain*		400 .	$88\frac{8}{9}$ *A*	Gain divided.
As	1125	. 250	::	350 .	$77\frac{7}{9}$ *B*	
				375 .	$83\frac{3}{9}$ Factor	

Proof of Factorſhip.

The Proof of nothing here need be ſpoken to, except the finding of the Eſtimation; foraſmuch as all the other Operations of this Chapter depend on *Proportions*, as the *Rule of Three*, *Fellowſhip*, &c. the Proof whereof hath been ſet forth at large where they have been handled.

In particular of finding the Eſtimation.

To prove the finding of the Eſtimation, add the Eſtimation to the whole Stock, as well of the Factor, if any be, as of the Merchant's; and take the Part thereof which the Factor was to have, and it will agree with the Eſtimation found: But where the Factor had Stock, this Part taken ſhall include his Stock with his Eſtimation, otherwiſe his bare Eſtimation only.

As in the firſt and laſt *Examples* of this Chapter.

Firſt,			*Laſt.*		
Stock	900 *l.*	Merchant.	Stock	400 *l.*	Merchant.
Eſtimation	450	Factor.	Stock	90	Factor.
$\frac{1}{3}$ Part)	1350	(450 *l.*	Eſtimation	110	Factor.
			$\frac{1}{3}$ Part)	600	(200 *l.* Stock and Eſtimation.

CHAP. XIV. *Falſhood or Poſition.*

Falſhood or Poſition.

THE Comparative Elements of the third Sort of Derivatives done with, yet remains for this Chapter the handling of the fourth Sort, (which have ſome peculiar Operation requiſite to their Reſolution) called *Falſhood* or *Poſition.*

Why ſo called.

Falſhood is not ſo called, becauſe it teacheth any Deceit, but for that by falſe Poſitions, or erroneous Numbers taken to work with, the Truth is found out. And though in ſome other of the foregoing Elements, the Numbers wrought with were not the true, yet were they in juſt Proportion to the True; but here they are taken at adventure; and if examining thereby the Queſtion, according to the Tenor thereof, your Poſition fall true, the Queſtion is anſwered: but becauſe ſometime falſe, it gave the Rule the Name of the *Rule of falſe Poſition.*

Of two ſorts.

This *Rule of falſe Poſition*, or *Falſhood*, is twofold, *viz.* *Single* and *Double.*

Single Poſition

How known.

The firſt is called *Poſition Single*, becauſe ſometimes at one working the Queſtion propounded may be reſolved, which happens when there is in the Propoſition ſome partition of Numbers in Parts proportional.

The

What to be done.

The Proceſs herein is thus; Imagine a Number at pleaſure, and proceed to work therewith according to the Purport of the Queſtion, as if it were the true Number; and what Proportion there is between the falſe Concluſion, and the falſe Poſition, ſuch Proportion hath the given Number to the Number ſought: Therefore the Number found by Argumentation ſhall be the firſt Number of the *Rule of Three*, and the Hypothetical or ſuppoſed Number ſhall be ſet in the ſecond Place, and in the third Place ſhall be the given Number.

Q. Of a Ship bought, what each paid.

Example 1. *A*, *B*, and *C*, conſent together to buy a Ship for 220 *l.* ſo that *B* muſt pay twice ſo much as *A*, and *C* four times ſo much as *B*: How much muſt each Man pay?

Anſwer.

Anſw. I ſuppoſe *A* paid 8 *l.* then muſt *B* pay 16 *l.* and *C* four times ſo much, which is 64 *l.* But all theſe Numbers added together make no more than 88 *l.* and there ſhould be 220 *l.* yet by this Number ſuppoſed I proceed to work, ſaying,

If 88 *l.* come of 8 *l.* of what comes 220 *l*?

Where in the Work is gained 20 *l.* for *A*; then muſt *B* pay 40 *l.* and *C* 160 *l.* all which added together, produce 220 *l.* the Number propounded.

l. *Poſition falſe.* *l.* *Concluſion true.*
As 88 . 8 *l.* :: 220 . 20 *l.* *A*

Poſition 8
Double 16
Quadruple 64
88

220
88)1760(20

20 *l.* *A*
40 . *B*
160 . *C*
220 Proof.

Q. Of Taffety and Sattin, the Price.

Example 2. A Mercer buyeth 30 Yards of Taffety, and 40 Yards of Sattin for 330 *s.* but every Yard of Sattin coſt twice ſo much as a Yard of Taffety: what did a Yard of Taffety coſt?

Anſwer.

Anſw. Suppoſe a Yard of Taffety coſt 4 *s.* then muſt a Yard of Sattin coſt 8 *s.* at which Rates 30 Yards of Taffety will coſt 120 *s.* and 40 Yards of Sattin 320 *s.* but 120 and 320 make 440, which ſhould be but 330. I therefore ſay, As 440 . 4 :: 330 . 3 . So is 3 *s.* gotten for a Yard of Taffety, and then the Yard of Sattin ſhall coſt 6 *s.*

s. *Poſition.* *s.* *Concluſion.*
If 440 come of 4 *s.* of what comes 330? *facit* 3 *s.*

4
Poſition 4 Doubled 8 440)1320(3 *s.* doubled 6 *s.*

Taffety 30 Sattin 40 Taffety 30 Sattin 40
120 + 320 = 440 Proof 90 + 240 = 330 *s.*

If the Queſtion have Fractions.

If the Queſtion hath a Fraction or more therein, it is beſt, for more facility in proceeding, to take ſuch a Number for the *Poſition* as may be equally parted by the Parts expreſt in the Queſtion.

Q. Of a Captain and his Party, how many.

Example 1. A Captain ordered upon Service with a Party, being demanded how many Souldiers he had in his Party? anſwered, that $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{4}{5}$ of them added together, made 245: how many Men had he?

Anſwer.

Anſw. I ſuppoſe 30, (a Number that will be equally parted by 2, 3, and 5) and adding the $\frac{1}{2}$ which is 15, with the $\frac{1}{3}$ which is 10, and $\frac{4}{5}$ which is 24, find but 49, and it ſhould be 245: Therefore tho I have erred in ſuppoſing, yet I commit the Work to the *Rule of Three*, and obtain 150 the Number ſought.

Men. *Poſition.* *Men.* *Concluſion.*
If 49 come of 30: of what comes 245? *facit* 150.

245
Poſition 30 49)7350(150 Souldiers.
Half 15
Third 10
4 Fifths 24
49

$\frac{1}{2}$ 75
$\frac{1}{3}$ 50
$\frac{4}{5}$ 120
245 Proof.

Q. Of Sheep left when ſome gone.

Example 2. A Man having 1000 Sheep ſold at one Time, $\frac{1}{2}$ ſo many as he now hath, and at another Time $\frac{1}{3}$ ſo many; and at another Time loſt $\frac{1}{4}$ ſo many: The Queſtion is, how many Sheep he hath yet remaining?

Anſwer.

Anſw. Suppoſe he had 12 Sheep left, the Half which is 6, added to $\frac{1}{3}$ which is 4, and $\frac{1}{4}$ which is 3, make with the 12 remaining but 25 inſtead of 1000; then by the Analogy is found 480 Sheep to be left.

As 25 . 12 :: 1000 . 480 . Sheep left.

$$\begin{array}{r} 1000 \\ 25)\overline{12000}(480 \end{array}$$

Poſition	12	$\frac{1}{2}$	240	. Sold.
$\frac{1}{2}$	6	$\frac{1}{3}$	160	. Sold.
$\frac{1}{3}$	4	$\frac{1}{4}$	120	. Loſt.
$\frac{1}{4}$	3		1000	. Proof.
	25			

If a Number propounded may be left or taken out of the Whole.

When in the Queſtion ſome Number is propounded to be left or taken out of the Whole, then the Parts propoſed may be added together as Fractions, and taken out of the whole Integer: Or the whole Numbers by Argumentation ſubſtracted out of the *Poſition*, and the reſt ſerve for the Analogy as before; only in the former Way the Number ſo propoſed to be left or taken out of the Whole, ſhall be the ſecond Number.

Q. Of a Man's Eſtate.

Example 1. A Man having ſpent 50 *l.* had yet $\frac{3}{4}$ and $\frac{1}{5}$ of his Eſtate remaining: what was his Eſtate at firſt?

Anſwer.

Anſw. 1000 *l.* For if I add $\frac{3}{4}$ and $\frac{1}{5}$, they make $\frac{19}{20}$, which taken from $\frac{20}{20}$, the whole Subſtance, there remaineth $\frac{1}{20}$; which if it be 50 *l.* ſhall make the Whole 1000 *l.*

Or if I ſuppoſe 20 *l.* then $\frac{3}{4}$ is 15, and $\frac{1}{5}$ is 4; which added are 19, and ſubſtracted from 20, the *Poſition* leaves 1; And if 1 come of 20, then 1000 ſhall come of 50.

$$\frac{3}{4} + \frac{1}{5} = \frac{19}{20} \qquad \text{If } \frac{1}{20} \text{ be } \overset{l.}{50} \text{ what ſhall } 1\,? \; \textit{facit } \overset{l.}{1000}$$

$$\frac{20}{20} - \frac{19}{20} = \frac{1}{20} \qquad \frac{1}{20})\frac{50}{1}(\frac{1000}{1}$$

$\frac{3}{4}$	750
$\frac{1}{5}$	200
Spent	50
	1000 Proof.

Otherwiſe.

Poſition	20
$\frac{3}{4}$	15
$\frac{1}{5}$	4
Total	19
Remain	1

As 1 . $\overset{l.}{20}$ :: 50 . $\overset{l.}{1000}$

20

1000 *l.*

Q. Of Stock, what it was.

Example 2. One having ſpent $\frac{2}{3}$ and $\frac{1}{5}$ of his Stock, had only 36 *l.* remaining; what was his Stock?

Anſwer.

Anſw. 270 *l.* Here the Fractions added make $\frac{13}{15}$, which taken from the whole $\frac{15}{15}$, leaves $\frac{2}{15}$; and this being equal to 36, makes the whole 270.

Or by ſuppoſing 15, then $\frac{2}{3}$ is 10, and $\frac{1}{5}$ is 3, which make 13; that taken from 15 leaves 2: therefore as 2 to 15, ſo is 36 to 270.

$$\frac{2}{3} + \frac{1}{5} = \frac{13}{15} \qquad \text{If } \frac{2}{15} \text{ be } \overset{l.}{36} : \text{what ſhall } 1\,? \; \textit{facit } \overset{l.}{270}$$

$$\frac{15}{15} - \frac{13}{15} = \frac{2}{15} \qquad \frac{\overset{1}{\ddot{2}}}{15})\frac{\overset{18}{\ddot{36}}}{1}(\overset{l.}{\frac{270}{1}}$$

$\frac{2}{3}$	180
$\frac{1}{5}$	54
Left	36
	270 Proof.

Otherwiſe.

Poſition 15
$\frac{2}{3}$ 10
$\frac{1}{5}$ 3
Total 13
Remain 2

As 2 . 15 *l.* :: 36 . 270 *l.*
36
90
45
2)540(270 *l.*

If a Number may be ſet by.

Sometime a Number in the Queſtion propounded abideth unalterable by the Fractions given, and ſo may be ſubſtracted from the Sum given, and ſet by for a Time till Operation be made with the reſt, and then reſtored again.

Q. Of Cattel, what Number.

Example. A Graſier ſaid of his Stock, If I had as many more Head of Cattel as I have with the half, third and fourth Parts thereof, and 1 overplus, I ſhould have juſt 630: how many Cattel had he?

Anſwer.

Anſw. I ſet by the 1, and ſo remaineth 629; when I have found therefore a Number, which being twice taken with the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ Parts thereof make 629, I add the 1 thereto: As ſuppoſing 12, the Parts whereof accordingly taken, and added to the double thereof, make 37, by which is found 204 the Number deſired.

Poſition 12
Double 24
$\frac{1}{2}$ 6
$\frac{1}{3}$ 4
$\frac{1}{4}$ 3
37

Poſition.
As 37 . 12 :: 629 . 204 Cattel.
37)7548(204

204
102 $\frac{1}{2}$
68 $\frac{1}{3}$
51 $\frac{1}{4}$
1 added.
630 Proof.

Two Numbers, yet ſingle Poſition.

Sometime two Numbers are demanded, and in the Work two Numbers are taken, yet the Reſolution by *Single Poſition*, becauſe one of the Numbers ſo taken proves true, and not ſuppoſitory only.

Q. Of two Numbers. Anſwer.

Example. What two Numbers are they, whoſe $\frac{2}{3}$ of the one is $\frac{3}{4}$ of the other?

Anſw. I take 12, a Number that hath ſuch Parts in it; $\frac{2}{3}$ of 12 is 8: Then ſeeking what Number 8 is $\frac{3}{4}$ of, I ſuppoſe 20; but $\frac{3}{4}$ of 20 is 15, and I ſhould have but 8: Nevertheleſs the *Analogy* will hold; If 15 come of the *Poſition* 20, then will 8 come of $10\frac{2}{3}$ the other Number deſired: So ſhall 12 and $10\frac{2}{3}$ be the Numbers demanded.

$\frac{2}{3}$ of 12 = 8
Poſition 20
$\frac{3}{4}$ 15

As 15 . 20 :: 8 . $10\frac{2}{3}$
8
15)160($10\frac{2}{3}$

Proof.
$\frac{2}{3}$ of 12 = 8 = $\frac{3}{4}$ of $10\frac{2}{3}$
$\frac{1}{3}$ 4 — $\frac{1}{2}$ $5\frac{1}{3}$
$\frac{1}{3}$ 4 — $\frac{1}{4}$ $2\frac{2}{3}$
8 — 8

Double Poſition, how known.

If there be no Partition in the Numbers to make a Proportion, then muſt be uſed the *Rule of Double Poſition.*

What to be done.

In *Double Poſition* make a Suppoſition twice, proceeding therein according to the State of the Queſtion; and if either Number of them ſuppoſed happen to reſolve the Queſtion, the Work is done: But if not, obſerve the Errors, and whether they be greater or leſſer than the Reſolution requireth, and mark the Errors accordingly with the Signs + or —.

Errors to be marked.

Multiplied croſſwiſe into the Poſitions.

Then multiply contrarywiſe the one *Poſition* by the other Error: And if the Errors be both too great or both too little, ſubſtract the one Product from the other, and the one Error from the other, and divide the Difference of the Products by the Difference of the Errors.

Errors like or unlike, and then what to do.

But if the Errors be unlike, as the one + and the other —, add the Products, and divide the Sum thereof by the Sum of the Errors added together: For the Proportion of the Errors, is the ſame with the Proportion of the Exceſſes or Defects of the Numbers ſuppoſed, to the Numbers ſought.

Example

Q. Of the Number of Tenements.

Example 1. *A* and *B* diſcourſing of their Tenements; *A* ſays to *B*, If I had 2 of your Tenements I ſhould have double the Number you have: To whom *B* replies, If I had 2 of your Tenements I ſhould be equal with you: How many Tenements had each of them?

Anſwer.

For Anſwer, ſuppoſe *A* had 16, to which 2 added makes 18, which is double the Number of 9; but having taken 2 from thence, it appeareth by this Suppoſition *B* had 11; wherefore 2 taken from 16 and added to 11, make *B* 13, and leave *A* 14. But they ſhould be equal, therefore the *Poſition* is erroneous, and the Error too much by 1.

Again, ſuppoſe *A* had 20, to which 2 being put, it's 22, and double the Number of 11; but from thence 2 being taken, *B* by this Suppoſition muſt have 13. Now take 2 from 20 and put to 13, it's 15 for *B*, and leaves 18 for *A*, which is not equal; therefore I have erred again by 3 too much.

Errors both +.

Then multiplying 16 the firſt *Poſition* by 3 the ſecond Error, and likewiſe 20 the ſecond *Poſition* by 1 the firſt Error, the Product 20 is taken from the Product 48, becauſe the Errors are both +, and the Remainer 28 is the Dividend; and abating 1 the leſſer Error from 3 the Greater, there is 2 for the Diviſor; the Quotient of which Diviſion will be 14, the Number ſought for *A*, and then by Conſequence muſt *B* have 10: for 2 taken from 10, and added to 14, makes 16, which is double to 8; and 2 taken from 14, and put to 10, makes both 12 alike.

Firſt. *Second.*
Poſition 16 X 20 Poſition.
Error 1+ 3+ Error.
20 48
Products 48 — 20 = 28 (14 *A*.
Errors 3 — 1 = 2 (10 *B*.

Proof.
14+2=16
10—2= 8 Half.
14—2=12
10+2=12 Equal.

Reſolution by other Poſitions, where the Errors are both —.

If the Suppoſitions had been 12 and 10, ſuppoſing 12 for *A*, then muſt *B* have 9; and taking 2 from 9 to put to 12, makes *A* 14, which is double to 7 for *B*; but taking 2 from 12 to put to *B*, makes *B* 11, and leaves but 10 for *A*, which is an Error too little by 1. And ſuppoſing 10 for *A*, then muſt *B* have 8; and taking 2 from 8 to put to 10, makes 12 for *A*, double to 6 for *B*; but then taking 2 from 10 to put to 8 makes *B* 10, and leaves *A* but 8, which is an Error too little by 2, for they ſhould be equal: And becauſe the Errors are both alike, the Work is as before.

Firſt. *Second.*
Poſition 12 X 10 Poſition.
Error 1— 2— Error.
10 24

Products 24—10=14 (14 *A*.
Errors 2— 1= 1 (10 *B*.

Reſolution by other Poſitions, where the Errors are + and —.

But if 20 and 10 had been the *Poſitions*, then the Errors as before being found unlike, that is 3 + and 2 —, the Total of the Products muſt be the Dividend, and the Total of the Errors the Diviſor.

Firſt. *Second.*
Poſition 20 X 10 Poſition.
Error 3+ 2— Error.
30 40

Products 30+40=70 (14 *A*.
Errors 3+ 2= 5 (10 *B*.

Q. Of 3 Debts, what they were.

Example 2. *A*, *B*, and *C*, were indebted to *D*, who hath forgotten their Particular Debts, but remembreth the Debt of *A* and *B* added together made 50 *l.* and *C* and *B* together owed 80 *l.* and the Debt of *A* and *C* together was 70 *l.* The Queſtion is, what each Man's particular Debt was?

Anſwer.

Anſw. Suppoſe *A* did owe 15 *l.* then muſt *B* owe 35 *l.* (for both their Debts made 50 *l.*) And if *B* owe 35 *l.* then did *C* owe 45 *l.* (becauſe both their Debts made 80 *l.*) But then the firſt Man's 15, and the third Man's 45, make together but 60 *l.* and they ſhould be 70 *l.* So is the Error 10 —.

Then ſuppoſe *A* did owe 26 *l.* it will follow that *B* owed 24 *l.* and *C* 56 *l.* But now 56 and 26 ariſe to 82 *l.* and it ſhould be but 70 *l.* ſo here is an Error of 12 +.

Now multiplying croſſwiſe as before, the Products 260 and 180 added, the Errors being unlike, make 440 to be divided by 22 the Sum of the Errors. So is 20 *l.* found the Debt of *A*, and conſequently *B* 30 *l.* and *C* 50 *l.*

Firſt Poſition. *A* & *B.* *Conſequence.* *B* & *C.* *Conſequence.* *A* & *C.*
A 15 + *B* 35 = 50 *B* 35 + *C* 45 = 80 *A* 15 + *C* 45 = 60
True Debts 70 Error 10 —

Second Poſition. *A* & *B.* *Conſequence.* *B* & *C.* *Conſequence.* *A* & *C.*
A 26 + *B* 24 = 50 *B* 24 + *C* 56 = 80 *A* 26 + *C* 56 = 82
True Debts 70 Error 12 +

Firſt. *Second.*
Poſition 15 X 26 Poſition.
Error 10 — 12 + Error.
260 180

Products 260 + 180 = $\frac{440}{22}$ (*l.* 20 *A.*
Errors 10 + 12 = 22

Proof.

A 20 + *B* 30 = 50 *B* 30 + *C* 50 = 80 *A* 20 + *C* 50 = 70

Example 3. A Parcel of Linen Cloth, *viz.* Lockram and Canvas, was ſold to the Number of 30 Ells for 51 *s.* The Canvas was rated at 18 *d.* the Ell, and the Lockram at 2 *s.* how many Ells of each ſort were there?

Q. *Of Lockram and Canvas, how much of each.*

Anſw. By ſuppoſing 10 Ells of Canvas, the Error will be 4 +: And by ſuppoſing 6 Ells, the Error will be 6 +; the Products therefore of 6, the *Poſition* into 4 and of 6, the Error into 10, taken one from the other, leave 36 the Dividend, and the Difference of the Errors 2 is the Diviſor: So is there found to be 18 Ells of Canvas, and by Conſequence 12 Ells of Lockram.

Anſwer.

Firſt Poſition. *s.* *d.* *s.*
Canvas 10 Ells × 1 : 6 = 15
Lockram 20 Ells × 2 : 0 = 40
30 55
Paiment 51
Error 4 +

Second Poſition. *s.* *d.* *s.*
Canvas 6 Ells × 1 : 6 = 9
Lockram 24 Ells × 2 : 0 = 48
30 57
Paiment 51
Error 6 +

Firſt. *Second.*
Poſition 10 X 6 Poſition.
Error 4 + 6 + Error.
24 60

Products 60 — 24 = $\frac{36}{2}$ (18
Errors 6 — 4 = 2

Proof.
s. *d.* *s.*
Canvas 18 Ells × 1 : 6 = 27
Lockram 12 Ells × 2 : 0 = 24
30 51

Example 4. A Carpenter was to build a Piece of Work in 30 Days, and by Agreement was to receive for every Day he wrought 3 *s.* but for every Day he did not work within that Time, he was to be amerced 2 *s.* When the Work was done he received but 25 *s.* for his Work: The Queſtion is, how many Days he worked, and how many Days he plaid?

Q. *Of a Carpenter's Work and Play.*

Anſw. By the Suppoſitions of 20 and 12, the Errors will be 15 + and 25 —; the Products added make 680, and the Errors added 40: So by Diviſion is found he worked 17 Days and plaid the reſt.

Anſwer.

Firſt Poſition.

$$\begin{array}{llcl} & & s. & s. \\ \text{Wrought} & 20 \text{ Days} & \times\, 3 = & 60 \\ \text{Played} & \underline{10} \text{ Days} & \times\, 2 = & \underline{20} \\ & 30 & & 40 \\ & & \text{Received} & \underline{25} \\ & & \text{Error} & 15\,+ \end{array}$$

Second Poſition.

$$\begin{array}{llcl} & & s. & s. \\ \text{Wrought} & 12 \text{ Days} & \times\, 3 = & 36 \\ \text{Played} & \underline{18} \text{ Days} & \times\, 2 = & \underline{36} \\ & 30 & & 00 \\ & & \text{Received} & \underline{25} \\ & & \text{Error} & \underline{25\,-} \end{array}$$

$$\begin{array}{lcccl} & \textit{First.} & & \textit{Second.} & \\ \text{Position} & 20 & \times & 12 & \text{Position.} \\ \text{Error} & \underline{15\,+} & & \underline{25\,-} & \text{Error.} \\ & \underline{180} & & \underline{500} & \end{array}$$

$$\begin{array}{lcl} \text{Products } 180 + 500 = & \dfrac{680}{40} & \Big(17 \\ \text{Errors } 15 + 25 = & & \end{array}$$

Proof.

$$\begin{array}{lclr} & & & s. \\ \text{Working Days} & 17 & \text{at } 3s. \text{ a Day} = & 51 \\ \text{Playing Days} & \underline{13} & \text{at } 2s. \text{ a Day} = & \underline{26} \\ & \underline{30} & & \underline{25} \text{ Received.} \end{array}$$

Notes. 1. *Suppoſe Number eaſy to be parted.* *Note* 1. As in *Single*, ſo in *Double Poſition*, though the Number ſuppoſed be never ſo falſe, Reſolution may be had thereby: Yet for more eaſe in the Work, ſuppoſe a Number likely or apt to be parted equally into ſo many Parts as are neceſſary to the Reſolution of the Queſtion.

2. *Second Poſition Homogeneal to the firſt.* 2. Let the ſecond *Poſition* always be *Homogeneal* to the firſt, that is, belong both to one Man, one Thing, &c. For though the Operator is at liberty to ſuppoſe for which he will of the Poſitives ſought in the Queſtion; yet if he ſuppoſe firſt for one, and then for another, the Reſolution will be confuſed.

3. *Both Errors if equal & unlike in Signs.* 3. If both the Errors be equal in Numbers, and yet their Signs unlike, half of both the *Poſitions* is the Sum deſired.

Q. *Of Apples given 3 Maids, how many.* *Example.* A Man having a certain Number of Apples, met three Maids who deſired ſome of his Apples; whereupon he giveth *A* $\frac{1}{4}$ of his Apples, and ſhe giveth him three Apples again. To *B* he giveth $\frac{1}{3}$ of his remaining Apples, and ſhe giveth him two Apples again. To *C* he giveth $\frac{1}{7}$ of his remaining Apples, and ſhe giveth him one again; ſo had he 13 Apples left: how many Apples had he at firſt?

Anſwer. *Anſw.* 20: For $\frac{1}{4}$ thereof is 5, and 3 returned again leaves 18; of which $\frac{1}{3}$ is 6, and 2 returned leaves him 14; of which $\frac{1}{7}$ is 2, and 1 returned leaves 13.

And if the Suppoſitions made for Reſolution be 16 and 24, the Errors will be $1\frac{5}{7}$ — and $1\frac{5}{7}$ +: So 20 the half of 40 (which is the Total of 16 and 24) is the Number ſought.

4. *Double Poſition moſt uſeful.* 4. All the Propoſitions reſolved by *Single Poſition*, will be reſolved by *Double*; and ſeveral by other of the Comparative Elements foregoing.

Example by Specificks *of the fourth Sort, as well as Poſition.*

Q. *Of filling a Ciſtern, the Time.* At *Bronze* is a Statue in form of a Lion ſtanding upon a Fountain, with this *Epigram*; If I let the Water paſs out of my right Eye, I can fill the Ciſtern (holding 732 Gallons) in 2 Days: If I let it paſs out of my left Eye, I can fill it in three Days: If it paſs out of my Feet, the Ciſtern will be filled in 4 Days: But if it paſs out of my Mouth, I can fill the Ciſtern in 6 Hours: In what time then ſhall I fill it, if I pour forth Water by all the Paſſages at once?

Anſwer. *Anſw.* In 4 Hours and 44 Minutes *ferè*: For in that time cometh out of his Mouth 576 Gallons, out of his right Eye 72, out of his Left 48, and out of his Feet 36.

By Poſition.

$$\begin{array}{lll} 1.\ \text{Poſition 3 Hours.} & \text{Then as} \left\{\begin{array}{lr} & \textit{Hours.} \\ \text{Right Eye} & 48 \\ \text{Left Eye} & 72 \\ \text{Feet} & 96 \\ \text{Mouth} & 6 \end{array}\right\} . \begin{array}{ccc} \textit{Gallons.} & & \textit{Hours.} \\ 732 & :: & 3 \end{array} . & \left\{\begin{array}{r} \textit{Gallons.} \\ 45\frac{3}{4} \\ 30\frac{1}{2} \\ 22\frac{7}{8} \\ \underline{366} \\ \underline{465\frac{1}{8}} \end{array}\right. \\ \quad \text{Error } 266\frac{7}{8}\, - & & \end{array}$$

$$732 - 465\tfrac{1}{8} = 266\tfrac{7}{8}$$

2. Poſition 4 Hours.
Error $111\frac{5}{6}-$

$$\text{Then as}\left\{\begin{array}{lr} & \textit{Hours.}\\ \text{Right Eye} & 48\\ \text{Left Eye} & 72\\ \text{Feet} & 96\\ \text{Mouth} & 6\end{array}\right\} : \overset{\textit{Gallons.}}{732} :: \overset{\textit{Hours.}}{4} . \left\{\begin{array}{r}\textit{Gallons.}\\ 61\\ 40\frac{2}{3}\\ 30\frac{1}{2}\\ 488\\ \hline 620\frac{1}{6}\end{array}\right.$$

$$732 - 620\frac{1}{6} = 111\frac{5}{6}$$

$$266\frac{7}{8} - 111\frac{5}{6} = 155\frac{1}{24}$$

$$\begin{array}{r} 4\\ \hline 1064\\ 3\frac{1}{2}\\ \hline 1067\frac{1}{2}\end{array} - \begin{array}{r} 3\\ \hline 333\\ 2\frac{1}{2}\\ \hline 335\frac{1}{2}\end{array} = 732 \qquad \begin{array}{l} 732 \times 24 = \\ 155 \times 24 + 1 = \end{array} \begin{array}{r} |2684\\ \hline 17568\\ \hline 3721\end{array}\Big(4\tfrac{44}{61} \text{ Hours.}$$

By* Ratio's *after the manner of* Specificks : *Thus,

By Specificks.

$$\begin{array}{ccccccccc} & & & & 1990656 & & & & \\ \hline \textit{Right Eye.} & & \textit{Left Eye.} & & \textit{Feet.} & & \textit{Mouth.} & & \\ 48 & \times & 72 & \times & 96 & \times & 6 & = & 1990656\\ \ddot{1} & & \ddot{1} & & \ddot{1} & & \ddot{1} & & \overline{}\Big(4\tfrac{44}{61} \text{ Hours.}\\ \hline 41472 & + & 27648 & + & 20736 & + & 331776 & = & 421632\end{array}$$

***Example by* Poſition *and* Equation of Paiment.**

A is indebted to *B* 800 *l.* to be paid at 4 Months; 1 Month being paſt, he payeth 200 *l.* and $1\frac{1}{2}$ Month after the firſt Month he payeth 300 *l.* more; when ſhall the other 300 *l.* be paid?

Q. *Of a Debt paid, part before due, when the reſt.*

Anſw. 5 Months after the laſt Paiment, that is, $3\frac{1}{2}$ Months after the end of the 4 Months.

Anſwer.

By Poſition.

	l.			
Debt	800	multiplied by the Time of Paiment	$800 \times 4 =$	3200
Paid	200	multiplied by the Time of Paiment	$200 \times 1 =$	200
Reſt	600	multiplied by the Time of Paiment	$600 \times 2\frac{1}{2} =$	1500
Paid	300	more, ſo remaineth yet 300, which ſuppoſe *A* kept 2 Months	$300 \times 2 =$	600
				2300
		Suppoſe *A* kept the 300 *l.* 4 Months;	Error	900 −
		Then inſtead of the 600 above, muſt be 1200 added to 1500 and 200, which		2900
			Error	300 −

$$900 - 300 = 600$$

$$\begin{array}{r} 4\\ \hline 3600\end{array} - \begin{array}{r} 2\\ \hline 600\end{array} = 3000 \qquad \frac{3000}{600}\Big(5 \text{ Months after } 2\tfrac{1}{2} \text{ Months.}$$

By* Equation of Paiment *thus.

By Equation of Paiment.

$$\begin{array}{l} \qquad\ \ \textit{l.}\\ \text{Debt } \underline{800} \text{ in 4 Months.}\\ \text{Paid}\left\{\begin{array}{l} 200 \times 1 = 200\\ \underline{300} \times 2\frac{1}{2} = 750\end{array}\right\}\dfrac{950}{\text{Paid } 500}\left(\begin{array}{l} \underline{4 \text{ Months.}}\\ 1\frac{9}{10} \text{ Time equated.}\\ \hline 2\frac{1}{10} \text{ Difference.}\end{array}\right.\\ \text{Reſt} \quad 300\end{array}$$

$$\text{As } 300 . 2\tfrac{1}{10} :: 500 . 3\tfrac{1}{2}$$

$$\begin{array}{r} 2\frac{1}{10}\\ \hline 1000\\ 50\\ \hline 300)\overline{1050}\Big(3\frac{1}{2} \text{ Months after 4 Months.}\end{array}$$

Endleſs were the Queſtions that might be propounded for Reſolution by *Poſition*; and ſeveral Authors are found ſtored with much variety of ſuch Propoſitions,

Other Queſtions choſen out of other Books.

ons, beſides thoſe already anſwered, ſome of the choice Ones here ſelected may ſerve inſtead of the reſt.

1. Of the Travel of two Poſts. *Example* 1. There are two Towns diſtant one from the other 200 Miles, from whence two Poſts that depart upon one Day from the one Town unto the other, and one goeth two Miles a Day more than the other, they meet in five Days; how many Miles doth each Poſt travel in a Day?

Anſwer. *Anſw.* The one went 19, and the other 21 Miles in a Day; which in 5 Days made the One to have gone 95, and the Other 105, and together 200 Miles.

2. Of 3 Silver Cups, with a Cover. *Example* 2. A Goldſmith hath three Silver Cups with a Cover of 18 Ounces, and the ſecond Cup weigheth half as much as the Firſt and third Cups. If the Cover be put to the firſt Cup, it weighs as much as all the three Cups; and if joined to the Second, it will weigh as much as the ſecond Cup twice, and the third Cup once: But if put to the third Cup, it weigheth twice as much as the firſt and ſecond Cups: what then was the Weight of each Cup?

Anſwer. *Anſw.* The firſt Cup weighed 6 ℥, the Second 8 ℥, and the Third 10 ℥, that is together 24 ℥: And ſo much is the Cover and firſt Cup 18 + 6. The Cover and ſecond Cup is 26, that is 18 + 8, which is equal to the third Cup, and double the Second 10 + 8 + 8. And the Cover and third Cup is 28 the double of 14, the Weight of the firſt and ſecond Cups.

3. Of Crowns caught up in a Difference. *Example* 3. Two Partners had in Accompt between them 400 French Crowns, whereof one ſhould have 230, and the other 170: But in parting them they fell ſo at Variance, that he had moſt that could catch moſt. Yet afterward being reconciled, they agreed that he that had moſt ſhould lay down $\frac{1}{2}$ of them he had, and he that had leaſt ſhould lay down $\frac{1}{3}$, and the Sum of both ſhould be equally divided between them, and ſo ſhould each Man have his due: The Queſtion is, how many of the Crowns each Man caught up?

Anſwer. *Anſw.* One caught up 280, and the other 120: for $\frac{1}{2}$ of 280 is 140, and there remaineth 140; and $\frac{1}{3}$ of 120 is 40, and there remaineth 80; this 40 and that 140 added are 180, the Half whereof is 90 for each; and this added to the Remains, makes for one 230, that is 90 + 140, and for the other 170, that is 90 + 80.

4. Of Money and Cloth, how much. *Example* 4. A Merchant buying Cloth, findeth if he take 12 Cloths, he ſhall want 42 *l.* to pay for them; but if he take 9 Cloths, then he hath 84 *l.* too much: How much Money had he, how much did a Cloth coſt, and how many Cloths bought he?

Anſwer. *Anſw.* He had 462 *l.* which will pay for 11 Cloths at 42 *l.* a Cloth.

5. Of the Money Partners had. *Example* 5. *A*, *B*, and *C*, buy a Ship for 200 *l.* If *A* have $\frac{1}{2}$ of what *B* pays, then he can pay for the Ship alone: And if *B* have $\frac{1}{4}$ of *C*, or *C* have $\frac{1}{3}$ of *A*, then they reſpectively can pay for the Ship: what Monies had each of them?

Anſwer. *Anſw.* *A* had 120 *l.* which lacking 80 *l.* of 200 *l.* *B* muſt be double to 80, that is 160 *l.* this wanting 40 *l.* of 200 *l.* *C* muſt have 4 times 40, that is 160 *l.* as *B* had. And then 160 *l.* and 40 *l.* which is $\frac{1}{3}$ of *A*, will make up 200 *l.* alſo.

6. Of Hiero's *Crown.* *Example 6. Hiero* King of *Syracuſe* in *Sicily*, had cauſed to be made a Crown of Gold to be offered for his good Succeſs in Wars; in making whereof his Goldſmith fraudulently took out a Portion of the Gold, and put in Silver for it, yet ſo that there was nothing thereof to be ſeen or abated of the full Weight. The King ſuſpecting the Fraud, propounds the Doubt to *Archimedes*; *viz.* How he might diſcover the Fraud without breaking the Crown. *Archimedes* not knowing preſently how to anſwer the King's Deſire; a while after as he chanced to enter into a Bath, he obſerved, as his Body entred into the Bathing-Veſſel, the Water ran over; and thereupon apprehending a Reaſon of Solution to the King's Queſtion, as the Story reports, was ſo rejoiced, that forgetting he was naked, ran Home, crying, ἕυρηκα, ἕυρηκα, *I have found, I have found:* And cauſed two maſſy Pieces, one of Gold and the other of Silver, to be prepared, of the ſame Weight with the Crown; and conſidering that Gold being heavier, and more compact by Nature than Silver, and ſo occupying leſs room, if it were put into a Veſſel brim full of Water, would cauſe leſs Water to run over than a Maſs of the ſame Weight of Silver would do: Whereupon trying both, he noted the Quantities of Water at each Time ſo run over, and learned thereby what Proportion in Quantity is between Gold and Silver of equal Weight. And then putting the Crown into the Veſſel brim full of Water, (as before) marked how much Water run over then; and comparing it with the Water that ran over when the Gold was put in, noted how much it did exceed that; and likewiſe comparing it to the Water that ran over when the Silver was put in, marked how much it was leſs than that; and by theſe Proportions found out the juſt Quantity of Gold that was taken out of the Crown, and how much Silver was put in inſtead thereof.

Vitruvius who writeth the Hiſtory, doth not declare the Particulars: But ſuppoſe the Crown to weigh 8 ℔, and ſo the other Maſſes, and imagine when the Gold was put in, there ran over 2 ℔ of Water; and when the Silver was put in, there ran over $3\frac{1}{2}$ ℔; and when the Crown was put in, there ran over but $2\frac{1}{4}$ ℔. Now ſuppoſing there was 2 ℔ of Silver in the Crown, then muſt there be 6 ℔ of Gold: And accordingly by the *Rule of Three*,

℔ *Gold* ℔ *Water.* ℔ *Gold* ℔ *Water.*
As 8 2 :: 6 $1\frac{1}{2}$
6
$8)\overline{12}(1\frac{1}{2}$

℔ *Silver.* ℔ *Water.* ℔ *Silver.* ℔ *Water.*
As 8 $3\frac{1}{2}$ :: 2 $\frac{7}{8}$
2
6
1
$8)\overline{7}(\frac{7}{8}$

$2\frac{3}{8}$ Sum.
$2\frac{1}{4}$ when the Crown was put in.
$\frac{1}{8}$ Error +

Suppoſing again the Silver 1 ℔, then muſt the Gold be 7 ℔; and the Proportions by the *Rule of Three*, thus:

℔ *Gold.* ℔ *Water.* ℔ *Gold.* ℔ *Water.*
As 8 2 :: 7 $1\frac{3}{4}$
7
$8)\overline{14}(1\frac{3}{4}$

℔ *Silver.* ℔ *Water.* ℔ *Silver.* ℔ *Water.*
As 8 $3\frac{1}{2}$ :: 1 $\frac{7}{16}$
$\frac{8}{1})\frac{7}{2}(\frac{7}{16}$

$2\frac{3}{16}$ Sum.
$2\frac{1}{4}$ when the Crown was put in.
$\frac{1}{16}$ Error −

1
Products.
$\frac{2}{1}\times\frac{1}{16}=\frac{1}{8}$
8
$\frac{1}{1}\times\frac{1}{8}=\frac{1}{8}$
$\frac{1}{4}$

Products $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$
Errors $\frac{1}{8}+\frac{1}{16}=\frac{3}{16}$ $\frac{3}{16})\frac{1}{4}(\frac{4}{3}(1\frac{1}{3}$ ℔ Silver.
4 1

So had the Crown, if 8 ℔ in all $\left\{\begin{array}{l}1\frac{1}{3} \text{ ℔ Silver}\\ 6\frac{2}{3} \text{ ℔ Gold}\end{array}\right\}$ By the Suppoſitions aboveſaid.

Yet *Archimedes* needed not to have the Maſſes of equal Weight with the Crown; for by other Quantities of the ſame Metal, though of unequal Weight, the Solution may be had, as *Record* hath well obſerved.

Proof of Poſition of both ſorts.

The Proof of all the Works wrought by *Poſition*, is when the Number deſired is found, to add, ſubſtract, multiply, or divide thereby according to the tenor of the Queſtion: and if in Concluſion there be Agreement in all things with the *Data* in the Propoſition, the Work is right; as by the Proof of ſeveral of the Works before in this Chapter is fully approved.

CHAP. XV. *Proportions doubled.*

Figural Proportions.

Hitherto this ſecond Part of the fourth Book hath ſpoken of plain *Disjunct Proportions*; now therefore a little is to be ſeen of *Figural*, which are ſo named, becauſe the Proportion between the Numbers given and required, lies properly between them as they are, or are to be Figurate, or to the Reſolution make uſe of Figural Numbers: So as being Figurate to the Square Quantity, they are called *Doubled Proportions*; if cubed, *Tripled Proportions*, &c.

Doubled Proportions how taken.

Doubled Proportions; are not here to be ſtrictly taken only for the Proportions about Squares, but take in among them other Plain or Superficial Figures, as Circles, Triangles, *&c.* and conſider as well the *Proportions* between the *Parts* of ſuch Figures one to another, as the *Proportions* between the *Data* and *Quæſita* in Propoſitions concerning them.

Circle, the Principal Parts, and Analogies thereof.

The principal Parts of a Circle, wherein an Analogy is requiſite, are three; The *Diameter*, the *Circumference* (called alſo *Peripherie* and *Perimeter*) and the *Area*: And becauſe the Product of half the Circumference into half the Diameter is the *Area*, accompting the Diameter to the Circumference, according to the aforementioned *Archimedes*, or *Oughtred* from *Ludolph van Ceulen*, mentioned before alſo in *Figural Numbers*, Chap. 2.

The Analogies *are:*

As 7, to 22 .
or 1, to 3,1416 } So is the Diameter to the Circumference.

As 14, to 11 .
or 1, to 0,7854 } So is the Square of the Diameter to the *Area*.

As 88, to 7 .
or 1, to 0,0795775 } So is the Square of the Circumference to the *Area*.

Propoſitions.

Q. Of the Circumferance and Area.

A Circle whoſe Diameter is 21: What ſhall be the Circumference, and what the *Area* by *Archimedes*?

Anſwer.

Anſw. 66 Circumference, and $346\frac{1}{2}$ *Area*.

As 7 . 22 :: 21 . 66 Circumference.
As 14 . 11 :: 441 (Q. 21) . $346\frac{1}{2}$ *Area*.

Q. Of the Diameter & Area.

A Circle whoſe Circumference is 66: what ſhall be the Diameter, and what the *Area* by *Archimedes*?

Anſwer.

Anſw. 21 Diameter, and $346\frac{1}{2}$ *Area*.

As 22 . 7 :: 66 . 21 . Diameter.
As 88 . 7 :: 4356 (Q. 66) . $346\frac{1}{2}$ *Area*.

Q. Of the Diameter and Circumference.

A Circle whoſe *Area* is $346\frac{1}{2}$: what ſhall be the Diameter, and what the Circumference by *Archimedes*?

Anſwer.

Anſw. 21 Diameter, and 66 Circumference.

As 11 . 14 :: $346\frac{1}{2}$. 441 (Q. 21) . Diameter.
As 7 . 88 :: $346\frac{1}{2}$. 4356 (Q. 66) . Circumference.

Triangles, *their Sides and Angles*, &c.

Touching *Triangles*, their principal Parts are their Sides, Angles and *Areas*; but (that *Geometry* be not too far intruded upon, to which properly theſe Speculations belong) it will be enough to ſay ſomething here of the Sides and Angles of Right-angled Triangles only.

Of a Right Angle.

The Doctrine of Triangles acquaints us, that in all plain Right-angled Triangles, the Square of the Side that ſubtendeth the Right-angle, is equal to the two Squares of both the other Sides. Wherefore in ſuch Propoſitions, where the *Data* and Number ſought repreſent the three Sides of ſuch a Triangle, the Reſolution of either in queſtion is accordingly to be found.

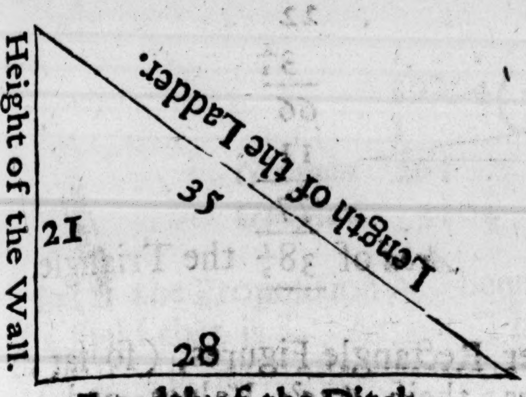
Q. Of the length of Scaling Ladders.

Example 1. The Walls of a City 21 Feet high, which hath a Ditch at the bottom 28 Feet broad, are to be ſcaled; and the ſcaling Ladders are commanded to be made a Foot longer than will reach from the outermoſt Brink of the Ditch, to the Top of the Wall: how long muſt the Ladders be?

Anſwer.

Anſw. 36 Feet long: For the Squares of 21 and 28, which are 441, and 784 added, make 1225, whoſe Square Root is 35 the length, that will reach from the Brink of the Ditch to the Top of the Wall; to which 1 Foot added, becauſe the Ladders were to be ſo much longer, makes the whole length of the Ladder 36 Feet.

Height



Breadth of the Ditch.

$21 \times 21 = 441$
$28 \times 28 = 784$ } = $\cancel{1}\cancel{2}2\cancel{5}$ (35
3
9
$\cancel{3}2\cancel{5}$
1 added.

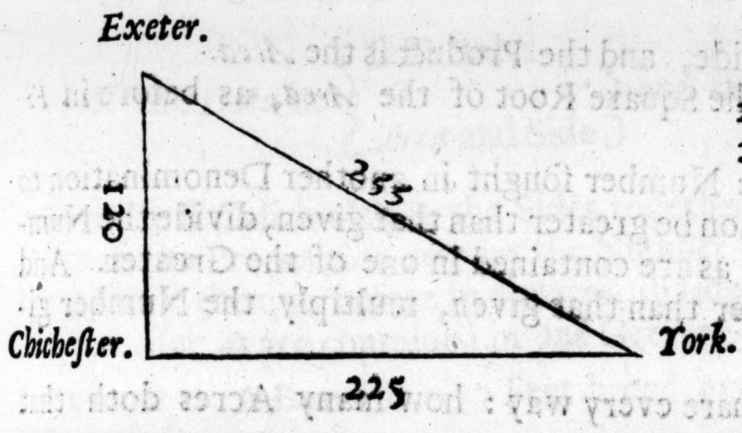
Length of the Ladders 36

Example 2. There are two Towns, as ſuppoſe *Chicheſter* and *York*, which lie South and North, diſtant one from the other 225 Miles; a third Town, as *Exeter* lying plain Weſt from *Chicheſter*, is diſtant from *York* 255 Miles: how far is *Chicheſter* diſtant from *Exeter* in a right Line?

Q. Of the Diſtance of two Cities.

Anſw. 120 Miles: For here the number ſought being not repreſented by the *Hypotenuſal* Line, but by one of the Sides, containing the Right Angle; the Square of the other Side containing the ſame Angle, is to be taken from the Square of the ſubtending Side, that is, 50625, the Square of 225 from 65025 the Square of 255; ſo is the Remain 14400, the Square of 120.

Anſwer.



$255 \times 255 = 65025$
$225 \times 225 = 50625$
14400 (120 Miles.
$\cancel{1}44$

The *Area* of ſuch Triangles being always half the Product of the 2 Sides, containing the Right Angle: If it were inquired how many Acres of Ground there were in the Triangle laſt mentioned, conſidering that one Engliſh Mile contains 5280 Feet or 320 Perches, and one Acre 160 Square Perches, the Sides 120 and 225 multiplied by 320, and the Products one into another, and the Half divided by 160, make ſuch a Triangle to contain 8640000 Acres of Land.

Q. Of the Area of ſuch a Triangle.

Anſwer.

Sides of the Triangle	120		225 Miles.	
Perches in an Engliſh Mile	320		320	
	2400		4500	
	360		675	Product.
Perches in the Sides	38400	×	72000	= 2764800000
			Half	1382400000

$\cancel{1}\cancel{0}6$
$\cancel{1}\cancel{3}8\cancel{2}\cancel{4}00000$ (8640000 Acres.
$\cancel{1}\cancel{6}\cancel{0}$

Sometime it is needful to turn a Circle into a Rectangle-Triangle, which is commonly performed thus; Make the Perpendicular of the Triangle equal to the Semidiameter of the Circle, and the Ground or Baſe-Line of the Triangle equal to the *Peripherie* of the Circle.

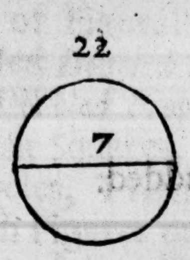
To turn the Triangle into a Circle.

Example. If a Circle have the Diameter 7, and *Peripherie* 22: what ſhall the Sides of a Rectangle-Triangle be, that is equal in *Area* to the Circle, according to *Archimedes*?

Example.

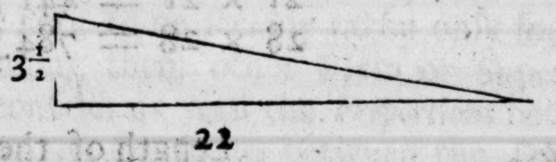
Anſw. The Perpendicular $3\frac{1}{2}$ and the Baſe 22.

Anſwer.



$$\begin{array}{r} 11 \\ 3\frac{1}{2} \\ \hline 33 \\ 5\frac{1}{2} \\ \hline 38\frac{1}{2} \end{array}$$

Area of $38\frac{1}{2}$ the Circle.



$$\begin{array}{r} 22 \\ 3\frac{1}{2} \\ \hline 66 \\ 11 \\ \hline 77 \\ \hline 38\frac{1}{2} \end{array}$$

Area of $38\frac{1}{2}$ the Triangle.

Quadrangles, *what conſidered in them.* *Quadrangles*, whether exact Squares, or other Rectangle Figures, (ſo far as concerns *Arithmetick*) have conſidered, their Sides, their *Area's*, Values, and Alteration of their Forms: But becauſe all theſe, more or leſs, have been touched before, as among ***Figural Numbers*** in the ſecond Part of the ſecond Book, or in the ***Indirect Rule of Three***, or ***Specificks***, in this ſecond Part of the fourth Book, they muſt be more ſparingly remembred now.

Of their Sides and Area's. Touching the Sides and *Area*'s of *Quadrangles*.

Square. *Of the* Square.

$$\text{If the } \textit{Data} \text{ be the} \left\{ \begin{array}{l} \text{Side} \\ \textit{Area} \end{array} \right\} \text{and the } \textit{Quesita} \text{ the} \left\{ \begin{array}{l} \textit{Area.} \\ \text{Side.} \end{array} \right.$$

For the Area. To reſolve the Firſt, ſquare the Side, and the Product is the *Area*.

For the Side. To reſolve the Second, extract the Square Root of the *Area*, as before in *Figural Numbers*.

If another Denomination be ſought. But if the Propoſition require the Number ſought in another Denomination to that given; Then if the Denomination be greater than that given, divide the Number found by ſo many of the Leſſer, as are contained in one of the Greater. And if the Denomination required be leſſer than that given, multiply the Number given or found accordingly.

Q. Of the Acres in a Field. *Example* 1. A Field is 36 Rods Square every way: how many Acres doth that Field contain?

Anſwer. *Anſw.* $8\frac{1}{10}$ Acres: Here, becauſe Acre is an higher Denomination than Rod, after 36 the Side is multiplied into it ſelf, the Product 1296 being the *Area* in Rods, is divided by 160 the Rods in an Acre, and the Quotient is $8\frac{1}{10}$ as above.

$$\begin{array}{l} \text{Side } 36 \times 36 = 1296 \\ \text{Perches in an Acre } \quad \overline{160} \left(8\frac{1}{10} \text{ Acres.} \right. \end{array}$$

Ex. *If the Yards or Feet therein be ſought.* But if the Propoſition had demanded, how many Yards or Feet, *&c.* there had been, then theſe Denominations being leſſer than Rods, either 36 muſt be multiplied by $5\frac{1}{2}$ for Yards, and by $16\frac{1}{2}$ for Feet, and the Products ſquared; or 1296 muſt be multiplied by $30\frac{1}{4}$ for Yards, and by $272\frac{1}{4}$ for Feet, becauſe ſo many make a Square Rod.

Anſwer.

$$\begin{array}{r} 36 \\ 5\frac{1}{2} \\ \hline 180 \\ 18 \\ \hline 198 \end{array} \times 198 = \begin{array}{r} 1296 \\ 30\frac{1}{4} \\ \hline 38880 \\ 324 \\ \hline 39204 \end{array} \text{ Yards.} \qquad \begin{array}{r} 36 \\ 16\frac{1}{2} \\ \hline 216 \\ 36 \\ 18 \\ \hline 594 \end{array} \times 594 = \begin{array}{r} 1296 \\ 272\frac{1}{4} \\ \hline 2592 \\ 90724 \\ 25922 \\ 3 \\ \hline 352836 \end{array} \text{ Feet.}$$

$$5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4} \qquad 16\frac{1}{2} \times 16\frac{1}{2} = 272\frac{1}{4}$$

Q. Of the Rods in one Side. *Example* 2. A Square Field containeth 39204 Yards: how many Rods doth one Side contain?

Anſwer. *Anſw.* 36 Rods: For Rod being an higher Denomination than Yard, 198 the Square Root of 39204 extracted, is divided by $5\frac{1}{2}$, or 5,5 the Yards in one Rod, and the Quotient is 36 Rods as before.

$\overset{\cdot}{3}9\overset{\cdot}{2}0\overset{\cdot}{4}\big(\overset{\cancel{3}}{\cancel{1}}\overset{\cancel{3}}{\cancel{9}}\,8{,}0\big(\,36$ Rods.
1 $\cancel{5},\cancel{5}$
Gnomon 261 $\cancel{5},\cancel{5}$
Gnomon 3104

But if the Propoſition had been to know how many Rods or Yards in the Side of a Field that is $8\frac{1}{10}$ Acres: Then, becauſe Rods and Yards are leſſer Denominations, $8\frac{1}{10}$ is multiplied by 160 for Rods, and 4840 for Yards, becauſe ſo many are in 1 Acre, and the Root of each ſhall be the Side in thoſe Denominations.

Ex. If the Rods or Yards of the Side be ſought, and Acres given.

Anſwer.

$$\begin{array}{r l} 160 & \\ 8\frac{1}{10} & \\ \hline 1280 & \\ 16 & \sqrt{} \\ \hline \dot{1}2\dot{9}6 & \big(\ 36 \text{ Rods.} \end{array} \qquad \begin{array}{r l} 4840 & \\ 8\frac{1}{10} & \\ \hline 38720 & \\ 484 & \sqrt{} \\ \hline \dot{3}9\dot{2}0\dot{4} & \big(\ 198 \text{ Yards.} \end{array}$$

Of Oblongs.

Oblongs.

Such Rectangle Figures as have their oppoſite Sides parallel.

If the *Data* be { Both Sides / *Area* and Side } and the *Queſita* { *Area.* / Other Side.

In the firſt Caſe multiply the Sides together.

For the Area.

But if the Propoſition require the Number ſought in another Denomination to that given: then, as before in Squares, divide or multiply accordingly by ſo many of the Leſſer as are contained in one Greater.

If another Denomination be ſought.

Example 1. A Board is 1,17 Feet broad, and 16,32 Feet long: how many ſquare Feet doth it contain?

Q. *Of the Feet in a Board.*

Anſw. Almoſt 19,1 Feet, as the Product of 16,32 by 1,17 makes appear.

Anſwer.

$$\begin{array}{r r l} & 16{,}32 & \text{Length.} \\ & 1{,}17 & \text{Breadth.} \\ \hline & 11424 & \\ & 1632 & \\ & 1632 & \\ \hline \textit{Area} & 19{,}0944 & \text{Square Feet.} \end{array}$$

Example 2. A tiled Roof hath the Breadth $16\frac{1}{4}$ Feet, and the Length 47 Feet: how many Squares of Tiling doth the whole Roof contain?

Q. *Of the Squares of Tiling in a Roof.*

Anſw. 15,275 Squares, or $15\frac{11}{40}$: Here the Length 47 being doubled (for both Sides) is 94; which multiplied by $16\frac{1}{4}$, or 16,25, the Product 1527,5 is divided by 100 (or 10 × 10) the Feet in a Square.

Anſwer.

$$\begin{array}{r l} 16{,}25 & \text{Breadth.} \\ 94 & \text{Double length.} \\ \hline 6500 & \\ 14625 & \\ \hline 1527{,}50 & \text{Feet.} \end{array} \qquad \frac{1527{,}5\cancel{00}}{100}\Big(\ 15{,}275 \text{ Squares.}$$

Example 3. A Pavement broad 17,35 Feet, and long 30,5: how many Square Yards doth it contain?

Q. *Of the Yards in a Pavement.*

Anſw. A ſmall matter above 58,797 Yards: For the Product of the Length into the Breadth 529,175 divided by 9, (or 3 × 3) the ſquare Feet in 1 Yard give 58,797, and 2 is left remaining on the Diviſion.

Anſwer.

17,35 Breadth.
30,5 Length.
8675
52050
529,175 Feet.

77 86(2
529,175
9 (58,797 Yards.

If the Demand were for Inches.

But if in any of theſe *Examples*, the Demand had been to know the Inches, then foraſmuch as Inches are of leſſer Denomination than the Feet given in the Propoſition, the Feet found ſhould have been multiplied by the Inches in 1 ſquare Foot, that is (12 × 12 or) 144, Or the Sides given multiplied by 12, and then multiplied one into the other, would have produced the ſame Effect.

For the Side unknown.

In the ſecond Caſe, when the *Area* and Side is given, to find the other Side, divide the *Area* by the given Side. But oftentimes the *Area* is given implicitly, and to be underſtood in the Denomination demanded.

Q. Of Length to make a Foot of Board.

Example 1. A Board is 6 Inches broad: how much Length thereof ſhall make a ſquare Foot.

Anſwer.

Anſw. 24 Inches, or two Feet: For either 144 the Inches in 1 ſquare Foot divided by 6, or 1 Foot by $\frac{1}{2}$ in Fractions, effecteth the Deſire; and the Analogy in all ſuch Queſtions is reciprocal, and reſolved by the *Indirect Rule of Three*, as in Plain Proportions before.

$$\text{As}\ \underbrace{\overset{\textit{Breadth.}}{12}\ .\ \overset{\textit{Length.}}{12}}\ ::\ \overset{\textit{Breadth.}}{6}\ .\ \overset{\textit{Length.}}{24}$$

6) 144 (24 Inches.

Or as 1 Breadth . 1 Length :: $\frac{1}{2}$ Breadth . 2 Length.

$$\frac{1}{2}\Big)\frac{1}{1}\Big(\frac{2}{1}\ \text{Feet.}$$

Q. Of Length to make a Square of Tiling.

Example 2. A Roof $16\frac{1}{4}$ Feet broad: how much Length thereof ſhall make a Square?

Anſwer.

Anſw. $6\frac{2}{13}$ Feet?

$$\text{As}\ \underbrace{\overset{\textit{Breadth.}}{10}\ .\ \overset{\textit{Length.}}{10}}\ ::\ \overset{\textit{Breadth.}}{16\tfrac{1}{4}}\ .\ \overset{\textit{Length.}}{6\tfrac{2}{13}}$$

$$\frac{65}{4}\Big)\frac{100}{1}\Big(\frac{400}{65}\Big(6\tfrac{2}{13}\ \text{Feet.}$$

Q. Of Length to make a Yard of Pavement.

Example 3. A Pavement $16\frac{1}{2}$ Feet broad: how much Length thereof ſhall make a Square Yard?

Anſwer.

Anſw. $\frac{6}{11}$ of a Yard.

$$\text{As}\ \underbrace{\overset{\textit{Breadth.}}{3}\ .\ \overset{\textit{Length.}}{3}}_{9}\ ::\ \overset{\textit{Breadth.}}{16\tfrac{1}{2}}\ .\ \overset{\textit{Length.}}{\tfrac{6}{11}}$$

$$\frac{\overset{11}{\cancel{33}}}{2}\Big)\frac{\overset{3}{\cancel{9}}}{1}\Big(\frac{6}{11}\ \text{Yard.}$$

Duplicate Ratio's.

Among like Plains in a Duplicate *Ratio*, that is, as the Squares of their Homologal Sides: If 3 Numbers be given, in which as the Square of the Firſt is to the Square of the Second, ſo the Third to a Number ſought. Then as the firſt Square to the third Number; ſo is the ſecond Square to the Number ſought.

Q. Of the Number of paving Tiles.

Example 1. Two like Rectangled *Area's* or Plains, the Greater in Length 40 Feet, the Leſſer 24, each paved with paving Tiles, the Greater hath 1200 Tiles: how many hath the Leſſer?

Anſwer.

Anſw. 432: Becauſe 1600, the Square of 40 to 576 the Square of 24, is as 1200 to 432, that is as 25 to 9.

$$\begin{array}{lllllll} & (Q.\,40) & & & (Q.\,24.) & & \\ \text{As} & 16{,}00 & . & 12{,}00 & :: & 576 & . \; 432 \text{ Tiles.} \end{array}$$

$$\begin{array}{r} 576 \\ 12 \\ \hline 1152 \\ 576 \\ 16)\overline{6912}(432 \end{array}$$

Example 2. How many Acres of Wood-land, meaſured with a Perch of 18 Feet, are there in 73 Acres of Land meaſured with a Perch of $16\frac{1}{2}$ Feet? *Q. Of Acres Woodland Meaſure.*

Anſw. 61,34, and ſomewhat over: For 18 and $16\frac{1}{2}$ reduced to their leaſt Terms, are as 12 to 11: Wherefore, *Anſwer.*

$$\begin{array}{llllll} & (Q.\,12) & & (Q.\,11) & & \\ \text{As} & 144 & . \; 73 & :: & 121 & . \; 61{,}34 \text{ Acres.} \end{array}$$

$$\begin{array}{r} 73 \\ \hline 363 \\ 847 \\ \hline 8833 \end{array}$$

$$\begin{array}{l} \cancel{4}\cancel{8} \\ \cancel{1}\cancel{9}\cancel{9}\,8|4 \\ 88\cancel{3}\cancel{3},\cancel{0}\,\cancel{0}\,(\,61{,}34 \\ \cancel{1}\cancel{4}\cancel{4} \\ \cancel{1}\cancel{4}\cancel{4} \\ \cancel{1}\cancel{4}\cancel{4} \\ \cancel{1}\cancel{4}\cancel{4} \end{array}$$

Concerning the Values of Quadrangles in exchange or otherwiſe, the Analogy lies between the two *Area's*, Direct if the Price be the ſecond Number, but Reciprocal if the Quantity. *For the Value.*

Example 1. A Merchant hath 40 Yards of Cloth of $1\frac{1}{4}$ Yard broad, and would exchange for another Piece of $\frac{3}{4}$ Broad: how many Yards of this ought he to have for the other? *Q. Of Cloth of one Breadth for another.*

Anſw. $66\frac{2}{3}$ Yards: For the 40 multiplied by $1\frac{1}{4}$, being the Content of 1 Piece, ſhall be divided by $\frac{3}{4}$, as in the *Indirect Rule of Three.* *Anſwer.*

$$\begin{array}{llllll} & (40\times1\frac{1}{4}) & & & & \\ \text{As} & 50 & . \; 1 & :: & \frac{3}{4} & . \; 66\frac{2}{3} \text{ Yards.} \end{array}$$

$$\frac{3}{4}\Big)\frac{50}{1}\Big(\frac{\overset{\cancel{2}(2}{\cancel{2}\cancel{0}\cancel{0}}}{\cancel{3}}\Big(66\frac{2}{3}$$

Example 2. How many Rods of Land, 4 Rods broad, ſhall I have for a Piece of Land that is 3 Rods broad and 12 Rods long? *Q. Of a Piece of Land for another.*

Anſw. 9 Rods in length. *Anſwer.*

$$\begin{array}{llllll} & (3\times12) & & & & \\ \text{As} & 36 & . \; 1 & :: & 4 & . \; 9 \text{ Rods.} \end{array}$$

Example 3. A Gentleman beſpeaks a Piece of Wainſcot of 36 Feet 3 Inches long, and 8 Feet 4 Inches broad; and agreeth to pay for it by the Yard ſquare, at 10*s.* every Square: what doth this Piece of Work come to? *Q. Of the Price of a Piece of Wainſcot.*

Anſw. 16*l.* : 15*s.* : $7\frac{7}{9}$*d.*: For the Length multiplied into the Breadth, is $302\frac{1}{12}$ ſquare Feet: And 9 ſquare Feet making 1 ſquare Yard, the Analogy is; As 9 to 10*s.* ſo is $302\frac{1}{12}$, to 16*l.* : 15*s.* : $7\frac{7}{9}$*d.*: Or reducing the ſquare Feet into ſquare Yards, it is, As 1 . 10*s.* :: $33\frac{61}{108}$. 16*l.* : 15*s.* : $7\frac{7}{9}$*d.* *Anſwer.*

Length. *Breadth.*

$36\frac{1}{4} \times 8\frac{1}{3}$ Or, $\frac{145}{4} \times \frac{25}{3} \Big\} = \frac{3625}{12}$

As 9 . 10 s. :: $\frac{3625}{12}$. 16 l. : 15 s. : $7\frac{7}{9}$ d.

$\frac{9}{1}\Big)\frac{36250}{12}\Big(\frac{3625(0}{108}\Big(\frac{33|\ 5\ s.}{}$ (7

l. $16:15\frac{3}{5}\frac{5}{4}$

Or (6 38(1 $\frac{3625}{12\times9} = \frac{}{108}\Big($ *Square Yards.* $33\frac{61}{108}$.

As 1 . 10 s. :: $33\frac{61}{108}$. 16 l. : $15\frac{3}{5}\frac{5}{4}$ s.

Q. *Of a Piece of Heben Wood, the Price.* *Example* 4. If a Foot Square of *Heben* Wood be worth 7 *s.* 3 *d.* what will that Piece be worth that is $9\frac{1}{2}$ Feet long, and $4\frac{1}{3}$ Feet broad, but of equal Thickneſs with the other?

Anſwer. *Anſw.* 14 *l.* 18 *s.* $5\frac{1}{2}$ *d.* For $9\frac{1}{2}$ the Length, multiplied into $4\frac{1}{3}$ the Breadth, produce $41\frac{1}{6}$ ſquare Feet; the Reſt is reſolved by the *Rule of Three Direct.*

Length. *Breadth.*

$9\frac{1}{2} \times 4\frac{1}{3}$ Or, $\frac{19}{2} \times \frac{13}{3} \Big\} = \frac{247}{6}$

Sq.Foot. *s.* *Sq.Feet.* *l.* *s.* *d.*

As 1 . $7\frac{1}{4}$ :: $\frac{247}{6}$. $14:18:5\frac{1}{2}$

2(1 230(1 $\frac{7163}{24}\Big(\frac{29{,}8\frac{11}{24}\ s.}{}\times12 = \frac{13(2}{24}\Big(5\frac{1}{2}\ d.$ (1

l. $14:18:5\frac{1}{2}$

Concerning the Forms of Quadrangles. About the Alteration of the Forms of *Quadrangles*, beſides what may be underſtood in ſome of the Premiſſes, more may be ſeen in ſome Military Propoſitions, wherein ſome of the other Propoſitions are alſo mixed.

Military Propoſitions. Military Propoſitions principally concern the Forming of Battels.

Battels, how conſidered. Battels are conſidered in reſpect to the Number of Men, or Form of the Ground.

Square Battel of Men. Square Battel of Men, is that which hath an equal Number of Men in Rank and File, or (as ſome phraſe it) *Front* and *Flank*.

Square Battel of Ground. Square Battel of Ground, is that which hath the Rank as long as the File, though the Men in Rank be more than in File.

Of the firſt four Sorts. In reſpect of Men, there is a fourfold Variety, *viz.* a Square Battel, a Double Battel, a Battel of the Grand Front, which is called a *Quadruple Battel*, and a Battel of any Proportion of the Number in Rank to the Number in File.

1. *Square Battel of Men.* 1. For a Square Battel of Men, extract the Square Root out of the whole Number of Men, the ſame ſhall be the Number of Souldiers to be ſet in Rank, and likewiſe in File.

Q. *Of two Brigades, how many the Front, &c.* *Example* 1. A Captain-General hath under his Command two Brigades, one of *Poles* conſiſting of 7225, and the other of *Switzers* conſiſting of 6084 Men; and upon occaſion of Service would form each of them into Square *Battalia*; and again, all of them into one Square Battel: how many Men ſhall be in the Front, and accordingly in the Flank of every of them?

Anſwer. *Anſw.* Of the *Poles* 85, of the *Switzers* 78. And in Square Battel of the Total 115; and there will be 84 Men ſpare.

```
 8             11                              12|84
 .  .           .  .                          .  .  .
7225( 85      6084( 78        7225           13309 ( 115
64            49              6084           1
 80           112            -----            21
  25             64           13309           1125
                             -----
```

Q. *Of an Army marching, what in Rank and File.* *Example* 2. An Army marching in a narrow Paſſage, were 20 in Rank, and 1620 in File; and when they come to pitch the Field, they would embody into a ſquare Form: how many muſt there be in Rank and File?

Anſw. 180: For multiplying 1620 by 20, the whole Number of the Men is found to be 32400; which is a Square Number, and hath 180 for the Root. Anſwer.

$$\begin{array}{r} 1620 \\ 20 \\ \hline 32400 \\ \hline \end{array} \qquad \begin{array}{l} \cancel{2}\cancel{6} \\ \dot{\cancel{3}}\cancel{2}\dot{\cancel{4}}0\dot{0}\Big(180 \\ 1 \\ 16 \\ \;64 \end{array}$$

2. For a double Battel of Men, extract the Square Root of half the Number of Men, and the ſame doubled ſhall be the Number of Souldiers to be ſet in a Rank. *2. Double Battel.*

Example. 1458 ſhall give 54, that is double of 27, the Root of 729, the half of 1458. *Example.*

3. For a Quadruple Battel, extract the Square Root of a Quarter of the Number of Men, and the ſame quadrupled, ſhall be the Number of Souldiers to be ſet in a Rank. *3. Quadruple Battel.*

Example. 1024 ſhall give 64, that is Quadruple of 16, the Root of 256, the Quarter of 1024. *Example.*

4. For a Battel required of any other Form, that is, if a *Ratio* be given according to which the Number of Men in Rank ſhall be to the Number of Men in File; Multiply the 2 Terms of the *Ratio* given: Then as the Product is to the Square of the Term which is for the Rank; or as the Term which is for the File, is to the Term which is for the Rank; ſo is the whole Number of Souldiers, to the Square of the Number of Men to be placed in a Rank. *4. Battel of any other Form.* Analogies.

For in *Species* FR . Rq :: F . R.

Example. 1944 Souldiers are to be marſhalled, ſo that the Number of the Rank be to the Number of the File, as 8 to 3; that is, for 8 in Rank, 3 in File: what ſhall then be the Number of Men in the Rank? *Example.*

Anſw. 72: For the Terms of 8 multiplied into 3 is 24, and the Square of 8 is 64: Therefore as 3 to 8, or as 24 to 64; ſo is 1944 to 5184, which is the Square of 72. Anſwer.

$$\begin{array}{lllll} & \textit{File.} & \textit{Rank.} & \textit{Number.} & \underline{\textit{Q}}.\ \textit{Rank.} \\ \text{As} & 3\ . & 8\ :: & 1944\ . & 5184 \end{array}$$

$$\begin{array}{r} 8 \\ \hline 3\Big)\underline{15552}\Big(\end{array} \begin{array}{l} \overset{2}{\dot{\cancel{5}}}\cancel{1}\dot{8}4\Big(\overset{\surd}{72}\ \text{Rank.} \\ 49 \\ \;284 \end{array}$$

And for Proof thereof, If there be 72 in Rank, which is 9 times 8, then there muſt be 27 in File, which is 9 times 3: And 72 multiplied into 27, produce 1944 the Number given. *Proof thereof.*

In reſpect of the Form of Ground, the Battel is either a Square form of Ground, or longer one way than the other. For the Diſtance or Order of Souldiers marſhalled in Array, may be diſtinguiſhed either into *Open-Order*, or *Order*. *Battels of the ſecond Sort.*

Open-Order, (as *Barriffe* in his *Military Diſcipline* tells us) is when the very Centers of their Places are diſtant 6 Feet aſunder, both in Rank and File. Open-Order, *what.*

Order, is when the Centers of their Places are diſtant 3 Feet, but ſome will have it 3 Feet in Rank and 6 Feet in File, which laſt *Order* and whatſoever *Order* elſe there is, in which the Diſtance of the Ranks one from another is greater than the Diſtance of the Files, cauſeth that a Square of Men maketh not a Square of Ground, but the Ground is longer on the File than on the Rank. So, Order, *what.*

Diſtance, in Rank . R. in File . F. Number, in Rank . R. in File . F.

1. If then it be a Square Battel of Ground, the Centers of the Diſtances being 3 Feet in Rank and 6 in File, becauſe 3 is the Half of 6, the *Ratio* of the Diſtances is as 1 to 2. and ſeeing the Number in Rank, to the Number in File is reciprocal to the Diſtances, the *Ratio* of the Number of Men in Rank, to the Number of Men in File, ſhall be as 2 to 1. and the Rule may be as in the 4*th* ſort of Battels abovementioned: As the Term of the File to the Term of the Ranks; *Square Battel of Ground.* Analogies.

So is the whole Number of Souldiers, to the Square of the true Number of the File.

$$RF . Fq :: R . F :: F . R.$$

Example. *Example.* 1352 Souldiers are to be ſet in a Square of Ground, that their Diſtances may be $3\frac{1}{2}$ Feet in a Rank, and 7 Feet in a File: what ſhall the File be?

Anſwer. *Anſw.* The *Ratio* of the Rank to the File ſhall be reciprocally, as 7 to $3\frac{1}{2}$, that is, as 2 to 1. Therefore,

$$\begin{array}{ccccccc} F & . & R & :: & N & . & Rq. \\ 1 & . & 2 & :: & 1352 & . & 2704 \end{array}$$

$$1\,)\overset{2}{\overline{\dot{2}7\dot{0}4}}(\overset{\surd}{52}\ \text{File.}$$

Proof thereoſ. So the File being 52, the Rank muſt be half ſo much, which is 26, and $52 \times 26 = 1352$, a Proof of the Truth thereof.

Battel when the Ground is a long Square. 2. If a Battel, wherein the Diſtance in Rank is unequal to that in File, that is, be longer one way than the other, according to any *Ratio* given, there is to be conſidered a double *Ratio*, one reciprocal in reſpect of the Diſtances, the other according to the Form of the Ground: wherefore to find the *Ratio* of Men in Rank to the Men in File, *Analogies.* multiply the 2 Terms of the Rank for the Rank, and the 2 Terms of the File for the File; and then the Rule ſhall be as in the 4*th* ſort of Battels above.

Example. *Example.* 10290 Souldiers are to be ſet in *Battalia*, ſo that they may ſtand only 3 Feet aſunder in Rank, and 7 Feet in File, and the Length of the Ground for the Rank to the Length of the Ground for the File, ſhall have the *Ratio* of 5 to 2: what number of Men ſhall be in the Front?

Anſwer. *Anſw.* 245. For the *Ratio* of the Rank to the File, is as 7 to 3, in reſpect of the Diſtances, but in reſpect of the Ground, the *Ratio* of the Rank to the File, is as 5 to 2. So the like Terms multiplied, make 35 and 6. Then as 6 . 35 :: 10290 . 60025 . whoſe Square Root is 245 the Number of Men to be ſet in Rank, briefly expreſſed in Species thus;

$$Ff . Rr :: N . Rq \qquad \text{or, } Rr. Ff :: N . Fq.$$

Proof thereoſ. And for Proof of the Truth thereof, ſeeing for every time 35 in the Rank (that is 7×5) there muſt be 6 in the File (that is 3×2) there being then 7 times 35 in 245, there muſt be 7 times 6, that is 42, in the File, and $42 \times 245 = 10290$ the whole Number of Souldiers.

How to enlarge or diminiſh a Plot of Ground. Hence ariſe Propoſitions, touching enlarging of a Plot of Ground for a Camp, Building, &c. or the contrary.

Example 1. *Example* 1. If 1000 Souldiers may be lodged in a Square of 300 Feet; how many muſt be the ſide of a Square which will ſerve to lodg 5000?

Anſwer. *Anſw.* Almoſt 671, for the Analogy is thus.

$$\begin{array}{llll} \text{N.S} & \text{Q:G} & \text{N.S} & \text{Q:G} \\ \text{As } 1000 \ . & \underbrace{300 \times 300}_{} & ::\ 5000 \ . & 450000 \end{array}$$

$$\begin{array}{l} \quad 90000 \qquad\quad (1 \\ \quad\ \ 5000 \qquad\quad \cancel{9}\cancel{6}(1 \\ 1000\,)\underline{\overline{450000,000}}(\ \dot{4}\cancel{5}\cancel{0}\ \dot{\cancel{0}}(0\dot{0}\,(\overset{\surd}{670}+ \\ \qquad\qquad\qquad\qquad\ \ 36 \\ \qquad\qquad\qquad\qquad\ \ 889 \end{array}$$

Example 2. *Example* 2. An Architect projecting a Building, at firſt layeth out a Plot of Ground of 58 Feet ſquare every way; but afterwards enlarging his Intentions, findeth it neceſſary to take in double the Ground: what muſt then the Side of that Square be that is double to the former?

Anſwer. *Anſw.* 82 Feet and a ſmall matter over: For ſeeing the Ground is to be doubled, the Square of 58 is to be doubled, which is 6728, and the greateſt ſquare Root therein 82, as before.

$$\text{Root } 58 \times 58 = \overset{\textit{Square.}}{3364} \times 2 = \overset{\textit{Doubled.}(4\ \surd}{6728}\Big(\ 82\ \text{Side.}$$

$$\begin{array}{c} 64 \\ 324 \end{array}$$

But if the Propoſition had been to double the Side of the Ground, then the Square muſt have been the Square of 116 the Double of 58, which is four times as much as the former.

$$\text{Root } 58 \times 2 = 116 \times 116 = \overset{\textit{Square.}}{13456}\ \textit{Area.}\ \frac{13456}{3364}\Big(\ 4$$

In all theſe Operations, ſo far as concerns the Extraction of Roots, or *Rule of Three*, have the Proof of their Work already ſpoken to in other Chapters; the reſt proper to this Chapter, hath the Proof in the Anſwers to the Propoſitions particularly illuſtrated.

Proof of doubled Proportions.

CHAP. XVI. *Proportions tripled, &c.*

AFTER the Proportions about Plain Figures, come thoſe uſed about Bodies, or Solids.

Tripled Proportions.

Proportions about Bodies taking in the *Ratio* of the ſeveral Parts, are of themſelves enough to fill a large Treatiſe, eſpecially if the word Body be largely taken.

Proportions of Bodies large.

Albertus Dureus hath wrote a whole Book of the Meaſures of Man's Body, and *Pythagoras* calls Man *the Meaſure of all Things*, becauſe he is the moſt Perfect among all ſublunary Bodies; and according to the Maxim of Philoſophers, that which is moſt Perfect, and the firſt in Rank, meaſureth all the reſt. And we know that the Meaſures of an Inch, Foot, Cubit, Pace, *&c.* have taken their Names and Quantities from Humane Bodies: Yea, ſuch is the admirable Symmetry and Concordancy in the Parts of Man's Body, that ſome well-proportioned Works have been faſhioned and compoſed after his Proportion. As *Noah*'s Ark was in Length 300 Cubits, in Breadth 50, and in Height 30; ſo that the Length doth contain the Breadth 6 times, and ten times the Depth: and ſuch Proportion will be found in the Body of a Man being meaſured.

Of Man's Body.

Meaſures taken from thence.

Proportions of Noah'*s Ark.*

Of Man's Body.

1. The Length of a Man well made (which is commonly called his Height) is equal to the Diſtance, from the one end of his longeſt Finger to the other, when the Arms are extended as far as they may.

Obſervations of Man's Body.

2. The Breadth of a Man (or the Space which is from one ſide to another) makes the ſixth part of all the Body, taken in Length or Height.

3. The Thickneſs of the Body (taken from the Belly to the Back) is the tenth Part of the whole Body, ſome ſay the ninth.

4. The Length of the Face, is equal to the Length of the Hand, taken from the Wriſt to the extremity of the longeſt Finger.

5. The Height of the Brow, the length of the Noſe, the ſpace between the Noſe and the Chin, the Length of the Ears, and the greatneſs of the Thumb, are perfectly equal the one to the other.

6. If a Man have his Feet and Hands extended or ſtretched forth in Form of St. *Andrew*'s Croſs, placing one Foot of a Pair of Compaſſes upon his Navel, there may be deſcribed a Circle which will paſs by the ends of his Fingers and Toes; and drawing Lines by the Terms of them, there may be inſcribed a Square within a Circle.

But theſe and ſuch like are ſet aſide, the intent of this Chapter being to call over thoſe Proportions, converſant about inanimate Bodies or Geometrical Figures that have Solidity; and among them, ſuch as are moſt uſeful in meaſuring of Timber, Stone and other Solids, Queſtions of Gunnery, Pyrotechnie, Gauging

What Bodies are here intended.

ing of Veſſels, and other Mathematical Propoſitions; moſt of which depend upon the Knowledg of the Cube and Globe, or Sphere.

Of Globes and Cubes.

Globes have their capacities one to another, as the Cubes of their Diameters; for by *Euclid* 18 *Prop.* 12 Book, they bear triple that Proportion their Diameters do. And Cubes by *Euclid*, 19 *Prop.* 8 Book, bear triple Proportion in compariſon of their Sides. Hence like *Solids*, are ſaid to be in a Triplicate *Ratio*, when they are as the Cubes of their Homologal Sides; and ſo not unfitly may this Chapter, concerned in the Proportions of ſuch Bodies, bear the Title of *Tripled Proportions*.

Triplicate Ratio's, what.

This agreement between Cubes and Globes, occaſions the Reſolutions to ſeveral Propoſitions concerning them, to agree alſo in the Method of Reſolution.

Propoſitions relating to theſe and other Bodies, commonly concern either their Solidity, Gravity, Value, or Form, or a Mixture of one with the other; ſome more particularly to one Body, and others to another.

Globe or Sphere.

Particularly of the Globe *or* Sphere.

Principal Parts.

Becauſe the Product of the *Axis* into the *Periphery* is the *Superficies*, and that again by $\frac{1}{6}$ of the *Axis* is the *Solidity*: Therefore,

If the *Data* be the {*Axis*, *Superficies*, *Axis*, *Solidity*,} and the *Quæſita* be the {*Superficies*, *Axis*, *Solidity*, *Axis*,}

According to *Archimedes* and *Oughtred* before mentioned,

Analogies therein.

The Analogies *are:*

As 7, to 22,
or 1, to 3,1416 } So is the Square of the *Axis*, to the *Superficies*.

Contrary, as 22, to 7,
or 1, to 0,31831 } So is the *Superficies*, to the Square of the *Axis*.

As 21, to 11,
or 1, to 0,5236 } So is the Cube of the *Axis*, to the *Solidity*.

Contrary, as 11, to 21,
or 1, to 1,90986 } So is the *Solidity*, to the Cube of the *Axis*.

Propoſitions.

Q. Of the Superficies and Solidity.

A Sphere whoſe *Axis* is 14; what is the Superficies thereof? and what the *Solidity* by *Archimedes*?

Anſwer.

Anſw. 616 Superficies, and 1437$\frac{1}{3}$ *Solidity*.

(Q. 14)

As 7 . 22 :: 196 . 616 *Superficies*.

(C. 14)

As 21 . 11 :: 2744 . 1437$\frac{1}{3}$ *Solidity*.

Q. *Of the Axis.*

A *Sphere*, whoſe *Superficies* is 616: what is the *Axis* by *Archimedes*?

Anſwer.

Anſw. 14 *Axis*.

As 22 . 7 :: 616 . 196 (Q. 14) *Axis*.

Q. *Of the Axis.*

A *Sphere*, whoſe *Solidity* is 1437$\frac{1}{3}$: what is the *Axis* by *Archimedes*?

Anſwer.

Anſw. 14 *Axis*.

As 11 . 21 :: 1437$\frac{1}{3}$. 2744 (C. 14) *Axis*.

Cube.

Particularly of the Cube.

Principal Parts.

If the *Data* be the {Side or Root, Solidity} and the *Quæſita* be the {Solidity. Side or Root:}

For the firſt, multiply the ſide cubically, and you have the Solidity.

For the Solidity.

For the ſecond, extract the Cube Root of the Solidity, as before in Figural Numbers.

For the Root.

But if the Propoſition require the Number ſought, in another Denomination to that given: then, as in the Chapter laſt before about Squares and long Squares, ſo here in Cubes and long Cubes, *&c.* divide for a Greater, and multiply for a leſſer Denomination.

If another Denomination be ſought.

Example 1. A piece of Timber is 18 Inches Square: how many ſolid Feet are therein?

Q. *Of the Feet in a Piece of Timber.*

Anſw. $3\frac{3}{8}$ Feet: for ſeeing Feet are an higher Denomination than Inches, the Cube of 18, is divided by the ſolid Inches in one Foot, that is, 1728 (or 12 × 12 × 12). And if 18 be turned into $1\frac{1}{2}$ Feet, the Triple *Ratio* will effect as much.

Anſwer.

$$\begin{array}{lcl} & & (648 \\ \text{Side } 18 \times 18 \times 18 & = & \dfrac{5832}{1728}\Big(3\tfrac{3}{8}\text{ Feet} \\ \text{Solid Inches in 1 Foot} & & \end{array} \qquad \frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}=\frac{\overset{(3}{27}}{8}\Big(3\tfrac{3}{8}$$

Example 2. A Cubical Body is $1\frac{1}{2}$ Foot Square every way: how many ſolid Inches are therein?

Q. *Of the Inches in a Cube.*

Anſw. 5832 Inches: For Inches being leſſer than Feet, the $3\frac{3}{8}$ Feet, Solid Content, are multiplied by 1728, or $1\frac{1}{2}$ Foot is reduced into Inches, and multiplied cubically.

Anſwer.

$$\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}=\frac{27}{8}\Big(3\tfrac{3}{8}\text{ Feet.}$$

$$\begin{array}{rr} & 1728 \\ & 3\tfrac{3}{8} \\ \hline & 5184 \\ & 648 \\ \hline 18\times18\times18 = & \underline{5832}\text{ Inches.} \end{array}$$

Example 3. A Rock exactly cubical, containeth 1728 ſolid Feet: how many Inches does one Side contain?

Q. *Of Inches in the Side of a Rock.*

Anſw. 144 Inches: The Cube Root of 1728 Feet extracted, is 12 Feet; but Inches being of leſſer Denomination, 12 is multiplied by 12, the Inches in a Foot.

Anſwer.

$$\begin{array}{l} .\,1\,.\quad\surd \\ 1728\big(\,12\times12=144\text{ Inches.} \\ 16 \\ \;\;128\end{array}$$

Example 4. A Mount of Cubick Form containeth 2985984 ſolid Inches: how many Feet doth one Side contain?

Q. *Of Feet in the Side of a Mount.*

Anſw. 12 Feet: The Cube Root of 2985984 is 144, to be divided by 12, becauſe Feet deſired is the higher Denomination.

Anſwer.

$$\begin{array}{l} 1241 \\ \dot{2}98\dot{5}98\dot{4}\Big(\overset{\surd}{\dfrac{144}{12}}\Big(12\text{ Feet.} \\ 1 \\ 1744 \\ \;\;241984 \end{array}$$

Thoſe Bodies that are cubical, yet not exact Cubes, order as Cubes.

Cubical Bodies.

Example 1. A Tree hewed Square, 2 Feet broad, $1\frac{1}{2}$ Foot thick, and 3 Yards long: how many ſolid Feet are therein?

Q. *Of Feet in a Tree.*

Anſw. 27 Feet: For $2\times1\frac{1}{2}$ produces 3 Feet for the *Area* of the Plain; which multiplied into the Length 9 Feet, every Yard being 3, makes the Product 27 as before for the Solidity. And in like manner the Solidity of other Bodies, not regular Cubes, is to be found.

Anſwer.

$$\begin{array}{lrcl} \text{Feet} & 2\times1\tfrac{1}{2} & = & 3 \\ \text{Yards} & 3\times3 & = & \underline{9} \\ & & & \underline{27}\text{ Feet ſolid.}\end{array}$$

Q. Of Feet in a Plank. *Example* 2. A Plank 20 Inches broad, $2\frac{1}{2}$ Inches thick, and 3 Yards long: how many ſolid Feet are therein?

Anſwer. *Anſw.* $3\frac{1}{8}$ Feet: For 50 the *Area* of the Plain, made by the Breadth and Thickneſs, multiplied into the Length, which is 108 Inches, gives the Solidity in Inches 5400; this divided by 1728 the ſolid Inches in a Foot, gives in the Quotient $3\frac{1}{8}$ as before.

$$\begin{array}{llcl} \text{Inches} & 20 \times 2\tfrac{1}{2} & = & 50 \\ \text{Yards} & 3 \times 36 & = & \underline{108} \\ & \text{Solid} & & \underline{5400}\ \text{Inches.} \end{array} \qquad \begin{array}{c} (216 \\ \hline \cancel{5}40\cancel{0} \\ \hline \cancel{1}728 \end{array}\Big(\ 3\tfrac{1}{8}\ \text{Feet.}$$

What Part of ſuch Bodies, &c. In theſe and ſuch other Bodies, if it be deſired to know what Portion of Length there muſt be to any Breadth and Thickneſs given to make a Foot cubical, or ſuch other ſolid Content: then by the *Area* made of the Breadth and Thickneſs, divide the Cubick Foot, or other Solid Content; for the Analogy is reciprocal, as in the *Indirect Rule of Three.*

Q. Of Length for a Foot. *Example* 1. A Piece of Timber is 8 Inches broad, and 9 Inches thick: how much in Length ſhall make a Foot thereof?

Anſwer. *Anſw.* 24 Inches, or 2 Feet: For the *Area* of 8 × 9, is 72; which dividing 1728, the ſolid Inches in 1 Foot, the Quotient is 24: Or by the *Indirect Rule of Three,* As 144 to 12; ſo 8 × 9 to 24.

$$8 \times 9 = \frac{\overset{\cancel{2}8}{\cancel{1}7\cancel{2}8}}{7\cancel{2}}\Big(\ 24\ \text{Inches. Or as}\ \underbrace{\overset{(12\times12)}{144}\ .\ 12}_{72)\underline{1728}(24}\ ::\ \overset{(8\times9)}{72}\ .\ 24\ \text{Inches.}$$

Q. Of the Length for a Yard of a Wall. *Example* 2. A Bricklayer hath undertaken to build a Brick-Wall of 3 Feet thick and 4 Feet high, to be paid for his Work by the Yard cubick; and would know how much Length ſhall make a cubical Yard of that Wall?

Anſwer. *Anſw.* $2\frac{1}{4}$ Feet: For 27 the Yard cubick divided by 12, the Product of the Height and Thickneſs, gives $2\frac{1}{4}$ in the Quotient.

$$\begin{array}{rcl} 3 \times 3 \times 3 & = & \overset{(\cancel{3}}{\cancel{2}7} \\ \hline 3 \times 4 & = & \cancel{1}\cancel{2} \end{array}\Big(\ 2\tfrac{1}{4}\ \text{Feet: Or as}\ \underbrace{\overset{(3\times3)}{9}\ .\ 3}_{12)\underline{27}(2\frac{1}{4}}\ ::\ \overset{(3\times4)}{12}\ .\ 2\tfrac{1}{4}\ \text{Feet.}$$

Jointly, of the Gravity, Value *and* Form.

For the Weight, Value & Form. *Analogy between the Weight of Bodies of the ſame Kind.* The *Gravity* or Weight of Bodies of the ſame Kind, is according to the Cubes of their Diameters or Axis in Spherical Bodies, and of their Sides in cubical Bodies in direct Proportion.

Q. Of the Weight of a Bullet. *Example* 1. Suppoſe a Bullet 8 Inches Diameter weigh 30 ℔: what ſhall a Bullet of the ſame Kind weigh that is but 4 Inches Diameter?

Anſwer. *Anſw.* $3\frac{3}{4}$ ℔: Here the Cube of 8 to 30, is as the Cube of 4 to $3\frac{3}{4}$: Or in leſs Terms, the Cube of 2 to 30, is as the Cube of 1 to $3\frac{3}{4}$: For 8 to 4, is as 2 to 1.

$$\frac{8}{4} = \frac{2}{1} \quad \text{As}\ \overset{(C.8)}{512}\ .\ \overset{℔}{30}\ ::\ \overset{(C.4)}{64}\ .\ \overset{℔}{3\tfrac{3}{4}} \qquad \text{Or, As}\ \overset{(C\,2)}{8}\ .\ \overset{℔}{30}\ ::\ \overset{(C.1)}{1}\ .\ \overset{℔}{3\tfrac{3}{4}}$$

$$512)\overset{64}{\underline{1920}}(3\tfrac{3}{4} \qquad\qquad \frac{\overset{(6}{\cancel{3}\cancel{0}}}{8}\Big(\ 3\tfrac{3}{4}$$

Q. Of the Weight of a Bullet. *Example* 2. If a Gun of $5\frac{1}{2}$ Inches Diameter in the Mouth, ſhoot a Bullet of $20\frac{5}{6}\frac{1}{4}$ ℔: what ſhall that Bullet weigh that ſerveth for a Gun of 8 Inches Diameter?

Anſwer. *Anſw.* 64 ℔: Here the Cubes may be reduced into Halfs; that is, 8 into 16, and $5\frac{1}{2}$ into 11: Or elſe the *Analogy* ſet as in Fractions;

As

(C.11) ℔ (C.16) ℔

As 1331 . $20\frac{51}{64}$ :: 4096 . 64

$20\frac{51}{64}$

81920

3264

1331)85184(64

(C.$5\frac{1}{2}$) ℔ (C.8) ℔

Or, As $\frac{1331}{8}$. $\frac{1331}{64}$ :: 512 . 64

And by cancelling the like Numerators 1331, after the manner of Fractions, it ſhall be,

As $\frac{1}{8}$. $\frac{1}{64}$ :: 512 . 64

Example 3. Let a Cube of Metal of 4 Inches ſquare, weigh 8 ℔: what ſhall a Cube of the ſame Metal weigh that is 5 Inches ſquare?

Q. Of the Weight of a Cube.

Anſw. $15\frac{5}{8}$ ℔: For the Cube of 4 to 8, is as the Cube of 5 to $15\frac{5}{8}$.

Anſwer.

(C.4) ℔ (C.5) ℔

As 64 . 8 :: 125 . $15\frac{5}{8}$

8

64)1000($15\frac{5}{8}$.

Example 4. When a Cube of Metal, that is $3\frac{1}{2}$ Inches every way, weigheth 64 Ounces *Troy* Weight: what ſhall a Cube of the ſame Metal weigh that is $5\frac{1}{4}$ Inches?

Q. Of the Weight of a Cube.

Anſw. 216 Ounces *Troy*: For the Cube of 14 the Quarters in $3\frac{1}{2}$, is to 64, as the Cube of 21 the Quarters in $5\frac{1}{4}$ is to 216.

Anſwer.

(C.14) ℥ (C.21) ℥

As 2744 . 64 :: 9261 . 216

64

37044

55566

592704

164

4396

592704(216

2744

2744

2744

Contrariwiſe; If the Gravity of one Body be given, to find the Side of another Cubical, or the Axis of a Spherical Body: Then as the Roots of the given Numbers one to the other; ſo ſhall the Side or Axis given be, to that required: Or as the given Numbers one to another; ſo ſhall the Cube of the Side or Axis given be, to that required.

Analogy to find the Side or Axis.

Example 1. If a Cube of 64 Ounces *Troy*, have the Side $3\frac{1}{2}$ Inches: what ſhall the Side of that Cube be that weigheth 216 Ounces *Troy*?

Q. Of the Side of a Cube.

Anſw. $5\frac{1}{4}$ Inches: The Cube Roots of 64 and 216, which are 4 and 6, ſet in leſs Terms are 2 and 3: Therefore as 2 to $3\frac{1}{2}$; ſo is 3 to $5\frac{1}{4}$.

Anſwer.

Cubes . 64 . 216

Roots . 4 . 6

Leaſt Terms . 2 : 3

Inches. *Inches.*

As 2 . $3\frac{1}{2}$ :: 3 . $5\frac{1}{4}$

$\frac{2}{1}$)$\frac{21}{2}$($\frac{21}{4}$

Example 2. In caſe a Bullet of 4 Inches in the Axis weigh $3\frac{3}{4}$ ℔ *Averdupois*: what ſhall be the Axis of a Bullet of the ſame Kind that weigheth 30 ℔?

Q. Of the Axis of a Bullet.

Anſw. 8 Inches: Where the firſt and third Numbers not being cubical, 4 is cubed; and after *Multiplication* and *Diviſion*, the Root of the Quotient is extracted.

Anſwer.

(C.4)

As $3\frac{3}{4}$. 64 :: 30 . 8 Inches.

30

$\frac{15}{4}$)$\frac{1920}{1}$($\frac{7680}{15}$(512($\sqrt{}$ 8

Among like Solids in a Triplicate *Ratio*, that is as the Cubes of their Homologal Sides: If 3 Numbers be given, in which as the Cube of the Firſt is to the Cube of the Second; ſo the Third to a Number ſought: Then as the firſt Cube to the third Number; ſo is the ſecond Cube to the Number ſought.

Triplicate Ratio's.

Example

Q. *Of Powder to charge a Gun.* *Example* 1. If $\frac{43}{100}$ ℔ of Gunpowder ſuffice to charge a Gun, whoſe Concave Diameter is $1\frac{1}{2}$ Inch: how many Pounds of the ſame Powder will ſuffice to charge a Gun whoſe Concave Diameter is 7 Inches?

Anſwer. *Anſw.* $43{,}7\frac{1}{135}$ ℔: For as the Cube of $1\frac{1}{2}$, or 1,5 to the Cube of 7; ſo is 0,43 to 43,7 and $\frac{1}{135}$ of a Prime.

```
        (C.1,5)           (C.7)
As      3,375 . 0,43 :: 343 . 43,7 1/135 Gunpowder.
                343              (2
               -----          22 365(5    ℔
                 129         147,490 0(  43,7 1/135
                172            3,375
               129              3,375
               ------            3,375
               147,49
```

Q. *Of Powder to charge a Gun.* *Example* 2. Suppoſe 43,7 ℔ of Gunpowder are ſufficient to charge a Gun, whoſe Diameter in the Concave is 7 Inches: And there is another Sort of Gunpowder more ſtrong, that is to the former as 5 to 2: how much of this will ſuffice to charge a Gun of 4 Inches Diameter?

Anſwer. *Anſw.* 3,26 ℔ and ſomewhat more: As appears, firſt, by the Proportions of the Powder, ſeeking how much of that ſtrong Powder is enough to charge a Gun of 7 Inches Diameter; and then, by a ſecond Work, with the Proportions of the Diameters.

```
            ℔               ℔             (C.7)   ℔     (C.4.)   ℔
As   5 .  43,7  ::  2  .  17,48      As  343 . 17,48 :: 64 . 3,26 54/343
             2     32                          64         21(5
           ----   87,40(  ℔                  -----        89 1(4   ℔
           87,4   -----  17,48                69 92     1118,7 2( 3,26 54/343
                    5                       1048 8      --------
                                            -------       343
                                            1118,72
```

Proportions of Magnitudes and Gravities of Bodies, a curious Enquiry. The Proportions as well of the Magnitudes, as of the Gravities of Bodies one to another, is a nice Inquiſition; and among Authors ſome difference is found, according as the Experiments of the one have been more or leſs curious than the other.

What by Van Etten. *Henry van Etten*, in his *Mathematical Recreations*, acquaints us, That a Quantity of Water to an equal Quantity of Metal, is in Proportion as,

Water.	*Tin.*	*Iron.*	*Copper.*	*Silver.*	*Lead.*	*Gold.*
10	75	81	91	104	$116\frac{1}{2}$	$187\frac{1}{2}$

What by Alſted. Herewith agreeth the Learned *Alſted*, ſave that *Iron* he makes to be but $80\frac{22}{37}$, and adds, Oil 9, Honey 15, and Quickſilver 150.

So as a Magnitude of Gold, weighing $187\frac{1}{2}$ ℔. ſhall have an equal magnitude of Silver weigh but 104 ℔. and the like of the reſt. But for that there is not only difference in the Weight of Freſh Water and Salt, but between Freſh Waters themſelves, ſome being more Mineral than others, according to the Tincture they receive by the Mines, along which they glide in the Bowels of the Earth; therefore ſome accompt this but an uncertain Baſis, and as unſtable as the Water it ſelf.

What by Mr. Oughtred, *according to* Ghetald. Mr. *Oughtred* (often already mentioned) in his *Circles of Proportion*, pag. 67, 68, *&c.* tells us, That pieces of Metal, if they be of equal Magnitude, have their Weights in direct Proportion; but if of equal Weight, they have their Magnitudes in Proportion reciprocal. And inſerts the Proportions of their Weights, according to the Experiments of *Marinus Ghetaldus*, in his Tractate, called *Archimedes Promotus*, thus:

☉	Gold	3990	Wherefore	☉ . ☿ :: 7 . 5 :: 3990 . 2850.
☿	Quickſilver	2850		☉ . ♄ :: 38 . 23 :: 3990 . 2415.
♄	Lead	2415		☉ . ☽ :: 57 . 31 :: 3990 . 2170.
☽	Silver	2170		☉ . ♀ :: 19 . 9 :: 3990 . 1890.
♀	Braſs	1890		☉ . ♂ :: 19 . 8 :: 3990 . 1680.
♂	Iron	1680		☉ . ♃ :: 95 . 37 :: 3990 . 1554.
♃	Tin	1554		

Therefore if 4 Pieces of Metals, whereof the Third is of the ſame kind with the firſt, and the fourth of the ſame kind with the ſecond, are Proportional; their Gravities alſo are Proportional.

And again, if there be 4 Pieces of Metals, whereof the Third is of the ſame kind with the Firſt, and the Fourth of the ſame kind with the Second, and the Firſt and Second of equal Greatneſs, and the Third and Fourth of equal Weight; the Gravities of the Firſt and Second ſhall be reciprocal to the Magnitudes of the Third and Fourth.

Weight of a Cylinder of Tin of an Inch long, &c.

The aforeſaid *Ghetald* (uſing the antient Roman Foot, which by his Accompt ſeems very little leſs than ours) found a Cylinder of Tin of 1 Inch thick and long, to weigh 1824 Grains, whereof $\frac{2}{3}$ is 1216 for the Weight of a Sphere of that Thickneſs; becauſe every Sphere is $\frac{2}{3}$ of a Cylinder that hath the Height and Diameter of the Baſe, the ſame with the *Axis* of the *Sphere*.

Weight of a Sphere.

(C. 10) (C. 12)
Then as 1000 . 1216 :: 1728 . 2101,248. the Weight of a Sphere whoſe Diameter is $\frac{1}{10}$ of a Foot; or ſet according to the manner of other Proportions, the Queſtion will ſtand thus: If a Sphere whoſe Diameter is $\frac{1}{12}$ of a Foot weigh 1216 Grains, what ſhall that Sphere weigh, whoſe Diameter is $\frac{1}{10}$ of a Foot? which in reaſon becauſe biggeſt muſt weigh moſt; and ſo the Cube of 10 being leſs than the Cube of 12, will be Diviſor as before.

Weight of a Cube.

And by the ſame Accompt, a cubed Inch of Tin weigheth 2322,3887324; and a cubed tenth Part of a Foot 4013,0877296.

How thereby to find the Weight of a Sphere of another Axis,

Accordingly to find the Weight of a Sphere of Tin of any other *Axis*, multiply the Cube of the *Axis* given, by 1216, if it be in Inch-meaſure, or by 2101,248, if it be by Decimal Parts of a Foot; and the Product will be the Weight of that Sphere.

And the Weight of a Sphere of another Metal.

But to find the Weight of a Sphere of any other Metal at any Diameter, aſſigned either in Inch-meaſure, or Decimal Parts of a Foot: ſeek the Weight of a Sphere of Tin at that Diameter given; and then as the proportional Number of Tin is to the Weight of the Sphere of Tin; ſo is the Proportional Number of the other Metal, to the Weight of the Sphere propoſed.

Example in Iron for the Weight.

Example. Suppoſe a Sphere of Iron have the Diameter 3 Inches: what ſhall the Weight be?

Anſw. 35494,054 Grains, and ſome ſmall Overplus.

(C. 1) (C. 3)
As 1 . 1216 :: 27 . 32832. Grains in the Sphere of Tin.
♃ ♂
As 1554 . 32832 :: 1680 . 35494,054. Grains in the Sphere of Iron.

To find the Diameter or Axis of a Sphere of other Metal.

And to find the Diameter of a Sphere of any Metal in Inch-meaſure, or Decimal Parts of a Foot by the Weight, ſeek the Cube of the Diameter of a Sphere of Tin of that Weight: and then as the Proportional Number of the other Metal, is to the Cube of the Diameter found; ſo is the Proportional Number of Tin, to the Cube of the Diameter of the Sphere propoſed.

Example in Iron for the Axis.

Example. A Sphere of Iron weigheth 35494,054 Grains: what is the Diameter thereof?

Anſw. 3 Inches.

Cube *Cube*
As 1216 . 1 :: 35494,054 . 29,189189144, *&c.*
♂ ♃
As 1680 . 29,18918914 :: 1554 . 27 Root 3.

Of the Value of Bodies.

The Value of all Bodies, is accompted according to their Solidity or Gravity; ſo as the Solidity or Gravity gotten, the Value is to be had by the *Rule of Three Direct*.

Q. Of the Worth of an Iron Bullet.

Example 1. At 15 *l.* the Tun, what is an Iron Bullet worth of 3 Inches Diameter?

Anſwer.

Anſw. $\overset{s.}{0,618967}$. or 7 Pence Farthing and ſomewhat above: For by the Work above, the Weight of ſuch a Bullet is found to be 35494,054 Grains; and then if one Tun coſt 15 *l.* or 1 Hundred 15 *s.* which is all as one, becauſe 20 Hundred are 1 Tun, the Weight of the Bullet, ſhall give the Sum aforeſaid.

Grains in 1 C. *s.* *Grains in the Bullet.* *s.*
As 860160 . 15 :: 35494,054 . 0,618967, *&c.*

Q. Of the Worth of a Piece of Timber.

Example 2. A Piece of Timber is 2 Feet broad, 3 Feet thick, and 6 Feet long: what doth it come to at 20 *s.* a Tun?

Anſwer.

Anſw. 18 *s.* For the Solidity found as before to be 36 ſolid Feet; and 40 Feet being a Tun, it ſhall be worth 18 *s.*

2 × 3 × 6 = 36. As 40 . $\overset{s.}{20}$ :: 36 . $\overset{s.}{18}$.

Of the Form of Bodies, 2 Things.

Touching the Form of Bodies, Propoſitions of two ſorts are uſual.

1. To increaſe or diminiſh the Body, and yet keep the ſame Form: Or,
2. To alter the Form, and yet keep the ſame Solidity or Gravity.

1. To increaſe or leſſen, &c.

To increaſe a Body Spherical or Cubical, Double or Triple, *&c.* as is deſired, the Solidity; and to diminiſh the ſame, do the contrary.

Q. Of the Axis of a Globe, and Side of a Cube, doubled.

Example 1. A Globe whoſe *Axis* is 14, and a Cube of the ſame ſide, are both to be doubled: what ſhall the *Axis* of the One, and the Side of the other ſo doubled be?

Anſwer.

Anſw. The deſired *Axis* and Side ſhall be between 17 and 18, becauſe the Solidity of the Globe, whoſe *Axis* is 14, being according to *Archimedes* 1437$\frac{1}{3}$, doubled is 2874$\frac{2}{3}$; and this Solidity ſhall have for the *Axis* the Cube Root of 5488, which is the Double of 2744 the Cube of 14.

Cube.
Root 14 × 14 × 14 = 2744
Doubled $\underline{5488}$

(5
2~~9~~(7
4~~3~~1(5
~~5~~~~4~~88(17$\underline{575}$
1
21
147
343

Periphery.
Axis 14 × 44 = 616 Superficies.
$\frac{1}{6}$ of the Axis $\underline{2\frac{1}{3}}$
1232
$\underline{205\frac{1}{3}}$
1437$\frac{1}{3}$ Solidity.
Doubled $\underline{2874\frac{2}{3}}$

As 11 . 21 :: 2874$\frac{2}{3}$. 5488 (C. 17$\underline{575}$)
$\underline{21}$
2874
5748
$\underline{14}$
11)60368(5488

Q. Of the Axis of a Globe, and Side of a Cube halfed.

Example 2. A Globe and *Cube*, whoſe Axis and Side is 14, are both to be made but half ſo much: what then ſhall the *Axis* of the one, and the Side of the other be?

Anſwer.

Anſw. 11 and a ſmall matter over: For ſo half 2744 the Cube of 14, and half 1437$\frac{1}{3}$ the ſolid Sphere of 14, according to *Archimedes*, will both give.

Root 14. Cube $\underline{2744}$ Axis 14 . Solidity $\underline{1437\frac{1}{3}}$

Half $\underline{1372}$ Half $\underline{718\frac{2}{3}}$

$$\begin{array}{l}(41\\ 1372\,(\,11\frac{41}{\ }\\ 1331\end{array}$$

As 11 . 21 :: $718\frac{2}{3}$. 1372 (C. $11\frac{41}{\ }$

$$\begin{array}{r}718\frac{2}{3}\\ 21\\ \hline 718\\ 1436\\ 14\\ 11\,)\,\overline{15092}\,(\,1372.\end{array}$$

And thus the Book called, *The Treaſure of Travellers*, teacheth in the Building of Ships, how to increaſe or diminiſh their Burden, by increaſing or diminiſhing accordingly their Length of the Keel, Breadth at the Main-Beam, and Depth in the Hold: and alſo to fit their Cables, Ropes, Maſts, *&c.* though as to the Tackle and Rigging of Ships, Nautical Experience gives the beſt Directions. *How to increaſe or leſſen the Burden of Ships.*

So if a Ship of 60 Tuns, have the Length of the Keel 32 Feet within the Poſt; and another were to be built of 120 Tuns, or of 30 Tuns: Then the Cube of 32, which 32768 doubled for the Greater, and halfed for the Leſſer, and the Roots accordingly taken, the Keel of the Greater will be ſomewhat above 40 Feet, and of the Leſſer 25 Feet and better. *Example.*

Keel 32. Cube 32768, Doubled 65536, Halfed 16384.

$$\begin{array}{l}1\\ 65536\,(\,40\frac{1536}{\ }\\ 64\end{array}\qquad\begin{array}{l}8|\underline{759}\\ 16384\,(\,25\frac{759}{\ }\\ 8\\ 7625\end{array}$$

In all Cubical Bodies, the doubling of one Side doubles the whole Body, and halfing of one Side accordingly leſſens the Body by half: So as in Bodies that are not exact Cubes, if the Increaſe or Decreaſe be only deſired; then their Solidity is to be increaſed or decreaſed accordingly: But if their Form be to be kept, then the beſt way is to increaſe or decreaſe one Side. *What the doubling or halfing of the Side of Cubical Bodies effects.*

As if a Piece of Timber 2 Feet broad, three Feet thick, and 6 Feet long, be doubled; then the Solidity 36 doubled ſhall be 72 equal to a Piece of 4 Feet broad, 3 Feet thick, and 6 Feet long; or 2 Feet broad, 6 Feet thick, and 6 Feet long; or 2 Feet broad, 3 Feet thick, and 12 Feet long. *Example in the Double.*

Breadth.		*Thickneſs.*		*Length.*		*Solidity.*
2	×	3	×	6	=	36
				Doubled		72

$$\left.\begin{array}{l}4\times3\times6\\ 2\times6\times6\\ 2\times3\times12\end{array}\right\}=72$$

Likewiſe if the ſame Piece of Timber were to be halfed, Then as the Half of 36 is 18; ſo ſhall be the Product of half one of the Sides multiplied into the reſt. *Example in the Half.*

Breadth.		*Thickneſs.*		*Length.*		*Solidity.*
2	×	3	×	6	=	36
				Half		18

$$\left.\begin{array}{l}1\times3\times6\\ 2\times1\frac{1}{2}\times6\\ 2\times3\times3\end{array}\right\}=18$$

To alter the Form, and keep the Solidity or Gravity, is moſt uſual in irregular Bodies, to reduce them to Cubes or Spheres; which when their Solidity or Gravity is had thereby, the Axis of the one, or Side of the other, is to be had as in the *Examples* before. *2. How to alter the Form, and keep the Solidity or Gravity.*

And not only Irregulars, but others alſo, as Cones, Cylinders, Pyramids, Priſms, *&c.* may be thus reduced without any Difficulty, when their Solidity or Gravity is found; but in getting thereof they differ one from another, as is well known by the Rudiments of *Geometry*, to which their Meaſure properly belongs: Yet it may not be unprofitable here to ſee of ſome of them, ſome of their *Analogies*:

Of

Cone.

Of a Right Cone.

Becauſe a Sphere is equal to two *Cones* that have their Height, and the Diameter of their Baſe the ſame with the Axis of the Sphere: The *Analogies*, by *Archimedes*, common to the *Cone*, are:

Analogies thereof.

As 7 to 22; ſo is half the Diameter multiplied in the Side, to the Conical Superficies.

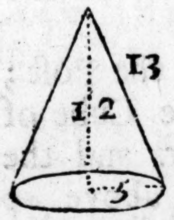
As 14 to 11; ſo is the Square of the Diameter multiplied in $\frac{1}{3}$ of the Axis, to the Solidity.

To the Conical Superficies, if the *Area* of the Baſe be added, the Total ſhall be the Superficies of the whole *Cone*.

Example.

Wherefore if a right *Cone* have the Diameter at the Baſe 10, and the Height or Axis 12; then ſhall the Side be 13, the Conical Superficies $204\frac{2}{7}$, the Total Superficies $282\frac{6}{7}$, and the Solidity $314\frac{2}{7}$.

The Measure of a Cone.



$$5 \times 5 = 25$$
$$12 \times 12 = \underline{144}\ \surd$$
$$\underline{169}(\ 13 \text{ Side.}$$

As $7 . 22 :: 10 . 31\frac{3}{7}$ *Periphery.*

$5 \times 15\frac{5}{7} = 78\frac{4}{7}$ *Area* of the Baſe.

As $7.22::\overset{(5\times13)}{65}.204\frac{2}{7}$ Conical Superficies.
$\underline{78\frac{4}{7}}$ *Area* of the Baſe added.
$\underline{282\frac{6}{7}}$ Total Superficies.

As $14 . 11 :: \overset{(Q.10\times4)}{400} . 314\frac{2}{7}$ Solidity.

$$\begin{array}{r} 11 \\ \hline 14)\underline{4400}(\ 314\frac{2}{7} \end{array}$$

Variety.

So if the Side and half the Periphery be multiplied, the Product is the Conical Superficies.

And if $\frac{1}{3}$ of the *Area* at the Baſe be multiplied into the Height, or $\frac{1}{3}$ of the Height into the *Area*, the Solidity is had.

$$13 \times 15\frac{5}{7} = 204\frac{2}{7} \text{ Conical Superficies.}$$

$$\left.\begin{array}{c} 26\frac{4}{21} \times 12 \\ \text{or} \\ 78\frac{4}{7} \times 4 \end{array}\right\} = 314\frac{2}{7} \text{ Solidity.}$$

Cylinder.

Of a Right Cylinder.

Becauſe $\frac{2}{3}$ of a *Cylinder* is equal to a Sphere, that hath the Height and Diameter of the Baſe the ſame with the Axis of a Sphere: The Common *Analogies* by *Archimedes* in the *Cylinder* are;

Analogies thereof.

As 7, to 22; ſo is the Diameter multiplied in the Axis, to the Superficies-Cylindrical: to which add the *Area* of both the Baſes for the Total Superficies.

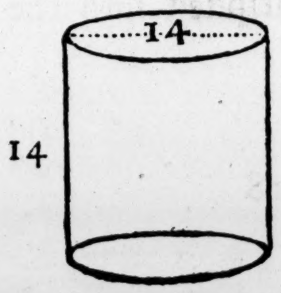
As 14, to 11; ſo is the Square of the Diameter multiplied in the Side to the Solidity.

Or as 88, to 7; ſo is the Square of the *Periphery* multiplied in the Side to the Solidity.

Example.

Therefore if a Right *Cylinder* have the Diameter 14, and the Height or Axis as much; then ſhall the *Area* of each Baſe be 154, the Cylindrical Superficies 616, the Total Superficies 924, and the Solidity 2156.

The Measure of a Cylinder.



As $7 . 22 :: \overset{D}{14} . 44$ *Periphery.*
Half $7 \times 22 = 154$ *Area* of the Baſe.
Doubled 308 Baſes.

As $7 . 22 :: \overset{D}{14} \times \overset{L}{14} . 616$ Cylindrical Superficies.
$\underline{308}$ Baſes added.
$\underline{924}$ Total Superficies.

$$\left.\begin{array}{l} \text{As } 14 . 11 :: \overset{(Q.14D)}{196} \times \overset{L}{14}. \\ \text{Or as } 88 . 7 :: \overset{(Q.44P)}{1936} \times \overset{L}{14}. \end{array}\right\} 2156 \text{ Solidity.}$$

Variety.

So alſo if the *Periphery* be multiplied into the Height or Axis; the Product is the Cylindrical Superficies.

And if the *Area* of one Baſe be multiplied into the Axis, the Solidity is found.

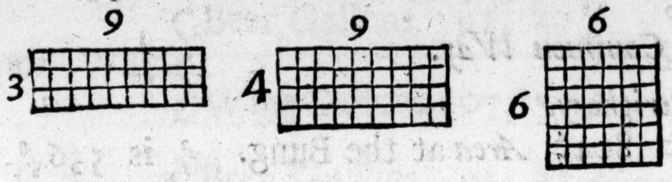
$$44 \times 14 = 616 \text{ Cylindrical-Superficies.}$$
$$154 \times 14 = 2156 \text{ Solidity.}$$

Hereby appeareth, 1. That it is not ſafe to judg the Value of Bodies by their Surface or Perimeter, but by their Solidity or Gravity, as was ſaid before: For Plain Figures, called *Ipſoperimeters*, and alſo Bodies of Equal Surface, may be vaſtly different in their *Area's* and Solid Contents; as thoſe of different Sides and Diameters may have their *Area's* and Solidities equal.

How to value Ipſoperimeters *and Bodies of equal Surfaces.*

For let there be among plain Rectangle Figures, one whoſe Sides are 3 and 9, another 4 and 9, and a Square 6 and 6; it is evident that the Sides of the Firſt are Ipſoperimetral with the Sides of the Square, that is, both 24: Yet the *Area* of the one is but 27, a Quarter leſs than 36 the Square *Area*. And though the *Perimeter* of the Second be 26, when the *Perimeter* of the Square is but 24; yet the *Area* of the ſecond Figure is 36, equal to the *Area* of the Square.

Examples in Plains.

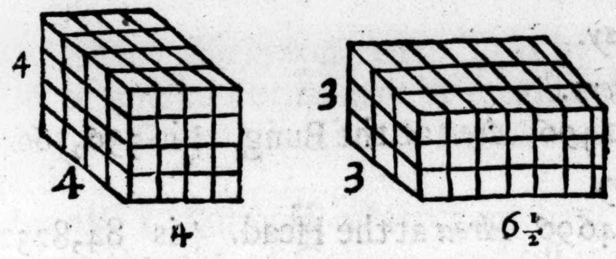


$$6 + 6 \times 2 = 24$$
$$6 \times 6 = 36$$

Perimeter 24	*Perimeter* 26	*Perimeter* 24
Area 27	*Area* 36	*Area* 36

So in Solids, if one Body whoſe Length is $6\frac{1}{2}$ Feet, Breadth and Thickneſs alike 3 Feet, be examined and compared with the Cube of 4 Feet; their Surfaces will be equal, but their Solidities as different as $58\frac{1}{2}$ from 64.

Examples in Solids.



Cube.

Superficies $16 \times 6 = 96.$
Solidity $4 \times 4 \times 4 = 64.$

Long Cube.

Solidity $3 \times 3 \times 6\frac{1}{2} = 58\frac{1}{2}$
Superficies $\left\{\begin{matrix} 3 \times 3 \times 2 \\ 6\frac{1}{2} \times 3 \times 4 \end{matrix}\right\} = 96.$

Again, let a Cube whoſe ſide is 14, be compared with a Sphere of the ſame Diameter, and a Cone and Cylinder of equal Diameter and Height; both the Superficies and Solidities will be thus different.

		Superficies	Solidities
14	Side or Root of the Cube,	1176	2744
	Diameter or Axis of the Sphere,	616	$1437\frac{1}{3}$
	Height and Diameter of the Cone,	$498\frac{6}{31}$	$718\frac{2}{3}$
	Height and Diameter of the Cylinder,	924	2156

Wherefore it is evident, if *Sempronius* borrow of *Caius* a Sack of Corn 4 Feet high, and 2 Feet over, and pay him again with 2 Sacks of Corn, each 4 Feet high and 1 Foot over; yet hath *Caius* been paid but one Quarter of his Corn.

Caius *paid too little.*

As $7 . 22 :: 2 . 6\frac{2}{7}$
$1 \times 3\frac{1}{7} = 3\frac{1}{7}$
4
Lent $12\frac{4}{7}$

As $7 . 22 :: 1 . 3\frac{1}{7}$ Paid.
$\frac{1}{2} \times 1\frac{4}{7} = \frac{11}{14} \times 4 = \frac{44}{14}(3\frac{1}{7}$
Paid $3\frac{1}{7}$ due $9\frac{3}{7}$.

2*ly*. It is evident, that if a Body partake of different Forms, the true Meaſure of that Body muſt be mixt accordingly; as in the Gaging of Veſſels will be further clear.

How to meaſure Bodies of mixt Forms.

A Wine or Beer Veſſel is in form of a *Spheriode*, partly like a Sphere, and partly *Oval* or *Cylindrical:* Wherefore meaſure the two Diameters, *viz.* that at the Head, and that at the Bung, and alſo the Length within their Veſſel, either in Inches or in Decimal Parts of a Foot, and by the Diameters find out the Circles. Then,

Spheroide, *what.*

To gage Veſſels.

Add together 2 third Parts of the greater Circle, and 1 third Part of the leſſer Circle, and multiply the Total by the Length; and this ſhall produce the Content or Solidity.

To find $\frac{2}{3}$ and $\frac{1}{3}$ of any Circle, the Analogies by the Common Way of *Archimedes*, and alſo by Mr. *Oughtred*, from *Van Ceulen*, are thus:

Analogies.

For $\frac{2}{3}$ As 21, to 11 . Or as 1, to 0,5236 } So is the Square of the Diameter, to the $\frac{2}{3}$ of the Circle.

For $\frac{1}{3}$ As 42, to 11 . Or as 1, to 0,2618 } So is the Square of the Diameter, to the $\frac{1}{3}$ of the Circle.

Example.

Example. A Veſſel whoſe Diameter at the Bung is 32 Inches, and at the Head 18 Inches, is in Length 40 Inches: what Content ſhall this Veſſel be of?

Anſw. By the common Proportions $24849\frac{11}{21}$ Solid Inches, and by the other 24839,584 Solid Inches; as by the following Operations of both appears.

Common Way.

Solidity by Archimedes.

As 7 . 22 :: 32 . $100\frac{4}{7}$ *Periphery.*
16 × $50\frac{2}{7}$ = $804\frac{4}{7}$ *Area* at the Bung. $\frac{2}{3}$ is $536\frac{8}{21}$
As 7 . 22 :: 18 . $56\frac{4}{7}$ *Periphery.*
9 × $28\frac{2}{7}$ = $254\frac{4}{7}$ *Area* at the Head. $\frac{1}{3}$ is $84\frac{6}{7}$
Total $621\frac{5}{21}$
Length 40
Solidity $24849\frac{11}{21}$

(Q.32.)
As 21 . 11 :: 1024 . $536\frac{8}{21}$ }
Or, (Q.18) } as above.
As 42 . 11 :: 324 . $84\frac{6}{7}$ }

Other Way.

Solidity by Van Ceulen.

As 1 . 3,1416 :: 32 . 100,5312 *Periphery.*
16 × 50,2656 = 804,2496 *Area* at the Bung. $\frac{2}{3}$ is 536,1664
As 1 . 3,1416 :: 18 . 56,5488 *Periphery.*
9 × 28,2744 = 254,4696 *Area* at the Head. $\frac{1}{3}$ is 84,8232
Total 620,9896
Length 40
Solidity 24839,5840

(Q.32)
As 1 . 0,5236 :: 1024 . 536,1664 }
Or, (Q.18) } as above.
As 1 . 0,2618 :: 324 . 84,8232 }

Content how otherwiſe to be found.

The Content of a Veſſel in Cubick Inches is alſo to be had thus; to 2 Squares of the Diameter at the Bung, add the Square of the Diameter at the Head; and dividing the Total into 3 Parts, ſay, As 452 to 355; ſo is $\frac{1}{3}$ of the ſaid Total multiplied by the Length, to the Content in Cubick Inches.

And ſo 18 × 18 = 324 And 32 × 32 = 1024
Double of 1024 is 2048
3)2372($790\frac{2}{3}$ × 40 = $31626\frac{2}{3}$
And as 452 . 355 :: $31626\frac{2}{3}$. 24839,&c. as before.

To turn the Solidity into Concave Meaſure. Example.

Then having the Cubick Inches, or Cubick tenth Parts of a Foot in one Gallon of Wine or Beer, the Content of the Veſſel in Gallons may be found.

For there being in 1 Foot 1000 Cubick 10*th* Parts, that is 10 × 10 × 10; And 1728 Cubick Inches, that is 12 × 12 × 12: And as Mr. *Oughtred* and others write, 231 Cubick Inches in a Wine Gallon, and $272\frac{1}{4}$ in a Beer Gallon; and the *Ratio* between them in leſſer Terms, as 14 to $16\frac{1}{2}$: If Cubick Inches be given, the *Analogy* is,

(C.12) (C.10)
As 1728 . 272,25 :: 1000 . 157,5521— Cubick 10*th* Parts in a Beer Gallon.
As 1728 . 231 :: 1000 . 133,6805+ Cubick 10*th* Parts in a Wine Gallon.

But if Cubick 10*th* Parts be given, the *Analogy* is contrary.

(C.10) (C.12)
As 1000 . 157,5521— :: 1728 . 272,25. Cubick Inches in a Beer Gallon.
As 1000 . 133,6805+ :: 1728 . 231. Cubick Inches in a Wine Gallon.

Wherefore if the Solidity of the Veſſel aforeſaid were deſired in Cubick 10*th* Parts: Then, by the

Common Way.

As 1728 . 24849,5238 :: 1000 . 14380,5114, *&c.*

Other Way.

As 1728 . 24839,584 :: 1000 . 14374,7592, *&c.*

And if the Content of the ſame Veſſel were deſired in Gallons Wine-meaſure or Beer-meaſure; then accordingly diviſion is to be made by 231 for Wine, if Cubick Inches, or by 133,6805, if Cubick 10*th* Parts be given: And by $272\frac{1}{4}$ or 272,25 for Beer, if Cubick Inches be given; or by 157,5521, if Cubick 10*th* Parts: So will be found in the ſaid Veſſel by the

$$\text{Common Way}\left\{\begin{array}{ll}\text{Beer Gallons} & 91,\ 27\\ \text{Wine Gallons} & 107,\ 57\end{array}\right\}\text{and ſomewhat above.}$$

$$\text{Other Way}\left\{\begin{array}{ll}\text{Beer Gallons} & 91,\ 23\\ \text{Wine Gallons} & 107,\ 53\end{array}\right\}\text{and ſomewhat above.}$$

☞ By one of the Officers of Exciſe, I have heard that their Gage, or Rule by which they meaſure the Brewers Tuns and Veſſels, is computed after the Rate of 282 Cubick Inches to a Gallon of Beer; which if ſo, ſeems to be for allowance of Lees, *&c.*

How to meaſure a mixt Body in Higher Proportions.

3*ly*. Hence alſo is apparent, That by the help of Figural Proportions, not only Bodies of mixed Forms may be meaſured, but alſo one and the ſame Body, whoſe Solidity or Gravity is mixed, may be meaſured, and in higher Proportions than tripled.

Q. Of Wine mixt with Water four times.

Example 1. Suppoſe a Merchant have a Piece of Wine of 128 Gallons, and draw out thereof 16 Gallons, and fill it up again with Water: And again draw out 16 Gallons, and fill it again with Water, and do the like the third and fourth Time: How much Wine and Water were then in the Veſſel?

Anſwer.

Anſw. $75\frac{1}{32}$ Gallons of Wine, and the reſidue Water.

Here after the Numbers 128 and 16 are ſet in their leaſt Terms, that is, 8 and 1, the Leſſer is taken from the Greater, ſo it is 8 and 7; and the like will be if 16 be taken from 128, and the Remain 112 abbreviated with 128, both which are to be Figurate to the 4*th* Quantity, becauſe the Mixture was fourfold, (for always the Figuration muſt be according to the Mixture) and then the Analogy is,

As the greater Figural Number to the whole Quantity: So is the leſſer Figural Number to the Quantity deſired.

$$\begin{array}{lllllll} & \text{(QQ. 8)} & & & \text{(QQ. 7)} & & \\ \text{As} & 4096 & . & 128\ :: & 2401 & . & 75\frac{1}{32}\ \text{Wine.}\\ & & & & \text{Complement.} & & \underline{52\frac{31}{32}\ \text{Water.}}\\ & & & & & & 128\ \text{Gallons.}\end{array}$$

Q. Of Silver mixt with Copper.

Example 2. If a Goldſmith have an Ingot of Silver 12 Penny Weights Fine, weighing 8 Marks, and cut off a Mark thereof, and melt with the reſidue a Mark of Copper; and from this mingled Maſs cut off another Mark, and put thereto again a Mark of Copper, and do the like the third Time: The Queſtion is, how much a Mark as it is mix'd will hold?

Anſwer.

Anſw. $8\frac{5}{128}$ Penny Weights fine.

For here the Proportion between the Cubes of 8 and 7, becauſe the Mixture is but triple, guide the Analogy.

$$\begin{array}{lll} \text{Whole} & 8 & \text{Marks.}\\ \text{Cut off} & \underline{1} & \text{Mark.}\\ \text{Difference} & 7 & \end{array} \qquad \begin{array}{lllll} & \text{(C. 8)} & & \text{(C. 7)} & \\ \text{As}\ 512 & . & 12\ :: & 343 & .\ 8\frac{5}{128}\ \text{Fine.}\end{array}$$

These Mixtures better reſolved here than by Alligation; alſo their Value.

Thus ſuch Mixtures are more eaſily reſolved than by *Alligation*, as in that Chapter was before noted: and hereby alſo the Value of ſuch Mixtures are as eaſily diſcovered.

Example

Q. Of the Worth of mixed Wine and Water.

Example 1. A Merchant had 128 Gallons of Wine in a Cask, worth 5 *s.* the Gallon, and draweth out thereof 16 Gallons, and filleth up the Cask again with Water; and then draweth out 16 Gallons more, and filleth up again the Cask with Water: what will a Gallon of this mixture be worth?

Anſwer.

Anſw. $3\frac{53}{64}$ *s.* for the Analogy is, as the whole Quantity Figurate, according to the Number of Mixtures, is to the higheſt Price; ſo is the Difference between the whole Quantity and the firſt Draught ſo figurate as the other, to the Price of the Mixture.

Whole. *Firſt Draught.* *Difference.*

$$128 \text{ ——— } 16 \text{ ==== } 112 \text{ abbreviated } \frac{112}{128} = \frac{7}{8} \text{ Figurate twice } \frac{49}{64}.$$

$$\text{Therefore as } 64 \;.\; \overset{s.}{5} \;::\; 49 \;.\; \overset{s.}{3\frac{53}{128}}.$$

Q. Of the Worth of Wine mixt with other Wine.

Example 2. If a Merchant have 128 Gallons of Wine, worth 5 *s.* a Gallon, and draw out thereof 16 Gallons, and fill up the Veſſel again with Wine of 4 *s.* the Gallon, and afterwards draw out 16 Gallons, and fill up the Cask again with the Wine of 4 *s.* the Gallon, and ſo do the third time: what ſhall a Gallon of this mingled Wine be worth?

Anſwer.

Anſw. $4\frac{343}{512}$ *s.* After 16, the firſt Draught, is taken from 128 the whole Quantity, and the Remain abbreviated therewith to their leaſt Terms, and each of them figurate according to the Mixture, becauſe the Wine put in was of Value as well as that drawn out, the *Analogy* here may be,

As the greater Figural Number, to 1; ſo is that greater Figural Number multiplied by the loweſt Price given, and added to the leſſer Figural Number, to the Price deſired.

$$128 \text{ — } 16 = 112 \text{ Abbreviated } \frac{112}{128} = \frac{7}{8} \text{ Figurate thrice } \frac{343}{512}.$$

$$\text{And } 512 \times 4 + 343 = 2391$$

$$\text{Therefore } 512 \;.\; 1 \;::\; 2391 \;.\; 4\frac{343}{512} \text{ Price deſired.}$$

Proof of this Work.

And the Truth hereof may appear, if, according to the Work of *Alligation* before, it be examined: for thereby being found upon the third Mixture in the Veſſel $85\frac{3}{4}$ of the higheſt Price, and $42\frac{1}{4}$ of the loweſt Price, and each of them multiplied reſpectively by their ſeveral Prices, and the Total of the Products divided by the whole Quantity ſo mixed, will bring forth in the Quotient the mean Price as before.

$$\begin{array}{lllll} & & \textit{Prices.} & & \\ \text{Gallons}\left\{\begin{array}{l} 42\frac{1}{4} \\ \underline{85\frac{3}{4}} \end{array}\right. & \begin{array}{l}\times\\ \times\end{array} & \begin{array}{l} 4 \\ \underline{5} \end{array} & \begin{array}{l} \text{=====} \\ \text{=====} \end{array} & \begin{array}{l} 169 \\ \underline{428\frac{3}{4}} \end{array} \\ \text{And as } 128 & . & 1 & :: & 597\frac{3}{4} \;.\; 4\frac{343}{512}. \end{array}$$

$$\frac{128}{1}\Big)\frac{2391}{4}\Big(\frac{2391}{512}\Big(\overset{s.}{4}\frac{343}{512}.$$

Proof of tripled Proportions.

The Operations of this Chapter being compoſed of *Figurals* and *Plain Proportions*, have their Proofs accordingly ſpoken to in other Places. And by reverſing the Queſtions and Varieties of Work, the Truth of the Concluſions will ſufficiently appear without further Aſſay or Example.

Partis ſecundæ Libri quarti

FINIS.

The Third PART of the Fourth BOOK.

CHAP. I.

Of PROPORTIONS Continued.

Continual Proportions *their Comparative Elements.*

SImple *Diſjunct Proportions* have at large, with their *Comparative Elements*, been unravelled in the foregoing Part; therefore this will concern it ſelf with *Continual Proportions.*

Arithmetical and Geometrical, how different.

Continual Proportions, as before in the firſt Chapter of this Fourth Book, were obſerved to be both *Arithmetical* and *Geometrical*, that having the Difference or Exceſs, and this the *Ratio* between every two Numbers or Terms equal. As,

Examples of both.

Arithmetical	1 .	2 .	3 .	4 .	5 .	6.	&c.	*Difference*	1.
	1 .	3 .	5 .	7 .	9 .	11.	&c.	*Difference*	2.
	2 .	4 .	6 .	8 .	10 .	12.	&c.	*Difference*	2.
Geometrical	1 .	2 .	4 .	8 .	16 .	32.	&c.	*Ratio*	2.
	1 .	3 .	9 .	27 .	81 .	343.	&c.	*Ratio*	3.
	2 .	8 .	32 .	128 .	512 .	2048.	&c.	*Ratio*	4.

Both called Progreſſion, *and why.*

Both theſe are called *Progreſſion*, becauſe the Numbers go forward from the firſt Term, and have their Progreſs in an orderly Series, increaſing by an equal and continued *Difference* in the one, and *Ratio* in the other.

Wherein they agree. *How placed.*

They both agree alſo in their placing or ſetting down, either the firſt Term to be ſet to the left Hand, and the reſt in order to the Right, as above: Or elſe the firſt Term at top, and the other underneath in order, as in *Addition of Integers.*

Antecedent and Conſequent. Extreams and Means.

Yet further, the firſt Term in both is called the *Antecedent*, and all the reſt in reference thereto *Conſequents:* And moreover the firſt and laſt Terms in both are the *Extreams*, and all the other *Means.*

Agreement with Diſjunct *Proportions.*

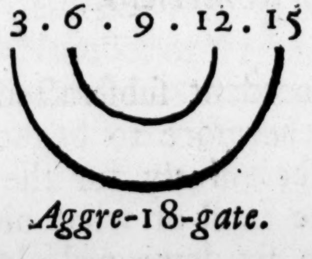
As they herein agree among themſelves, ſo in ſome things they agree with *Diſjunct Proportions*; that is to ſay, *Arithmetical Continued*, with *Arithmetical Diſjunct*; and *Geometrical Continued*, with *Geometrical Diſjunct.*

1. *Total of the Extreams and Means equal.*

1. Numbers in *Arithmetical Proportion Continued*, or *Diſjunct*, have the *Aggregate* of the *Extreams*, equal to the *Aggregate* of the *Means.*

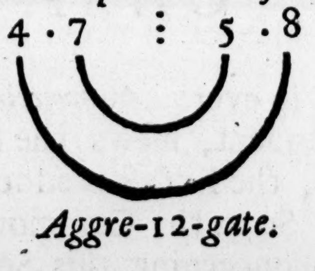
Example.

Arith. Proportion Continued.



Aggre-18-gate.

Arith. Proportion Diſjunct.



Aggre-12-gate.

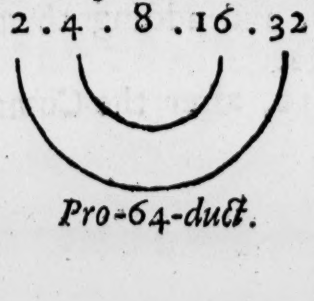
For in the *Diſjunct*, as 4+8, ſo 7+5 are 12. And in the *Continued*, as 3+15 are 18, ſo alſo 6+12; and ſo likewiſe 9, the odd Mean added to himſelf, makes alſo 18.

2. *Product of the Extreams and Means equal.*

2. Numbers in *Geometrical Proportion Continued*, or *Diſjunct*, have the *Product* of the *Extreams* equal to the *Product* of the *Means.*

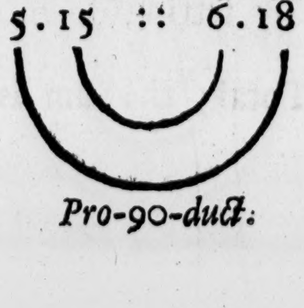
Example.

Geom. Proportion Continued.



Pro-64-duct.

Geom. Proportion Diſjunct.



Pro-90-duct.

For $\left.\begin{array}{l}2\times 32\\ 4\times 16\end{array}\right\}=64$ Alſo $\left.\begin{array}{l}5\times 18\\ 15\times 6\end{array}\right\}=90$

And ſo 8 the odd Mean multiplied by himſelf, produceth 64.

3. Four Proportionals added to or taken from 4 others, the Conſequence.

3. If 4 Numbers in *Arithmetical Proportion Continued* or *Disjunct*, be added to, or ſubſtracted from 4 other alike Proportional, the Totals and Remains will reſpectively be like Proportionals; and in the Totals the Exceſs will alſo be added, but in the Remains diminiſhed.

Example.

	Arith. Proportion continued.			*Arith. Proportion disjunct.*		
	1 . 4 . 7 . 10	Exceſs	3	3 . 5 : 7 . 9	Exceſs	2
	4 . 8 . 12 . 16	Exceſs	4	2 . 4 : 5 . 7	Exceſs	2
Totals	5 . 12 . 19 . 26		7	5 . 9 : 12 . 16		4
Remains	3 . 4 . 5 . 6		1	1 . 1 : 2 . 2		0

4. Four Proportionals multiplied or divided by 4 others, the Conſequence.

4. If 4 Numbers in *Geometrical Proportion continued*, or *disjunct*, be multiplied or divided by 4 other Numbers reſpectively proportional, the Products and Quotients ſhall be accordingly proportional; and the *Ratio* in the Products will likewiſe be multiplied, but in the Quotients divided.

Example.

	Geom. Proportion continued.			*Geom. Proportion disjunct.*		
	1 . 2 . 4 . 8	Ratio	2	7 . 28 :: 9 . 36	Ratio	4
	3 . 6 . 12 . 24	Ratio	2	8 . 32 :: 9 . 36	Ratio	4
Products	3 . 12 . 48 . 192		4	56 . 896 :: 81 . 1296		16
Remains	3 . 3 . 3 . 3		1	$1\frac{1}{7}$. $1\frac{1}{7}$ :: 1 . 1		1

Computation of Continual Proportions, and their Iſſues.

Wherein further both Sorts of *Continued Proportions* agree, or diſagree with themſelves, or *Disjunct Proportions*, may be obſerved in uncovering their ſeveral *Comparative Elements*. Thoſe *Arithmetical* in the next Chapter, and thoſe *Geometrical* in the Fifth following, between which is placed the Computation of what iſſues from the Firſt, and afterwards the Proceeds of the Second in the Order following, *viz.*

Continued Proportions	*Progreſſion Arithmetical*,	Chap. 2.	*Tranſpoſition*,	Chap. 3.
			Technologie,	Chap. 4.
	Progreſſion Geometrical,	Chap. 5.	*Tranſmutation*,	Chap. 6.
			Anatociſm,	Chap. 7.

CHAP. II. *Progreſsion Arithmetical.*

Arithmetical Progreſſion. *New Proportionals how gotten. Common Way of proceeding.*

BEcauſe in every *Arithmetical Progreſſion*, the Antecedent ſubſtracted from the Conſequent, ſhews the Exceſs or Difference; therefore to beget new Proportionals, the Exceſs is added to the Antecedent ſucceſſively for the new Conſequents. So as the Common Way, and indeed the moſt of what old Authors have left concerning this Sort of *Progreſſion*, is, to ſet down orderly all the Increaſe in their due Places, and then add them together as in common Addition of *Integers*.

Q. Of Sattin ſold encreaſing the Price of every Yard.

For Example-ſake: Suppoſe a Merchant ſelleth 15 Yards of Sattin, to be paid for the firſt Yard 4 *s.* for the ſecond Yard 6 *s.* and ſo for every Yard an orderly Increaſe of 2 *s.* And it were demanded, what the 15 Yards of Sattin did amount to?

Anſwer.

Anſw. By ſetting down orderly all the Terms, and adding them together, into one Total; the Sum is found $\overset{s.}{270}$; or $\overset{l.}{13}$: $\overset{s.}{10}$, after the Common Way: Thus, *Terms or Places.*

1.2.3.4.5.6.7.8.9.10.11.12.13.14.15

First Term, 4.6.8.10.12.14.16.18.20.22.24.26.28.30.32 Laſt Term.

Exceſs or Difference 2. Total or Sum 270 *s.* or 13 *l.* 10 *s.*

Or thus;

Yards of Sattin	1	4 *s.*	Firſt or leaſt Term.
	2	6	
	3	8	
	4	10	
	5	12	
	6	14	
	7	16	Exceſs or Difference 2.
	8	18	
	9	20	
	10	22	
	11	24	
	12	26	
	13	28	
	14	30	
Terms of Places	15	32	Laſt or greateſt Term.
		270	Total or Sum.

How the Terms are made.

In this Example, the firſt Term, and as it were the Root, is 4, the ſecond is made of the firſt and one Difference, the third of the firſt and 2 Differences, and generally every Term is made of the firſt and the Sum of the Differences, the Number of which is leſs by 1 than the Number of Terms, and ſo the thirteenth Term ſhall be 4 and twelve Differences, that is, $4 + 24$, ſeeing in this Example the Difference is 2.

The Number of Differences is noted by $T - 1$.
The Sum of the Differences by $TX - X = \omega - \alpha$.

What to be noted.

But for a more orderly Proceeding in the Computation of this firſt ſort of Progreſſion, two things in general are to be noted.

1. That there are 5 Principals in every *Arithmetical Progreſſion.*
2. That any 3 of the 5 being given, the other 2 may be found.

What the five Principals are.

The 5 Principals are.

1. The firſt Term and his Species.

1. The firſt or leaſt Term of any *Progreſſion*, which in the Example above is 4, this is noted in Species with the Greek α, being the firſt ſmall Letter of their Alphabet called *Alpha.*

2. The laſt Term and his Species.

2. The laſt or greateſt Term noted in Species with the Greek ω, being the laſt ſmall Letter of their Alphabet called *Omega*, and in the Example above is 32.

3. The Number of Terms and Species thereof.

3. The Number of Terms or Places in the whole *Progreſſion*, this in the Inſtance above is 15, and in Species noted commonly with the Capital Roman T.

4. The Exceſs or Difference, and his Species.

4. The common Difference or Exceſs (*ſometime called Increaſe*), whoſe Note in Species is X, another Capital Letter of the Roman Alphabet, in the Example above is 2.

5. The Total or Sum, and his Species.

5. The Total or Sum of all the Terms in the *Progreſſion*, which above in the Example is 270; for this the Common Note in Species is Z, the laſt Capital Roman Letter in their Alphabet.

How by any 3 of theſe the other may be found.

The Invention of any 2 of the 5 by 3 given, conſtitutes 20 Propoſitions, ſet down in Species by Mr. *Oughtred*, *p.* 78, 79, and 80, *of his Clavis*, that is 4 Propoſitions for every of the 5 Principals: So as every Queſtion may be varied 20 ſeveral ways, though the thing ſought be one of thoſe 5.

To find the firſt Term.

1*ſt*. To find the firſt Term (or α) of any *Arithmetical Progreſſion.*

1. Data. ω.T.X.

1. If the laſt Term, the Number of Terms, and the Exceſs be given, that is, ω.T.X. Then $\omega + X - TX = \alpha$.

Rule.

That is, from the Sum of the ſecond Principal added to the Fourth, ſubſtract the Product of the third Principal, multiplied by the Fourth.

Example.

As in the former Inſtance 32.15.& 2. to find 4.

$$\begin{array}{lll} 32\ (\omega) & 15\ (T) & 34\ \text{Total.}\\ \underline{\ \ 2\ (X)} & \underline{\ \ 2}\ (X) & \underline{30}\ \text{Product.}\\ \underline{34}\ (\omega+X) - & \underline{30}\ (TX) = & \underline{\ 4\ }\ (\alpha) \end{array}$$

2. Data.ω.T.Z. 2. If the laſt Term, the Number of Terms, and the Sum be given, that is, ω . T . Z .

$$\text{Then } \frac{2\,Z}{T} - \omega = \alpha.$$

Rule. That is, divide the double of the fifth Principal by the Third, and from the Quotient take the Second.

Example. As in the former Inſtance 32 . 15 . and 270 . to find 4.

$$\begin{array}{llll} 270\ (Z) & & \overset{9}{} & \\ \underline{\ \ \ 2} & & \cancel{5}\cancel{4}0\big(& 36\\ \underline{540}\ (2Z) & (T) & \overline{15} & \underline{32}\ \ (\omega)\\ & & & \ \ 4\ \ (\alpha) \end{array}$$

3. Data.ω.X.Z. 3. If the laſt Term, the Exceſs and the Sum be given, that is, ω . X . Z.

$$\text{Then } \tfrac{1}{2}X \pm \sqrt{:\omega q + \omega X + \tfrac{1}{4}Xq - 2ZX} = \alpha.$$

$$\text{As } \alpha \text{ ſhall happen to be} \left\{\begin{array}{l}\text{greater}\\ \text{leſſer}\end{array}\right\} \text{than } \tfrac{1}{2}X$$

Rule. That is, add together the Square of the ſecond Principal, with the Product of the ſecond multiplied into the Fourth, and the fourth Part of the Square of the Fourth; and from the Total, take the Product of the fourth and fifth doubled: to the Square Root of the Remain, add half the Fourth, except the firſt Term (or Number ſought) be leſs than half the Fourth, and then this Root is to be taken from half the Fourth.

Example. As in the former Inſtance 32 . 2. and 270 . to find 4.

$$\begin{array}{llllll} 32\ (\omega) & & 2\ (X) & & 270\ (Z) & \\ \underline{32} & & \underline{2} & & \underline{\ \ \ 2}\ (X) & \\ 64 & 32\ (\omega) & 4\ (Xq) & & 540\ (ZX) & \\ \underline{96} & \underline{\ \ 2}\ (X) & & & \underline{\ \ \ 2} & \\ 1024\,(\omega q) + & 64\,(\omega X) + & 1\,(\tfrac{1}{4}Xq) = & 1089 - & 1080\,(2ZX) = & 9\big(\begin{array}{l}\surd\\ 3\\ \underline{1\ (\tfrac{1}{2}X)}\\ \underline{4\ (\alpha)}\end{array} \end{array}$$

The half of X is here added, becauſe 4 the First Term is greater than 2 the Exceſs.

4. Data. T.X.Z. 4. If the Number of Terms, the Exceſs, and the Sum be given, that is, T . X . Z .

$$\text{Then } \frac{2\,Z}{2\,T} - \frac{T\,X}{2} + \frac{X}{2} = \alpha.$$

Rule. That is, divide the double of the fifth Principal, by the double of the Third; to the Quotient add half the Fourth, and from the Total, ſubſtract half the Product of the Third multiplied into the Fourth.

Example. As in the former Inſtance 15 . 2 . and 270 . to find 4.

$$\begin{array}{llll} 270\ (Z) & 15\ (T) & & 15\ (T)\\ \underline{\ \ \ 2} & \underline{\ \ 2} & \frac{540}{30}\Big(\begin{array}{l}18\\ 1\ (\tfrac{1}{2}X)\end{array} & \underline{\ \ 2\ (X)}\\ \underline{540}\,(2Z) & \underline{30}\,(2T) & & \underline{30}\,(TX)\\ & & 19 - & 15 = 4\ (\alpha) \end{array}$$

To find the laſt Term. 2*dly*. To find the laſt Term (or ω) of any *Arithmetical Progreſſion*.

1. Data.α.T.X. 1. If the firſt Term, the Number of Terms, and the Exceſs be given, that is, α . T . X .

$$\text{Then } \alpha + TX - X = \omega. \qquad \text{or } T - 1 \times X + \alpha = \omega.$$

Rule. That is, to the Product of the third Principal multiplied by the Fourth, add the firſt and ſubſtract the Fourth; or multiply the Number of Terms lacking 1, by the fourth Principal, and to the Product add the Firſt.

As in the former Inſtance 4 . 15 . and 2 . to find 32. **Example.**

$$\begin{array}{l} 15\ (T) \\ \underline{2}\ (X) \\ 30\ (TX) \\ \underline{4}\ (\alpha) \\ \underline{34}\ (TX+\alpha) \\ \underline{2}\ (X) \\ \underline{32}\ (\omega) \end{array} \qquad \begin{array}{l} 15-1=14\ (T-1) \\ \underline{2}\ (X) \\ 28\ (T-1\times X) \\ \underline{4}\ (\alpha) \\ \underline{32}\ (\omega) \end{array}$$

2. If the firſt Term, the Number of Terms, and the Sum be given, that is, α . T . Z. **2. Data. α.T.Z.**

$$\text{Then } \frac{2Z - T\alpha}{T} = \omega.$$

That is, double the fifth Principal, and take out thereof the Product of the third multiplied by the Firſt; then divide the Remainder by the Third. **Rule.**

As in the former Inſtance 4 . 15 . and 270 . to find 32. **Example.**

$$\begin{array}{ll} 270\ (Z) & 15\ (T) \\ \underline{\ \ 2\ \ } & \underline{4}\ (\alpha) \\ \underline{540}\ (2Z) - & \underline{60}\ (T\alpha) = \frac{480}{(T)\ 15}\Big(\ 32\ (\omega) \end{array}$$

3. If the firſt Term, the Exceſs, and the Sum be given, that is, α . X . Z. **3. Data. α.X.Z.**

$$\text{Then } \sqrt{:\alpha q - \alpha X + \tfrac{1}{4}Xq + 2ZX:} - \tfrac{1}{2}X = \omega.$$

That is, Square the firſt Principal, double the Product of the fifth multiplied by the Fourth, and add them together with the fourth Part of the Square of the Fourth: From the Total, take the Product of the firſt multiplied by the Fourth, and from the Square Root of the Remain take half the Fourth. **Rule.**

As in the former Inſtance 4 . 2 . & 270 . to find 32. **Example.**

$$\begin{array}{lllll} & & XX & \overset{Z\ \ \ X}{\underline{270\times 2}} & \\ 4\ (\alpha) & 4\ \ (\alpha) & \underline{2\times 2} & 540\ (ZX) & \\ \underline{4} & \underline{2}\ (X) & \underline{4}(Xq) & \underline{2} & \sqrt{} \\ \underline{16}(\alpha q) & \underline{-8}(\alpha X)+ & \underline{1}(\tfrac{1}{4}X)+ & \underline{1080}\ (2ZX) = 1089 & \Big(\ 33 \\ & & & & \underline{1\ (\tfrac{1}{2}X)} \\ & & & & \underline{32\ (\omega)} \end{array}$$

4. If the Number of Terms, the Exceſs, and the Sum be given, that is, T . X . Z. **4. Data. T.X.Z.**

$$\text{Then } \frac{2Z}{2T} + \frac{TX}{2} - \frac{X}{2} = \omega.$$

That is, divide the fifth Principal doubled, by the double of the Third; to the Quotient add half the Product of the Third multiplied by the Fourth, and from the Total take half the Fourth. **Rule.**

As in the former Inſtance 15 . 2 . and 270 . to find 32. **Example.**

$$\begin{array}{llll} 270\ (Z) & 15\ (T) & 15\ (T) & \\ \underline{\ \ 2\ \ } & \underline{\ 2\ } & \underline{2}\ (X) & \frac{540}{30}\Big(\ 18 \\ \underline{540}\ (2Z) & \underline{30}\ (2T) & 30\ (TX) & \quad \underline{15} \\ & & 15\ (2) & \quad 33 \\ & & & \quad \underline{1\ (\tfrac{1}{2}X)} \\ & & & \quad \underline{32}\ (\omega) \end{array}$$

3*dly*. To find the Number of Terms (or T) of any *Arithmetical Progreſſion.* ***To find the Number of Terms.***

1. If the firſt Term, the laſt Term, and the Exceſs be given; that is, α . ω . X. **1. Data. α.ω.X.**

$$\text{Then } \frac{\omega - \alpha}{X} + 1 = T.$$

Rule. That is, ſubſtract the firſt Principal from the Second, divide the Remain by the Fourth, and to the Quotient add an Unit.

Example. As in the former Inſtance 4 . 32 . & 2 . to find 15 .

$$\overset{\omega}{32} - \overset{\alpha}{4} = \frac{28}{(X)\ 2}\Big(\ 14 + 1 = 15\ (T)$$

2. Data. α. ω. Z. 2. If the firſt Term, the laſt Term, and the Sum be given; that is, α . ω . Z.

$$\text{Then } \frac{2\ \ Z}{\omega + \alpha} = T.$$

Rule. That is, divide the Double of the fifth Principal, by the Firſt added to the Second.

Example. As in the former Inſtance 4 . 32 . & 270 . to find 15.

$$\overset{\alpha}{4} + \overset{\omega}{32} = 36\Big)\overset{2\,Z}{540}\Big(\ 15\ (T)$$

3. Data. α. X. Z. 3. If the firſt Term, the Exceſs, and the Sum be given; that is, α . X . Z.

$$\text{Then } \sqrt{:}\ \frac{\alpha q - \alpha X + \tfrac{1}{4}Xq + 2ZX:}{Xq} - \alpha + \tfrac{1}{2}X = T.$$

Rule. That is, ſquare the firſt Principal, double the Product of the fifth multiplied by the Fourth, and add them together with the fourth Part of the Square of the Fourth; From the Total take the Product of the Firſt multiplied by the Fourth, divide this Remainer by the Square of the Fourth; and from the Square Root of the Quotient take the Firſt, and add half the Fourth.

Example. As in the former Inſtance 4 . 2 . & 270 . to find 15 .

$$\begin{array}{llll} 4\ (\alpha) & 4\ (\alpha) & \overset{X\,X}{2\times 2} & \overset{Z\ \ X}{270\times 2} \\ \underline{4} & \underline{2\ (X)} & \underline{4}\ (Xq) & 540\ (ZX) \\ & & & \underline{2} \end{array}$$

$$\underline{16}\ (\alpha q) \underline{- 8}\ (\alpha X) \underline{+ 1}\ (\tfrac{1}{4}Xq) + \underline{1080}\ (2ZX) = \frac{1089}{(Xq)\ 4}\Big(\overset{\surd}{\frac{33}{2}}$$

$$\frac{33 - \overset{\alpha}{4} + \overset{\frac{1}{2}X}{1}}{2} = 15\ (T)$$

4. Data. ω. X. Z. 4. If the laſt Term, the Exceſs, and the Sum be given; that is, ω . X . Z.

$$\text{Then } \frac{\omega + \tfrac{1}{2}X}{X} \mp \sqrt{:}\ \frac{\omega q + \omega X + \tfrac{1}{4}Xq - 2ZX}{Xq} = T.$$

$$\text{As } \alpha \text{ ſhall happen to be } \left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\} \text{ than } \tfrac{1}{2}X.$$

Rule. That is, ſubſtract the Product of the fourth and fifth Principals doubled, from the Product of the Second and Fourth, added to the Square of the Second and Fourth Part of the Square of the Fourth; divide the Remainer by the Square of the Fourth, and take the Square Root of the Quotient from the Quotient of the Second added to half the Fourth, and divided by the Fourth; except the firſt Term be leſs than half the Fourth, for then this Root is to be added.

Example. As in the former Inſtance, 32 . 2 . & 270 . to find 15.

$$\begin{array}{lllll} 32\ (\omega) & 32\ (\omega) & & \overset{X\,X}{2\times 2} & \overset{Z\ \ X}{270\times 2} \\ \underline{1\ (\tfrac{1}{2}X)} & \underline{32} & & \underline{} & \underline{} \\ \underline{33}\ (\omega + \tfrac{1}{2}X) & 64 & 32\ (\omega) & & 540 \\ 2\ (X) & \underline{96} & \underline{2}\ (X) & \underline{4}\ (Xq) & \underline{2} \end{array}$$

$$\frac{1024\,(\omega q) + 64\,(\omega X) + 1\,(\tfrac{1}{4}Xq) - 1080\,(2ZX)}{4\,(Xq)} = \tfrac{9}{4}\Big(\overset{\surd}{\tfrac{3}{2}}$$

$$\tfrac{33}{2} - \tfrac{3}{2} = 15\ (T)$$

The Root ½ is ſubſtracted here, becauſe 4, the
First Term, is greater than 2 the Exceſs.

4*thly*. To find the Exceſs (or X) of any Arithmetical Progreſſion. *To find the Exceſs.*

1. If the firſt Term, the laſt Term, and the Number of Terms be given; that is, α. ω. T. 1. Data. α. ω. T.

$$\text{Then } \frac{\omega - \alpha}{T - 1} = X.$$

That is, Take the firſt Principal from the Second, and divide the Remainer by the Third made leſs by an Unit. Rule.

As in the former Inſtance, 4 : 32. & 15. to find 2. Example.

$$\begin{array}{rcl} 15\ (T) & & 32\ (\omega) \\ 1 & & 4\ (\alpha) \\ \hline 14 &) & 28\ (\ 2\ (X) \end{array}$$

2. If the firſt Term, the laſt Term, and the Sum be given; that is, α : ω : Z. 2. Data. α. ω. Z.

$$\text{Then } \frac{\alpha q - \alpha q}{2Z - \omega - \alpha} = X.$$

That is, Subſtract the Square of the firſt Principal from the Square of the Second, and divide the Remainer by double the Fifth, lacking the First and Second. Rule.

As in the former Inſtance, 4. 32. & 270. to find 2. Example.

$$\begin{array}{r} 32\ (\omega) \\ 32 \\ \hline 64 \\ 96 \\ \hline 1024\ (\omega q) \\ 4 \times 4 = 16\ (\alpha q) \\ \hline 1008 \\ \hline 504 \end{array}$$

$$\begin{array}{c} 270 \\ 2 \\ \hline 540 \end{array} (2Z) - \begin{array}{c} 32 + 4 \\ \hline 36 \end{array} (\omega + \alpha) = = = = 504\ (\ 2\ (X)$$

3. If the firſt Term, the Number of Terms, and the Sum be given; that is, α. T. Z. 3. Data. α. T. Z.

$$\text{Then } \frac{2Z - 2T\alpha}{Tq - T} = X.$$

That is, Take the Product of double the firſt Principal multiplied by the Third from the Fifth doubled, and divide the Remain by the Remain of the Third ſubſtracted from his Square. Rule.

As in the former Inſtance, 4. 15. & 270. to find 2. Example.

$$\begin{array}{c} 270\ (Z) \\ 2 \\ \hline 540 \\ \hline 540 \end{array} \quad - \quad \begin{array}{c} 15(T) \times 4(\alpha) \\ \hline 60 \\ 2 \\ \hline 120 \end{array} (2T\alpha) = = = 420$$

$$\begin{array}{c} T \quad T \\ 15 \times 15 = 225\,(Tq) - 15\,(T) = 210 \end{array} \quad \frac{420}{210}\ (\ 2\ (X)$$

4. If the laſt Term, the Number of Terms, and the Sum be given, that is, ω. T. Z. 4. Data. ω. T. Z.

$$\text{Then } \frac{2\,T\omega - 2\,Z}{Tq - T} = X.$$

That is, double the Product of the third Principal multiplied by the Second, and ſubſtract therefrom the double of the Fifth: divide the Remain by the Remain of the Third ſubſtracted from his Square. Rule.

As in the former Inſtance 32. 15. and 270. to find 2. Example.

$$\begin{array}{l}
32\ (\omega)\\
\underline{15\ (T)}\\
160\\
\underline{32}\\
480 \qquad\qquad 270\ (Z)\\
\underline{\quad 2} \qquad\qquad\quad \underline{\quad 2}\\
\underline{960}\ (2\,T\omega) - \underline{540}\ (2Z) = = = = \dfrac{420}{210}\Big(\ 2\ (X)\\
\qquad\qquad \overset{T}{15}\times\overset{T}{15} = 225\ (Tq) - 15\ (T) = 210
\end{array}$$

To find the Sum. 5*thly*. To find the Sum (or Z) of any *Arithmetical Progreſſion*.

1.Data. α.ω.T. 1. If the firſt Term, the laſt Term, and the Number of Terms be given, that is, α . ω . T.

$$\text{Then } T\omega + T\alpha = 2Z \quad \text{or} \quad \frac{T\omega + T\alpha}{2} = Z$$

Rule. That is, the Product of the third Principal multiplied by the ſecond, and added to the Product of the Third multiplied by the Firſt, ſhall be equal to twice the Sum: or if the two Products be halfed, or if half the firſt and ſecond be multiplied by the Third, or the whole firſt and ſecond by half the Third, the Sum will be had.

Example. As in the former Inſtance, 4 . 32 . and 15. to find 270.

$$\begin{array}{l}
15\ (T)\\
\underline{32\ (\omega)}\\
30 \qquad\qquad 15\ (T)\\
\underline{45} \qquad\qquad\ \underline{4\ (\alpha)}\\
\underline{480}\ (T\omega) + \underline{60}\ (T\alpha) = \dfrac{540}{2}\Big(\ 270\ (Z)
\end{array}
\qquad
\begin{array}{l}
\overset{\omega}{32} + \overset{\alpha}{4} = \underline{36} \times \overset{\frac{1}{2}T}{7\tfrac{1}{2}} = 270\ (Z)\\
\qquad\qquad\ \ 18\\
\qquad\qquad\ \ \underline{15}\\
\qquad\qquad\ \ 90\\
\qquad\qquad\ \ \underline{18}\\
\qquad\qquad\ \ \underline{270}\ (Z)
\end{array}$$

2.Data. α.ω.X. 2. If the firſt Term, the laſt Term, and the Exceſs be given, that is, α . ω . X.

$$\text{Then } \frac{\omega q - \alpha q}{X} + \omega + \alpha = 2Z.$$

Rule. That is, ſubſtract the Square of the firſt Principal from the Square of the Second, divide the Remain by the Fourth; to the Quotient add the Firſt and Second, and take half the Total.

Example. As in the former Inſtance, 4 . 32 . and 2 . to find 270.

$$\begin{array}{l}
32\ (\omega)\\
\underline{32}\\
64 \qquad\qquad 4\ (\alpha)\\
\underline{96} \qquad\qquad \underline{4}\\
\underline{1024}\ (\omega q) - \underline{16}\ (\alpha q) = = \dfrac{1008}{2}\Big(\ 504 + \overset{\omega}{32} + \overset{\alpha}{4} = \dfrac{540}{2}\Big(\ 270\ (Z)\\
\qquad\qquad\qquad\qquad\quad (X)
\end{array}$$

3.Data. α. T.X. 3. If the firſt Term, the Number of Terms, and the Exceſs be given; that is, α . T . X. Then TX—X+2α in T==2Z.

Rule. That is, multiply the third and fourth Principals; from the Product take the Fourth, to the Remain add double the Firſt, multiply the Total by the Third, and half the Product.

Example. As in the former Inſtance 4 . 15 . and 2 . to find 270.

$$\begin{array}{l}
15\ (T)\\
\underline{2\ (X)}\\
\underline{30}\ (TX) - \overset{X}{2} = 28 + \overset{2\alpha}{8} = 36 \times \overset{T}{15} = \dfrac{540}{2}\Big(\ 270\ (Z)
\end{array}$$

4.Data.ω.T.X. 4. If the laſt Term, the Number of Terms, and the Exceſs be given, that is, ω . T . X. Then 2ω+X—TX in T==2Z.

That

Rule. That is, from the doubled Sum of the ſecond Principal added to the Fourth, take the Product of the third multiplied by the Fourth; multiply the Remain by the third and half the Product.

Example. As in the former Inſtance 32 . 15 . and 2 . to find 270.

$$\begin{array}{rlrl}
32 & (\omega) & & \\
2 & & & \\ \cline{1-1}
64 & & 15 & (T) \\
2 & (X) & 2 & (X) \\ \cline{1-1}\cline{3-3}
\underline{66} & (2\omega+X) - & \underline{30} & (TX) = 36\times\overset{T}{15} = \frac{540}{2}\Big(\ 270\,(Z)
\end{array}$$

What further to be noted.

Touching theſe Principals and Propoſitions, two things more come under farther obſervance.

1. Both the two unknown found by three given.

Firſt, That by every three of the 5 Principals given, both the other 2 wanting are to be found.

As *Data.*	*Queſita.*			*Propoſitions.*		
α . ω . T	Z & X	1	of the	5 . & 1	of the	4
α . ω . X	T & Z	1		3 . & 2		5
α . ω . Z	T & X	2		3 . & 2		4
α . T . X	ω & Z	1		2 . & 3		5
α . T . Z	ω & X	2		2 . & 3		4
α . X . Z	ω & T	3		2 . & 3		3
ω . T . X	α & Z	1		1 . & 4		5
ω . T . Z	α & X	2		1 . & 4		4
ω . X . Z	α & T	3		1 . & 4		3
T . X . Z	α & ω	4		1 . & 4		2

2. The Data and Queſita in every Queſtion.

Secondly, That all Queſtions duly propounded in *Arithmetical Progreſſion*, give 3 of the 5 Principals, and require ſometime one, ſometime both the other, and ſometime one or other of the middle Terms; ſometime the Increaſe is inverted, and ſometime one Queſtion includes another, ſo as 5 Caſes will compleat all needful to this ſort of *Progreſſion*.

Five Caſes.

1. If but one ſought, then as before. Alteration for Fractions and Decimals, &c.

Caſe 1. When but one of the 5 is ſought, the Propoſition under which the Reſolution falls, is to be uſed, as by the Examples above is largely to be ſeen; alteration only to be made for Fractions and Decimals, where occaſion requires, according to their Nature and Uſe.

2. If 2 be ſought, after one found the eaſieſt to be choſen.

Caſe 2. When 2 of the Principals are required in the Queſtion, after one is found, the other is to be ſought; and becauſe the Propoſitions get 2 of the Principals, and ſome of the Propoſitions are more eaſy than others, ſometimes it happens that the Principal found in the Work of the Firſt, will more eaſily procure the other ſought, and ſo may be uſed as if one of the *Data*.

Q. Of Spice ſold, what paid for the laſt Pound, and for the Whole.

Example 1. A Grocer ſelleth 80 ℔. of Spice conditionally, to be paid for the firſt Pound 2 *d.* for the ſecond 5 *d.* and ſo increaſing by 3: The Queſtion is, what was paid for the laſt Pound, and for the whole 80 ℔?

Anſwer.

Anſw. For the laſt Pound 239 Pence, and for the whole 9640, or 40 *l.* 3 *s.* 4 *d.* In this Queſtion are,

The *Data* α . T . X
2 . 80 . 3
Reſolution.
Queſita ω (2) Principal . 1 } of the { 2 } Propoſitions.
Z (5) Principal . 3 } { 5 }

Wherefore

$$\begin{array}{rl}
80 & (T) \\
3 & (X) \\ \hline
240 & \\
2 & (\alpha) \\ \hline
242 & (TX+\alpha) \\
3 & (X) \\ \hline
\underline{239} & (\omega)
\end{array}
\qquad\qquad
\text{And}\quad
\begin{array}{rl}
80 & (T) \\
3 & (X) \\ \hline
240 & \\
3 & (X) \\ \hline
237 & (TX-X) \\
4 & (2\alpha) \\ \hline
\underline{241} & \times 80 = \frac{19280}{2}\Big(\ 9640\,(Z)
\end{array}$$

Q. Of how many Men a Sum was received, and what each paid.

Example 2. If a Man receive 84 *l.* of certain Men, by an orderly Increaſe, remembring only the Paiment of the firſt Man to be 9 *l.* and the laſt 33 *l.* and would know of how many Men he received the ſaid 84 *l.* and what each Man paid one more than another: what ſhall the Anſwer be?

Anſwer.

Anſw. Of 4 Men, and each Man paid 8 *l.* more than the other.

$$\text{Here are}\left\{\begin{array}{l} \textit{Data}\;.\;\alpha\;.\;\omega\;.\;Z\\ \qquad\quad 9\,.\,33\,.\,84 \qquad\qquad\qquad\quad \textit{Reſolution.}\\ \textit{Queſita}\left\{\begin{array}{l}T\\X\end{array}\right\}\begin{array}{l}(3)\ \text{Principal}\,.\,2\\(4)\ \text{Principal}\,.\,2\end{array}\Big\}\text{of the}\left\{\begin{array}{l}3\\4\end{array}\right\}\text{Propoſitions.}\end{array}\right.$$

Wherefore

$$\overset{\alpha}{9}+\overset{\omega}{33}=42)\,\overset{2Z}{168}\,(4\,(T)$$

And

$$\begin{array}{r l} 33 & (\omega)\\ 33 & \\ \hline 99 & \\ 99 & \\ \hline 1089 & (\omega q)\\ 9\times 9=\!=81 & (\alpha q) \end{array}$$

$$\begin{array}{r}84\,(Z)\\ 2\\ \hline 168\,(2Z)\end{array}\;\begin{array}{c}33+9\\ \hline -42\,(\omega+\alpha)\end{array}=\!=\;\begin{array}{c}1008\\ \hline 126\end{array}\Big(\,8\,(X)$$

Q. Of Gain in the firſt and laſt Months.

Example 3. Suppoſe a Man gain every Month in the Year 30 *s.* more than he did the firſt Month, and at 12 Months End found the Whole to amount to 280 *l.* what were his Gains the firſt and laſt Months of the 12?

Anſwer.

Anſw. The firſt Month 15 *l.* : 1 *s.* : 8 *d.* : And the laſt Month 31 *l.* : 11 *s.* : 8 *d.*

$$\text{Here are}\left\{\begin{array}{l} \textit{Data}\;.\;T\,.\,X\,.\,Z.\\ \qquad\quad 12\,.\,30\,.\,5600\ \text{the Shillings in 280}\ l.\\ \qquad\qquad\qquad\qquad\qquad\quad \textit{Reſolution.}\\ \textit{Queſita}\left\{\begin{array}{l}\alpha\\ \omega\end{array}\right\}\begin{array}{l}(1)\ \text{Principal}\,.\,4\\(2)\ \text{Principal}\,.\,4\end{array}\Big\}\text{of the}\left\{\begin{array}{l}1\\2\end{array}\right\}\text{Propoſitions.}\end{array}\right.$$

Wherefore

$$\begin{array}{r}5600\,(Z)\\ 2\\ \hline 11200\,(2Z)\end{array}\quad\begin{array}{r}12\,(T)\\ 2\\ \hline 24\,(2T)\end{array}\quad\begin{array}{c}11200\\ \hline 24\end{array}\Big(\begin{array}{r}466\frac{2}{3}\\ 15\,(\frac{1}{2}X)\\ \hline 481\frac{2}{3}\end{array}\quad\begin{array}{r}12\,(T)\\ 30\,(X)\\ \hline 360\,(TX)\\ 180\end{array}\;\overset{s.}{=\!=301\tfrac{2}{3}}\,(\alpha)$$

And

$$\begin{array}{r}5600\,(Z)\\ 2\\ \hline 11200\,(2Z)\end{array}\quad\begin{array}{r}12\,(T)\\ 2\\ \hline 24\,(2T)\end{array}\quad\begin{array}{r}12\,(T)\\ 30\,(X)\\ \hline 360\,(TX)\\ \hline 180\quad 2\end{array}\quad\begin{array}{c}11200\\ \hline 24\end{array}\Big(\begin{array}{r}466\frac{2}{3}\\ 180\ \ \frac{1}{2}X\\ \hline 646\frac{2}{3}\end{array}\overset{s.}{-15=631\tfrac{2}{3}}\,(\omega)$$

Q. Of the Number and Ages of Children.

Example 4. A Man had divers Children; the Youngeſt 6 Years old, and the Eldeſt 40, and every one elder than the other by 2 Years: how many Children had he, and what was the Sum of all their Ages?

Anſwer.

Anſw. He had 18 Children, and their Ages together were 414 Years.

$$\text{Here are}\left\{\begin{array}{l} \textit{Data}\;.\;\alpha\,.\,\omega\,.\,X\\ \qquad\quad 6\,.\,40\,.\,2 \qquad\qquad\qquad\quad \textit{Reſolution.}\\ \textit{Queſita}\left\{\begin{array}{l}T\\Z\end{array}\right\}\begin{array}{l}(3)\ \text{Principal}\,.\,1\\(5)\ \text{Principal}\,.\,2\end{array}\Big\}\text{of the}\left\{\begin{array}{l}3\\5\end{array}\right\}\text{Propoſitions.}\end{array}\right.$$

Wherefore

$$\overset{\omega}{40}-\overset{\alpha}{6}=\frac{34}{(X)\ 2}\Big(17+1=18\,(T)$$

And

$$\begin{array}{r}40\,(\omega)\\ 40\\ \hline 1600\,(\alpha q)\end{array}\begin{array}{r}6\,(\alpha)\\ 6\\ \hline -36\,(\alpha q)\end{array}=\frac{1564}{(X)\ 2}\Big(782+\overset{\omega}{40}+\overset{\alpha}{6}=\frac{828}{2}\Big(414\,(Z)$$

In all theſe Examples the Demands being double, after the firſt is found, he may be taken with two other of the *Data* for finding the ſecond Demand: As in the laſt Example after T, the Number of Terms was found to be 18; Reſolution of the ſecond Demand Z might have been found by $\alpha . T . X .$ or $\alpha . \omega . T .$ or $\omega . T . X .$ as well as by $\alpha . \omega . X .$ the firſt *Data*. And if by $\alpha . \omega . T .$ that is the firſt Propoſition of the 5*th* Principal, the Work had been ſhorter than that above; and the like is to be underſtood of others.

Data.

$$\begin{array}{l} \alpha . \omega . T \\ 6 . 40 . 18 \end{array} \qquad \begin{array}{l} \overset{\omega}{40} + \overset{\alpha}{6} = \underline{46} \qquad \overset{}{T} \qquad \textit{Quæsita.} \\ (\tfrac{1}{2})\ 23 \times 18 = 414\ (Z) \end{array}$$

Caſe 3. When together with a Queſtion in *Progreſſion* is involved, explicitly or implicitly, another Queſtion, whoſe Reſolution belongs to ſome other Element of Numbers, Operation is to be made accordingly.

3. If one Queſtion be included in another.

Example. If 100 Eggs be placed, every one a Yard diſtant from other in length, and the firſt a Yard diſtant from a Basket: whether one might gather up the Eggs one after another, ſtill returning to the Basket to put them in, before one can run 4 Miles.

Q. Of gathering up 100 Eggs.

Here are given in this Queſtion the firſt Principal α, which is 2 Yards, (*viz.* a Yard from the Basket to the firſt Egg, and as much back again to the Basket) the third Principal T, that is, the 100 Eggs, and the fourth Principal X or 2 Yards, (*viz.* a Yard forth and a Yard back): And Z the fifth Principal is deſired, that is, the Number of Yards he that doth gather up the Eggs runneth in all forward and backward. And then becauſe another Queſtion is included, that is, whether this Number of Yards will reach in Length 4 Miles or not? the Number found is to be compared with the Yards in 4 Miles, and allowing 1760 Yards to an Engliſh Mile, Z or 10100 Yards found to be run in gathering up the Eggs, is ſeen to amount to 5 Miles and almoſt 3 quarters of a Mile more.

Anſwer.

Data

$\alpha . T . X .$

2 . 100 . 2

$$\begin{array}{r} 100\ (T) \\ \underline{2\ (X)} \\ 200 \\ \underline{2\ (X)} \\ 198 \\ \underline{4\ (2\alpha)} \\ \underline{202} \times \underline{100} = \dfrac{20200}{2} \end{array} \Bigg(\begin{array}{l} \textit{Yards.} \\ 10100 (Z) \end{array}$$

Quæſita *Yards.* *Mile.* *Yards.* *Miles.*

As $1760 . 1 :: 10100 . 5\frac{65}{88}$.

Miles.

$4 \times 1760 = 7040.$

Caſe 4. When the Increaſe is inverted, that is, turned into Decreaſe, and the Queſtion propounded with every Term ſucceeding the Firſt leſs than the Firſt; then accompt the laſt Term the Firſt, and the Firſt the Laſt, till the Work of the Propoſition be ended.

4. If the Increaſe be inverted.

Example. A Scout-Maſter-General being commanded to diſcover the Quarters of an Enemy, returned this Accompt; If moving (ſaith he) from the Place where we now are the firſt Day 30 Furlongs, the ſecond Day 28, the third Day 26, and ſo every Day leſſening 2 Furlongs, in the 15*th* Day we ſhall come to the Enemy; but they would meet in 9 days: how far ſhall they march in a Day one day with another, to overtake the Enemy in their Quarters in 9 days?

Q. Of marching to overtake an Enemy.

Here the firſt Work being to find out the Diſtance of the two Armies, or Z the 5*th* Principal, to the finding whereof is given α, the firſt day's march, 30 Furlongs; T the 15 days, and X the Difference of their March, which is 2 decreaſing; ſo is the increaſe of the *Progreſſion* inverted, wherefore α the 30 ſhall be ω the 15*th* Term, and that Term 2 (implicit in the Queſtion) ſhall be inſtead of α. Then either by $\alpha . \omega . T .$ or $\alpha . T . X .$ or $\omega . T . X .$ may Z be found, which will be 240 Furlongs; and then the other Queſtion will be reſolved by dividing 240 by 9, and their March thereby found to be $26\frac{2}{3}$ Furlongs in a Day.

Anſwer.

Data.

$\omega . T . X .$

30 . 15 . 2

Quæſita.

$$\begin{array}{r} 30\ (\omega) \\ \underline{2} \\ 60 \\ \underline{2\ (X)} \\ \underline{62}\ (2\omega + X) \end{array} - \begin{array}{r} 15\ (T) \\ \underline{2\ (X)} \\ \underline{30}\ (TX) \end{array} = 32 \times \overset{T}{15} = \dfrac{480}{2} \Bigg(\dfrac{\overset{Z}{240}}{9} \Bigg(26\tfrac{2}{3}\ \text{Furlongs.}$$

5. If a middle Term be ſought.

Caſe 5. When any middle Term (that is, intermediate between the firſt and laſt Terms) is demanded, the ſame is to be found according to the *Data*. For ſeeing the Number of ſuch middle Term may be repreſented by T, the whole Number of Terms in a *Progreſſion*, the Sum of ſuch Term may be repreſented by ω the laſt Term of the *Progreſſion*: ſo as with little alteration all the Propoſitions before, for finding the ſecond Principal ω, may ſerve to find the middle Term deſired.

Examples.

As if α and X be given, then by the firſt Propoſition of the ſecond Principal, take an Unite from the Term deſired, and multiply the Remain by the Exceſs, and to the Product add the firſt Term.

And ſo in the foregoing Inſtance, if 4 and 2 be given to find the 12*th* Term of that *Progreſſion*, that is, 26: An Unite taken from 12, and the Remain 11 multiplied by 2, is 22, to which 4 added, the Total is 26 deſired.

But if Z be one of the *Data*, let it be underſtood to be the Sum of the *Progreſſion* to that middle Term deſired, and not the Total Sum of the whole *Progreſſion*; and then Operation may be made therewith, as in the other 3 Propoſitions of the ſecond Principal.

And if ω, or the laſt Term of the whole *Progreſſion* be given, for one of the *Data*; then invert the Terms in the *Progreſſion*, and multiplying 1 leſs than the Term deſired by the Exceſs, ſubſtract the Product from ω.

So the 12*th* Term of the former Inſtance ſhall be the 4*th* Term, from which 1 taken, and the reſt 3 multiplied by 2, the Exceſs ſhall be 6, which taken from 32, the Remain will be 26 as before.

Terms	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	
	4.	6.	8.	10.	12.	14.	16.	18.	20.	22.	24.	26.	28.	30.	32.	
	15.	14.	13.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.	inverted.

Proof of Arithmetical Progreſſion.

The Proof of the Operations in this ſort of *Progreſſion*, is by placing every Increaſe in its due Seat or Term, and by common Addition of Integers to collect the Numbers into one Total, as in the firſt Example of this Chapter.

CHAP. III. *Tranſpoſition.*

Tranſpoſition what, and the Sorts.

THE firſt-born of *Arithmetical Progreſſion* is *Tranſpoſition*, which is an orderly diſpoſing of ſome Parts of a Number, ſo as there may be an equal Difference between the Parts ſo placed, or a diſpoſing of a Number ſo that the Parts deſired may be taken, and the other left.

The firſt Sort what it doth reſemble. How wrought.

The former ſort reſembles *Diviſion*, in which though the Dividend be taken a thouſand Times by the Diviſor, yet the Diviſor continues intire. This *Tranſpoſition* is to be done with due obſervation of the Parts: for if the Parts placed be ſome Aliquot Part of a Number, and the Places be equal to the Diviſor, that Aliquot Part is the deſired Number; but if 2 or more different Parts be taken, then half the Number of the Diviſor of one ſort, and half the Number of the Diviſor of the other ſort of Parts, muſt be taken.

Examples.

Examples of both follow.

Q. Of Souldiers, how diſpoſed, That a like Number may face the Side of a Fort, when ſome are entertained, and others diſcharged.

A certain Paſſage of Square Form had 4 Gates oppoſite one to the other, that is, in the middle of each ſide one; and there were appointed 9 Men to defend each Front thereof, ſome at the Gates, and the other at each Corner or Angle: ſo each Angle ſerved to aſſiſt two Faces of the Square if need required. Now this Square Paſſage being thus mann'd with 9 at each Side, it happened that 4 Souldiers coming by, deſired of the Governor to be entertained into Service, who told them he could not admit of more than 9 upon each Side of the Square: to whom one of the Souldiers being skilled in the Art of Numbers replied, If he would take them into Pay, they would place themſelves amongſt the reſt, and yet keep ſtill the Order of 9 for each Face of the Square; to which the Governor agreed, and they were admitted. But afterwards liking not their Service, they indeavoured to remove themſelves and alſo draw away each Man his Comrade, yet would leave 9 to defend each Side of the Paſſage: and how may this be?

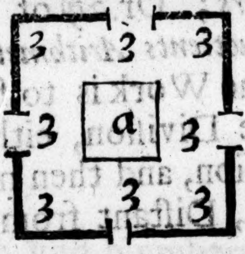
Anſwer.

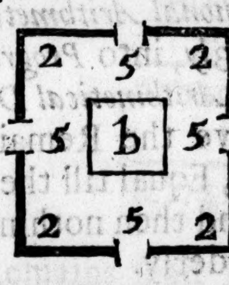
Anſw. The Square having 4 Gates and 4 Angles, that is, 8 in all, 3 of each Side including the Angles, this 8 multiplying 3, produceth 24 for the whole

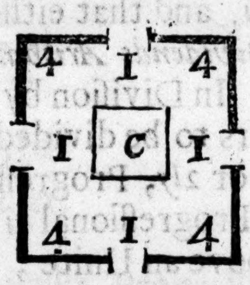
Number, whoſe Aliquot Part deſired is 3; and ſo many were placed at each Gate and Angle before the 4 Souldiers were admitted, as at *a*.

Then each of the 4 placing themſelves at the Gates, removed from every Angle to every Gate one Man, as at *b*. where the whole Number being 28, that is, 4 times 7, and 4 being half the Number of 8 the Diviſor, muſt be now taken, becauſe 8 will not evenly divide 28; and ſo it muſt be, that 7 ſhall be the Number belonging both to a Gate and an Angle, that is, 5 to one and 2 to the other, and 4 times 5 and 4 times 2 make up the 28.

But then 8 being taken away, and 20 Souldiers only left, which 20 will not be evenly divided by 8, but by 4 the half thereof; and therefore 5 the Quotient ſhall be the Number belonging both to an Angle and a Gate, that is, 4 to one, and 1 to the other, as at *c*. So as upon their departure, 2 were removed from the Gates, and placed at the Angles.



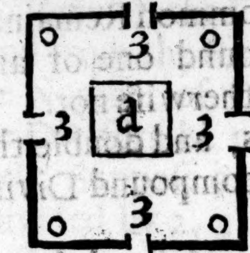


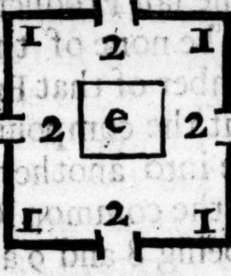


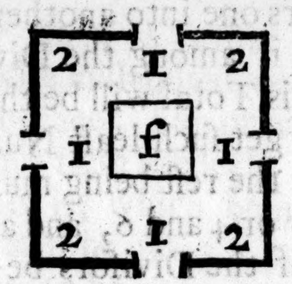
And the like is to be obſerved on the contrary, when the Parts of one and the ſame Number are to be ſo tranſpoſed, that their Fronts ſhall be more or leſs.

Example, where with a like Number the Side may have more or leſs.

As the Number 12 may be ſo parted and placed about a Square, that the Fronts every Way ſhall have 3, 4, or 5, as at *d. e. f.*







The other ſort of *Tranſpoſition* to diſpoſe a Number ſo that the Parts deſired may be choſen, is uſed in Lottery, where ſometime the 4, 5, 7, 20, *&c.* Perſon or Thing is reſpected to be ſecured, and this is performed thus: Take as many Units or Ciphers as there be Perſons or Things on which the Lot is to paſs, and diſpoſe them on a Paper in a Row; then ſet a Prick on the reſpected Place as the Lot is to fall upon, whether the 3, 4, 5, *&c.* and ſo count along on the remaining Places, ſtill pricking the alloted Place till the Complement of the Lottery be marked out; and then you ſhall ſee how to place the Perſons or Things, that the Lot may fall where deſired.

The other Sort of Tranſpoſition *uſed in Lottery.*

Example 1. If 24 Perſons have done a Villany, and 6 of them more guilty than the reſt, and being apprehended and found Guilty at Trial, the Judg determined by a Lot, that every 7*th* Man ſhould die, till the Number of 6 were executed: how then ſhould the Men be diſpoſed, that ſo counting by 7, the Lot of Eſcape ſhould always fall upon one of the 6 that were moſt notoriouſly guilty?

Q. Of Malefactors condemned, which to allot to ſuffer.

Anſw. By ſetting down 24 Ciphers, and telling along by 7 and 7, the 6 Men deſigned to die are ſeen to be ſet in the 4, 7, 12, 14, 20 and 21 Places.

Anſwer.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.

0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.

1 2 3

4 5 6

Example 2. A Ship at Sea wanting Proviſion, having 15 Chriſtians and as many Turks, and to preſerve all from famiſhing, reſolve to die to feed each other, yet ſo as by Lot, and that counting by 9 and 9 the ninth Perſon ſhould die: Now the Maſter being a Chriſtian, and deſirous to ſave the Chriſtians, diſpoſed them ſo as the ninth Perſon ſtill fell out to be a Turk: how may this be done?

Q. Of Chriſtians and Turks, how to ſave the Chriſtians.

Anſw. 4 Chriſtians 5 Turks, 2 Chriſtians 1 Turk, and ſo along according to the Vowels in *Populeam virgam mater Regina ferebat*, counting *a* 1, *e* 2, *i* 3, *o* 4, *u* 5. Or by the former way;

Anſwer.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.25.26.27.28.29.30.

14. 4 . 7 . 12. 1 10 5 2. 8 13.15.11 6. 3 9

Proof of Tranſ-*poſition.*

Tranſpoſition of both ſorts, carries along with the Demonſtration above ſuch Evidence of the Truth of the Work, that nothing can be added for Proof more convincing.

CHAP. IV. *Technology.*

Technology, *how called.*

What it doth import.

THE latter Iſſue of *Arithmetical Progreſſion* called *Technology*, goes commonly under the Names of Sports and Paſtimes; but the Learned *Alſted* entitles it *Technology*, which imports as much as an Artificial diſcourſe or diſcovery of Numbers or other things concealed.

How defined by Alſted, *and the Sorts.*

Geyſius, and *Alſted* from him, defines *Technology* to be a *Progreſſional Arithmetical Diviſion*, and that either by *Progreſſional Arithmetical Diviſors*; Or 2*ly* of *Progreſſional Dividends Arithmetically*: Or 3*ly*, into *Progreſſional Quotients Arithmetically*.

In the firſt Sort ſix things.

§. 1. In Diviſion by *Progreſſional Arithmetical Diviſors*, the Work is to find ſuch Numbers to be divided as will leave the Remains of the Diviſion, either firſt Equal; or 2*ly*, Progreſſional; or 3*ly*, Equal till the laſt Diviſion, and then nothing; or 4*ly*, Progreſſional till the Laſt, and then nothing; or 5*ly*, Diſtant from the Diviſor above an Unite; or 6*ly*, Diſorderly.

In all which there are many Numbers of ſuch Properties, ſo as the Enquiry is ſometimes for the leaſt Number of that Property, and ſometime for any ſuch Number at random.

1. *To find Remains equal.*

1*ſt*. To find Numbers whoſe Remains ſhall be equal continually, multiply all the Diviſors one into another, and to the laſt Product add the common Remain.

The leaſt Number of ſuch Property.

And if among the Diviſors there be none of them compound one of another, then this Total will be the leaſt Number of that Property, otherwiſe not. Therefore to get ſuch leaſt Number, omit the compound Diviſors, and double the Product of the reſt being multiplied one into another, if the compound Diviſors be 4 or 6, or 4 and 6, and add thereto the common Remain.

But if the Diviſors be 8, 9, 10, being 8 and 9 are doubly compound, beſides the doubling multiply by 6, and ſo proceed accordingly for other Compounds: And having gotten the leaſt Number, add him, lacking the Remain, to himſelf ſucceſſively, and other like Numbers will be produced.

Q. *Of a Number whoſe Remain* 1, *Diviſors* 2, 3, 4, 5, 6, 7.

Example. One trying whether a Number was Prime or Compound, found that by Diviſion with 2 there was 1 remaining, and ſo likewiſe dividing by 3, 4, 5, 6 and 7, there was ſtill left 1 for the Remain: what was that Number?

Anſwer.

Anſw. If the Number queſited be intended the leaſt of that ſort, it ſhall be 421; but if any Number of that Property, add to 421 continually 420, and other like Numbers will be produced.

Here the Diviſors 2, 3, 4, 5, 6, 7, multiplied one into another, produce 5040, to which 1 the common Remain added, the Total is 5041: but this is not the leaſt Number of that Property, becauſe among the Diviſors 4 is compound of 2, and 6 of 3, wherefore both 4 and 6 being omitted, the Product of the reſt is 420, to which 1 the Remain added, makes 421 for the leaſt Number of that Property.

$$2\times\overset{6}{3}\times\overset{24}{4}\times\overset{120}{5}\times\overset{720}{6}\times 7 = 5040$$

1 Remain added.

5041

$$\overset{\cancel{1}}{\frac{\cancel{5}\cancel{0}\cancel{4}1}{\cancel{2}}}\Big(2520 \quad \overset{\cancel{2}\cancel{2}}{\frac{\cancel{5}\cancel{0}\cancel{4}1}{\cancel{3}}}\Big(1680 \quad \overset{\cancel{1}\cancel{2}}{\frac{\cancel{5}\cancel{0}\cancel{4}1}{\cancel{4}}}\Big(1260 \quad \frac{\cancel{5}\cancel{0}\cancel{4}1}{\cancel{5}}\Big(1008 \quad \overset{\cancel{2}}{\frac{\cancel{5}\cancel{0}\cancel{4}1}{\cancel{6}}}\Big(840 \quad \overset{\cancel{1}}{\frac{\cancel{5}\cancel{0}\cancel{4}1}{\cancel{7}}}\Big(720$$

$$2\times\overset{6}{3}\times\overset{30}{5}\times 7 = 210$$

2

420 Doubled.

1 Remain added.

421 The leaſt Number of that Property:

$$\frac{\cancel{4}\cancel{2}1}{\cancel{2}}\Big(210 \quad \frac{\cancel{4}\cancel{2}1}{\cancel{3}}\Big(140 \quad \frac{\cancel{4}\cancel{2}1}{\cancel{4}}\Big(105 \quad \frac{\cancel{4}\cancel{2}1}{\cancel{5}}\Big(84 \quad \frac{\cancel{4}\cancel{2}1}{\cancel{6}}\Big(70 \quad \frac{\cancel{4}\cancel{2}1}{7}\Big(60$$

If

Example in more Diviſors.

If the Diviſors propoſed be 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11 . then the leaſt Number which divided by them will ſtill leave 1 remaining, is found to be 47721. For the Uncompound or Prime Numbers, 2 . 3 . 5 . 7 . 11, multiplied together, make 2310, which doubled for 4 and 6 makes 4620; and then for 8, 9 and 10, the other 3 Compounds multiplied by 6, that is 2 × 3, produceth 27720; to which 1 the common Remain added, is 27721: And by Addition of 27720 thereto, other-like Numbers will be produced.

$$\overset{}{2}\times\overset{6}{3}\times\overset{30}{5}\times\overset{210}{7}\times 11 \;=\; \begin{array}{r|c|c} 2310 & 1 & 2 \\ 2 & 4\,.\,6 & \\ \hline 4620 & 1\quad 2 & 3 \\ 6 & .\,8\,.\,9\,. & 10 \\ \hline 27720 & & \\ 1 & \text{Remain added.} & \\ \hline 27721 & \text{Leaſt Number.} & \end{array}$$

2. To find Remains Progreſſional.

2*ly*. To find Numbers whoſe Remains ſhall be Arithmetically Progreſſional: Multiply as before the Diviſors, if they be all Prime, one into another; and from the laſt Product ſubſtract the Difference between the Diviſor and Remainer given, and this Remain ſhall be the leaſt Number of that Property; to which if the Product be added, other-like Numbers will be produced.

The leaſt Number of ſuch Property.

And if ſome of the Diviſors be compound one of another, then as before omit them; and from the Product of the reſt doubled, or otherwiſe multiplied, according to the Number of Compounds as before, take the Difference: For otherwiſe if all the Diviſors be multiplied, the Number will be higher than the leaſt.

Q. Of a Number whoſe Remains are 1, 2, 3, &c. Diviſors 2, 3, 4, &c. Anſwer.

Example. Suppoſe one deſire to know what Number that is, which being ſeverally divided by 2, 3, 4, 5, 6, 7, the reſpective Remains will be 1,2,3,4,5,6.

Anſw. The leaſt Number of that Property will be found to be 419; to which 420 added ſucceſſively, other Numbers will be produced.

Here 4 and 6 among the Diviſors, being compound of 2 and 3 as before, they are omitted; and from 420 the double Product of the reſt, 1 is ſubſtracted, ſo is 419 obtained.

$$\begin{array}{rl} 2\times\overset{6}{3}\times\overset{30}{5}\times 7 = 210 & \\ 2 & \\ \hline 420 & \text{Doubled.} \\ 1 & \text{Difference ſubſtracted.} \\ \hline 419 & \text{The leaſt Number of that Property.} \end{array}$$

$$\overset{(1}{\frac{419}{2}}\Big(209 \quad \overset{\cancel{1}2(2}{\frac{4\cancel{1}9}{\cancel{3}}}\Big(139 \quad \overset{(3}{\frac{\cancel{4}\cancel{1}\cancel{9}}{\cancel{4}}}\Big(104 \quad \overset{(4}{\frac{\cancel{4}\cancel{1}\cancel{9}}{\cancel{5}}}\Big(83 \quad \overset{\cancel{5}(5}{\frac{4\cancel{1}\cancel{9}}{\cancel{6}}}\Big(69 \quad \overset{6(6}{\frac{\cancel{4}\cancel{1}\cancel{9}}{\cancel{7}}}\Big(59$$

$$\begin{array}{rl} 2\times\overset{6}{3}\times\overset{24}{4}\times\overset{120}{5}\times\overset{720}{6}\times 7 = 5040 & \\ 1 & \text{Difference ſubſtracted.} \\ \hline 5039 & \end{array}$$

$$\overset{\cancel{1}\,\cancel{1}(1}{\frac{\cancel{5}\cancel{0}\cancel{3}\cancel{9}}{\cancel{2}}}\Big(2519 \quad \overset{\cancel{2}\cancel{2}\cancel{2}(2}{\frac{\cancel{5}\cancel{0}\cancel{3}\cancel{9}}{\cancel{3}}}\Big(1679 \quad \overset{\cancel{1}\cancel{2}(3}{\frac{\cancel{5}\cancel{0}\cancel{3}\cancel{9}}{\cancel{4}}}\Big(1259 \quad \overset{(4}{\frac{\cancel{5}\cancel{0}\cancel{3}\cancel{9}}{\cancel{5}}}\Big(1007 \quad \overset{\cancel{2}\cancel{5}(5}{\frac{\cancel{5}\cancel{0}\cancel{3}\cancel{9}}{\cancel{6}}}\Big(839 \quad \overset{\cancel{1}\cancel{6}(6}{\frac{\cancel{5}\cancel{0}\cancel{3}\cancel{9}}{\cancel{7}}}\Big(719$$

Example in more Diviſors.

If the Diviſors propoſed were 2, 3, 4, 5, 6, 7, 8, 9, 10, 11; then multiplying 2, 3, 5, 7, 11, and doubling the Product, and multiplying the Double by 6 as before; and from this Product 27720 taking 1, the leaſt Number of that Property is found to be 27719: And if 27720 be added, other-like Numbers are produced.

$$\overset{}{2}\times\overset{6}{3}\times\overset{30}{5}\times\overset{210}{7}\times 11 = 2310\quad \begin{array}{ccc} 1 & & 2 \\ 2\,. & 4\,. & 6 \end{array}$$

$$\begin{array}{llll} 4620 & 1 & 2 & 3 \\ 6\,. & 8\,. & 9\,. & 10 \\ \hline 27720 & & & \\ \quad 1 & \text{Difference ſubſtracted.} & & \\ \hline 27719 & \text{Leaſt Number.} & & \end{array}$$

3. To find Remains equal till the laſt.

3*ly*. To find Numbers whoſe Remains ſhall be equal till the laſt Diviſion, and then 0 left; multiply all the Diviſors except the laſt one into another, and to the Product add the common Remain.

The leaſt Number of ſuch Property.

But if among the Diviſors more be compound one of another than 4, the leaſt of that Nature, this Number will not be the leaſt of that Property; but the Product of the Prime Diviſors doubled as before, for 4 and 6, and for the next 3 multiplied by 6, and ſo according to the Number of Compounds; and then dividing thereby the Number above-gotten, will leave the leaſt Number remaining.

Q. Of Eggs broken.

Example. A Maid carrying Eggs to Market, met with an unruly Fellow who broke them; and afterward by determination of the Juſtice, was enforced to pay for them; and thereupon the Maid being demanded, how many Eggs ſhe had? anſwered, That counting them by 2 and 2, there remained 1; and ſo likewiſe by 3 and 3, and 4 and 4, and 5 and 5, and 6 and 6; but when ſhe counted them by 7, there reſted 0: How many Eggs had ſhe at leaſt?

Anſwer.

Anſw. 301: For the Product of all the Diviſors, except 7, is 720; to which 1 added is 721, a Number of like Property, but not the leaſt of that Property: Wherefore becauſe there were 2 compound Numbers, if the double Product of all the Prime Diviſors, which is 420, be ſubſtracted from 721, the Remain will be 301. And by addition of 420 to 301, other-like Numbers will be produced.

$$2\times\overset{6}{3}\times\overset{24}{4}\times\overset{120}{5}\times 6 = 720 \qquad 2\times\overset{6}{3}\times\overset{30}{5}\times 7 = 210\times 2 = 420$$

$$\begin{array}{l} \quad\;\, 1 \;\text{Remain.} \\ \hline 721 \end{array} - 420 = 301 \text{ Leaſt Number deſired.}$$

$$\overset{\cancel{1}}{\frac{\cancel{3}01}{\cancel{2}}}(150 \quad \frac{\cancel{3}\cancel{0}1}{\cancel{3}}(100 \quad \overset{\cancel{2}}{\frac{\cancel{3}\cancel{0}1}{\cancel{4}}}(75 \quad \frac{\cancel{3}\cancel{0}1}{\cancel{5}}(60 \quad \frac{\cancel{3}\cancel{0}1}{\cancel{6}}(50 \quad \overset{\cancel{2}(0}{\frac{\cancel{3}\cancel{0}\cancel{1}}{7}}(43$$

Example in more Diviſors.

And if the leaſt Number were deſired, which being divided by 2, 3, 4, 5, 6, 7, 8, 9 and 10, would ſtill leave 1 remaining, but would be evenly divided by 11; then 3628801, the Product of all the firſt 9 Diviſors with the common Remain, divided by 27720, the Product of the Prime Diviſors doubled and multiplied by 6, the Remain will be 25201, the leaſt Number deſired of that Property.

$$2\times\overset{6}{3}\times\overset{24}{4}\times\overset{120}{5}\times\overset{720}{6}\times\overset{5040}{7}\times\overset{40320}{8}\times\overset{362880}{9}\times 10 = 3628800$$

$$\begin{array}{r} 1 \text{ Remain.} \\ \hline 3628801 \\ \hline \end{array}$$

$$2\times\overset{6}{3}\times\overset{30}{5}\times\overset{210}{7}\times\overset{2310}{11}\times\overset{4620}{2}\times 6 = 27720$$

```
   (25
   856|2
   3628 8|01 (130
   2772 2 20
   2777
    27
```

4. To find Remains Progreſſional till the laſt.

The leaſt Number of ſuch Property.

4*ly*. To find Numbers whoſe Remains ſhall be *Arithmetically Progreſſional* till the laſt Diviſion, and then 0 left; becauſe all Numbers that will be evenly divided by a greater Compound Number, will be evenly divided by the Leſſer whereof he is ſo compound, it follows therefore properly that the laſt Diviſor given ought to be a Prime Number, and then the Rule may be thus: For Diviſors to 4 multiply the Prime 2 into himſelf and abate 1, ſo ſhall 3 be divided by 2 and leave 1, by 3 and leave 0; for Diviſors to 5, multiply the Prime Numbers 2 and 3 into themſelves, and ſquare the Product, from which ſubſtract 1; ſo ſhall 35 be divided by 2 and leave 1, by 3 and leave 2, by 4 and leave 3, by 5 and leave 0. But for Diviſors higher than 5, multiply all the Diviſors except the two laſt one into another, and from the Product take the Difference as aforeſaid, and this for 7 gets the leaſt Number of that Property; and to get the leaſt Number of ſuch Property for higher Diviſors, multiply only the Prime Numbers, and for 4 and 6 double the Product for 8, 9, 10, the next 3 Compounds; multiply the laſt Product by 6, and ſo for 13 the like; and this dividing the former Number ſhall leave the leaſt Number remaining.

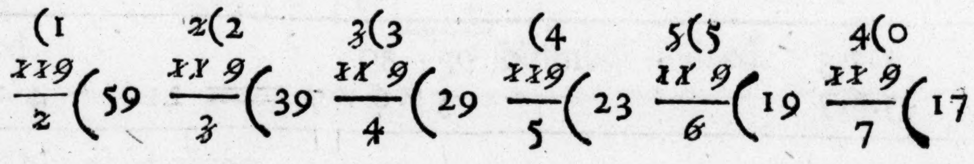
Q. Of Numbers divided by 2, 3, 4, &c. whoſe Remains 2,3,4; &c. at laſt 0.

Example. There are Numbers which being divided by 2, 3, 4, 5, 6, 7, and 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, will leave their Remains orderly in *Arithmetical Progreſſion* till the laſt Diviſion, upon which 0 will remain: what are the leaſt Numbers of thoſe Properties?

Anſwer.

Anſw. For 7 the leaſt Number is 119, which is the Product of the firſt 4 Diviſors lacking 1, and by continued Addition of 420 thereto, other like Numbers are produced: but for 11, the leaſt Number is 2519, and by Addition of 27720 thereto, other like Numbers are produced.

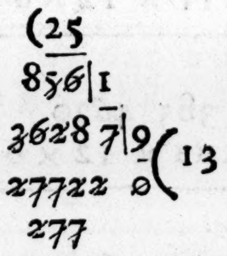
6 24
2 × 3 × 4 × 5 = 120
1
119



6 30 210
2 × 3 × 5 × 7 × 2 = 420 Number to be added.

6 24 120 720 5040 40320
2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 = 362880
1
362879

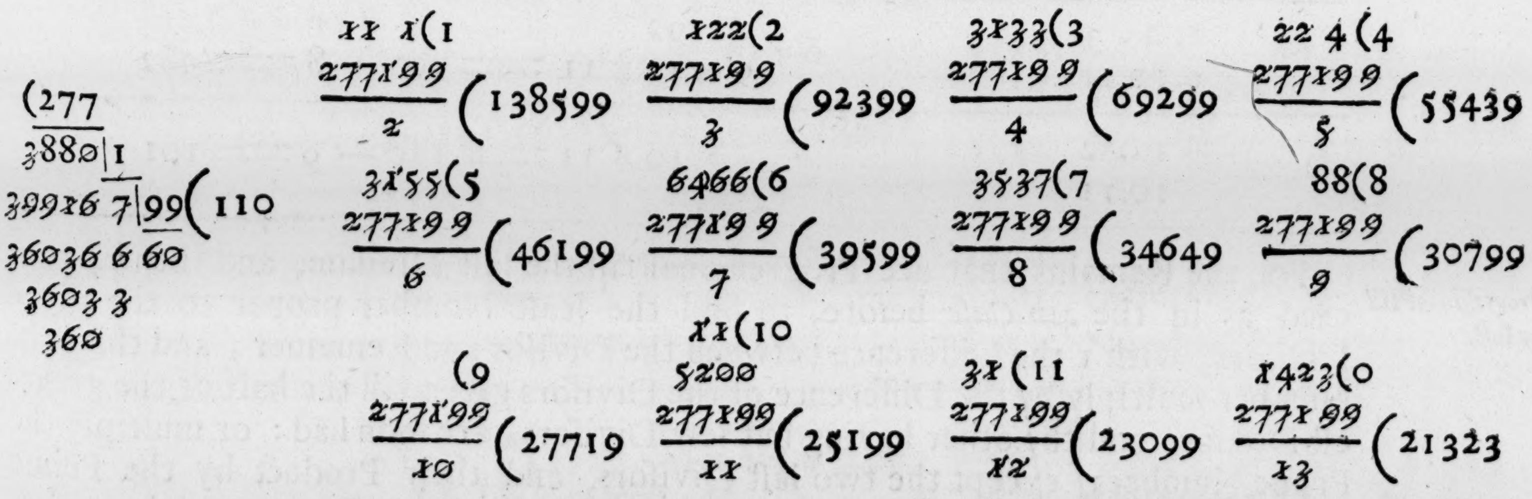
6 30 210 2310 4620
2 × 3 × 5 × 7 × 11 × 2 × 6 = 27720 Number to be added.



And if the leaſt Number were deſired, which would evenly be divided by 13; but by all the other Numbers from 1, would leave the Remains in *Arithmetical Progreſſion*; this accordingly will be found to be 277199: to which if 360360 be continually added, other-like Numbers will be produced.

6 24 120 720 5040 40320 362880 3628800
2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 = 39916800
1
39916799

6 30 210 2310 30030 60060
2 × 3 × 5 × 7 × 11 × 13 × 2 × 6 = 360360 Diviſor.



5. To find Remains Progreſſional, &c. but diſtant above 1 from the Diviſor.

5*ly*. To find Numbers that leave the Remains Progreſſional throughout, or Progreſſional till the laſt Diviſion, and then 0; but diſtant from the Diviſor above 1, as ſometimes 2, 3, 4, &c.

Progreſſional throughout.

Proceed for the Remains Progreſſional throughout, as in the ſecond *Caſe* before, only in getting the leaſt Numbers after the Prime Numbers are multiplied, their Product is to be multiplied by the Prime Numbers omitted from the Unit to the firſt Diviſor multiplied one into another; and for higher Diviſors than 7 by 6 beſides. And for eaſe it may be obſerved, that the leaſt Numbers to the half of the greateſt Diviſor, is but to abate the Difference between the Diviſor and Remainer from the leaſt Number that hath the Difference but an Unit. Other Neceſſaries may be diſcerned by the *Examples* following.

Examples to 7.

Examples to 7.

Remains. 12 60 360 *Difference.* *Numbers deſired.*
1.2.3.4.5 $3 \times 4 \times 5 \times 6 \times 7 = 2520 - 2 = 2518$. Greater.
3.4.5.6.7 $3 \times 5 \times 7 \times 4 = 420 - 2 = 418$. Leaſt.
Diviſors. 15 105

20 120
1.2.3.4 $4 \times 5 \times 6 \times 7 = 840 - 3 = 837$. Greater.
4.5.6.7 $5 \times 7 \times 12 = 420 - 3 = 417$. Leaſt.
35

1.2.3 30
5.6.7 $5 \times 6 \times 7 == 210 - 4 = 206$. Leaſt.

1.2 $6 \times 7 == 42 - 5 = 37$. Leaſt.
6.7

Examp. to 11.

Examples to 11.

1.2.3.4.5.6.7. 8 . 9 Greater Number 19958398
3.4.5.6.7.8.9.10.11 15 105 1155 4620 Leaſt Number.
$2 \times 2 = 4$ $3 \times 5 \times 7 \times 11 \times 4 \times 6 = 27720 - 2 = 27718$

1.2.3.4.5.6. 7 . 8 6652797
4.5.6.7.8.9.10.11 35 385 4620
$2 \times 2 \times 3 = 12$ $5 \times 7 \times 11 \times 12 \times 6 = 27720 - 3 = 27717$

1.2.3.4.5. 6 . 7 1663196
5.6.7.8.9.10.11 35 385 4620
$2 \times 2 \times 3 = 12$ $5 \times 7 \times 11 \times 12 \times 6 = 27720 - 4 = 27716$

1.2.3.4. 5 . 6 332635
6.7.8.9.10.11 77 4620
$2 \times 2 \times 3 \times 5 = 60$ $7 \times 11 \times 60 \times 6 = 27720 - 5 = 27715$

1.2.3. 4 . 5 55434
7.8.9.10.11 77 4620
$2 \times 2 \times 3 \times 5 = 60$ $7 \times 11 \times 60 \times 6 = 27720 - 6 = 27714$

1.2. 3 . 4 7913
8.9.10.11 660
$2 \times 2 \times 3 \times 5 = 60$ $11 \times 60 \times 6 = 3960 - 7 = 3953$

1. 2 . 3 90
9.10.11 $9 \times 10 \times 11 == 990 - 8 == 982$

1 . 2 $10 \times 11 == 110 - 9 == 101$
10.11

Progreſſional till the laſt.

For the Remains that are Progreſſional till the laſt Diviſion, and then 0, proceed as in the *4th Caſe* before, to find the leaſt Number proper to the given Diviſors, with 1 the Difference between the Diviſor and Remainer; and this leaſt Number multiply by the Difference of the Diviſors given, till the half of the greateſt Diviſor, and the other having but few Diviſors, are ſoon had: or multiply the Prime Numbers, except the two laſt Diviſors, and their Product by the Prime Numbers omitted from the Unit, multiplied with the Difference, and 2 and 6, according to the Compounds as aforeſaid, taking in 4 among the Prime Diviſors for 7; but halfing the Difference, and next omitting it in higher Diviſors in this laſt Multiplication, when in 7 the firſt Diviſor is next above the greater Half of the greater or laſt Diviſor, and in higher Diviſors than 7 a Place further, and from the Product take the Difference. What elſe needful may be obſerved in the Inſtances following.

Examples to 7.

Examples to 7.

Remains. 12 60 Difference 2 × 2 omitted.
1.2.3.4.0 3 × 4 × 5 × 4 = 240
3.4.5.6.7 2 2
Diviſors. omitted. 238 Number deſired.

Remains. 20 Difference 3 × 2 × 3 omitted.
1.2.3.0 4 × 5 × 18 = 360
4.5.6.7 2 + 3 3
Diviſors. omitted. 357 Number deſired.

Remains. Difference 4 ½ 2 × 2 × 3 omitted.
1.2.0 5 × 12 = 60
5.6.7 2 + 3 4
Diviſors. omitted. 56 Number deſired.

1.0
6.7 is 7 the Number given.

Alſo 119 being found by the 4*th Caſe* before to be the Number ſought, where 1 is the Difference between the Diviſor and Remainer: If therefore 119 be multiplied by the Differences 2 and 3, there will be produced 238 and 357, as before.

Examples to 11.

Examp. to 11.

1.2.3.4.5.6.7.8.0 15 105 420 840
3.4.5.6.7.8.9.10.11 3 × 5 × 7 × 4 × 2 × 6 = 5040
2 Difference.
2 omitted × 2 Difference = 4 5038 Number.

1.2.3.4.5.6.7.0 35 630 1260
4.5.6.7.8.9.10.11 5 × 7 × 18 × 2 × 6 = 7560
3 Difference.
2 + 3 omitted × 3 Difference = 18 7557 Number.

1.2.3.4.5.6.0 35 840 1680
5.6.7.8.9.10.11 5 × 7 × 24 × 2 × 6 = 10080
4 Difference.
2 + 3 omitted × 4 Difference = 24 10076 Number.

1.2.3.4.5.0 1050 2100
6.7.8.9.10.11 7 × 150 × 2 × 6 = 12600
5 Difference.
2 + 3 + 5 omitted × 5 Difference = 150 12595 Number.

1.2.3.4.0 1260 2520
7.8.9.10.11 7 × 180 × 2 × 6 = 15120
6 Difference.
2 + 3 + 5 omitted × 6 Difference = 180 15114 Number.

1.2.3.0 300
8.9.10.11 150 × 2 × 6 = 1800
2 + 3 + 5 + 7 omitted. 7 Difference.
So 2 × 3 × 5 × 5 (omitting 7) = 150 1793 Number.

1.2.0 60
9.10.11 30 × 2 × 6 = 360
2 + 3 + 5 + 7 omitted. 8 Difference.
So 2 × 3 × 5 (omitting 7) = 30 352 Number.

1.0

1.0
10.11 is 11 the Number given.

Alſo 2519 by the *4th Caſe* before, being found to be the Number ſought, where 1 is the Difference; if then 2519 be multiplied by the Differences 2, 3,4,5 and 6, the Numbers will be found as above 5038. 7557. 10076. 12595. and 15114.

Other Diverſities may be obſerved in higher Diviſors, but for that ſeldom more than 11 are propounded in any Queſtion, this may ſuffice.

6. To find Remains diſorderly.

6ly. To find the Numbers which divided by the given Diviſors leave the Remains diſorderly, let it be obſerved, that the greateſt Diviſor is the Exceſs of an *Arithmetical Progreſſion*, and the Remain of that Diviſor, if any, the firſt Term thereof if 0 remain; the greateſt Diviſor is both the firſt Term and the Exceſs, and the Number ſought is the laſt Term of the *Progreſſion:* add therefore the Exceſs to the firſt Term continually, till proving every Term by all the Diviſors, a Number be found that will leave the Remains propoſed.

Q. Of a Sum divided to 4 Children, &c.

Example 1. A Man diſtributeth a Sum of Money among his 4 Children equally, and hath 3 *l.* left for himſelf: but if that Sum had been equally parted among 7, he would have had 5 *l.* left: what was that Sum?

Anſwer.

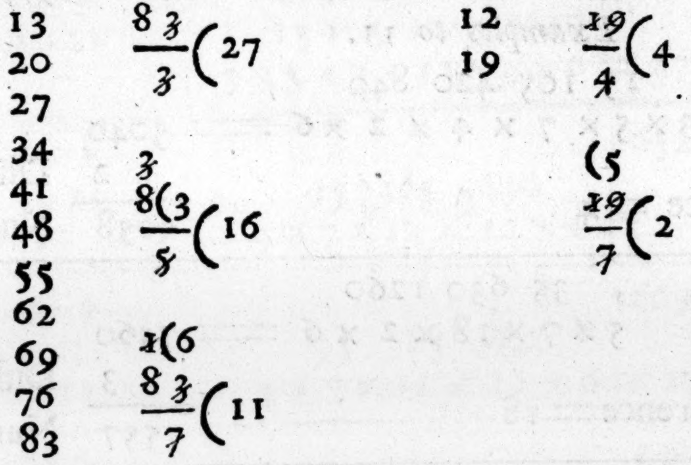
Anſw. The leaſt Number of that Property is 19, which is found by adding 7 the greateſt Diviſor to 5 the firſt Number, and again, to 12 the ſecond Term, becauſe 12 divided by 4 leaves 0.

Q. Of a Number thought on.

Example 2. One thinking on a Number, deſires me to tell him what it is, whereupon I bid him divide it by 3, 5, 7, and tell me the Remains, which done, he declares them to be 2, 3, 6: what then is the leaſt Number of that Property?

Anſwer.

Anſw. 83: For by adding 7 the Exceſs to 6, the firſt Term ſucceſſively, I find no Numbers till 83 of that Property.



Other Examples.

To get other Numbers of like Property, multiply all the Diviſors given one into another, and add the Product to the leaſt Number found as above. So in the firſt Example 28, that is 4×7, added to 19; and in the ſecond Example 105, that is 3 × 5 × 7; added to 83, ſhall produce other Numbers of like ſort.

The Reaſon of leaving the Way uſed by others.

The *Appendix* added by *Alſted* out of *Geyſius*, directeth to get the leaſt Numbers by multiplying the Remains into certain Multiplicands, and dividing the Total of the Products by the Diviſors multiplied one into another, the leaſt Number ſhall be left remaining: but becauſe ſometime the leaſt Number will be more than the Product of all the Diviſors, that way is not ſo generally approved. For 32 divided by 4 and 7, leaves the Remains 0, 4; but 28, that is 4 × 7, dividing any Number, can never have 32 left a Remain: wherefore though the Way here above mentioned be tedious in many Diviſors, yet being general and holding true in all Caſes, it is to be choſen before the other, which by reaſon of the wonderful Variety without multiplicity of Rules and Exceptions, cannot be made good but in ſome particular Caſes.

In diviſion of Progreſſional Dividends, four things.

§. 2. In Diviſion of *Progreſſional Dividends*, the work is to divide ſo that the Quotients may be *Arithmetically Progreſſional* in their Natural Order, but the Remains Nothing or Equal, Ordinate or Perturbate; in which latter only the Quotients are often interrupted in their *Progreſſion*.

1. To find Nothing remain.

In every *Arithmetical Progreſſion*, the Remains will be 0 when the firſt Term is equal to the Difference: For then the Difference will exactly divide all the Terms, as 3, 6, 9, 12, 15, *&c.* this is a Natural *Progreſſion*; and 3 the firſt Term being equal to the Difference, dividing all the Terms, the Quotients will be in their Natural Order, 1, 2, 3, 4, 5, and 0 remain.

(o

Example.

(0 (0 (0 (0 (0

$\frac{3}{3}$(1 $\frac{6}{3}$(2 $\frac{9}{3}$(3 $\frac{12}{3}$(4 $\frac{15}{3}$(5

2. To find equal Remains.

In an *Arithmetical Progreſſion*, when the Difference will not exactly divide the Terms, this is called ſometimes an *Artificial Progreſſion*, and ſhall have the Quotients *Progreſſional*, but all the Remains equal; as 3, 5, 7, 9, 11, *&c.* divided by 2, the Exceſs ſhall have the common Remain 1, but the Quotients 1, 2, 3, 4, 5, in their Natural Order.

Example.

(1 (1 (1 (1 (1

$\frac{3}{2}$(1 $\frac{5}{2}$(2 $\frac{7}{2}$(3 $\frac{9}{2}$(4 $\frac{11}{2}$(5

3. To find ordinate Remains.

In Diviſion of an *Arithmetical Progreſſion*, when the Diviſor equal to the Number of Terms ſhall be but an Unit greater than the firſt Term in a Natural *Progreſſion*, the Remains ſhall be Ordinate or Retrograde, *Arithmetically Progreſſional*; but the Quotients orderly, as 5, 10, 15, 20, 25, 30, divided by 6, makes the Quotients 0, 1, 2, 3, 4, 5, but the Remains 5, 4, 3, 2, 1, 0.

Example.

(4 (3 (2 (1 (0

$\frac{5}{6}$(0 $\frac{10}{6}$(1 $\frac{15}{6}$(2 $\frac{20}{6}$(3 $\frac{25}{6}$(4 $\frac{30}{6}$(5

4. To find perturbate Remains.

In Diviſion of an *Arithmetical Progreſſion*, when the Diviſor equal to the Number of Terms ſhall be more than an Unit greater than the firſt Term in a Natural *Progreſſion*, the Remains ſhall be Pertubate, yet have all the Natural Numbers to the Diviſor, as 5, 10, 15, 20, 25, 30, 35, divided by 7; the Remains ſhall be 5, 3, 1, 6, 4, 2, 0, which are all the Numbers to 7.

Example.

(3 (1 (6 (4 (2 (0

$\frac{5}{7}$(0 $\frac{10}{7}$(1 $\frac{15}{7}$(2 $\frac{20}{7}$(2 $\frac{25}{7}$(3 $\frac{30}{7}$(4 $\frac{35}{7}$(5

In diviſion into Progreſſional Quotients, two things.

§. 3. The Diviſion into *Progreſſional Quotients Arithmetically*, is not properly where the Dividends are *Progreſſional*, as in the laſt Section, though *Geyſius* placeth the 2 laſt ſorts above here: but the Work is to find a Number which being divided, and the Remains continually by certain Diviſors given, the Integers in the Quotients ſhall be in their Natural *Progreſſion* deſcending to 1. As to divide 110 by 24, and the Remain 14 by 4, and the Remain 2 by 1; the Quotients will be 4, 3, 2.

In this Diviſion is conſiderable the Invention of $\left\{\begin{array}{l}\text{Diviſors.}\\ \text{Dividends.}\end{array}\right.$

1. To find the Diviſors.

For the finding of Diviſors, let be noted.

1ſt, The laſt Diviſor will always be 1.

2ly, Therefore to 1 add the firſt Quotient, and the Total is the *Penult Diviſor*.

3ly, To the Product of the firſt Quotient by the *Penult Diviſor*, add the ſecond Quotient and 1 for the *Antepenult Diviſor*.

4ly, To the Product of the firſt Quotient by the *Antepenult Diviſor*, add the Product of the ſecond Quotient into the *Penult Diviſor*; and the third Quotient and 1 for the *Proantepenult Diviſor*.

Example.

As if the Quotients were deſired to be 5, 4, 3, 2; then ſhall

1 be the *Ultimate Diviſor.*
$5 + 1 = 6$ *Penult Diviſor.*
$5 \times 6 + 4 + 1 = 35$ *Antepenult Diviſor.*
$5 \times 35 + 4 \times 6 + 3 + 1 = 203$ *Proantepenult Diviſor.*

2. To find the Dividends.

For the finding of the Dividends.

Add together the ſeveral Products of the Diviſors, multiplied by their reſpective Quotients.

Example.

As if a Number be deſired, which divided by 203, the Quotient ſhall be 5; and the Remain of that Diviſion divided by 35, the Quotient ſhall be 4; and the Remain of that Diviſion divided by 6, the Quotient ſhall be 3; and the Remain of that Diviſion divided by 1, the Quotient ſhall be 2: Then ſhall 1175 be obtained thus.

$$\begin{array}{rcr} 203\times 5 & = & 1015\\ 35\times 4 & = & 140\\ 6\times 3 & = & 18\\ 1\times 2 & = & 2\\ \cline{3-3} & & 1175\\ \cline{3-3} \end{array}$$

$$\begin{array}{l}(160\\ \hline \cancel{1}\cancel{1}7\cancel{5}\\ \hline \cancel{2}\cancel{0}\cancel{3}\end{array}(5 \qquad \begin{array}{l}(2\\ \cancel{1}\cancel{6}(0\\ \hline \cancel{3}\cancel{5}\end{array}(4 \qquad \begin{array}{l}(2\\ \cancel{2}\cancel{0}\\ \hline \cancel{6}\end{array}(3 \qquad \begin{array}{l}(0\\ \cancel{2}\\ \hline \cancel{1}\end{array}(2$$

Q. Of diſcovering three Things, hidden.

Suppoſe *A*, *B*, *C* have three Things, as a Ring, a Thimble, and a Bodkin; which they hiding, exchange one with another: how may the Order of their hiding be diſcovered?

Rule, how called.

This Rule is called *Bis cæca*, or *twice Blind*, becauſe it is grounded on blind or unknown Proportions, and therefore requires the more careful Obſervation of theſe ſix Particulars.

What to be done.

1. Progreſſionals taken.

1. For the Number of Things ſo hidden or concealed, take Numbers Progreſſional, as 1, 2, 3, *&c.* So let the Ring be 1, the Thimble 2, and the Bodkin 3.

2. Diviſors found.

2. According to the third Section above, find the peculiar Diviſors for thoſe Progreſſional Quotients, always 1 leſs than the Quotients; which in the Example above ſhall be 4, 1; that is, 1 the Laſt, and 3 + 1 the *Penult Diviſor*.

3. Multiplicands found.

3. Multiplicands are to be found in a certain Order, thus; The laſt Multiplicand is taken at pleaſure; from which Number ſubſtract the Diviſors in their order, and the Remains ſhall be the other Multiplicands in their order: As in the Example above, taking 6 for the laſt Multiplicand for *C*, then ſhall the other Multiplicands be 2 and 5, that is, 6 — 4, and 6 — 1: So ſhall the three Multiplicands be, for *A* 2, for *B* 5, for *C* 6.

4. Progreſſionals multiplied.

4. Cauſe every one of the Perſons hiding the Things, to multiply the Progreſſional Number of the Thing they have ſo hidden by their proper Multiplicands, and to tell you the Sum of the Products. So if *A* have the Thimble, or 2; and *B* the Ring, or 1; then ſhall *C* have the Bodkin, or 3: And then multiplying 2 by 2, and 5 by 1, and 3 by 6, the Product of *A* is 4, *B* 5, *C* 18, and the Total 27.

5. Total of the Products ſubſtracted.

5. Having the Total, ſubſtract the ſame from the Product of the laſt Multiplicand by the Sum of the Progreſſionals: So 6 being the laſt Multiplicand in the above-mentioned Example, it ſhall be multiplied in 6 the Progreſſionals; that is, 1 + 2 + 3 from this Product 36; the Total 27 ſubſtracted, leaves 9 remaining.

6. Remain divided, gives diſcovery.

6. This Remain divided by the firſt Diviſor, the Quotient ſheweth the Thing hid by *A*; the Remain of this Diviſion divided by the next Diviſor, ſheweth the Thing hid by *B*: And ſo ſucceſſively for all the reſt till the laſt, where there are many Diviſors, which laſt Thing ſhall be the Complement. Wherefore in the former Example, 9 the Remain divided by 4 for *A*, the Quotient ſhall be 2, and ſhew *A* to have the Thimble; the 1 remaining divided by 1, ſhall ſhew *B* to have the Ring, or firſt Thing; and by Conſequence the other Thing remaining, which is the Bodkin, muſt be in the Cuſtody of *C*.

Example in four Progreſſionals.

Example in four Things hidden.

Perſons *A*, *B*, *C*, *D*. Things hid 1, 2, 3, 4.

Diviſors 24 . 5 . 1. that is, 1 the laſt Diviſor; 5, which is 4 + 1, the *Penult Diviſor*; and 24 made of 4×5 + 3 + 1, the *Antepenult Diviſor*.

The laſt Multiplicand ſuppoſe 12; then the other Multiplicands ſhall be

$$\begin{array}{rrrrrrrr} & 12 & & 12 & & 12 & & 12\\ & 24 & & 5 & & 1 & & 0\\ \cline{2-2}\cline{4-4}\cline{6-6}\cline{8-8} \text{for } A\,— & 12 & B & 7 & C & 11 & D & 12\\ \cline{2-2}\cline{4-4}\cline{6-6}\cline{8-8} \end{array}$$

$$\begin{array}{lrrrrrrrrrr} \text{Suppoſing } A \text{ hath hid} & 2 & . & B & 3 & . & C & 1 & . & D & 4.\\ \text{Then the Products are}\,— & 12 & & & 7 & & & 11 & & & 12\\ \cline{2-2}\cline{5-5}\cline{8-8}\cline{11-11} \text{For } A\,— & 24 & . & B & 21 & . & C & 11 & . & D & 48\\ \cline{2-2}\cline{5-5}\cline{8-8}\cline{11-11} \end{array}$$

The Total 21 + 11 + 48 — 24 = 56.

The laſt Multiplicand 12. Progreſſionals 1, 2, 3, 4 = 10

Product of 12 × 10 = 120 — 56 = 64.

Then dividing 64 by 24, the Quotient 2 ſhews *A* to have the ſecond Thing hid; the Remain 16 divided by 5, gives 3 in the Quotient for *B*, and denotes he the third Thing hid; the Remain 1 divided by 1, ſhews *C* hath the firſt Thing; and by Conſequence *D* muſt have the fourth Thing.

$$\begin{array}{c} (\underline{16} \\ \cancel{6}\cancel{4} \\ \hline \cancel{2}\cancel{4} \end{array}\Big(\,2\ A. \qquad \begin{array}{c} (\underline{1} \\ \cancel{1}\cancel{6} \\ \hline \cancel{5} \end{array}\Big(\,3\ B. \qquad \begin{array}{c} \cancel{1} \\ \hline \cancel{1} \end{array}\Big(\,1\ C. \qquad D\ 4.$$

Hence the Ground of other Methods, to find Things hid, and Numbers thought on.

The Things premiſed in the foregoing Sections of this Chapter being the Ground, as well of diſcovering the Dividends by the Diviſors and Remains, concealing the Quotients, as by the natural Progreſſion of Numbers from an Unit, and the Products of them by certain Multiplicands in one Sum, (concealing the Order of Multiplication) to find the Order of Multiplication, gave being principally to the ſeveral Inventions of other Methods for finding Numbers thought upon, and Things hidden. Some Examples whereof follow.

To find a Number thought on.

1. *To find a Number thought upon, or to tell a Man how many ſingle Pence, or Pieces of Money he hath in his Purſe.*

Bid the Perſon that thinketh, that he quadruple the Number thought on, and to the Product add 6, 8, 10, or any other Number at pleaſure, and tell you the Half of the Total: For then if you take away half the Number which you willed him to add, there will remain double the Number at firſt thought upon.

Example.

The Number thought, ſuppoſe —— 6
The Quadruple thereof —— 24
To which if 10 be added, it makes —— 34
The Half of it is —— 17
From which 5 half the Number added being ſubſtracted, there remaineth —— } 12
One half of which returneth the firſt Number 6

Otherwiſe.

Or bid him that thinketh, to double his Number; to which let him add 5, and then multiply the Total by 5; and having the Product, cut off the Right-hand Figure, or Cipher, and from the Reſidue ſubſtract 2.

Example.

The Number thought upon —— 8
The double of it —— 16
Addition of 5 makes it —— 21
Multiplied by 5, produceth —— 105
Right-hand Figure cut off, leaves —— 10
From which 2 ſubſtracted, reſts —— 8

Otherwiſe.

Otherwiſe, bid him that thinketh to double the Number, and add 4 thereto, and multiply the Total by 5, and add to the Product 12, and multiply that Total by 10, and declare to you this laſt Product: Then from the ſame withdraw 320, and the Remainder in the hundred Place ſhall be the Number deſired.

Example.

The Number thought —— 6
Doubled is 12, to which 4 added is —— 16
Multiplied by 5 produceth 80, and 12 added is - 92
That multiplied by 10, produceth —— 920
From which 320 ſubſtracted, leaves —— 600
The Remainder in the hundred Place is —— 6

Otherwiſe.

Otherwiſe, bid him that thinketh, to triple his Number thought: then aſk him if it be Even or Odd; if Odd, give him 1 to make it Even: and if 1 be given, reſerve 1 in your Mind; and after the Number tripled is made Even, let him caſt away half, and then triple that half: Then aſk him again, if it be Even or Odd: if Odd, give him 1 again to make it Even; and for this 1 given upon the laſt tripling, reſerve 2 in your Mind, (ſo that if you give 1 at both triplings, then have you 3 reſerved in Mind); afterwards let him caſt away half, and tell you how many Nines he can give you out of the other half; and for every 9 accompt 4, to which add the reſerved Numbers if the Triples fell odd.

Example.

The Number thought ———————— 15
The Triple ———————————— 45
Becauſe it is Odd, 1 is added, and it is —— 46 1 Reſerved.
The Half ————————————— 23
The Triple ———————————— 69
Becauſe it is Odd, 1 is added, and it is —— 70 2 Reſerved.
The Half ————————————— 35
Out of 35 can be given but 3 Nines, which at 4 for 9, returneth —— } ——— 12
15 Total.

Otherwiſe.

Otherwiſe, bid him that thinketh that he break the Number in two Parts, and ſquare each part, and add the Products together; and then let him multiply the Parts one by another, and add the Product doubled to the former, and tell you the whole Sum; then take the Square Root thereof for the deſired Number.

Example.

The Number thought ———————— 7
The Parts ſuppoſe 3 and 4
The Square of { 3 is 9, 4 is 16 } together —25
The Parts 3 and 4 multiplied make —12
Doubled is 24, added to 25 is ——49
The Square Root thereof is ——— 7

Otherwiſe.

Otherwiſe, bid the Party that thinketh, that multiply the Number thought by what Number you pleaſe; then bid him divide the Product by another Number, and multiply that Quotient by ſome other Number; and that Product again divide by ſome other as often as you will: and in like manner take a Number at Pleaſure, and ſecretly multiply and divide by the ſame Multipliers and Diviſors as oft and in the Order he did, then bid him divide the laſt Quotient by the Number firſt thought, and in like manner do you; ſo will the Quotients be both alike; a Thing which ſeems admirable to thoſe ignorant of the Cauſe: then bid him to his Quotient add his Number thought on, and demand the Sum (as if you knew nothing of his Quotient), from which ſubſtract your Quotient, and you have your deſire.

Example.

The Number thought ———————— 8
Divided by 4, the Quotient is ——— 2
Which multiplied by 8, is ———— 16
This divided by 4, is ————— 4
Which multiplied by 5, is ———— 20
This divided by the firſt 8, is ——— $2\frac{1}{2}$

$$\frac{8}{\cancel{4}}\Big(\begin{matrix}2\\ \underline{8}\end{matrix}$$

$$\frac{\cancel{16}}{\cancel{4}}\Big(\begin{matrix}4\\ \underline{5}\end{matrix}$$

$$(4$$

$$\frac{\cancel{20}}{8}\Big(\,2\tfrac{1}{2}$$

In like manner, if 12 be taken and divided and multiplied by the ſame Numbers, and the laſt Product divided by 12, the Quotient will be $2\frac{1}{2}$ as the other.

$$\frac{\cancel{12}}{\cancel{4}}\Big(\,3\times 8=\frac{\cancel{24}}{\cancel{4}}\Big(\,6\times 5=\overset{(6}{\frac{\cancel{30}}{12}}\Big(\,2\tfrac{1}{2}.$$

To find a Number thought on, without asking any Queſtions.

2. *To find a Number thought upon without asking any Queſtions, certain Operations being done.*

Bid him that thinketh add half of his Thought-Number thereto; and if it be an odd Number and cannot be halfed evenly, let him take the bigger half to add to it, and if ſo, keep 3 in Mind; then bid him take half the whole Sum and add thereto, and if it be an odd Number again as before, let him take the bigger Half, and for this odd one reſerve 2 in Mind, ſo as if both be odd, 5 muſt be reſerved: when this is done, let him ſubſtract from the laſt Total double the Number thought upon, and from the Remainer will him to caſt away half if he can; if it be odd, then reject 1, which 1 reſerve, and ſo perpetually halfing it till he come to 1: for then mark how many halfs there were after the double Number was ſubſtracted, and for the firſt half accompt 2, for the ſecond 4, for the third 8, *&c.* and add unto theſe Numbers the Units rejected upon the laſt halfings (noting the firſt

1

1 rejected is but 1, the ſecond is to be reckoned 2, the third 4, *&c.*) multiply this Sum by 4, and from the Product deduct the Numbers reſerved upon the firſt halfings, if any.

Examples in even Numbers.

Example in even Numbers.

The Number thought ——	8		The Number thought ——	18	
The half added 4, is ——	12		The half added 9, is ——	27	
The half of 12 is 6, added is	18		The bigger half added makes ——	41	2 Reſerved.
Double of 8, is ——	16		Double of 18, is ——	36	
Subſtracted, leaves ——	2		Subſtracted, leaves ——	5	
The half ——	1		The firſt half ——	2	1 Rejected.
			The ſecond half ——	1	
For this one halfing is accompted 2, which multiplied by 4, returneth the firſt Number	8		For this ſecond halfing is accompted 4, which with 1 rejected is 5; this multiplied by 4 is 20, and the 2 reſerved deducted, there reſts	18	

Examples in Odd Numbers.

Example in odd Numbers.

The Number thought ——	7		The Number thought ——	17	
Greater half 4 added, is-	11	3 Reſerv'd.	Greater half 9 added, is -	26	3 Reſerv'd.
Greater half 6 added, is-	17	2 Reſerv'd.	Half 13 added, is ——	39	
Double of 7, is ——	14		Double of 17, is ——	34	
Subſtracted, leaves ——	3		Subſtracted, leaves ——	5	
Leſſer half ——	1	1 Rejected.	Leſſer half ——	2	1 Rejected.
			Second half ——	1	
For this half 2 being taken, and the Rejected 1 is 3, multiplied by 4 is 12, from whence the Reſerved 5 deducted, leaves	7		For this half 4 being taken, and the Rejected 1 is 5, multiplied by 4 is 20; from whence the Reſerved 3 deducted, leaves	17	

3. *Two Numbers being propoſed unto two Perſons, to tell which of thoſe Number is taken by each of the Parties.*

To find which of two Numbers taken.

Admit the two Numbers propoſed be one Even and the other Odd, as 11 and 12; and when they have privately accepted which they pleaſe thereof, triple the one and double the other, and add their Products together: Contrariwiſe, double that Number you tripled, and triple that Number you doubled, and add their Products together, the one will be Even and the other Odd; then bid A to triple the Number he took, and bid B to double the Number he took, and add both their Products together; and if A can half the Sum evenly, he took the Even Number, otherwiſe B accepted thereof.

Numbers propoſed, 11, 12.

Example.

A taketh ——	11	A taketh ——	12
B ——	12	B ——	11
A tripleth ——	33	A tripleth ——	36
B doubleth ——	24	B doubleth ——	22
Total Odd ——	57	Total Even ——	58

4. *To diſcover whether Even or Odd Numbers be taken when they are not propoſed.*

To find if the Number taken be Even or Odd.

Bid A to triple his Number and B to double his, and add both the Products together, and if they can give you the Even half, A took an Even Number, and B an Odd; if not, underſtand the contrary.

Example.

A		B			A		B		
4		3			3		4		
3		2	Even.		3		2	Odd.	
12	+	6	= 18		9	+	8	= 17.	

5. *To know ſeveral Numbers thought upon by one or ſundry Perſons.*

To find ſeveral Numbers thought on.

Bid them add the firſt and ſecond together, and tell you the Total; likewiſe the Total of the Second and Third, and ſo further if there be more Numbers, and then tell you the Total of the firſt and laſt Numbers added together: For then by the Rule of *Falſhood* or *double Poſition*, as before in the ſecond Part of this 4th Book, *Chap.* 14. Reſolution may be had.

For Odd.

Or if the many Numbers thought on be Odd, as 3 Numbers, 5, 7, *&c.* having the Totals given as before, place them in order, and add together all thoſe that ſtand in the odd Places, *viz.* the Firſt, Third, Fifth, *&c.* keep this Number apart from which you muſt make Subſtraction. In like manner add all thoſe Numbers together which are in the even Places, *viz.* the Second, Fourth, *&c.* and ſubſtract this Total from the former, the Remain ſhall be double the firſt Number; which found, the reſt are eaſily known, becauſe the Sum of the Firſt and Second is given.

For Even.

But if the many Numbers thought on be Even, as 4 Numbers, 6, 8, *&c.* then having the Totals of each two Numbers as before, with the Total of the ſecond and laſt Numbers, add the Numbers in the odd Places, except the Firſt, and take the Sum from the Sum of the Numbers in the even Places, the Remain ſhall be double the ſecond Number thought upon: And this being known, the reſt are eaſily obtained.

Example in Odd.

Example in five Numbers.

Suppoſe the Numbers thought ——— 3 . 6 . 4 . 8 . 10
Firſt and Second, are ——— 9
Second and Third ——— 10
Third and Fourth ——— 12
Fourth and Fifth ——— 18
Firſt and Laſt ——— 13
Numbers in the odd Places, are 9 . 12 . 13. together ——— 34
Numbers in the even Places, are 10 . 18. together ——— 28
Difference ——— 6
The Half is the firſt Number 3.
This taken from 9, the Firſt and Second leaves 6, *&c.*

Example in Even.

Example in ſix Numbers.

Suppoſe the Numbers thought ——— 4 . 6 . 7 . 9 . 10 . 11
Firſt and Second, are ——— 10
Second and Third ——— 13
Third and Fourth ——— 16
Fourth and Fifth ——— 19
Fifth and Sixth ——— 21
Second and laſt ——— 17
Numbers in the even Places are 13 . 19 . 17 . together ——— 49
Numbers in the odd Places are, 16 . 21 . together ——— 37
Difference ——— 12
The Half is the ſecond Number 6.
This taken from 10, the Firſt and Second, leaves 4, *&c.*

To find Digits thought on, or Points of Dice.

6. *To declare one or more of the Digits thought upon, or the Points caſt by two or more Dice.*

Let the firſt Number be doubled, and thereto add 5; multiply the Sum by 5, then add 10, and the next Number thought upon; multiply this Sum by 10, and thereunto add the next Number, and ſo proceed. Now if he give you the laſt Sum, then if he thought but upon one Number, ſubſtract only 35 from it, and the Remain in the Place of Tens is the Number thought upon: If he thought on 2, then take 350, and the Remainers in the 100 and 10 Places are the deſired Numbers, *&c.* And becauſe the Right-hand Figures, where above 3 are thought upon, will be the ſame Numbers thought upon, the more to conceal the Secret, you may ask for half the laſt Sum, or bid him put 14, 15, or ſome other Number thereto, which afterward may be eaſily deducted.

Numbers thought, ſuppoſe ——— 4.6.8.7.9. **Example.**
Double of 4 is ——— 8
Addition of 5 makes it ——— 13
Multiplication of 5 produceth ——— 65
Addition of 10 makes it ——— 75 — 35 = 40
Next Number thought added ——— 81
Sum multiplied by 10, is ——— 810 — 350 = 460
Next Number 8, added, is ——— 818
Sum multiplied by 10, is ——— 8180 — 3500 = 4680
Next Number 7 added, is ——— 8187
Sum multiplied by 10, and 9 added, makes it ——— } 81879 — 35000 = 46879

7. *To tell what Numbers remain, after certain Operations done, without asking any Queſtions.* ***To find what Remains, &c.***

Let him that thinketh on a Number, multiply it by what Number you pleaſe: to the Product bid him add another Number, (which you muſt be ſure may be equally divided by that Number before multiplied by) then let him divide the Sum by the Number he firſt multiplied by, and from the Quotient ſubſtract the Number thought; and this Remainer ſhall be equal to your Quotient, if you divide that Number which was added by that which multiplied.

Number thought, ſuppoſe ——— 8 **Example.**
Multiplied by 5, is ——— 40
Adding 20, it is ——— 60
Which divided by 5, gives ——— 12 So 20, divided by 5, yieldeth 4
Number thought taken away, leaves remaining ——— } ——— 4

8. *A Ring hidden among 9 or 10 Perſons; how to diſcover the Perſon that hath the Ring, and upon which Hand, Finger, and Joint.* ***To find what Hand, Finger and Joint a Ring is on.***

When the Ring is diſpoſed among the Company, you being abſent, cauſe the Perſons to ſit down in a Row; and let one of them who is privy to the Ring's Diſpoſal, double the Number of the Perſon, and thereto add 5; then let him multiply the Addition by 5, to the Product bid him add the Number of the Finger: And laſtly, to the right Hand adjoin the Number of the Joint, and add to the whole Sum, for Secrecy-ſake, 7, 8, 9, *&c.* which done, require the Total; whence take the Number laſt added, and from the Reſidue ſubſtract 250, and you ſhall have 3 Figures left; the Firſt whereof to the left Hand ſhall ſignify the Perſon, the middle Number the Finger, and the Third the Joint. And if after Subſtraction of 250 there reſt 0 in the Place of Tens, then is the Ring on the Tenth or little Finger of the left Hand; and ſo muſt 1 be abated from the Place of Hundreds.

Suppoſe *A*, *B*, *C*, *D*, *E*, *F*, or 6 Perſons. **Example.**
And *E* the 5*th* Perſon had the Ring on his left Hand, on the middle Joint of the 9*th* Finger, accompting from the Thumb of the Right.
Then 5 doubled is 10, and 5 added makes ——— 15
This multiplied by 5, produceth ——— 75
The Finger added, it is ——— 84
The Joint 2 adjoined, makes it ——— 842
And if for Secrecy 7 be added, the Total is ——— 849—257=592
From this given Total 7, and 250 taken, there remaineth 592, repreſenting ——— } *Perſon.* *Finger.* *Joint.* the 5, the 9, the 2.

Otherwiſe, obſerving an Order of the Perſons, and likewiſe of their Hands as before, calling the Right the Firſt, and the left the Second; after the Number of the Perſon is doubled, 5 added, and that Sum multiplied by 5 as before, bid the Party add 10, and the Number of the Hand; which Sum bid him alſo to multiply by 10, and then add the Number of the Finger; and then again multiply by 10, and add the Number of the Joint, and what Number you pleaſe afterward for ſecrecy-ſake: Then demanding the Total, deduct from thence the Number laſt added, and 3500, and the remaining Figures repreſent the Perſon, Hand, Finger and Joint deſired. ***Otherwiſe.***

Example.

Suppoſe the *6th* Perſon hath the Ring on the left Hand, third Finger, and ſecond Joint.
Then 6 doubled is 12, and 5 added makes —— 17
This multiplied by 5 is 85, and 10 added is —— 95
The Hand added is 97, and multiplied by 10 is —— 970
The Finger added is 973, and multiplied by 10 is — 9730
The Joint added, makes the Total —— 9732
From this 3500 taken, leaves —— 6232
Repreſenting the *6th* Perſon, ſecond Hand, third Finger, and ſecond Joint.

To find the Points caſt on Dice.

9. *Two or more Dice being caſt, how by Art to diſcover the Number of Points that may ariſe.*

Suppoſe one had caſt three Dice, bid him add the Points that were upmoſt together, and put one of the Dice apart with the ſame Side upmoſt; let him add to the Sum the Points under the other two, then bid him caſt thoſe two Dice, and add the Points caſt to the former Sum; and put one of the two Dice away, not changing the Side; and marking the Points under the other Dye, add it to the former Sum: Laſtly, throwing that one Dye, whatever appears upwards, add it to the former Sum, and let the Dice remain without Alteration. This done, coming to the Table, note what Points appear upward on the three Dice; to which add 3 times 7, (for every Dye caſt, always 7 is to be added) and this addition of 7 for every Dye, and the Points lying almoſt on the Dice, ſhall be equal to all the Operations made by the other Party privately.

Example.

As if three Dice being caſt, there ſhould appear 4, 6, 3, which added together make 13; then laying by one of the Dice, as ſuppoſe 4, and adding to 13 the Points under 6 and 3, which will be 1 and 4, (for always the Point, and his Oppoſite, or that above and underneath, makes 7) the Sum will be 18: Then throwing the 2 Dice, ſuppoſe there appear 5 and 2, which makes (being added) 25; and laying by 5, and adding the Point under 2, which will be 5, it makes the former Sum 30: Then throwing the one Dye, ſuppoſe there appear 4, then is the Sum 34. So the Points lying upward, are 4, 5, 4; which when I find, I add for each Dye 7, and that 21 with 13, the Points lying upward, make up 34 as before.

To find the Number of Pieces of Money, &c. in the Hand.

10. *If one hold in each Hand as many pieces of Money, Stones,* &c. *as in the other, how to find the Number.*

Bid him that holds the ſame, that he put out of one Hand into the other what Number you think convenient (provided it may be done): then bid him take out of the Hand that he put the Number into as many as remain in the other Hand, and put into that Hand; for then be aſſured that in the Hand which was put the firſt taking away, there will be found juſt double the Number taken away at firſt.

Example.

As admit in each Hand were 10 pence, then ſuppoſe 4 were taken out of the right Hand and put into the Left; then was there 14 in the Left, and but 6 remaining in the Right; then if 6 be taken from 14 and put to the other Hand, there will be left but 8 the Double of 4, the Number firſt ſubſtracted.

To find how many Counters, &c. three Perſons have taken.

11. *Three Perſons having taken Counters, Cards or other Things; to find how many each hath taken.*

Cauſe the third Party to take a Number which may be divided by 4; and as often as he takes 4, let the Second take 7, and the Firſt 13; then cauſe them to put all together and declare to you the Total, which divide by 3, and the Quotient is the Double of the third Number, or Sum which the third Perſon did take.

Otherwiſe.

Or cauſe the Firſt to give to the Second and Third as many as each of them hath; then let the Second give to the Firſt and Third as many as each of them hath; Laſtly, let the Third give to the Firſt and Second as many as each of them hath, and ask how much one of them hath (for then they will have all alike); ſo half that Number is the Number the third Perſon had at firſt; which once known, all is ſoon known.

Example the firſt Way.

Example by the firſt way: Suppoſe the Third took 8 Counters, which is twice 4, then muſt the Second take 14, which is ſo many times 7, and ſo conſequently the Firſt 26, which is twice 13; all added together is 48, which divided by 3 yieldeth 16, the half of which 8 is the third Perſon's Number.

Example

	A	*B*	*C*
Example by the latter way: Numbers ſuppoſed to be taken :	26	14	8
A giving to *B* and *C* like Sums, that is, *B* 14 and *C* 8, they have	4	28	16
B then giving to *A* and *C* like Sums, that is, *A* 4 and *C* 16, they have	8	8	32
C then giving to *A* and *B* like Sums, that is, *A* 8 and *B* 8, they have	16	16	16

Example the latter Way.

12. *Three Cards choſen out of a Pack, to find how many Points they contain if the Pack be full.*

To find the Points in three Cards.

Let him that hath choſen the 3 Cards, accompt the Points in each Card, and bid him take as many Cards as will make up 15 to each Number of the Points on the Cards firſt choſen, then will him to give you the remaining Cards; for 4 of them being rejected, the reſt ſhew the Number of Points that were on the 3 Cards ſo choſen.

Example.

As if the Cards were 7, 6, 5: now 7 wanting 8 of 15, therefore he took 8 Cards to make up the Points on the firſt Card 15; ſo is there 9 Cards taken from 52 (the whole Pack) for the firſt Card, then 6 wants 9 of 15; therefore taking 9 Cards with the Card of 6 Points, that is, 10 more from the whole Pack, ſo will but 33 Cards remain of the 52; then 5 doth want 10 of 15, therefore 11 Cards more taken from 33 there will be left but 22; which Number of Cards given, ſubſtract 4 from, and the Remain 18 is the Number of the Points 7, 6, 5, on the Cards firſt choſen.

If more than 3 Cards be taken.

But when 4, 5, 6, or more Cards be choſen, and the Number of Cards out of which they are taken be more than 52, and the Term be 15, 14, 12, or ſuch like; then multiply the Term by the Number of Cards taken at firſt, and to the Product add the Number of Cards ſo taken; then this Total ſubſtract from the whole Number of Cards, the Remain ſhall be the Number which muſt be taken from the Cards which remain to make up the Game: if there remain 0 after Subſtraction, then the Number of Cards which remain, do declare the Number of Points in the Cards firſt choſen: if the Subſtraction cannot be made, then ſubſtract the Number of Cards from that Number, and the Remainer added to the Cards that did remain, will be the Number of Points in the Cards firſt taken.

Examples.

As if the Cards were 8, 6, 3, 2, and the Term given 14. I ſee the firſt or 8 wants 6 of 14, the ſecond or 6 wants 8 of 14, and 3 wants 11 of 14, and 2 wants 12, which taken, the Party delivers you the reſt of the Cards; and if the Pack were whole, they are but 11; for 7, 9, 12 and 13 are 41: then do you multiply 14 by 4 (the Number of the Cards choſen at firſt) which makes 56, to which 4 (the ſaid Number of Cards) added is 60, from which 52 ſubſtracted, leaves 8, which with 11 is 19, and ſo many were the Points of the Cards, *viz.* 8, 6, 3, 2.

Admit the Cards were 7, 10, 5, 8, and the Term given 12, then doth 7 want 5 of 12, ten 2, five 7 and eight 4, which with the 4 Cards firſt taken make 22, this taken from 52 leaves 30: Now multiplying 12 by 4, is produced 48, to which 4 added, the Total is 52, which ſubſtracted from 52 leaves 0: therefore doth 30 (the Cards left) repreſent the Numbers firſt taken, *viz.* 7, 10, 5, 8, which together make up 30.

Again, ſuppoſe 3 Cards be taken, as 7, 9, 5, their differences to 15 are 8, 6, 10, which Numbers of Cards taken from 52, the Remain is but 25; then multiply 15 by 3, to the Product 45 add 3 (for the firſt Cards taken) it is 48, which taken from 52, there reſts 4; this taken from 25 (the remaining Cards) leaves 21 the Sum of 7, 9, 5.

13. *Three Things and three Perſons propoſed; to find which of them hath either of the three Things.*

To find which of 3 Perſons hath things hidden.

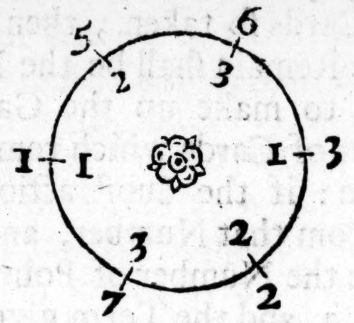
Suppoſe the Perſons be *A. B. C.* and the 3 Things be a Ring, a Thimble, and a Bodkin, and the Perſons mutually conſent to change the Things among themſelves; then take 24 Counters (if more be taken the Matter may ſeem the more ſecret, but no more are needful) and lay them before the Parties, and cauſing them to ſit or ſtand in a Row, give *A* with the Ring 1 Counter, *B* with the Thimble 2 Counters, and *C* with the Bodkin 3 Counters, leaving the reſt of the Counters with them: retire apart, and let them change the Things, but not the Counters given them, and bid them, that he that hath the Ring after the Change, take up ſo many Counters as you gave him at firſt, and him that then ſhall have the Thimble to take up 2 for every Counter given him at firſt, and he that ſhall have the Bodkin to take up for every one given him at firſt 4: Then returning, conſider the remaining Counters left out of the 24; for if the Parties have followed your Directions, there will be left either 1, 2, 3, 5, 6 or 7, and no other Number. Now if 1 remain, then hath there been no Exchange, but as you delivered them ſo they remain; if 2 remain, then hath *A* the Thimble, *B* the Ring, and *C* the Bodkin: If 3 remain, then hath *A* the Ring, *B* the Bodkin, and *C* the Thimble, and ſo further as is expreſſed in the following Table.

By the Table.

Remaining Counters.	*Perſons.*	*Things hid.*	*Remaining Counters.*	*Perſons.*	*Things hid.*
1	*A*	Ring.	5	*A*	Thimble.
	B	Thimble.		*B*	Bodkin.
	C	Bodkin.		*C*	Ring.
2	*A*	Thimble.	6	*A*	Bodkin.
	B	Ring.		*B*	Ring.
	C	Bodkin.		*C*	Thimble.
3	*A*	Ring.	7	*A*	Bodkin.
	B	Bodkin.		*B*	Thimble.
	C	Thimble.		*C*	Ring.

Otherwiſe by the Circle.

Otherwiſe, the ſame may be wrought without the Table by the help of a Circle, ſo divided into 6 Parts, wrote with 1 within, and 1 without; 2 within, and 5 without, *&c.* as followeth,



So if the Number of Counters remaining of the 24, be found in the upper Semicircle without; then that which is oppoſite within ſhews the Firſt, and the next the Second, *&c.* as counting the Ring, Thimble and Bodkin, 1, 2, and 3: And ſuppoſe 6 Counters remain, 6 being found without at top, the oppoſite Figure within is 3; which imports that the firſt Man hath the Bodkin, or third Thing; and ſo going to the left Hand, the ſecond Man hath the Ring, or firſt Thing, *&c.* But if the Remainer be found at the Bottom, as 2, then the Oppoſite within is 2, declaring the firſt Man to have the ſecond Thing, or Thimble; and ſo going backward, contrary to the Former, the ſecond Man hath the firſt Thing, or the Ring, *&c.* As if the former Table were thus figured as at *A*, or in Tranſmutations of Names as at *B*.

Table how otherwiſe ſet.

A

1	*A*	1	5	*A*	2
	B	2		*B*	3
	C	3		*C*	1
2	*A*	2	6	*A*	3
	B	1		*B*	1
	C	3		*C*	2
3	*A*	1	7	*A*	3
	B	3		*B*	2
	C	2		*C*	1

B

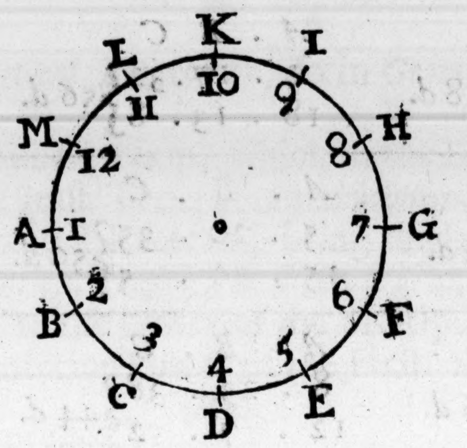
1	1	*A*	5	1	*B*
	2	*B*		2	*C*
	3	*C*		3	*A*
2	1	*B*	6	1	*C*
	2	*A*		2	*A*
	3	*C*		3	*B*
3	1	*A*	7	1	*C*
	2	*C*		2	*B*
	3	*B*		3	*A*

To find which Number on a Circle was thought on.

14. *Many Numbers diſpoſed circular (or otherwiſe) to find which of them any one thinks upon.*

Suppoſe that having ranked 10 or 12 Things in a Row, as A, B, C, D, E, F, G, H, I, K, L, M, or Circular, (as the Figure ſheweth); and one hath thought upon G, which is the 7*th*, then ask the Party at what Letter he will begin to accompt, (for accompt he muſt, otherwiſe it cannot be done); which ſuppoſe at K, that is the 10*th* Place; to which 10 add the Number of the Circle 12, which together is 22; ſo bid him to accompt 22 from K backwards, beginning his Accompt with that Number he thought, ſo ſhall he accompt I to be 8, H 9, &c. And ſo will the Accompt of 22 end on the Letter G exactly, the Thing or Number ſought.

Example.



To divide Liquors equally without Fuſion.

To cloſe up this Chapter, it is worthy note; 1*ſt*. That when a Queſtion is propounded, of dividing equally among a Number of Perſons, certain Liquors in Cask without Fuſion; if the Number of Veſſels in which the Liquors are, will not be equally divided by the Number of Perſons, without a Remain, the Reſolution is impoſſible: but if 0 remain upon the Diviſion, break the Quotient into as many ſeveral Parts as Partners; ſo as being added together, they will make the Whole, and according to theſe Parts ſhall the Cask be divided.

Q. Of 21 Cask with Wine divided.

Example. A, B, and *C,* bought a Quantity of Wines, each paid alike, and was to have alike: It happened (ſome being ſold) there was to be parted among them at laſt 21 Hogſheads, of which 7 were full, 7 half full, and 7 empty: how muſt they ſhare the Veſſels with the Wine in them, that each of them may have as many Veſſels, and as much Wine one as another?

Anſwer.

Anſw. Becauſe 21 will be equally divided by 3, the Quotient 7 is to be broken into Parts, as 3, 3, 1, or 2, 2, 3; either of which Partitions make 7, and each of the Parts leſs than the Half: So may *A* have 3 Veſſels full, 3 empty, and 1 half full; or 2 full, 2 empty, and 3 half full: then will he have 7 Veſſels and $3\frac{1}{2}$ Hogſheads of Wine in Quantity, *B* the like; and then will be left for *C*, after the firſt Diviſion, 1 full Veſſel, 1 empty, and five half full; or after the ſecond Diviſion, 3 full, 3 empty, and 1 half full. And becauſe this, as other-like Queſtions, will be truly reſolved by more Diviſions than one, the Propoſition, however placed by ſome, is moſt proper for *Technology*.

	Full.	*Empty.*	*Half.*		*Full.*	*Empty.*	*Half.*
A	3	3	1	*A*	2	2	3
B	3	3	1	*B*	2	2	3
C	1	1	5	C	3	3	1

To equalize the Sums for which divers Things ſold at different Rates.

2*ly*. That in Queſtions about equalizing the Sums for which different Numbers of Commodities may be ſold at different Rates, beſides a Progreſſion given, an occult Progreſſion in the Propoſition is neceſſary to be diſcovered for the Reſolution; which is often done by adding the different Rates, and thereby dividing the leaſt Term of the given Progreſſion, and by the Integers in the Quotient multiplying the higheſt Rate for the Increaſe of the Progreſſion ſought from an Unit. Yet it is queſtionable whether it will hold generally; but the Artiſt when it fails, may ſupply the defect by his Ingenuity.

Q. Of Apples ſold differently, yet the Sums alike.

Example. Suppoſe 3 Women carry Apples or Oranges to Market; as *A* 20, *B* 30, and *C* 40: how can it be that they can ſell as many for a Penny the one as the other, and when they had ſold all, have one as much Money as another and no more, the Rates they ſell at increaſing as 3 to 1?

Anſwer.

Anſw. 3 and 1 added make 4, which dividing 20 give 5; this multiplied into 3 the higheſt Rate, produceth 15 for the Increaſe of the Progreſſion ſought: So if *A* ſold 1 Apple for 1 Penny, then adding 15 to 1, *B* ſhall ſell 16 for Pence apiece; and adding 15 to 16, *C* ſhall ſell 31 for Pence apiece; and the reſt of their Apples being all ſold at 3 Pence apiece, ſhall make up each of their Monies 4 *s.* 10 *d.* or 58 *d.*

If the Sum be not propounded, the Reſolution divers.

But it is to be further noted, that if the whole Sum of the Money made be not propounded in the Queſtion, theſe as other Propoſitions in *Technology*, are capable of being truly reſolved by ſeveral Numbers, even as many as the laſt Parcel at the higheſt Price is more than an Unit, if the firſt Parcel at the loweſt Price begin at the Unit. And therefore in the former Example, becauſe 31 wants 9 of 40, the Queſtion may be reſolved 9 ways: Nevertheleſs the Sum of Money will be different in every of them, as here followeth.

Examples.

Price	*A*	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*	
1*d*	1	16	31	58 *d.*	2	17	32	56 *d.*	3	18	33	54 *d.*
3	19	14	9		18	13	8		17	12	7	
d.	*A*	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*	
1	4	19	34	52 *d.*	5	20	35	50 *d.*	6	21	36	48 *d.*
3	16	11	6		15	10	5		14	9	4	
d.	*A*	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*	
1	7	22	37	46 *d.*	8	23	38	44 *d.*	9	24	39	42 *d.*
3	13	8	3		12	7	2		11	6	1	

Proof of Technology. The Practice of moſt of the Works in this Chapter carry the Evidence of their Proof in their Operation, the Concluſions proving themſelves true, ſo as other Demonſtrations would be vain.

CHAP. V. *Progreſſion Geometrical.*

Geometrical Progreſſion. Beſides what hath been ſaid of this ſort of *Progreſſion*, *Chap.* 1. *Part* 1. and *Chap.* 1. *Part* 3. of this 4*th* Book, it is neceſſary to add, that in every *Geometrical Progreſſion*, the Conſequent divided by the Antecedent, ſhews the *Ratio* in the Quotient; ſo as to beget new Proportionals of this ſort, is to multiply the Antecedent by the *Ratio*: but this is to be underſtood of Integers in Fractions, and Decimals the contrary. *New Proportionals, how gotten.*

Ratio, *improperly called the* Exceſs. The *Ratio* (called in ſome Authors the *Exceſs*, but improperly, the Exceſs being the Difference of 2 Numbers in *Arithmetical Progreſſion*) is alike between all the Terms; ſo as if the ſecond Number contain the firſt 2, 3, or 4 times or more, then ſhall the Third contain the Second ſo many times alſo, and the Fourth the Third, *&c.*

To double it, &c. what. To double, triple or multiply how often ſoever any *Ratio* is, ſo often to put together the Space or Diſtance between the Terms.

Common way of Proceeding. The common Proceedings by moſt in this *Progreſſion* is by the firſt Term, the *Ratio*, and the Number of Terms, to place all the Terms orderly as they increaſe, and collect their Total by common Addition.

Q. Of Wheat ſold, increaſing the Price of every Buſhel. *Example.* A Farmer ſelleth a Quarter of Wheat (containing 8 Buſhels) to be paid for the firſt Buſhel 2 Farthings, for the ſecond 8 Farthings, and ſo increaſing by a fourfold Proportion or *Ratio*: the Queſtion is, what was to be paid for the Wheat by that Agreement?

Anſwer. *Anſw.* 43690 Farthings or 45 *l.* 10 *s.* 2 *d.* ½. For by ſetting down orderly, as at *A.* *B.* or *C.* all the Terms, and adding them together, the Sum is ſo found.

	d			*B*	*q*
A. Leaſt Term or Extream	0½	1	Buſhel of Wheat.		2
	2	2			8
	8	3			32
	2:8	4			128
Ratio 4.	10:8	5			512
	2:2:8	6			2048
	8:10:8	7			8192
Greateſt Term or Extream	34:2:8	8	Terms or Places.		32768
Total or Sum	45:10:2½				43690

q 4) 43690 (2 q (1092(2 d / 12 (91,0 / 45:10:2½

C

Terms or Places.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .

Leaſt Term 2.8.32.128.512.2048.8192.32768. Greateſt Extream.

Ratio 4. Sum 43690 *q.*

And in Species thus, $\alpha \,.\, \beta \,.\, \frac{\beta q}{\alpha} \,.\, \frac{\beta c}{\alpha q} \,.\, \frac{\beta qq}{\alpha c} \,.\, \frac{\beta qc}{\alpha qq} \,.\, \frac{\beta cc}{\alpha qc} \,.\, \frac{\beta qqc}{\alpha cc} \,.\, \frac{\therefore}{\because}$

In the Computation of this ſecond ſort of *Progreſſion* is to be obſerved as in the Sections following. *What to be noted.*

§ 1. That as in *Arithmetical Progreſſion*, ſo in *Geometrical*, there are 5 principal Things, *viz.* *1ſt. The five Principals.*

1. The firſt Term or leaſt Extream, which in the Example above is 2, and noted in Species with α as the leaſt Term of an *Arithmetical Progreſſion.* *1. The firſt Term, &c.*

2. The laſt Term or greateſt Extream, being the laſt of the *Progreſſion*, noted alſo ſometime as in *Arithmetical Progreſſion* with ω: but if all the Terms be ſet down, then it is according to the Power of the Multiplication of the ſecond Term, to be divided by the next inferior Power of the Firſt. So as in the former Example 32768 the laſt Term is expreſſed by $\frac{\beta qqc}{\alpha cc}$. *2. The laſt Term, &c.*

3. The Number of Terms or Places in the whole *Progreſſion*, which in the Inſtance above is 8, and in Species noted commonly as in *Arithmetical Progreſſion* by *T*, and ſometimes by *N*. *3. The Number of Terms, and Species thereof.*

4. The *Ratio* marked ſometime with *R*, ſometime with *r*, ſometime with *Rat* and ſometime with $\frac{R}{S}$ or R to S; and when α is 1, is always equal to β, and ſo ſometime marked therewith. This in the foregoing Example is *Quadruple*, or as 4 to 1; the Number of *Ratio*'s in every *Progreſſion*, or the Number of Spaces or Diſtances between the firſt and laſt Terms is always leſs by 1 than the Number of Terms, and ſo marked with T—1, or N—1. *4. The Ratio, and ſeveral Species thereof.*

5. The Sum or Total of all the Terms in the *Progreſſion*, which above in the Example is 43690. and for this the Note in Species is Z, in common with *Arithmetical Progreſſion.* *5. The Sum or Total, &c.*

§. 2. By any 3 of the 5 Principals, the other two may be found. As, *2ly. To find the two unknown.*

1. *To find the firſt Term* (or α) *of a Geometrical Progreſſion.* *To find the firſt Term.*

1. If the laſt Term, the Number of Terms, and the *Ratio* be given; that is, ω . T . R. and in the former Inſtance 32768 . 8 . and 4. to find 2. *1. Data. ω. T. R.*

Then either 1*ſt*, divide the ſecond Principal by the Fourth ſo many times lacking one, as there be Units in the Third: *Rules.*

Or, 2*ly*, divide the ſecond Principal by the Fourth, exalted to the next inferior Power of the Third, taking the Third for an Index.

Examples.

$$\frac{\omega}{R}\text{ to } T - 1 = \alpha \qquad \begin{array}{l} \quad\;\; 1 \,.\; 2 \,.\; 3 \,.\; 4 \,.\; 5 \,.\; 6 \,.\; 7 = 8 - 1 \\ 4)\,32768\,(8192\,(2048\,(512\,(128\,(32\,(8\,(2 \end{array}$$

$$\frac{\omega}{R \text{ figurat to } T - 1} = \alpha \qquad \begin{array}{r} \frac{32768}{4.16.64.256.1024.4096.16384}\Big(2 \\ 1.2.3.4.5.6.7 = 8 - 1 \end{array}$$

2. If the laſt Term, the Number of Terms, and the Sum be given; that is, ω . T . Z. and in the former Inſtance 32768 . 8 . & 43690 . to find 2. *2. Data. ω. T. Z.*

Then without reſpect to the third Principal, take the greateſt Common Diviſor between the Second and Fifth, marked ſometimes in Species with M: So 2 ſhall be the Number ſought, becauſe it is the greateſt Common Diviſor between 32768 and 43690. *Rule.*

Example.

$$M\Big)\frac{\omega}{Z} = \alpha \qquad \begin{array}{l} 43690\,(1 \\ \underline{32768}\,(3 \\ \underline{10922} \\ \quad 2 \end{array} \qquad 2\Big)\frac{32768}{43690}\Big(\frac{16384}{21845}$$

3. If the laſt Term, the *Ratio*, and the Sum be given; that is, ω . R . Z. and in the former Inſtance 32768 . 4 . & 43690 . to find 2. *3. Data. ω. R. Z.*

Then becauſe the ſecond and fifth Principals are given, take the greateſt Common Diviſor between, as before in the next precedent Propoſition. *Rules*

Or, (2.) multiply the ſecond Principal by the Fourth, divide the Product by the Fourth, lacking an Unit: From the Quotient take the Fifth, and multiply the Remain by the Diviſor: So ſhall 32768 be multiplied by 4, and the

Product 131072 be divided by 3 from the Quotient $43690\frac{2}{3}$; if 43690 be taken, and the Remain $\frac{2}{3}$ multiplied by 3, produceth 2.

Example.

$$\frac{\omega R}{R-1} - Z.\times R - 1 = \alpha \qquad \begin{array}{l} \quad 32768 \\ \quad\quad\; 4 \\ 3)\overline{\underline{131072}}(\,43690\tfrac{2}{3} - 43690 = \tfrac{2}{3} \times 3 = 2 \end{array}$$

4. Data. T.R.Z. 4. If the Number of Terms, the *Ratio*, and the Sum be given; that is, T . R . Z . and in the former Inſtance 8 . 4 . & 43690 . to find 2.

Rule. Then from the fourth Principal multiplied into it ſelf, according to the Units in the Third, ſubſtract 1; multiply the Remain by 1 leſs than the Fourth, ſometime called the *Antecedent of the Ratio*, and multiply 1 leſs than the Fourth by himſelf, marked in Species ſometime by *D*, and called the *Difference of the Terms of the Ratio*: Then as the Product of the Remain, by the Antecedent of the *Ratio*, is to the Product of that Antecedent into himſelf; or as the fourth Principal figurate to T—1, is to the Fourth lacking 1; ſo is the fifth Principal given to the Firſt required.

Example.

$$\begin{array}{ll} & 1.\;2.\;3.\;4.\;5.\;6.\;7.\;8 \\ \text{R figurate to T.}-1 \text{ in R}-1.\,\text{D} :: \text{Z}.\,\alpha & 4.16.64.256.1024.4096.16384.65536 \\ & (65535\times 3)\;(3\times 3) \qquad\qquad\qquad\qquad\quad 1 \\ \qquad\qquad \text{Or,} & \text{As } 196605\,.\,9 :: 43690\,.\,2 \qquad \overline{\underline{65535}} \\ \text{R figurate to T.}-1.\,\text{R}-1 :: \text{Z}.\,\alpha & \text{Or, } 65535\,.\,3 :: 43690\,.\,2 \end{array}$$

To find the laſt Term. 2. *To find the laſt Term* (or ω) *of a Geometrical Progreſſion.*

1. Data. α.T.R. 1. If the firſt Term, the Number of Terms, and the *Ratio* be given; that is, α . T . R . and in the former Inſtance, 2 . 8 . and 4 . to find 32768.

Rules. Then either, (1.) multiply the firſt Principal figurately by the Fourth, according to the Units in the Third lacking 1; and ſo ſhall 2 be multiplied into 4, till the Index be 7, that is T—1.

Or, (2.) figurate the fourth Principal to the Third lacking 1, and multiply by the Firſt.

Examples.

$$\begin{array}{ll} & 1.\;2.\;3.\;4.\;5.\;6.\;7 \\ \alpha \text{ in R to T} - 1 = \omega & 2.8.32.128.512.2048.8192.32768. \\ & 1.\;2.\;3.\;4.\;5.\;6.\;7 \\ \text{R figurate to T}-1\times\alpha = \omega & 4.16.64.256.1024.4096.16384 \\ & \qquad\qquad\qquad\qquad\qquad\quad 2 \\ & \qquad\qquad\qquad\qquad\qquad \overline{\underline{32768}} \end{array}$$

2. Data. α.T.Z. 2. If the firſt Term, the Number of Terms, and the Sum be given; that is, α . T . Z . and in the former Inſtance. 2 . 8 . and 43690, to find 32768.

Rule. Then divide the fifth Principal by the Firſt, and from the Quotient take the greateſt Figural Number, whoſe Index is 1 leſs than the Third, and multiply this Figural Number by the Firſt: So ſhall 43690, divided by 2, give in the Quotient 21845; out of which the greateſt ſecond Surſolid, whoſe Index is 7, (that is, 1 leſs than 8) that can be taken, is 16384; which multiplied by 2, produceth 32768 the ſecond Principal deſired.

Example.

$$\frac{Z}{\alpha} - \square\; \begin{array}{l}\text{Power} \\ \text{Index}\end{array} \Big\}\, T-1 \qquad \frac{43690}{2}\Big(\begin{array}{l} 21845 \\ 16384 \times 2 = 32768 \end{array}$$

$$\textit{Ergo}, \text{Power} \times \alpha = \omega \qquad \begin{array}{c} qqc \\ 7 = 8 - 1 \end{array}$$

3. Data. α.R.Z. 3. If the firſt Term, the *Ratio*, and the Sum be given; that is, α . R . Z . and in the former Inſtance, 2 . 4 . and 43690, to find 32768.

Rule. Then multiply the fifth Principal by the Fourth lacking an Unit, to the Product add the Firſt, and divide the Total by the Fourth. So ſhall 43690, multiplied by 3, and 2 added, make the Total 131072, which divided by 4, gives in the Quotient 32768.

Example.

$$\frac{\underline{ZR}-1+\alpha}{R} = \omega \qquad \begin{array}{l} 43690 \\ \qquad\;\; 3 \\ \hline \underline{131070} + 2 = \dfrac{131072}{4}\Big(\,32768 \end{array}$$

4. Data. T.R.Z. 4. If the Number of Terms, the *Ratio*, and the Sum be given; that is, T.R.Z. and in the former Inſtance 8 . 4 . and 43690, to find 32768.

Then

Rule.

Then by the *Data* find the firſt Principal, according to the *4th* Propoſition of the *1ſt* above-mentioned; and then take the Figurate Number of the Fourth Principal, according to the Units in the Third, lacking 1, and the *Analogy* will be; As 1 to this Figurate *Ratio*; ſo ſhall the firſt Principal found, to the Second required.

Example.

R figurate to T.—1 in R—1. D :: Z.α 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8

Or R figurate to T.— 1. R — 1 :: Z.α 4.16.64.256.1024.4096.16384.65536

$$\begin{array}{r} 65536 \\ 1 \\ \hline 65535 \\ \hline \end{array}$$

As 196605 . 9 :: 43690.2.

Or, 65535 . 3 :: 43690.2.

1. R figurate to T — 1 :: α . ω As 1 . 16384 :: 2.32768.

3. *To find the Number of Terms* (or T) *of a Geometrical Progreſſion.*

To find the Number of Terms.

1.Data. $\alpha.\omega$.R.

1. If the firſt Term, the laſt Term, and the *Ratio* be given; that is, α . ω . R. and in the former Inſtance, 2.32768, & 4. to find 8.

Rule.

Then multiply the firſt Principal by the Fourth ſucceſſively, till the Second be produced, and to the Number of the ſeveral Multiplications add 1: So 2 multiplied by 4, ſhall in 7 Multiplications produce 32768; therefore to 7 add 1, and 8 is obtained.

Example.

$$\left.\begin{array}{l}\text{N of Mult. of} \\ \alpha\,\text{R to}\,\omega + 1\end{array}\right\} = \text{T} \qquad \begin{array}{l} 2.8.32.128.512.2048.8192.32768 \\ \quad 1 . 2 . 3 . 4 . 5 . 6 . 7 + 1 = 8 \end{array}$$

Rule.

Or if the ſecond Principal be divided by the Firſt, and the Fourth be figurate to that Quotient, the Index of this figurate *Ratio* ſhall be the Number of Terms lacking 1.

Example.

2) 32768 (16384

α . 1 :: ω . R figurate to T—1 4.16.64.256.1024.4096.16384

1 . 2 . 3 . 4 . 5 . 6 . 7 + 1 = 8

2.Data. $\alpha.\omega$.Z.

2. If the firſt Term, the laſt Term, and the Sum be given; that is, α . ω . Z. and in the former Inſtance, 2.32768. & 43690, to find 8.

Rule.

Then without reſpect to the fifth Principal; as the firſt Principal to 1, ſo ſhall the ſecond Principal be to the *Ratio* multiplied into it ſelf, according to the Diſtance of the Term given from the firſt Term. From hence therefore, if a Root be extracted of the higheſt Power therein, the Index of that Power and 1 more, ſhall be the Number ſought: So ſhall 2 to 1, be as 32768 to 16384; therefore 16384 being the ſecond Surſolid of 4, hath the Index 7, to which 1 added is 8.

Example.

α . 1 :: ω . R figurate to T—1. As 2 : 1 :: 32768 . 16384.

Ergo, √ qqc . 16384 (4

7 + 1 = 8

Rule.

Or divide the fifth Principal by the ſecond, and by the Remain of that Diviſion, the Remain of the firſt taken from the Second; then by the Sum of both the Quotients multiply the Firſt till the Second be produced, and to the Number of the ſeveral Multiplications add an Unit. So ſhall 43690 divided by 32768 give 1 in the Quotient, and 10922 Remain, which ſhall be Diviſor to 32766, that is the Remain after 2 is taken from 32768: this laſt Diviſion giveth 3 in the Quotient, to which 1 added is 4, this multiplied into 2 the firſt 7 times, produceth 32768; therefore 7 and 1 make 8 the Number deſired.

Example.

$$\frac{Z}{\omega} + \frac{\omega - \alpha}{\text{Remain}} \left.\begin{array}{l}\text{N. of Mult. of} \\ \times\,\alpha\,\text{to}\,\omega + 1\end{array}\right\} = \text{T} \qquad \begin{array}{c} 10922 \\ \hline \cancel{43690} \\ \hline \cancel{32768} \end{array}\Big(\,1 \qquad \begin{array}{r} 32768 \\ 2 \\ \hline 32766 \\ \hline \end{array} \quad \begin{array}{c} \cancel{32766} \\ \hline \cancel{10922} \end{array}\Big(\,3$$

1 + 3 = 4 × 2.8.32.128.512.2048.8192.32768

1 . 2 . 3 . 4 . 5 . 6 . 7 + 1 = 8

3.Data. α.R.Z.

3. If the firſt Term, the *Ratio*, and the Sum be given; that is, α . R . Z . and in the former Inſtance 2, 4, and 43690, to find 8.

Rule.

Then multiply the firſt Principal by the Fourth, till a Number be produced next greater than the Fifth, and the Number of Multiplications ſhall be the Third. So 2 by 4 multiplied till 131072 be produced, the Number of the ſeveral Multiplications will be 8, the Number of Terms ſought.

Example.

$$\left.\begin{array}{l}\text{N of Mult. of}\\ \alpha\ \text{R to } \Box - \text{Z}\end{array}\right\} = \text{T} \qquad \begin{array}{l} 2\times 4 = 8.32.128.512.2048.8192.32768.131072\\ \qquad\qquad 1.\ 2\ .\ 3\ .\ 4\ .\ 5\ .\ 6\ .\ 7\ .\ 8\end{array}$$

Rule. Or by Analogy, the firſt Principal to the Fifth, ſhall be as the Difference of the Terms of the *Ratio* to the *Ratio* figurate according to the Units in the Third lacking 1, and the Remain multiplied by the Antecedent of the *Ratio*; or as the *Ratio* lacking 1, to the *Ratio* figurate to T lacking 1; therefore if from the Quotient of this Number and 1 added, divided by the ſame Antecedent, if the upper Way be taken, or the Quotient and 1 of the lower way a Root be extracted of the higheſt Power therein, the Index of this Root ſhall be the Number of Terms.

Example.

$\alpha\ .\ \text{Z} :: \text{D}\ .\ \text{R}$ figurate to $\text{T}. - 1$ in $\text{R} - 1$. $\qquad 4 - 1 = 3 \times 3 = 9$.

Or, $\alpha\ .\ \text{Z} :: \text{R} - 1\ .\ \text{R}$ figurate to $\text{T}. - 1$.

As $2\ .\ 43690 :: 9\ .\ 196605$.

Or $2\ .\ 43690 :: 3\ .\ 65535$.

Ergo $3)\ 196605\ (65535 + 1 = 65536$.

And $\surd$qcc $65536\ (4$. Index 8.

4.Data. ω.R.Z. 4. If the laſt Term, the *Ratio*, and the Sum be given; that is, ω. R. Z. and in the former Inſtance 32768. 4. and 43690. to find 8.

Rule. Then multiply the greateſt common Diviſor between the ſecond and fifth Principals by the Fourth till the Second be produced; and to the Number of the ſeveral Multiplications add an Unit. So 2 the common Diviſor between 32768 and 43690, ſhall be multiplied by 4 the *Ratio* 7 times, which will produce 32768: therefore 1 added to 7 gives 8, the Number of Terms in this Example.

Example.

$$\text{M}\Big)\frac{\omega}{\text{Z}} = \alpha \qquad\qquad 2\Big)\frac{32768}{43690}\Big(\frac{16384}{21845}$$

$$\textit{Ergo},\ \left.\begin{array}{l}\text{N of Mult. of}\\ \alpha\ \text{R to}\ \omega + 1\end{array}\right\} = \text{T} \qquad \begin{array}{l} 2\times 4 = 8.32.128.512.2048.8192.32768\\ \qquad\qquad 1.\ 2\ .\ 3\ .\ 4\ .\ 5\ .\ 6\ .\ 7 + 1 = 8\end{array}$$

Otherwiſe. Or, becauſe this Common Diviſor is the firſt Term, any of the Ways in the ſecond and third Propoſitions precedent by $\alpha\ .\ \omega\ .\ \text{Z}$. or $\alpha\ .\ \text{R}\ .\ \text{Z}$. may be uſed.

To find the Ratio.

4. *To find the* Ratio (or R) *of a Geometrical Progreſſion.*

1.Data. α. ω. T. 1. If the firſt Term, the laſt Term, and the Number of Terms be given; that is, $\alpha\ .\ \omega\ .\ \text{T}$. and in the former Inſtance, 2. 32768. and 8. to find 4.

Rule. Then divide the ſecond Principal by the Firſt, and from the Quotient extract a Root, whoſe Index ſhall be leſs than the Third by an Unit: For the *Analogy* is, As the Firſt to 1; ſo is the Second to the *Ratio* figurate to the Diſtance of the Term given from the Firſt. Wherefore 16384, the Quotient of 32768 divided by 2, ſhall be a ſecond Surſolid, which hath 7 for the Index, that is 1 leſs than 8, and 4 for the Root the *Ratio* deſired.

Example.

$$\surd\,\frac{\omega}{\alpha}\ \text{Index T} - 1 = \text{R} \qquad \frac{32768}{2}\Big(\ 16384\left(\begin{array}{l}\surd\text{qqc}\ .\ 4\\ \text{Index}\ .\ 7 = 8 - 1\end{array}\right.$$

$$\begin{array}{ll} \alpha.1 :: \omega\ .\ \text{R figurate to T} - 1 & \quad 4.16.64.256.1024.4096.16384.\\ 2.1 :: 32768\ .\ 16384 & \quad 1.\ 2.\ 3.\ 4\ .\ 5\ .\ 6\ .\ 7\end{array}$$

2. Data. α. ω. Z. 2. If the firſt Term, the laſt Term, and the Sum be given; that is, $\alpha\ .\ \omega\ .\ \text{Z}$. and in the former Inſtance, 2. 32768. and 43690, to find 4.

Rule. Then divide the fifth Principal by the Second, and by the Remain of that Diviſion, divide the Remain of the Firſt taken from the Second, and add both the Quotients together: So ſhall 1, the Quotient of 43690, divided by 32768, and 3 the Quotient of 32766, (that is, $\omega - \alpha$) divided by 10922, (the Remain of the firſt Diviſion) make together 4 the *Ratio* ſought.

Example.

$$\frac{\text{Z}}{\omega} + \frac{\omega - \alpha}{\text{Remain.}} = \text{R} \qquad \begin{array}{c} 10922\\ \hline \cancel{43690}\\ \hline \cancel{32768}\end{array}\Big(\ 1 \qquad \begin{array}{r} 32768\\ 2\\ \hline 32766\\ \hline\end{array} \qquad \frac{\cancel{32766}}{\cancel{10922}}\Big(\ 3 \qquad 1 + 3 = 4$$

3.Data. α. T.Z. 3. If the firſt Term, the Number of Terms, and the Sum be given; that is, $\alpha\ .\ \text{T}\ .\ \text{Z}$. and in the former Inſtance, 2.8. and 43690, to find 4.

Rule. Then divide the fifth Principal by the Firſt, and from the Quotient take the greateſt figural Number, whoſe Index is 1 leſs than the Third, the Root of this figural Number ſhall be the *Ratio*. So 43690 divided by 2, giveth 21845; out of which the greateſt ſecond Surſolid, whoſe Index is 7, (that is 1 leſs than 8) that can be taken is 16384, the Root whereof is 4.

Z

$$\frac{Z}{\alpha} - \square - \left.\begin{matrix}\text{Power}\\ \text{Index}\end{matrix}\right\} T-1 \qquad \frac{43690}{2}\Big(\begin{matrix}21845\\ 16384\end{matrix} \left\{\begin{matrix}\sqrt{qqc} & .\ 4\\ \text{Index} & .\ 7=8-1\end{matrix}\right.$$ Example.

Ergo, $\sqrt{\ }\text{Power} = R$

4.16.64.256.1024.4096.16384.
1 . 2 . 3 . 4 . 5 . 6 . 7 .

4. If the laſt Term, the Number of Terms, and the Sum be given; that is, ω . T . Z . and in the former Inſtance, 32768 . 8 . and 43690, to find 4. 4. Data. ω. T. Z.

Rule. Then by the greateſt common Diviſor, between the ſecond and fifth Principals, divide the Second: And from the Quotient extract a Root, whoſe Index ſhall be leſs than the Third by an Unit. So 2 the common Diviſor between 32768 and 43690, ſhall divide 32768, and the Quotient 16384 ſhall be a ſecond Surſolid, whoſe Index is 7, that is 1 leſs than 8, and 4 the Root is the deſired *Ratio.*

$$M\Big)\frac{\omega}{Z} = \alpha \qquad 2\Big)\frac{32768}{43690}\Big(\frac{16384}{21845}$$ Example.

$$\textit{Ergo}\ \sqrt{\ }\frac{\omega}{\alpha}\,\text{Index } T-1 = R \qquad 16384\left\{\begin{matrix}\sqrt{qqc} & .\ 4\\ \text{Index} & .\ 7=8-1\end{matrix}\right.$$

5. *To find the Sum* (or Z) *of a Geometrical Progreſſion.*

To find the Sum.

1. If the firſt Term, the laſt Term, and the Number of Terms be given; that is, α . ω . T . and in the former Inſtance 2.32768, and 8, to find 43690. 1. Data. α. ω. T.

Rule. Then divide the ſecond Principal by the Firſt; from the Quotient extract a Root, whoſe Index is leſs than the Third by an Unit; and by an Unit leſs than this Root divide the Remain of the Firſt taken from the Second, and to this Quotient add the Second; ſo ſhall 32768, divided by 2, give 16384: the Index leſs than 8 by 1 is 7; the ſecond ſurſolid Root therefore of 16384 is 4, from which 1 taken, leaves 3 for Diviſor to 32766, the Remain of 32768; when 2 the Firſt is ſubſtracted, the Quotient of this Diviſion 10922 and 32768, make up 43690 the deſired Sum.

$$\frac{\omega-\alpha}{\sqrt{\ }\frac{\omega}{\alpha}\,\text{Index}-1\ T-1} + \omega = Z \qquad \frac{32768}{2}\Big(\begin{matrix}16384\ \sqrt{qqc}\ 4-1=3\\ 32768-2=32766\end{matrix}\Big)32766\Big(\begin{matrix}10922\\ \underline{32768}\\ \underline{43690}\end{matrix}$$ Example.

2. If the firſt Term, the laſt Term, and the *Ratio* be given; that is, α . ω . R . and in the former Inſtance, 2.32768 and 4, to find 43690. 2. Data. α. ω. R.

Rules. Then multiply the ſecond Principal by the Fourth; from the Product take the Firſt, and divide the Remain by 1 leſs than the Fourth.

Or, ſubſtract the Firſt from the Second, and divide the Remain by 1 leſs than the Fourth, and to the Quotient add the Second.

So by the firſt of theſe ſhall 32768 be multiplied by 4; and 2 taken from the Product, leaves 131070 to be divided by 3: And by the latter way 32766 ſhall be divided by 3; and to the Quotient 10922 ſhall be added 32768, and both ways bring forth 43690.

$$\frac{\omega R-\alpha}{R-1} = Z \qquad 32768\times 4 = 131072 - 2 = \frac{131070}{4-1=3}\Big(43690$$ Examples.

$$\frac{\omega-\alpha}{R-1} + \omega = Z \qquad 32768 - 2 = \frac{32766}{4-1=3}\Big(10922 + 32768 = 43690$$

And by way of *Analogy*: As the Difference of Terms of the *Ratio*, to the *Ratio* multiplied into it ſelf according to the Units in the Third lacking 1, and the Remain multiplied by the Antecedent of the *Ratio*; or the *Ratio* lacking 1 to the *Ratio* figurate to T—1: So ſhall the Firſt be to the Fifth: Or the ſame *Ratio* ſo figurate and multiplied as aforeſaid, multiplied into the Firſt, and divided by the ſaid Difference, ſhall be equal to the Fifth. Otherwiſe.

α . 1 :: ω . R figurate to T−1.
For ſeeing, As 2 . 1 :: 32768 . 16384 4—1=3×3=9 Example.

Ergo, D. R figurate to T.—1 in R—1 :: α . Z .
9 . 196605 :: 2 . 43690 .

Or, R—1 . R figurate to T.—1 :: α . Z .
3 . 65535 :: 2 . 43690 .

3. Data. α.T.R. 3. If the firſt Term, the Number of Terms, and the *Ratio* be given; that is, α . T . R . and in the former Inſtance, 2 . 8 . and 4, to find 43690.

Rule. Then multiply the firſt Principal by the Fourth continually, ſo many Times as there be Units in the Third; and from the laſt Product take the Firſt, and divide the Remain by 1 leſs than the Fourth. So 2 by 4 ſhall be multiplied 8 times; and taking 2 from 131072 the laſt Product, and the Remain 131070 divided by 3, giveth 43690 as deſired.

Example.

$$\frac{\alpha \text{ in R to T}-\alpha}{R-1}=Z$$

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
2×4=8.32.128.512.2048.8192.32768.131072
2
4—1=3)131070(43690

Rule. Or if the Fourth be multiplied into himſelf, according to the Units in the Third lacking 1, theſe Products multiplied by the Firſt, ſhall be equal to the Fifth lacking the Firſt.

Example.

$$\left.\begin{array}{l}\text{Sum of}\\ \text{R figurate to T}-1\times\alpha+\alpha\end{array}\right\}=Z$$

4+16+64+256+1024+4096+16384=21844
2
43688
Ergo, 43688+2=43690

4. Data. ω.T.R. 4. If the laſt Term, the Number of Terms, and the *Ratio* be given; that is, ω . T . R . and in the former Inſtance 32768, 8 and 4, to find 43690.

Rule. Then continue the Diviſion of the ſecond Principal by the Fourth, ſo many times as there be Units in Third lacking 1: and take this laſt Quotient from the Product of the Fourth multiplied into the Second; this Remain divide by 1 leſs than the Fourth, ſo 32768 divided 7 times by 4 bringeth 2 in the Quotient, which taken from 131072, and the Remain divided by 3, giveth the Sum deſired 43690.

Example.

$$\frac{\omega R-\frac{\omega}{R}\text{ to T}-1}{R-1}=Z$$

1 . 2 . 3 . 4 . 5 . 6 . 7 .
4) 32768 (8192 (2048 (512 (128 (32 (8 (2.
32768×4=131072—2=131070 (43690
4—1=3

Data and Quesita in every Queſtion. Here alſo, as before in *Arithmetical Progreſſion*, is to be obſerved, that by every 3 of theſe Principals both the other 2 may be found; and alſo that in all Queſtions duly propounded, 3 of the 5 are given to find ſometime the one, ſometime both the other: *Common Sort of Queſtions.* For though the common Sort of Queſtions, as the ſelling of an Horſe by the Nails in his Shoes, a Coat by the Buttons, or an Houſe by the Doors, &c. be all alike to the former Example firſt mentioned; *Every Queſtion may be varied 20 ways.* yet may one Queſtion be varied in the propounding 20 ſeveral Ways, according to theſe precedent Propoſitions, and by them accordingly may Reſolution be had. *Alterations of Fractions and Decimals.* But all theſe Calculations are for whole Numbers; if Fractions or Decimals be uſed, Alterations muſt be made according to their Nature and Uſe.

3ly. To find any middle Term, 3 Caſes. §. 3. Next after the finding of the 5 Principals, it is neceſſary to know how to find any of the middle Terms or Means that lie between the two Extreams, in effecting of which there are 3 Caſes conſiderable.

1. If the leaſt Extream be an Unit. *Caſe* 1. If the leaſt Extream of the *Progreſſion* be an Unit, then ſquare the Term given, and the Product ſhall be the Double of the given Term lacking one: as if I ſquare the third Term, I get thereby the Fifth; So if the fourth Term be ſquared, the ſeventh is produced.

If two of the middle Terms be given to find another, then add the Number of the Terms given, and the Sum lacking 1 ſhall be the Number of the Term produced by multiplying the Terms one into another: As to find the 8*th* Term, the fourth and fifth Terms ſhall be multiplied one into another, for 4 and 5 makes 9, that is one more than 8.

Examples.

Example.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .
Example. 1 . 4 . 16 . 64 . 256 . 1024 . 4096 . 16384.

$$\begin{array}{r} 16\times16 \\ \hline 96 \\ 16 \\ \hline 256 \\ \hline \end{array} \quad \begin{array}{l} 3+3=6 \\ \\ \\ \\ 5=3+3-1 \end{array} \qquad \begin{array}{r} 64\times256 \\ \hline 1024 \\ 1536 \\ \hline 16384 \\ \hline \end{array} \quad \begin{array}{l} 4+5=9 \\ \\ \\ \\ 8=4+5-1 \end{array}$$

2. If the leaſt Term be no Unit, yet may be divided to 1.

Caſe 2. If the leaſt Term be not an Unit, and yet by the *Ratio* of the *Progreſſion* may be divided to the Unit; then any one Term ſquared produceth juſt the Double of the given Term; and two of the Terms multiplied one into another, ſhall produce the Term the Number of their Terms added point at.

Examples.

As to find the 8*th* Term, multiply the 4*th* Term into it ſelf: and to get the 7*th* Term, let the Third and Fourth be multiplied, for 3 and 4 make 7.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .
Example. 3 . 9 . 27 . 81 . 243 . 729 . 2187 . 6561 .

$$\begin{array}{r} 81\times81 \\ \hline 81 \\ 648 \\ \hline 6561 \\ \hline \end{array} \quad \begin{array}{l} 4+4=8 \\ \\ \\ \\ 8=4+4 \end{array} \qquad \begin{array}{r} 27\times81 \\ \hline 567 \\ 162 \\ \hline 2187 \\ \hline \end{array} \quad \begin{array}{l} 3+4=7 \\ \\ \\ \\ 7=3+4 \end{array}$$

3. If the leaſt Term be no Unit, nor can be divided to 1.

Caſe 3. If the leaſt Term be not an Unit, nor by the *Ratio* of the *Progreſſion* can be brought to an Unit; then the beſt way to avoid multiplicity of Rules, is to proceed by the following Analogy, which indeed will ſerve in all Caſes, that is, As an Unit to the *Ratio* multiplied into it ſelf, according to the Diſtance of the Term ſought from the firſt Term; ſo is the firſt Term, to the Term ſought.

Example.

As to know the 8*th* Term of a *Geometrical Progreſſion*, whoſe leaſt Extream is 2, and *Ratio* 3, the Diſtance between the firſt Term and the 8*th* Term is 7, the *Ratio* figurate to 7 is 2187, which multiplied by 2 is 4374 the 8*th* Term deſired.

1 : R figurate to T—1 :: α . ω
1 . 2187 :: 2 . 4374.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 .
2 . 6 . 18 . 54 . 162 . 486 . 1458 . 4374.
3 . 9 . 27 . 81 . 243 . 729 . 2187 .

Lemma's of Mr. Briggs.

Here among theſe *Geometrical Proportions* may have Place the *Lemma*'s ſet down by Mr. *Briggs* in his Latin Piece, that demonſtrate the Original of *Logarithms*, making the Index of Unity 0, though they might have been placed before among Figural Numbers or *Logarithms*.

1. Lemma.

Lemma 1. If in a Rank or Series of Numbers continually Proportional from an Unit, (as thoſe following in *A*) any two of them be taken, as *C* 8 and *D* 32, with their Indices 3 and 5; two other Series of Numbers continually Proportional from an Unit, being made from the 2 Numbers taken out of the firſt Series, *viz.* from 8 and 32, as at *E* and *F*, the firſt Numbers in each Series next to the Unit being the former Numbers marked with *C* and *D*; and both Series being continued to 3 (the Index of the Number *C* in the Column *A*) ſhall be the Index of a Proportional Number in the Column *F*; and 5 in the like manner (the Index of the Number *D*) and the Index of a Number in the Column *E*; the laſt Proportional Number in the Columns *E* and *F* ſhall be equal to the Number found in the firſt Series marked with *A*, whoſe Index is 15, the Product of the 2 Indices to the laſt Numbers in the Columns *E* and *F*.

Q		*A*	*B*	*P*	*E*	*M*
0		1	0	0	1	0
		2	1		8	1
		4	2		64	2
	C	8	3	1	512	3
		16	4		4096	4
1	*D*	32	5		32768	5
		64	6	2		
		128	7		*F*	*N*
		256	8		1	0
		512	9	3	32	1
2		1024	10		1024	2
		2048	11		32768	3
		4096	12	4		
		8192	13			
		16384	14			
3		32768	15	5		

A The firſt Series of Proportional Numbers.
B The Indices of that Series of Numbers.
E The ſecond Series of Numbers given.
M The Indices of that Series, to which thoſe in *P* are equal.
F The third Series, whoſe Indices are equal to thoſe in *Q*.

2. Lemma. *Lemma* 2. If in a Series of Numbers continually proportional from an Unit, any one of them be divided continually by his Side or Root as often as it can, the Number of Diviſions ſhall be the Index of the Number divided, ſhewing the Diſtance of that Number from the Unit, or the Number of Intervals between Unity and the Dividend.

As of 729 by the Root 3, the Index is 6.

Indices 0 . 1 . 2 . 3 . 4 . 5 . 6 .
Proportionals 1 . 3 . 9 . 27 . 81 . 243 . 729. 3) 729 (243 (81 (27 (9 (3 (1
1 . 2 . 3 . 4 . 5 . 6

3. Lemma. *Lemma* 3. In a Series of Numbers continually proportional, any two of them being given, with the Index of one of them, to find the Index of the other, or its diſtance from Unity; as the given Numbers 8 and 32 in the Series above marked with *A* in the firſt *Lemma*, and the Index of the greater 5; Let then another Series of Numbers continually proportional be made by the Multiplication of 8, whoſe Index is ſought by it ſelf, and of the Product by it ſelf continually, till you come to a Number whoſe Index is equal to 5, the Index given at *E*; The laſt Product by the firſt *Lemma*, ſhall be the ſame with the Product made by the continual Multiplication of 32, till the Index of the Product in this laſt Series be the Index ſought: And therefore if the Number 32768 be divided by 32, the Side in the third Series, according to the Directions of the ſecond *Lemma*, the Quotient will be 3, *viz.* 1024 . 32 . 1 . And ſo 3 ſhall be the Index, as well of this Product 32768 in this third Series, as of 8 in the firſt Series, from whence the 2 given Numbers were taken.

4. Lemma. *Lemma* 4. In a Series of Numbers continually proportional from Unity; If one Number multiply another, the Product will be one of the Numbers in that Series continued, and the Index thereof will be the Sum of the Indices given: As in the Series at *A*, if 4 multiply 256, the Product will be 1024, and the Index thereof 10, *viz.* the Sum of 2 & 8, the Indices annexed to 4 and 256.

5. Lemma. *Lemma* 5. If one Number be multiplied by another, the Number of Places in the Product ſhall be equal to the Number of Figures in both the Factors, unleſs the Product made of the firſt Figures toward the left Hand, in both the Numbers given, may be expreſſed by one Digit; as often as this ſhall happen, the Number of Places in the Product will be leſs by 1 than the Number of Figures in both the Factors: As if 68 be multiplied by 26, the Product 1768 is expreſſed with 4 Figures; but if 68 be multiplied by 14, the Product 952 is expreſſed by 3 Figures.

Thus

Logarithms diſcovered.

Thus came to be diſcovered the Number of Places, or Decimal Indices in every Figural Number which are the Logarithms, as was ſaid before in the firſt Chapter of *Logarithms.*

4ly. To find out Means. Two Caſes.

§. 4. After finding out the middle Terms in any *Geometrical Progreſſion*, is convenient to know how to find one or more mean Proportionals between two Numbers given: In finding which are conſtituted 2 *Caſes.*

1. *If one Mean be ſought.*

Caſe 1. If one Mean Proportional be deſired between two given Numbers.

Then multiply them together, and take the Square Root of the Product.

Common Way.

As to find a Mean Proportional between 4 and 9, they produce, being multiplied, 36; therefore 6 the Square Root thereof ſhall be the Proportional Mean between 4 and 9.

So between 6 and 24, is 12 found to be the Mean Proportional.

Example.

$$\underline{4\times 9} = \underline{36}\big(6\surd \qquad\qquad \underline{6\times 24} = \underline{144}\big(12\surd$$

$$\underline{4\ .}\ \underline{6\ .\ 9} \qquad\qquad\qquad \underline{6\ .}\ \underline{12\ .\ 24}$$

How by Logarithms.

Becauſe Extraction of Roots is very eaſy by Logarithms, as for the Square Root to divide by 2, for the Cube by 3, &c. and oftentimes Fractions are given, or happen in the Work of Mean Proportionals: It is beſt for the Artiſt to work by Logarithms. Wherefore,

If the Data are alike.

1. When the Logarithms of the Numbers propounded are of the ſame Kind, add them together, and half the Sum ſhall be the Logarithm of the Mean Proportional required, and of the ſame Kind with the Logarithms of the Numbers given.

Examples.

Examples.

Integers, The Mean Proportional between 4 and 25, is 10: becauſe 10 is the Square Root of 100, the Product of 4 by 25.

Fractions, The Mean Proportional between $\frac{1}{4}$ and $\frac{4}{25}$ is $\frac{1}{5}$: for the Fractions multiplied are $\frac{4}{100}$, the $\surd 3$ whereof is $\frac{2}{10}$ or $\frac{1}{5}$,

Decimals, The Mean Proportional between 0,25 (that is $\frac{1}{4}$) and 0,16 (that is $\frac{4}{25}$) is 0,20 (that is $\frac{2}{10}$ or $\frac{1}{5}$) as before. Their Logarithms follow.

Integers. *Fractions.*

$$\begin{array}{rl} 0{,}60205{,}99913\ . & \text{Log. of } 4 \\ 1{,}39794{,}00087\ . & \text{Log. of } 25 \\ \hline 2{,}00000{,}00000\ . & \text{Log. of } 100 \\ \hline 2)\ 1{,}00000{,}00000\ . & \text{Log. of } 10 \\ \hline \end{array} \qquad \begin{array}{rl} -0{,}60205{,}99913\ . & \text{Log. of } \frac{1}{4} \\ -0{,}79588{,}00174\ . & \text{Log. of } \frac{4}{25} \\ \hline -1{,}39794{,}00087\ . & \text{Log. of } \frac{4}{100} \\ \hline 2)\ -0{,}69897{,}00043\ , & \text{Log. of } \frac{1}{5} \\ \hline \end{array}$$

Decimals.

$$\begin{array}{rl} -1{,}39794{,}00087\ . & \text{Log. of } 0{,}25'' \\ -1{,}20411{,}99827\ . & \text{Log. of } 0{,}16'' \\ \hline -2{,}60205{,}99914\ . & \text{Log. of } 0{,}04 \\ \hline 2)\ -1{,}30102{,}99957\ . & \text{Log. of } 0{,}2 \\ \hline \end{array}$$

If the Data are unlike.

2. When the Logarithms of the Numbers propounded are of divers Kinds, and the defective Log. the Log. of a common Fraction; then take the leſſer Log. out of the Greater, and one half of the Remain ſhall be the Logarithm of the Mean Proportional, and of the ſame kind with the greater Logarithm.

Examples.

As the Mean Proportional between $\frac{3}{4}$ and 12 ſhall be 3, becauſe the Product $\frac{36}{4}$ hath the Square Root $\frac{6}{2}$ or 3.

And the Mean Proportional between $\frac{1}{25}$ and 4, ſhall be $\frac{2}{5}$; for ſo is the Square Root of their Product $\frac{4}{25}$.

$$\begin{array}{rl} 1{,}07918{,}12460\ . & \text{Log. of } 12 \\ -0{,}12493{,}87366\ . & \text{Log. of } \frac{3}{4} \\ \hline 0{,}95424{,}25094\ . & \text{Log. of } \frac{36}{4} \\ \hline 2)\ 0{,}47712{,}12547\ . & \text{Log. of } 3 \\ \hline \end{array} \qquad \begin{array}{rl} -1{,}39794{,}00084\ . & \text{Log. of } \frac{1}{25} \\ 0{,}60205{,}99913\ . & \text{Log. of } 4 \\ \hline -0{,}79588{,}00171\ . & \text{Log. of } \frac{4}{25} \\ \hline 2)\ -0{,}39794{,}00085\ . & \text{Log. of } \frac{2}{5} \\ \hline \end{array}$$

But if the Defective Log. be the Log. of a Decimal Fraction, then add the Logarithms together, and divide the Sum by 2, with the Addition and Diviſion proper thereto, ſet forth at large in the Part treating of Logarithms.

As the Mean Proportional between 4 and $\frac{1}{25}$ turned into a Decimal, (that is, 0,04'') is $4'$, equal to the former $\frac{2}{5}$.

0,60205,99913 . Log. of 4.
—2,60205,99913 . Log. of 0,04"
—1,20411,99826 . Log. of 0,16"
2)—1,60205,99913 . Log. of 0,4

Obſervations.

Mean Proportionals under this firſt Caſe thus found, make evident,

1. What the third Proportional is equal to. How noted in Species.

1. That among Numbers geometrically continually Proportional, every third Proportional ſhall be equal to the Square of the Second divided by the Firſt; which is the reaſon of their being marked in Species as before, by $\alpha . \beta . \frac{\beta q}{\alpha}$. As 4 . 12 . 36 . for the Square of 12 is 144; which divided by 4, giveth 36.

2. If the Firſt be a Square, ſo the Third.

2. That if the Firſt be a ſquare Number, ſo ſhall the Third, becauſe the Product of the Extreams ſhall be equal to the Product of the Mean multiplied into himſelf; as 4 and 36 be both Squares, and 4 multiplied into 36, ſhall be equal to 12 x 12, that is 144.

3. Complement between ſingle Squares is the Mean.

3. That the Complement between ſingle Squares, is the Mean Proportional: As if the Segments of the Side 12 be 10 and 2, then 20 the Complement is the Mean Proportional between 100 and 4: For $100 + {}^{20}_{20} + 4$, the Squares of the Segments and the two Complements are equal to the Square of the whole Line 144.

2.
If more Means than 1 be ſought. To get 2.

Caſe 2. If two or more mean Proportionals be deſired between 2 Numbers given.

Then for 2 Means, multiply the greater Extream by the Square of the Leſſer, and the Cube Root of the Product is the leſſer of the two Means ſought: contrariwiſe, multiply the Square of the greater Extream by the Leſſer, and extract the Cube Root of the Product for the greater Proportional Mean. Or if the greater Extream be divided by the Leſſer, the Cube Root of the Quotient ſhall be the *Ratio* to multiply the leaſt Extream, *&c.*

Example.

As if between 2 and 54, two proportional Means be ſought, the Leſſer will be 6 and the Greater 18: For 54 multiplied by 4, the Square of 2, produceth 216, whoſe Cube Root is 6 the leſſer Mean: and 2916 the Square of 54 multiplied by 2 produceth 5832, whoſe Cube Root is 18 the greater Mean; ſo if 2 divide 54 the Cube Root of 27, the Quotient is 3, which multiplying 2 is 6, and 6 is 18.

Leaſt Extream . 2. Square. 4. $54 \times 4 = 216$ (6. Leaſt Mean.
Greateſt Extream. 54. Square. 2916. $2916 \times 2 = 5832$ (18. Greateſt Mean.
And 2) 54 (27 √ ϕ 3 × 2 = 6 $3 \times 6 = 18$

Proportionals 2 . 6 . 18 : 54 . Triple Ratio.

To get 3.

To get 3 Means between 2 Numbers given, proceed by the firſt Caſe to get the middle Proportional between the 2 given Extreams; then between that middle Proportional and the leaſt Extream, a mean Proportional is to be found in like manner, and alſo between the greateſt Extream and that middle Proportional.

Example.

As between 2 and 512 to get 3 Means Pproportional, the middle Proportional is found to be 32, by multiplying 2 in 512 and taking the ſquare Root of the Product: Likewiſe between 2 and 32, the Mean is found to be 8, and between 32 and 512 the Mean is 128.

Extreams { 2, 512
$2 \times 512 = 1024$ (32 √ ʒ middle Proportional.
$2 \times 32 = 64$ (8 √ ʒ Proportional between 2 and 32.
$32 \times 512 = 16384$ (128 √ ʒ Proportional between 32 & 512.

Proportionals 2 . 8 . 32 . 128 . 512 . Quadruple Ratio.

To get many.

When many Means are deſired to avoid multiplicity of Rules, the beſt way is to get the *Ratio:* for ſeeing the two Extreams and Number of Terms are given, the *Ratio* is obtained by the firſt Propoſition for finding the fourth Principal before ſpoken to; and when the *Ratio* is gotten, the leaſt Extream multiplied continually thereby, produceth the Means deſired.

Example in 4, the common Way.

As to get 4 proportional Means between 2 and 2048, the Extreams given are the firſt and laſt Terms of a *Geometrical Progreſſion,* between which if 4 Means be gotten, the whole Number of Terms will be 6; therefore the Quotient of 2048 divided by 2, which is 1024, ſhall be a Surſolid Number whoſe Index is 5, that is 1 leſs than 6: the Number of Terms and the Surſolid Root of 1024 which is 4, ſhall be the *Ratio* ſought, by which 2 ſucceſſively multiplied ſhall produce the Means deſired.

$$\sqrt{}\ \frac{\omega}{\alpha}\text{Index } T-1=R \qquad 2)\,2048\,(\,1024 \quad \begin{matrix}\sqrt{qc}:4.\\ \text{Index } 5=6-1\end{matrix}$$

Proportionals . 2 . 8 . 32 . 128 . 512 . 2048 . Ratio 4.

How by Logarithms.

To avoid the Fractions that may ariſe in this Caſe as in the Firſt, the Practice may be by *Logarithms*, which thoſe expert therein will find eaſy and commodious. Wherefore,

If the Data *are alike, both* Integers.

1. When the Logarithms of the Numbers given are both of the ſame kind, and the Numbers Integers, take the Log. of the leſſer Number out of the Log. of the Greater, and the third Part of the Remainder added to the Log. of the Leſſer, ſhall give the Log. of the leſſer mean Proportional required; and if to this Log. the third Part be added, the Sum ſhall be the Log. of the other Mean Proportional required.

Example. 2 Mean Proportionals between 2 and 54, are 6 and 18, as before was found.

1,73239,37598 . Log. of 54
0,30102,99957 . Log. of 2 $\left(\frac{54}{2}\right.$ 27 $\sqrt{\Phi}$. 3.
3) 1,43136,37641 : Log. of 27
0,47712,12547 . Third part of the Difference, or $\sqrt{\Phi}$.
0,77815,12504 . Log. of 6, The leſſer Mean.
1,25527,25051 . Log. of 18, The greater Mean.

Both Common Fractions.

But when the Logarithms are both of common Fractions, take the Log. of the greater Fraction from the Log. of the Leſſer: and the third Part of the Remainer added to the Log. of the greater Fraction, ſhall be the Log. of the greater Mean; and being added to this, ſhall make the Log. of the leſſer Mean.

Example.

As to find 2 Means between $\frac{4}{45}$ and $\frac{3}{10}$, the Log. of $\frac{3}{10}$ ſhall be taken from the Log. of $\frac{4}{45}$, and the third Part of the Difference added to the Log. of $\frac{3}{10}$, ſhall be the Log. of $\frac{1}{5}$ the greater Mean; and the ſame third Part added to the Log. of $\frac{1}{5}$, ſhall be the Log. of $\frac{2}{15}$ the leſſer Mean.

Fractions —1,05115,25225 . Log. of $\frac{4}{45}$ the leſſer Fraction.
—0,52287,87453 . Log. of $\frac{3}{10}$ the greater Fraction.
—0,52827,37772 . Log. of $\frac{8}{27}$
3) —0,17609,12591 . Third part of the Difference.
—0,69897,00044 . Log. of $\frac{1}{5}$ the greater Mean.
—0,87506,12635 . Log. of $\frac{2}{15}$ the leſſer Mean.

Proportionals $\frac{3}{10}$. $\frac{1}{5}$. $\frac{2}{15}$. $\frac{4}{45}$. *Ratio* $\frac{2}{3}$.

Both Decimals.

Thus alſo working with the *Logarithms* of Decimals will the Effects be anſwerable, always uſing the Subſtraction and Diviſion proper to Decimal Logarithms.

Example.

As to find 2 Means between 0,048 and 0,75, the Log. of 0,75 taken from the other, leaves the Log. of 0,064, whoſe third Part or Cube Root is 4′, the Log. of which added to the Log. of 0,75, gives the Log. of 0,3 for the greater Mean, and again, added to this Log. gives the Log. of 0,12 for the leſſer Mean.

Decimals —2,68124,12374 . Log. of 0,048‴ . The leſſer Decimal.
—1,87506,12634 . Log. of 0,75″ . The greater Decimal.
—2,80617,99740 . Log. of 0,064‴ .
3) —1,60205,99913 . Third part of the Difference.
—1,47712,12547 . Log. of 0,3′ . The greater Mean.
—1,07918,12460 . Log. of 0,12″ . The leſſer Mean.

Proportionals 0,75″ . 0,3′ . 0,12″ . 0,048‴ . *Ratio* 0, 4′.

If the Data *are unlike, and one a common Fraction.*

2. When the Logarithms of the Numbers given are of divers Kinds, and the defective Log. is the Log. of a common Fraction, then add them together; and if out of the third Part of the Sum you may ſubſtract the Log. of the Fraction given, the Remain of this Subſtraction ſhall be the abundant Log. of the leſſer Mean ſought, to which add the third Part aforeſaid, and the Sum ſhall be the abundant Log. of the other Mean required.

But if when the given Logarithms are added, you may ſubſtract the third Part of the Sum out of the Log. of the given Fraction; then the Remainder of this Subſtraction ſhall be the Defective Log. of the leſſer Mean ſought: Out of which Log. if again you may ſubſtract the ſaid third Part, the Remain thereof ſhall be the Defective Log. of the other Mean required.

And if you may ſubſtract the Log. of the firſt Proportional found as above, out of the ſaid third Part, the Remainder thereof ſhall be the abundant Log. of the latter Proportional required.

Examples. As between $\frac{3}{4}$ and 48 will be found 3 the leſſer Proportional, and 12 the Greater. And betwen $\frac{1}{4}$ and 2 are found 2 Means, $\frac{1}{2}$ the Leſſer, and 1 the Greater.

—0,12493,87366	. Log. of $\frac{3}{4}$	—0,60205,99913	. Log. of $\frac{1}{4}$
1,68124,12374	. Log. of 48	0,30102,99957	. Log. of 2
1,80617,99740	. Log. of 64	0,90308,99870	. Log. of 8
3)0,60205,99913	. 3*d* Part of the Sum.	3)0,30102,99956	. 3*d* Part of the Sum.
0,47712,12547	. Log. of 3. —⊐	—0,30102,99956	. Log. of $\frac{1}{2}$. —⊐
1,07918,12460	. Log. of 12. ⊏—	0,00000,00000	. Log. of 1. ⊏—
Prop. $\frac{3}{4}$. 3 . 12 . 48 .	Ratio 4.	Prop. $\frac{1}{4}$. $\frac{1}{2}$. 1 . 2 .	*Ratio* 2.

One a Decimal. When the Defective Log. given is the Log. of a Decimal, then ſubſtract the Log. of the Decimal from the Log. of the Integer, and take the third Part of the Difference from the Log. of the Integer, the Remain ſhall be the greater Mean ſought. Take again the third Part out of the Log. of the Mean thus found, and this Remain ſhall be the Log. of the leſſer Mean required, always uſing the Subſtraction proper to Decimal Logarithms.

As in the laſt Examples, and two others that follow.

Examples.

1,68124,12374	. Log. of 48	0,30102,99957	. Log. of 2.
—1,87506,12634	. Log. of 0,75″	—1,39794,00087	. Log. of 0,25″
1,80617,99740	. Log. of 64	0,90308,99870	. Log. of 8.
3)0,60205,99913	. Third Part	3)0,30102,99957	. Third Part.
1,07918,12461	. Log. of 12 ⊏—	0,00000,00000	. Log. of 1. ⊏—
0,47712,12548	. Log. of 3 —⊐	—1,69897,00043	. Log. of 0,5′ —⊐
Prop. 0,75 . 3 . 12 . 48 .	*Ratio* 4.	Prop. 0,25 . 0,5 . 1 . 2 .	*Ratio* 2.

0,82930,37728	. Log. of 6,75″	0,00000,00000	. Log. of 1.
—1,39794,00087	. Log. of 0,25″	—1,09691,00130	. Log. of 0,125‴
1,43136,37641	. Log. of 27.	0,90308,99870	. Log. of 8.
3)0,47712,12547	. Third Part.	3)0,30102,99956	. Third Part.
0,35218,25181	. Log. of 2,25″ ⊏—	—1,69897,00044	. Log. of 0,5′ ⊏—
—1,87506,12634	. Log. of 0,75″ —⊐	—1,39794,00088	. Log. of 0,25″ —⊐
Prop. 0,25 . 0,75 . 2,25 . 6,75 .	*Ratio* 3.	Prop. 0,125 . 0,25 . 0,5 . 1 .	*Ratio* 2.

Many Means gotten in like manner. In like manner obſerving the Nature of the given Logarithms, whether of *Integers*, *Fractions*, or *Decimals*, and accordingly taking their Sum or Difference, may as many Means be gotten as ſhall be deſired: Thus,

Divide this Sum or Difference by 1 more than the intermediate Numbers ſought; this Quotient, which will always be the Log. of the *Ratio*, multiply ſeverally by 2, 3, 4, *&c.* till you come to the Diviſor, and theſe Products ſeverally muſt be added to, or ſubſtracted from one of the Logarithms of the given Numbers, or one of them therefrom as the Caſe requires, and the Remains ſhall be the Logarithms of the Means required.

Example in 5 Means. As to get 5 intermediate Numbers between $\frac{3}{4}$ and 48 the Sum; but if $\frac{3}{4}$ be turned into a Decimal, then the Difference of their Logarithms is to be divided by 6, that is 1 more than the Means ſought: the Quotient of this Diviſion taken from the Log. of 48, ſhall leave the Log. of 24; double the Quotient taken away, ſhall leave the Log. of 12; and ſo for the reſt as below appeareth: And the like will be effected, if the Log. of $\frac{3}{4}$ be taken ſeverally from theſe Products.

And other Proportionals. And if A, the ſixth Part of the Logarithms of the Extreams, be multiplied by 4, and the Product 4 A, or P, be added to the Log. of 48, the Sum ſhall be the 4*th*

Proportionl above 48. But if the Log. of $\frac{3}{4}$ be added to P, the Sum ſhall be the 4*th* Proportional below $\frac{3}{4}$, and ſo Numbers above and under each Extream are found.

Extreams	B	—0,12493,87366	. Log. of	$\frac{3}{4}$.	I
	H	1,68124,12374	. Log. of	48.	K
	6 A 6)	1,80617,99740	. Log. of	64	the Sum L
	1 A	0,30102,99957	. Log. of	2.	M
	2 A	0,60205,99914	. Log. of	4.	N
	3 A	0,90308,99871	. Log. of	8.	O
	4 A	1,20411,99828	. Log. of	16.	P
	5 A	1,50514,99785	. Log. of	32.	Q
Means	C	0,17609,12589	. Log. of	$1\frac{1}{2}$.	R
	D	0,47712,12546	. Log. of	3.	S
	E	0,77815,12503	. Log. of	6.	T
	F	1,07918,12460	. Log. of	12.	V
	G	1,38021,12417	. Log. of	24.	W

α I . R . S . T . V . W . K ω

$\frac{3}{64}$. $\frac{3}{32}$. $\frac{3}{16}$. $\frac{3}{8}$. $\frac{3}{4}$. $1\frac{1}{2}$. 3 . 6 . 12 . 24 . 48 . 96 . 192 . 384 . 768 .

For $4A + B = \frac{I}{P} = \alpha$ Log. —1,32905,87194 $\frac{1}{16}$) $\frac{3}{4}$ ($\frac{3}{64}$

And $4A + H = KP = \omega$ Log. 2,88536,12202 48×16=768

Obſervations.

The finding of Mean Proportionals under this ſecond Caſe in the manner aforeſaid, makes evident,

1. Complements between Cubes, the 2 Means.

1. That the greater Complement between the Cubes of the Segments of any Line is the greater Mean, and the leſſer Complement the leſſer Mean. As if the Segments of 12 be 10 and 2, their Cubes are 1000 and 8, the greater Baſe 100, the Leſſer 4, the Altitude of the Greater 10, of the Leſſer 2, the greater Complement is made of 100 by 2, the Leſſer of 4 by 10: So are the Cubes of the Segments with the 3 greater and 3 leſſer Complements, equal to the Cube of the whole Line 1728.

Greater Cube.	*Greater Compl.*	*Leſſer Compl.*	*Leſſer Cube.*	
1000 .	200 .	40 .	8	
	200	40		
	200	40		
1000 +	600 +	120 +	8	=1728

2. Product of 3 Proportionals, the Cube of the Mean.

2. That the Product of 3 *Geometrical Proportionals* continued, is the Cube of the Mean: So 8 multiplied into 40, and the Product 320 into 200, ſhall be 64000 the Cube of 40.

3. Product of the Extreams by the Squares alternate, are the Cubes of the Means.

3. That if 4 Numbers be Geometrically continued in Proportion the Product of the Extreams by their Alternate Squares, are the Cubes of the Means, the Greater of the Greater, and the Leſſer of the Leſſer: As 1000 multiplied into 64 (the Square of 8) is 64000 the Cube of 40, the leſſer Mean: And 8 multiplied into 1000000 (the Square of 1000) is 8000000, the Cube of 200 the greater Mean.

4. Analogy in 4 Proportionals of this ſort.

4. That in 4 ſuch Proportionals, as the Firſt to the Fourth; ſo is the Cube of the Firſt to the Cube of the Second.

As 8 . 1000 :: 512 . 64000.

5. Such Proportionals like Solids, their leaſt Terms are Cubes.

5. That if the Extreams of ſuch 4 Proportionals are like Solids, being reduced to their leaſt Terms they will be Cubes. As 16 . 24 . 36 . 54 . here 16 and 54 are like Solids, becauſe made of 3 ſides Proportional, (as 2×4×2=16 / 3×6×3=54) reduced to their leaſt Terms, are 8, 12, 18, 27, where 8 and 27 are Cubes.

5ly. *The State of the Queſtion duly conſidered.*

Operation as the Data *requires. Principal occult to be diſcovered.*

§. 5. In *Geometrical Progreſſion*, as before in other Places hath been warned, the Artiſt muſt carefully conſider the State of the Queſtion, and upon what point the Reſolution depends, and when Decimals or Fractions are given or happen in the Work, to work accordingly; and if one Queſtion be included in another, to reſolve them orderly: Alſo when ſome one of the 3 Principals are not very obvious in the Queſtion propounded, but given under ſome Occult Phraſes; the skill of the Artiſt is concerned to diſcover it, and ſometime to diſpoſe the firſt Perſon

Perſons to be orderly placed. in the Queſtion to the laſt Place in the Progreſſion, or ſuch other Term as the State of the Queſtion requireth.

Q. Of a Sum divided to 6 Men. *Example* 1. A certain Sum of Money was to be divided among 6 Men, ſo as *A* was to have $\frac{1}{3}$ of the whole, *B* $\frac{1}{3}$ of the remainder, and ſo to every one of the reſt, and at laſt there was left 4 *l.* The Queſtion is, what each Man received, and what was the Sum divided amongſt them?

Anſwer. To which for Anſwer, upon peruſal of the Queſtion there appeareth to be clearly given the third Principal 6, and occultly the leaſt Extream and *Ratio*, (*viz.* the firſt and fourth Principals) to find the fifth and each Particular Term: Wherefore to make the leaſt Extream plain, and accordingly the *Ratio*, I conſider that to the 4 *l.* remaining muſt be put 2, that it being divided into 3 Parts, and one third Part taken away, no more than 4 may be left, and this third Part of 6 being 2, muſt be the ſixth Man's Portion, and accordingly the leaſt Extream of the *Progreſſion*; which 6 is alſo ſoon perceived to be the Remainder before, and conſequently but $\frac{2}{3}$ Parts of a former Remain, that of neceſſity muſt be 9, and ſo the fifth Man's Portion found to be 3 the $\frac{1}{3}$ of 9; and 3 in compariſon to 2 the ſixth Man's Portion, is in the *Ratio Seſquialter:* Wherefore multiplying by the *Ratio* $1\frac{1}{2}$ or otherwiſe proceeding by the former Propoſitions, the ſeveral Terms and Sum with the Remain added, may be obtained for a full Reſolution.

Remains.			*Terms.*	*Perſons.*	*Portions.*		
l.	*s.*	*d.*			*l.*	*s.*	*d.*
4	00	00	1	6 . *F*	2	00	00
6	00	00	2	5 . *E*	3	00	00
9	00	00	3	4 . *D*	4	10	00
13	10	00	4	3 . *C*	6	15	00
20	05	00	5	2 . *B*	10	02	06
30	07	06	6	1 . *A*	15	03	09

Sum of the Progreſſion 41 : 11 : 03 Money Received.
Remain added 4 : 00 : 00
Sum to be divided 45 : 11 : 03

Q. Of Alms to 4 Beggars. *Example* 2. A Beggar meeting a Gentleman on the Road, requeſteth an Alms: to whom the Gentleman replies, he hath but little Money about him, but if the Beggar will give him as much as he hath, he will give him 6 Pence: whereunto the Beggar at firſt unwilling, being afterward aſſured he ſhould thereby be a Gainer did conſent, and received his Alms. The Gentleman afterward meeting with the ſecond, third and fourth Beggars, did likewiſe, and then had left himſelf no Money at all, having given the laſt Beggar but 4 Pence: The Queſtion is, what Money he had at firſt, and what were the Alms he gave?

Anſwer. *Anſw.* He had at firſt $5\frac{1}{2}$ *d.* ſo the firſt Beggar received one half Penny for an Alms, the ſecond a Penny, the third two Pence, and the laſt as much.

In the ſtating this Queſtion, as in the former, the third Principal 4 is given plain, the leaſt Extream and *Ratio* more occult; yet the laſt Beggar's Alms being aſcertained to be 4 *d*, it is ſufficient to declare them, ſeeing by the Form of the Queſtion it appeareth to have been doubled, therefore the half of Neceſſity muſt be the Remain after the third Beggar had his 6 *d* deducted, which 6 *d.* and 2 *d.* make 8 *d.* that had been doubled of the former Remain. The like is to be underſtood of the Reſt, whereby the *Ratio* of the Alms is found to be double except the laſt, which is diſtracted on purpoſe to make the Queſtion intricate and uncouth: and ſometimes ſuch Queſtions as theſe receive an eaſier Reſolution by the Rule of falſe *Poſition* than by theſe Rules of *Progreſſion*, though they ſeem hence to take their Foundation.

Terms of the *Progreſſion*	1	2	3	4
Beggars in their Order	4	3	2	1
Money received by the Beggars	4	6	6	6
Money in the Almoner's Purſe, and doubled by the Beggars	2	4	5	$5\frac{1}{2}$
Proper Alms received by the Beggars	2	2	1	$0\frac{1}{2}$

$$\text{For } 5\tfrac{1}{2} \times 2 = 11 - 6 = 5 \times 2 = 10 - 6 = 4 \times 2 = 8 - 6 = 2 \times 2 = 4$$

6ly. Mean Proportionals helpful to diſcover the diſtinct Quantities in mixt Bodies.

§. 6. The finding of Mean Proportionals beſides many other Uſes, is neceſſary to the diſcovery of the true and diſtinct Quantities of each ſort in mixed Bodies, and reſolveth Queſtions of that Nature, placed by ſome under *Alligation*, but cannot thereby be well reſolved without borrowing help from hence.

Q. Of the Weight of Silver mixt.

Example 1. Let an Ingot of the fineſt Silver, from which a Mark being cut off, be melted with a Mark of Copper inſtead thereof; and afterward a Mark of that mixt Metal cut off, and the Remainder melted with another Mark of Copper: Again, the third time a Mark of this Mixture being cut off, and the Remainder melted with another Mark of Copper; and finding by the Saie, that this laſt Mixture is $8\frac{5}{128}$ Ounces fine: The Queſtion is, how many Marks the Ingot weighed?

Anſwer.

Anſw. 8 Marks; for the fineſt Silver (as the Ingot was before Allay) being 12 Ounces fine, two Mean Proportionals are to be ſought between the ſame and $8\frac{5}{128}$ (the fineneſs of the Ingot after Allay), which to do, firſt $8\frac{5}{128}$ and 12 muſt be reduced to one Denominator, and then leaving the Denominators, the Proportions will reſpect their Numerators only. And as the Differences between the Proportionals reſpect the ſeveral Allays, ſo ſhall the Difference between the 2 greateſt Proportionals, reſpect the greateſt Fineneſs and leaſt Allay; wherefore the Analogy is, as that Difference to 1, ſo is the greateſt Proportional to the Weight deſired.

$8\frac{5}{128}$ and $\frac{12}{1}$ reduced, are $\frac{1029}{128}$ and $\frac{1536}{128}$.

Proportionals . 1029 . 1176 . 1344 . 1536.

1536 — 1344 = 192 . Then as 192 . 1 :: 1536 . 8 Marks.

Q. Of Wine mixt with Water, how much drawn out at a time.

Example 2. A Merchant hath a Piece of Wine of 128 Gallons, out of which he draweth certain Gallons, and filleth up the Veſſel again with Water; the ſecond Time he draweth out as much as he did at the Firſt, and filleth it up again with Water; and the like he doth the third and fourth Time, and in the end findeth that there was left in the Veſſel $75\frac{1}{32}$ Gallons of Wine, beſides the Water that was put in: how many Gallons was drawn out at a Time?

Anſwer.

Anſw. 16 Gallons: Here the Draught being repeated 4 times, 3 Mean Proportionals are to be gotten, (for they are always to be 1 leſs than the Mixtures) 128 and $75\frac{1}{32}$ then reduced as before, ſhall have their Numerators 2401 and 4096, between which the 3 Proportionals are 2744 . 3136 . 3584. Then becauſe the Inquiry here, is not as in the laſt Queſtion for the Quantity at firſt, but for the Draught out of that Quantity, the Analogy is, As the Denominator to 1, ſo is the Difference between the higheſt Proportionals to the Number ſought.

$75\frac{1}{32}$ and $\frac{128}{1}$ reduced, are $\frac{2401}{32}$ and $\frac{4096}{32}$

Proportionals 2401 . 2744 . 3136 . 3584 . 4096.

4096 — 3584 = 512. Then as 32 . 1 :: 512 . 16 Gallons.

Of the Wine drawn out every time.

And in like manner, if the Quantity of Wine drawn out every Time in the 16 Gallons were deſired: by an orderly deduction of the leſſer Proportional out of the next Greater, and dividing the Difference by the common Diviſor 32, the Quotients will determine the Deſire.

Gall.

So 3584—3136=448 Therefore 32) 448 (14 . 2 Draught.

And 3136—2744=392 Therefore 32) 392 ($12\frac{1}{4}$. 3 Draught.

And 2744—2401=343 Therefore 32) 343 ($10\frac{23}{32}$. 4 Draught.

If the Price of the Wine be given, to find the Quantity.

And if the Prices of the Wine before and after Mixture had been given, to find the Quantity: (as ſuppoſe a Gallon before mixture were worth 5 *s.* and after Mixture 2 *s.* $\frac{3813}{4096}$ to know how many Gallons in the whole Piece): then after the Mean Proportionals are gotten between the 2 Prices, the Analogy is, As the Difference between the 2 higheſt Prices to the firſt Draught, ſo is the higheſt Price to the whole Quantity.

$2\frac{3813}{4096}$ and $\frac{5}{1}$ reduced, are $\frac{12005}{4096}$ and $\frac{20480}{4096}$

Proportionals . 12005 . 13720 . 15680 . 17920 . 20480.

Prices . $2\frac{3813}{4096}$. $3\frac{179}{512}$. $3\frac{53}{64}$. $4\frac{3}{8}$. 5. and $5 - 4\frac{3}{8} = \frac{5}{8}$

Then as $\frac{5}{8}$. 16 :: 5 . 128 . Gallons in the whole.

Proof of Geometrical Progreſſion.

The ordinary Proof of *Geometrical Progreſſion*, is according to the firſt Example of this Chapter, to place every Term in order, and collect the Sum by common *Addition*; what other Operations the Sections include, carry Evidence of their truth in their Operation, or may be examined one by another, where there are varieties of Reſolution.

CHAP. VI. *Tranſmutation.*

Tranſmutation *whence, and what it ſerves for.*

GEometrical *Progreſſion* begat *Tranſmutation*, which ſerveth to ſhew what Number of Changes may be made by any Number of Perſons or other things in their Places or Poſitions: As to know what Number of Changes in the Sound may be made by 5,6,7,8, or more Bells, or different Notes upon 4, 6, 8, or 10 ſtringed Inſtruments, and how often the Gammaut may be varied, *&c.*

What it is like.

Tranſmutation, ſometime like *Multiplication*, regardeth the Place of the Numbers, or Numbers themſelves, to increaſe the one by the other; and ſometime the Place in which the Perſon or Numbers ſtands, is not ſo much reſpected as the Variety and Multiplicity of Changes.

Changes, the Sorts.

Changes are Simple or Intermixt.

Simple Changes, what.

Simple Changes are (without relation to any Conſort) the Number of the ſeveral Poſitions or Places that a Company of Perſons, or other things, may be ſet or placed in order one beſide another; and every time ſome one or other of the Perſons or Things changing their Station, is removed into the Place of another, and all of them never found ſtanding alike.

To find ſuch.

To get ſuch a Number, multiply every Number from the Unit ſucceſſively into each other's Product unto the Number aſſigned.

Q. Of the Sittings of 7 Scholars.

Example. Seven Scholars taken out of a Free-School to be ſent to the Univerſity, were to be entertained at ſome College at Commons for a certain Sum of Money, with two Meals a Day, ſo long and no longer than that ſitting altogether on a Form at every Meal, they might ſit diverſly, and never the whole 7 to be alike in Situation: The Queſtion is, how long they were to ſtay there, or how many ſeveral Poſitions or Sittings there might be made by them in an unlike Poſiture?

Anſwer.

Anſw. 5040 Sittings, which at 2 Meals a Day, amounts to almoſt 7 Years, fully to 6 Years 330 Days; for 1 multiplied by 2 is 2, and 2 by 3 is 6, and 6 by 4 is 24, and 24 by 5 is 120, and 120 by 6 is 720, and 720 by 7 is 5040.

$$\text{And}\ \frac{\overset{\textit{Sittings.}}{5040}}{\underset{\textit{Meals.}}{2}}\left(\begin{array}{c}(330\\ \hline \cancel{2}\cancel{5}\cancel{2}\cancel{0}\\ \hline \cancel{3}6\cancel{5}\end{array}\right(\ 6\ \text{Years.}$$

A prodigious Number of Changes by the 24 Letters.

Hence it is no marvel that by 24 Letters there ariſeth and is made ſuch variety of Languages in the World, and ſuch a prodigious Number of Words in each Language; ſeeing the Diverſity of Syllables produceth that Effect, by the interchangable placing of Conſonants ſingle, double, treble, *&c.* with and among the Vowels. The Number of ſimple Changes in 24 by multiplying every Product ſucceſſively into the next Number, is found to be 620448401733239439360000.

Intermixt Changes, what.

Changes intermixt, or Counterchanges, are the Number of Varieties which a Company may make amongſt themſelves, part with part, or ſome of them to match the reſt in Conſort, and regard their Companions, and not the Places they ſtand in as the ſimple Changes did, which changed none of their Company, but the Place they ſtood in.

To find ſuch.

To get the Number of ſuch a Variety, let the whole Number given be broken into as many Parts as there be Units therein and under; or beſide thoſe Parts, ſet the Complement of the Units that make up the whole Number; then get the Varieties that any of thoſe Parts make to match with one of the Complements, and multiply this Number by the whole Number given.

Varieties like the Unciæ in the Table for Extraction of Roots.

The Varieties that the Parts make with 1 of the Complements, are like the Numbers in the Table for Extraction of Roots; for 1 makes no Changes, 2 makes but 2 ſimple Changes and none intermixt; 3 therefore begins the Dance: And of 3, if any 2 be given to match with the other 1, the Changes will be but 3, becauſe 3 times 1 is 3: But if 1 of 3 be to match with any of the other 2, there will be 6 Changes; for 1 may be joined with the Second, or with the Third, and ſo the Second with the Firſt or Third, and the Third with the Firſt or Second.

Thus any Number lacking 1, in oppoſition to any 1 thereof, can make but ſo many intermixt Changes as there be Units in the whole Number given: but in oppoſition to 2, 3, or more, order their Changes accordingly, by multiplying the whole Number into thoſe Tabellary Numbers, only as the given Number lacks 1, take the Numbers in the Table to 1 leſs than the Number given: As for 4, they muſt be 3, 3; for 5 the next Numbers, 4, 6, 4; and for 6 the next, 5, 10, 10, 5, *&c.* that is ſo many of the Tabellary Numbers as will ſerve for the Number of Parts.

Example.

Example. Suppoſe in 5 Numbers any 4 of them will find out the other 1, or any 3 of them the other 2, or any 2 of them the other 3, or any 1 of them the other 4: how many intermixt Changes there muſt be in any or all theſe Caſes is the Queſtion?

For Reſolution, 5 diſpoſed with his Parts and Complements, ſtand thus:

Parts 4 . 3 . 2 . 1
Complements 1 . 2 . 3 . 4

Then 4 to 1 cannot change; therefore 1 multiplied by 5, the Number makes 5, the Variety of Rules or Operations that muſt be in that Caſe.

But if any 3 of 5 be given, to find any of the other 2; then any 4 Numbers conſorting but 3 together at a time, making 4 Changes, this Tabellary Number 4 is to be multiplied by 5 the whole Number: So ſhall 20 be the Number of Changes or Rules neceſſary for the finding any of the other 2 deſired; as was before evident both in the Chapters of *Arithmetical* and *Geometrical Progreſſion*.

If 2 of 5 be given, to find any of the other 3: Then becauſe any 2 of 4 aſſociating together will make 6 Changes, this Tabellary Number 6 ſhall be multiplied by 5 the whole Number, and the Product 30 ſhall in this Caſe be the Number of Changes or Rules requiſite.

Laſtly, If any 1 of 5 will find out the reſt, then doth 1 match himſelf 4 times; This Tabellary Number 4 therefore is to be multiplied by 5, and the Product 20 is the Number of Changes deſired; and the whole Proceſs appears thus:

	5
Parts of 5, the whole Number given ————————	4 . 3 . 2 . 1
Complements of the Parts to 5 ————————————	1 . 2 . 3 . 4
Varieties of the Parts to 1, or Tabellary Numbers ———	1 . 4 . 6 . 4
Products of the Varieties by the whole Number ———	5 . 20 . 30 . 20

What Weights will weigh certain even Number of Pounds.

Hence notice came to be taken, how many Weights were neceſſary to weigh any aliquot Number of Pounds by: As to weigh any even Number of Pounds between 1 ℔ and 40 ℔; take 4 Weights in triple Proportion continued, as 1, 3, 9, 27, which make up the Total 40; and to weigh thereby 21 ℔, put 9 to the thing weighed, and 3 and 27 to the Counter-poiſe: For $3+27=30-9=21$, *&c.*

So to weigh any aliquot Number of Pounds, between 1 ℔ and 121 ℔, take 5 Weights in like Proportion, as 1, 3, 9, 27, 81, which make together 121.

Proof of Tranſmutation.

The beſt Demonſtration to prove the Truth of both Sorts of Changes, is to take Letters or Species, and place them orderly in their Places or Counter-places as the Caſe requires. So the ſimple Changes in 4 is ſeen to be 24, and the Intermixt 4, 12, 12, as hereafter followeth.

Simple Changes in 4.

	a . b . c . d	b . a . c . d	c . a . b . d	d . a . b . c
	a . b . d . c	b . a . d . c	c . a . d . b	d . a . c . b
1 . 2 . 3 . 4	a . c . b . d	b . c . a . d	c . b . a . d	d . b . a . c
1 . 2 . 6 . 24	a . c . d . b	b . c . d . a	c . b . d . a	d . b . c . a
	a . d . b . c	b . d . a . c	c . d . a . b	d . c . a . b
	a . d . c . b	b . d . c . a	c . d . b . a	d . c . b . a

Intermixt Changes in 4.

a . b . c . d
a . b . d . c
a . c . d . b
b . c . d . a

4
Parts —— 3 . 2 . 1
Complem's. 1 . 2 . 3
Varieties – 1 . 3 . 3
Products – 4 . 12 . 12

a . b .			b
a . c .	d	a .	c
b . c .			d
a . b .			a
a . d .	c	b .	c
b . d .			d
a . c .			a
a . d .	b	c .	b
c . d .			d
b . c .			a
b . d .	a	d .	b
c . d .			c

CHAP. VII. *Anatociſm.*

Anatociſm.

TO cloſe up this Part of continued Proportions, comes in the latter Brood of *Geometrical Progreſſion*, a Child more like the Father than the former of *Tranſmutation.*

Whence derived. How taken. Includes both Intereſt and Annuities.

Anatociſm, derived from ἀνὰ and τίκτω, importeth a bringing forth, renewing or increaſing, is here taken for the annual Increaſe of Intereſt or Uſury; and under the Title of *Intereſt* and *Annuities* commonly paſſes in moſt Authors, becauſe Propoſitions of both have their Reſolutions near of kin to each other, and are proper enough to be joined together.

Intereſt what. Principal what.

Intereſt is the Sum reckoned for the Loan or Forbearance of ſome Principal Sum lent for a certain Time, according to ſome certain Rate, and therefore called *Principal*, becauſe it is the Sum that procreates the Intereſt, or from which the Intereſt is reckoned.

Intereſt the ſorts.

Intereſt is twofold, *Simple* and *Compound.*

Simple Intereſt what, and how found.

Simple Intereſt is counted from the Principal only; and ſo the *Rule of Three*, or *Rule of five Numbers* before ſeen, being ſufficient to find out at any Rate, and for any Time whatſoever, any Portion of *Simple Intereſt* required, no further inquiry is to be made thereabouts here.

Compound Intereſt, what.

Compound Intereſt is that which is counted from the Principal and Simple Intereſt forborn, called alſo *Intereſt upon Intereſt.*

Annuities, what. Why the Caſes thereof differ from Intereſt.

Annuities, are yearly Paiments, or Annual Rents, paiable Half-yearly or Quarterly: And their Caſes differ ſomewhat from thoſe of Intereſt, becauſe if an Annuity be forborn, the Paiments increaſe as well as the Intereſt; but if a Sum of Money be lent, that Principal only is to be reſtored with the Intereſt.

What to be noted in both.

In the Queſtions and Caſes concerning *Intereſt* and *Annuities*, is to be noted;

1. The 5 Principals of a Geom. Prog. to be well known.

1*ſt*. That the Learner be well acquainted with the five Principals of *Geometrical Progreſſion*, and the way to find any 2 of them by the other 3, as before to be ſeen in the 5*th* Chapter of this Part.

2. How to conſtitute a Progreſſion by any Ratio.

2*ly*. How by any ſort of *Ratio*, as well as Multiplex exampled in there, a Progreſſion may be conſtituted, which is to be done by this *Analogy*: As the Antecedent of the *Ratio*, is to the Conſequent thereof; ſo is the firſt Term of a *Progreſſion Geometrical*, to the Second; and by conſequence ſo the Second to the Third, the Third to the Fourth, *&c.*

Examples.

As if the *Ratio* were 2 to 5, beginning at 8: Then as 2 . 5 :: 8 . 20 the ſecond Term. And as 2 . 5 :: 20 . 50, the third Term, *&c.*

So if the *Ratio* be at 4, 5, 6, 7, or 8 *per Cent.* As ſuppoſe 8, and the firſt Term of the Progreſſion be 5; Then as 100 . 108 :: 5 . 5,4. And if of both Examples Progreſſions be inſtituted, ſeven Terms thereof will be, as hereunder follow.

1	2 . 5 :: 8 . 20	1	100 . 108 :: 5 . 5,4.
2	20	2	5,4
3	50	3	5,832
4	125	4	6,29856
5	312,5	5	6,8024448
6	781,25	6	7,346640384
7	1953,125	7	7,93437161472

3*ly*. The *Ratio* of any former Term in a Series of ſuch continued Proportionals, unto any of the Terms following, is equal to the *Ratio* of the firſt Term to the Second, multiplied into it ſelf, according to the Diſtance of that latter Term from the former. 3. *What the Ratio of one Term to another is.*

As the *Ratio* of 5,4 to 6,8024448, which is the Third from it, is as the *Ratio* of 100 to 108 triplicated, or as the Cube of 100 to the Cube of 108. Example.

(C.100) (C.108)
1000000 . 1259712 :: 5,4 . 6,8024448.

4*ly*, It is moſt convenient, firſt to reduce the *Ratio* of Intereſt, ſo that the Antecedent may be 100 or 1; and to uſe Decimals for common Fractions and Logarithms for the reſt of the Work, becauſe otherwiſe many laborious Extractions of Roots are required. 4. Ratio *of the Intereſt, how beſt to be reduced.*

As if the *Ratio* be of Pence $\overset{d.}{14,4}$. or of Shillings $\overset{s.}{1,2}$. then for $\overset{l.}{1}$ the *Ratio* will be as 100 to 106, commonly called 6 *per Cent.* or 6 on the 100: For $\overset{d.}{240}$, (the Pence in a Pound Sterling) to $\overset{d.}{254,4}$. (the Pence a Pound, and the Intereſt of that Pound in a Year at the ſame Rate): Or $\overset{s.}{20}$, to $\overset{s.}{21,2}$. is as 100, to 106; and 100 . 106 :: 1 . 1,06 . So is 1 . 1,06 α . ϐ . and ϐ is procreated of the Penſion or Paiment α whatſoever it be at that Rate in one whole Year. Example.

5*ly*, If the Paiment be half Yearly or Quarterly, that is, in Days 182,5 . or 91,25 . For ϐ, otherwiſe *R*, commonly called the Rate when the firſt Term is an Unit: let the Log. of the Yearly Procreat be multiplied accordingly by $\frac{1}{2}$ or $\frac{1}{4}$ or $\frac{182,5}{365}$ or $\frac{91,25}{365}$: So the Quotient dividing the Numerator by the Denominator, ſhall be the Log. of the *Ratio* for half Years or Quarters: or in Decimals take √q1,06. or √qq 1,06 inſtead of ϐ. For it is vulgarly taken amiſs, when half the yearly Intereſt is taken for the half Year, and the Quarter thereof for a Quarter of a Year: And though *Simple Intereſt* be commonly ſo taken, yet in *Compound Intereſt* it is not ſo, becauſe the Increaſe is continual from the firſt beginning to the end of 3 Months, and ſo to 6 Months, and 12 Months. Then if the Increaſe in 3 or 6 Months ſhould come to the Quarter or Half of the Whole, it would come to more than the whole at the Years end. 5. *To get the Log. of the Rate for the Paiment Yearly, Half-yearly,* &c. *Vulgar Error.*

As let the *Ratio* be as 100 to 106, the Log. of 100 is 2,00000,00000; the Log. of 106 is 2,02530,58653, the Difference between them is 0,02530,58653, and this is the Log. of the Rate for 1 Year: And in like manner the Difference between the Log. of any Principal Money, and the Log. of that Principal, and the Increaſe or Decreaſe added together, ſhall be the Log. of the Rate for 1 Year. And accordingly this Difference multiplied or divided as the Caſe may require by the Time or Term, be it Years, Months, Weeks or Days, may not unfitly be termed the Log. for the Time, ſeeing it performs the Office thereof: ſo for 2 Years it muſt be multiplied by 2, for 3 Years by 3; if for $\frac{1}{2}$ a Year by $\frac{1}{2}$, for $\frac{1}{4}$ by $\frac{1}{4}$, for a Month by $\frac{1}{12}$, for a Week by $\frac{7}{365}$, for a Day by $\frac{1}{365}$, according to the Time required. Example.

Rate at 6 l. per Centum.

2,00000,00000 . Log. of 100 *l.* Principal.
2,02530,58653 . Log. of 106 . Principal and Intereſt.
0,02530,58653 . Difference or Log. of the Rate for 1 Year.
0,01265,29326 . Log. of the Rate for $\frac{1}{2}$ Year.
0,00632,64663 . Log. of the Rate for $\frac{1}{4}$ Year.
0,00210,88221 . Log. of the Rate for a Month.
0,00048,53179 . Log. of the Rate for a Week.
0,00006,93311 . Log. of the Rate for a Day.

6*ly*,

6. The Number of Ratio's, and Number of Terms to be well obſerved.

6ly, Becauſe in theſe *Progreſſions*, the Number of *Ratio*'s is leſs by 1 than T, the Number of Terms or Paiments, the Number of *Ratio*'s ſhall be T—1. And if α be the firſt Paiment and 1, then the Log. of β multiplied by T—1 ſhall be the Log. of ω the laſt Term; and the Log. of β multiplied by T, ſhall be the Log. of $\beta\omega$, that is, of β multiplied into himſelf according to the Number of Paiments, and $\beta-1$ ſhall be the Intereſt of the firſt Paiment: But if α be no Paiment but the Principal Stock put out to Intereſt, then ſhall $\beta\omega$ be the laſt Paiment for the Increaſe of α forborn ſo long time.

Example.

As ſuppoſe a *Progreſſion*, whoſe *Ratio* as 1, to 1,06, were inſtituted thus:

α . 1.
β . 1,06
βq . 1,1236
βc . 1,191016
βqq . 1,26247696
Z . 5,63709296

Then if β be the firſt Paiment, the ſame Number ſhall be both of the Rates and Paiments, of which the Fourth ſhall be 1,26247696 for ω, and accordingly T ſhall be 4: but if α be the firſt Paiment, the ſame Number ſhall be both of the Terms and Paiments, of which the Fourth ſhall be 1,191016, for ω, leſs than the other by one Degree, which being multiplied by β, ſhall give the Sum 1,26247696 for $\beta\omega$. and ſo ſhall T. be 5.

And by Logarithms, if α be the firſt Payment.

0,02530,58653 . Log. of $\beta = 1,06$
4 . T—1
0,10122,34612 . Log. of $\omega = 1,26247696$
0,02530,58653 . Log. of $\beta = 1,06$
5 . T
0,12652,93265 . Log. of $\beta\omega = 1,33822,55776$.
The 5*th* Term if β be the Firſt.

7. Whence ariſe the 2 firſt Theorems.

7ly, Wherefore $\beta\omega$ being procreated of α the Penſion, or 1 *l.* let out at Intereſt according to T, there ariſe hence theſe two Theorems, and by marking *Quantamlibet Summam librarum*, or any Sum of Money imployed with Q℔. may briefly be expreſſed in Species thus:

Species Q℔. for what uſe.

l. *Let out.*

Theorem 1. *Theor.* 1. 1. $\beta\omega$ :: Q℔ . Q℔ with the Gain according to T.

Theorem 2. *Theor.* 2. $\beta\omega$. 1 *l.* :: Q℔ and Gain according to T. To the preſent Value.

Example.

For in the foregoing Inſtance, becauſe 1 to 1,26247696 is as 1 to the 1 *l.* let out, and 0,26247696 the Gain let run for 4 Years: It ſhall be on the contrary, that 1,26247696 to 1. ſhall be as 1 *l.* let out, and the Intereſt let run for 4 Years, to 1 *l.* the preſent Value.

Otherwiſe.

But if β be the firſt Paiment, then ſhall $\beta\omega$ accordingly be ω the laſt Paiment, α being none, or β multiplied into the laſt Paiment where α is the Firſt; and then the Theorems ſhall be thus:

Let out.

Theorem 1. *Theor.* 1. 1 *l.* ω :: Q℔ . Q℔ with the Gain according to T.

Theorem 2. *Theor.* 2. ω. 1 *l.* :: Q℔ and Gain according to T. To the preſent Value.

8. Whence two other Theorems ariſe.

8ly, Foraſmuch as $\frac{\beta\omega-\alpha q}{\beta-\alpha}$ that is, $\frac{\beta\omega-1}{\beta-1} = Z$ the Sum of all the Terms of the *Progreſſion* (whoſe laſt is ω), and is therefore the Procreat of 1 *l.* Paiment let run according to the Number of Terms T. Hence ariſe 2 other Theorems.

Theorem 3. *Theor.* 3. $\beta-1$. $\beta\omega-1$ :: Q℔. Penſion let run according to T.
To the Penſions with the Intereſt to be paid in the End.

Theorem 4. *Theor.* 4. $\beta\omega-1$. $\beta-1$ :: Q℔ to come according to T.
To the equivalent Penſion to be paid according to T.

Example.

For in the Inſtance above, ſeeing ($\beta-1$ that is) 0,06 dividing ($\beta\omega-1$ that is) 0,33822,55776, the Quotient ſhall be (Z that is) 5,63709296, the Sum of the Progreſſion. And becauſe 0,06 to 0,33822,55776, is as 1 *l.* Penſion let run 5 Years to 5,63709296, the Penſions and Intereſt for forbearance then due: It ſhall be on the contrary, that 0,3382255776, to 0,06, ſhall be as 5,63709296, (that is 1 *l.* future according to T) to the equivalent Penſion, *viz.* 1 *l.*

And according to the firſt Paiment ſo is ω, and $\beta\omega$ to be accompted, and theſe Theorems to be underſtood and altered, as was above obſerved in the two firſt Theorems.

9thly, Seeing $\beta\omega$ is procreate of 1 *l.* let out according to T: And $\frac{\beta\omega - 1}{\beta - 1}$ is the Procreate of the Penſion of 1 *l.* let run according to T; which in ready Money equalleth the Price of the Penſion: *9. Whence ariſe two other Theorems.*

Say $\beta\omega \,.\, 1\,l. :: \frac{\beta\omega - 1}{\beta - 1} \,.\, \frac{\beta\omega - 1}{\beta - 1 \text{ in } \beta\omega}$. Whence therefore according to T, ſhall $\frac{\beta\omega - 1}{\beta - 1}$ the Price of the Penſion be procreated. Hence alſo ariſe 2 Theorems.

Theor. 5. $\beta - 1$ in $\beta\omega \,.\, \beta\omega - 1 ::$ Qt Penſion according to T. Theorem 5.
To the Price of the ſame in ready Money.

Theor. 6. $\beta\omega - 1 \,.\, \beta - 1$ in $\beta\omega ::$ Qt Preſent. Theorem 6.
To the Penſion to be bought according to T.

For in the former Inſtance, $\beta - 1$ (being 0,06) multiplied in $\beta\omega$, (which is 1,3382255776) makes 0,080293534656. And becauſe 0,080293534656 to 0,3382255776 (that is $\beta\omega - 1$) is as 1 *l.* Penſion in 5 Years to 4,212364— (the Price thereof in ready Money); it ſhall be on the contrary, that 0,3382255776, to 0,080293534656, ſhall be as 4,212364— paid preſent for the Annuity of 1 *l.* bought for 5 Years. Example.

And in theſe, as in the 2 Theorems laſt before, let ω and $\beta\omega$ be underſtood according to the firſt Paiment: And ſeeing 1 *l.* is taken for Qt in every of the ſix Theorems, and Explanation thereof, for Annuities of greater Value, multiply the 4*th* Number, or Number found, by the *Analogy*, by the Number of Pounds employed: Or let the Third in the *Analogy* be the ſame Number. And for Example-ſake, further followeth the Work of a Penſion for 10 Years, payable Half-yearly, at the Rate of 6 *per Cent.* that is 1 to 1,06, and T. the Number of Paiments 20. ☞ *Farther Example.*

Log. of 1,06, is . 0,02530,58653 .
in $\frac{1}{2}$
0,01265,29326$\frac{1}{2}$. Log. of $\beta = 1{,}0295$, &c.
20 . T
0,25305,86530 . Log. of $\beta\omega = 1{,}7908$, &c.
—2,47129,17111 . Log. of $\beta - 1 = 0{,}0295$, &c.
—2,72435,03641 . Log. of $\beta - 1$ in $\beta\omega$.
— 1,89812,15755 . Log. of $\beta\omega - 1 = 0{,}7908$, &c.

It is therefore —1,89812,15755 .
—2,72435,17111 .
1,17376,98644 . Log. of the Price 14,922 + for 1 *l.* Penſion.

And —2,72435,17111
—1,89812,15755 .
—2,82623,01356 . Log. of the Penſion 0,067023, for 1 *l.* Price.

To theſe found Logarithms, add the Logarithm of Qt.
Or multiply thoſe Values found by Qt.

10thly, Queſtions both of Intereſt and Annuities, as others in *Geometrical Progreſſion*, (upon whom their Reſolution depends) may be varied many ways: So that the finding of the five Principals of a *Geometrical Progreſſion*, before ſeen in the Chapter thereof, and the true knowledg of the ſix Theorems laſt above-mentioned, are ſufficient to reſolve any Propoſitions concerning them; the Principal Varieties whereof follow, firſt diſtinctly, and afterward intermixt; and in both are chiefly wrought by Logarithms, all other Ways for any conſiderable Time being tedious and troubleſome. *10. Queſtions may be varied many ways. On what the Work depends. Beſt done by Logarithms.*

Touching Compound-Intereſt diſtinctly.

Of Compound Intereſt, what to be noted.

Compound-Intereſt reſpecteth four Things: By any three of which given, the other may be found out.

1. *The Sum to be received.*

What it agreeth and anſwereth to, and how called.

1. The Sum or Amountment of any principal Sum lent or forborn, for a certain Time or Term, to be received with Intereſt upon Intereſt, after a certain Rate: This anſwereth to ω, or the Second of the five Principals in a *Geometrical Progreſſion*, and agreeth with the firſt Theorem, and is ſometime called the *Profit* or *Loſs*.

2. *The Sum lent.*

What it agreeth and anſwereth to, and how called.

2. The principal Sum lent, or ready Money to be paid for a Sum of Money to be forborn a certain Time or Term, with Intereſt upon Intereſt, after a certain Rate: This anſwereth to α, or the firſt of the five Principals in a *Geometrical Progreſſion*, and agreeth with the ſecond Theorem, and is ſometime called *Abatement* or *Rebatement*, and ſometime *Diſcount*.

3. *The Rate.*

What it anſwereth to.

3. The Rate, according to which the principal Sum lent or forborn increaſeth or decreaſeth, as the Queſtion concerneth Profit or Loſs. This anſwereth to R, or the Fourth of the five Principals in a *Geometrical Progreſſion*.

4. *The Time or Term.*

What it anſwereth to.

4. The Time or Term in which the Increaſe or Decreaſe is to be, whether Years, Quarters, Months, Days, or what elſe. This marked by T, or N, anſwereth to the Third of the five Principals in a *Geometrical Progreſſion*.

Prop. 1. *To find the Sum to be paid.*

Q. *Of* 200 *l. forborn* 7 *Years, what it comes to.*

Propoſition 1. To know what Sum ought to be paid for a Sum forborn a certain Time, or Term, at any Rate, with Compound Intereſt.

As to know what Sum ſhall be paid for 200 *l.* forborn 7 Years, with Intereſt upon Intereſt, after the Rate of 6 *l. per Cent. per Annum:* Or what comes 200 *l.* to in 7 Years, by Intereſt upon Intereſt, at the Rate of 6 on the 100?

Anſwer.

Anſw. l. 300,726, and ſomewhat more: For ſeeing here is given α. T. R. to find ω. that is α. 200. T. 7. and R. as 1 to 1,06, the 300,726, *&c.* may be found by the former Rules in *Progreſſion*, only there the *Ratio* was figurate to T—1, but here to T, as by the Decimals appeareth.

By Decimals.

Thus 1,06 R
1,1236
1,191016
1,26247696
1,3382255776
1,418519112256
1,50363025899136
200 α

300,72605179827200 ω l

By the firſt Theorem.

And by the firſt Theorem.

1 . 1,50363025899136 :: 200 . 300,72605179827200.

Or thus the common Way, *By the Common Way.*

	2 00	α	
	1,06	R	
	1200		
	2000		
	212,00	1	
	1,06		
	127200		
	212000		
	224,7200	2	
	1,06		
	13483200		
	22472000		
	238,203200	3	
	1,06		
	1429219200		
	2382032000		T
	252,49539200	4	
	1,06		
	151497235200		
	252495392000		
	267,6451155200	5	
	1,06		
	16058706931200		
	267645115 52000		
	283,7038224 51200	6	
	1,06		
	17022229347072 00		
	2837038224512000		
ω	300,72605179827200	7	

The Work by Logarithms is both eaſy and ſpeedy thus:

To the Product of the Log. of the Rate (which as aforeſaid is the Difference between the Log. of the Principal Money, and the Log. of the Principal and Simple Intereſt in 1 Year) multiplied by the Term or Time (called the *Log. for the Time*) add the Log. of the Sum ſo lent or forborn. *By Logarithms.*

	2,00000,00000	. Log. of 100
	2,02530,58653	. Log. of 106
Difference	0,02530,58653	. Log. of 1,06 . R. or the Rate for 1 *l.* for 1 Year.
	7	. T.
Product	0,17714,10571	. Log. of 1,5036, *&c.* or Log. for the Time.
	2,30102,99957	. Log. of 200 . α
Sum	2,47817,10528	. Log. of 300,726, *&c.* ω.

If the ſame Sum of 200 *l.* were forborn only for half a Year, then half the Difference or Log. of the Rate ſhall be the Log. for the Time, which added to the Log. of 200, ſhall be the Log. of 205,91, *&c.* and not full 206 *l.* as is commonly reckoned at *Simple Intereſt.* *Example for half a Year.*

0,01265,29326 . Log. for $\frac{1}{2}$ Year.
2,30102,99957 . Log. of 200
2,31368,29283 . Log. of 205,91, *&c.*

If the Sum of the former Example be forborn 3 Years 5 Months and 4 Days: then firſt the Log. of the Rate is to be multiplied for the 3 Years by 3, and then by $\frac{5}{12}$, or the Log. of the Month by 5, and then by $\frac{4}{365}$, or the Log. of the Day by 4; and to theſe is to be added the Log. of 200, as before. *Example for 3 Years, 5 Months and 4 Days.*

0,02530,58653 . Log. of the Rate in 1 Year.
3 . Years.
0,07591,75959 . Log. for 3 Years.
0,01054,41105 . Log. for 5 Months.
0,00027,73244 . Log. for 4 Days.
0,08673,90308 . Log. for the Time.
2,30102,99957 . Log. of 200.
2,38776,90265 . Log. of 244,213, *&c.*

If the Rate be unuſual, or the Sum uneven.

If occaſion be to accompt the Intereſt at an unuſual Rate, or the Sum for which the Intereſt be reckoned be an uneven Sum; there is no Difference in the Method of proceeding from the former, but alike eaſy by Logarithms.

Example.

As if 56 *l.* 13 *s.* 6 *d.* be forborn 4 Years and 8 Months, and Compound Intereſt be reckoned for the forbearance after the Rate of 5 *l.* 10 *s. per Cent.* the Sum to be paid will be *l.* 72,7617, *&c.*

2,02325,24596 . Log. of 105,5. or 105 *l.* 10 *s.*
2,00000,00000 . Log. of 100.
0,02325,24596 . Log. of 1,055. the Rate.
0,09300,98384 . Log. for 4 Years.
0,01550,16397 . Log. for 8 Months, or $\frac{2}{3}$ of a Year.
0,10851,14781 . Log. for the Time.
1,75339,15288 . Log. of 56,675 . or 56 *l.* 13 *s.* 6 *d.*
1,86190,30069 . Log. of 72,7617, *&c.*

Example in a more unuſual Rate.

If the Rate be more unuſual, as at 5 *l.* 10 *s.* for 56 *l.* 13 *s.* 6 *d. per Annum*, and it be deſired to know what 98 *l.* 10 *s.* 6 *d.* would amount to after that Rate in 3 Years an half and 10 Days; the Operation will be in like manner, but otherwiſe than by the uſe of Logarithms extraordinary Difficult.

56 : 13 : 6 1,79361,57939 . Log. of *l.* 62,175, or 62 *l.* 03 *s.* 6 *d.*
5 : 10 : 0 1,75339,15288 . Log. of 56,675, or 56. 13. 6.
62 : 03 : 6 0,04022,42651 . Log. of the Rate for 1 Year.
0,12067,27953 . Log. for 3 Years.
0,02011,21325 . Log. for $\frac{1}{2}$ Year.
0,00110,20346 . Log. for 10 Days.
0,14188,69624 . Log. for the Time.
1,99354,64435 . Log. of 98,525 . or 98 *l.* 10 *s.* 6 *d.*
2,13543,34059 . Log. of 136,5945, *&c.*

Prop. 2. To find the ready Money.

Prop. 2. To know what Sum in ready Money ought to be paid for a Sum of Money to be received after a certain Time or Term, by *Compound Intereſt.*

Q. Of l. 300,726 &c. payable 7 Years hence, what now worth.

As to know what *l.* 300,726, *&c.* to be paid 7 Years hence is worth ready Money, at the Rate of 6 *l. per Cent. per Annum.* Or what Sum in 7 Years did amount to *l.* 300,726, *&c.* by Intereſt upon Intereſt, after the Rate of 6 on the 100.

Anſwer.

Anſw. 200 *l.* For this being the Converſe of the 1*ſt Prop.* here is ω . T . R . given to find α, that is, ω . 300,726, *&c.* T . 7 . and R . as 1 to 1,06, and ſo as before noted *Prop.* 1. only figurating the *Ratio* to T, may be found by the former Rules in *Progreſſion*, or the ſecond Theorem: For 1,50363025899136 . 1 :: 300,726051798272 . 200. But the work as aforeſaid, being moſt eaſy by Logarithms, the Rule thereof is thus:

By Progreſſion. By the ſecond Theorem. By Logarithms.

Subſtract the Log. for the Time out of the Log. of the Sum to be paid.

2,02530,58653 . Log. of 106.
2,00000,00000 . Log. of 100.
0,02530,58653 . Log. of 1,06, the Rate for 1 *l.* in 1 Year.
7 . Years T.
0,17714,10571 . Log. for the Time, or 1,50363, *&c.*
2,47817,10528 . Log. of 300,726, *&c.* ω.
2,30102,99957 . Log of 200 . α.

For other Rates or Times.

Thus whatever Rate or Time of forbearance it be, when the Log. for the Time is gotten, let it be taken from the Log. of the Sum to be paid, if Increaſe be accompted.

As

Example for 3 Years, 5 Months, 4 Days.

As *l.* 244,213, *&c.* will be payable at 3 Years 5 Months and 4 Days hence; yet it may be received preſently, if allowance be made for the Time or Term, after the Rate of *6 per Cent. per Annum:* what will be payable in ready Money?

Anſw. 200 *l.* For the Log. for the Time 0,08673,90308, taken from the Log. of 244,213, *&c.* will leave the Log. of 200.

0,02530,58653 .	Log. of the Rate in 1 Year.
3 .	Years.
0,07591,75959 .	Log. for 3 Years.
0,01054,41105 .	Log. for 5 Months.
0,00027,73244 .	Log. for 4 Days.
0,08673,90308 .	Log. for the Time.
2,38776,90265 .	Log. of 244,213, &c.
2,30102,99957 .	Log. of 200.

Example in Loſs.

But in all the Queſtions found in other Authors under this Propoſition, if there have been a Loſs or Decreaſe, the Log. for the Time ſhall be added to the Log. of the Sum of Money not loſt.

As 200 *l.* is remaining at the end of 7 Years, by Loſs or Diſcount, after the Rate of *6 per Cent. per Annum:* what was the Sum at the beginning of the 7 Years?

Anſw. l. 300,726, &c.

0,02530,58653 .	Log. of the Rate in 1 Year.
7 .	Years.
0,17714,10571 .	Log. for the Time.
2,30102,99957 .	Log. of 200.
2,47817,10528 .	Log. of 300,726, &c.

This ſecond Propoſition converſe to the firſt.

And becauſe this Propoſition is converſe to the Firſt, (as before noted) the Reſolution of this laſt Queſtion is done by the firſt Propoſition. And if under the firſt Propoſition ſuch a Queſtion as this had been ſet, *viz. l.* 300,726, &c. is to be received at the end of 7 Years; but loſing the Compound Intereſt at 6 *per Cent.* it will be paid preſently: what ſhall be paid for the ſame at preſent? The Work by this ſecond Propoſition, as above, will appear to be 200 *l.* So as Queſtions like this laſt are proper to this Propoſition, and thoſe like the other to the firſt Propoſition, however tranſpoſed in other Books.

What Queſtions are proper to both.

Prop. 3. *To find the Rate.*

Prop. 3. To know after what Rate any Sum hath increaſed or decreaſed by Compound Intereſt in a certain Time or Term.

Q. Of the Rate by which 200 l. in 7 Years increaſed to 300,725 &c. Anſwer.

As to know after what Rate 200 *l.* in 7 Years will increaſe to *l.* 300,726, &c. Or ſuppoſe 200 *l.* by Intereſt upon Intereſt in 7 Years did amount to *l.* 300,726, &c. what was the *Ratio* compared to 100?

By Progreſſion.

Anſw. 6 l. per Cent. per Annum. And ſo by the former Rules in *Progreſſion* may be found, here being given α . ω . T . to find R . that is α . 200 . ω. 300,726, &c. and T . 7 . For 200 : 1 :: 300,726, &c. 1,503630, &c. differing from the way there only in this, that here the Number found by the *Analogy*, is the *Ratio* figurate to T, thereto T—1, as before noted in the two precedent Propoſitions.

By Logarithms.

By Logarithms the Work is thus. Divide the Difference of the Logarithms of the two Sums propounded, by the Term or Time given; to the Quotient add the Log. of the intended or ſuppoſed Antecedent of the Rate, and the Total ſhall be the Log. of the Antecedent and Conſequent: Then deducting the abſolute Number of the Antecedent, from the abſolute Number of the other, the Remain ſhall be the Rate deſired.

2,47817,10528 .	Log. of *l.* 300,726, &c. ω.
2,30102,99957 .	Log. of 200. α.
7) 0,17714,10571 .	Difference, or Log. for the Time.
0,02530,58653 .	Quotient, or Log. of the Rate.
2,00000,00000 .	Log. of 100. Antecedent of the Rate.
2,02530,58653 .	Log. of 106, Antecedent and Conſequent.
	6. on 100 the Rate.

If the Time be an Half-year, Quarter, &c.

If the Term or Time given, beſides the whole Number of Years, be ſome odd Parts of a Year, as half a Year, a Quarter, Months, Days, or ſuch-like, then reduce them all into one Denomination; and as a Fraction, with 365 (the Days in a Year) *&c.* abbreviate them to their leaſt Terms; and thereby, or rather by the Decimal thereof, divide the Log. of the Difference as above.

Example.

As if it were deſired to know after what Rate 200 *l.* in 3 Years, 5 Months, and 4 Days, will increaſe to *l.*244,213, *&c.* Becauſe the Time reduced into Days will not be abbreviated, but both Terms thereof large, the Decimal is rather to be choſen, which is 3,42762556; by which the Difference divided maketh the Quotient, and ſo the Rate as above.

2,38776,90265 . Log. of *l.*244,213, &c.
2,30102,99957 . Log. of 200.
0,08673,90308 . Difference.
3,42762556) 0,02530,58653 . Quotient.
2,00000,00000 . Log. of 100, Antecedent.
2,02530,58653 . Log. of 106, Antecedent and Conſequent.
6. Rate.

Prop.4. To find the Time or Term.

Prop. 4. To know in what Time or Term a Sum of Money, will increaſe or decreaſe by Compound Intereſt, to a Sum propounded according to a certain Rate.

Q. *Of the Time in which 200 l. increaſed to 300,726, &c.*

As to know in what Time 200 *l.* after the Rate of 6 *per Cent. per Annum*, by Intereſt upon Intereſt, will increaſe to *l.* 300,726, *&c.* or when will *l.* 300,726, *&c.* be payable for 200 *l.* lent, after the Rate of 6 *l.* on the 100 for a Year?

Anſwer.

By Progreſſion.

Anſw. At the end of 7 Years after the Loan, as may be found by the former Rules in *Progreſſion*, with a little Difference, for here is given α . ω . R. to find T. that is, α . 200 . ω . 300,726, *&c.* and R . 1,06. So 200 multiplied by 1,06 till 300,726, *&c.* be produced, the Number of Multiplications 7, is the Number deſired; For 1 needs not be added as in *Progreſſion*. The Work is ſhort by Logarithms, thus.

By Logarithms.

Divide the Difference of the Logarithms of both Sums given by the Log. of the *Rate*; and the Quotient ſhall be the Number deſired.

2,47817,10528 . Log. of *l.* 300,726, *&c.* ω
2,30102,99957 . Log. of 200 . α
0,17714,10571 . Difference, or Log. for the Time.
Log. of the Rate 0,02530,58653 (7 Years T.

If any Remain be on Diviſion.

If after the Diviſion any Thing remain, divide that Remain by the Log. of the Rate for a Month: And if upon that Diviſion any thing remain, divide it by the Log. of the Rate for a Day.

Example.

As 200 *l.* hath increaſed to *l.*244,213, *&c.* and I deſire to know in what Time it hath ſo increaſed by Compound Intereſt, after the Rate of 6 *per Centum per Annum.*

Anſwer.

Anſw. In 3 Years, 5 Months, and 4 Days: For after the Firſt Diviſion, which bringeth 3 Years in the Quotient, there is left remaining 1082,14349; this divided by 210,88221, the Log. of the Rate for a Month, bringeth 5 in the Quotient, and leaveth remaining 27,73244; which divided by 6,93311, the Log. of the Rate for a Day, giveth 4 in the Quotient.

2,38776,90265 . Log. of *l.* 244,213, &c.
2,30102,99957 . Log. of 200.
0,08673,90308 . Difference.

(1082 14349 (27 73244
~~8673,90308~~ (3 Years . ~~1082,14349~~ (5 Months . ~~27,73244~~ (4 Days.
~~2530,58653~~ ~~210,88221~~ ~~6,93311~~

Touching Annuities diſtinctly.

Annuities reſpect 5 Things, by any 3 whereof the other may be found as before in *Progreſſion.*

Of Annuities, what to be noted.

1. The Sum, or Arrerages of the Rent of a Leaſe, or Annuity, or Penſion forborn a certain Term or Time, to be paid with Intereſt upon Intereſt after a certain Rate. This anſwereth to Z, or the fifth of the 5 Principals in a *Geometrical Progreſſion*, when the Annuity is propoſed as the firſt; otherwiſe it anſwereth to ω or the ſecond Principal, when the ready Money or Price is given for the Firſt, and agreeth with the third of the 6 Theorems in this Chapter.

1. The Arrerages. *What it agreeth and anſwereth to.*

2. The Sum or Price, which in ready Money will buy or purchaſe an Annuity for a certain Time or Term, after a certain Rate by Intereſt upon Intereſt; this anſwereth to α, or the firſt of the 5 Principals in a *Geometrical Progreſſion*, where the Arrerages are propoſed as the Second, and agreeth with the 5*th* of the ſix Theorems abovementioned.

2. The Price. What it agreeth and anſwereth to.

3. The Annuity or Penſion that any Sum of ready Money will buy or purchaſe for a certain Time, after a certain Rate with Intereſt upon Intereſt. This anſwereth to α, or the firſt of the 5 Principals in a *Geometrical Progreſſion*, where the Arrerages are propoſed as the Fifth, and agreeth both with the 4*th* and laſt of the aforeſaid 6 Theorems.

3. The Annuity or Penſion. What it agreeth and anſwereth to.

4. The Time or Term any Annuity or Penſion, after a certain Rate by Compound Intereſt, may be detained to amount to a Sum propounded; This marked by T or N, anſwereth to the Third of the 5 Principals of a *Geometrical Progreſſion.*

4. The Time or Term. What it anſwereth to.

5. The Rate by which any Annuity or Penſion by Compound Intereſt did increaſe to a Sum propounded; this anſwereth to R. or the fourth of the 5 Principals of a *Geometrical Progreſſion.*

5. The Rate. What it anſwereth to.

And according to the different State of the Queſtion, under one or other of theſe Claſſes, is Reſolution attainable by ſome or other of the Rules proper thereto: For as before in *Progreſſion*, one of theſe 5 being ſought, the Queſtion may be varied 4 ſeveral Ways; and where the *Data* are all of thoſe 5 Principals, the Rules there may be uſed with what alteration is neceſſary for Fractions and Decimals: otherwiſe becauſe the Second of theſe is none of the former, ſome new Rules will be neceſſary, wherefore all the Varieties ſhall be exampled in.

Propoſition 1. To know what Sum of Money ought to be paid for the Arrerages of Rent reſerved upon a Leaſe, or for an Annuity or Penſion forborn a certain Time or Term, after a certain Rate by Compound Intereſt.

Prop. 1. *To find the Arrerages.*

The *Data* for the Reſolution of Queſtions under this Claſſis, will be either,

Data *under the firſt Prop.*

2 . 3 . 4 . The Price, Annuity, and Term.
2 . 3 . 5 . The Price, Annuity, and Rate.
2 . 4 . 5 . The Price, Term, and Rate.
or 3 . 4 . 5 . The Annuity, Term, and Rate.

The laſt being eaſier than the firſt, ſhall be firſt in Example, and the reſt in their retrograde Order.

Of theſe 4, the laſt eaſieſt.

Variety 4. If an Annuity of 5 *l. per Annum* be detained 7 Years: what will the Amount thereof be by Intereſt upon Intereſt, after the Rate of 6 *l. per Cent. per Annum?* or what are the Arrerages of an Annuity of 5 *l.* a Year with the Compound Intereſt, after the Rate of 6 on the 100 for a Year forborn 7 Years?

Variety 4. Q. *Of* 5 *l.* per Annum, *forborn 7 Years, what it comes to.*

Anſw. l. 41,969188249280: for ſeeing here is given the Annuity, Term and Rate, or α . T . R . 3 of the 5 Principals of a *Geometrical Progreſſion*, to find Z. that is, α . 5 . T . 7 . and R . 6 on the 100, which is as 1 to 1,06 by the Rules in the third Propoſition for finding the 5*th* Principal there, 41,969, &c. is found thus:

Anſwer. *By Progreſſion.*

5

	5 α 1,06 R	1	
	30 50		
1	5,30 1,06	2	
	3180 5300		
2	5,6180 1,06	3	
	337080 561800		
3	5,955080 1,06	4	
	35730480 59550800		
4	6,31238480 1,06	5	
	3787430880 6312384800		
5	6,6911278880 1,06	6	
	401467673280 669112788800		
6	7,092595561280 1,06	7	T
	42555573367680 70925955612800		
7	7,51815129495680	8	

Collection of the Terms.

5,
5,30
5,6180
5,955080
6,31238480
6,6911278880
7,092595561280
Z 41,969188249280

,06) 5 α
2,51815129495680 (41,969188249280.

Otherwise. Otherwiſe by figurating the *Ratio* as in *Progreſſion* alſo mentioned, but different herein, that here an Unit the Antecedent of the *Ratio* is to be added to the Sum, as,

1,
1,06
1,1236
1,191016
1,26247696
1,3382255776
1,418519112256
1,50363025899136

Sum of the firſt 7 Terms with 1, 8,393837649856
5 α Annuity.
41,969188249280 Z Arrerages.

And the third Theorem thus;

By the third Theorem. 0,06 . 0,50363025899136 :: 1 . 8,393837649856
5 α
41,969188249280 Z

By Logarithms. But as before in Intereſt, ſo in Annuities, the Work is eaſy and ſpeedy in Logarithms, thus;

To the Log. for the Time, add the Log. of ſo much Principal Money as at Simple Intereſt in 1 Year will raiſe ſuch an Annuity, and the Sum will be the Log. of that Principal Money, and the Arrerages; then ſubſtract the Principal out thereof, and the Reſidue is the Sum deſired.

The Principal Money that at Simple Intereſt, after any Rate, will raiſe the Annuity propounded, is gotten either by the *Golden Rule direct* without, or rather with Logarithms, as in the former Inſtance. If 6 *l.* require 100 *l.* Principal to raiſe the ſame, then ſhall 5 *l.* require 83 *l.* 6 *s.* 8 *d.*

As 6 . 100 :: 5 . 83 : 6 : 8.
Or, 2,00000,00000 . Log. of 100
0,69897,00043 . Log. of 5
2,69897,00043 . Log. of 500
0,77815,12504 . Log. of 6
1,92081,87539 . Log. of 83,33333, &c.

Then the reſt of the Reſolution is thus:

2,02530,58653 . Log. of 106
2,00000,00000 . Log. of 100
0,02530,58653 . Log. of 1,06 . Rate . R or ϐ
7 . Years . T
0,17714,10571 . Log. of 1,50363, *&c.* Time or ϐω
1,92081,87539 . Log. of 83,33333, *&c.* Principal in 1 Year, to raiſe 5 *l.* at Simple Intereſt.
2,09795,98110 . Log. of 125,30251, *&c.* Principal & Arrerages.
41,96918, *&c.* Z . Arrerages.

Or if the Log. of the Rate lacking 1, be taken from the Log. for the Time lacking 1, and to the Remain the Log. of the Annuity be added, the Sum will be the Log. of the Arrerages deſired. *Otherwiſe.*

—1,70211,18155 . Log. of 0,50363, *&c.* Time—1. or ϐω—1
—2,77815,12504 . Log. of 0,06 Rate—1. or ϐ—1
0,92396,05651 . Log. of 8,39383, *&c.*
0,69897,00043 . Log. of 5 α . Annuity.
1,62293,05694 . Log. of 41,96918, *&c.* Z . Arrerages.

If the Rents or Annuities be payable Half-yearly or Quarterly, as moſt commonly they are; then after the Principal Money that will raiſe ſuch a Paiment is gotten, the reſt of the Work differs nothing from the former. *If the Paiments by Half-yearly or Quarterly.*

To get this Principal Money, let the Log. of the Rate be halfed or quartered, according as the Rent or Annuity becomes payable; and for this Half or Quarter, get the *Geodetical* or Decimal as of another Log. Or commonly thus, enter the Table of Logarithms next after the Antecedent of the *Ratio* (neglecting the Index) with the halfed or quartered Log. till the next leſſer Log. to that propounded be found; and there by the Difference of the next Leſſer to the next Greater, and the next Leſſer to the propounded Log. the *Geodeticals* or Decimals ſought may be found, as in the Chapter of the *Reduction of Logarithms* was taught. And to the left Hand of this *Geodetical* or Decimal thus found out, is to be prefixed the abſolute Number of the leſſer Log. in the Table exceeding the Antecedent of the *Ratio* lacking the ſame Antecedent: And this ſhall be the firſt Number of the *Rule of Three*, to find the Principal Money required, to which the Antecedent ſhall be the Second, and the Paiment propounded the Third: Or if Operation be made by Logarithms, the Log. of that *Geodetical* or Decimal, with the abſolute Number prefixed as aforeſaid, ſhall be the Log. to be ſubſtracted from the Sum of the other two Logarithms. *How to get the Principal Mony.*

Example. The Rent of a Leaſe of 5 *l. per Annum*, payable Half-yearly, is in Arrear 7 Years: what do the Arrerages thereof amount to, after the Rate of 6 *l. per Cent. per Annum* by Compound Intereſt? *Q. Of 5 l. per Ann. payable Half-yearly, what the Arrears in 7 Years.*

Anſw. *l.* 42,5873, *&c.* For here the Log. of the Rate as before being 0,02530,58653 (becauſe the Paiments are Half-yearly) is to be halfed; and with this half, which is 0,01265,29326, if the Table of Logarithms be entred *Anſwer.*

next after 100 (the Antecedent of the *Ratio)* it will be found to fall between the Logarithms of 102 and 103, and to be more than the Log. of 102, the Index neglected 0,00405,27608: If therefore Ciphers be adjoined as many as ſhall be neceſſary, and Diviſion made by 423,70529 (the Difference between the Log. of 102 and 103) the Decimal 9565, *&c.* will be gotten to be placed on the right Hand of 2, the Abſolute Number exceeding the Antecedent of the *Ratio*; and having gotten the Decimal, the Shillings, Pence, *&c.* therein, are ſoon found as before in *Reduction of Decimals.* If otherwiſe, the Learner be more expert at Common Fractions, then the Difference between the leſſer Log. found as aforeſaid, and the Log. propounded, multiplied by 20, and divided by the Difference between the leſſer and greater Logarithms found in the Table, giveth the Quotient in Shillings; the Remain of this Diviſion, if any, multiplied by 12, and divided by the former Diviſor, giveth the Quotient in Pence, *&c.*

```
   2,02530,58653 . Log. of 106
   2,00000,00000 . Log. of 100
2)0,02530,58653 . Log. of 1,06.          Difference.
  0,01265,29326 . Log. of 1,029565, &c.  Half.
   2,00860,01718 . Log. of 102
   2,01283,72247 . Log. of 103
   0,00405,27608 . Difference of the Half and Log. of 102.
   0,00423,70529 . Difference between the Log. of 102 and 103.
```

```
        (1970                              2 prefixed is l. 2,9565,&c.
      213822(1                                               20
     275605 47(1                                    s.  19 | 1300
    23 9413 1 956(5                                        |   12
  405,27608,0000,&c.(,9565,&c.                             | 2600
  423,7052 9999                                         1  | 300
   423,705 222                                      d.  1  | 5600
    423,70 55                                              |    4
     423,70                                         q.  2  | 2400
```

Or for the *Geodetical* thus:

```
                   (551210
405,27608        3868 4687(9              5512109       (23774779
       20        8105,52160(19s.               12       661 45308(1 d.
8105,52160       423,705299              661145308      423,70529
                  423,7052
```

This Principal Money found (omitting the Remain of the firſt Diviſion as inconſiderable) the reſt of the Reſolution to the laſt foregoing Queſtion, is as followeth.

```
      As 2,9565 . 100 :: 2,5 . 84,5594, &c.  by Decimals.
    Thus 2,00000,00000 . Log. of 100.        by Logarithms.
         0,39794,00087 . Log. of 2,5.
         2,39794,00087 . Log. of 250.
         0,47077,78833 . Log. of 2,9565 . or 2 l. 19 s. 1 1/2 d.
         1,92716,21254 . Log. of 84,5594, &c.
         0,17714,10571 . Log. for the Time 14 half Years, or 7 Years.
         2,10430,31825 . Log. of 127,1461, &c.
                                     42,586 &c. Z.

Or thus —1,70211,18155 . Log. of 0,50363, &c.  Time—1 . or βω—1.
        —2,47077,78833 . Log. of 0,029565, &c. Rate—1 . or β—1.
         1,23133,39322 . Log. of 17,03467, &c.
         0,39794,00087 . Log. of 2,5            α .
         1,62927,39409 . Log. of 42,586, &c.    Z .
```

As well in Annuities as before in Intereſt, the Log. of the Rate if unuſual, and the Log. for the Time if odd Months, Days, *&c.* once gotten, the reſt of the Work is in like manner, ſo that Examples thereof need not be added here.

Unuſual Rates and uneven Times, as in Intereſt.

Variety 3. If an Annuity to endure 7 Years be ſold for *l.* 27,9119071981 3863, *&c.* ready Money: what will the Arrerages thereof being forborn all the Term amount to, at the Rate of *6 l. per Cent. per Annum*, by compound Intereſt?

Variety 3.
Q. *Of an Annuity ſold, what the Arrears.*

Anſw. l. 41,969188249280. Here being given the Price, the Term and Rate, becauſe the Price or ready Money propounded, is by the firſt Propoſition of Intereſt as α and Z, the Arrerages ſought as ω to α, Reſolution may be had by the firſt Theorem, ſeeing the Amountment of a Sum of Money, purchaſing an Annuity in any Number of Years by Intereſt upon Intereſt, ſhall equal the Arrerages of that Annuity ſo long detained: And becauſe this variety gives α. T. R. to find ω. by the Rules in the firſt Propoſition, for finding the ſecond Principal of a *Geometrical Progreſſion*, the ſaid 41,969, *&c.* may thereby be obtained, only differing in figurating the Rate as before noted in Intereſt, but briefly by Logarithms thus.

Anſwer.
By the firſt Theorem.
By Progreſſion.
By Logarithms.

Add the Log. for the Time, to the Log. of the Price or ready Money, and the Sum ſhall be the Log. of the Arrerages deſired.

1,44578,95123 . Log. of 27,9119, *&c.* Price.
0,17714,10571 . Log. of 1,5036, *&c.* Time as before.
1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.

Variety 2. The Arrerages of an Annuity of 5 *l. per Annum*, after the Rate of *6 l. per Cent. per Annum* by Compound Intereſt, are valued to be worth *l.* 27,9119, *&c.* in ready Money: how much do thoſe Arrerages amount to?

Variety 2.
Q. *Of Arrerages of* 5 *l.* per Ann. *valued ready Money.*

Anſw. l. 41,9691, *&c.* as before: here are given the Price, the Annuity, and the Rate; ſo the Work without Logarithms is thus.

Anſwer.

Get (as before under the fourth *Variety*) the Principal Money, that at Simple Intereſt after the Rate propounded, will raiſe ſuch an Annuity; from this Number take the given Price, and by the Remainder divide the Number ſo gotten as aforeſaid, then multiply this Quotient by the Price, and the Product is the deſired Arrerages.

By Decimals.

But to avoid thoſe long and tedious Multiplications and Diviſions that are in Decimals, Logarithms are uſed thus.

After the Principal Money, that at Simple Intereſt will raiſe the Yearly Annuity propounded, is gotten, taken there from the given Price, and the Log. of the Remainder out of the Log. of that Principal, then to the remaining Log. add the Log. of the Price; and the Sum is the Log. of the Arrerages.

By Logarithms.

1,92081,87539 . Log. of *l.* 83,33333, *&c.* Principal, as before.
27,91190, *&c.* Price.
1,74367,76968 . Log. of 55,4214, *&c.* Remainer.
0,17714,10571 . Log. of 1,5036, *&c.* Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.
1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.

Variety 1. An Annuity of 5 *l. per Annum* for 7 Years, was purchaſed for *l.* 27,9119, *&c.* ready Money: how much was the Arrerages reckoned to amount to by Compound Intereſt, if all had been let alone till the end of the Term?

Variety 1.
Q. *Of* 5 *l.* per Ann. *for 7 Years bought, what the Arrears.*

Anſw. l. 41,9691, *&c.* as before: The Rate being a principal Key in this Work, and none of the *Data* in this Variety, but the Price, Annuity, and Term; the Log. for the Time (being as aforeſaid, the Log. of the Rate multiplied by the Term) cannot well be had. And becauſe the Price is to be multiplied by the Rate figurate according to the Term, or their Logarithms added together; it is a moſt difficult Propoſition for a young Scholar to reſolve ſuch a Queſtion by the former Rules. But it is to be conſidered, ſeeing 5 is the firſt Term of one *Progreſſion*, and 27,9119, *&c.* the firſt Term of another, which ſhall have the *Ratio* and Number of Terms alike to the firſt *Progreſſion* that beginneth with 5, and the Number of Terms in both muſt be 7, and the Arrerages ſought muſt be the Sum of all the Terms beginning with 5, and the laſt Term only of the other; by neceſſary Conſequence 27,9119, *&c.* muſt be multiplied by the ſeventh Power of ſome Root, which ſhall make the ſeventh Term of that Progreſſion equal to the Total of all the 7 Terms of the Progreſſion that beginneth with 5: And becauſe in this Example no other Number can do this but 1,503630258991 36, which is

Anſwer.
A difficult Propoſition.
By Progreſſion.

the

By Logarithms.

the *7th* Power of the Root 1,06, therefore 27,91190, *&c.* multiplied thereby, or their Logarithms added together, the Product of their Multiplications anſwering to the Sum of their Logarithms will be 41,9691882 4928, which Number ſhall be Z, if 5 be α, and ſhall be ω if 27,911907198, *&c.* be α the Terms of both 7, and the *Ratio* 1,06. This may a little illuſtrate the Matter, but the beſt way is to proceed as above in the ſecond *Variety.*

1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. of 1,50363, *&c.* Time.
1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.

Prop. 2. *To find the Price.*

Propoſition 2. To know what an Annuity or Penſion for a certain Time or Term, after a certain Rate by Compound Intereſt is worth, to buy or ſell in ready Money.

Data *under the ſecond Prop.*

The *Data* for the Reſolution of Queſtions under this Claſſis will be either,

1 . 3 . 4 . The Arrerages, Annuity and Term.
1 . 3 . 5 . The Arrerages, Annuity and Rate.
1 . 4 . 5 . The Arrerages, Term and Rate.
Or 3 . 4 . 5 . The Annuity, Term and Rate.

Theſe ſet as the former.

The Varieties here are ſet as the former in their retrograde Order.

Variety 4. Q. *Of* 5 *l.* per An. *for 7 Years, what worth. By the 5th Theorem.*

Variety 4. What is an Annuity of 5 *l.* a Year for 7 Years, worth in ready Money at 6 *l. per Cent. per Annum* by Compound Intereſt?

Anſw. l. 27,91190, *&c.* For the Annuity, Term and Rate being given, and ſeeing in this Example βω is 1,5036302589 9136 and β—1 is 0,06, according to the *5th* Theorem it ſhall be, That,

β—1 in βω . βω—1 :: *Ann.* . *Price.*
0,0902178155394816 . 0,5036302589 9136 :: 5 . 27,91190, *&c.*

By Progreſſion.

And becauſe by the *Data* α . 5 . T . 7 . and R . 1,06 . Z may be found according to the third Propoſition for finding the *5th* Principal of a *Geometrical Progreſſion,* and the *4th* Variety of the firſt Propoſition here; then Z which will be 41,9691, *&c.* being divided by R. figurate to T. which is 1,50363, *&c.* giveth in the Quotient 27,91190, *&c.* as before.

By Logarithms.

But the work is moſt expeditious by Logarithms thus: Out of the Log. of the Principal Money, which at Simple Intereſt will in 1 Year raiſe ſuch an Annuity after the Rate propounded, ſubſtract the Log. for the Time, and the abſolute Number of the remaining Log. out of that Principal.

1,92081,87539 . Log. of *l.* 83,33333, *&c.* as before *Prop.* 1.
0,17714,10571 . Log. for the Time there alſo.
1,74367,76968 . Log. of 55,4214, *&c.*
27,9119, *&c.* Price.

Otherwiſe.

Or if from the Log. of the Annuity, added to the Log. for the Time lacking 1, be taken the Log. of the Rate lacking 1, added to the Log. for the Time; the Remain ſhall be the Log. of the deſired Number, agreeable to the Work of the *5th* Theorem above-mentioned.

0,69897,00043 . Log. of 5 . Annuity α.
—1,70211,18155 . Log. of 0,50363, *&c.* Time—1 . or βω—1.
0,40108,18198 . Sum . βω—1 in α.
—2,77815,12504 . Log. of 0,06 . Rate—1 . or β—1.
0,17714,10571 . Log. of 1,50363, *&c.* Time or βω.
—2,95529,23075 . Sum β—1 in βω.
1,44578,95123 . Log. of 27,91190, *&c.* Price.

Otherwiſe.

Alſo if the Arrerages be firſt found, the Log. for the Time taken out thereof, ſhall leave the Log. of the Price remaining.

1,62293,05694 . Log. of 41,9691, *&c.* Arrerages, as before *Prop.* 1.
0,17714,10571 . Log. for the Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

If the Paiment be Half-yearly. **Example.**

If this Annuity had been payable half Yearly, then the Principal Money, as appeareth under the former Propoſition, being *l.* 84,5594, *&c.* the Price will be augmented to *l.* 28,32258, *&c.* as by the firſt work of Logarithms (moſt uſed becauſe the ſhorteſt) here appeareth.

1,

1,92716,21254 . Log. of *l.* 84,55944, *&c.* Principal.
0,17714,10571 . Log. for the Time.
1,75002,10683 . Log. of 56,23686, *&c.*
28,32258, *&c.* Price.

And if the Paiment be Quarterly, when the Principal Money is had that will raiſe ſuch a Quarterly Paiment at Simple Intereſt, the reſt of the Work is alike.

If Quarterly.

Variety 3. The Arrears of an Annuity detained 7 Years after the Rate of 6 *l.* *per. Cent. per Annum* by Compound Intereſt, do amount to 41,969188, *&c.* what is it worth in ready Money?

Variety 3.
Q. *Of Arrears in 7 Years, what worth.*

Anſw. l. 27,91190, *&c.* The *Data* here being the Arrerages, Term and Rate; and becauſe the Arrerages or Z may be accompted ω, or the laſt Term of a *Geometrical Progreſſion*, when the Price which is here the Number ſought ſhall be α or the firſt Term; and ſo by the Rules in the firſt Propoſition, for finding the firſt Principal of a *Geometrical Progreſſion*, the *Ratio* being figurate to T as aforeſaid, may 27,91190, *&c.* be found by ω . 41,969, *&c.* T . 7 . and R . 1,06 . the *Data*: And this is alſo agreeable to the ſecond Theorem.

Anſwer.
By Progreſſion.
By the ſecond Theorem.

Wherefore by Logarithms, if the Log. for the Time be taken from the Log. of the Arrerages; the Remain ſhall be the Log. of the ready Money or Price deſired.

By Logarithms.

1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
0,17714,10571 . Log. for the Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

Variety 2. If the Arrerages of an Annuity of 5 *l. per. Annum*, after the Rate of 6 *l. per. Cent. per Annum*, come to *l.* 41,96918, *&c.* what ready Money will purchaſe the ſame?

Variety 2.
Q. *Of Arrears of* 5 *l.* per Ann. *what worth.*

Anſw. l. 27,9119, *&c.* The *Data* here are the Arrerages, Annuity and Rate; ſo the Work without Logarithms is thus:

Anſwer.

Get (as before) the Principal Money, that at Simple Intereſt after the Rate propounded will raiſe ſuch an Annuity, to this add the Arrerages, and divide the Sum by the Principal, then by the Quotient divide the Arrerages, and this laſt Quotient is the Price deſired.

By Decimals.

Accordingly the Rule of working by Logarithms is framed for avoiding the tedious Multiplications and Diviſions thus:

By Logarithms.

After the Principal Money, that at Simple Intereſt after the Rate propounded will raiſe the Annuity, is found and added to the Arrerages, from the Log. of the Sum take the Log. of that Principal Money, and take the remaining Log. from the Log. of the Arrerages.

1,92081,87539 . Log. of *l.* 83,3333, *&c.* Principal.
41,9691, *&c.* Arrerages.
2,09795,98110 . Log. of 125,3024, *&c.* Sum.
0,17714,10571 . Log. for the Time.
1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
1,44578,95123 . Log. of 27,9119, *&c.* Price.

Variety 1. How much ready Money will purchaſe an Annuity of 5 Pounds *per Annum* for ſeven Years, whoſe Arrerages forborn ſo long will amount to *l.* 41,96918, *&c.*

Variety 1.
Q. *Of Arrears of* 5 *l.* per Ann. *7 Years, what worth.*

Anſw. l. 27,91190, *&c.* Here are given the Arrerages, Annuity and Term: And becauſe the Arrerages are as Z in a *Geometrical Progreſſion*, when the Annuity is α, by α . 5 . T . 7 . & Z . 41,9691, *&c.* the Rate may be found according to the third Rule in *Progreſſion Geometrical*, for finding the 4*th* Principal; but with ſome Variation, becauſe of the Decimal in the *Data* here, (the former being fitted for Integers): For by reaſon of the Retrogreſſion of Figurate Decimals towards the right Hand, contrary to Integers which increaſe to the Left, it is very difficult to find out the higheſt Figurate Decimal (according to that Rule) to be taken out of the Quotient of Z divided by α, ſeeing greater Numbers muſt be left than thoſe taken away. Yet inaſmuch as the Quotient of Z divided by α, both here and there, is the Sum of a *Progreſſion*, whoſe firſt Term or α is 1, and ſo conſequently is 1, and the Sum of all the Terms of the *Ratio*, multiplied into it ſelf figurately according to the Number of Terms lacking 1, the Contrariety lies

Anſwer.
By Progreſſion.
A difficult Propoſition.

only in this, That in Integers from the Quotient muſt be taken the higheſt Power contained therein, whoſe Index was T—1: in Decimals to the Quotient muſt be added ſuch a Number as will make the Quotient a Power according to T, lacking 1, (in the left-hand Place); and inſtead there of *Addition*, let this Number found in the Quotient be multiplied by one or more of the Digits, till the Power deſired be produced; and the Number ſo multiplying, ſhall be the Decimal Root to be added to the Antecedent of the *Ratio*, and the Antecedent ſhall always be the Root of 1, and ſo many Ciphers as there be Decimals.

Wherefore dividing 41,969188249280 (or Z) by 5 (that is α) the Quotient is 8,393837649856: So ſhall this Number be the Sum of a Progreſſion beginning at 1, and the Arrears of 1 *l.* Annuity let run 7 Years, and contain therein a Zenzicube Number, whoſe Index is 6, (or T—1) but muſt be exalted to T, (that is, to a Power whoſe Index is 7) lacking only the Left-hand Unit. And becauſe the Decimals in the Quotient are 12, and the Number of Terms of that Power but 6, it is eaſy to diſcern the Root was Seconds, and that the 7*th* Power in Decimal Seconds with 1 Integer, cannot have leſs than 15 Places. Now if 8,393837649856 be multiplied by 2″. 3″. 4″. 5″. or any other Digit except 6″, and 1 prefixed to the left Hand, the Product will be no qqc, or Power of the 7*th* Quantity, but multiplied by 6″, produceth ,503630258899136, which with 1 prefixed, and no other, is the Number ſought, that is, 1,503630258899136, the ſecond Surſolid, or 7*th* Power of the Root 1,06, to which the Antecedent is 1,00, the √qqc of 1,00000000000000, or which is all one therewith, the √cc of 1,000000000000.

By Logarithms. Thus then having found the *Ratio* thereby, with the other *Data*, the Price may be found by any of the other 3 Varieties for this ſecond Propoſition. And ſeeing this Figural Power found out, is always the Rate multiplied into it ſelf according to T, if Z be divided thereby, the Quotient will be the Price: And accordingly the Log. thereof taken from the Log. of Z, will leave remaining the Log. of the ready Money, or Price deſired. So as upon the whole Matter it is moſt eaſy to proceed, as in the ſecond Variety above.

1,62293,05694 . Log. of *l.* 41,969188, *&c.* Z.
0,17714,10571 . Log. of 1,503630, *&c.* Time.
1,44578,95123 . Log. of 27,911907, *&c.* Price.

Prop. 3. *To find the Annuity.* *Propoſition* 3. To know what Annuity or Penſion any Sum of ready Money will buy, for a certain Time or Term, after a certain Rate, by Compound Intereſt?

Data under the third Prop. The *Data* for the Reſolution of Queſtions under this Claſſis, will be either,

1 . 2 . 4 . The Arrerages, Price and Term.
1 . 2 . 5 . The Arrerages, Price and Rate.
1 . 4 . 5 . The Arrerages, Term and Rate.
Or 2 . 4 . 5 . The Price, Term and Rate.

How ſet. The Varieties of this Claſſis being alike eaſy, are ſet in their proper Order.

Variety 1. *Q. Of Arrears for 7 Years, and Price, what the Annuity.* *Variety* 1. An Annuity run in Arrears for 7 Years, will come to *l.* 41,96918, *&c.* and may be bought for *l.* 27,91190, *&c.* what was the Annuity?

Anſwer. *Anſw.* 5 *l.* a Year: For ſeeing the Arrerages of every Annuity, divided by the Price or ready Money, giveth in the Quotient the *Ratio* figurate according to the Number of Terms; and here being given the Arrerages, Price and Term, from the Quotient may be extracted the Septupled Root: And then by the *Analogy*, according to the 4*th* or 6*th* Theorems, 5 may be found. *By the 4th and 6th Theorems.* For the Quotient of 41,96918, *&c.* divided by 27,91190, *&c.* being 1,503630258899136, and the Septupled Root thereof 1,06, it ſhall by the

4th Theor. ßω—1 ß—1 *Arrerages.* *Annuity.*
0,503630258899136 . 0,06 :: 41,969188249280 . 5 .

6th Theor. ßω—1 ß—1 in ßω *Price.* *Annuity.*
0,503630258899136 . 0,090217815539481 6 :: 27,9119, *&c* . 5 .

By Logarithms. And accordingly by Logarithms, take the Log. of the Price from the Log. of the Arrerages, the Remainder ſhall be the Log. for the Time; that is, the Log. of the Rate multiplied by the Term, which divided by the Term, ſhall give in the Quotient the Log. of the Rate. And then the Log. of the Rate lacking 1, added to the Log. of the Arrears; and from thence the Log. for the Time lacking 1 ſubſtracted, leaves remaining the Log. of the Annuity: Or elſe the Log. of the Rate lacking 1, and the Log. for the Time added to the Log. of the ready Money; and from thence the Log. for the Time lacking 1 ſubſtracted, ſhall leave remaining the Log. of the Annuity.

1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. for the Time 1,50363, *&c.*
7) 0,02530,58653 . Log. of the Rate 1,06.

The former way —1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
—2,77815,12504 . Log. of 0,06 R—1.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,50363, *&c.* Time—1.
0,69897,00043 . Log. of 5 Annuity.

The latter way —2,77815,12504 . Log. of 0,06 R—1.
0,17714,10571 . Log. of 1,50363, *&c.* Time.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,50363, *&c.* Time—1.
0,69897,00043 . Log. of 5. Annuity.

Variety 2. The Arrears of an Annuity to be forborn, at the Rate of 6 *l. per Cent. per Annum*, by Compound Intereſt, amounting to *l.* 41,969188, *&c.* are ſold for *l.* 27,91190, *&c.* what was the Annuity? *Variety 2. Q. Of Arrears ſold, what the Annuity.*

Anſw. 5 *l.* a Year as before: Here being given the Arrerages, Price and Rate, the Arrerages 41,9691, *&c.* divided by 27,9119, *&c.* as in the laſt Variety, giveth the Power of the Root figurate according to T, as aforeſaid in the Quotient, that is 1,50363, *&c.* and then Reſolution may be had, as in the firſt Variety of this third Propoſition, according to the Theorems, or by Logarithms. *Anſwer. By the Theorems and Logarithms.*

1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
1,44578,95123 . Log. of 27,9119, *&c.* Price.
0,17714,10571 . Log. of 1,5036, *&c.* Time.

The former way 1,62293,05694 . Log. of 41,9691, *&c.* Arrerages.
—2,77815,12504 . Log. of 0,06 R—1.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,5036, *&c.* Time—1.
0,69897,00043 . Log. of 5. Annuity.

The latter way —2,77815,12504 . Log. of 0,06. R—1.
0,17714,10571 . Log. of 1,5036, *&c.* Time.
1,44578,95123 . Log. of 27,9119, *&c.* Price.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,5036, *&c.* Time—1.
0,69897,00043 . Log. of 5. Annuity.

Variety 3. An Annuity detained 7 Years, with the Compound Intereſt, after the Rate of 6 *l. per Cent. per Annum*, did amount to *l.* 41,9691, *&c.* how much was the Annuity? *Variety 3. Q. Of Arrears in 7 Years, what the Annuity.*

Anſw. 5 *l.* a Year. Seeing the *Data* here being the Arrerages, Term and Rate anſwering to T. R. and Z. of a *Geometrical Progreſſion*, and the Annuity ſought, as α the firſt Term, that is T. 7. R. 1,06, and Z. 41,9691, *&c.* the Rules in the 4*th* Propoſition for finding the firſt Principal there, will give Reſolution here. And ſeeing the Rate ſeptupled according to T, is 1,50363025899136, by the 4*th* Theorem; *Anſwer. By Progreſſion. By the 4th Theorem.*

ℜω—1 . ℜ—1 :: *Arrerages.* . *Annuity.*
0,50363025899136 . 0,06 :: 41,969188249280 . 5

And accordingly by Logarithms: If from the Log. of the Arrerages added to the Log. of the Rate lacking 1, be taken the Log. for the Time lacking 1, the Remainder ſhall be the Log. of the Annuity. *By Logarithms.*

1,

1,62293,05694 . Log. of 41,96918, &c. Arrerages.
—2,77815,12504 . Log. of 0,06 R—1.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,50363, &c. Time—1.
0,69897,00043 . Log. of 5. Annuity.

Otherwiſe. And becauſe the Arrears of any Annuity, divided by the Annuity it ſelf, always giveth in the Quotient the Arrears of 1 *l.* Annuity let run according to T: And contrary, the Arrears of 1 *l.* Annuity for the Term propounded, dividing the Arrears given, ſhall be the Annuity, and the Difference of their Logarithms accordingly: If then by the firſt Propoſition be gotten the Arrears of 1 *l.* Annuity forborn for the Term given, let the Log. thereof be taken from the Log. of the Arrerages propounded, and the Remainder is the Log. of the Annuity deſired.

The Principal Money to raiſe 1 *l.* Annuity at Simple Intereſt, after the Rate propounded by the 4*th* Variety of the firſt Propoſition, is found to be 16 *l.* 13 *s.* 4 *d.* and in Decimals 16,6666, &c. the Reſidue of the Work follows.

1,22184,87496 . Log. of *l.* 16,66666, &c. or 16 *l.* 13 *s.* 4 *d.*
Principal, to raiſe 1 *l.* Annuity at Simp. Intereſt
0,17714,10571 . Log. for the Time.
1,39898,98067 . Log. of 25,06050, &c.
Arrerages 8,39383, &c. of 1 *l.* Annuity 7 Years.

1,62293,05694 . Log. of 41,96918, &c. Arrerages given.
0,92396,05651 . Log. of 8,39383, &c. Arrerages of 1 *l.* Annuit.
0,69897,00043 . Log. of 5. Annuity.

Variety 4. *Q. Of the Price to buy: what Annuity.* Anſwer. *By the ſixth Theorem.*

Variety 4. What yearly Annuity will *l.* 27,91190, &c. in ready Money purchaſe for 7 Years, after the Rate of 6 *l.* on the 100 for a Year, by Compound Intereſt?

Anſw. 5 *l.* a Year: For here are given the Price, Term and Rate. So by the 6*th* Theorem, If the Price be multiplied by the Product of the Rate lacking 1, led into the Rate figurate according to the Term, and this Product be divided by the ſaid Rate ſo figurate lacking 1, the Quotient ſhall be the Annuity deſired, as before under the firſt Variety of this Propoſition.

$\beta\omega$—1 β—1 in $\beta\omega$ *Price.* *Annuity.*
As 0,503630258991 36 . 0,09021781553948 16 :: 27,9119, &c. . 5 .

By Logarithms. And by Logarithms, the Log. for the Time lacking 1, ſhall be taken out of the Log. for the Time added to the Log. of the Rate lacking 1, and the Log. of the Price.

0,17714,10571 . Log. of 1,50363, &c. Time or $\beta\omega$.
—2,77815,12504 . Log. of 0,06 R—1 . or β—1.
1,44578,95123 . Log. of 27,91190, &c. Price.
0,40108,18198 . Sum.
—1,70211,18155 . Log. of 0,50363, &c. Time—1 . or $\beta\omega$—1.
0,69897,00043 . Log. of 5. Annuity.

Otherwiſe. Or if by the ſecond Propoſition be gotten the ready Money, that after the Rate propounded will buy 1 *l.* Annuity yearly for the Term given, then ſubſtract the Log. thereof out of the Log. of the ready Money given, and you have the Log. of the Annuity; becauſe the Price propounded, divided by the Annuity, will always bring in the Quotient the Price of 1 *l.* Annuity to continue according to T. And contrary, the Price propounded divided by the Price of 1 *l.* Annuity according to T, ſhall give the Annuity; and accordingly the Difference of their Logarithms.

The Principal Money to raiſe 1 *l.* Annuity at Simple Intereſt, after the Rate propounded, being found as before to be 16 *l.* 13 *s.* 4 *d.* the Work appeareth thus:

1,22184,87496 . Log. of 16,66666, &c. or 16 *l.* 13 *s.* 4 *d.* Principal in 1 Year, to raiſe 1 *l.* Annuity.
0,17714,10571 . Log. for the Time.
1,04470,76925 . Log. of 11,08428, &c.
Ready Money 5,58238, &c. to buy 1 *l.* Annuity 7 Years.

1,44578,95123 . Log. of 27,9119, &c. Price given.
0,74681,95080 . Log. of 5,5823, &c. Price of 1 *l.* Annuity.
0,69897,00043 . Log. of 5, Annuity.

And in all the Varieties of this Claſſis, the like may be done for Annuities after other Rates, and for Half-yearly and quarterly Paiments if occaſion be. *For Half-yearly and quarterly Paiments.*

Propoſition 4. To know in what Time or Term a Sum of Money propounded will be paid by an Annuity according to a certain Rate given: or how long an Annuity was detained to increaſe to a Sum propounded, after a certain Rate, by Compound Intereſt. *Prop. 4. To find the Time or Term.*

The *Data* for the Reſolution of Queſtions under this Claſſis, will be either, *Data under the 4th Prop.*

1 . 2 . 3 . The Arrerages, Price and Annuity.
1 . 2 . 5 . The Arrerages, Price and Rate.
1 . 3 . 5 . The Arrerages, Annuity and Rate.
Or, 2 . 3 . 5 . The Price, Annuity and Rate.

The Varieties here, as the laſt Precedent, follow in their proper order. *How placed.*

Variety 1. An Annuity of 5 *l.* a Year, if the Arrerages by Compound Intereſt be forborn a certain Time or Term, will amount to *l.* 41,96918, &c. and may be bought for *l.* 27,91190, &c. ready Money: what is the Time or Term of that Annuity? Variety 1. Q. *Of 5 l. per Ann. ſuch Arrears and Price, what the Time.*

Anſw. 7 Years. The *Data* here, are the Arrerages, Price and Annuity: And becauſe the Arrerages divided by the Price, give in the Quotient always the Rate multiplied into it ſelf, according to the Term, and the Difference of their Logarithms is the Log. thereof; it follows, that the Index of the Higheſt Power of the Quotient ſhall be the Term deſired. So the Quotient of their Diviſion being 1,503630258991 36, is a Power of the 7*th* Quantity ariſing from the Root 1,06. Anſwer. *By Decimals.*

And accordingly the Log. of the Annuity added to the Log. of the ſaid Power lacking 1, and from the Sum the Log. of the Arrerages ſubſtracted, leaveth the Log. of the Rate lacking 1: wherefore the Difference between the Log. of the Rate, and the Antecedent thereof, dividing the Difference between the Logarithms of the Arrerages and Price, ſhall give the Term deſired. *By Logarithms.*

1,62293,05694 . Log. of 41,9691, &c. Arrerages.
1,44578,95123 . Log. of 27,9119, &c. Price.
0,17714,10571 . Log. of 1,5036, &c. Time. βω.
—1,70211,18155 . Log. of 0,5036, &c. βω—1.
0,69897,00043 . Log. of 5. Annuity.
0,40108,18198 . Sum.
1,62293,05694 . Log. of 41,9691, &c. Arrerages.
—2,77815,12504 . Log. of 0,06. R—1 or β—1.
0,02530,58653 . Log. of 1,06. R.
0,00000,00000 . Log. of 1,00. Antecedent.

,17714,10571 (*Years.*
2530,58653 (7 . Term.

Variety 2. The Arrerages of an Annuity, accompted by Compound Intereſt, after 6 *l. per Cent. per Ann.* were *l.* 41,9691, &c. and purchaſed for *l.* 27,9119, &c. For what Term of Years was the Annuity to endure? Variety 2. Q. *Of Arrears and Price, what the Time at ſuch Rate.*

Anſw. 7 Years. In this Variety the *Data* being the Arrerages, Price and Rate, and the Price being as α, the Arrerages as ω, and the Rate R of a *Geometrical Progreſſion* to find T, the Queſtion may be reſolved by the Rules in the firſt Propo- Anſwer. *By Progreſſion.*

Propoſition for finding the third Principal of ſuch a Progreſſion, obſerving only that the Number found out here will be the Rate figurate according to T, but there lack 1.

$$\begin{array}{lllll} & \alpha & 1 & \omega & \text{R figurate to T.} \\ \text{For} & 27{,}9119, \&c. & 1 :: & 41{,}9691, \&c. & 1{,}5036302589913 6. \end{array}$$

And ſo this Number being a Power, (as was noted in the laſt Variety above) the Index of the higheſt Power therein ſhall be the Term deſired, and the Rate being the Root thereof, dividing it till the Quotient be an Unit, ſhall ſhew the Index by the Number of Diviſions, to which it will always be equal.

By Logarithms. But ſhortning the Work by Logarithms, the Proceſs is thus; take the Log. of the Price from the Log. of the Arrerages, and divide the Log. remaining by the Log. of the Rate.

1,62293,05694 . Log. of 41,9691, &c. Arrerages.
1,44578,95123 . Log. of 27,9119, &c. Price.
0,17714,10571 . Log. of 1,5036, &c. Time.
0,02530,58653 . Log. of 1,06. Rate.

$$\frac{,17714,10571}{2530,58653}\left(\begin{array}{l}\textit{Years.}\\ 7 \text{ . Term.}\end{array}\right.$$

Variety 3. Q. Of 5 *l.* per Ann. *and Arrerages, in what Time.*

Variety 3. If an Annuity of 5 *l.* yearly detained, with the Compound Intereſt, after the Rate of 6 *l.* *per Cent. per Annum*, increaſe to *l.* 41,9691, &c. how long was it detained?

Anſwer. By *Progreſſion.*

Anſw. 7 Years: Here the *Data* are the Arrerages, Annuity and Rate anſwering to α . R . Z of a *Geometrical Progreſſion* to find T. that is α . 5 . R . 1,06 . and Z . 41,96918, &c. So by the Rules in the third Propoſition, for finding the third Principal of ſuch a Progreſſion, the Queſtion will be anſwered.

For 5 . 41,9691, &c. :: 0,06 . 0,50363025899136.

And this Number found being the Ratio figurate to T—1, anſwering to the Log. for the Time lacking an Unit; if 1 be added, it ſhall be a Power, the Index whereof ſhall be the deſired Term.

By Logarithms. And ſo to the abſolute Number of the remaining Log. when the Log. of the Annuity is taken from the Sum of the Log. of the Arrerages, and the Rate lacking 1, add 1, and divide the Log. thereof by the Log. of the Rate.

1,62293,05694 . Log. of 41,9691, &c. Arrerages Z.
—2,77815,12504 . Log. of 0,06 R—1 or ϐ—1
0,40108,18198 . Sum.
0,69897,00043 . Log. of 5. Annuity α.
—1,70211,18155 . Log. of 0,5036, &c. Time—1, or ϐω—1.
0,17714,10571 . Log. of 1,5036, &c. Time, or ϐω.

Rate . 0,02530,58653 (*Years.* 7 . Term.

Variety 4. Q. *Of* 5 *l.* per Ann. *bought for l.*27,9119,&c. *for what Time.*

Variety 4. If *l.* 27,9119, &c. were to be paid by 5 *l. per Annum*: in what Time will it be paid, with allowance of 6 *l.* *per Cent. per Annum*?

Anſwer. *By Decimals.*

Anſw. In 7 Years: For here being given the Price, Annuity and Rate, the Work, without Logarithms, is to get the Principal Money, which at Simple Intereſt, after the Rate propounded, will raiſe ſuch an Annuity, and take the Price out thereof, and by the Remainder divide the ſame Principal, this being a Figural Number, whereof the Rate is the Root, divided by the Root, till the Root be brought in the Quotient, the Number of the ſeveral Diviſions ſhall be the deſired Term.

By Logarithms. And much ſooner by Logarithms, for after finding the Principal Money, which at Simple Intereſt in one Year, after the Rate propounded, will raiſe the Annuity; from the ſame Principal take the given Price, and the Log. of the Remain from the Log. of that Principal, and divide the remaining Log. by the Log. of the Rate.

1,92081,87539 . Log. of 83,3333, *&c.* Principal to raiſe 5 *l.* a Year.
27,9119, *&c.* Price.
1,74367,76968 . Log. of 55,4214, *&c.*
0,17714,10571 . Log. of 1,5036, *&c.* Time, or $\beta\omega$.
Years.
Rate. 0,02530,58653 (7 . Term T.

In like manner may the Term be found for Half-yearly Paiments, Quarterly, or others, and at other Rates as the Caſe may require, under all the Varieties of this Claſſis.

For other Rates, and half-yearly and quarterly Paiments.

Propoſition 5. To know after what Rate, by Compound Intereſt, any Annuity did increaſe to a Sum propounded; or the Rate the Compound Intereſt is reckoned at by the ready Money the Annuity is ſold for to endure a certain Time or Term.

Prop. 5. *To find the Rate.*

The *Data* for the Reſolution of Queſtions under this Claſſis, will be either

1 . 2 . 3 . The Arrerages, Price and Annuity.
1 . 2 . 4 . The Arrerages, Price and Term.
1 . 3 . 4 . The Arrerages, Annuity and Term.
or 2 . 3 . 4 . The Price, Annuity and Term.

Data under the *fifth Prop.*

The firſt two of theſe Varieties agree in Facility, as the two latter in Difficulty, with ſome of the former.

Set in order, but the latter 2 harder than the firſt.

Variety 1. The Arrears of an Annuity of 5 *l.* a Year to endure a certain Time, with Intereſt upon Intereſt, were reckoned to amount to *l.* 41,96918, *&c.* and may be bought for *l.* 27,9119, *&c.* after what Rate was the Intereſt reckoned?

Variety 1. Q. *Of* 5 *l.* per Ann. *ſuch Arrears and Price, what the Rate.*

Anſw. After 6 *l. per Cent. per Annum.* The *Data* here, *viz.* the Arrerages, Price and Annuity, being the ſame as in the firſt Variety of the 4*th* Propoſition; the Work is the ſame as there, till the laſt Diviſion to find the Term.

By Logarithms.

1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. of 1,50363, *&c.* Time. $\beta\omega$
—1,70211,18155 . Log. of 0,50363, *&c.* $\beta\omega - 1$.
0,69897,00043 . Log. of 5. Annuity.
0,40108,18198 . Sum.
1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
—2,77815,12504 . Log. of 0,06. R—1, or $\beta - 1$.
1,00. Antecedent added.
1,06. Rate, or β.

Variety 2. For *l.* 27,9119, *&c.* ready Money, one may buy 7 Years Arrerages of an Annuity; which caſt up by Compound Intereſt, amounts to *l.* 41,9691, *&c.* after what Rate is the Intereſt accompted?

Variety 2. Q. *Of an Annuity for 7 Years, ſuch Arrears and Price, what the Rate.*

Anſw. After 6 *l.* on the 100 for a Year. Here the *Data* being the Arrerages, Price and Term, the ſame as in the firſt Variety of the third Propoſition, the Work ſhall be as the former Part thereof, ſeeing the Rate was there found to get the Annuity thereby.

By Logarithms.

1,62293,05694 . Log. of 41,96918, *&c.* Arrerages.
1,44578,95123 . Log. of 27,91190, *&c.* Price.
0,17714,10571 . Log. for the Time 1,50363, *&c.*
7) 0,02530,58653 . Log. of the Rate 1,06.

And this agreeth with the Rules in the firſt Propoſition for finding the 4*th* Principal of a *Geometrical Progreſſion*, ſeeing the Price is as α, the Arrerages as ω. So by α . ω . T. may R be found, with this only Difference, that here the Index of the Quotient is T, but there T—1.

By Progreſſion.

Variety 3. An Annuity of 5 *l.* a Year, with the Compound Intereſt thereof in 7 Years, amounted to *l.* 41,96918, *&c.* after what Rate on the 100 did it increaſe?

Variety 3. Q. *Of* 5 *l.* per Annum 7 *Years ſuch Arrears, what the Rate.*

Anſw.

Anſwer. *Anſw.* As before, after 6 *l.* on the 100 for a Year: The Arrerages, Annuity and Term here are given, and are the ſame *Data* as in the firſt Variety of the ſecond Propoſition: And becauſe there the *Ratio* was gotten before the Price could be found, the Work here will be the ſame for the *Ratio*, and by Logarithms thus after this way.

By Logarithms. Take the Log. of α, the Annuity from the Log. of Z the Arrerages, and the Remainder is the Log. of the Arrerages of 1 *l.* let run according to T; to which a Log. being added, that will make the Sum the Log. of a Power lacking 1, whoſe Index ſhall be T, which in this Example will be done by the Log. of 0,06 the *Ratio* lacking 1, and the Sum will be the Log. of 0,5036302589913б, which is βω—1. And then without farther Work, add 1 the Antecedent to the Abſolute Number of the Log. ſo added, and you have the Rate 1,06. See more at the end of the next Variety.

1,62293,05694 . Log. of 41,96918, &c. Arrerages.
0,69897,00043 . Log. of 5. Annuity.
0,92396,05651 . Log. of 8,39383, &c. Arrerages of 1 *l.* in 7 Years.
—2,77815,12504 . Log. of 0,06 R—1, or β—1.
—1,70211,18155 . Log. of 0,50363, &c. Time—1, or βω—1.

Ergo, to 0,06
add 1,00 Antecedent.
1,06 Rate.

Variety 4. Q. Of 5 *l.* per Ann. 7 *Years, ſuch Price, what the Rate.* *Variety* 4. If *l.* 27,91190, &c. in ready Money, purchaſe an Annuity of 5 *l.* a Year, to endure for 7 Years: after what Rate on the 100 is the Compound Intereſt thereof reckoned?

Anſwer. *Anſw.* After 6 *l. per Cent. per Annum.* For ſeeing the *Data* here, *viz.* the Price, Annuity and Term, agree with the *Data* of the firſt Variety of the firſt Propoſition, and the *Ratio* there was found thereby before the Arrerages could be found, the Work to get the ſame here will be the ſame, and ſo needs no Repetition, but is a moſt difficult Propoſition to reſolve, as there was obſerved,

By Logarithms. becauſe it is hard to find a Number to multiply the ready Money by, that ſhall produce the Arrerages of the Annuity forborn to T. which in this Example muſt be 1,5036302589913б, and no other; and accordingly the Log. thereof added to the Log. of the Price, maketh the Sum the Log. of 41,96918, &c. the Arrerages, or Z of a Progreſſion, where 5 is α; and the laſt Term of a Progreſſion or ω, where 27,9119, &c. is α. But both in this and the laſt Variety, it is beſt to divide by the Time the Log. for the Time, which may be gotten by the firſt or ſecond Varieties of the firſt and ſecond Propoſitions of Annuities.

1,44578,95123 . Log. of 27,9119, &c. Price.
0,17714,10571 . Log. of 1,5036, &c. Time.
1,62293,05694 . Log. of 41,9691, &c. Arrerages.
&c.

Touching Intereſt and Annuities intermixt.

Of Intereſt and Annuities mixt.

Propoſitions many. Becauſe the Principal Propoſitions about Intereſt are 4, and about Annuities are 5, that is 9 in all as aforeſaid, ſuch a vaſt Number of Propoſitions concerning them, as well amongſt themſelves as mixed one with another, according to the foregoing Chapter of *Tranſmutation*, might be thence deduced, as would weary

Thoſe moſt uſeful. the Reader: But the moſt uſeful are ſuch as follow, and relate more eſpecially to the ſecond and third Propoſitions of Annuities, or the firſt or ſecond of Intereſt already handled.

Prop. 1. To put a Value on Wares ſold for Time. *Prop.* 1. To put a Value, preſent or future, on Commodities ſold according to the Time given for Paiment.

For the Future. By the firſt Propoſition of Intereſt, add the Log. for the Time, to the Log. of the Value in ready Money, and the Sum is the Log. of the future Value.

Q. Calico what worth at 9 Months. *Example.* Calico is worth 25 *d.* the Yard ready Money: what is it worth to be paid at 9 Months end, accompting 10 *per Cent. per Annum* by Compound Intereſt?

Anſw. $26\frac{3}{4}$ *d.* and ſomewhat more. Anſwer.

0,04139,26852 . Log. of 1,10. R.
0,03104,45139 . Log. for the Time $\frac{3}{4}$, or 9 Months.
1,39794,00087 . Log. of 25.
1,42898,45226 . Log. of 26,85, *&c.*

For the preſent. On the contrary, by the ſecond Prop. of Intereſt, take the Log. for the Time from the Log. of the future Value, and the Difference is the Log. of the Preſent.

Wherefore the Log. 0,03104,45139, taken from the Log. of 26,85, *&c.* ſhall leave the Log. of 25.

Prop. 2. *To ballance an Accompt.*

Prop. 2. To ballance an Accompt preſent or future, of ſeveral Merchants, where divers Sums of Money are paid and received by each of them, to be reckoned with the Compound Intereſt.

For the future. According to the Times of the ſeveral Receipts and Paiments by the firſt Propoſition of Intereſt, the Charge and Diſcharge of every Perſon is to be gotten, by adding the Log. for the Time between the Accompt, and ſuch Receipt or Paiment accordingly, to the ſame Receipt or Paiment for the future Ballance of the Accompt.

Q. *Of divers Sums paid and received between* A *and* B, *which in debt.*

Example. A and *B* have paid divers Sums of Money one to and for another; and received of and for one another divers other Sums between the 25*th* Day of *March* 1611, and the 27*th* Day of *March* 1613, according to their particular Accompts following: they agree to clear Accompts, and allow each to other 10 *per Cent. per Annum,* Intereſt upon Intereſt; and that their Reckoning ſhall conclude upon the 27*th* Day of *March* 1613, and demand which of them is indebted to the other, and how much?

Particular Accompt of A.

The Accompt of A.

		l.			l.
Mar. 27. 1611.	Received of *B* ———	200	*June* 27. 1611.	Paid to *B* ———	100
Sept. 27.	Received for Accompt partable in halves between them ———	260		More then paid for Accompt partable in halves between them --	200
Dec. 27.	Received of *B* ———	300	*June* 27. 1612.	Paid to *B* ———	300
Mar. 27. 1612.	Received for Accompt partable in thirds, *viz.* $\frac{1}{3}$ for himſelf, and $\frac{2}{3}$ for *B* ———	210	*Sept.* 27.	Paid for Accompt, partable in Thirds, *viz.* $\frac{1}{3}$ for himſelf, and $\frac{2}{3}$ for *B* ———	300
Sept. 27.	Received of *B* ———	200	*Dec.* 27.	Paid to *B* ———	300

Particular Accompt of B.

The Accompt of B.

		l.			l.
June 27. 1611.	Received of *A* ———	100	*Mar.* 27. 1611.	Paid to *A* ———	200
Sept. 27.	Rec. for Accompt partable in Thirds, *viz.* $\frac{1}{3}$ for *A* & $\frac{2}{3}$ for himſelf-	450	*June* 27.	Paid for Accompt partable in halves between them ———	300
June 27. 1612.	Received of *A* ———	300	*Dec.* 27.	Paid to *A* ———	300
Sept. 27.	Received for Accompt partable in halves between them ———	300	*June* 27. 1612.	Paid for Accompt partable in Thirds, *viz.* $\frac{1}{3}$ for *A* & $\frac{2}{3}$ for himſelf-	300
Dec. 27.	Received of *A* ———	300	*Sept.* 27.	Paid to *A* ———	200

Accompt of A *caſt up.*

The Charge of *A.*	The Diſcharge of *A.*
0,08278,53704. *Log. of* R *in* 2 *Years.*	0,07243,71991. *Log. of* R *in* $1\frac{3}{4}$ *Year.*
2,30102,99957. *Log. of* 200 *received.*	2,00000,00000. *Log. of* 100 *paid.*
2,38381,53661. *Log. of* 242.	2,07243,71991. *Log. of* 118,150, &c.
0,06208,90278. *Log. of* R *in* $1\frac{1}{2}$ *Year.*	0,07243,71991. *Log. of* R *in* $1\frac{3}{4}$ *Year.*
2,11394,33523. *Log. of* 130 *received.*	2,00000,00000. *Log. of* 100 *paid.*
2,17603,23801. *Log. of* 149,979, &c.	2,07243,71991. *Log. of* 118,150, &c.
0,05174,08565. *Log. of* R *in* $1\frac{1}{4}$ *Year.*	0,03104,45139. *Log. of* R *in* $\frac{3}{4}$ *Year.*
2,47712,12547. *Log. of* 300 *received.*	2,47712,12547. *Log. of* 300 *paid.*
2,52886,21112. *Log. of* 337,957, &c.	2,50816,57686. *Log. of* 322,229, &c.
0,04139,26852. *Log. of* R *in* 1 *Year.*	0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.*
2,14612,80357. *Log. of* 140 *received.*	2,30102,99957. *Log. of* 200 *paid.*
2,18752,07209. *Log. of* 154.	2,32172,63383. *Log. of* 209,761, &c.
0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.*	0,01034,81713. *Log. of* R *in* $\frac{1}{4}$ *Year.*
2,30102,99957. *Log. of* 200 *received.*	2,47712,12547. *Log. of* 300 *paid.*
2,32172,63383. *Log. of* 209,761, &c.	2,48746,94260. *Log. of* 307,234, &c.
0,07243,71991. *Log. of* R *in* $1\frac{3}{4}$ *Year.*	0,06208,90278. *Log. of* R *in* $1\frac{1}{2}$ *Year.*
2,17609,12591. *Log. of* 150 *paid by* B.	2,17609,12591. *Log. of* 150 *received by* B.
2,24852,84582. *Log. of* 177,226, &c.	2,23818,02869. *Log. of* 173,053, &c.
0,03104,45139. *Log. of* R *in* $\frac{3}{4}$ *Year.*	0,02069,63426. *Log. of* R *in* $\frac{1}{2}$ *Year.*
2,00000,00000. *Log. of* 100 *paid by* B.	2,17609,12591. *Log. of* 150 *received by* B.
2,03104,45139. *Log. of* 107,409, &c.	2,19678,76017. *Log. of* 157,321, &c.

Collection of the Charge.	*Collection of the Diſcharge.*
242.	118,1509, *&c.*
149,9796, *&c.*	118,1509, *&c.*
337,9575, *&c.*	322,2298, *&c.*
154.	209,7617, *&c.*
209,7617, *&c.*	307,2341, *&c.*
177,2264, *&c.*	173,0534, *&c.*
107,4099, *&c.*	157,3213, *&c.*
1378,3351	1405,9021

Total Charge of *A*, *l.* 1378,3351.
Reſt due to *A*, 27,5670.
1405,9021. Total Diſcharge of *A*.

Accompt of B *caſt up.*

And by like Examination of the Accompt of *B*, there will be found in his Hands the Sum of *l.* 27,5670, *&c.* due to *A* as aforeſaid, ſeeing the Charge of the one is the Diſcharge of the other.

The Charge of B.		*The Diſcharge of* B.	
D$^{r.}$ $1\frac{3}{4}$ Year by 100 ℞ of A.	118,1509	C$^{r.}$ 2 Years to 200 P^{d} to A.	242.
$1\frac{1}{2}$ Year by 150 ℞ for A.	173,0534	$1\frac{3}{4}$ Year to 150 P^{d} for A.	177,2264
$0\frac{3}{4}$ Year by 300 ℞ of A.	322,2298	$1\frac{1}{4}$ Year to 300 P^{d} to A.	337,9575
$0\frac{1}{2}$ Year by 150 ℞ for A.	157,3213	$0\frac{3}{4}$ Year to 100 P^{d} for A.	107,4099
$0\frac{1}{4}$ Year by 300 ℞ of A.	307,2341	$0\frac{1}{2}$ Year to 200 P^{d} to A.	209,7617
$1\frac{3}{4}$ Year by 100 P^{d} by A.	118,1509	$1\frac{1}{2}$ Year to 130 ℞ by A.	149,9796
$0\frac{1}{2}$ Year by 200 P^{d} by A.	209,7617	1 Year to 140 ℞ by A.	154.
B Debitor	1405,9021	*B* Creditor	1378,3351

Total Diſcharge of *B*, *l.* 1378,3351.
Reſt in the Hands of *B*, 27,5670
1405,9021 Total Charge of *B*.

So to ballance this Accompt, it appears that *B* is indebted to *A* *l.* 27,5670, or 27 *l.* 11 *s.* 4 *d.* &c. due on the Foot of his Accompt, to be paid *March* 27. 1613.

Ballance B *indebted to* A.

On the contrary, to ballance an Accompt at preſent, when the Paiments are to be made afterward, by the ſecond Prop. of Intereſt take the Log. for the Time gotten, as aforeſaid, from the Log. of the Monies ſo to be paid or received.

For the preſent.

Example. *A* owes to *B* 800 *l.* to be paid at 4 Years end: And *B* is indebted to *A* 900 *l.* to be paid in 6 Years, that is to ſay, at every 2 Years end 300 *l.* Now they agree to clear their Debts preſently, allowing each other 8 *per Cent. per Annum*, Intereſt upon Intereſt: The Queſtion is, which of them muſt pay Money to the other, and how much?

Q. *Of the Debts of* A *and* B *to be paid in Time, how to clear at preſent.*

The Charge of A.

2,90308,99870 . Log. of 800.
0,13369,50220 . Log. of R in 4 Years.

2,76939,49650 . Log. of 588,023.

Collection of the Diſcharge.

257,201, *&c.*
220,509, *&c.*
189,051, *&c.*

666,761, *&c.*

The Diſcharge of A.

2,47712,12547 . Log. of 300.
0,06684,75110 . Log. of R in 2 Years.

2,41027,37437 . Log. of 257,201.

2,47712,12547 . Log. of 300.
0,13369,50220 . Log. of R in 4 Years.

2,34342,62327 . Log. of 220,509.

2,47712,12547 . Log. of 300.
0,20054,25330 . Log. of R in 6 Years.

2,27657,87217 . Log. of 189,051.

Total Charge of *A* *l.* 588,023, *&c.*
Reſt due to *A* 78,738, *&c.*
666,761, *&c.* Total Diſcharge of *A*.

And by like examination of *B* his Accompt, ſo much over-ballance will be found in his Hands as is due to *A* on the Foot of his Accompt; and therefore *B* muſt pay to *A*, to ballance the Accompt, *l.* 78,738, *&c.* or 78 *l.* 14 *s.* 9 *d.* *&c.* in ready Money.

Ballance B *indebted to* A.

Prop. 3. To diſcover what ready Money is to be paid for a Leaſe, when certain Years of the Term are of greater Value than the Reſt, and ſo conſequently to bring ſeveral Annuities iſſuing out of the ſame Lands into one Paiment.

Prop 3. *To find the ready Money paid for a Leaſe, when part of the Term of more Value than the reſt: Or to bring divers Annuities into one.*

Suppoſe the greateſt Annuity or Annual Profit, to begin and continue for the whole Term or Number of Years propounded: Then according to the 4*th Variety* of the Second *Propoſition* of Annuities, find what ſuch an Annuity or Rent is worth in ready Money. And alſo in like manner having ſubſtracted the leſſer Rent or Annuity from the Greater, find what the Difference of the Rents or Annuities to continue for the Term of the leaſt Annuity is worth in ready Money: Then abate this ready Money from the former, and the remainder is the ready Money that ought to be paid for the Leaſe, or both Annuities; and with this ready Money, according to the 4*th* Variety of the third Propoſition of Annuities, is to be found the Annuity or Rent to continue for the whole Term in lieu of the other.

Example. Certain Lands ſtand charged with the Paiment of 40 *s. per Annum* for 13 Years, and afterwards for paiment of 10 *l. per Annum* for 17 Years: Or ſuppoſe a Leaſe for 30 Years to come, whereof the preſent yearly Profit all Out-Rents paid be 2 *l.* but after 13 Years expired, it will be worth 10 *l.* Yearly, all out-Rents paid. If this Leaſe be offered to ſale, what may be given for it, reckoning 6¼ *per Cent. per Annum*, Intereſt upon Intereſt? Or if the Owner of the Lands agree with the Party to whom the aforeſaid Annuities are to be paid, to reduce both into one for the whole 30 Years; the Queſtion is, what that new Annuity ought to be at the Rate aforeſaid by Compound Intereſt?

Q. *Of Lands charged with* 2 *Annuities, what worth to be ſold for the Term of a Leaſe,* &c.

Anſw. The Purchaſe of the Leaſe will be found worth in ready Money, *l.* 64,245, *&c.* which will buy an Annuity of 4 *l.* 15 *s.* 10 *d.* and ſomething above, for 30 Years.

Anſwer.

6,

0,02632,53434 . Log. of R. 6¼, or 1,0625.
30 . T.
0,78976,03020 . Log. for the whole Time.
2,20411,99827 . Log. of 160 Principal, to raiſe 10 *l.* the greateſt Annu.
1,41435,96807 . Log. of 25,963, *&c.*
Ready Money 134,037—to buy 10 *l. per Annum* 30 Years.

Now ſeeing for 13 Years of theſe 30, the Annuity or Annual Profit is but 2 *l. per Annum*, this is to be taken from 10 *l. per Annum*, and there remaineth 8 *l. per Annum*; and the 134,037—before found being too much, ſo much as 8 *l. per Annum* is worth in ready Money for 13 Years, therefore in like manner the Price thereof is found.

0,02632,53434 . Log. of R as aforeſaid.
13 . T.
0,07897,60302
0,26325,3434
0,34222,94642 . Log. for 13 Years.
2,10720,99696 . Log. of 128 Principal. to raiſe 8 *l.* the Difference of Ann.
1,76498,05054 . Log. of 58,208, *&c.*
Ready Money 69,792— to buy 8 *l. per Annum* 13 Years.

This *l.* 69,792—taken from *l.* 134,037—leaves *l.* 64,245, *&c.* for the Value of the Leaſe, which will buy an Annuity of *l.* 4,7931, *&c.* or 4 *l.* 15 *s.* 10 *d*, &c. *per Annum* for 30 Years.

1,20411,99827 . Log. of 16 Principal, to raiſe 1 *l.* Annuity at Simple Intereſt, after the Rate of 6¼ *per Cent.*
0,78976,03020 . Log. for the Time of 30 Years.
0,41435,96807 . Log. of 2,5963, *&c.*
Ready Money 13,4037—to buy 1 *l.* Annuity for 30 Years.

1,80783,93335 . Log. of 64,245, *&c.* Price given.
1,12722,46987 . Log. of 13,4037, *&c.* Price of 1 *l.* Annuity.
0,68061,46348 . Log. of 4,7931, *&c.* New Annuity to continue 30 Years.

When the ſeveral Annuities are payable together.

If the ſeveral Annuities be iſſuant together, then get the Price of them ſeverally, and add them together for the Sum to buy the Annuity deſired, and the Total taken from the Price of the Whole, ſhews the Price of the Purchaſe ſo charged with the Annuities.

Q. Of a Farm to be ſold, charged with 2 Annuities, what the Purchaſer may keep in hand to ſatisfy for them. Anſwer.

Example. A Farm is to be ſold for 680 *l.* which ſtands charged with the Paiment of 30 *l. per Annum*, viz. 7 *l.* 10 *s. per* Quarter for 2 Years: And alſo 9 *l. per Annum*, viz. 2 *l.* 5 *s. per* Quarter for 7 Years: how much Money may the Purchaſer retain in his Hands of the 680 *l.* to ſatisfy the Paiments abovementioned, reckoning after 6¼ *per Cent. per Annum* by Compound Intereſt?

Anſw. l. 107,0339, *&c.* that is, for the firſt Annuity *l.* 56,0804, *&c.* and for the other *l.* 50,9535, *&c.* So this *l.* 107,0339 taken from 680 *l.* leaves *l.* 572,9661 to be paid for the Purchaſe as charged with the ſaid Annuities. And if the Queſtion were, what new Annuity might have been granted in lieu of the other two, this *l.* 107,0339 being found to be the ready Money both the ſaid Annuities are worth, the Work will be reſolved as before by the 4*th* Variety of the third Prop. of Annuities.

0,02632,53434 . Log. of 1,0625 . R . as before.
2 . T.
0,05265,06868 . Log. for the Time 2 Years or 8 Quarters.
2,69125,06683 . Log. of 491,1913, *&c.* Principal to raiſe 7 *l.* 10 *s.* in 1 Quart.
2,63859,99815 . Log. of 435,1109, *&c.*
Ready Money 56,0804, *&c.* to buy 30 *l. per Annum* for 2 Years payable quarterly.

0,

0,02632,53434 . Log. of 1,0625 . R . as before.
7 . T.
0,18427,74038 . Log. for Time 7 Years, or 28 Quarters.
2,16837,19230 . Log. of 147,3573, &c. Principal to raiſe 2 *l.* 5 *s.* in 1 Quarter.
1,98409,45192 . Log. of 96,4038, &c.
Ready Money 50,9535, &c. to buy 9 *l.* *per Annum* for 7 Years, payable quarterly.

Prop 4. *To exchange one Annuity for another.* *With Money to boot.*

Prop. 4. To exchange one Annuity for another, with or without Money to boot.

With Money is, when the Terms of both Annuities are equal, but the Annuities are one greater than the other, then abate the leſſer Annuity from the Greater; and as aforeſaid, by the 4*th* Variety of the ſecond Propoſition of Annuities, find what the Remainder eſteemed as an Annuity for the Time given is worth in ready Money, and ſo much is to be given with the leſſer Annuity for the Purchaſe of the Greater.

Q. *Of* 70 *l.* per Ann. *exchanged for* 150 *l.* per Ann. *what Money to be given.*

Example. *A* hath an Annuity of 70 *l.* *per Annum*, and *B* another of 150 *l.* both to endure 15 Years: They agree to exchange, and that *A* ſhall pay to *B* ſo much ready Money as will countervail the Difference of the Annuities: how much ready Money ſhall *B* receive of *A*, after the Rate of 6 *per Cent.* *per Annum* by Compound Intereſt?

Anſwer.

Anſw. *l.* 776,9799, &c. For 70 taken from 150, leaves 80; and 80 *l.* *per Annum* for 15 Years is worth ſo much ready Money.

0,02530,58653 . Log. of 1,06 . R.
15 . T.
0,37958,79795 . Log. for the Time.
3,12493,87366 . Log. of 1333,3333, &c. Principal to raiſe 80 *l.* *per An.*
2,74535,07571 . Log. of 556,3534, &c.
Ready Money 776,9799, &c. to buy 80 *l.* *per Annum* for 15 Years.

Without Money to boot.

Exchange of one Annuity for another without Money, muſt ballance the Inequality of the Annuities with more or leſs Time: Wherefore as before, get the ready Money that will buy each Annuity, and then by the 4*th* Variety of the 4*th* Propoſition of Annuities, ſee how many yearly, half-yearly, or quarterly Paiments accordingly the ready Money of one Annuity will buy of the other.

Q. *Of* 20 *l.* per Ann. *exchanged for* 30 *l.* per Ann. *what Term*

Example. *A* hath an Annuity of 20 *l.* *per Annum* for 21 Years, yearly to be paid, and will exchange with *B* for an Annuity of 30 *l.* *per Annum*: how long ſhall the Annuity of 30 *l.* *per Annum* be paid to *A*, that neither of them loſe, reckoning 6 *l.* *per Cent.* *per Annum*, Intereſt upon Intereſt?

Anſwer.

Anſw. 10 Years *A* ſhall receive 30 *l.* *per Annum*, but for the 11*th* Year no more than 27 *l.* 7 *s.* 4¾ *d.*

0,02530,58653 . Log. of 1,06. R.
21 . T.
0,53142,31713 . Log. for the Time.
2,52287,87453 . Log. of 333,3333, &c. Principal to raiſe 20 *l.* *per Ann.*
1,99145,55740 . Log. of 98,0518, &c.
Ready Money 235,2815, &c. to buy 20 *l.* *per Ann.* for 21 Years.

2,69897,00043 . Log. of 500 Principal, to raiſe 30 *l.* *per Annum.*
235,2815, &c.
2,42278,42932 . Log. of 264,7185, &c.
0,27618,57111

(23127058
27618,57111 | 1 (*Years.* 10
2530,58653

Then if 2530,58653 anſwer to 1 Year, or 365 Days, 2312,70581 the Remain ſhall anſwer to 333 Days, and part of a Day. And if 365 Days pay 30 *l.* then 333 Days ſhall pay *l.* 27,3698, &c. or 27 *l.* 7 *s.* 4¾ *d.*

Hence

In like ſorts Debts by a new Agreement to be paid at other Times.

Hence alſo after the manner of Annuities, a Debt to be paid at certain Times, and afterwards agreed to be paid at other Times, with Allowance or Diſcount of Compound Intereſt, the Paiments are found.

Q. *Of* 5000 *l. payable in* 5 *Years, what to be paid in every* 3 *Months.*

Example. One oweth 5000 *l.* to be paid in 5 Years, *viz.* every Year 1000 *l.* but afterwards agreeth with his Creditor to pay the Whole in 20 equal Paiments, *viz.* every 3 Months one Paiment: The Queſtion is, what each Paiment ſhall be, with Allowance of Intereſt upon Intereſt, after the Rate of 10 *l. per Cent. per Annum?*

Anſwer.

Anſw. l. 241,2561, *&c.* every Quarter during the 5 Years: For 1000 *l.* a Year for 5 Years being found to be worth, in ready Money, *l.* 3790,7868, *&c.* that ready Money will buy ſuch a quarterly Paiment as aforeſaid, by the 4*th* Variety of the third Propoſition of Annuities.

1,61752,36773 . Log. of 41,4499, *&c.* Principal to raiſe 1 *l. per* Quarter.
0,20696,34260 . Log. for the Time 5 Years, or 20 Quarters.
1,41056,02513 . Log. of 25,7371, *&c.*
Ready Money 15,7127, *&c.* to buy 1 *l.* Annuity for 5 Years.

3,57872,93597 . Log. of 3790,7868, *&c.* Price given.
1,19625,10297 . Log. of 15,7127, *&c.* Price of 1 *l.* Annuity.
2,38247,83300 . Log. of 241,2561, *&c.* New Annuity.

If the Debt be payable at once.

But ſuppoſing by the new Agreement, the whole Debt had been agreed to be diſcharged in one Paiment, as at the end of 3 Years, with Allowance of Intereſt as aforeſaid: Then after the ready Money found, as aforeſaid, by the 4*th* Variety of the ſecond Propoſition of Annuities, the reſt of the Reſolution is wrought by the firſt Propoſition of Intereſt, and that one Paiment found to be *l.* 5045,5372, *&c.*

Example.

0,04139,26852 . Log. of 1,10 . R.
3 . T.
0,12417,80556 . Log. for the Time.
3,57872,93597 . Log. of 3790,7868, *&c.* ready Money.
3,70290,74153 . Log. of 5045,5372, *&c.* ω.

Prop. 5. *To leſſen the Rent or Fine.*

Prop. 5. To leſſen the yearly Rent of an Houſe or Land by increaſing the Fine, or to leſſen the Fine by increaſing the Rent.

The Rent.

For leſſening the Rent by the 4*th* Variety of the ſecond Propoſition of Annuities, find what the Sum diminiſhed, eſteemed as an Annuity, is worth ready Money, and add this ready Money to the Fine, and the Aggregate is the new Fine.

Q. *Of* 10 *l.* per Ann. *leſſened to* 4 *l. what Fine.*

Example. A Leaſe is to be ſold for 30 Years, whereof the Fine is 100 *l.* and the Rent 10 *l.* yearly. The Leſſee is deſirous to pay leſs Rent, and to increaſe the Fine or Income: If therefore the yearly Rent be decreaſed to 4 *l. per Annum*, what Fine ſhall he pay after 10 *l. per Cent. per Annum* by Compound Intereſt.

Anſwer.

Anſw. l. 156,5614, *&c.* For ſeeing the 10 *l.* yearly Rent decreaſed to 4 *l.* is diminiſhed 6 *l. per Annum*, the ready Money that will buy an Annuity of 6 *l. per Annum*, at the Rate and for the Time aforeſaid, is found to be *l.* 56,5614, *&c.* which added to 100 *l.* makes *l.* 156,5614, *&c.* as before.

0,04139,26852 . Log. of 1,10. R.
30 . T.
1,24178,05560 . Log. for the Time.
1,77815,12504 . Log. of 60 Principal to raiſe 6 *l. per Annum.*
0,53637,06944 . Log. of 3,4385, *&c.*
Ready Money 56,5614, *&c.* to buy 6 *l.* Annuity for 30 Years.
Fine 100.
Total 156,5614, *&c.* Fine to be paid.

For leſſening the Fine by the *4th* Variety of the third Propoſition of Annuities, find what Annuity may be bought for that Quantity of the Fine deſired to be abated, and add that Annuity to the Rent, and the Aggregate is the new Rent. *The Fine.*

Q. Of 100 *l. Fine leſſened to* 60 *l. what Rent.*

Example. If a Leaſe of an Houſe for 21 Years be worth 100 *l.* Fine, and 10 *l. per Annum* Rent: of how much yearly Rent ought it to be, to bring the Fine down to 60 *l.* reckoning 10 *l. per Cent. per Annum*, Intereſt upon Intereſt?

Anſwer.

Anſw. l. 14,6250, *&c.* For ſeeing the Fine of 100 *l.* decreaſed to 60 *l.* the Leſſee abateth 40 *l.* of his Fine. And this 40 *l.* at the Rate, and for the Term aforeſaid, will buy an Annuity of *l.* 4,6250, *&c.* which added to 10 *l.* makes the Rent *l.* 14,6250, *&c.* as before.

0,04139,26852 . Log. of 1,10 . R.
21 . T.
0,86924,63892 . Log. for the Time.
1,00000,00000 . Log. of 10 Principal to raiſe 1 *l.* Annuity.
0,13075,36108 . Log. of 1,3513, *&c.*
Ready Money 8,6486, *&c.* to buy 1 *l.* Annuity for 21 Years.

1,60205,99913 . Log. of 40. Price given.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
0,66511,41800 . Log. of 4,6250, *&c.* New Annuity.
Rent 10.
Total 14,6250, *&c.* Rent to be paid yearly.

In like manner Exchanges of Fines for Rents, or Rents for Fines, or Fines and Rents for Rents or Fines, are performed: And the Annuity for the one, and the ready Money for the other found out accordingly. ☞

Prop. 6. To find what Annuity for any Term one may buy for a Sum of Money, when the Annuity beginneth preſently, and the Buyer hath Time for the Payment of his Money.

Prop. 6. *To find what Annuity to begin preſently, may be bought for a Sum paid afterward.*

This *Prop.* by the ſecond *Prop.* of Intereſt, firſt findeth what the Purchaſer's Money is worth to be paid preſently; and then by the *4th Variety* of the third *Prop.* of *Annuities*, findeth what Annuity to endure for the Time propounded may be bought for that ready Money.

Q. Of an Annuity for 10 *Years bought for* 500 *l. paid* 4 *Years after.*

Example. What Annuity to endure for 10 Years, and begin preſently, may I grant for 500 *l.* to be received at 4 Years end, reckoning Intereſt upon Intereſt, after the Rate of 10 *l. per Cent. per Annum?*

Anſwer.

Anſw. l. 55,5786, *&c.* For 500 *l.* due at 4 Years end, being worth in ready Money *l.* 341,5067, *&c.* will buy at the Rate aforeſaid an Annuity of *l.* 55,5786, *&c.* for 10 Years.

0,04139,26852 . Log. of 1,10 . R.
4 . T.
0,16557,07408 . Log. for the Time 4 Years.
2,69897,00043 . Log. of 500 . ω.
2,53339,92635 . Log. of 341,5067, *&c.* α.

0,41392,68520 . Log. for the Time 10 Years.
1,00000,00000 . Log. of 10 Principal to raiſe 1 *l.* Annuity.
0,58607,31480 . Log. of 3,8554, *&c.*

Ready Money 6,1445, *&c.* to buy 1 *l.* Annuity for 10 Years.

2,53339,92635 . Log. of 341,5067, *&c.* Price above found.
0,78849,14961 . Log. of 6,1445, *&c.* Price of 1 *l.* Annuity.
1,74490,77674 . Log. of 55,5786, *&c.* Annuity deſired.

But if the preſent Value of a Leaſe be given, and the Term begin afterward, the preſent Worth is only to be valued according to the future beginning of the Term. *If the Term begin after.*

Example.

Q. Of the Reverſion of a Leaſe, what worth.

Example. If a Leaſe for 30 Years to begin preſently, be worth 1000 *l.* what is it worth in Reverſion, to begin at 7 Years end, after the Rate of 10 *l. per Cent. per Annum,* Intereſt upon Intereſt?

Anſwer.

Anſw. l. 513,1581, *&c.* Becauſe the Purchaſer pays his Money 7 Years before he enters upon the Land, the Drift is to know what 1000 *l.* due at 7 Years end, after the Rate aforeſaid, is worth at preſent; which by the ſecond *Prop.* of Intereſt, is diſcovered to be *l.* 513,1581, *&c.* and no more is the preſent Value of that Leaſe.

0,04139,26852 . Log. of 1,10 . R.
7 . T.
0,28974,87964 . Log. for the Time.
3,00000,00000 . Log. of 1000 . ω.
2,71025,12036 . Log. of 513,1581, *&c.* α.

Prop. 7. To countervail Fines and Rents, and the Leaſe in Reverſion.

Prop. 7. To countervail a preſent Fine, with an Annual Rent, beginning and continuing with a Leaſe, when the Leaſe in Reverſion is offered to ſale for ready Money.

Here is to be found what the Fine will amount to, being put out at Intereſt for ſo many Years as are to come between the Bargain and the beginning of the new Leaſe, according to the firſt Propoſition of Intereſt: and then find what Annuity to endure for the like Term, with the Leaſe, may be bought for that Fine ſo increaſed by the 4*th Variety* of the third Propoſition of Annuities; and that ſame Annuity is the Annual Rent deſired.

Q. Of the Rent upon renewing a Leaſe inſtead of a Fine.

Example. Suppoſe I have 10 Years to come of an old Leaſe, and would renew it for 21 Years after the Expiration of the Old; for which I am demanded 100 *l.* to be paid preſently, and inſtead thereof I offer ſuch a Rent during the new Leaſe, as ſhall countervail the ſaid preſent Fine, after the Rate of 10 *per Cent. per Annum* by Compound Intereſt: what ſhall that Rent be?

Anſwer.

Anſw. l. 29,9903, *&c.* For the 100 *l.* paid 10 Years before-hand at the Rate propounded, will produce *l.* 259,3742, *&c.* which Sum will at the ſame Rate, purchaſe a Rent of *l.* 29,9903, *&c.* for 21 Years.

2,00000,00000 . Log. of 100. α.
0,41392,68520 . Log. for the Time 10 Years, as before.
2,41392,68520 . Log. of 259,3742, *&c.* ω.

1,00000,00000 . Log. of 10 Principal to raiſe 1 *l.* Annuity.
0,86924,63892 . Log. for the Time 21 Years, as before.
0,13075,36108 . Log. of 1,3513, *&c.* as before.
Ready Money 8,6486, *&c.* to buy 1 *l.* Annuity for 21 Years.

2,41392,68520 . Log. of 259,3742, *&c.* Price as before.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
1,47698,10407 . Log. of 29,9903, *&c.* Annuity or Rent required.

Prop. 8. *To find an Annuity anſwerable to another, and Money to begin preſently.*

Prop. 8. To get the Annuity for a certain Term, which another Annuity for a leſſer or greater Term with ſome Money will buy to begin preſently.

This Propoſition is almoſt like the 4*th* before: For firſt muſt be found, what the Buyer's Annuity is worth ready Money, and thereto add the Money to be paid with the leſſer Annuity, or the greater Annuity with the leſſer Time, as the Caſe requires; and then find what Annuity that Sum will buy for the Term propounded, the one by the 4*th* Variety of the ſecond Propoſition of Annuities, and the other by the 4*th* of the Third.

Q. Of an Annuity for 21 Years, bought for 200 l. and 50 l. per Annum.

Example. What Annuity to endure for 21 Years is worth 200 *l.* ready Money, and an Annuity of 50 *l. per Annum* for 7 Years, reckoning 10 *l. per Cent. per Annum,* Intereſt upon Intereſt?

Anſwer.

Anſw. l. 51,2708, *&c. per Annum.* For ſuch an Annuity for 21 Years, at the Rate of 10 on the 100, will be purchaſed for the Sum of *l.* 443,4209, *&c.* that is, 200 *l.* in preſent Money, and *l.* 243,4209, *&c.* the Value of the Annuity of 50 *l. per Annum* for 7 Years.

2,69897,00043 . Log. of 500 Principal to raiſe 50 *l.* Annuity.
0,28974,87964 . Log. for the Time 7 Years as before.
2,40922,12079 . Log. of 256,5790, *&c.*

Ready Money 243,4209, *&c.* to buy 50 *l. per Ann.* for 7 Years.

2,64681,62080 . Log. of 443,4209, *&c.* Price as before.
0,93694,58113 . Log. of 8,6486, *&c.* Price of 1 *l.* Annuity.
1,70987,03967 . Log. of 51,2708, *&c.* Annuity deſired.

Prop. 9. *To find what a longer or ſhorter Term is worth, not altering the Rent.*

Prop. 9. To know by the Price or Fine of a Leaſe to endure for a certain Term, what a greater or leſſer Term of the ſame is worth, not altering the Rent.

If the Fine be the Price of the leſſer Term, and the greater be ſought.

When the Fine given is the Price of the leſſer Term, and the Value of the Greater required; then firſt by the 4*th* Variety of the third Propoſition of Annuities, find what Annuity to endure with the firſt Leaſe may be bought for the Fine: And then by the 4*th* Variety of the ſecond Propoſition of Annuities, find what this laſt found Annuity eſteemed with the ſecond Leaſe is worth in ready Money, and this ready Money is the Value of the Leaſe for the greater Term.

Q. *Of a Leaſe, with* 100 *l. Fine for* 10 *Years, what Fine for* 20 *Years.*

Example. If a Leaſe of an Houſe for 10 Years be worth 100 *l.* Fine, and 10 *l.* a Year Rent: What Fine is a Leaſe of the ſame Houſe worth for 20 Years, not altering the Rent, after the Rate of 10 *l. per Cent. per Annum* by Compound Intereſt?

Anſwer.

Anſw. l. 138,5504, *&c.* becauſe at the Rate aforeſaid 100 *l.* will buy an Annuity of *l.* 16,2741, *&c.* for 10 Years; and this Annuity in 20 Years is worth as before *l.* 138,5504, *&c.* ready Money.

1,00000,00000 . Log. of 10 Principal to raiſe 1 *l.* Annuity.
0,41392,68520 . Log. for the Time 10 Years.
0,58607,31480 . Log. of 3,8554, *&c.*

Ready Money 6,1445, *&c.* to buy 1 *l.* Annuity for 10 Years.

2,00000,00000 . Log. of 100 Price given.
0,78849,14961 . Log. of 6,1445, *&c.* Price of 1 *l.* Annuity.
1,21150,85039 . Log. of 16,2741, *&c.* Annuity for 10 Years.

2,21150,85039 . Log. of 162,7415, *&c.* Principal to raiſe *l.* 16,2741, *&c.* Annuity.
0,82785,37040 . Log. for the Time 20 Years.
1,38365,47999 . Log. of 24,1910, *&c.*

Ready Money 138,5504, *&c.* to buy *l.* 16,2741, *&c.* for 20 Years.

If the Fine be the Price of the greater Term, and the Leſſer be ſought.

On the contrary, when the Fine given is the Price of the greater Term, and the Value of the Leſſer is required: Then firſt find, as above, what Annuity to endure with the greater Term may be bought for the Fine; and afterward find what this laſt found Annuity is worth in ready Money, according to the leſſer Term, and this ready Money is the Value of the Leaſe for that leſſer Term.

Q. *Of* 360 *l. paid for a Leaſe of* 50 *Years, ſurrendred at* 11 *Years, how much to be paid back.*

Example. A for a Leaſe of 50 Years paid 360 *l.* and after 11 Years agreed with his Landlord to ſurrender his Leaſe, conditionally, that he might receive ſo much as the Reſidue of the Years in the Leaſe will come to, after the Rate of 5 *l. per Cent. per Annum*, and reckoning the Rent half-yearly: how much therefore ſhall the Landlord pay back of the 360 *l.*?

Anſwer.

Anſw. l. 335,5696, *&c.* For after finding the Principal Money, that at 5 *l. per Cent. per Annum* Simple Intereſt, will raiſe 1 *l.* Annuity in half a Year, to be *l.* 40,4740, *&c.* the Fine 360 *l.* working as before, is found to buy an Annuity of *l.* 9,74433, *&c.* half-yearly, which for 39 Years (that is the Reſidue of 50 after expiration of 11) is worth in ready Money *l.* 335,5696, *&c.* as aboveſaid, and plainly appeareth by the Operations following.

½) 0,02118,92991 . Log. of 1,05. R.
0,01059,46495½ . Half.

2,00000,00000 . Log. of 100.
0,39282,35307 . Log. of 2,4707, *&c.*
1,60717,64693 . Log. of 40,4740, *&c.*

1,60717,64693 . Log. of 40,4740, *&c.* Principal to raiſe 1 *l.* Annuity at Simple Intereſt in half a Year.
1,05946,49550 . Log. for the Time 50 Years.
0,54771,15143 . Log. of 3,5294, *&c.*

Ready Money 36,9445, *&c.* to buy 1 *l.* Annuity for 50 Years, payable half-yearly.

2,55630,25008 . Log. of 360 Price given.
1,56755,03222 . Log. of 36,9445, *&c.* Price of 1 *l.* Annuity.
0,98875,21786 . Log. of 9,7443, *&c.* Half-yearly Annuity for 50 Years.
2,59592,86479 . Log. of 394,3925, *&c.* Principal to raiſe *l.* 9,7443, *&c.* Half-yearly.
0,82638,26649 . Log. for the Time 39 Years.
1,76954,59830 . Log. of 58,8228, *&c.*

Ready Money 335,5696, *&c.* to buy *l.* 9,7443, *&c.* Half-yearly for 39 Years.

Prop. 10. *To turn a Rate on the Hundred, to a Purchaſe by the Year, &c. Analogies. For the Rate on the Hundred.*

Prop. 10. To reduce a Rate on the Hundred to a Purchaſe by the Year, or the contrary.

To the Reſolution of theſe ſerve the following Analogies, *viz.*

For the former; As the Rate, to an Unit: ſo is an Hundred, to the Number of Years deſired.

For the latter; As the Number of Years propounded, to an Unit: ſo is an Hundred, to the Rate deſired.

As if I would know what Rates on the Hundred Lands bought at 16 or 20 Years Purchaſe is equivalent to:

Then as 16 . 1 :: 100 . 6¼ }
or 20 . 1 :: 100 . 5 } by the Latter.

For the Years Purchaſe.

Or on the contrary; How many Years Purchaſe the Lands are bought for, when the Money is reckoned at 6¼ *l.* or 5 *l. per Cent. per Annum?*

Then as 6,25 . 1 :: 100 . 16 }
or 5 . 1 :: 100 . 20 } by the Former.

Whereby it appears that Land bought for 16 Years Purchaſe, is all one with 6¼ *per Cent. per Annum*; and Land bought for 20 Years Purchaſe, is alike as if the Purchaſe-Money were reckoned at 5 *l.* on the 100.

If the Enquiry be which is moſt.

When the Enquiry is, Which is moſt, ſo many Years Purchaſe, or ſuch a Rate, both are to be gotten and compared together.

Q. *Of* 1000 *l. paid for* 70 *l.* per Ann. 7 *Years before any Profit, if more or leſs than* 20 *Years purchaſe.* Anſwer.

Example. September the 29*th*, 1643, A Merchant paid 1000 *l.* for certain Lands, which was out in Leaſe till *September* the 29*th*, 1650; during which Leaſe the Merchant ſhall receive but a Pepper-corn a Year, but afterward he will receive 70 *l. per Annum*: whether now doth this Merchant pay more or leſs than 20 Years Purchaſe?

Anſw. More than 20 Years Purchaſe, by 7 *l.* 2 *s.* +: For 20 Years Rent at 70 *l. per Annum*, is but 1400 *l.* and 1000 *l.* paid 7 Years before any Rent received, makes the Arrerages, with Intereſt upon Intereſt, after the Rate of 5 *l. per Cent. per Annum*, (that is all one with 20 Years Purchaſe) amount to the Sum of 1407 *l.* 2 *s.* + by the firſt Propoſition of Intereſt.

0,02118,92991 . Log. of 1,05 . R.
7 . T.
0,14832,50937 . Log. for the Time.
3,00000,00000 . Log. of 1000. α.
3,14832,50937 . Log. of 1407,1004, *&c.* ω.

70 *l.*
20
1400

When the Enquiry is, Whether more or leſs be taken than a Rate propounded, or than ſo many Years Purchaſe, both being gotten and compared together, as laſt above-mentioned, the Difference will appear on which Side it is.

If the Enquiry be which is moſt of a Rate, or Year's Purchaſe.

Example. If a Man diſburſe 500 *l.* and at 12 Months end receive in part 80 *l.* and at the end of every Half-year afterward 40 *l.* till 10 Years in all be expired: doth he take more or leſs than 10 in the 100 Intereſt upon Intereſt?

Q. *Of* 500 *l. received firſt*, 80 *l. and* 40 *l. half-yearly*, &c. *whether* 10 *l. in the* 100 *Intereſt.*

Anſw. He takes more than 10 in the 100 by *l.* 4,53977, *&c.* For ſeeing he is a Year before he receive any, the Year's Intereſt of 500 *l.* at 10 *l. per Cent. per Annum*, is 50 *l.* which added to 500 *l.* makes 550 *l.* out of which 80 *l.* taken, (which was then paid in) leaves 470 *l.* the Compound Intereſt whereof, at the Rate propounded for the other 9 Years, amounts with the Principal by the firſt Propoſition of Intereſt, but to *l.* 1108,23567, *&c.* And the Rent of 40 *l.* every Half-year at the ſame Rate for nine Years, or eighteen Half-years, amounts to *l.* 1112,77544, *&c.* by the firſt Propoſition of Annuities.

Anſwer.

0,04139,26852 . Log. of 1,10 R.
9 . T.
0,37253,41668 . Log. for the Time.
2,67209,78579 . Log. of 470.
3,04463,20247 . Log. of 1108,2356, *&c.*

0,37253,41668 . Log. for the Time.
2,91352,44915 . Log. of 819,4538, *&c.*
3,28605,86583 . Log. of 1932,2292, *&c.*
1112,7754, *&c.*

When the Enquiry is, which is beſt, ſuch a Sum or ſuch an Annuity, the ready Money of the Annuity gotten is to be compared with the Sum propounded.

If the Enquiry be, whether a Sum or an Annuity be beſt.

Example. A offers *B* 50 *l.* for an Annuity of 10 *l. per Annum* for 8 Years: whether were *B* beſt to accept thereof, accompting the Compound Intereſt at 10 *l. per Cent. per Annum*?

Q. *Of* 10 *l. per* Ann. *for* 8 *Years, or* 50 *l. be beſt.*

Anſw. No; becauſe an Annuity of 10 *l.* yearly for 8 Years, at the Rate propounded, is worth in ready Money *l.* 53,349, *&c.* by the firſt Propoſition of Annuities, which is *l.* 3,349, *&c.* more than 50 *l.*

Anſwer.

0,04139,26852 . Log. of 1,10. R.
8 . T.
0,33114,14816 . Log. for the Time.
2,00000,00000 . Log. of 100 Principal to raiſe 10 *l.* Annuity.
1,66885,85184 . Log. of 46,6507, *&c.*
53,3492, *&c.*

In like manner when the Difference between ſo many Years Purchaſe, and the Money paid, or to be paid for the Purchaſe, is ſought; both are gotten, and the one ſubſtracted from the other.

If the Difference of the Years Purchaſe & the Price be ſought.

Example. Upon *May*-Day, *Anno Regni Elizabethæ* 30, *A* ſold to *B* certain Lands, that yielded 33 *l.* 16 *s.* 8 *d. per Annum* Rent: At which Sale *A* and *B* agreed thus, That *A* ſhould pay to *B* 50 *l.* upon the firſt Day of *November* next following: And that then *B* ſhould ſurrender the Lands back again to *A* (in nature of a Mortgage) for the Paiment of 100 Marks *per Annum* for eight Years, *viz.* at the end of every Year, ſtill upon the firſt Day of *November*, 66 *l.* 13 *s.* 4 *d.*

Example in fulfilling an Order of the High Court of Chancery.

Before the Day appointed for the firſt Paiment *B* died, ſo as the Surrender was not made, nor any of the aboveſaid Paiments paid: Wherefore upon Suit in *Chancery*, it was ordered by the Lord Keeper, that there ſhould be an Accompt made, and the Queſtions in the Accompt to be, what the Land was worth at the Sale, after the Rate of 20 Years Purchaſe; and what the Paiments above-mentioned were worth in ready Money at the ſame Time, reckoning the Intereſt at 10 *per Centum per Annum*; And then look how much the Land was worth more than the Paiments, ſo much ſhould the Heirs of *B* pay to *A*: how much therefore by this Order ought to be paid to *A*?

The Caſe.

Anſw. l. 289,8833, *&c.* For ſuch is the Difference between *l.* 676,6666, *&c.* the Worth of the Land at 20 Years Purchaſe, and *l.* 386,7833, *&c.* the Sum of the ready Money of the Annuity added to the 50 *l.* and taking from the Total one

What is to be paid according to the Order.

one half Year's Intereſt, becauſe by the Agreement the 50 *l.* was not to be paid, nor the Annuity to begin till one half Year after the Bargain was made.

l. : *s.* : *d.*
33 : 16 : 08 . Yearly Rent of the Land.
20 . Years.
676 : 13 : 04 . Sum of 20 Years Purchaſe.

0,04139,26852 . Log. of 1,10. R.
$8\frac{1}{2}$.
0,33114,14816
0,02069,63426

Marks
2,82390,87366 . Log.of 666,6666, *&c.* Principal to raiſe 100 Annu.
0,33114,14816 . Log. for the Time 8 Years.
2,49276,72550 . Log. of 311,0049, *&c.*

Marks
Ready Money 355,6617, *&c.* to buy an Annuity of 100 for 8 Years.
50 added.

2,60816,40598 . Log. of 405,6617, *&c.*
0,02069,63426 . Log. for the Time $\frac{1}{2}$ Year.
2,58746,77172 . Log. of 386,7833, *&c.*
676,6666, *&c.*—386,7833, *&c.* = 289,8833, *&c.*

Proof of Anatociſm.

The Varieties of Queſtions are even numberleſs; but what hath been ſaid in this Chapter, is ſufficient for a through underſtanding of all *Anatociſm:* And ſince both the way of Work by *Decimals*, and alſo by *Logarithms* hath been ſeen, and ſeveral of the Operations and Propoſitions converſe one to another; the Truth of the Concluſions is ſufficiently proved thereby, and to give farther Example here, will be altogether ſuperfluous.

Partis tertiæ Libri quarti

FINIS.

The

The Fourth PART of the Fourth BOOK.

CHAP. I.

Of EQUATIONS.

Equations cloſe the Work.
They are Compound Proportions, or Proportions of Equality.

NOthing remains now to finiſh the whole Work, but to overlook *Equations*, which in the Chapter of *Ratio's* before are called *Compound Proportions*, to diſtinguiſh them from others; and ſometimes *Proportions of Equality*, becauſe the Number or Numbers, Magnitude or Magnitudes, between which Compariſon is made, are equal one to the other in Value, though of different Denominations.

Wherein their Computation conſiſts.
The ſeveral Names.

The whole Computation of *Equations*, conſiſteth in the Invention, Reduction and Reſolution of an *Equation*; and is called (though by ſome, with reference only to the latter) the *Art of Equation*, *Rule of Equation*, *Rule of Coſs*, *Rule of Quantity*, *Almucabula*, (ſigniſying an hidden or ſecret Tradition) *Algeber*, or *Algeber's Rule*, another Arabick word taken by ſome for the Name of the Inventor, from the Article *Al* and *Geber* a Man; and by ſome that it was *Geber* the old Arabian: But others will have it only for a Name of ſingular Excellence, *quaſi Ars magiſtralis*, it being of ſuch Perfection, that it performs not only what may be done by other Rules of Proportion, *Alligation*, *Falſe Poſition*, and the Rules of *Archindus*, and ſix Quantities of *Cataym*, (as *Digges* in his *Stratioticos* tells us) but with much Facility, Evidence and Demonſtration.

Excellency thereof.

Algebra from whence.
How to be wrote.
Geber not the Inventor.

From this *Algeber* came the Name *Algebra*, now moſt common thereto: Nevertheleſs neither the one nor the other of them is the right Word, but *Algiebar*, as Dr. *Dee* in his Math. Preface, *&c.* proves from *Avicen*, tranſlated by *Andreus Alpagus*, (one very ſkilful in the Arabick Tongue) who calls The Science of working *Algiebar* and *Almachabel*, The Science of finding an unknown Number, by adding of a Number, *Diviſion* and *Equation*: Nor could *Geber* (however ſkilled therein, and other profound Knowledg) be the Inventor, ſince as that Learned Doctor ſaith, *Diophantus*, a Greek Philoſopher and Mathematician, before the Time of *Geber*, wrote 13 Books thereof.

Parallels of what uſe.
How called.
What to be underſtood thereby.

The Numbers or Quantities compared one to another, are ſet one on the one Side, and the other on the other, of a pair of Parallels or Gemowe Lines, called the *Sign of Equation*, and ſometime Metonymically called *Equation* it ſelf, there being no two things more alike or equal, than ſuch two right Lines ═══ equally extended, whereby is ſaved the oft repetition of the words *equal to*: As if I would ſay, 1 *s.* is equal to 12 *d.* or 3 Crowns are equal to 15 *s.* they are ſet thus:

Example.

$$\overset{s.}{1} = \overset{d.}{12} \qquad \overset{\triangle}{3} = \overset{s.}{15}$$

So ſhall one Shilling be underſtood to be equal in Value to twelve Pence, and 3 Crowns equivalent to 15 Shillings, though their Denominations differ, and neither 1 nor 12, nor yet 3 and 15 are alike.

The Sorts of Equations.

Equations, in the Chapter of *Ratio's* before-mentioned, were there divided into two principal Sorts, *viz.* *Pure* and *Mixt*, as one Number or Magnitude compared to another, or many others to one or to many, and both Sorts there obſerved principally to converſe with Contract Numbers: Not but that ſometime Abſtract Numbers are uſed, as well as and with thoſe that are Contract; but if the Denominations in Contract, or the Abſtract Numbers themſelves on both Sides of the *Equation*, be alike, the *Equation* is either Nugatory or Impoſſible. For when the *Equation* ſhall be identical in Numbers and Denominations, it is nugatory or trifling; as to ſay, 3 ʒ = 3 ʒ, is all one as to ſay, 3 Squares is as much as 3 Squares. And when the Denominations are identical, and the Numbers annexed different, the *Equation* is impoſſible, for to ſay 2 ʒ = 3 ʒ, is as much as to ſay 2 = 3, which

Abſtract Numbers uſed how.
Nugatory and Impoſſible Equations, *what.*

which is impoſſible. So as if one Number or Magnitude be valued with another, their Denominations muſt be different; but one greater Abſtract Number may be compared to divers leſſer Numbers, and one Contract Number of higher Power or Demonination with divers lower.

One ſort of Equation reduced to another.

Oftentimes by *Reduction*, (as afterwards in the Chapter thereof may be ſeen) *Equations* of the latter Sort are reduced from one Form to another: For ſome of the Numbers on the one Side may be abated, or elſe carried over to the other Side of the *Equation*; ſo as one Number may be left alone at one Side, which is therefore called, the *Solitary Side of the Equation.*

Solitary Side, what.

Equality of a Number to its Parts abſtract, what helped thereby.

Although *Equations* are moſt frequent with Contract Numbers, yet the Equality conſidered between Abſtract Numbers, (as leſſer Numbers may be Parts of the Greater) hath not been without ſome Effect, and among others in giving Light to the Way of parting equally an even Magnitude by 2 odd Parts thereof: And hence this Problem gained Solution, *viz.*

To part one Veſſel of Liquor by unequal Veſſels.

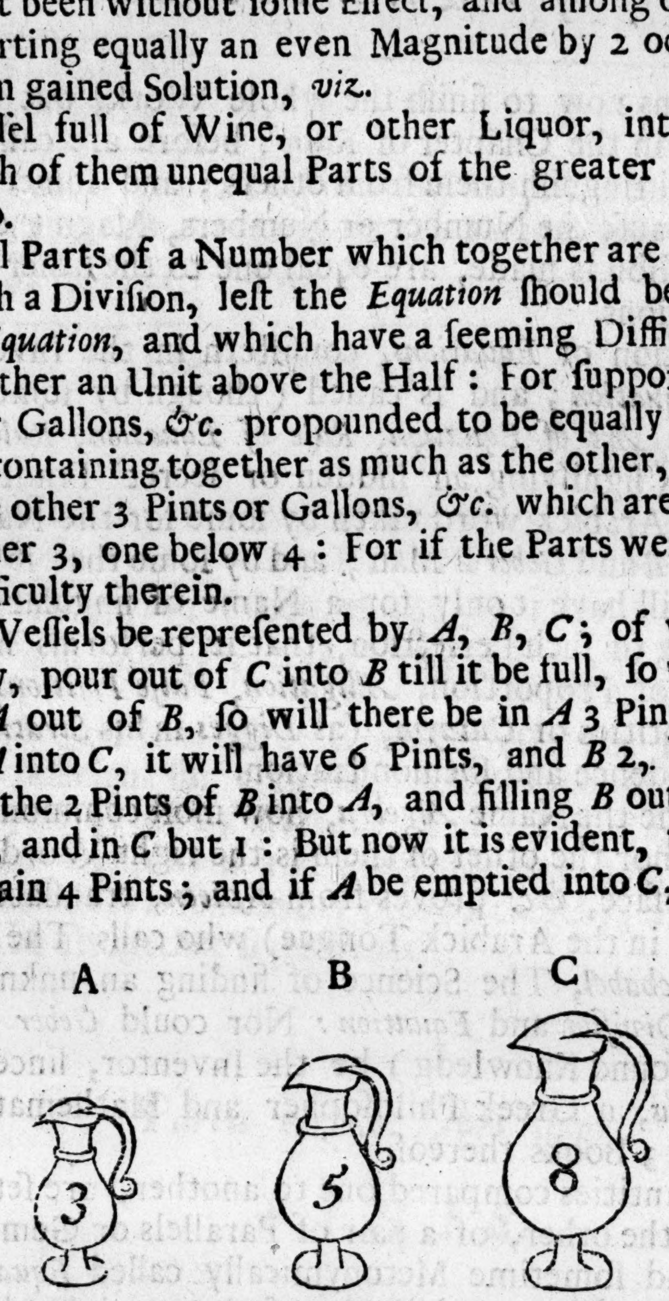
To part a Veſſel full of Wine, or other Liquor, into two equal Parts, by other Veſſels, each of them unequal Parts of the greater Veſſel, and both together equal thereto.

What Parts to be taken.

Neverthel[e]ſs all Parts of a Number which together are equal thereto, are not taken to make ſuch a Diviſion, leſt the *Equation* ſhould be nugatory: But thoſe moſt apt for the *Equation*, and which have a ſeeming Difficulty, as one an Unit below, and the other an Unit above the Half: For ſuppoſe a Veſſel full of Wine contain 8 Pints or Gallons, *&c.* propounded to be equally divided by two other Veſſels unequal, containing together as much as the other, theſe two Veſſels muſt be one 5, and the other 3 Pints or Gallons, *&c.* which are one above 4, the Half of 8, and the other 3, one below 4: For if the Parts were 1 and 7, or 2 and 6, there were no difficulty therein.

Example.

Let then the 3 Veſſels be repreſented by *A*, *B*, *C*; of which *C* being full, and the other 2 empty, pour out of *C* into *B* till it be full, ſo will there be left 3 Pints in *C*; then fill *A* out of *B*, ſo will there be in *A* 3 Pints, in *B* 2, and in *C* 3: Then emptying *A* into *C*, it will have 6 Pints, and *B* 2, but *A* none. Furthermore, emptying the 2 Pints of *B* into *A*, and filling *B* out of *C*, there will be in *A* 2 Pints, in *B* 5, and in *C* but 1: But now it is evident, that if from *B* you fill *A*, yet will *B* retain 4 Pints; and if *A* be emptied into *C*, there will be 4 Pints alſo.



Equations chiefly are uſed in Contract Numbers, but not to be confounded therewith.

However the chief exerciſe of *Equations* be among Contract Numbers, and of them eſpecially the 3 latter, *viz. Coſſicks*, *Surds*, and *Species*; yet I ſee little reaſon to crowd all the Simple Elements thereof into the Accompt of *Equations*, and leſs to ſubject all Figural Numbers, and their Operations, under the Title of *Algebra*, as ſome have done, but confuſedly.

Nor becauſe *Equations* frequently deal with *Coſſicks*, (whence the Rule for reſolving an *Equation* came to be called the *Rule of Coſs*) will it be reaſonable to ſubjugate the whole Proceſs of *Equations* with *Coſſicks*, as is done by others; ſince *Equations* are as familiar to *Species* as to them.

How ſome ſort Equations.

The late uſe of *Species* hath occaſioned ſome to make three Sorts of *Equations.*

1. *Pure.*

The firſt Pure or Abſolute, wherein is no Power on either Side of the *Equation.*

2. *Powerful.*

2. A powerful *Equation*, wherein is ſome Figural Number or other.

3. *Affected. Reckoned here only of 2 ſorts, Pure and Affected.*

3. A mixt or affected *Equation*, made up of both, and of many Numbers: But ſeeing all *Equations* by Coſſical Reduction may be brought to two Numbers or more, thoſe brought thereby to 2 only, are here reckoned for pure *Equations*, though the one thereof be a Power, and all others are to be counted for mixt *Equations.*

Both ſorts, how conſidered.

As well pure *Equations* as thoſe mixt, (now commonly known by the Name of *Affected*, and ſometime *Adfected*) when made up with Powers, whether Coſſical or others, come under farther obſervation.

1. Quantities orderly diſtant, none omitted.

1. When the Quantities are orderly different one from the other, without omiſſion of any Quantity between them.

Examples.

Examples. *Pure.*	√	*Affected.*	√
1 ℨ = 3 N	3	1 ʒ = 2ℨ + 3N	3
2 ʒ = 4 ℨ	2	1 φ = 6ʒ — 5ℨ	5
4 φ = 12 ʒ	3	3 φ = 2ʒ + 10ℨ — 4 N	2

In all theſe Examples, one Denomination immediately ſucceeds the other; for between φ and ʒ. ʒ and ℨ. ℨ and N. no Quantity intervenes.

2. Quantities orderly diſtant, ſome omitted.

2. When the Quantities are orderly different one from the other, but between them are 1, 2, 3, or more Quantities omitted.

Examples.

Examples. *Pure.*	√	*Affected.*	√
1 ʒʒ = 4 ʒ	2	1 ʒʒ = 12 ʒ + 64 N	4
1 ſs = 16 ℨ	2	1 ʒφ = 108 ʒ — 3 ʒʒ	3
1 ʒφ = 32 ℨ	2	1 φφ = 7ʒφ + 9 φ — 8N	2

In the firſt Pure, between the Quantities ʒʒ and ʒ, is one Quantity, *viz.* φ omitted. In the next, between ſs and ℨ, three Quantities are omitted, *viz.* ʒʒ φ. and ʒ. And in the other Inſtance are omitted four Quantities, ſs. ʒʒ. φ. and ʒ. between ʒφ and ℨ.

In the firſt Affected is omitted only one Quantity, that is φ, between ʒʒ and ʒ, and ℨ between ʒ and N. In the next alſo one Quantity only is omitted, *viz.* ſs between ʒφ and ʒʒ, and φ between ʒʒ and ʒ. But in the laſt Inſtance two Quantities are omitted, that is ʒʒʒ and Bſs, between φφ and ʒφ, and ſs and ʒʒ between ʒφ and φ, and ʒ and ℨ between φ and N.

3. Quantities in Affected between the Parts omitted.

3. When the Quantities omitted in an *Affected Equation*, are between the Sides or Parts of the *Equation*, but on either Side none.

Examples.

Examples.

1 ʒφ == 30 ℨ + 4 N √ 2
2 ʒʒ + 3 φ == 82 ℨ — 3 N √ 3

Here between ʒφ on the ſolitary Side, and ℨ on the other Side in the firſt Example, are four Quantities omitted, but between ℨ and N none. And in the ſecond Inſtance, between φ and ℨ is but one Quantity omitted, and none on either Side between ʒʒ and φ, or between ℨ and N.

4. Quantities in an Affected between the Parts none, but on either ſide omitted.

4. When the Quantities omitted in an *Affected Equation* are on either Side, but none between the Sides or Parts.

Examples.

Examples.

1 ʒφ == 1 ſs + 32 N √ 2
4 φφ — 3 Bſs == 10 ʒφ + 1024 N √ 2

Between ʒφ and ſs in the firſt Example, no Quantity is wanting, but ʒʒ. φ. ʒ. and ℨ. are omitted between ſs and N. And in the other Example Bſs at one Side the *7th* Quantity, and ʒφ the *6th* Quantity at the other Side, ſhew none omitted; but between φφ and Bſs one Quantity is omitted, and between ʒφ and N five Quantities are wanting.

5. Quantities omitted diſorderly.

5. When the Quantities omitted are diſorderly between, and on both Sides of the *Equation*.

Examples.

1 φφ + 2ʒ == 20ſs — 4ℨ — 112 N √ 2
3 ʒφ — 100ℨ == 16ʒʒ + 618N — 1φ √ 3

The firſt Example wants 6 Quantities between φφ and ʒ of the one Side, and 3 Quantities between ſs and ℨ of the other Side. And ʒφ is diſtant from ℨ in the latter Example 4 Quantities on the one Side, and φ from N 2 Quantities on the other Side.

Proof of Equations.

The Truth of theſe, and other *Equations*, may be tried, having the Root as *Coſſicks* before noted, by taking Abſtract Numbers. For in the laſt Example, ſeeing the Root is 3, the ʒφ thereof ſhall be 729, the ʒʒ 81, and the Cube 27: So ſhall 3 ʒφ be 2187; from which 100 ℨ, that is 300 taken, there ſhall remain for the left Side of the *Equation* 1887. And ſo 16ʒʒ is 1296, to which 618 added, makes 1914; from whence 27 the Sum of 1 φ abated, leaves 1887 for the right Side of the *Equation* equal to the other.

$$2187 - 300 = 1296 + 618 - 27 = 1887.$$

CHAP. II. *Invention of Equations.*

Invention of Equations firſt to be done. Firſt Equation *what, and why ſo called.* Invention, *&c. how called. What the Steps thereto.*

WHEN a Queſtion is propounded to be reſolved by an *Equation*, the *Equation* is firſt to be found, and ſuch an one too as is apt and pertinent to the Queſtion. This *Equation* found, (becauſe many times altered by *Reduction*, as in the next Chapter) is ſometime called the *Firſt Equation* in reſolving the Queſtion; and the finding thereof, here called *Invention*, is by ſome called *Compoſition*.

He that would arrive at the end of *Equations*, will find it convenient, for the *Invention* of an *Equation*, to proceed gradually by theſe two following Steps.

1. To be ſtored with Analytical Proviſion.

1. Let him be well ſtored with Analytical Proviſion, to uſe as occaſion ſerves: For as in examining a Propoſition of 4 Proportionals, it is neceſſary to know, That the Rectangle of the Means is equal to that of the Extreams: So in a Work of *Equations*, it is as well beneficial as neceſſary, to know both how to deduce and draw an *Equation* out of a Propoſition, and out of the firſt and moſt eaſy *Equations*, (which are nothing but the ſimple Affections or Expoſitions of the Terms) to draw forth, by meet and proper Conſequences, other *Equations* equal to the Firſt, but in other Terms.

Example by two Numbers given, what may be found thereby.

For ſuppoſe of two Numbers given, A be the Greater, E the Leſſer, Z the Sum, X the Difference, *&c.* as before ſhewed in *Species*, it will follow by *Oughtred*, Chap. 11. of his *Clavis*:

1. By A and E, what found.

1. That by the *Species* for any two Numbers given, (whereof one the Greater, and the other the Leſſer) may be found the 13 following Concluſions.

Data.	*Queſita.*	*Reſolution.*
Greater A.	Sum.	$Z=A+E.$
	Difference.	$X=A-E.$
	Rectangle.	$P=\text{Æ}$, or $AE.$
	Sum of the Squares.	$\underset{,}{Z}=Aq+Eq.$
	Difference of the Squares.	$\underset{,}{X}=Aq-Eq.$
	Sum of the Sum and Difference.	$Z+X=2A.$
	Difference of the Sum and Difference.	$Z-X=2E.$
	Rectangle of the Sum and Difference.	$ZX=Aq-Eq$, or $\underset{,}{X}.$
	Square of the Sum.	$Zq=Aq+2\text{Æ}+Eq$, or $\underset{,}{Z}+2\text{Æ}$
	Square of the Difference.	$Xq=Aq-2\text{Æ}+Eq$, or $\underset{,}{Z}-2\text{Æ}$
	Sum of the Squares of the Sum and Difference.	$Zq+Xq=2Aq+2Eq$, or $2\underset{,}{Z}.$
	Difference of the Squares of the Sum & Difference.	$Zq-Xq=4\text{Æ}.$
Leſſer E.	Square of the Rectangle.	$Pq=\text{Æ}q$, or $AqEq.$

2. By Z and A or E, what found.

2. That by the Species for the Sum of any two Numbers, (whereof one the Greater, and the other the Leſſer) and one of thoſe Numbers, may be found the other, the Difference, the Rectangle, the Sum of the Squares, and the Difference of the Squares.

Data.	*Queſita.*	*Reſolution.*
Greater A	Leſſer.	$E = Z-A.$
	Difference.	$X = 2A-Z.$
	Rectangle.	$P = ZA-Aq.$
	Sum of the Squares.	$\underset{,}{Z} = Zq-2ZA+2Aq.$
	Difference of the Squares.	$\underset{,}{X} = 2ZA-Zq.$
Sum Z		
	Greater.	$A = Z-E.$
	Difference.	$X = Z-2E.$
	Rectangle.	$P = ZE-Eq.$
	Sum of the Squares.	$\underset{,}{Z} = Zq-2ZE+2Eq.$
Leſſer E	Difference of the Squares.	$\underset{,}{X} = Zq-2ZE.$

3. That by the Species for the Difference of any 2 Numbers, (whereof one the Greater and the other the Leſſer) and one of thoſe Numbers, may be found the other, the Sum, the Rectangle, the Sum of the Squares, and the Difference of the Squares.

3. By X & A or E, what found.

Data.	*Quæſita.*	*Reſolution.*
Greater A	Leſſer.	$E = A - X.$
	Sum.	$Z = 2A - X.$
	Rectangle.	$P = Aq - XA.$
	Sum of the Squares.	$Z = 2Aq - 2XA + Xq.$
Difference X.	Difference of the Squares.	$X = 2XA - Xq.$
	Greater.	$A = E + X.$
	Sum.	$Z = 2E + X.$
	Rectangle.	$P = Eq + XE.$
	Sum of the Squares.	$Z = 2Eq + 2XE + Xq.$
Leſſer E	Difference of the Squares.	$X = 2XE + Xq.$

4. That by the Species for the Rectangle of any 2 Numbers (whereof one the Greater and the other the Leſſer) and one of thoſe Numbers, may be found the other, the Sum, the Difference, the Sum of the Squares, and the Difference of the Squares.

4. By P & A or E, what found.

Data.	*Quæſita.*	*Reſolution.*
Greater A.	Leſſer.	$E = \frac{P}{A}.$
	Sum.	$Z = \frac{Aq + P}{A}.$
	Difference.	$X = \frac{Aq - P}{A}.$
	Sum of the Squares.	$Z = \frac{Aqq + Pq}{Aq}.$
Rectangle P.	Difference of the Squares.	$X = \frac{Aqq - Pq}{Aq}.$
	Greater.	$A = \frac{P}{E}.$
	Sum.	$Z = \frac{P + Eq}{E}.$
	Difference.	$X = \frac{P - Eq}{E}.$
	Sum of the Squares.	$Z = \frac{Pq + Eqq}{Eq}.$
Leſſer E.	Difference of the Squares.	$X = \frac{Pq - Eqq}{Eq}.$

5. That by the Species for the *Ratio* of any 2 Numbers (whereof one the Greater, and the other the Leſſer) and one of thoſe Numbers, may be found the other, the Sum, the Difference, the Rectangle, the Sum of the Squares, and the Difference of the Squares.

5. By RS & A or E, what found.

Data.	*Quæſita.*	*Reſolution.*
Greater A	Leſſer.	$E = \frac{SA}{R}$.
	Sum.	$Z = \frac{RA + SA}{R}$.
	Difference.	$X = \frac{RA - SA}{R}$.
	Rectangle.	$P = \frac{SAq}{R}$.
	Sum of the Squares.	$Z = \frac{RqAq + SqAq}{Rq}$.
	Difference of the Squares.	$X = \frac{RqAq - SqAq}{Rq}$.
Ratio R. S		
	Greater.	$A = \frac{RE}{S}$.
	Sum.	$Z = \frac{RE + SE}{S}$.
	Difference.	$X = \frac{RE - SE}{S}$.
	Rectangle.	$P = \frac{REq}{S}$.
	Sum of the Squares.	$Z = \frac{RqEq + SqEq}{Sq}$.
Leſſer E	Difference of the Squares.	$X = \frac{RqEq - SqEq}{Sq}$.

Many more may be deduced thence.
Value not altered by different Expreſſions.

And not only theſe, but from theſe a Multitude of other Propoſitions may be deduced by Conſequence, and yet the different Terms of Expreſſion alter nothing of the equal Value of ſuch Concluſions; as ſuppoſing according to the firſt of theſe Propoſitions A be 3, and E 2. then,

Examples.

$$\text{Becauſe} \left\{ \begin{array}{l} Z + X = 2A \\ 5 + 1 = 6 \\ Z - X = 2E \\ 5 - 1 = 4 \\ Zq - Xq = 4Æ \\ 25 - 1 = 24 \end{array} \right\} \text{ſhall} \left\{ \begin{array}{l} \frac{1}{2}Z + \frac{1}{2}X = A \\ 2\frac{2}{1} + \frac{1}{2} = 3 \\ \frac{1}{2}Z - \frac{1}{2}X = E \\ 2\frac{1}{2} - \frac{1}{2} = 2 \\ \frac{1}{4}Zq - \frac{1}{4}Xq = Æ \\ \frac{25}{4} - \frac{1}{4} = 6 \end{array} \right\}$$

$$\text{And} \left\{ \begin{array}{l} Z = A + E = 2A - X = 2E + X = \frac{Aq + P}{A} = \frac{P + Eq}{E}, \ \&c. \\ 5 = 3 + 2 = 6 - 1 = 4 + 1 = \frac{9 + 6}{3} = \frac{6 + 4}{2}, \ \&c. \\ X = A - E = 2A - Z = Z - 2E = \frac{Aq - P}{A} = \frac{P - Eq}{E}, \ \&c. \\ 1 = 3 - 2 = 6 - 5 = 5 - 4 = \frac{9 - 6}{3} = \frac{6 \quad 4}{2}, \ \&c. \end{array} \right.$$

Authors where Store of ſuch Proviſion is.

Deductions of this ſort being endleſs, to endeavour their recital here were an *Herculean* Task: He that deſires to ſee more, let him read *Bachetus* before *Diophantus*; *Billy* before his Continual Proportionals, Mr. *Thomas Harriot* his *Artis Analyticæ Praxis*, Mr. *Jonas Moore* in the *9th* and *10th* Chapters of his Second Book, Mr. *Richard Balam* the *16th* Chapter of his *Algebra*; and above all that profound Analyſt Mr. *William Oughtred* in the *18th* Chapter of his *Clavis Mathematicæ limata*, whom none hath outdone.

2. Step to find the Equation *to ſuppoſe in* Coſſicks 1 ℨ, *in* Species A *for that ſought. When a to be taken.*

2. Being well provided of ſuch Store as aforeſaid, and grounded in the common Principles of *Arithmetick* and *Geometry*, (becauſe ſometime the Uſe of ſome known Problem will much expedite and eaſe the Work) the next Step of the Artiſt is to ſuppoſe the Number or Magnitude ſought in any Queſtion to be 1 ℨ working in *Coſſicks*, or in *Species* A, and the given Magnitudes Conſonants. But if A be any of the *Data*, let the Capital Letter be changed for *a*, or ſome other not given, for diſtinction ſake.

How to proceed with the Suppoſition.

And with this Suppoſition, proceed to examine the Queſtion according to the Tenor thereof (as though this were given, and you were proving the Truth of it) till you bring it to an *Equation.*

Q. Of a Wager, how much offered to be laid.

Example. Two Men in a Controverſy are upon a Wager, one offers to lay 20 *l.* the other to lay as much Money as will make the Sum of both when added together as much as the Product, if one Number be multiplied by the other: how much then did he offer to lay on the Wager againſt the other 20 *l?*

Reſolution.

Here ſuppoſing he offered to lay 1ȥ; this according to the Queſtion muſt be added to 20, and multiplied by 20; when added, the Total is 1ȥ+20, when multiplied, the Product is 20ȥ; and ſo I have gotten an *Equation* that is 1ȥ+20=20ȥ. And this being the firſt *Equation*, and capable of *Reduction*, as in the next Chapter, by taking 1ȥ at the one ſide from 20ȥ at the other; ſo will there be left 20=19ȥ another *Equation*, and be reſolved by Conſequence, if 19ȥ be 20, that 1ȥ ſhall be $1\frac{1}{19}$ *l.*

What to be done if more be ſought than one thing.

If more than one Suppoſition.

Sometime for eaſe in working the Queſtion, A may be joined with ſome Number, and ſometime the Numbers or Magnitudes ſought are more than one; and then oftentimes one being found, the other is known by Conſequence. But if it be neceſſary to ſuppoſe more than once, let the Suppoſitions be different Letters if the Work be in *Species*; and as many Letters as are uſed for Roots, ſo many ſeveral *Equations* to find out the Value of thoſe Letters in Relation to A the firſt Suppoſition, which is to be kept diſtinct from others. And ſometime A being ſuppoſed, thoſe other ſought Numbers or Magnitudes ſhall by Conſequence be drawn from A before the firſt *Equation* be found: but till *Reduction* be learned, one Example more may ſuffice here.

Q. Of the Number of Bottles of Wine.

Example. A Vintner ſold 30 Bottles of Wine for 210 Pence, whereof ſome were White, others Claret; but he ſold one Bottle of White for 5 *d.* and a Bottle of Claret for 8 *d.*: how many Bottles of each ſort were there?

Reſolution in Species.

By ſuppoſing for the White.

Here the Numbers given being thus noted, *viz.* 30 *B*, 5 *C*, 8 *D*, 210 *F*, as in divers other Queſtions where 2 Things are ſought, the Suppoſition may be for either. And if A be ſuppoſed for the White, the Value of A when found, ſhall be the Number of thoſe Bottles: But if A be ſuppoſed for the Claret, the Number of thoſe Bottles ſhall be denoted thereby; wherefore ſuppoſing A to be the Number of the Bottles of White-wine, then by conſequence becauſe there were 30 Bottles in all, the Number of the Bottels of Claret muſt be $B-A$; then each part multiplied by their Rates reſpectively, it will follow the White to be CA, and the Claret to be $DB-DA$, and both together to be equal to F. So will the firſt *Equation* be found, and ſtand thus, $CA+DB-DA=F$, which by *Reduction* as in the next Chapter may be brought to $A=\frac{DB-F}{D-C}$. So if from DB 240 be taken F 210, and the reſidue 30 divided by 3, that is, $D-C$, the Value of A is found to be 10 for the Number of Bottles of White-wine; and then by conſequence there muſt be 20 Bottles of Claret, becauſe the whole were 30; and 10 at 5 *d.* a Bottle, and 20 at 8 *d.* a Bottle make together 210 *d.*

By ſuppoſing for the Claret.

In like manner, If A had been ſuppoſed for the Claret, then ſhould the White be $B-A$, which ſeverally multiplied by their Roots, makes the Product of the Claret to be DA, and the Product of the White to be $CB-CA$, and both together the firſt Equation $DA+CB-CA=F$: And by the Reduction of the next Chapter brought to $A=\frac{F-CB}{D-C}$. So from F 210, ſhall 150 the Product of C into B be taken, and the Remainder 60 divided by 3, that is, $D-C$, or $8-5$, and the Quotient 20 ſhall be the Bottles of Claret.

Reſolution in Coſſicks.

And thoſe leſs accuſtomed to Species, may work with Coſſicks thus: If 1ȥ be ſuppoſed for the Number ſought, either of White or Claret, then the other by Conſequence ſhall be 30—1ȥ; and accordingly each multiplied by the Prices reſpectively ſhall produce, if 1ȥ be ſuppoſed for the White, 5ȥ+240—8ȥ; but if 1ȥ be ſuppoſed for the Claret, 8ȥ+150—5ȥ; and either of them muſt be equal to 210: And when reduced, as taught in the next Chapter, the one will be 3ȥ=30, or 1ȥ=10 of the White; and the other will be 3ȥ=60, or 1ȥ=20 of the Claret, as before.

Suppoſition is always the true Number ſought.

From hence it appears, that this Suppoſition 1ȥ or A, is always the true Number or Magnitude ſought, and no ſuch falſe Poſition, as before in the *Rule of Falſhood* was taken: But it is obſerved, that as to thoſe that know not the Cauſe, it

was

Difference herein from Falſhood.

was ſomewhat ſtrange there, out of Falſe Poſitions to procure True Concluſions, (ſeeing commonly Erroneous Premiſſes have ſuch Concluſions); ſo here it ſeems much more ſtrange and admirable, that a right Number ſhall be taken at firſt touch before any abſolute Knowledg can be had what it is: And on this Accompt *Record* ſays, it may be called the *Rule of Dark* or *Strange Poſition.*

Rule of dark or ſtrange Poſition.

Proof of the Invention of Equations.

The Different work by Coſſicks and Species, ſeeing both concur in the Concluſion, may ſerve for a Proof of the Truth of both Operations.

CHAP. III. *Reduction of Equations.*

Reduction of Equations.

THE firſt *Equation* found as ſuggeſted, either by the Senſe of the Queſtion, or by ſome known Theorem pertinent thereto, according to the former Chapter; the next Work is to reduce or order this *Equation*, ſo that it may be fit for Reſolution.

How called by Balam.

Balam calls this *Reduction*, *Equative Inference*, Chap. 13. But ſeeing, according to the Nature of all *Reduction*, the Terms of the *Equation* are only altered thereby, but the ſame Equality always kept, it may very properly be called *Reduction.*

The ſorts of Reduction.

Equations are reduced ſeveral ways.

1. *By the Simple Elements of Numbers; as*

Firſt, By ſome of the Simple Elements of Numbers, as *Addition*, *Subſtraction*, *Multiplication* and *Diviſion:* For according to the common Axiom, equal things added to or taken from, or multiplying or dividing equal things, ſhall make the Totals, Remains, Products or Quotients, accordingly equal.

Addition *to take off Fractions, or increaſe the* Data.

Addition is uſed on both ſides of an *Equation*, to clear the Species or Quantities of Fractions, or increaſe the *Data* with New Quantities.

Exampl s.

Examples. $5ʒ + 2\frac{1}{2}N = 6ꝝ + 10\frac{1}{2}N$ · √2.

$5Aq + 2\frac{1}{2}B = 6A + 10\frac{1}{2}B$

In both, if $\frac{1}{2}$ be added on either ſide, they ſtand thus:

$5ʒ + 3N = 6ꝝ + 11N.$ $5Aq + 3B = 6A + 11B.$

So becauſe $A+E=Z$, if B be added, it ſhall be that $A+E+B=Z+B$.

Subſtraction *to take away Fractions, and double Denominations.*

Subſtraction is uſed as well to clear the *Equation* of double Denominations, as of Fractions: And ſo not only $\frac{1}{2}$ may be taken from either Side of the former Example, and the *Equations* ſtand thus:

$5ʒ + 2N = 6ꝝ + 10N.$ $5Aq + 2B = 6A + 10B.$

Examples.

But becauſe the $2\frac{1}{2}$ and $10\frac{1}{2}$, on both Sides of the *Equation* are of one Denomination, *viz.* Abſolute Numbers, the leſſer of them (ſeeing their Signs are alike, that is, both +) muſt be taken from the greater; and ſo $2\frac{1}{2}$ taken from $10\frac{1}{2}$, leave the *Equations* thus:

$5ʒ = 6ꝝ + 8N.$ $5Aq = 6A + 8B.$

If doubled at one Side.

If any Denominations on the one Side of the *Equation* be doubled, they muſt be added or ſubſtracted to leſſen the Terms.

Example.

Example. $1ʒʒ = 2φ + 1ʒ + 2ꝝ + 4N - 3ʒ$ √2.

Becauſe ʒ is found with + and — on one Side of the *Equation*, + 1ʒ ſhall be taken from — 3ʒ, and the Remain — 2ʒ, ſet down in the *Equation* thus:

$1ʒʒ = 2φ + 2ꝝ + 4N - 2ʒ.$

To leſſen the Value.

Subſtraction is alſo uſed to leſſen the Value of the firſt given Species by new Ones. And ſo if $A + E = Z$ and B be taken away, then ſhall $A+E-B=Z-B$.

Multiplication *to turn Fractions into Integers &c This called* Converſion *and* Iſomeria.

Multiplication is uſed to turn Fractions, or Heterogeneal Numbers, into Integers or Homogeneal; this is ſometime called *Converſion*; and by *Vieta*, *Iſomeria*, and is thus done: Multiply croſs-wiſe after the manner of Fractions, all the other Species or Magnitudes, into the Denominator of each other's Fraction except his own, and theſe Products ſhall make the New *Equation.* And ſeeing the Work is as *Reduction* of common Fractions, as occaſion ſerves, the greateſt Common Diviſor may be uſed.

If one Side be an Integer, and the other a Fraction; then multiply the Integer by the Denominator of the Fraction, and ſet this Product againſt the Numerator of the Fraction for the new *Equation*.

Examples.

Examples. $\frac{3\phi+3\text{ʒ}}{3\mathfrak{z}}=6\text{N}.$ $A+B=\frac{D}{C}.$

Theſe converted by *Multiplication*, ſhall be thus:

$3\phi+3\text{ʒ}=18\mathfrak{z}.$ $AC+BC=D.$

Thus $A-C=\frac{Aq+Bq}{D}+B+C$ converted, is $DA-DC=Aq+Bq+DB+DC.$

So where the Fractions are on both Sides of the *Equation*.

Examples. $\frac{1\mathfrak{z}+2\text{N}}{3\phi}=\frac{1\text{ʒ}-2\mathfrak{z}+3\text{N}}{3\text{ʒ}+6\text{N}}.$ $A+\frac{B}{DC}=B+\frac{5A}{G}.$

Converted by *Multiplication*, ſhew themſelves thus:

$3\text{ſs}+9\phi-6\text{ʒʒ}=3\phi+6\text{ʒ}+6\mathfrak{z}+12\text{N}.$ $DCGA+BG=BGDC+5DCA.$

Diviſion is uſed, either to depreſs an *Equation* in Numbers or Quantities, or both, and ſo reduce them to their leaſt Terms; and therefore is this often called *Depreſſion*, and is the *Hypobibaſmus* of *Vieta*: Or elſe when the Magnitude ſought, or his higheſt Power in the *Equation* is joined with ſome other Species, then muſt the reſt of the Species be divided by that Species ſo adjoined, and the higher Power cleared thereof: And this latter Sort is *Vieta* his *Paraboliſmus*.

Diviſion *to reduce them to their leaſt Terms, &c. This called* Depreſſion, Hypobibaſmus, *and* Paraboliſmus.

Examples of the firſt ſort are performed, in all reſpects, like *Reduction* of Compound Coſſical Fractions and Species.

Examples of Hypobibaſmus.

As $3\text{ſs}=6\text{ʒʒ}+6\text{ʒ}+12\mathfrak{z}-6\phi$ $\sqrt{2}.$

Depreſſed in Numbers. *Depreſſed in Quantities.*

$1\text{ſs}=2\text{ʒʒ}+2\text{ʒ}+4\mathfrak{z}-2\phi.$ $1\text{ʒʒ}=2\phi+2\mathfrak{z}+4\text{N}-2\text{ʒ}.$

And thus the Coſſical Examples laſt above-mentioned, after Converſion, may be alſo depreſſed; that is,

$3\phi+3\text{ʒ}=18\mathfrak{z}.$ to $1\text{ʒ}+1\mathfrak{z}=6\text{N}.$ $\sqrt{2}.$

And the other in Numbers only to

$1\text{ſs}+3\phi-2\text{ʒʒ}=1\phi+2\text{ʒ}+2\mathfrak{z}+4\text{N}.$

Examples in Species. $3Ac+3Aq=18A.$

Depreſſed in Numbers. *Depreſſed in Quantities.*

$Ac+Aq=6A.$ $Aq+A=6.$

And depreſſed in the Quantities only as,

$Ac+BAq=DA.$ to $Aq+BA=D.$

Examples of the latter ſort, or Paraboliſm.

Examples of Paraboliſmus.

$AqB=D+C$, divided by B is, $Aq=\frac{D+C}{B}.$

And ſo $AqB-DA=R$, divided by B is, $Aq-\frac{D}{B}\times A=\frac{R}{B}.$

And $DAq+DqA=Zc$, divided by D is, $Aq+DA=\frac{Zc}{D}.$

Secondly, *Equations* are reduced by Tranſpoſition or Tranſlation, that is, removing ſome Species or Quantities from one Side of the *Equation* to the other; in which Removal the Signs ſhall be changed, for + at one Side of the *Equation* ſhall be — at the other. This ſort of *Reduction* is uſed, when the ſame Denominations or Quantities are on both Sides of the *Equation*, to leſſen the Terms, and when one Side is to be left ſolitary.

2. *Reduction by Tranſpoſition or Tranſlation.*

Thus in the firſt Example of the former Chapter, where the firſt *Equation* found was $1\mathfrak{z}+20=20\mathfrak{z}$, ſeeing $\mathfrak{z}$ on both Sides the *Equation* with +, the leſſer is abated out of the greater; ſo $1\mathfrak{z}$ taken from $20\mathfrak{z}$, the Remain $19\mathfrak{z}$ anſwered at one Side to the 20N at the other Side.

Example to leſſen the Terms.

But if it had been 22N — 1ȥ = 20ȥ, then the Signs being contrary, — 1ȥ at one Side ſhall be added to + 20ȥ at the other Side; and the *Equation* would have been by this Tranſpoſition, adding 1ȥ to each Side, 22N = 21ȥ.

Example to leave one Side ſolitary.

So to leave one Side ſolitary in Tranſpoſition of the next *Equation* there, the Signs are accordingly changed, for +C is made —C, and —D is made +D; and becauſe +DB was tranſpoſed with the ſame Sign he had before, the Sign of F remaining on that Side he was before, is changed from + to —.

$$CA + DB - DA = F \text{ tranſpoſed is, } A = \frac{DB - F}{D - C}.$$

In like manner all other Tranſpoſitions are performed.

As 1ſs + 3φ — 2ʒʒ = 1φ + 2ʒ + 2ȥ + 4N tranſpoſed, ſhall be
1ſs = 2ʒʒ + ʒ + 2ȥ + 4N — 2φ.

And $DA - DC = Aq + Bq + DB + DC$ ſhall be $DA = Aq + Bq + DB + 2DC$.

If one Side be left empty, what to be done.

Sometime in *Tranſpoſition*, one ſide of the *Equation* is left empty, and then the Quantities with + ſhall be equal with them that are —; wherefore the Defect is eaſily ſupplied, for it never happeneth but when the Quantities are joined with contrary Signs.

Example.

Example. 4ʒ = 5ʒ + 10N — 7ȥ; here tranſpoſing becauſe ʒ is on both ſides, and taking 4ʒ from 5ʒ, the *Equation* will be left thus, 0 = 1ʒ + 10N = 7ȥ: but now becauſe 7ȥ are —, they are equal to 1ʒ + 10N, and the *Equation* may be ſet thus, 7ȥ = 1ʒ + 10N, or by placing the higheſt Power therein Solitary, 1ʒ = 7ȥ — 10N.

3. Reduction by Exaltation, when one Side is a Power.

Thirdly. An *Equation* is ſometime reduced by *Exaltation*, which is uſed when ſome *Coſſick* or *Species* on the one ſide of the *Equation* is referred to the Power of a Number on the other ſide; for then muſt the plain Part of the *Equation* be exalted to that Power by ſquaring, cubing, *&c.* that plain Part, and cancelling the Coſſick Character on the other Part.

Example.

Example. If 1ȥ + 4N = √ʒ36, then ſhall 1ȥ + 4N be ſquared on the one ſide and ſet againſt 36 on the other Side, and √ʒ cancelled. And ſo 1ȥ + 4N = √ʒ 36 reduced, ſhall be 1ʒ + 8ȥ + 16N = 36N, and tranſlated thus, 1ʒ + 8ȥ = 20N, or thus, 1ʒ = 20N — 8ȥ.

Examples in Species.

$B + C = \sqrt{A}$ exalted, ſhall be $Bq + 2BC + Cq = A$.
And $B = \sqrt{AB} - D$ tranſpoſed, is $B + D = \sqrt{AB}$.
And then exalted ſhall be $Bq + 2BD + Dq = AB$.
And if A be left Solitary $\frac{Bq + 2BD + Dq}{B} = A$.

Proof of **Reduction** *of* **Equations.**

That all theſe ſorts of *Reduction* keep the Numbers and Magnitudes ſtill in equality, may be proved by reducing the *Coſſicks* and *Species* into *Integers*; and then comparing them before *Reduction* with thoſe reduced, the Truth of the ſeveral Operations will appear.

CHAP. IV. *Reſolution of Equations.*

Reſolution *of* Equations.

What to be ſet by themſelves.

AFTER the Invention and Reduction of the *Equation*, as in the two Chapters laſt before, the *Equation* is to be reſolved, being ſo found and ordered that the known Magnitudes may be on the one ſide, and the unknown on the other, if *Species* be uſed, but if *Coſſicks*, the higheſt Quantity ſet Solitary.

Reſolution, what.

Root of an E*quation improperly called ſo.*

How gotten.

The Reſolution of an *Equation* is to find out the Value of the Root ſuppoſed, often called, but improperly, the Root it ſelf, becauſe the Root of every *Coſſick* Number muſt be a *Coſſick* Number, and ſuch as by Multiplication will make that Rooted Number, which this ſo called Root will not.

The Value of this Suppoſititious Root is to be procured or extracted as the *Equation* is Pure or Affected.

Touching Pure Equations.

Of pure Equations.

1. *To get the Value of* 1 ℨ, *when the Quantities are orderly diſtant, and none omitted.*

1. When the Quantities or Magnitudes compared one to another, are orderly different each from other without omiſſion of any Quantity between them, then divide the Number of the loweſt Denomination by the Number of the higheſt Denomination, and the Quotient ſhall be the Value of 1 ℨ. But oftentimes the Reduction of the firſt found *Equation* brings the Numbers ſo low, that Diviſion is needleſs, becauſe 1 ℨ is one part of the *Equation.*

Q. *Of the Value of a Cloak.*

Example 1. A Gentleman hireth a Servant to ſerve him a Year for 24 *s.* and a Cloak; but at 8 Months end they fall at variance, and the Gentleman diſchargeth his Servant, and giveth him 13 *s.* and the Cloak: what was the Cloak rated at?

Reſolution.

Anſw. 9 *s.* Here ſuppoſing the Value of the Cloak ſought to be 1 ℨ or A, or keeping the *Species* for Cloak; if 8 Months give 1 Cloak and 13 *s.* then ſhall 12 Months give $1\frac{1}{2}$ Cloak and $19\frac{1}{2}$ *s.* and this ſhall be equal to the whole Year's Wages, and being ſet thus $1\frac{1}{2}$Cl $+$ $19\frac{1}{2}$ *s.* $=$ 1 Cl $+$ 24 *s.* by Tranſpoſition of both Species will be reduced to $\frac{1}{2}$Cl $=$ $4\frac{1}{2}$ *s.* And ſo by conſequence without Diviſion it is eaſily perceived, if half a Cloak be worth 4 *s.* 6 *d.* the whole Cloak ſhall be valued at 9 *s.* and ſuch will be the Quotient if $4\frac{1}{2}$ be divided by $\frac{1}{2}$.

Q. *Of the Age of* Alexander, &c.

Example 2. *Alexander* being asked how old he was, anſwered, he was 2 Years elder than *Epheſtio*; yea ſaid *Epheſtio*, and my Father is as old as we both and 4 Years more; and my Father, ſaid *Alexander*, having all thoſe Years, was 96 Years of Age: the Queſtion is, how old each of them was?

Anſwer.

Here ſuppoſing the Age of *Epheſtio* (who was the Youngeſt) to be 1 ℨ, then *Alexander* being 2 Years Elder, muſt be 1 ℨ $+$ 2N. and then muſt the Father of *Epheſtio* be 2 ℨ $+$ 6N (that is, 4 Years more than them both) all which added together make up the Age of *Alexander*'s Father, that is, 4 ℨ $+$ 8N, and is to be equal to 96.

This *Equation* 4 ℨ $+$ 8N $=$ 96N reduced by *Tranſpoſition* to 4 ℨ $=$ 88N, if depreſſed, or the 88 divided by 4, makes 1 ℨ $=$ 22N for the Age of *Epheſtio.*

Suppoſition.		*Proof.*
1 ℨ	Age of *Epheſtio* ————————	22 Years.
1 ℨ $+$ 2	Age of *Alexander* ————————	24
2 ℨ $+$ 6	Age of *Epheſtio*'s Father ————————	50
4 ℨ $+$ 8	Age of *Alexander*'s Father ————————	96

2. *To get the Value of* 1 ℨ *when the Quantities are omitted.*

2*ly*. When the Quantities or Magnitudes compared one to another are diſtant one from another by 1, 2, 3, or more Quantities or Powers, after *Reduction* of the *Equation* firſt found as before, and Diviſion thereof (if occaſion be) from the Quotient, extract a Root according to the Quantities omitted; that is, if one Quantity be omitted, extract the Square Root, if 2 the Cube Root, if 3 the ſquared Square Root, &c.

Q. *Of the Length and Breadth of a Pavement.*

Example 1. A Floor paved with ſquare Bricks, is longer than it is broad by $\frac{1}{7}$; and the whole Pavement contained 3584 Bricks: what was the Length and Breadth thereof?

Reſolution.

Anſw. 56 Broad, and 64 Long: For ſuppoſing the Breadth which is the leſſer Quantity to be 1 ℨ, then ſhall the Length be $1\frac{1}{7}$ℨ: now multiplying the Breadth by the Length, (as in all ſuch Forms to find the *Area*) there is produced $\frac{8}{7}$ʒ, which according to the Propoſition muſt be equal to 3584, and by *Reduction* 8ʒ $=$ 25088, and again, 1ʒ $=$ 3136: and ſo if in this *Equation* 3584 be divided by $\frac{8}{7}$, will the Quotient be 3136; and becauſe between ʒ and N, one Denomination is omitted, that is ℨ, the Square Root of 3136 is taken, which is 56 for the Breadth, to which 8 that is $\frac{1}{7}$ of 56 added, makes 64 for the Length.

And if in this Inſtance 1 ℨ had been ſuppoſed for the Length, then ſhall the Breadth be $\frac{7}{8}$ℨ, and the Square $\frac{7}{8}$ʒ, and $\frac{7}{8}$ʒ $=$ 3584, which divided by $\frac{7}{8}$ is 4096, whoſe Square Root is 64, as before.

Q. *Of the Length, Breadth and Height of a Pile of Brick.*

Example 2. A Pile of Brick, whoſe Length was to the Breadth as 7 to 2, that is, *Tripla-ſeſquialter*, and the Height was 5 times as much as the Length; being ſold for 980 Crowns, the Owner received for every Yard ſo many Crowns as the Pile had Yards in Breadth: what then was the Length, Breadth, and Height of this Pile?

Reſolution. *Suppoſition for the Breadth.*

Anſw. Two Yards broad, 7 Yards long, and 35 Yards high. For here ſuppoſing the Breadth 1 ℨ, then was the Length $3\frac{1}{2}$ ℨ, and the Height $17\frac{1}{2}$ ℨ; all which multiplied one into another, make $\frac{245}{4}$ φ, and this ſhall be equal to all the Yards in the whole Pile: And ſeeing every Yard coſt as many Crowns as the Breadth contained Yards, if the Breadth be 1 ℨ, then did every Yard coſt 1 ℨ of Crowns; and then by the *Golden Rule*,

	Yard.		*Crowns.*		*Whole Yards.*		*Crowns.*
As	1	.	1 ℨ	::	$\frac{245}{4}$ Φ	.	$\frac{245}{4}$ ʒʒ.

So muſt $\frac{245}{4}$ ʒʒ = 980 Crowns, and conſequently 245 ʒʒ = 3920 Crowns; wherefore dividing 3920 by 245, the Quotient is 16; from whence the *Zenzizenzike* Root taken, becauſe three Quantities are omitted in this *Equation*, the Root 2 is the Breadth, therefore ſhall 7 be the Length, and 35 the Height, as before; each of which will be gotten in like manner, if Suppoſition be made for them.

For the Length and Height.

Suppoſition for the Length.	*Suppoſition for the Height.*
1 ℨ Length.	1 ℨ Height.
$\frac{2}{7}$ ℨ Breadth.	$\frac{1}{5}$ ℨ Length.
5 ℨ Height.	$\frac{2}{35}$ ℨ Breadth.
$\frac{10}{7}$ Φ Pile.	$\frac{2}{175}$ Φ Pile.
y △ y △	y △ y △
1 . $\frac{2}{7}$ ℨ :: $\frac{10}{7}$ Φ . $\frac{20}{49}$ ʒʒ.	1 . $\frac{2}{35}$ ℨ :: $\frac{2}{175}$ Φ . $\frac{4}{6125}$ ʒʒ.
$\frac{20}{49}$ ʒʒ = 980 △	$\frac{4}{6125}$ ʒʒ = 980 △.
20 ʒʒ = 48020 △	4 ʒʒ = 6002500 △.
1 ʒʒ = 2401 △ √ʒʒ *Length.* 7.	1 ʒʒ = 1500625 △ √ʒʒ *Height.* 35.

Proof thereof.

And that all theſe Varieties are true, appears, becauſe if every Yard of the whole Pile coſt 2 Crowns, that is as many as there were Yards in the Breadth, then ſhall the whole Pile coſt 980 Crowns, becauſe it contained 490 ſolid Yards.

	Breadth.		*Length.*		*Height.*		*Solid Yards.*		△		△
For	2	×	7	×	35	=	490	×	2	=	980

Falſhood *cannot do what* **Equations** *can.*

By the Work of theſe two laſt Examples of the Pavement and Pile it appeareth, That though all the Operations of *Falſhood* may be performed by *Equations*; yet ſuch Queſtions as theſe, which concern Figural Forms, will not be wrought right by *Falſhood*, as at leiſure the Artiſt may make experiment.

Several other Queſtions reſolved.

Under theſe two Sorts of *Reſolution* fall very many of the Examples, Queſtions and Propoſitions propounded in moſt Books of *Arithmetick*: Some of which follow, reſolved by way of *Coſſicks* as well as *Species*, and may ſerve to ripen the Apprehenſions of the Ingenious, as well in the Invention (the moſt difficult Part) of the *Equation*, as in the *Reſolution* of ſuch kind of Propoſals.

Q. *Of a Number multiplied by 6, and added to 8.*

1. What Number is that which multiplied by 6, and the Product added to 8, will be 48?

Reſolution.

Anſw. $6\frac{2}{3}$: For if multiplied by 6, and 8 be added, the Total is 48.

B 6. C 8. D 48. Then $A \times B + C = D$, or $AB + C = D$.

And $A = \frac{D - C}{B}$, or $\frac{48 - 8}{6} = \frac{40}{6} = 6\frac{2}{3}$.

1 ℨ × 6 = 6 ℨ. And 6 ℨ + 8 = 48. Then will 6 ℨ = 40
And 1 ℨ = $6\frac{2}{3}$.

Q. *Of 2 Parts of* 100, &c.

2. Which are the two Parts of 100, that the Quotient of the greater Part divided by 3, added to the Quotient of the leſſer Part divided by 5, ſhall together make 30?

Reſolution.

Anſw. 75 and 25: For 75 divided by 3 gives 25, and 25 divided by 5 gives 5, and 25 and 5 make together 30.

B 100. D 30. Greater Part $A = a$ Suppoſition.
Leſſer Part $E = B - a$ Conſequence.

Then $\frac{a}{3} + \frac{B - a}{5} = D$. And $5a + 3B - 3a = 15D$.

And $2a + 3B = 15D$. And $a = \frac{15D - 3B}{2}$ or $\frac{450 - 300}{2} = \frac{150}{2} = 75$.

If A be 1ℨ, then E 100 — 1ℨ.

$$\frac{1\text{ℨ}}{3} + \frac{100 - 1\text{ℨ}}{5} = 30. \quad \text{Then } 5\text{ℨ} + 300 - 3\text{ℨ} = 450.$$

And 2ℨ + 300 = 450. And 2ℨ = 150. And 1ℨ = 75.

3. Two Numbers, one Greater and the other Leſſer, or A and E, the Greater is as much as the Leſſer, and 4 more; and the Square of the Greater is equal to the Square of the Leſſer, and 32 more: what are thoſe Numbers?

Q. *Of 2 Numbers, what they are.*

Anſw. Greater 6, Leſſer 2: For 6 = 2 + 4; and 36 = 4 + 32.

Reſolution.

B 4. D 32. Greater ſuppoſe *a*. Leſſer conſequently *a*—B.
Then *a*q = *a*q — 2*a*B + Bq + D. And 2*a*B = Bq + D.

$$\text{And } a = \frac{\text{Bq} + \text{D}}{2\text{B}}. \quad \text{Or } \frac{16 + 32}{8} = \frac{48}{8} = 6.$$

A E B
1ℨ 1ℨ — 4 D
1ʒ = 1ʒ — 8ℨ + 16 + 32. Or 1ʒ = 1ʒ — 8ℨ + 48.
And 8ℨ = 48. And 1ℨ = 6.

And thus both in this and the next precedent Example, may the leſſer Number be found if the Suppoſition be therefore.

4. Two Numbers, or A and E, the Square of A lacking the Square of E is 45, and A lacking E is 5: what are thoſe Numbers?

Q. *Of 2 Numb. what they are.*

Anſw. A 7, and E 2: For 49 — 4 = 45. And 7 — 2 = 5.

Reſolution.

B 45. X 5. Greater ſuppoſed *a*. Leſſer conſequently *a*—X.
Then *a*q—*a*q—2*a*X + Xq = B. And 2*a*X = B + Xq.

$$\text{And } a = \frac{\text{B} + \text{Xq}}{2\text{X}}, \quad \text{Or } \frac{45 + 25}{10} = \frac{70}{10} = \overset{\text{E}}{7}$$

A E
1ℨ 1ℨ — 5
1ʒ — 1ʒ — 10ℨ + 25 = 45. And 10ℨ — 25 = 45.
And 10ℨ = 70. And 1ℨ = 7.

5. A Number joined with 18, and taken from 100, the Sum and Remain are found to be in a *Subtriple Ratio*, that is, as 1 to 3: what is that Number?

Q. *Of the Ratio of a Sum and Remain, what the Number.*

Anſw. 11½: For 29½ to 88½, is as 1 to 3.

Reſolution.

B 18. C 100. R 1. S 3. Suppoſition A.
Then B + A . C — A :: R . S. And becauſe the Product of the Extreams is equal to the Product of the Means, it ſhall be that RC — RA = BS + SA.

$$\text{And A} = \frac{\text{RC} - \text{BS}}{\text{R} + \text{S}}, \quad \text{Or } \frac{100 - 54}{1 + 3} = \frac{46}{4} = 11\tfrac{1}{2}.$$

1ℨ. Then 1ℨ + 18 . 100 — 1ℨ :: 1 . 3.
And 100 — 1ℨ = 3ℨ + 54. And 4ℨ + 54 = 100.
And 4ℨ = 46. And 1ℨ = 11½.

6. There is a Number from which if ⅖ be taken, the Number will be left as much under 100, as it was at firſt above: what is that Number?

Q. *Of a Number, &c.*

Anſw. 125, of which ⅖ is 50. And 125 — 50 = 100 — 25.

Reſolution.

$$\frac{\text{R}}{\text{S}}\ \frac{2}{5}. \text{ B } 100. \text{ Suppoſition A.}$$

$$\text{Then A} - \frac{\text{R}}{\text{S}} = \frac{\text{SA} - \text{RA}}{\text{S}}. \quad \text{And B} - \frac{\text{SA} - \text{RA}}{\text{S}} = \text{A} - \text{B}.$$

$$\text{And } 2\text{B} = \frac{2\text{SA} - \text{RA}}{\text{S}}. \quad \text{And } 2\text{BS} = 2\text{SA} - \text{RA}.$$

$$\text{And A} = \frac{2\text{BS}}{2\text{S} - \text{R}}. \quad \text{Or } \frac{200 \times 5}{2 \times 5 - 2} = \frac{1000}{8} = 125.$$

1ℨ greater than 100, ſo ſhall 1ℨ — ⅖ℨ be leſſer than 100.
And 100 being the Mean between them, 100 = 1ℨ — ⅕ℨ.
And ⅘ℨ = 100. And 1ℨ = 125.

Q. Of the Stock of two Merchants.

7. Two Merchants trade till they gain 150 *l.* whereof *A* having 200 *l.* in Stock more than *B*, taketh 100 *l.* what Money had each in Stock?

Reſolution.

Anſw. *A* 400 *l.* *B* 200 *l.* For ſeeing *A* of the 150 took 100, that is 50 *l.* more than the 50 *l.* left for *B*; it muſt follow, That if 50 *l.* Gain require 200 *l.* Stock, then 100 *l.* Gain ſhall require 400 *l.* Stock.

C 150. D 200. F 100. Suppoſe Stock of *B* to be a. Then *A* is $a+\mathrm{D}$.

And as $2a+\mathrm{D} \,.\, \mathrm{C} :: a \,.\, \frac{\mathrm{C}a}{2a+\mathrm{D}}$. And $\frac{\mathrm{C}a}{2a+\mathrm{D}}+\mathrm{F}=\mathrm{C}$.

And $\mathrm{C}a+2a\mathrm{F}+\mathrm{DF}=2a\mathrm{C}+\mathrm{DC}$. And $2a\mathrm{F}-\mathrm{C}a=\mathrm{DC}-\mathrm{DF}$.

And $a=\frac{\mathrm{DC}-\mathrm{DF}}{2\mathrm{F}-\mathrm{C}}$. Or $\frac{30000-20000}{200-150}=\frac{10000}{50}=200$.

And by *Coſſicks*, ſuppoſing the Stock of *B* to be $1\mathfrak{z}$: Then *A* is $1\mathfrak{z}+200$.

And as $2\mathfrak{z}+200 \,.\, 150 :: 1\mathfrak{z} \,.\, \frac{150\mathfrak{z}}{2\mathfrak{z}+200}$.

And $\frac{150\mathfrak{z}}{2\mathfrak{z}+200}+100=150$. And $150\mathfrak{z}+200\mathfrak{z}+20000=300\mathfrak{z}+30000$.

And $50\mathfrak{z}=10000$. And $1\mathfrak{z}=200$.

Q. Of Travel.

8. A Man travelling 9 Miles a Day, is followed by another from the ſame Place, that ſets forth 10 Days after, and went 14 Miles a Day: in how many Days will the latter overtake the former?

Reſolution.

Anſw. In 18 Days: For 5 Miles (that is 14 — 9) Gain in a Day will reach 90 Miles (that is 10×9) in 18 Days.

B 9. C 90. D 14. Suppoſition A.

Then $\mathrm{BA}+\mathrm{C}=\mathrm{DA}$. And $\mathrm{C}=\mathrm{DA}-\mathrm{BA}$.

And $\mathrm{A}=\frac{\mathrm{C}}{\mathrm{D}-\mathrm{B}}$. Or $\frac{90}{14-9}=\frac{90}{5}=18$.

$1\mathfrak{z}$. Then ſhall $9\mathfrak{z}+90=14\mathfrak{z}$. And $5\mathfrak{z}=90$.
And $1\mathfrak{z}=18$.

Q. Of the Days a Servant worked and played.

9. A Gentleman hired a Servant for a Year, upon Condition that for every Day he laboured, he ſhould have 1 *s.* and for every Day he was idle, he ſhould loſe or diſcount 8 *d.* Now at the Year's end the Maſter was by this Agreement to give the Servant nothing, nor was the Servant in the Maſter's Debt: what Days did the Servant work, and how many Days did he play?

Reſolution.

Anſw. 146 he wrought, and 219 he plaid: For 146 Shillings and 219 Eightpences are equal.

B 365. C 12. D 8. Greater ſuppoſed A. Leſſer then B—A.

Then $\mathrm{CA}=\mathrm{BD}-\mathrm{DA}$. And $\mathrm{CA}+\mathrm{DA}=\mathrm{BD}$.

And $\mathrm{A}=\frac{\mathrm{BD}}{\mathrm{C}+\mathrm{D}}$. Or $\frac{365\times8}{12+8}=\frac{2920}{20}=146$.

$1\mathfrak{z}$ Greater, $365-1\mathfrak{z}$ Leſſer. Then $12\mathfrak{z}=2920-8\mathfrak{z}$.
And $20\mathfrak{z}=2920$. And $1\mathfrak{z}=146$.

Q. Of two Sorts of Monies.

10. There are two Sorts of Monies, in Number 1000 Pieces, worth 80 *l.* whereof 10 of the firſt Kind, and 20 of the other are worth 1 *l.* what Number were there of both theſe Sorts ſeverally?

Reſolution.

Anſw. 600 of the one, and 400 of the other: For the Quotients of 600 divided by 10, and 400 by 20, make together 80.

B 1000. C 10. D 20. F 80. Greater ſuppoſed A. Leſſer then B—A.

Then $\frac{\mathrm{A}}{\mathrm{C}}+\frac{\mathrm{B}-\mathrm{A}}{\mathrm{D}}=\mathrm{F}$. And $\mathrm{DA}+\mathrm{BC}-\mathrm{CA}=\mathrm{DCF}$.

And $\mathrm{A}=\frac{\mathrm{DCF}-\mathrm{BC}}{\mathrm{D}-\mathrm{C}}$. Or $\frac{20\times10\times80-1000\times10}{20-10}=\frac{16000-10000}{10}$

And $\frac{6000}{10}=600$ of the Greater.

$1\mathfrak{z}$ Greater. Then $1000-1\mathfrak{z}$ the Leſſer.

Then $\frac{1\mathfrak{z}}{10}+\frac{1000-1\mathfrak{z}}{20}=80$. And $20\mathfrak{z}+10000-10\mathfrak{z}=16000$.

And $10\mathfrak{z}+10000=16000$. And $10\mathfrak{z}=6000$. And $1\mathfrak{z}=600$.

11. Suppoſe *Berwick* and *London* are diſtant 228 Miles, out of which 2 Foot-poſts take their Journeys, and meet the 12*th* Day; but the one went each day one Mile farther than the other: how many Miles did each of them go every day? *Q. Of two Poſts, how many Miles they travel.*

Anſw. 10 the one, and 9 the other. For the difference between them being 1, yet each of them ſeverally multiplied by 12, will make together 228. Reſolution.

X 1. B 12. C 228. Greater ſuppoſed A. Leſſer conſequently A—X.

Then $BA+BA-BX=C$. And $A=\frac{C+BX}{2B}$.

Or $\frac{228+12}{24}=\frac{240}{24}=10$ Greater. 9 Leſſer.

$1\mathfrak{z}$ Greater. $1\mathfrak{z}-1$ Leſſer.

Then as 1 Day . $1\mathfrak{z}$ :: 12 Days . $12\mathfrak{z}$.

And as 1 . $1\mathfrak{z}-1$ :: 12 . $12\mathfrak{z}-12$.

And $24\mathfrak{z}-12=228$.

And $24\mathfrak{z}=240$. And $1\mathfrak{z}=10$ Greater. 9 Leſſer.

12. Suppoſe the ſame Diſtance as above, and the one Poſt travel 10 Miles a Day from the one Place, and the other 9 from the other: when ſhall they meet? *Q. Of two Poſts, when they meet.*

Anſw. The 12*th* Day: For 10 multiplied by 12, and 9 by 12, make together 228. Reſolution.

B 10. C 9. D 228. Suppoſition A.

Then $BA+CA=D$. And $A=\frac{D}{B+C}$. Or $\frac{228}{19}=12$.

$1\mathfrak{z}$. Then $10\mathfrak{z}+9\mathfrak{z}=228$. And $19\mathfrak{z}=228$. And $1\mathfrak{z}=12$.

13. Three are in Company: The Stock of *A* and *B* together is 238 *l.* of *B* and *C* 470 *l.* of *A* and *C* 568 *l.* what Stock had each of them ſeverally? *Q. Of the Stock of 3 Merchants.*

Anſw. *A* 168 *l.* *B* 70 *l.* *C* 400 *l.* For $168+70=238$. And $70+400=470$. And $168+400=568$. Reſolution.

Perſons *A*, *B*, *C*. Stocks ſuppoſed *a*, *b*, *c*. D 238. F 470. G 568.

Then $a=D-b$. And $a=G-c$. Therefore $D-b=G-c$.

And $c=G+b-D$. And becauſe by the *Data* it appears that $c=F-b$. Therefore $G+b-D=F-b$. And $2b+G=F+D$.

And $2b=F+D-G$. And $b=\frac{F+D-G}{2}$. Or $\frac{470+238-568}{2}=\frac{140}{2}=70$.

In *Coſſicks:* If *b* be ſuppoſed $1\mathfrak{z}$, then *a* is $238-1\mathfrak{z}$. And *c* is $330+1\mathfrak{z}$. And $330+2\mathfrak{z}=470$. And $2\mathfrak{z}=140$. And $1\mathfrak{z}=70$.

14. One buying 100 Yards of Cloth, being demanded what a Yard coſt, anſwered, For how much leſs than 80 *s.* I bought 40 Yards, by ſo much leſs than 95 *s.* I bought 50 Yards: what was the Price of a Yard? *Q. Of the Price of a Yard of Cloth.*

Anſw. 1 *s.* 6 *d.* For 40 Yards at that Rate is 60 *s.* which is 20 leſs than 80; and 50 Yards at that Rate is 75 *s.* leſs than 95 by 20 alſo. Reſolution.

B 40. C 50. D 80. F 95. Suppoſition A.

Then $BA-D=CA-F$. And $F-D=CA-BA$.

And $A=\frac{F-D}{C-B}$. Or $\frac{95-80}{50-40}=\frac{15}{10}=1\frac{1}{2}$.

$1\mathfrak{z}$. Then $40\mathfrak{z}-80=50\mathfrak{z}-95$. And $40\mathfrak{z}=50\mathfrak{z}-15$.

And $10\mathfrak{z}=15$. And $1\mathfrak{z}=1\frac{1}{2}$.

15. A General marſhaling his Men in *Battalia*, intended a Square Form; and to that purpoſe, when he had placed a Number at random in the Front, found *Q. Of the Number of Souldiers, and the Fronts.*

100 Men over; and thinking to amend it by placing one more in the Front, found 201 Men wanting: how many Souldiers had he, and what were the Fronts?

Reſolution.

Anſw. The whole Number of Souldiers were 22600, the firſt Front 150, the other 151; for the Square of 150 is 22500, to which 100 added, makes 22600; and the Square of 151 is 22801, which is 201 above 22600.

B 100. C 201. Firſt Front ſuppoſed A. Then the Second A+1.
Square thereof Aq. Square thereof Aq+2A+1.
Then Aq+B=Aq+2A+1−C. And B=2A+1−C.
And 2A=B+C−1. And $A=\frac{B+C-1}{2}$. Or $\frac{100+201-1}{2}=\frac{300}{2}=150$.

1ʑ Firſt Front. Then the Second ſhall be 1ʑ+1.
Square of the Firſt 1ʒ. Square of the Second 1ʒ+2ʑ+1.
Then 1ʒ+100=1ʒ+2ʑ−200. And 100=2ʑ−200.
And 2ʑ=300. And 1ʑ=150.

Likewiſe if Suppoſition be made for the ſecond Front 1ʑ; Then the firſt Front ſhall be 1ʑ−1, and their Squares 1ʒ, and 1ʒ+1−2ʑ; and the Body of Men by the one ſhall be 1ʒ−201, and by the other 1ʒ+101−2ʑ; which *Equation* tranſpoſed 2ʑ=302; and 1ʑ=151.

Q. *Of Legacies to 4 Children.*

16. A poor Man dying which had 4 Children, bequeathed 72 Crowns to his 4 Children; ſo that the Second and Third ſhould have together 7 times as much as the Firſt; and the Portions of the Third and Fourth ſhould be five Times ſo much as the Second's Part, and the Firſt and Fourth ſhould have twice ſo much as the Third: how were the 72 Crowns to be divided?

Reſolution.

Anſw. A 4$\frac{1}{2}$. B 11$\frac{1}{4}$. C 20$\frac{1}{4}$. D 36. which together make 72. And 11$\frac{1}{4}$+20$\frac{1}{4}$=31$\frac{1}{2}$, that is 7 times 4$\frac{1}{2}$. And 20$\frac{1}{4}$+36=56$\frac{1}{4}$, that is 5 times 11$\frac{1}{4}$. And 4$\frac{1}{2}$+36=40$\frac{1}{2}$, that is twice 20$\frac{1}{4}$.

Perſons, A 1. B 2. C 3. D 4. Crowns *b*. Suppoſition for A if *a*. Then by Conſequence muſt B+C be 7*a*; and D the Reſidue be *b*−8*a*. And becauſe A+D=2C, therefore C=$\frac{b}{2}$−3$\frac{1}{2}$*a*; and becauſe C+D=5B, therefore 5B=1$\frac{1}{2}$*b*−11$\frac{1}{2}$*a*; or 5B=$\frac{3}{2}$*b*−$\frac{23}{2}$*a*. And B=$\frac{3}{10}$*b*−$\frac{23}{10}$*a*, and the Total ſhall be equal to *b*.

$$\begin{array}{rlcl} \text{That is A} = & a & & \\ \text{B} = & \frac{3}{10}b & - & 2\frac{3}{10}a. \\ \text{C} = & \frac{b}{2} & - & 3\frac{1}{2}a. \\ \text{D} = & \underline{b} & - & \underline{8a.} \\ \text{Total} & 1\frac{4}{5}b & - & 12\frac{4}{5}a = b. \end{array}$$

And $\frac{4}{5}b=\frac{64}{5}a$. And 4*b*=64*a*.

And *b*=16*a*. And *a*=$\frac{b}{16}$, or $\frac{72}{16}$=4$\frac{1}{2}$.
1ʑ ſuppoſed for A. Then B+C=7ʑ. And D the Reſt 72−8ʑ.
And the half of A and D is 36−3$\frac{1}{2}$ʑ for C. So ſhall the Part of B be $\frac{1}{5}$ of 108−11$\frac{1}{2}$ʑ, that is 21$\frac{3}{5}$−2$\frac{3}{10}$ʑ. And the Total collected thus,

A = 1ʑ
B = 21$\frac{3}{5}$ − 2$\frac{3}{10}$ʑ
C = 36 − 3$\frac{1}{2}$ʑ
D = 72 − 8ʑ
Total 129$\frac{3}{5}$ − 12$\frac{4}{5}$ʑ=72. And 12$\frac{4}{5}$ʑ=57$\frac{3}{5}$.

And 64ʑ=288. And 1ʑ=4$\frac{1}{2}$.

Q. *Of keeping Sheep.*

17. A Farmer agreeth with a Shepherd to keep 80 Sheep for a Year, and at 3 Months end delivereth the Shepherd 30 Sheep more; and 4 Months after that 3 Months bringeth to the Shepherd 30 Sheep more, ſaying to him, Keep me all theſe Sheep ſo long, till ſuch time as the Money I promiſed you at firſt be all earned: how long muſt he keep theſe 140 Sheep?

Reſolution.

Anſw. Nine Months the 80 Sheep, ſix Months the firſt 30, and two Months the laſt 30; which together are equivalent to the keeping of 80 Sheep twelve Months.

B 80. C 12. D 30. F 3. G 7. Suppoſing the Time of keeping 80 be A, then muſt the Time of keeping the firſt 30 be A−F; and of the latter be A−G.

And then $BA+DA-DF+DA-DG=BC$. And by tranſpoſition,

$BA+2DA=BC+DF+DG$. And $A=\frac{BC+DF+DG}{B+2D}$

Or $\frac{960+90+210}{80+60}=\frac{1260}{140}=9$.

For the 80 ſuppoſe the Time $1\mathfrak{z}$. Then muſt $1\mathfrak{z}-3$ be for the firſt 30.
And $1\mathfrak{z}-7$ for the laſt.
Then $1\mathfrak{z}\times 80$ is $80\mathfrak{z}$; and $1\mathfrak{z}-3\times 30$ is $30\mathfrak{z}-90$; and $1\mathfrak{z}-7\times 30$ is $30\mathfrak{z}-210$.
And theſe 3 Products together make $140\mathfrak{z}-300$, which are equal to 80×12, that is $140\mathfrak{z}-300=960$; and $140\mathfrak{z}=1260$; and $1\mathfrak{z}=9$.

Q. Of the Rate and Cuſtom of a Tun of Wine.

18. A Merchant hath laden two Ships with Wine, in the one 20 Tuns, and in the other 30 Tuns; and coming to the *Cuſtom-Houſe*, payeth for the Cuſtom of the former Cargo a Tun of Wine, and receiveth of the Officers in Money 60 *s.* and for the latter Cargo a Tun of Wine lacking 15 *s.* The Queſtion is, how the Wine was rated a Tun, and ſo conſequently how much he paid for the Cuſtom of a Tun of Wine?

Reſolution.

Anſw. The Tun of Wine was valued at 7 *l.* 10 *s.* ſo muſt two Tuns be 15 *l.* From which 3 *l.* 15 *s.* abated, leaves 11 *l.* 5 *s.* to be paid for Cuſtom of the whole 50 Tuns; and ſo for 1 Tun is 4 *s.* 6 *d.*

B 20. C 30. D 60. F 15. Suppoſition *A* for the Value of 1 Tun.

Then if B coſt $A-D$, ſhall C coſt $\frac{CA-CD}{B}$. And this ſhall be equal to $A-F$.

And $CA-CD=BA-BF$. And then ſhall $BA-CA=CD-BF$.

And $A=\frac{CD-BF}{C-B}$. Or $\frac{1800-300}{30-20}=\frac{1500}{10}=150$ *s.* or 7 *l.* 10 *s.*

$1\mathfrak{z}$. Then as $20 \,.\, 1\mathfrak{z}-60 :: 30 \,.\, 1\frac{1}{2}\mathfrak{z}-90$. And
$1\frac{1}{2}\mathfrak{z}-90=1\mathfrak{z}-15$; and $\frac{1}{2}\mathfrak{z}=75$; and $1\mathfrak{z}=150$.

Q. Of a Poſt carrying Letters.

19. A Poſt ſets out from *Amſterdam* to carry Letters to *Danske*; and at the ſame time the Merchant to whom the Letters were ſent, taketh Horſe in *Danske* to ride to *Amſterdam*, and they meet together on the Way. The Poſt delivereth his Letters, and the Merchant asking what he muſt have? he anſwereth, I have gone as much one Day as another, and ſhould have made my Journey in 20 Days, for which my Hire was 30 *s.* conditionally, that if I met you in the Way, that then proportionally for the Way you had travelled, I ſhould be cut off from my Wages: To which the Merchant replied, that he had rode every Day alike, and ſhould have been at his Journey's End in 16 Days: how much then ſhall the Merchant pay the Poſt?

Reſolution.

Anſw. $13\frac{1}{3}$ *s.* For having found the Days of the Poſt's Travel to be $8\frac{8}{9}$, ſuch a Proportion of 30 *s.* will belong to him.

B 20. C 16. Suppoſition for the Poſt's Days A. The whole Way D. F. 30.

Then as $C \,.\, D :: A \,.\, \frac{DA}{C}$. And as $B \,.\, D :: A \,.\, \frac{DA}{B}$

And $\frac{DA}{C}+\frac{DA}{B}=D$. And $\frac{BDA+CDA}{BC}=D$.

And $BDA+CDA=BCD$. And $A=\frac{BC}{B+C}$. Or $\frac{320}{36}=8\frac{8}{9}$ Days.

Then as $B \,.\, F :: A \,.\, \frac{FA}{B}$. Or as $20 \,.\, 30 :: 8\frac{8}{9} \,.\, 13\frac{1}{3}$ Shillings.

$1\mathfrak{z}$ ſuppoſed, for the Poſt's Days.
Then as $16 \,.\, 1 :: 1\mathfrak{z} \,.\, \frac{1}{16}\mathfrak{z}$.
And as $20 \,.\, 1 :: 1\mathfrak{z} \,.\, \frac{1}{20}\mathfrak{z}$.

Total $\frac{36}{320}\mathfrak{z}=1$ whole Journey.

And $36\mathfrak{z}=320$. And $1\mathfrak{z}=8\frac{8}{9}$.
And if 20 Days Hire be 30 *s.* what $8\frac{8}{9}$ Days? *Anſw.* $13\frac{1}{3}$ *s.*

Q. *Of two Ships ſailing.* 20. A Ship ſaileth out of the *Texel* to *Spain*, with ſuch a Wind that he might perform his Voyage in 15 days; but when 6 days were paſt, the Wind changing, the Ship ſailed backwards as much in 4 days, as it had done forward in one day. At the beginning of the ſecond Wind there departed another Ship from *Spain* towards the *Texel*, (being light loaden) ſailed forwards to the *Texel* as often 7 Leagues, as the other Ship backward from his Port 2 Leagues: in how much Time after the firſt 6 days, and how far from the *Texel* ſhall the Ships meet? and in what Time may the Ship from *Spain* arrive at the *Texel*, ſuppoſing the Parts 300 Leagues aſunder?

Reſolution. *Anſw.* In $14\frac{2}{5}$ days they meet; and the Ship from *Spain* had then ſailed 252 Leagues, which is 48 from the *Texel*; and in 17 days $\frac{1}{7}$ may arrive there: diſcovered thus,

B 15. C 6. D 1. F 4. G 2. H 7. K 300. Suppoſition for the meeting A.

Then as $B . K :: C . \frac{KC}{B}$. Leagues that the Ship coming from the *Texel* had made toward *Spain* with the firſt Wind.

And $K - \frac{KC}{B}$. ——— Leagues which that Ship had to ſail when the Wind changed.

Then as $B . K :: \frac{D}{F} . \frac{KD}{BF}$. Leagues which that Ship ſailed backward every day.

Then as $G . H :: \frac{KD}{BF} . \frac{HKD}{GBF}$. Leagues that the Ship coming from *Spain* made every day towards the *Texel*.

Then as $D . \frac{KD}{BF} :: A . \frac{KA}{BF}$. Leagues that the firſt Ship had gone backward at the day of meeting.

Then as $D . \frac{HKD}{GBF} :: A . \frac{HKA}{GBF}$. Leagues that the ſecond Ship had ſailed forward at the day of meeting.

And $\frac{HKA}{GBF} - \frac{KA}{BF} = K - \frac{KC}{B}$. And $\frac{HKA}{GF} - \frac{KA}{F} = KB - KC$.

And $\frac{HA}{GF} - \frac{A}{F} = B - C$. And $\frac{HA}{F} - \frac{AG}{F} = BG - CG$.

And $A = \frac{BFG - CFG}{H - G}$. Or $\frac{120 - 48}{7 - 2} = \frac{72}{5} = 14\frac{2}{5}$ days.

$1\mathfrak{z}$ Suppoſition. As $15 . 300 :: 6 . 120$. $\frac{KC}{B}$ as before. And $300 - 120 = 180$.

And as $15 . 300 :: \frac{1}{4} . 5 . \frac{KD}{BF}$. And as $2 . 7 :: 5 . 17\frac{1}{2} . \frac{HKD}{GBF}$, as before.

Then as $1 . 5 :: 1\mathfrak{z} . 5\mathfrak{z}$. So $5\mathfrak{z} + 180$ diſtance of the firſt Ship from *Spain*.
And $1 . 17\frac{1}{2} :: 1\mathfrak{z} . 17\frac{1}{2}\mathfrak{z}$, Run of the ſecond Ship towards the *Texel*.
Then $5\mathfrak{z} + 180 = 17\frac{1}{2}\mathfrak{z}$; and $12\frac{1}{2}\mathfrak{z} = 180$; and $1\mathfrak{z} = 14\frac{2}{5}$.

Wherefore by Conſequence if they meet in $14\frac{2}{5}$ days, then had the Ship from *Spain* ſailed 252 Leagues, becauſe ſhe ſailed $17\frac{1}{2}$ Leagues in a day; and ſuch a Courſe will run 300 Leagues in $17\frac{1}{7}$ days.

Q. *Of digging a Well.* 21. A Gentleman hired a Workman to dig a Well 12 Feet deep for 12 *s.* When the Workman had digged 8 Feet, they fall at Variance, and the Gentleman will pay him off: what muſt he give the Workman for his Work, conſidering the Labour at the Bottom is worth more than that at the Top?

Reſolution. *Anſw.* $5\frac{7}{13}$ *s.* For ſuch a Proportion of the 12 *s.* is the Sum of an *Arithmetical Progreſſion*, beginning with $\frac{2}{13}$, and continued to 8 Terms; which by an increaſe of $\frac{2}{13}$ to every Term, and continued to 12 Terms, will make the Sum thereof 12 *s.*

Here working in Numbers without Species, a *Progreſſion Arithmetical* is framed of 12 Terms from an Unit, whoſe Sum being 78, and the Sum of 8 Terms thereof but 36, the *Analogy* is, As the Sum of the greater Number of Terms, is to the whole Price: So is the Sum of the leſſer Number of Terms, to the Price deſired.

$$1+2+3+4+5+6+7+8+9+10+11+12 = 78$$
$$1+2+3+4+5+6+7+8 = 36$$

Wherefore as $\overset{Z}{78} \;.\; \overset{s.}{12} :: \overset{Z}{36} \;.\; \overset{s.}{5\frac{7}{13}}$ price of the 8 Feet.

Admit then in *Species* alike *Progreſſion* be framed, whoſe firſt Term be α: Then ſhall the Sum of 12 Terms be $78\,\alpha$, and the Sum of 8 Terms $36\,\alpha$: And let 12 *s.* the whole Price be B, and A ſuppoſed for the Number ſought, or Sum of 8 Terms. Then becauſe the Product of the Means is equal to the Product of the Extreams;

$$78\,\alpha\,A = 36\,\alpha\,B. \qquad \text{And } 78\,A = 36\,B.$$

$$\text{And } 13\,A = 6\,B. \quad \text{Or } \frac{6\times 12}{13} = \frac{72}{13} = 5\tfrac{7}{13}.$$

So by *Coſſicks* ſuppoſing $1\,\mathfrak{z}$ for the Sum of 8 Terms:
$78\,\mathfrak{z} = 36\times 12$. Or $78\,\mathfrak{z} = 432$. And $1\,\mathfrak{z} = 5\frac{7}{13}$.

Q. *Of digging a Well.*

22. A Workman is agreed with to make a Well of 12 Feet deep for 12 *s.* and after digging 8 Feet thereof, fell ſick, and deſired Money for what he had done: but becauſe the digging the lower Part is more labour than the upper, Allowance was agreed on, that every Foot ſhould be a Penny more than the other: what then ſhall the Workman receive for digging the firſt 8 Feet?

Reſolution.

Anſw. 6 *s.* 8 *d.* For ſuch is the Sum of 8 Terms of a *Progreſſion*, the Sum of 12 Terms whereof is 12 *s.* and the Exceſs 1 *d.*

The *Data* here being T 12. X 1. Z 12 *s.* or 144 *d.* by the former Rules in *Progreſſion* may be found α the firſt Term, or ω the laſt Term; and thereby the Sum of 8 Terms of ſuch a *Progreſſion* to be 6 *s.* 8 *d.* as before.

$$\text{For } \frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \underset{6\frac{1}{2}\,d.}{\alpha}. \quad \text{And } \frac{2Z}{2T} + \frac{TX}{2} - \frac{X}{2} = \underset{17\frac{1}{2}\,d.}{\omega}.$$

Otherwiſe let a *Progreſſion* be framed to 12 Terms, whoſe firſt Term may be ſuppoſed α, and the Exceſs 1. So ſhall the ſecond Term be $\alpha+1$, and the Third $\alpha+2$, *&c.* Then will the eighth Term be $\alpha+7$, and the 12*th* Term $\alpha+11$; and the Sum of 12 Terms $12\alpha+66$, and the Sum of 8 Terms thereof $8\alpha+28$. Now becauſe $12\alpha+66=144$. And $12\alpha=78$. And $1\alpha=6\frac{1}{2}$; therefore $8\alpha+28=80$, that is $52+28$.

Likewiſe by *Coſſicks* $12\,\mathfrak{z}+66=144$. And $1\,\mathfrak{z}=6\frac{1}{2}$.
Wherefore $8\,\mathfrak{z}+28=80$. For $8\times 6\frac{1}{2}=52$. And $52+28=80$.

Q. *Of paying a Reckoning.*

23. Four Men drink together, and play at Tables for the Wine, every Game 1 *d.* and after they had plaid a while, they found *A* had loſt moſt; whereupon he payeth a Pint of Wine of 5 *d.* upon the Reckoning; and beſides found his Loſs three times as much more as *B* who had loſt leaſt of all; and *C* had loſt 2 *d.* more than *B*, and *D* was 4 *d.* leſs indebted than *A.* Whereupon at laſt they plaid again who ſhould pay all, and it happened upon *B*: So beſides the 5 *d.* *A* had paid, *B* paid 27 Pence: how much ſhould each of them have paid at the firſt?

Reſolution.

Anſw. *A* 14 *d.* *B* 3 *d.* *C* 5 *d.* *D* 10 *d.* For taking 5 from 14, the Remain 9 is 3 times 3; and $3+2=5$; and $14-4=10$.

b 5. *d* 2. *e* 4. *f* 27. Suppoſe the Debt of *A* in all to be *a*.

Then ſubſtracting the 5 *d.* paid, the Remain ſhall be $a-b$. And this being 3 times ſo much as *B*, he ſhould therefore pay $\frac{a-b}{3}$, to which *d* added, makes the Debt of *C* to $\frac{a-b}{3}+d$. And becauſe *D* was 4 *d.* leſs indebted than *A*, therefore the Score of *D* was $a-e$. And then theſe added together muſt be equal to 27.

$$\left.\begin{array}{ll} A\,. & a-b \\ B\,. & \dfrac{a-b}{3}. \\ C\,. & \dfrac{a-b}{3}+d. \\ D\,. & a-e \end{array}\right\} \begin{array}{c} \textit{Total of their Debts.} \\ 2\tfrac{2}{3}\,a - 1\tfrac{2}{3}\,b + d - e = f. \end{array}$$

Then $8a-5b+3d-3e=3f$. And $8a=3f+5b+3e-3d$.

And $a=\frac{3f+5b+3e-3d}{8}$. Or $\frac{81+25+12-6}{8}=\frac{112}{8}=14$.

Alſo let the Debt of *A* be 1ζ. Then taking 5 *d.* thence, the reſt $1\zeta-5$ muſt be divided by 3. So is the Debt of *B* $\frac{1\zeta-5}{3}$. And *C* being 2 *d.* more, muſt be $\frac{1\zeta-5}{3}+2$. And *D* is indebted $1\zeta-4$. The Total of which 4 Sums is $2\frac{2}{3}\zeta-10\frac{1}{3}$; and equal to 27.

Then if $2\frac{2}{3}\zeta-10\frac{1}{3}=27$, ſhall $2\frac{2}{3}\zeta=37\frac{1}{3}$.

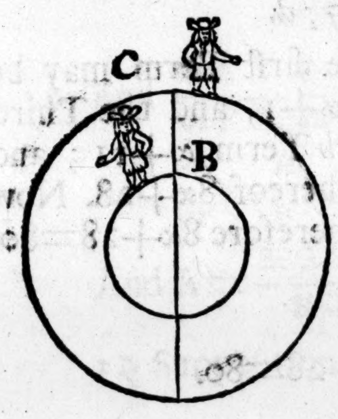
And $8\zeta=112$; and $1\zeta=14$, the whole Loſs of *A*.

Q. Of compaſſing Circles.

24. Let there be two Circles one within another, deſcribed upon the ſame Center (as below); and ſuppoſe one upon the outermoſt Circle in C goeth on the right Hand, and compaſſeth it about in 12 Hours: And another within upon the innermoſt Circle in B, at the ſame Time and in the ſame Proportion as the firſt, goeth on the left Hand, and compaſſeth it about in 3 Hours: when ſhall they come to be one under the other, or as near together as when they firſt began their Journies?

Reſolution.

Anſw. In $2\frac{2}{5}$ Hours: For if the greateſt be in compaſs 8 Miles, then ſhall he at C in that time go $1\frac{3}{5}$ Mile. And ſo muſt the ſmalleſt Circle at B be 2 Miles, (becauſe if he in the greateſt Circle go in 12 Hours 8 Miles, then he in the ſmalleſt muſt go in 3 Hours 2 Miles): and then if in 3 Hours he goeth within a Compaſs of 8 Miles, in $2\frac{2}{5}$ Hours he ſhall go within the Compaſs of $6\frac{2}{5}$ Miles; which added to $1\frac{3}{5}$ Mile, ſhall make together 8 Miles.



b 12. *d* 3. Suppoſition for the Time ſought A.

Then as $b \,.\, 1 :: A \,.\, \frac{A}{b}$. And as $d \,.\, 1 :: A \,.\, \frac{A}{d}$.

Both which Quotients added together, make $\frac{A}{b}+\frac{A}{d}$,

Equal to 1 Hour. And then reduced

$\frac{Ad+Ab}{bd}=1$. And $Ad+Ab=bd$. And $A=\frac{bd}{d+b}$.

Or $\frac{36}{12+3}=\frac{36}{15}=2\frac{2}{5}$.

The Time deſired 1ζ. Then as $12 \,.\, 1 :: 1\zeta \,.\, \frac{1}{12}\zeta$.

And as $3 \,.\, 1 :: 1\zeta \,.\, \frac{1}{3}\zeta$. And $\frac{1}{12}\zeta+\frac{1}{3}\zeta=\frac{5}{12}\zeta$.

And $\frac{5}{12}\zeta=1$. And then $5\zeta=12$. And $1\zeta=2\frac{2}{5}$.

Q. Of compaſſing Circles.

25. If in the Circles aforeſaid the two Men go both one Way, and keep going in like Pace and Proportion: when ſhall they come to the ſame Poſture they were in at firſt?

Reſolution.

Anſw. In 4 Hours: For if the greateſt Circle be 8 Miles, then ſhall the leaſt be 2 Miles: and if 12 Hours give 8 Miles, then 4 Hours ſhall give $2\frac{2}{3}$ Miles for him in C: and if 3 Hours give 2 Miles, then 4 Hours ſhall give $2\frac{2}{3}$ Miles for B, thereby ſhewing the one in his Circle hath gone as many Miles as the other in his Circle.

Here one of the Quotients is to be taken from the other, that is, $\frac{A}{b}$ from $\frac{A}{d}$; and then $\frac{A}{d}-\frac{A}{b}$ reduced, ſhall be $\frac{Ab-Ad}{db}=1$.

And $Ab-Ad=db$. And $A=\frac{db}{b-d}$. Or $\frac{36}{12-3}=\frac{36}{9}=4$.

So by *Coſſicks* $\frac{1}{3}\zeta-\frac{1}{12}\zeta=\frac{1}{4}\zeta$. And $\frac{1}{4}\zeta=1$. And $1\zeta=4$.

Q. Of two Servants, what Cloth they received of their Maſter.

26. A Merchant hath 2 Servants, to whom he delivereth together the Value of 300 *l.* in Linen Cloth; and one of them ſelleth his Part, and loſeth therein with his Charges $\frac{1}{8}$ Part of that he received of his Maſter, and with the reſt he buyeth Spices, and gaineth by them 42 *l.* The Second gaineth by his Cloth $\frac{1}{4}$ Part as much as he had received of his Maſter, out of which he ſpendeth 12 *l.*

And when they came home, they both pay their Maſter 330 *l.* how much Money in Cloth did each of them receive of their Maſter?

Anſw. *A* 200 *l.* and *B* 100 *l.* For 200 with 25 (that is $\frac{1}{8}$) loſs, is 175, to which 42 (Gain) added, is 217. And 100 with 25 (that is $\frac{1}{4}$) Gain, is 125; from which 12 (ſpent) being taken away, the Remain is 113. And $217+113=330$. Reſolution.

C 300. D 42. F 12. G 330. Suppoſe *A* received a.

Then taking away $\frac{1}{8}$ leaves $\frac{7}{8}a$; to which adding D, makes $D+\frac{7}{8}a$, that *A* muſt bring his Maſter. Now if *A* received of his Maſter a, then muſt *B* receive $C-a$; to which $\frac{1}{4}$ added, and his Expences deducted, makes him bring his Maſter home $1\frac{1}{4}C-1\frac{1}{4}a-F$, both which muſt be equal to G; wherefore ſhall $1\frac{1}{4}C+D-\frac{3}{8}a-F=G$; and $1\frac{1}{4}C-\frac{3}{8}a=G+F-D$; and $\frac{10C-3a}{8}=G+F-D$. And $10C-3a=8G+8F-8D$; and $3a=10C+8D-8G-8F$.

And conſequently $a=\frac{10C+8D-8G-8F}{3}$. Or $\frac{3000+336-2640-96}{3}=\frac{3336-2736}{3}=\frac{600}{3}=200$.

In like ſort by *Coſſicks*, if *A* receive $1\mathfrak{z}$, then $\frac{1}{8}$ taken away, and 42 added, makes him bring home $\frac{7}{8}\mathfrak{z}+42$. And if *A* receive $1\mathfrak{z}$, then *B* receives $300-1\mathfrak{z}$; to which $\frac{1}{4}$ added, and 12 taken away, makes him bring home $363-1\frac{1}{4}\mathfrak{z}$: both Sums of *A* and *B* added, makes $405-\frac{3}{8}\mathfrak{z}=330$. And $\frac{3}{8}\mathfrak{z}=75$; and $3\mathfrak{z}=600$; and $1\mathfrak{z}=200$.

27. If a Merchant buy Rie for 36 *s.* the Quarter: for how much ſhall he ſell it again that he may gain by 120 *s.* imployed therein, as much as he received for a Quarter? *Q. Of Rie, how ſold by the Quarter.*

Anſw. For $51\frac{3}{7}$ *s.* the Quarter: Seeing if 36 gain $15\frac{3}{7}$, 120 ſhall gain $51\frac{3}{7}$. Reſolution.

B 36. C 120. Suppoſition A.

Then as $B . A :: C . \frac{AC}{B}$. And $\frac{AC}{B}=C+A$.

And $AC=BC+BA$. And $AC-BA=BC$.

And $A=\frac{BC}{C-B}$. Or $\frac{4320}{120-36}=\frac{4320}{84}=51\frac{3}{7}$.

By *Coſſicks*, ſuppoſing $1\mathfrak{z}$: Then as $36 . 1\mathfrak{z} :: 120 . 3\frac{1}{3}\mathfrak{z}$.
And $3\frac{1}{3}\mathfrak{z}=120+1\mathfrak{z}$. And $2\frac{1}{3}\mathfrak{z}=120$; and $7\mathfrak{z}=360$.
And $1\mathfrak{z}=51\frac{3}{7}$.

28. A Merchant hath bought 7 Yards of Cloth, 8 Ells of Damask, and 9 Yards of Sattin together, for 74 *l.* Flemiſh: And of the ſame ſelleth 5 Yards of Cloth, 4 Ells of Damask, and 6 Yards of Sattin for 47 *l.* The Yard of Cloth coſt 4 *l.* what coſt the Ell of Damask, and Yard of Sattin ſeverally? *Q. Of Damask, Sattin & Cloth, the Price.*

Anſw. The Ell of Damask $2\frac{3}{4}$ *l.* and the Yard of Sattin $2\frac{2}{3}$ *l.* For $7\times4=28$. And $8\times2\frac{3}{4}=22$; and $9\times2\frac{2}{3}=24$; and $28+22+24=74$. Reſolution.

B 7. C 8. D 9. F 74. G 5. H 4. K 6. M 47.

Suppoſe the Ell of Damask coſt A.

Then Cloth coſt	BH.	Sold G for	GH.
Damask	CA.	H for	HA.
Sattin	F−BH−CA.	K for	$\frac{KF-KBH-KCA}{D}$.

Becauſe as $D . F-BH-CA :: K . \frac{KF-KBH-KCA}{D}$.

And then $GH+HA\frac{KF-KBH-KCA}{D}=M$.

And by Reduction $GHD+HAD+KF-KBH-KCA=MD$.
And likewiſe $HAD-KCA=MD+KBH-KF-GHD$.

And then $A=\frac{MD+KBH-KF-GHD}{HD-KC}$. Or $\frac{423+168-444-180}{36-48}=\frac{33}{12}=2\frac{3}{4}$.

In *Coſſicks*, ſuppoſing the Ell of Damask coſt $1\mathfrak{z}$.

Then Cloth coſt	28 *l.*	Sold 5 Yards for	20 *l.*
Damask	$8\mathfrak{z}$.	4 Ells for	$4\mathfrak{z}$.
Sattin the reſt	46 *l.* $-8\mathfrak{z}$.	6 Yards for	$30\frac{2}{3}$ *l.* $-5\frac{1}{3}\mathfrak{z}$.

Becauſe as $9 \,.\, 46\,l. - 8\text{ℨ} :: 6 \,.\, 30\tfrac{2}{3}\,l. - 5\tfrac{1}{3}\text{ℨ}$.
Then $20 + 4\text{ℨ} + 30\tfrac{2}{3} - 5\tfrac{1}{3}\text{ℨ}$, that is $50\tfrac{2}{3} - 1\tfrac{1}{3}\text{ℨ} = 47$.
And $1\tfrac{1}{3}\text{ℨ} = 3\tfrac{2}{3}$; and $4\text{ℨ} = 11$; and $1\text{ℨ} = 2\tfrac{3}{4}$.

Q. Of Sugar the Hundred Netto. 29. A Grocer buyeth 21 C. 98 ℔ of Sugar for 600 *l.* Flemiſh; and being allowed Tare upon the Hundred 10 ℔, the Hundred *Netto* coſteth him 30 *l.* Flemiſh: how many Pound is the Hundred *Netto* accompted?

Reſolution. *Anſw.* 102 ℔: For if 30 *l.* buy 112 ℔, then will 600 *l.* buy 2240 ℔; which at 102 for 1 C. makes 21 C. 98 ℔.

B 21. D 98. F 600. G 10. H 30. Suppoſition A.

Then as $H \,.\, A+G :: F \,.\, \frac{AF+GF}{H}$. And $\frac{AF+GF}{H} = B+D$.

And $AF+GF = BH+DH$. And $AF = BH+DH-GF$.

And $A = \frac{BH+DH}{F} - G$. Or $\frac{630}{600}C. + \frac{2940}{600}℔ - 10℔ = 1\frac{1}{20}C. + 4\frac{18}{20}℔ - 10℔$.

Or $1\frac{1}{20}C - 5\frac{2}{20}℔$. And $\frac{1}{20}C = \frac{102}{20}℔$; and $1C = 102℔$.

Suppoſing 1ℨ in *Coſſicks*; Then as $30\,l \,.\, 1C+10℔ :: 600\,l \,.\, 20C+200℔$.

And $21C+98℔ = 20C+200℔$; and $21C = 20C+102℔$; and $1C = 102℔$.

Q. Of Pitch, how much ſold. 30. Two Merchants ſold Pitch for 504 *l.* whereof *B* ſold 3 Tuns more than *A.* Whereupon *A* ſaid to *B*, I would have ſold all your Pitch for 288 *l.* And *B* ſaid to *A*, I would have ſold all your Pitch for 216 *l.* how many Tuns of Pitch did each of them ſell?

Reſolution. *Anſw. A* ſold 9 Tuns, and *B* 12 Tuns; which at the Rate of 504 *l.* for the whole 21 Tuns, is 24 *l.* a Tun, and makes 9 amount to 216 *l.* for *A*, and 288 *l.* for *B.*

C 504. D 3. F 288. G 216. Suppoſe *A* ſold *a*; then *B* ſells $a+3$.

And as $a \,.\, G :: a+D \,.\, \frac{Ga+DG}{a}$. And as $a+D \,.\, F :: a \,.\, \frac{Fa}{a+D}$.

Then both Quotients added are $\frac{Faq+Gaq+2DGa+DqG}{aq+Da} = C$.

And $Faq+Gaq+2DGa+DqG = Caq+CDa$. And becauſe by the *Data* it appears C was equal to F and G; therefore their Squares are alſo equal, and ſo omitted. And then $2DGa+DqG = CDa$; and $CDa-2DGa = DqG$.

And $a = \frac{DqG}{CD-2DG}$. Or $\frac{1944}{1512-1296} = \frac{1944}{216} = 9$.

In *Coſſicks*, ſuppoſing 1ℨ for *A*; Then muſt *B* ſell $1\text{ℨ}+3$.

And $1\text{ℨ} \,.\, 216 :: 1\text{ℨ}+3 \,.\, \frac{216\text{ℨ}+648}{1\text{ℨ}}$. And as $1\text{ℨ}+3 \,.\, 288 :: 1\text{ℨ} \,.\, \frac{288\text{ℨ}}{1\text{ℨ}+3}$.

Total $\frac{504\text{ʒ}+1296\text{ℨ}+1944}{1\text{ʒ}+3\text{ℨ}} = 504$. And by Reduction

$504\text{ʒ}+1296\text{ℨ}+1944 = 504\text{ʒ}+1512\text{ℨ}$. And $216\text{ℨ} = 1944$. And $1\text{ℨ} = 9$.

Q. Of Cloth ſhrunk. 31. Eight Ells of Cloth being $2\tfrac{1}{4}$ broad, after ſhrinking $3\tfrac{3}{4}$ Ells thereof make but $3\tfrac{3}{5}$ in length, and $2\tfrac{1}{2}$ Ells in breadth, make but $2\tfrac{1}{3}$. A ſecond ſort of Cloth being $1\tfrac{3}{4}$ Ell broad; when it is wet is no broader than $1\tfrac{1}{2}$, and the length of 6 Ells but $5\tfrac{1}{4}$: how much of this ſecond ſort of Cloth will ſerve to line the firſt 8 Ells after the ſhrinking..

Reſolution. *Anſw.* $12\frac{756}{2625}$ Ells; which will appear, if the length and breadth of each dry, or the length and breadth of each wet and ſhrunk, be multiplied one by the other, becauſe the Product of the one will be equal to the Product of the other.

As $\frac{8}{1} \times \frac{9}{4}$. Or $10\frac{2}{7} \times 1\frac{3}{4} = \frac{72}{4} = 18$.

Omitting the *Species*, for brevity-ſake, the Work by *Coſſicks* is thus:

As $3\frac{3}{4}$ dry . $3\frac{3}{5}$ wet :: 8 dry . $7\frac{17}{25}$ wet and ſhrunk in length.

As $2\frac{1}{2}$ dry . $2\frac{1}{3}$ wet :: $2\frac{1}{4}$ dry . $2\frac{1}{10}$ wet and ſhrunk in breadth.

$7\frac{17}{25}$ and $2\frac{1}{10}$, or $\frac{192}{25} \times \frac{21}{10} = \frac{4032}{250}$ Square Content.

Then for the length of the ſecond Cloth, ſuppoſe 1ℨ.

As 6 dry . $5\frac{1}{4}$ wet :: 1ℨ dry . $\frac{7}{8}$ℨ, wet and ſhrunk in length.

The breadth given $1\frac{1}{2}$ multiplied by $\frac{7}{8}$ℨ $=\frac{21}{16}$ℨ, Square Content.

Then $\frac{21}{16}$ℨ $=\frac{4032}{250}$. And 5250ℨ $=$ 64512.

And 1ℨ $= 12\frac{756}{2625}$.

32. If 6 Ells of Black, 5 Ells of Red and 7 *l.* be worth as much as 9 Ells of Black, and $3\frac{1}{3}$ Ells of Red: And if at the ſame Price 8 Ells of Black, and 7 of Red lacking 5 *l.* coſt as much as 6 Ells of Black, 5 Ells of Red and 9 *l.* how much ſhall 1 Ell of Black and 1 Ell of Red coſt ſeverally? *Q. Of the Price of Black and Red.*

Anſw. One Red 3 *l.* and 1 Black 4 *l.* Reſolution.

Here being two *Equations* given, there needs no Suppoſition.

Equations given.	*Reduced.*
$6B + 5R + 7 = 9B + 3\frac{1}{3}R$	$1\frac{2}{3}R + 7 = 3B.$
$8B + 7R - 5 = 6B + 5R + 9$	$1B + 1R = 7.$
Both reduced *Equations* added	$1B + 2\frac{2}{3}R + 7 = 3B + 7.$
This laſt *Equation* reduced is	$1\frac{2}{3}R = 1B.$
The ſecond reduced *Equation* added	$1B + 1R = 7.$
Total	$1B + 2\frac{1}{3}R = 1B + 7.$

Then $2\frac{1}{3}R = 7$. And $7R = 21$. And $1R = 3$.

And if 1R be 3, then 1B muſt be 4, becauſe by the ſecond reduced *Equation* both were equal to 7.

33. A King lying with a great Army before an Enemy, the Adverſary endeavoureth to corrupt one of the Heralds to declare the Strength of the Army: who willing to receive the Reward, dealeth ſubtilly, and deviſeth this Anſwer, (nevertheleſs true) Look, ſaith he, how many Dukes there are, and for each of them there are twice ſo many Earls, and under every Earl there are 4 times ſo many Souldiers as there be Dukes in the Field: And when the Muſter of the Souldiers was taken, the 200 Part of them was 9 times ſo many as the Number of the Dukes: how many then were there of each Sort? *Q. Of an Army, how many therein.*

Anſw. 15 Dukes, 450 Earls, (that is 15 times 15 twice); and the Number of Souldiers (being 4 times 15 multiplied by 450) is 27000, the 200 Part whereof (being 135) is 9 times 15 the Number of the Dukes. Reſolution.

B 200. Suppoſition for the Dukes A.

Then the Earls 2Aq; that is $A \times A + A \times A$.

And the Souldiers 8Ac, that is $2Aq \times 4A$.

Then $\frac{8Ac}{B} = 9A$. And $8Ac = 9BA$; and $\frac{Ac}{A} = \frac{9B}{8}$.

And $Aq = \frac{9B}{8}$. Or $\sqrt{q}\,\frac{1800}{8} = \sqrt{q}\,225 (15$.

Dukes ſuppoſe 1ℨ. Then Earls 2ʒ. And Souldiers 8φ.

Then $\frac{8\phi}{200} = 9$ℨ. And 8φ $=$ 1800ℨ. And 8ʒ $=$ 1800N.

And 1ʒ $=$ 225N; whoſe Square Root is 15.

34. Among 4 Walls, the 2 longeſt are in proportion to the ſhorteſt, as 5 to 3; and to the Height they be double Seſquialter: now multiplying the longeſt by the ſhorteſt, and the Total by the Height, there will ariſe 39930 Feet: what then is the Length and Height of each Wall? *Q. Of the length and height of four Walls.*

Anſw. The Height of the ſhorteſt is 22 Feet, and the longeſt being double *Seſquialter* (that is $2\frac{1}{2}$ as much) muſt be 55, and the Length of the ſhorteſt 33, that is as 5 to 3. Reſolution.

B 39930. Suppoſition for the Height of the ſhorteſt A.

Then the longeſt muſt be $2\frac{1}{2}$ A. And becauſe the longeſt to the ſhorteſt are as 5 to 3, the ſhorteſt muſt be $1\frac{1}{2}$ A. For $5 . 3 :: 2\frac{1}{2} . 1\frac{1}{2}$.

And $A \times 2\frac{1}{2}A \times 1\frac{1}{2}A = \frac{15}{4}Ac$. And $\frac{15}{4}Ac = B$.

Wherefore $Ac = \frac{4B}{15}$. or $\frac{159720}{15} = 10648$. And $\sqrt{c}\,10648 = 22$.

In

Becauſe as $9 . 46\,l. - 8ℨ :: 6 . 30\tfrac{2}{3}\,l. - 5\tfrac{1}{3}ℨ$.
Then $20 + 4ℨ + 30\tfrac{2}{3} - 5\tfrac{1}{3}ℨ$, that is $50\tfrac{2}{3} - 1\tfrac{1}{3}ℨ = 47$.
And $1\tfrac{1}{3}ℨ = 3\tfrac{2}{3}$; and $4ℨ = 11$; and $1ℨ = 2\tfrac{3}{4}$.

Q. Of Sugar the Hundred Netto. 29. A Grocer buyeth 21 C. 98 ℔ of Sugar for 600 *l.* Flemiſh; and being allowed Tare upon the Hundred 10 ℔, the Hundred *Netto* coſteth him 30 *l.* Flemiſh: how many Pound is the Hundred *Netto* accompted?

Reſolution. *Anſw.* 102 ℔: For if 30 *l.* buy 112 ℔, then will 600 *l.* buy 2240 ℔; which at 102 for 1 C. makes 21 C. 98 ℔.

B 21. D 98. F 600. G 10. H 30. Suppoſition A.

Then as $H . A+G :: F . \dfrac{AF+GF}{H}$. And $\dfrac{AF+GF}{H} = B+D$.

And $AF+GF = BH+DH$. And $AF = BH+DH-GF$.

And $A = \dfrac{BH+DH}{F} - G$. Or $\tfrac{630}{600}C. + \tfrac{2940}{600}℔ - 10℔ = 1\tfrac{1}{20}C. + 4\tfrac{18}{20}℔ - 10℔$.

Or $1\tfrac{1}{20}C - 5\tfrac{2}{20}℔$. And $\tfrac{1}{20}C = \tfrac{102}{20}℔$; and $1C = 102℔$.

Suppoſing 1ℨ in *Coſſicks*; Then as $30\,l . 1C + 10℔ :: 600\,l . 20C + 200℔$.

And $21C + 98℔ = 20C + 200℔$; and $21C = 20C + 102℔$; and $1C = 102℔$.

Q. Of Pitch, how much ſold. 30. Two Merchants ſold Pitch for 504 *l.* whereof *B* ſold 3 Tuns more than *A*. Whereupon *A* ſaid to *B*, I would have ſold all your Pitch for 288 *l.* And *B* ſaid to *A*, I would have ſold all your Pitch for 216 *l.* how many Tuns of Pitch did each of them ſell?

Reſolution. *Anſw.* *A* ſold 9 Tuns, and *B* 12 Tuns; which at the Rate of 504 *l.* for the whole 21 Tuns, is 24 *l.* a Tun, and makes 9 amount to 216 *l.* for *A*, and 288 *l.* for *B*.

C 504. D 3. F 288. G 216. Suppoſe *A* ſold *a*; then *B* ſells $a+3$.

And as $a . G :: a+D . \dfrac{Ga+DG}{a}$. And as $a+D . F :: a . \dfrac{Fa}{a+D}$.

Then both Quotients added are $\dfrac{Faq+Gaq+2DGa+DqG}{aq+Da} = C$.

And $Faq+Gaq+2DGa+DqG = Caq+CDa$. And becauſe by the *Data* it appears C was equal to F and G; therefore their Squares are alſo equal, and ſo omitted. And then $2DGa+DqG = CDa$; and $CDa - 2DGa = DqG$.

And $a = \dfrac{DqG}{CD-2DG}$. Or $\dfrac{1944}{1512-1296} = \dfrac{1944}{216} = 9$.

In *Coſſicks*, ſuppoſing 1ℨ for *A*; Then muſt *B* ſell $1ℨ+3$.

And $1ℨ . 216 :: 1ℨ+3 . \dfrac{216ℨ+648}{1ℨ}$. And as $1ℨ+3 . 288 :: 1ℨ . \dfrac{288ℨ}{1ℨ+3}$.

Total $\dfrac{504ʒ + 1296ℨ + 1944}{1ʒ + 3ℨ} = 504$. And by Reduction

$504ʒ + 1296ℨ + 1944 = 504ʒ + 1512ℨ$. And $216ℨ = 1944$. And $1ℨ = 9$.

Q. Of Cloth ſhrunk. 31. Eight Ells of Cloth being $2\tfrac{1}{4}$ broad, after ſhrinking $3\tfrac{3}{4}$ Ells thereof make but $3\tfrac{3}{5}$ in length, and $2\tfrac{1}{2}$ Ells in breadth, make but $2\tfrac{1}{3}$. A ſecond ſort of Cloth being $1\tfrac{3}{4}$ Ell broad; when it is wet is no broader than $1\tfrac{1}{2}$, and the length of 6 Ells but $5\tfrac{1}{4}$: how much of this ſecond ſort of Cloth will ſerve to line the firſt 8 Ells after the ſhrinking..

Reſolution. *Anſw.* $12\tfrac{756}{2625}$ Ells; which will appear, if the length and breadth of each dry, or the length and breadth of each wet and ſhrunk, be multiplied one by the other, becauſe the Product of the one will be equal to the Product of the other.

As $\tfrac{8}{1} \times \tfrac{9}{4}$. Or $10\tfrac{2}{7} \times 1\tfrac{3}{4} = \tfrac{72}{4} = 18$.

Omitting the *Species*, for brevity-ſake, the Work by *Coſſicks* is thus:

As $3\tfrac{3}{4}$ dry . $3\tfrac{3}{5}$ wet :: 8 dry . $7\tfrac{17}{25}$ wet and ſhrunk in length.

As $2\tfrac{1}{2}$ dry . $2\tfrac{1}{3}$ wet :: $2\tfrac{1}{4}$ dry . $2\tfrac{1}{10}$ wet and ſhrunk in breadth.

$7\tfrac{17}{25}$ and $2\tfrac{1}{10}$, or $\tfrac{192}{25} \times \tfrac{21}{10} = \tfrac{4032}{250}$ Square Content.

Then for the length of the ſecond Cloth, ſuppoſe 1ℨ.
As 6 dry . $5\frac{1}{4}$ wet :: 1ℨ dry . $\frac{7}{8}$ℨ, wet and ſhrunk in length.
The breadth given $1\frac{1}{2}$ multiplied by $\frac{7}{8}$ℨ $=$ $\frac{21}{16}$ℨ, Square Content.
Then $\frac{21}{16}$ℨ $= \frac{4032}{250}$. And 5250ℨ $=$ 64512.
And 1ℨ $= 12\frac{756}{2625}$.

32. If 6 Ells of Black, 5 Ells of Red and 7 *l.* be worth as much as 9 Ells of Black, and $3\frac{1}{3}$ Ells of Red: And if at the ſame Price 8 Ells of Black, and 7 of Red lacking 5 *l.* coſt as much as 6 Ells of Black, 5 Ells of Red and 9 *l.* how much ſhall 1 Ell of Black and 1 Ell of Red coſt ſeverally? **Q. *Of the Price of Black and Red.***

Anſw. One Red 3 *l.* and 1 Black 4 *l.* **Reſolution.**

Here being two *Equations* given, there needs no Suppoſition.

Equations given.	*Reduced.*
$6B + 5R + 7 = 9B + 3\frac{1}{3}R$	$1\frac{2}{3}R + 7 = 3B.$
$8B + 7R - 5 = 6B + 5R + 9$	$1B + 1R = 7.$
Both reduced *Equations* added	$1B + 2\frac{2}{3}R + 7 = 3B + 7.$
This laſt *Equation* reduced is	$1\frac{1}{3}R = 1B.$
The ſecond reduced *Equation* added	$1B + 1R = 7.$
Total	$1B + 2\frac{1}{3}R = 1B + 7.$

Then $2\frac{1}{3}R = 7$. And $7R = 21$. And $1R = 3$.

And if 1R be 3, then 1B muſt be 4, becauſe by the ſecond reduced *Equation* both were equal to 7.

33. A King lying with a great Army before an Enemy, the Adverſary endeavoureth to corrupt one of the Heralds to declare the Strength of the Army: who willing to receive the Reward, dealeth ſubtilly, and deviſeth this Anſwer, (nevertheleſs true) Look, ſaith he, how many Dukes there are, and for each of them there are twice ſo many Earls, and under every Earl there are 4 times ſo many Souldiers as there be Dukes in the Field: And when the Muſter of the Souldiers was taken, the 200 Part of them was 9 times ſo many as the Number of the Dukes: how many then were there of each Sort? **Q. *Of an Army, how many therein.***

Anſw. 15 Dukes, 450 Earls, (that is 15 times 15 twice); and the Number of Souldiers (being 4 times 15 multiplied by 450) is 27000, the 200 Part whereof (being 135) is 9 times 15 the Number of the Dukes. **Reſolution.**

B 200. Suppoſition for the Dukes A.
Then the Earls 2Aq; that is $A \times A + A \times A$.
And the Souldiers 8Ac, that is $2Aq \times 4A$.

Then $\frac{8Ac}{B} = 9A$. And $8Ac = 9BA$; and $\frac{Ac}{A} = \frac{9B}{8}$.

And $Aq = \frac{9B}{8}$. Or $\sqrt{q}\,\frac{1800}{8} = \sqrt{q}\,225 (15$.

Dukes ſuppoſe 1ℨ. Then Earls 2ʒ. And Souldiers 8φ.

Then $\frac{8φ}{200} = 9$ℨ. And 8φ $=$ 1800ℨ. And 8ʒ $=$ 1800N.

And 1ʒ $=$ 225N; whoſe Square Root is 15.

34. Among 4 Walls, the 2 longeſt are in proportion to the ſhorteſt, as 5 to 3; and to the Height they be double Seſquialter: now multiplying the longeſt by the ſhorteſt, and the Total by the Height, there will ariſe 39930 Feet: what then is the Length and Height of each Wall? **Q. *Of the length and height of four Walls.***

Anſw. The Height of the ſhorteſt is 22 Feet, and the longeſt being double *Seſquialter* (that is $2\frac{1}{2}$ as much) muſt be 55, and the Length of the ſhorteſt 33, that is as 5 to 3. **Reſolution.**

B 39930. Suppoſition for the Height of the ſhorteſt A.
Then the longeſt muſt be $2\frac{1}{2}$ A. And becauſe the longeſt to the ſhorteſt are as 5 to 3, the ſhorteſt muſt be $1\frac{1}{2}$ A. For $5 . 3 :: 2\frac{1}{2} . 1\frac{1}{2}$.
And $A \times 2\frac{1}{2}A \times 1\frac{1}{2}A = \frac{15}{4}Ac$. And $\frac{15}{4}Ac = B$.
Wherefore $Ac = \frac{4B}{15}$. or $\frac{159720}{15} = 10648$. And $\sqrt{c}\,10648 = 22$.

In *Coſſicks* $1ℨ$. Then $1ℨ \times 2\frac{1}{2}ℨ \times 1\frac{1}{2}ℨ = \frac{15}{4}\phi$.
And $\frac{15}{4}\phi = 39930$. And $15\phi = 159720$. And $1\phi = 10648$.
And $\sqrt{\phi}\ 10648 = 22$.

Q. Of Stock traded with. 35. Traffique is made at *Danske*, and Gain thereupon in 100 *l.* as many Pounds as there were in Stock at firſt; after which Traffique with the Gain only is made at *Hamburgh*, where the Gain of 100 *l.* is as much as the Gain at *Danske*, and the Gain at *Hamburgh* being found to be $16\frac{1}{5}$ *s.* the Queſtion is, what was the Stock at firſt?

Reſolution. *Anſw.* 30 *l.* For if 100 *l.* gain 30, then 30 ſhall gain at *Danske* 9 *l.* And if 100 *l.* gain 9, then 9 ſhall gain $16\frac{1}{5}$ *s.*

Here to make the Denominations agree, $16\frac{1}{5}$ *s.* is to be turned into a part of a Pound, and is $\frac{81}{100}$ *l.*

$$B\ 100.\ \frac{R}{B}\ \frac{81}{100}.$$ Suppoſe the Stock A.

Then $B . A :: A . \frac{Aq}{B}$. Gain at *Danske*.

And $B . \frac{Aq}{B} :: \frac{Aq}{B} . \frac{Aqq}{Bc}$. Gain at *Hamburgh*.

Then $\frac{Aqq}{Bc} = \frac{R}{B}$. And $Aqq = \frac{BcR}{B}$. or $\frac{81000000}{100} = 810000$.

And $\sqrt{qq}\ 810000 = 30$.

Suppoſing the Stock in *Coſſicks* $1ℨ$.

Then $100 . 1ℨ :: 1ℨ . \frac{1ʒ}{100}$. Gain at *Danske*.

And $100 . \frac{1ʒ}{100} :: \frac{1ʒ}{100} . \frac{1ʒʒ}{1000000}$. Gain at *Hamburgh*.

And $\frac{1ʒʒ}{1000000} = \frac{81}{100}$. And $1ʒʒ = 810000$; and the Squared ſquare Root of 810000 is 30. $\dot{8}1\dot{0}0\dot{0}0$ ($\dot{9}0\dot{0}$ (30.

Q. Of the Number of Merchants and their Gain. 36. Certain Merchants make a Company, and every one putteth in Stock 150 times ſo many Pounds as there are Merchants, and they gain as much on the Hundred as the Number of Merchants are; and if $9\frac{1}{2}$ be taken out of the Gain, and $9\frac{1}{2}$ be added to the Gain, and the Remain and Total be multiplied together, the Product will be 1550 *l.* how many Merchants were there, and how much did they gain?

Reſolution. *Anſw.* 3 Merchants; 450 *l.* each Man's Stock (that is 3 times 150) and the Gain $40\frac{1}{2}$ *l.* For $40\frac{1}{2} - 9\frac{1}{2} = 31$; and $40\frac{1}{2} + 9\frac{1}{2} = 50$; and $50 \times 31 = 1550$.

Suppoſe the Number of Merchants in *Coſſicks* $1ℨ$, (omitting the Work in *Species*) ſo is every Man's Stock $150ℨ$; and the whole Stock $150ʒ$.

And as $100 . 1ℨ :: 150ʒ . \frac{150}{100}\phi$. the whole Gain.

Then $\frac{3}{2}\phi - \frac{19}{2} \times \frac{3}{2}\phi + \frac{19}{2} = 1550$. that is, $\frac{9ʒ\phi - 361}{4} = 1550$.

And $9ʒ\phi - 361 = 6200$. And $9ʒ\phi = 6561$; and $1ʒ\phi = 729$, whoſe Zenzicube Root is 3.

Touching Mixt or Affected Equations.

Of Affected Equations. It is to be noted, that the *Equation* firſt found may be reduced to 3 ſorts of Numbers or more.

Brought to 3 Numbers. An *Equation* conſiſting of 3 Numbers or Magnitudes, muſt have one a Power, another a Root, and the third an Abſolute Number; and of theſe by the Signs + and — there are 3 ſorts, in every of which the greateſt Denomination is ſet Solitary as equal to the other two.

As $ʒ = ℨ + N$. $ʒ = N - ℨ$. $ʒ = ℨ - N$.

And accordingly with the Antients, the Extraction, or rather the finding the Value of the Root, is of 3 Varieties; but of late both the Extraction of the Roots of theſe and others conſiſting of more than three Magnitudes, comprehended under one and the ſame Method, is moſt ingeniouſly taught by the aforeſaid Mr. *Oughtred*, as ſhall be ſhewn hereafter. Notwithſtanding becauſe very many Propoſitions of ordinary uſe are reduced to *Equations* of 3 Numbers, and the antient Way of finding the Value of the Root thereof is by ſome accompted the moſt eaſy, that ſhall be firſt examined.

Reſolution the old Way.

Reſolution of ſome Affected Equations *the antient Way.*

In 3 Numbers.

When an *Equation* conſiſts of 3 Numbers or Magnitudes, either they are in their Natural Order, as aforeſaid, without omiſſion of any Denomination between them, or they are not,

1. If in a natural Order, and none omitted, the firſt Sort.

1ſt. If they are in their Natural Order, and without ſuch Omiſſion, and the *Equation* be of the firſt ſort that ʒ = ℞ + N. or ʒ = N + ℞ which is all one: The Rule is, multiply half the middle Quantity ſquarely; to the Product add the Abſolute Number, out of the Total take the Square Root, and thereto add the former half of the middle Quantity, and this laſt Total ſhall be the Value of one Root of that *Equation*.

Examples.

As ſuppoſe 1ʒ = 4℞ + 21N. or 1ʒ = 90N + 9℞.

In the firſt, the half of 4 (belonging to ℞ the middle Quantity) is 2, which ſquared is 4, to which 21 added makes 25, whoſe Square Root is 5, to which 2 the half added, makes the Total 7, for the Value of the Root ſought.

For 1ʒ = 4℞ + 21 N. The Root being 7, ſhall make 49 = 28 + 21.

In the other Inſtance $4\frac{1}{2}$℞ or $\frac{9}{2}$℞, the half of 9℞ ſquared, produceth $\frac{81}{4}$; this added to 90 or $\frac{360}{4}$, makes $\frac{441}{4}$, the Square Root whereof is $\frac{21}{2}$, to which $\frac{9}{2}$ added makes $\frac{30}{2}$ or 15 for the Value of 1℞, and appears true.

Seeing if 15 be 1℞, then 9℞ is 135, whereunto 90 being added makes the Total 225, and ſo much is the Square of 15.

If ſome Quantity omitted.

But when the *Equation* of this ſort omits ſome Denomination orderly between the Quantities, then after the Root found in like manner as above, from this ſo found Root or Value, extract a Root according to the Number of Quantities omitted, *viz.* if 1 the Square Root, if 2 the Cube Root, *&c.*

Examples.

As ſuppoſe 1ʒʒ = 2ʒ + 8N. Or 1ʒɸ = 7ɸ + 8 N.

In the firſt the Value of the Root gotten as above, will be 4, which becauſe one Denomination is omitted muſt be a Square Number, and the Root thereof 2 the Number ſought.

Thus $\frac{1}{2}$ of 2 is 1, and 1 ſquared is 1, to which 8 added is 9, whoſe Root is 3, to which 1 the half added makes 4, and √ʒ 4 = 2.

And 1ʒʒ = 2ʒ + 8N. The Root being 2 ſhall make 16 = 8 + 8.

In the other Inſtance, the Root gotten as aforeſaid will be 8, out of which the Cube Root muſt be extracted, becauſe 2 Quantities were orderly omitted in the *Equation*; and this Cube Root 2 ſhall be the Value of the Root ſought, whereof the *Equation* is framed.

Becauſe $\frac{1}{2}$ of 7ɸ the Middle Quantity is $\frac{7}{2}$, which ſquared is $\frac{49}{4}$, to which 8 added that is $\frac{32}{4}$, makes $\frac{81}{4}$, the Square Root whereof is $\frac{9}{2}$, to which the half $\frac{7}{2}$ added, makes $\frac{16}{2}$ or 8, and √ɸ8 = 2.

And if 1℞ be 2, then the Cube will be 8, and 7ɸ are 56, which with 8 N makes 64, a Zenzicube Number, and hath 2 for the Root.

2. If in a natural Order, and none omitted, the ſecond Sort.

2ly. Equations affected of the ſecond Sort, (that is) conſiſting of 3 Numbers in their natural Order, without omiſſion of any Denominations between them, and have their Middle Quantity joined with the Sign, as ʒ = N − ℞, have the Rule to find the Value of the Root in all reſpects, ſave one like the firſt Sort of *Affected Equations*; which Difference is only inſtead of adding half the Middle Quantity to the Root: here it is to be ſubſtracted, and the Remainder ſhall be the Value of the Root deſired.

Examples.

As ſuppoſe 1ʒ = 60N − 7℞. Or 1ʒ = 153N − 8℞.

In the firſt, the half of 7, which is $3\frac{1}{2}$ or $\frac{7}{2}$ ſquared, is $\frac{49}{4}$; whereto 60, or $\frac{240}{4}$ added, makes $\frac{289}{4}$ the Square Root, whereof is $\frac{17}{2}$; from whence $\frac{7}{2}$ taken, leaves $\frac{10}{2}$ or 5 for the Value of the Root deſired.

For 1ʒ = 60 N − 7℞: The Root being 5, makes 25 = 60 − 35.

In the other Inſtance, half 8 is 4, the Square 16; to which 153 added, is 169; the Square Root whereof is 13; from whence 4 taken, leaves 9 for the Value of 1ℨ.

So ſhall the Square be 81, and ſo much is 153 lacking 72 that is 8ℨ.

If ſome Quantity omitted.

But when in *Equations* of this ſort ſome Quantities are orderly omitted between the Quantities propounded; then after the Root or Value rather found as before, out thereof let there be a Root extracted, according to the Number of Quantities omitted, as before noted in the former ſort of *Equations*.

Examples.

As ſuppoſe 1ʒʒ = 117N — 4ʒ. Or 1ʒφ = 810N — 3φ.

In the firſt of theſe, ſquaring 2 the Half of 4, there ariſeth 4, which added to 117 maketh 121, the Square Root of which is 11; whence if 2 the firſt Half be ſubſtracted, there reſteth 9; which becauſe one Denomination is omitted, the Square Root thereof 3 is the Number deſired.

In the other $\frac{3}{2}$ ſquared is $\frac{9}{4}$, which added to 810, makes $\frac{3249}{4}$, the Square Root of which is $\frac{57}{2}$; if then $\frac{3}{2}$ be abated, there remaineth $\frac{54}{2}$, or 27, which is a Cube Number, and the Cube Root thereof 3 is the Value deſired.

So the Root being 3 { 1ʒʒ = 117N — 4ʒ / 1ʒφ = 810N — 3φ } is as { 81 = 117 — 36. / 729 = 810 — 81. }

2. If in a natural Order, and none omitted, the third ſort.

3*ly*. *Affected Equations* of the third Sort, to wit, conſiſting of 3 Quantities in their natural Order, without omiſſion of any Denominations, and having their leaſt Quantity joined with —, as ʒ = ℨ — N, have 2 Roots or Values, and therefore are called *Ambiguous Equations:* For finding both which, the Rule is thus; Square half the middle Quantity as before, from thence ſubſtract the ſmalleſt Quantity, extract the Square Root of the Reſidue, and add this Root to the Half firſt taken for the Value of one, and take it away from the Half for the Value of the other Root: And either of theſe Numbers ſhall be as the Root of the *Equation*, and one of them ſhall ſerve to reſolve the *Equation*, as occaſion ſhall require; and which of the two is neceſſary, by the Frame of the Queſtion is directed.

For though no Number can have 2 Square Roots, or 2 Cube Roots, &c. yet one Number may have both a Square Root and a Cube Root. Yea, as before hath been ſeen, one Number may have ſeveral Roots of different Denominations, as 64 hath a Square Root 8, a Cube Root 4, and a Zenzicube Root 2. And ſo in *Equations* of this kind, 2 Roots or a double Value may be had, unleſs the Half of the Middle Quantity be equal to the Abſolute Number, for then is the Moiety or half of the Middle Quantity the Number ſought. And theſe 2 Roots ſo called, muſt be ſuch, as being added together, will make the Number of the Middle Quantity, and multiplied together, will make the Number of the leaſt Quantity, and ſo may be found without farther Operation.

Examples.

As ſuppoſe 1ʒ = 12ℨ — 32N.

The Parts of 32 examined, are found to have no more to his Compoſition likely to this purpoſe, than 2, 4, 8, 16; but if 2 and 16 be taken, their addition 18 will be greater than 12: wherefore 4 and 8 making 12 by *Addition*, and 32 by *Multiplication*, ſhall be the 2 deſired Numbers.

And this is agreeable to the Rule: For the half of 12 is 6, whoſe Square is 36, from which is to be taken 32 of the Reſidue; 4 the Square Root is 2, which either taken from 6 leaves 4 for the one, or added to 6 makes 8 for the other Root or Value required.

So 1ʒ=12ℨ—32N, let the Root be $\frac{4}{8}$ and 16=48—32. / 64=96—32.

If ſome Quantity omitted.

But when the *Equation* of this ſort omits ſome Quantities orderly, there happeneth oftentimes but one Root in *Integers*, and not two: From which Root or Value, as in the others before, muſt be extracted a Root according to the Denominations omitted.

Examples.

As ſuppoſe 1ʒʒ=24ʒ—135N. Or 1ʒφ=12φ—32N.

In the firſt 12 ſquared is 144; from which if 135 be taken, the Square Root of the remaining 9 is 3; to which if 12 be added it makes 15, which is no Square

Number; but if 3 be abated from 12, the Remainder 9 is a Square Number, and hath 3 for his Root.

In the other 6 times 6 is 36, whence 32 abated leaves 4, whoſe Square Root is 2; to which if 6 be added ariſeth 8, the Cube Root whereof is 2 for the Number deſired; but if 2 be abated from 6, there remaineth 4, which is no Cube Number, nor hath a Cube Root in whole Numbers.

Thus the Root being $\begin{matrix} 3 \ . & 1\text{ʒʒ} = 24\text{ʒ} - 135\text{N} \\ 2 \ . & 1\text{ʒ}\varphi = 12\varphi - 32\text{N} \end{matrix}$ is as $\begin{matrix} 81 = 216 - 135. \\ 64 = 96 - 32. \end{matrix}$

Becauſe in every Operation where an *Equation* ariſeth, it is to be noted, That according to the Suppoſition for Reſolution of the Propoſal, ſo will the *Equation* be produced of the one ſort or other; and when *Equations* of this ſort happen, wherein is a double Valuation of the Root, the form of the Queſtion ſheweth which of the 2 Roots is to be taken, unleſs the Propoſition may be truly reſolved by both, as ſometimes it cometh to paſs.

Queſtions wherein the firſt ſort of Affected Equations *come to be reſolved.*

Ex. *Where the firſt Sort are reſolved.*

1. A Merchant buyeth 45 *C.* and 88 ℔. of Pepper, the Tare is 12 ℔. *per Cent.* the Hundred coſt $6\frac{3}{5}$ *l.* and the whole 274 *l.* how many Pound is the Hundred accompted?

Q. *Of the Pounds in an Hundred of Pepper.*

Anſw. 132 ℔.

Reſolution.

Here ſuppoſing the Hundred to be 1 ℨ, the Tare abated makes it 1 ℨ — 12.

Then as $1\text{ℨ} \,.\, 1\text{ℨ} - 12 :: 45\text{ℨ} + 88 \,.\, \dfrac{45\text{ʒ} - 452\text{ℨ} - 1056}{1\text{ℨ}}$.

And then as $1\text{ℨ} \,.\, 6\frac{3}{5} :: \dfrac{45\text{ʒ} - 452\text{ℨ} - 1056}{1\text{ℨ}} \,.\, \dfrac{297\text{ʒ} - 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}}{1\text{ʒ}}$.

Wherefore $\dfrac{297\text{ʒ} - 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}}{1\text{ʒ}} = 274$.

And $274\text{ʒ} = 297\text{ʒ} - 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}$. And $23\text{ʒ} = 2983\frac{1}{5}\text{ℨ} - 6969\frac{3}{5}$.
And $1\text{ʒ} = 129\frac{81}{115}\text{ℨ} + 303\frac{3}{115}$. And $1\text{ℨ} = 132$ ℔.
For $\frac{1}{2}$ of $129\frac{81}{115}$ is $64\frac{98}{115}$ or $\frac{7458}{115}$ ſquared is $\frac{55621764}{13225}$. and $303\frac{3}{115}$ or $\frac{4007520}{13225}$ added is $\frac{59629284}{13225}$ the $\sqrt{\text{ʒ}}$ whereof is $\frac{7722}{115}$ whereto $\frac{7458}{115}$ added makes the Total $\frac{15180}{115}$ or 132.

Proof.

Proof.

If 132 ℔. abate 12 ℔. for Tare, then 1 *C.* ſhall be but 120 ℔.
And as 1 *C* . 120 ℔ :: 45 *C* + 88 ℔ . 5480 ℔.
And as 132 ℔ . $6\frac{3}{5}$ *l* :: 5480 ℔ . 274 *l.*

2. Two Men have Silks to ſell, *viz.* *A* hath 40 Ells, and *B* 90; and the Silk of *A* being not ſo fine as the Silk of *B*, he ſelleth in every Angel more by $\frac{1}{3}$ of an Ell than *B* doth, and both their Monies made 42 Angels: now how much did each of them ſell for an Angel?

Q. *Of Silks, how much ſold for an Angel.*

Anſw. *A* ſold $3\frac{1}{3}$ Ells, *B* ſold 3 Ells for an Angel.

Reſolution.

Here ſuppoſing the leaſt Quantity which is *B*, to be 1 ℨ;
Then muſt *A* ſell $1\text{ℨ} + \frac{1}{3}$ Ell: and each whole Parcel divided thereby,

is for *B* $\dfrac{90}{1\text{ℨ}}$ for *A* $\dfrac{40}{1\text{ℨ} + \frac{1}{3}\text{N}}$ or reduced $\dfrac{120}{3\text{ℨ} + 1\text{N}}$

And theſe two Numbers declaring the Angels each Man received;

It follows that $\dfrac{120}{3\text{ℨ} + 1\text{N}} + \dfrac{90}{1\text{ℨ}} = 42$. And reduced $\dfrac{390\text{ℨ} + 90}{3\text{ʒ} + 1\text{ℨ}} = 42$.
And $390\text{ℨ} + 90\text{N} = 126\text{ʒ} + 42\text{ℨ}$. And $126\text{ʒ} = 348\text{ℨ} + 90\text{N}$.
And $21\text{ʒ} = 58\text{ℨ} + 15\text{N}$. And $1\text{ʒ} = \frac{58}{21}\text{ℨ} + \frac{15}{21}\text{N}$.
And $1\text{ℨ} = 3$. For $\frac{1}{2}$ of $\frac{58}{21}$ is $\frac{29}{21}$, ſquared is $\frac{841}{441}$; whereto $\frac{15}{21}$ or $\frac{315}{441}$ added, is $\frac{1156}{441}$, the $\sqrt{\text{ʒ}}$ whereof is $\frac{34}{21}$, to which $\frac{29}{21}$ added makes $\frac{63}{21}$ or 3.

Proof.

Proof.

Ells *Ells*

A $3\frac{1}{3}$ or $\frac{10}{3}$) 40 ($\frac{120}{10}$ (12 Angels. B 3) 90 (30 Angels.

And $12 + 30 = 42$.

Ex. *Where the ſecond Sort are reſolved.*

Queſtions wherein the ſecond ſort of Affected Equations *come to be reſolved.*

Q. *Of the Stock and Gain of 3 Merchants.*

1. Three traffique together, *A* putteth in Stock 14 *l.* leſs than *B*, and *B* and *C* together put in 148 *l.* they gain 42 *l.* more than their Stock, of which *A* taketh $60\frac{20}{33}$ *l.* what was each Man's Stock and Part of the Gain?

Reſolution.

Anſw. The Stock of *A* 50 *l.* *B* 64 *l.* *C* 84 *l.* Gains in all 240 *l.* of which to *A* $60\frac{20}{33}$ *l.* *B* $77\frac{19}{33}$ *l.* *C* $100\frac{9}{11}$ *l.*

Here ſuppoſing the Stock of *A* to be 1ȥ; then muſt the Stock of *B* be $1\text{ȥ} + 14$; and *B* and *C* making up 148, the Stock of *C* muſt be $134 - 1\text{ȥ}$; and all theſe three added, make $1\text{ȥ} + 148$ the whole Stock, which with 42 more, that is $1\text{ȥ} + 190$, is all the Gains.

Then as $1\text{ȥ} + 148 \,.\, 1\text{ȥ} + 190 :: 1\text{ȥ} \,.\, \frac{1\text{ʒ} + 190\text{ȥ}}{1\text{ȥ} + 148}$. Gain of *A*

And then $\frac{1\text{ʒ} + 190\text{ȥ}}{1\text{ȥ} + 148} = 60\frac{20}{33}$. And $33\text{ʒ} + 6270\text{ȥ} = 2000\text{ȥ} + 296000$.

And $33\text{ʒ} = 296000 - 4270\text{ȥ}$. And $1\text{ʒ} = \frac{296000}{33} - \frac{4270}{33}\text{ȥ}$.

And $1\text{ȥ} = 50$

For $\frac{1}{2}$ of $\frac{4270}{33}$ is $\frac{2135}{33}$, ſquared is $\frac{4558225}{1089}$, to which $\frac{296000}{33}$, that is $\frac{9768000}{1089}$, added is $\frac{14326225}{1089}$. the $\sqrt{\text{ʒ}}$ whereof is $\frac{3785}{33}$, from which $\frac{2135}{33}$ taken, the Remainder is $\frac{1650}{33}$ or 50.

Proof.

Proof.

The Stock of *A* leſs by 14 than *B*; if therefore it be 50, that of *B* muſt be 64, that is $50 + 14$. And becauſe *B* and *C* made 148, if *B* be 64, then muſt *C* be 84, for $84 + 64 = 148$: and the Gains being 42 more than the Stock, muſt be 240, for $50 + 64 + 84 + 42 = 240$; and then by the *Rules of Fellowſhip*,

$$\text{If } 198 \text{ gain } 240, \text{ the Gain of} \begin{cases} 50 & \\ 64 & \text{is} \\ 84 & \end{cases} \begin{matrix} 60\frac{20}{33} & A. \\ 77\frac{19}{33} & B. \\ 101\frac{9}{11} & C. \end{matrix}$$

Q. *Of the Days in which a Journey is performed.*

2. A Traveller hath a Journey to go of 2955 Miles, and the firſt Day he goeth $1\frac{1}{2}$ Mile, and every Day afterwards increaſeth his Journey by $\frac{1}{6}$ of a Mile as in an *Arithmetical Progreſſion*: in how many Days ſhall he finiſh his Journey?

Reſolution.

Anſw. In 180 Days.

Here are given the firſt Term of the *Progreſſion* $1\frac{1}{2}$, the Exceſs $\frac{1}{6}$ and the Sum 2955, to find the Number of Terms; for which ſuppoſing 1ȥ, then ſhall all the Exceſſes or Number of Spaces be $1\text{ȥ} - 1\text{N}$, by which the Exceſs multiplied, the Product is $\frac{1\text{ȥ} - 1\text{N}}{6}$ the Sum of all the Exceſſes; to which the firſt Term added which is $1\frac{1}{2}$, or reduced to like Denomination $\frac{9}{6}$, ſo will the Total be $\frac{1\text{ȥ} + 8\text{N}}{6}$ the laſt Term of that *Progreſſion*; which gotten, becauſe $\alpha + \omega$ in $\frac{1}{2}\text{T} = \text{Z}$, the firſt $\frac{9}{6}$ is added to the laſt Term $\frac{1\text{ȥ} + 8\text{N}}{6}$ and the Total $\frac{1\text{ȥ} + 17\text{N}}{6}$ multiplied by $\frac{1}{2}\text{ȥ}$, (ſeeing the whole Number of Places is ſuppoſed to be 1ȥ) the Product is $\frac{1\text{ʒ} + 17\text{ȥ}}{12}$, the Sum of the *Progreſſion*.

So is $\frac{1\text{ʒ} + 17\text{ȥ}}{12} = 2955$. And $1\text{ʒ} + 17\text{ȥ} = 35460$.

And $1ʒ = 35460 - 17ℨ$. And $1ℨ = 180$.

For $\frac{1}{2}$ of 17 is $8\frac{1}{2}$, or $\frac{17}{2}$ ſquared $\frac{289}{4}$, to which $\frac{141840}{4}$ added, makes $\frac{142129}{4}$ whoſe Square Root is $\frac{377}{2}$, from whence $\frac{17}{2}$ taken, there remaineth $\frac{360}{2}$, or 180.

Proof. Proof.

Seeing $\alpha\,1\frac{1}{2}$. $X\,\frac{1}{6}$. $Z\,2955$, are given thereby, the Rules before in *Progreſſion* direct to the finding of T the Number here ſought.

For $\sqrt{:}\,\dfrac{\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX}{Xq} : -\alpha + \frac{1}{2}X = T$. That is in Numbers

$$\sqrt{:}\,\dfrac{\frac{9}{4} - \frac{1}{4} + \frac{1}{144} + \frac{5910}{6}}{\frac{1}{36}} : -\frac{3}{2} + \frac{1}{12} = 180.$$

$$\text{Or } \sqrt{:}\,\frac{1}{36}\Big)\frac{324 - 36 + 1 + 141840}{144} : -\frac{3}{2} + \frac{1}{12} = 180.$$

$$\begin{array}{r} 141840 \\ 1 \\ 324 \\ \hline 142165 \\ -36 \\ \hline \end{array}$$

$$\frac{1}{36}\Big)\frac{142129}{144}\Big(\sqrt{q}\,\frac{5116644}{144}\Big(\frac{2262}{12}. \text{ And } \frac{2262 - \frac{3}{2} + \frac{1}{12}}{12} = 180.$$

$$\text{And } \frac{2262 - \frac{36}{2} + \frac{12}{12}}{12} = 180. \text{ And } \frac{377 - 18 + 1}{2} = \frac{360}{2} = 180.$$

Queſtions, wherein the third Sort of Affected Equations *come to be reſolved.* Ex. *Where the third Sort are reſolved.*

1. Two Numbers added together make 8, but added ſeverally, with their reſpective Squares and Cubes, make 194: what are thoſe Numbers? Q. *Of 2 Numbers, what they are.*
Anſw. 3 and 5.

Here ſuppoſing one of the two Numbers to be $1ℨ$, then muſt the other be $8 - 1ℨ$. Theſe with their Squares and Cubes are thus: Reſolution.

$1ℨ$	Numbers	$8 - 1ℨ$.
$1ʒ$	Squares	$1ʒ + 64 - 16ℨ$.
$1φ$	Cubes	$24ʒ + 512 - 1φ - 192ℨ$.

$$\begin{array}{r} 1φ + 1ʒ + 1ℨ. \\ -1φ + 25ʒ - 209ℨ + 584. \\ \hline \end{array}$$

Total of both $26ʒ - 208ℨ + 584$.

And then $26ʒ - 208ℨ + 584 = 194$. And $26ʒ = 208ℨ - 390$.
And $1ʒ = 8ℨ - 15$. And $1ℨ = 3$ or 5.

For $\frac{1}{2}$ of $8ℨ$ is 4, ſquared is 16; from whence 15 ſubſtracted, there is left 1, whoſe Square Root is 1, which added to 4 makes 5, or taken from 4 leaves 3, for the Value of either Root; both which are neceſſary to the Solution of this Queſtion.

Proof. Proof.

$$\overset{ℨ}{3} + \overset{ʒ}{9} + \overset{φ}{27} = 39. \text{ And } \overset{ℨ}{5} + \overset{ʒ}{25} + \overset{φ}{125} = 155.$$
$$\text{And } 39 + 155 = 194.$$

2. A Man owed a Sum of Money, which might be divided into two ſuch Parts, that being multiplied would make 24, and their Cubes added together 280: what was the Debt? Q. *Of a Debt, what it was.*
Anſw. 10 *l.* Reſolution.

Here ſuppoſing one Part of the two to be $1ℨ$, then muſt the other be $\frac{24}{1ℨ}$.

The Cubes of both which are,

$$\overset{\surd}{1\text{ℨ}}.\ \overset{q}{1\text{ʒ}}.\ \overset{c}{1\phi}.\quad \overset{\surd}{\frac{24}{1\text{ℨ}}}.\ \overset{q}{\frac{576}{1\text{ʒ}}}.\ \overset{c}{\frac{13824}{1\phi}}.\quad \text{And } \frac{1\phi}{1} + \frac{13824}{1\phi} \text{ added are } \frac{1\text{ʒ}\phi+13824}{1\phi}$$

And this equal to 280. Wherefore $1\text{ʒ}\phi = 280\phi - 13824$.
And $1\text{ℨ} = 4$ or 6.

For $\frac{1}{2}$ of 280 is 140, ſquared is 19600; out of which 13824 taken, the Remain is 5776, whoſe Square Root is 76; which added to 140, maketh 216, and abated from it, leaveth 64; out of both which the Cube Root extracted, becauſe there are 2 Quantities omitted in the *Equation*, the Roots deſired are 6 and 4.

Proof.

Proof.

Parts. *Debt.*
$6+4=10$. $6\times 4=24$.
Cubes. *Sum.*
$216+64=280$.

Ex. *Where the Equation is according to the Suppoſition.*

Queſtions wherein according to the Suppoſition, ſo the Equation *happens to be of one ſort or other.*

Q. *Of a Number, what it was.*

1. A Number thought upon hath two Parts, of which the one is 4, and the other multiplied into it ſelf; and then alſo with 4 the two Products will be 117: what was that Number?

Reſolution. *Anſw.* 13.

Here ſuppoſing the Number to be 1ℨ, then 1 Part being 4, the other Part muſt be $1\text{ℨ} - 4$: This Part ſquared is $1\text{ʒ} - 8\text{ℨ} + 16$. And the ſaid unknown Part (that is $1\text{ℨ} - 4$) multiplied by 4, is $4\text{ℨ} - 16$; which added to the Square, makes $1\text{ʒ} - 4\text{ℨ}$; which being equal to 117, makes an *Equation* of the firſt Sort thus:

$1\text{ʒ} = 4\text{ℨ} + 117$. And $1\text{ℨ} = 13$.

For $\frac{1}{2}$ of 4 is 2, ſquared is 4, to which 117 added is 121, the Square Root of which is 11, and thereto 2 added is 13, the Number deſired.

But if Suppoſition be made for the unknown Part, then if that Part be 1ℨ, the whole Number ſhall be $1\text{ℨ} + 4$: And then multiplying this Part by it ſelf, and alſo by 4, the known Part there will ariſe 1ʒ and 4ℨ; which being equal to 117, makes an *Equation* of the ſecond Sort thus: $1\text{ʒ} = 117 - 4\text{ℨ}$. And $1\text{ℨ} = 9$, the unknown Part.

For $\frac{1}{2}$ of 4ℨ is 2, which ſquared is 4, this added to 117 is 121, whoſe Square Root is 11; from whence 2 taken, there remaineth 9 for the Part unknown.

Proof.

Proof.

Parts. *Whole Number.*
$4+9=13$.

One Part led into the other	$9\times 4 = 36$
One Part ſquared	$9\times 9 = 81$
	117 Total.

Q. *Of 3 Numbers, what the Leaſt and Mean.*

2. There are 3 Numbers in *Geometrical Proportion*, the greateſt Extream is $20\frac{1}{4}$, the leaſt Extream with the double of the middle Term makes 22: what is the leaſt Extream, and the middle Term?

Reſolution. *Anſw.* The leaſt Extream is 4, and the middle Term 9.

Here ſuppoſing the leaſt Extream to be 1ℨ: And becauſe the leaſt with the double of the middle Term muſt make 22, the middle Term ſhall be $11 - \frac{1}{2}\text{ℨ}$, for his double is $22 - 1\text{ℨ}$, which with 1ℨ is 22. Then becauſe the Product of the Mean multiplied into it ſelf is equal to the Product of the Extreams, the two Extreams multiplied, which are 1ℨ and $20\frac{1}{4}$, produce $\frac{81}{4}\text{ℨ}$. And the Mean that is $11 - \frac{1}{2}\text{ℨ}$, ſquared is $121 + \frac{1}{4}\text{ʒ} - 11\text{ℨ}$; which being equal to the other, the *Equation* ſtands thus, $121 + \frac{1}{4}\text{ʒ} - 11\text{ℨ} = \frac{81}{4}\text{ℨ}$. And by Reduction $484 + 1\text{ʒ} - 44\text{ℨ} = 81\text{ℨ}$. And again $1\text{ʒ} = 125\text{ℨ} - 484$, an *Equation* of the third Sort. And $1\text{ℨ} = 4$.

For $\frac{1}{2}$ of 125 is $\frac{125}{2}$, ſquared is $\frac{15625}{4}$; from which 484 that is $\frac{1936}{4}$ abated, the Remain is $\frac{13689}{4}$, whoſe Square Root is $\frac{117}{2}$; this abated from $\frac{125}{2}$, leaves $\frac{8}{2}$ or 4 for the Root deſired. And although by Addition of the Square Root to the Half, as in other *Equations* of the third Sort, that is $\frac{117}{2}$ to $\frac{125}{2}$, another Root will be obtained, to wit $\frac{242}{2}$, or 121; yet cannot this be the Root intended by the Propoſition, becauſe the leaſt Extream with the double of the middle Term was bounded to make but 22, when this Root 121 is much more of it ſelf. But if the middle Term be ſuppoſed $1ℨ$, then becauſe the double thereof with the leaſt Term muſt make 22, that leaſt Term muſt be $22-2ℨ$. And ſeeing the Square of the Mean $1ℨ$, that is $1ʒ$, muſt be equal to the Product of the two Extreams, that is $22 - 2ℨ \times 20\frac{1}{4}$, which is $445\frac{1}{2}-40\frac{1}{2}ℨ$, there ariſeth an *Equation* of the ſecond Sort, that is $1ʒ = 445\frac{1}{2} - 40\frac{1}{2}ℨ$. And $1ℨ = 9$.

For $\frac{1}{2}$ of $40\frac{1}{2}$ is $\frac{81}{4}$, whoſe Square is $\frac{6561}{16}$, to which adding $445\frac{1}{2}$, or $\frac{7128}{16}$, the Total is $\frac{13689}{16}$ the Square of $\frac{117}{4}$; from which $\frac{81}{4}$ abated, there remaineth $\frac{36}{4}$, or 9 for the middle Term.

Proof.

Proof.

Proportionals $4 . 9 : 20\frac{1}{4}$. And $4 + 9 + 9 = 22$.
Extreams $4 \times 20\frac{1}{4} = 81$. Mean $9 \times 9 = 81$.

Reſolution of Affected Equations, *by Mr.* Oughtred's *Way.*

Reſolution of Affected Equations by Mr. Oughtred.

Mr. *Oughtred* in the 16*th* Chapter of his *Clavis*, hath delivered two Rules for the Reſolution of every *Equation*, wherein are 3 Species or Quantities orderly aſcending, ſuch as thoſe herein already ſpoken to; and directs, that the Abſolute Number (being one of the three) ſhall be reckoned the Rectangle (or Product) of the two Magnitudes ſought, whether Root, Square, or Cube, *&c.* that is to ſay, ſuch as is the Power of the middle *Species*. And in the middle *Species*, if the higheſt be Negative, the Coefficient ſhall be counted the Sum of the ſought Magnitudes, and be manifeſt of both. But if the higheſt *Species* be Affirmative, the Coefficient ſhall be the Difference of the Magnitudes ſought; and the ſame *Species* ſhall be ſhewn of the Greater denied, or of the Leſſer affirmed.

And ſeeing (by the ſecond Chapter of this 4*th* Part in the *Invention of Equations* it appeareth) that the Sum and Rectangle of two Magnitudes given, the Difference is given; or the Difference and Rectangle given, the Sum is given; and by the Sum and Difference the Magnitudes themſelves are given. So as Z and X given, A and E may be found by the ſaid Rules, thus by him ſymbolized.

Two Rules in Species by Mr. Oughtred *for the former* 3.

$$1.\quad \tfrac{1}{2}Z \pm \sqrt{:}\,\tfrac{1}{4}Zq - \text{Æ}: (\tfrac{1}{2}X) = \substack{A.\\E.}$$

$$2.\quad \sqrt{:}\,\tfrac{1}{4}Xq + \text{Æ}: (\tfrac{1}{2}Z) \pm \tfrac{1}{2}X = \substack{A.\\E.}$$

Theſe two Rules being the ſame in effect with thoſe before handled in the Antient Coſſical way, for Reſolution of orderly *Affected Equations*, will need no Explanation here; and the rather, for that the ſame Author afterward, in a Tract of 28 *Sections*, or *Precepts*, hath taught the Inveſtigation of the Root of all ſorts of *Affected Equations*, as well diſordered as others, in one Method; which to avoid multiplicity of Rules, is chiefly to be choſen and followed.

Difference in the Reſolution from Coſſicks.

In this joint way of Work, contrary to that by *Coſſicks*, the Abſolute Number or Magnitude is ſet ſolitary, and not the higheſt; ſo as the three ſorts of *Equations* before

$$\begin{array}{llll} \text{Set thus} & ʒ = ℨ + N. & ʒ = N - ℨ. & ʒ = ℨ - N. \\ \text{are here} & Aq - A = N. & Aq + A = N. & A - Aq = N. \\ \text{And ſo} & Aq - XA = \text{Æ}. & Eq + XE = \text{Æ}. & ZA - Aq = \text{Æ}. \end{array}$$

The Number known to be ſet alone againſt all the other.

The Reaſon whereof is, becauſe every *Equation* may, by Reduction, be brought to an Abſolute Number or Magnitude, which being known, is certain and without Fiction or Figuration, and ſo may be ſet againſt all the other unknown Parts of the *Equation*. For it is evident in this *Equation*,
$8ʒφ = 10ſs + 20ʒʒ + 400φ + 31250N$. Or in *Species* tranſlated,
$8Acc - 10Aqc - 20Aqq - 400Ac = 31250$, that the eighth Part of
$10ſs + 20ʒʒ + 400φ + 31250N$ muſt be a Zenzicube Number. And that $20ʒʒ + 400φ + 31250N$ contain a certain Number of Surſolids: And alſo that $400φ + 31250N$, contain certain Zenzizenzikes. And ſo conſequently 31250 ſhall contain therein certain Cubes, as may be proved, the Root being 5.

The

28 Precepts of Mr. Oughtred *of Affected Equations.*

The Subſtance of thoſe 28 *Sections*, or *Precepts*, (ſave what concerns the uſe of *Logarithms*, already learned) follows in order as the Author hath left them; the Tranſlation whereof, with the following Examples, will be ſufficient, without Additional Illuſtration.

1. To conſtitute an Affected Equation.

1. The manner of conſtituting an *Affected Equation*; Let there be taken at pleaſure, for B, 3; for Cq, 16; for Dc, 125; for Fqq, 1296, *&c.* Neither is it material, whether the Numbers are truly figurate or not. And let there be conſtituted of theſe Coefficients, a *Surſolid Equation*, according to the manner of the *Analytical Table*, (in Chap. 11. *Figuration of Rational Species*) *viz.*

Example.

$Lqc - 5BLqq + 10CqLc - 10DcLq + FqqL = Gqc$; which in Numbers appointing L(the Root)47, ſhall be $1qc - 15qq + 160c - 1250q + 6480l = 170304782$: or omitting the Diſtinction of the *Unciæ*; For 15qq, ſay BLqq; for 160c, ſay CqLc; for 1250q, ſay DcLq; and for 6480l, ſay FqqL. For if L be 47, then ſhall $Lq = 2209$, and $Lc = 103823$, and $Lqq = 4879681$, and $Lqc = 229345007$.

The Practice of this Conſtitution.

BLqq 15×4879681	229345007 −73195215	Lqc.
CqLc 160×103823	156149792 +16611680	
DcLq 1250 × 2209	172761472 −2761250	
FqqL 6480 × 47	170000222 + 304560	
	170304782	Gqc.

2. Every one to be counted as the former Example.

2. Count every propounded *Equation* as this now found.

$1qc - 15qq + 160c - 1250q + 6480l = 170304782$.

Or the Numbers changed into *Species*.

$Lqc - BLqq + CqLc - DcLq + FqqL = Gqc$.

How to expreſs more Affections.

And if there were more *Species* of the Affections, conſequently they might be expreſſed by Hcc, Kqqc, Mqcc, Nccc, and ſo further.

3. To ſearch out the two Parts.

Example.

3. Of the Root L there ſhall by theſe be two Parts ſearched out, to wit, A the firſt Side, and E the ſecond Side, or whatſoever is ſubſequent; wherefore $L = A + E$, and all the Powers of L equally to the like Powers of $A + E$. As $Lq = Aq + 2AE + Eq$. And $Lc = Ac + 3AqE + 3AEq + Ec$, *&c.*

4. Heterogeneals not to be added or ſubſtracted, &c.

4. In the propounded *Equation*, the Power to be reſolved 170304782, or Gqc, is a Surſolid, of which kind alſo are the ſeveral Species of the Affections; for *Heterogeneals* cannot be added nor ſubſtracted among themſelves.

5. Two things to be conſidered.
(1) The Degree of Affection.
(2) The Coefficient.

5. Wherefore in the ſeveral Affections, 2 things are to be conſidered, The Degree of Affection, and the Coefficent; as in 15qq, the Degree of Affection is ſquared Square, and the Coefficient 15 Root: In 160c, the Degree of Affection is Cube, and the Coefficient 160 Square: In 1250q, the Degree of Affection is Square, and the Coefficient 1250 Cube: Laſtly, in 6480l the Degree of Affection is Root, and the Coefficient 6480 ſquared Square: and hence ariſe 2 Conſectaries for Extraction of the ſingular Sides.

6. Conſectary from thence.

Example.

6. The firſt Conſectary is, if the Root of the Coefficient according to his own kind multiplied into the Degree of Affection, ſhall multiply the ſame Coefficient, the Product ſhall be of the ſame kind with the Power to be reſolved: As in the precedent *Equation*, if the Side 15 multiplied ſquared-ſquaredly, be multiplied into 15; and if √q 160 cubed be multiplied into 160 ſquared; and if √c1250 ſquared be multiplied into the Cube of 1250; Laſtly if √qq 6480 be multiplied into the ſquared Square of 6480: of all theſe ſeveral Multiplications ſhall ariſe a Surſolid Number. And this Analytical Multiplication is the manner of reducing every Coefficient to the Species of the Power to be reſolved, moſt uſed in Extraction of every firſt Side.

7. Another Conſectary from the firſt.

7. From whence alſo moſt clearly appeareth, That if the Number ariſing of the Coefficients, in this manner reduced and compared, be leſs than the Power to be reſolved, the Side thereof alſo is leſs than the ſide of the Power to be reſolved; but if Greater, it is Greater; and if Equal, Equal. Therefore in this

Example. *Equation* $1qc - 15qq + 160c - 1250q + 6480l = 170304782$; or $170304782 + 15qq - 160c + 1250q - 6480l = 1qc$. If then the lateral Coefficient 15, and √q160, and √c1250, and √qq6480 be made Surſolids, they ſhall produce four Homogeneal Species of Affections, to wit, 7593.., 3238.., 1450.., 0581.., which by Logarithms is moſt eaſily done, and ſufficiently exact for the purpoſe.

1) 2) 3) 4) are the Dimenſions in the Coefficient.

	Logarithms.		*Coefficient Numbers.*
1) 5 ×	1,17609,12591		15 qq
	5,88045,62955		+7593 ..
2)	2,20411,99827		160 c
5 ×	1,10205,99913	12,65	
	5,51029,99565		−3238 ..
3)	3,09691,00130		1250q
5 ×	1,03230,33376	10,8 —	
	5,16151,66880		+1450 ..
4)	3,81157,50059		6480 l
5 ×	0,95289,37514	8,97	
	4,76446,87570		−0581 ..

The Species being gathered into one Sum, according to the Order of their Signs among thoſe propounded in the *Equation*, it ſhall be, that

$$170304700 + 759300 - 323800 + 145000 - 058100 = 1qc = 170827100.$$

which alſo in other *Equations* may likewiſe be done.

8. The ſecond Conſectary is, if the Power to be reſolved be divided by the Coefficient, the Quotient ſhall be referred to the ſame Degree of Affection; that is, the Quotient ſhall be the Side if the Affection be under the Side, or the Square if under the Square, and ſo of other Degrees: as in the former *Equation*, if 170304782 be divided by 15, the Quotient ſhall be ſquaredly Quadratical; if by 160, the Quotient ſhall be Cubical; if by 1250, the Quotient ſhall be Quadratical; and if by 6480, the Quotient ſhall be Lateral; wherefore not always the Quotient it ſelf, but for the moſt part the Root thereof, according to the Degree of Affection, ſhall be the ſingular ſide to be extracted. *8. The ſecond Conſectary.* Example.

9. In ſearching out the ſecond Figure of the Root, this ought to be remembred, that according to the Number of Figures in the Quotient, the Degree thereof ſhall be very nearly reckoned; as if the Quotient conſiſt in one Figure only, it may be a Side; if in 2, a Square; if in 3, a Cube, *&c.* And if the Quotient exceed 5, or 50, or 500, *&c.* it may be extended perhaps to the Degree following, eſpecially in the greater Affections: And theſe are the Laws of *Analytical Diviſion*. *9. What to be noted in finding the ſecond Figure of the Root.*

10. Neither in this ſort of *Multiplication* or *Diviſion*, ſhall there be need to run through the whole Power to be reſolved, with the whole Coefficient, but only to the next Point agreeing thereto. *10. Proceed only to the next Point.*

11. For in the Reſolution of *Affected Equations*, all the Pointings of the Degrees ought to be made in the Power to be reſolved, as in other Figural Numbers; thoſe of the higheſt Degrees above, and of the Reſidue beneath. Alſo the Coefficients, every one according to his own Kind, are to be pointed. The Points of the former Examples ſhall be thus: *11. How the Numbers are to be pricked.*

Example.

$$1qc - 15qq + 160c - 1250q + 06480l = 170304782$$

12. And regularly (eſpecially if the Coefficient be Negative) the Number of Points in all ought to be equal. Wherefore if the Power to be reſolved have more or fewer Points upon it ſelf than the Coefficient, ſo many Ciphers ſhall be ſet before that which is deficient, that the Points to both may be equal. And in getting the ſeveral Sides, the Point of the Coefficient proper to that Side, is to be accommodated to the like Point above of the Power to be reſolved: which ſhall *12. Points to be equal and void, Places ſupplied with Ciphers.*

be done, if the Unit's Place in the Coefficient be removed in order to the lower Points of the Power agreeable to his Degree.

13. If the Coefficient be a Fraction or Surd.

13. If any Coefficient be a Fraction, or Surd Side, let it be reduced to Integers with Decimal Parts.

14. Root to be extracted with Decimals.

14. And if need be, to purſue the Extraction of the Root in the Decimal Parts, adjoin as many Ciphers as ſhall be meet after the Separatrix, and mark them above and below with Points in like ſort.

15. The Table for the Diviſors and Gnomons.

Affirmative and Negative.

15. The following Table ſhews as well the Diviſors as the Gnomons, for finding the ſeveral Sides in *Affected Equations*; collected and continued out of the *Analytical Table*, before mentioned. And it is to be noted, That all the *Species* of every Coefficient are Affirmative, if the ſame be Affirmative; but Negative, if Negative.

For the firſt Side.	*For the ſeveral following Sides to compleat the Gnomon.*					
Aq	2AE .				Eq	= Cq
BA	BE .					
Ac	3AqE.	3AEq .			Ec	= Dc
BAq	B2AE.	BEq .				
CqA	CqE.					
Aqq	4AcE.	6AqEq .	4AEc .		Eqq	= Fqq
BAc	B3AqE.	B3AEq .	BEc .			
CqAq	Cq2AE.	CqEq .				
DcA	DcE.					
Aqc	5AqqE.	10AcEq .	10AqEc.	5AEqq.	Eqc	= Gqc
BAqq	B4AcE.	B6AqEq .	B4AEc.	BEqq.		
CqAc	Cq3AqE.	Cq3AEq .	CqEc.			
DcAq	Dc2AE.	DcEq .				
FqqA	FqqE.				&c.	

16. Diviſors to be orderly collected and added.

16. The Diviſors every where are taken of thoſe, which are had in the Meaſure given, diſpoſed and gathered together in due order, according to their Signs.

17. Ambiguous Equations.

17. If the higheſt Power of any *Equation* be Negative, that *Equation* is ambiguous.

18. The firſt of the Side whence.

18. The firſt ſingular Side is drawn out of theſe Rules, taken from the 2 Conſectaries in *Sect.* 6. and 8.

(1) When the Coefficient may be neglected.

(1.) If the Coefficient ſo far depart to the latter Part, that it ſcarce reach to the firſt Point of the Power to be reſolved, neither (alſo Analytically reduced) make any great change in it; it may altogether be neglected, in Extraction of the firſt ſingular Side.

(2) When devolved into the Conſequent Points.

(2.) If the Coefficient break forth in the fore part, and be Affirmative, it is to be devolved into the Conſequent Points, until a Place be made for Diviſion: By which Diviſion the Quotient found ſhall be referred to the Degree of Affection; which alſo in extracting the leſſer Root of an Ambiguous *Equation* ought to be underſtood.

(3) When Negative and of many Points.

(3.) But if it be Negative, and conſiſt of more Points than the Power to be reſolved, the deficient Places may be ſupplied with Cyphers prefixed; and for the firſt ſingular Side, the Root of the Coefficient it ſelf may be taken according to his kind.

(4) When the Points to both equal.

(4.) If the Points to both are equal, and the Numbers differ not much in the firſt Point, both of the Coefficient and of the Power to be reſolved; the Coefficient by his Root extracted according to the Species with which he is pointed, under the Point agreeable thereto, reduced to the Species of the Power (by Analytical Multiplication) may be added to the Power to be reſolved, if it be Negative, or taken away if it be Affirmative. For if $Ac + CqA = Dc$, then ſhall $Ac = Dc \mp CqA$; but if the greater Side of an Ambiguous *Equation* be ſought, the Power to be reſolved may be taken away from the Coefficient reduced: for if $CqA - Ac = Dc$, then ſhall $Ac = CqA - Dc$; then the Root of the Sum or Difference ſhall be the firſt Side to be extracted. And note that the greater Side of an Ambiguous *Equation* may be found ſometime by Diviſion, ſometime by Extraction of the Root of the Coefficient; but for the moſt part by Reduction of the Coefficient.

19. And by theſe Precepts diligently weighed at laſt, the firſt true ſingular Side ſhall be that, which firſt of all ſheweth ſuch a Diagonal; which together with the Coefficients (as the Condition of the *Equation* requireth) multiplied according to the precedent Table, and all gathered together into one Sum, (diligent reſpect every where had as well to the Signs as Places) bringeth forth a Number not greater than the Power to be reſolved, from whence it is to be ſubſtracted. And it is to be noted, that every Negative Number is leſs than any Affirmative, and than any leſſer Negative, as -4 is leſs than 1, and than -1; alſo that Subſtraction changeth the Sign of the Subtrahend; as from 4 take away 6, there reſteth $4-6$, that is -2; and from -4 take away -6, there reſteth $-4+6$, that is 2; again from 4 take away -6, there reſteth $4+6$, that is 10; wherefore in Extraction of the firſt ſingular Side, it is ſo often to be tried until the true Side be found, which by the next greater ſhall certainly be known.

19. *To find the firſt of the Side.*

20. In conſtituting the Diviſor for finding the ſecond Side, the Place of the Coefficient multiplied into every Degree, ought to be ordered according to the pointing of his own Degree, that is, the Place of the Coefficient under the Side ſhall be diſtant towards the left Hand, one Place from the Point or Place of the ſame Coefficient; the Place of the Coefficient under the Square, two Places; under the Cube, three, *&c.* And to avoid Confuſion, it will be profitable, in the Reſidue of the Power to be reſolved, to diſtinguiſh thoſe Points alone which ſerve to the preſent Root to be extracted.

20. *To get the Diviſor for the Second of the Side.*

21. Then the ſecond ſingular Side ſhall be thus gotten; Let the Diviſors of every Kind, out of the precedent Table, be gathered together into one Sum, and diſpoſed in due order, and the Reſidue of the Power to be reſolved divided by all that Diviſor. For the Quotient, according to the Laws of Analytical Diviſion (if need require it) weighed, ſhall give the ſecond ſingular Side to be gotten. But in this ſearch oftentimes great Difficulty happeneth, eſpecially if the Aggregate of the Negative Magnitudes dividing, be almoſt equal to the Aggregate of the Affirmatives, (ſo that the Diviſor may be leſs than the Reſidue of the Power to be reſolved); which difficulty notwithſtanding the ſagacious Analyſt will eaſily avoid.

21. *To find the Second of the Side.*

22. Let this Rule therefore be perpetual; That the true ſingular ſecond Side is that which firſt of all ſheweth ſuch a Gnomon, conſiſting of the Complements of every Kind, and multiplied Coefficients, as the Condition of the *Equation* requireth, according to the precedent Table, and all gathered together into one Sum, diligent reſpect had every where, as well to the Signs as Places; which Gnomon may not be greater than the Power to be reſolved, from whence it is to be ſubſtracted: Wherefore it is often to be tried, until the true Side be found; which alſo by the next greater will moſt certainly be known.

22. *To know the Second of the Side.*

23. All the ſingular Sides after the Second, by Simple Diviſion, are moſt eaſily obtained.

23. *To get the other Figures after the Second.*

24. If the Affections are compounded of Affirmatives and Negatives, the Antecedent Precepts are to be mixt with Diſcretion and Judgment: And in the Sides to be valued, always the greater Affection ſhall be conſidered before the Leſſer.

24. *If the Affections are mixt.*

25. But becauſe oftentimes above it is ſaid, it will be needful to try, which in many-fold Affections, and where the Degrees are lofty, will be exceeding laborious, I will add here for a Cloſe, two Manners of eaſing theſe Trials: One by Depreſſion, another by the Canon of Logarithms. But in both, if the *Equation* ſhall be ambiguous, all the Signs thereof ſhall be changed. Here alſo is to be noted, that every Negative Number is leſs than any Affirmative, and than any Leſſer Negative.

25. *Two ways.*
By Depreſſion.
By Logarithms.

26. The Invention of the Singular Sides by Depreſſion: If the firſt Side be ſought, all the Points after the Firſt, in the ſeveral *Species* of the given *Equation*, may be cut off by the Separatrix: Afterwards all the *Species* may be applied to the Side, that is, depreſſed by one Degree.

26. *Invention by Depreſſion.*

Example 1. *Example* 1. 1qq — 72c + 238600l = 8725815. This by Depreſſion ſhall be made 1c + 238,6 — 7,2q = L) 872,5.

Let A be 4, then ſhall 4) 872,5 (218,1 the juſt Number.

And + 64 + 238,6 — 115,2 = 187,4 leſs than the Juſt.

Let A be 5, then ſhall 5) 872,5 (174,5 the juſt Number.

And + 125 + 238,6 — 180,0 = 183,6 greater than the Juſt.

The true Side therefore A = 5 — 1, that is 4.

Example 2. *Example* 2. Of the *Ambiguous Equation*, 1c — 3257l = — 45744

This by Depreſſion ſhall be made 1q — 32,5 = L) — 45,7.

Let A be 4, then ſhall 4) — 45,7 (— 11,4 the juſt Number.

And + 16 — 32,5 = — 16,5 leſs than the Juſt.

Let A be 5, then ſhall 5) — 45,7 (— 9,1 the juſt Number.

And + 25 — 32,5 = — 7,5 greater than the Juſt.

The true Side therefore A = 5 — 1, that is 4.

For the ſecond of the Side. If the ſecond Side be ſought, all the Points after the ſecond may be cut off in the ſeveral *Species*; Afterwards all the *Species* may be applied to the Square, that is depreſſed by two Degrees. As in the firſt Example.

1qq — 72c + 238600l = 8725815. This by Depreſſion ſhall be made 1q + L) 238600 — 72l = Q) 8725815.

Let A be 47, then ſhall 2209) 8725815 (3949 the juſt Number.

And 2209 + 5077 — 3384 = 3896 leſs than the Juſt.

Let A be 48, then ſhall 2304) 8725815 (3787 the juſt Number.

And 2304 + 4971 — 3456 = 3819 greater than the Juſt.

The true Side therefore is 48 — 1, that is 47.

27. Omitted. 27. This being wholly about the Uſe of Logarithms, is omitted here.

28. Examples of Logarithms. 28. This contains nothing but Examples, wherein trial is made by Logarithms. In which Examples all the Points after the two firſt are cut off by the Seperatrix.

Example 1. 1qq — 72c + 238600l = 8725815 the juſt Number.

Let the two firſt ſingular Sides be ſought.

47 . 1,67209,78579	|—72|	|+238600|
C . 5,01629,35737	1,85733,24964	5,37767,04393
QQ. 6,68839,14316	5,01629,35737	1,67209,78579
	6,87362,60701	7,04976,82972
+4879...	—7475...	+11214...

And + 4870... + 11214... — 7475... = + 8618... leſs than the Juſt.

48 . 1,68124,12374	1,85733,24964	5,37767,04393
C . 5,04372,37122	5,04372,37122	1,68124,12374
QQ. 6,72496,49496	6,90105,62086	7,05891,16767
+5308...	—7962...	+11453...

And + 5308 ... + 11453 ... — 7962 ... = + 8799 ... greater than the Juſt.

The true Root therefore ſhall be 48 — 1, that is 47.

Example 2. 1c — 3257l = — 45744 the juſt Number.

Let the two firſt Singular Sides be ſought.

48 .	1,68124,12374	— 3257 .	3,51281,77586
C .	5,04372,37122		1,68124,12374
	+ 1106	— 1563 .	5,19405,89960

And + 1106 — 1563 = — 457. Leſs than the Juſt (at leaſt not Greater).

49 .	1,69019,60800	— 3257 .	3,51281,77586
C .	5,07058,82400		1,69019,60800
	+ 1176	— 1596 .	5,20301,38386

And + 1176 — 1596 = — 420. Greater than the Juſt.

The true Root therefore ſhall be 49 — 1, that is 48.

The ſecond Side may alſo be found by Logarithms, Depreſſion preceding.

As in Example, 1qq — 1246,00q = 08972,6256.

This depreſſed ſquaredly, ſhall be made 1q — 1246 = Q) 8972,6.

The two firſt Singular Sides may be ſuppoſed.

34 .	1,53147,89170	8972,6	3,95291,83073	
Q .	3,06295,78340		3,06295,78340	
	+ 1156	Value 7,76	0,88996,04733	The Juſt.

And + 1156 — 1246 = — 90. Leſs than the Juſt.

36 .	1,55630,25008	8972,6	3,95291,83073	
Q .	3,11260,50016		3,11260,50016	
	+ 1296	Value 6,92	0,84031,33057	The Juſt.

And + 1296 — 1246 = + 50. Greater than the Juſt.

Notes of the Author.

Upon theſe Examples, in the *26th* and *28th* Sections, the ſame Author afterwards hath added ſome Notes of Explanation; the Subſtance whereof is thus:

What called the juſt Number.

That is called *The juſt Number*, which ariſeth of the Application of the Power to be reſolved to the Degree of the ſuppoſed Side, by which Depreſſion is made: For this is the Meaſure to which all the other *Species* duly gathered together, ought to be equal. As in the firſt Example of the *26th* Section, 1c + 238,6 — 7,2q = L) 872,5. If for the firſt Side be ſuppoſed 5, it muſt be that C:5: + 238,6 — 7,2Q:5: = 872,5 divided by 5; that is 125 + 238,6 — (7,2×25) 180, to wit 183,6 to be equal to 174,5 the Juſt. But it is greater, and therefore the true Side is leſs than 5; therefore let 4 be again ſuppoſed, and make trial whether C:4: + 238,6 — 7,2 Q:4: be equal to 872,5 divided by 4.

Monitions.

But leſt in theſe Examples, as alſo in the following, theſe Trials be taken up by chance, it muſt be admoniſhed,

1. Monition.

1. If the Homogeneal Power of the Extracted Root, exceed the Power to be reſolved; or if the Magnitudes increaſing the Power to be reſolved, exceed them which they leſſen; The true Side A (for the moſt part) ſhall be leſs than the Side extracted, but otherwiſe greater: As in this *Equation*.

1c + 26,00col = 180931713

180,9 (4 The Side A.
26,0 Cq.

√q26 is 5, in 26 is made 130, taken from 180, there reſteth 50,C:3+: which is leſs than 180; wherefore the true Side A is greater than 3.

2. Monition. 2*ly*. If the Diviſors under the ſame Sign with the Reſidue of the Power to be reſolved, exceed them which are under a diverſe Sign, the true Side E (for the moſt part) ſhall be leſs than the Quotient; but otherwiſe greater: As in this *Equation*.

$$1568l - 1c = 21952.$$

The ſame alſo happeneth in *Ambiguous Equations*, when the Reſidue of the Power to be reſolved is Affirmative: As in this *Equation*.

$$6768l - 1c = 214273.$$

The Work of both.

21	952	(28 The two firſt Sides.
15	68	Cq
−8		Ac
+31	36	CqA
+23	36	*Subtrahend.*
R−1	408	
1	2	−3Aq
	6	−3A
−1	26	
+1	568	Cq
+	308	*Diviſor.*

The Sign R is −; But −1,26 is leſs than +1,568. Wherefore the true Side E is greater than the Quotient 4.

214	273	(47 The 2 firſt Sides.
67	68	Cq
−64		−Ac
+270	72	CqA
+206	72	*Subtrahend.*
R+ 7	553	
4	8	−3Aq
	12	−3A
− 4	92	
+ 6	768	Cq
+ 1	848	*Diviſor.*

The Sign is +; But the Diviſor out of the Degrees of the Side A Negative, is leſs than the Coefficient Affirmative Diviſor; that is −4,92, is leſs than +6,768. Wherefore the true Side E ſhall be greater than the Quotient 4.

3. Monition. 3*ly*. If after theſe Monitions ſome Doubt remain, trial by 5 ſhall be moſt fit to be begun; and from thence Inquiry to be continued by odd Numbers: Or the ſame may be done by Depreſſion, or by Logarithms.

16 Examples of the ſame Author, and his Notes thereupon.

The other Examples of the ſame Author, with his Notes thereupon.

Ex. 1. $1qc - 15qq + 160c - 1250q + 06480l = 170304782$

That is $Lqc - BLqq + CqLc - DcLq + FqqL = Gqc$.

1703	04782	(47
1	5	—B
1	250	—Dc
1	60	Cq
	6480	Fqq
1024		Aqc
102	40	CqAc
2	5920	FqqA
+1128	9920	
384	0	—BAqq
20	000	—DcAq
—404	000	
724	9920	*Subtrahend.*
R 978	05582	
128	0	5Aqq
6	40	10Ac
	160	10Aq
	20	5A
7	680	Cq3Aq
	1920	Cq3A
	160	Cq
	6480	Fqq
+142	50040	
38	40	—B4Ac
1	440	—B6Aq
	240	—B4A
	15	—B
1	0000	—Dc2A
	1250	—Dc
—40	87665	
+101	62375	*Diviſor.*
896	0	5AqqE
313	60	10AcEq
54	880	10AqEc
4	8020	5AEqq
	16807	Eqc
53	760	Cq3AqE
9	4080	Cq3AEq
	54880	CqEc
	45360	FqqE
+1333	62047	
268	80	—B4AcE
70	560	—B6AqEq
8	2320	—B4AEc
	36015	—BEqq
7	0000	—DC2AE
	61250	—DcEq
—355	56465	
978	05582	*Subtrahend.*

In this Example √qc1703 is 4+, by *Sect*. 18. *Rule* 1. For as it appeareth out of *Sect* 7. there is not made any notable Change in the firſt Point, by the Coefficients analytically reduced; wherefore the true Side A ſhall be 4.

The true Side E is leſs than the Quotient 9; becauſe the Diviſors under the Sign +, (which is the Sign of the Reſidue) exceed them which are under the Sign —. *Exam-*

Example 2.

1c + 420000l = 247651713

That is Lc + CqL = Dc.

247	651	713	(417
42	000	0	Cq
64			Ac
168	000	0	CqA
232	000	0	*Subtrabend.*
R 15	651	713	
4	8		3Aq
	12		3A
4	200	00	Cq
9	120	00	*Diviſor.*
4	8		3AqE
	12		3AEq
	1		Ec
4	200	00	CqE
9	121	00	*Subtrabend.*
R 6	530	713	
	504	3	3Aq
	1	23	3A
	420	000	Cq
	925	530	*Diviſor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
2	940	000	CqE
6	530	713	*Subtrabend.*

In this Example

42) 247 (6—, by *Sect.* 18. *Rule* 2. For 42 reduced Analytically by *Sect.* 6 and 8, it is made 252, greater than 247: And the true Side A is leſs than 6, becauſe C:6—: exceeds 247,6.

Example 3.

1c + 1007q = 247617936

That is Lc + BLq = Dc.

247	617	936	(417
10	07		B
64			Ac
161	12		BAq
225	12		*Subtrabend.*
R 22	497	936	
4	8		3Aq
	12		3A
8	056		B2A
	100	7	B
13	076	7	*Diviſor.*
4	8		3AqE
	12		3AEq
	1		Ec
8	056		B2AE
	100	7	BEq
13	077	7	*Subtrabend.*
R 9	420	236	
	504	3	3Aq
	1	23	3A
	825	74	B2A
	1	007	B
1	332	277	*Diviſer.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
5	780	18	B2AE
	49	343	BEq
9	420	236	*Subtrabend.*

In this Example

10) 247 (24 + = Q:5—: by *Sect.* 18. *Rule* 2. But 10 Q:5: = 250 ⊏ 247,6 by *Monition* 1.

Example 4.

1qq — 44299005l = 022252086

That is Lqq — DcL = Fqq.

0	2225	2086	(354
—44	2990	05	—Dc
+81			Aqq
—132	8970	15	—DcA
—51	8970	15	*Subtrahend.*
R 52	1195	3586	
10	8		4Ac
	54		6Aq
	12		4A
+11	352		
— 4	4299	005	—Dc
+ 6	9220	995	*Diviſor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+69	0625		
—22	1495	025	—DcE
+46	9129	975	*Subtrahend.*
R 5	2065	3836	
1	7150	0	4Ac
	73	50	6Aq
		140	4A
+ 1	7223	640	
—	4429	9005	—Dc
1	2793	7395	*Diviſor.*
6	8600	0	4AcE
	1176	00	6AqEq
	8	960	4AEc
		256	Eqq
+ 6	9784	9856	
— 1	7719	6020	—DcE
+ 5	2065	3836	*Subtrahend.*

In this Example,
√c44,3 is 3+, by *Sect.* 18. *Rule* 3. wherefore the true Side A is 3.

The true Side E is leſs than the Quotient 8— by *Monition* 2.

Example 5.

1qq — 124600q = 089726256

That is Lqq — CqLq = Fqq.

0	8972	6256	(354
— 12	4600		—Cq
+ 81			Aqq
— 112	1400		—CqAq
— 31	1400		*Subtrahend.*
R 32	0372	6256	
10	8		4Ac
	54		6Aq
	12		4A
+ 11	352		
7	4760	0	—Cq2A
	1246	00	—Cq
— 7	6006	00	
+ 3	7514	00	*Diviſor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+ 69	0625		
37	3800	0	—Cq2AE
3	1150	00	—CqEq
— 40	4950	00	
+ 28	5675	00	*Subtrahend.*
R 3	4697	6256	
1	7150	0	4Ac
	73	50	6Aq
		140	4A
+ 1	7223	640	
	8722	000	—Cq2A
	12	4600	—Cq
—	8734	4600	
+	8489	1800	*Diviſor.*
6	8600	0	4AcE
	1176	00	6AqEq
	8	960	4AEc
		256	Eqq
+ 6	9784	9856	
3	4888	000	—Cq2AE
	199	3600	—CqEq
— 3	5087	3600	
+ 3	4697	6256	*Subtrahend.*

In this Example,
√q12,4 is 3+, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 3.

The true Side E, is leſs than the Quotient 9— by *Monition* 2.

Example 6.

1qq — 340c = 621066096

That is Lqq — BLc = Fqq.

	6	2106	6096	(354
—	3	40		—B
+	81			Aqq
—	91	80		BAc
—	10	80		*Subtrahend.*
R	17	0106	6096	
	10	8		4Ac
		54		6Aq
		12		4A
+	11	352		
	9	180		—B3Aq
		3060		—B3A
		34	0	—B
—	9	4894	0	
+	1	8626	0	*Diviſor.*
	54	0		4AcE
	13	50		6AqEq
	1	500		4AEc
		625		Eqq
+	69	0625		
	45	900		—B3AqE
	7	6500		—B3AEq
		4250	0	—BEc
—	53	9750	0	
+	15	0875	0	*Subtrahend.*
R	1	9231	6096	
	1	7150	0	4Ac
		73	50	6Aq
			140	4A
+	1	7223	640	
	1	2495	00	—B3Aq
		35	700	—B3A
			340	—B
—	1	2530	7340	
+		4692	9060	*Diviſor.*
	6	8600	0	4AcE
		1176	00	6AqEq
		8	960	4AEc
			256	Eqq
+	6	9784	9856	
	4	9980	00	—B3AqE
		571	200	—B3AEq
		2	1760	—BEc
—	5	0553	3760	
+	1	9231	6096	*Subtrahend.*

In this Example, The Lateral Coefficient 3, 4, ſquared-ſquaredly, multiplied and encreaſed 6, 2, is made 140, QQ: 3 +: by *Sect.* 18. *Rule* 4. Wherefore the true Side A is 3. The true Side E is leſs than the Quotient 9—, by *Monition* 2.

Example 7.

1qq — 77108000l = 085530576

That is Lqq — DcL = Fqq

	0	8553	0576	(426
—	77	1080	00	—Dc
+	256			Aqq
—	308	4320	00	—DcA
—	52	4320	00	*Subtrahend.*
R	53	2873	0576	
	25	6		4Ac
		96		6Aq
		16		4A
+	26	576		
—	7	7108	000	—Dc
+	18	8652	000	*Diviſor.*
	51	2		4AcE
	3	84		6AqEq
		128		4AEc
		16		Eqq
+	55	1696		
—	15	4216	000	—DcE
+	39	7480	000	*Subtrahend.*
R	13	5393	0576	
	2	9635	2	4Ac
		105	84	6Aq
			168	4A
+	2	9741	208	
—		7710	8000	—Dc
+	2	2030	4080	*Diviſor.*
	17	7811	2	4AcE
		3810	24	6AqEq
		36	288	4AEc
			1296	Eqq
+	18	1657	8576	
—	4	6264	8000	—DcE
+	13	5393	0576	*Subtrahend.*

In this Example.
√c77 is 4, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 4.

Example 8.

32001 — 1c = 46577

That is CqL — Lc = Dc

An Ambiguous *Equation.*

46	577	(47 The greater Root.
32	00	Cq
— 64		—Ac
+128	00	CqA
+ 64	00	*Subtrabend.*
R— 17	423	
4	8	—3Aq
	12	—3A
— 4	92	
+ 3	200	Cq
— 1	720	*Diviſor.*
33	6	—3AqE
5	88	—3AEq
	343	— Ec
— 39	823	
+ 22	400	CqE
— 17	423	*Subtrabend.*

In this Example,

√q32, is 5,65 in 32 is made 180,8. lacking 46,5. there remaineth 144 . C: 5 : by *Sect.* 18. *Rule* 4. But 144 exceedeth 46,5. Wherefore the true Side A is leſs than 5 by *Monition* 1.

The true Side E is leſs than the Quotient 10, by *Monition* 2.

Example 9.

32001 — 1c = 46577

That is CqL — Lc = Dc

The ſame Ambiguous *Equation.*

46	577	(15,7 The leſſer Root.	
32	00	Cq	
— 1		—Ac	
+ 32	00	CqA	
+ 31	00	*Subtrabend.*	
R 15	577		
	3	—3Aq	
	3	—3A	
—	33		
+ 3	200	Cq	
+ 2	870	*Diviſor.*	
1	5	—3AqE	
	75	—3AEq	
	125	— Ec	
— 2	375		
+ 16	000	CqE	
+ 13	625	*Subtrabend.*	
R 1	952	000	
	67	5	—3Aq
		45	—3A
—	67	95	
+	320	0	Cq
+	252	05	*Diviſor.*
	472	5	—3AqE
	22	05	—3AEq
		343	— Ec
—	494	893	
+ 2	240	0	CqE
+ 1	745	107	*Subtrabend.*
R	206	893	000, &c.

In this Example,

The Solution is moſt eaſy by *Diviſion*, according to *Sect.* 18. *Rule* 3.

Example 10.

53q — 1c = 13254

That is BLq — Lc = Dc

An Ambiguous *Equation.*

13	254	(47 The greater Root.
5	3	B
—64		—Ac
+84	8	BAq
+20	8	*Subtrahend.*
R— 7	546	
4	8	—3Aq
	12	—3A
— 4	92	
4	24	B2A
	53	B
+ 4	293	
—	627	*Diviſor.*
33	6	—3AqE
5	88	—3AEq
	343	— Ec
—39	823	
29	68	B2AE
2	597	BEq
+32	277	
— 7	546	*Subtrahend.*

In this Example,

C: 5 : is 125, lacking 13, there remaineth 112, C: 5 — : by *Section* 18. *Rule* 4. But 112 exceedeth 13. Wherefore the true Side A is leſs than 5 by *Monition* 1.

The true Side E is leſs than the Quotient 12, by *Monition* 2.

Example 11.

53q — 1c = 13254

That is BLq — Lc = Dc

The ſame Ambiguous *Equation.*

13	254	(20,05, *&c.* The leſſer Root.		
5	3			B
— 8				—Ac
+21	2			BAq
+13	2			*Subtrahend.*
R	54	000	000	
	12	000	0	—3Aq
		6	00	—3A
—	12	006	00	
	21	200		B2A
		5	3	B
+	21	205	3	
	9	199	30	*Diviſor.*
	60	000	0	—3AqE
		150	00	—3AEq
			125	— Ec
—	60	150	125	
	106	000		B2AE
		132	5	BEq
+	106	132	5	
	45	982	375	*Subtrahend.*
R	8	017	625	000, &c.

In this Example,

The Solution is moſt eaſy by *Diviſion*, according to *Section* 18. *Rule* 3.

Example 12.

600341l — 1c = 1023768

That is CqL — Lc = Dc.

An Ambiguous *Equation.*

1	023	768	(236. The greater Root.
6	003	4	Cq
— 8			—Ac
+12	006	8	CqA
+ 4	006	8	*Subtrahend.*
R— 2	983	032	
1	2		—3Aq
	6		—3A
— 1	26		
+	600	34	Cq
—	659	66	*Diviſor.*
3	6		—3AqE
	54		—3AEq
	27		— Ec
— 4	167		
+ 1	801	02	CqE
— 2	365	98	*Subtrahend.*
R—	617	052	
	158	7	—3Aq
		69	—3A
—	159	39	
+	60	034	Cq
—	99	356	*Diviſor.*
	952	2	—3AqE
	24	84	—3AEq
		216	— Ec
—	977	256	
+	360	204	CqE
—	617	052	*Subtrahend.*

In this Example.

√q 6 is 2+, in 6 is made 12 lacking 1, there remaineth 11, C : 2,5 by *Sect.* 18. *Rule* 4. But 11 exceedeth 1; wherefore the true Side A a little leſs than 2+ by *Monition* 1.

The true Side E is leſs than the Quotient 5— by *Monition* 2.

Example 13.

600341l — 1c = 1023768

That is CqL — Lc = Dc.

The ſame Ambiguous *Equation.*

1	023	768			(17,13, &c. The leſſer Root.
	600	34			Cq
	—1				—Ac
+	600	34			CqA
+	599	34			*Subtrahend.*
R	424	428			
		3			—3Aq
		3			—3A
—		33			
+	60	034			Cq
	59	704			*Diviſor.*
	2	1			—3AqE
	1	47			—3AEq
		343			— Ec
—	3	913			
+	420	238			CqE
+	416	325			*Subtrahend.*
R	8	103	000		
		86	7		—3Aq
			51		—3A
—		87	21		
+	6	003	4		Cq
+	5	916	19		*Diviſor.*
		86	7		—3AqE
			51		—3AEq
			1		— Ec
—		87	211		
+	6	003	4		CqE
+	5	916	189		*Subtrahend.*
R	2	186	811	000	
		591	562	57	*Diviſor.*
	1	774	656	903	*Subtrahend.*
R		412	154	097	000, &c.

In this Example.

The Solution is moſt eaſy by *Diviſion*, according to *Section* 18. *Rule* 3.

Example 14.

1qq—72c+238600l=8725815,7056.

That is Lqq—BLc-+DcL=Fqq.

872	5815,	7056	(47,6
— 7	2		—B
+ 238	600		Dc
256			Aqq
954	400		DcA
+1210	400		
— 460	8		—BAc
+ 749	600		*Subtrahend.*
R 122	9815,	7056	
25	6		4Ac
	96		6Aq
	16		4A
23	8600		Dc
+ 50	4360		
34	56		—B3Aq
	864		—B3A
	72		—B
— 35	4312		
+ 15	0048		*Diviſor.*
179	2		4AcE
47	04		6AqEq
5	488		4AEc
	2401		Eqq
167	0200		DcE
+ 398	9881		
241	92		—B3AqE
42	336		—B3AEq
2	4696		—BEc
— 286	7256		
+ 112	2625		*Subtrahend.*
R 10	7190,	7056	
1	7698	808	*Diviſor.*
10	7190,	7056	*Subtrahend.*

In this Example,

QQ:7,2: is—2687; and √c238,6 is 6,2, whoſe QQ is + 1480. Then — 2687 +1480=—1207. This added to 872, giveth 2079, QQ:6+: by *Sect*.18.*Rule* 4. And becauſe—2687 to be added, is greater than + 1480 to be ſubſtracted, the true Side A ſhall be leſs than 6, by *Monition* 1. The true Side E is leſs than the Quotient 9, by *Monition* 2.

Example 15.

3l.—1c=1,258640782100.

That is CqL—Lc=Dc.

	1,	258	640	782	100	(0,4499,&c.
	3	3				Cq
—		64				—Ac
+	1	2				CqA
+	1	136				*Subtrahend.*
R		122	640	782	100	
		4	8			—3Aq
			12			—3A
—		4	92			
+		3				Cq
		25	08			*Diviſor.*
		19	2			—3AqE
		1	92			—3AEq
			64			— Ec
—		21	184			
+		12				CqE
+		98	816			*Subtrahend.*
R		23	824	782	100	
			580	8		—3Aq
			1	32		—3A
—			582	12		
+		3				Cq
		2	417	88		*Diviſor.*
		5	227	2		—3AqE
			106	92		—3AEq
				729		—Ec
—		5	334	849		
+		27				CqE
+		21	665	151		*Subtrahend.*
R		2	159	631	100	
			60	480	3	—3Aq
				13	47	—3A
—			60	493	77	
+			3			Cq
			239	506	23	*Diviſor.*
			544	322	7	—3AqE
			1	091	07	—3AEq
					729	—Ec
—			545	414	499	
+		2	7			CqE
+		2	154	585	501	*Subtrahend.*
R			5	045	599	000, &c.

In this Example,

Becauſe the leſſer Root of the Ambiguous *Equation* is ſought, the Coefficients, altho reduced, hinder not. The *Analyſis* ſhall be made by *Diviſion*, according to *Sect*.18.*Rule* 1.

Ex-

Example 16.

1qc — 5c + 5l = 1,147152872702092.

That is Lqc — CqLc + FqqL = Gqc.

1	14715	28727	02092	(0,2437 The Subtenſe of 14 Degrees.
+	5			Fqq
—	5			—Cq
	32			Aqc
1	0			FqqA
+1	00032			
—	40			—CqAc
	96032			*Subtrahend.*
R	18683	28727	02092	
	8	0		5Aqq
		80		10Ac
		40		10Aq
		10		5A
	5			Fqq
+	5008	8410		
	60			—Cq3Aq
	30			—Cq3A
		5		—Cq
—	630	5		
+	4378	3410		*Diviſor.*
	32	0		5AqqE
	12	80		10AcEq
	2	560		10AqEc
		2560		5AEqq
		1024		Eqc
	20			FqqE
+	20047	62624		
	240			—Cq3AqE
	480			—Cq3AEq
	32	0		—CqEc
—	2912	0		
+	17135	62624		*Subtrahend.*
R	1547	66103	02092	&c.

The farther *Diviſors* and Sums to be ſubſtracted, may in like ſort be gotten for the other Figures of the Root after 24.

In this Example,

The Author's Note being the ſame with that on the laſt, needs not be repeated here; nor yet his Rules for the *Geneſis* and *Analyſis* of the *Six Binomials*, Chap. 16. of his *Clavis*, as being more proper for *Trigonometry* than *Arithmetick*. Wherefore having now waded thus far into the Deeps of that Curious, but Myſterious Mathematician, as to untie the Knots of *Affected Equations*, it is high time to deſiſt: for whatſoever may ſeem to be omitted, the Ingenious may ſupply, by diligent Obſervation, and often Practice.

Side uſed for Root.

The mention of Side for Root in the *Sections*, and conſequently L or l, for *Latus* the Side or Root, inſtead of A the Suppoſititious Root in the *Examples*, being frequent in *Species*, needs no Memorandum; So as a Queſtion or two being added, wherein *Affected Equations* will ariſe; as well all this general Survey of *Equations*, as the whole Review of *Arithmetick*, may be ſhut up together.

Q. Of a Number, what it is.

1. There is a Number, whoſe Square abated by 16, and the firſt Number augmented by 8; and the Total of one multiplied by the Remainder of the other, will produce 2560: what is that Number?

Reſolution.

Anſw. Twelve; For the Square of 12 being 144, leſſened by 16, leaves 128; this multiplied by 12 and 8, that is 20, produceth 2560.

The Work by Coſſicks.

Suppoſe 1ℨ: Then the Square is 1ʒ, abating 16, the Remain is 1ʒ—16. And the Number increaſed by 8, is 1ℨ + 8. And 1ʒ — 16 × 1ℨ + 8, makes 1φ+8ʒ—16ℨ—128=2560. And by Reduction, 1φ+8ʒ—16ℨ=2688.

The Work by Species.

—B 16. C 8. D 2560. Suppoſe the Number A, then the Square is Aq: And from thence abating 16, leaves Aq—B; and to the Number adding 8, makes the Total A + C. Then multiplying Aq — B into A + C, there is produced Ac + AqC — BA — BC, which are equal to D.

And by Reduction, Ac + AqC — BA = D + BC.

Or ſet after the other Mode, Lc + BLq — CqL = Dc.

The Reſolution.

1c + 08q — 016l ==			
	2	688	(12 Root.
	—	8	B
	—	16	—Cq
	1		Ac
		8	BAq
	1	8	
	—	16	—CqA
	+1	64	*Subtrahend.*
	R 1	048	
		3	3Aq
		3	3A
		16	B2A
		8	B
	+	498	
	—	16	—Cq
	+	482	*Diviſor.*
		6	3AqE
		12	3AEq
		8	Ec
		32	B2AE
		32	BEq
	+1	080	
	—	32	—CqE
	1	048	*Subtrahend.*

Q. Of a Fat of Wine mixed with Water, what drawn out at a time.

2. Suppoſe out of a Fat of Wine of 360 Gallons, be drawn out a certain Number of Gallons; and as many of Water as were drawn out be put into the Fat; and the like be done the ſecond and third Times, and at laſt there be found to remain in the Fat of Wine (beſides the Water mixed therewith) 208⅓ Gallons: how much Wine was drawn out at each time?

Anſw. Sixty Gallons: As may be tried by ***Alligation***, and ***Tripled Proportions***, in the Second Part of this *4th* Book; and by the ***Mean Proportionals*** in the third Part, where a like Queſtion to this is reſolved. **Reſolution.**

To the Reſolution, 2 Proportionals gotten (that is 1 leſs than the Draughts) between the whole Quantity 360, and the Remainder given $208\frac{1}{3}$, and let B be 360 and D $208\frac{1}{3}$; the Proportionals in *Species* ſtand thus:

$$D.\ \sqrt{c}BDq.\ \sqrt{c}BqD.\ B.$$

Then ſuppoſing the Draught ſought to be A, it ſhall be that

$$A = B - \sqrt{c}BqD.$$

And by exalting (as was ſhewed in *Reduction*) the plain *Species* to an equal Power with the other in this *Equation*, B—A ſhall be cubed.

And ſo $Bc - 3BqA + 3BAq - Ac = BqD.$

Tranſlated $Ac - 3BAq + 3BqA = Bc - BqD.$

Reduced into Numbers $1c - 1080q + 388800l = 19656000$, and reſolved.

19	656	000	(60
—	108	0	—3B
+ 3	888	00	3Bq
	216		AC
23	328	00	3BqA
+23	544	00	
— 3	888	0	—3BAq
19	656	00	*Subtrahend.*
R		000	

Several of the Queſtions in this Chapter reſolved by ***Equations***, fall under ſome or other of the ***Rules of Proportions disjunct*** or ***continued***, before handled, whereby the Truth of the Operations here may be tried. But if not, the ***Reſolution of the Equations***, both by ***Coſſicks*** and ***Species***, where both are uſed, evidence the Truth of both Concluſions by their Agreement. And where ***Affected Equations*** are reſolved by this latter Way of Mr. ***Oughtred*** only, the Reſolution agreeing in all things with the Tenor of the Queſtion, is as in all other Works, a Proof ſufficient of the Truth thereof. ***Proof of the Reſolution of Equations.***

Partis quartæ & Libri quarti

FINIS.

AN APPENDIX

OF THE

Properties of ſome Numbers.

Abſtract Numbers of 3 ſorts. ALL Abſtract whole Numbers being expreſſed by 9, ſignifying Figures and the Cipher, (as in the beginning of this Treatiſe was noted) the Numbers made up thereby were divided into three Sorts, *Digits*, *Articles*, and *Mixt Numbers*, of all which ſome peculiar Properties may be obſerved; but leſt it prove tedious, Content ſhall be taken with the firſt 12.

Digits. *Whence ſo called.* A *Digit* is always wrote with one Figure; of which there being 9, with the Cipher 0, makes up the firſt Article 10, the Number of the Fingers in Latin known by the Name of *Digiti*, from whence the 9 Figures came to be called *Digits*.

Articles. An *Article* hath always a Cipher in the firſt Place.

Mixt. And a *Mixt Number* hath a Digit there.

Properties of the Unit. *One*, Though ſome differ about its being a Number, yet all agree it is the entire Foundation, and the Root and Meaſure of every Number, every Number meaſuring another ſo many times as there are Units therein: For every Number, whom beſides the Unit, no other Number meaſureth, is meaſurable by no other than the Unit. And as 1 is the Foundation of Number, ſo abides he firm and unalterable, he cannot break, nor will be broken by others, but remains whole. He neither multiplieth nor divideth, nor will be multiplied nor divided by himſelf; if he be ſquared, cubed, *&c.* he is ſtill no more than 1.

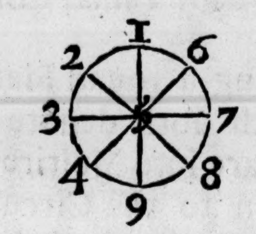
Of 2. *Two*, The only even prime Number (all other even Numbers being compound) meaſureth every even Number; and is the only Integer that multiplied by or added to himſelf, makes the Total equal to the Product; for 2 and 2 is 4, and no more is twice 2. He is ſometime called the *firſt Lineary Number*, becauſe a Line is bounded with two Points thus •—•.

Of 3. *Three*, Is the firſt Simple odd Number made by *Addition* of Units, and not by *Multiplication*; and the firſt that multiplies an even Number to make the Product even, and an odd Number odd: As twice 3 is 6, and 3 times 5 is 15. It compoundeth and meaſureth every Number, whoſe ſeveral Notes taken by themſelves, and added to themſelves, are numbred from the ſame Ternary; as 39.54. *&c.* It is ſometime called the *Muſical Number*, becauſe the third Concord is the Chief in Muſick: Sometimes it is called a *Syſtatical*, or *Subſtantial Number*, becauſe all Sublunary Bodies conſiſt of the three principal Subſtances, *Sal*, *Sulphur*, and *Mercury*. Alſo it is the firſt that diſpoſed in Units, hath Beginning, Middle, and End; and in *Geometry* may repreſent the three Angles of a Triangle, thus; ▽ and ſo came to be called the *firſt Figural Number*, becauſe a Triangle is the firſt Figure in *Geometry*.

Of 4. *Four*, Is the firſt even Compound Number, begotten by the Multiplication of 2 by 2, and the firſt proper Square Number, and repreſenteth the ſame, if the Units thereof be placed in oppoſition one to the other, as □. It meaſureth compounded, and numbereth every Number, whoſe Figure comprehended under the two firſt Sides, it can number: As becauſe it can meaſure 16, it ſhall meaſure 69816. It is called ſometime a *Worldly*, or *Mundane Number*, becauſe the Sublunary World conſiſts of 4 Elements.

Of 5. *Five* meaſureth and numbereth every Number, in whoſe firſt Place is 5 or 0; It is the ſecond Simple odd Number, and the firſt Circular Number; becauſe as a Circle turns to the Point whence it begun, ſo 5 multiplied by it ſelf, ends in 5: Wherefore whatever Equilateral figural Number hath 5 for his Root, in the firſt Place of his Quantity, will 5 be ſtill retained. Five is alſo called the *firſt Central Number*,

Number, becauſe 5 placed in the Center of a Circle, all the reſt of the Digits may be ſo diſpoſed about the Circle, that every two Oppoſites ſhall make 10, the firſt Article, and taking in the Central Number, make every way 15, thus:

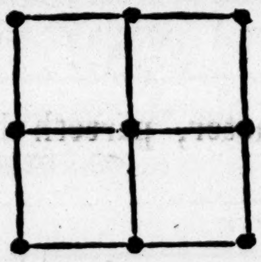


Six, Compoundeth and meaſureth every even Number, which 3 can meaſure. Of 6.
It is the ſecond even Compound and Circular Number, and the firſt perfect Number, whoſe even Parts are equal to himſelf, as $1 + 2 + 3 = 6$. Alſo Six is called the firſt *Pyramidal Number*; for the Units therein may be ſo placed, as to repreſent a Pyramis, thus:

Seven, The old *Magi* called a *Virgin Number*, ſuppoſing the Force thereof Of 7.
great, as a Virgin in her full ſtrength: But this Force is diſcerned in things concrete, and dependeth not on the Quantity of 7; and ſo every 7*th* Year bringing ſome change in Nature, is Climacterical. It is ſometime called the *Sacred and Quiet*, or *Sabbatory Number*, becauſe in Sacred Writ, the Seventh Day and Year were appointed to be reſted in.

Eight, Is the firſt proper Cubick Number, every Cube having 8 Corners. It Of 8.
compoundeth and numbereth every Number, whoſe Figure comprehended under the three firſt Figures it can number. It is a chief Note in Muſick, and taketh turns with 6 in the Termination of perfect Numbers, for they alternately end in 6 and 8. as 6. 28.96.8128. 130816. 2096128. 33550336. 536854528. &c.

Nine, Is the ſecond Square Number, the firſt compound odd Number, and the Of 9.
laſt Digit. What Number ſoever it is applied to, rejecting the Nines of that whole Number taken in Groſs, or the Nines of the ſeveral Parts taken Simple, will leave the Remains alike. As 27 makes 3 times 9: Wherefore whether the Nines of 27 be rejected, or of 2 and 7, the Remain will be all one (to wit 0.) The Units in 9 regularly diſpoſed, repreſent 4 the Square of 2. And by an orderly placing in every Point and Corner one of the Digits, the whole 9 will not only be taken up, but counted up any way as they ſtand, will make 15.



8	3	4
1	5	9
6	7	2

Scaliger called 9, the *chief and moſt perfect Number*; and *Exerc.* 365. *S.* 1. ſaith, It can be increaſed of none but the *Unit* to be made 10; and containeth in it ſelf all Species and Proportions of Quantity, as well Primary as Conſequent and Reſultant: For in it are *Length*, *Breadth*, *Depth*, *Perfect*, *Imperfect*, *Diviſible*, *Indiviſible*, *Triangle*, *Cube*, *Oblong*, *Plurilateral*, *Equality*, *Inequality*, *Abſolute*, *Comparate*, *Simple*, *Manifold*; and in *Specie* Double, *Seſquialter*, *Triple*, *Seſquitertia*, *Quadruple*, *Superpartiens*, &c.

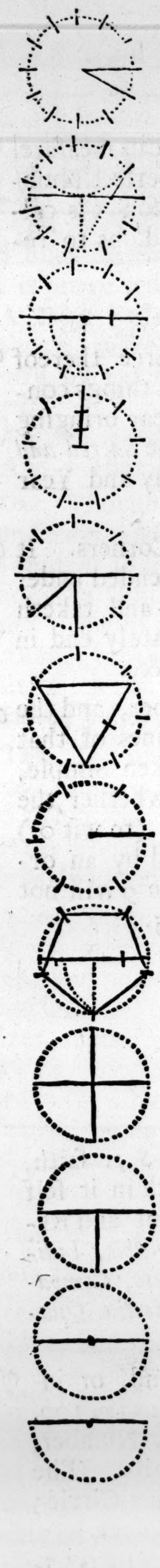
Ten, Is the firſt Article, and by adjoining Ciphers to the right Hand, or in- Of 10.
creaſing the Unit to the left Hand, other Articles will be produced, as 10. 100. 1000, *&c.* or 20, 30, 40, *&c.* It meaſureth and compoundeth every Number, in whoſe Right-hand Place is a Cipher. It comprehendeth all the Digits. The firſt round Number, becauſe now they begin with the Digits again as in a Circle; and hence by the *Pythagoreans* called *Circular*, and counted a perfect Number.

Eleven, multiplied by any Digit, beginneth and endeth alike; as 11 by 2, is Of 11.
22; by 3, is 33, *&c.*

Twelve, Is the Perimeter of a Triangle, whoſe *Area* is 6. For if 3, 4, 5, Of 12.
which make 12, be the Sides of a Triangle, half the Product of 3 × 4, which is 12, ſhall be the *Area*, as in Figural Numbers before was demonſtrated.

Sir Balt. Gerbier's Notes of the Circle.

In imitation of theſe 12 Notes, becauſe a Circle may be divided into ſuch Portions, as the *Polygones* of ſeveral regular Works may be noted thereby. The Diviſions of a Circumference in concordancy to 12, taken out of Sir *Baltaſar Gerbier*, with a little Alteration, ſhall ſerve for a final Concluſion of all this Work.



1. The Circle being a round Figure, repreſenteth a Cipher or 0. The Center of which doth denote the *Unit*, or 1. From whence infinite Deductions are into Multitude; as from the Center infinite Lines may be drawn to the Circumference: And by cutting the whole Circle into Parts, ſhall be infinite Sections beneath the Whole; as from 1 broken into Pieces, ariſeth infinite Fractions. And thus the Diameter terminating, the Circumference repreſents one half the Semicircle.

2. The Diameter drawn through the Center to the Circumference, divideth the Circle into two equal Parts.

3. A Perpendicular falling from the Circumference on the Diameter in the Center, parteth the Circumference into 3 Parts, of which the leſſer 2 are equal to the Third.

4. Two Diameters croſſing each other at Right Angles in the Center, divide the Circle into four equal Parts, called *Quadrants*.

5. From the half of the Semidiameter, to the half of an Arch drawn from the Circumference to the touch of the other Semidiameter; the diſtance of the Touch to the firſt half, gives a Fifth of the Circumference, or very near it.

6. The Semidiameter applied to the Circumference, is little leſs than the ſixth Part thereof.

7. Half one of the Sides of the greateſt Equilateral Triangle inſcribed in a Circle, ſhall equal the ſeventh Part of the Circumference pretty exactly.

8. The Quadrant equally biſſected, ſhall exactly part the Circumference into eight Parts.

9. Two thirds of the Semidiameter, parteth the Perimeter into 9 equal Diviſions.

10. From the half of the Semidiameter, to the half of an Arch drawn from the Circumference to the touch of the other Semidiameter, the Diſtance of the Touch to the Center will give nigh the Tenth.

11. The Length of a Line from the Point where two Circles cut each other to the Semidiameter, may be taken from the eleventh Part of the Perimeter.

12. The third Part of the Quadrant is the twelfth Part of the Circle, and of the Semicircle the Sixth.

Totius Operis Finis.

Soli Deo Gloria.

AN
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FINIS.

MVSEVM
BRITAN
NICVM

L

To the READER.

I Know no Reaſons that can be aſſigned for ſuppreſſing the Errata : *unleſs to impoſe on a Credulous Buyer, that the Impreſſion needs no Correction ; or that the Author (if found in an Error) may take Sanctuary under the Miſtakes of the Preſs. But neither of theſe Motives, have any Prevalence with me ; as being fully aſſured, that the Succeſs would prove as contemptible as the Deſign : When by the Diſcovery of ſome* Errata ; *the Reader would (as the natural Conſequence of ſuch an unjuſt Concealment) be tempted, to judge it unſafe to rely in any thing, either upon the Printer or the Author.*

I do therefore give this Advertiſement ; That though the diſtance of my habitation from London, *permitted me not to correct the Sheets as they were printed off. Yet I have (before the Publication) carefully examined the whole Impreſſion by the Original Manuſcript left by my Father at his death. And have exactly noted all the deviations of the Preſs, that can poſſibly miſlead the moſt unexperienced* Tyro. *So that I may perhaps incurr the cenſure of being unneceſſarily ſcrupulous ; in that, together with the Errors which are more material ; I have alſo inſerted thoſe that the meaneſt* Genius *will ſcarce think worth the trouble of noting with his Pen ; as being only literal.*

London March 26*th*.
1696.

S. Jeake.

ERRATA ſic corrigas.

P*Age* 1. *line ult. read* p. 66. *p.* 2. *l.* 24. *r.* Hypſometria, *l.* 35. *r.* Hypogeiodia, *p.* 3. *l.* 23. *r.* ס. Sixty, *l.* 24. *r.* טו. Fifteen, *l.* 55. *r.* XVIII *for* XXII, *p.* 4. *l.* 14. *r.* Affections, *l.* 39. *r.* Leſſe, *p.* 5. *l.* 35. *dele* the latter [into], *p.* 7. *l.* 1. *r.* Solid, & Marg. *l.* 1. *r.* 3. Solid, *p.* 8. l. 31. *r.* mixed either, *p.* 10. *l.* 17. *Lat. r.* Grana, *l.* 51. *r.* Æs, *p.* 11. *l.* 16. *r.* Imum, *p.* 12. *l. penult. r.* ⊏ next Greater, *p.* 13. *l.* 1. *r.* .. *l.* 54. *r.* Second Book, *p.* 14. *l.* 53. *r.* Quintillion, *p.* 16. *l.* 16. *r.* —Prime Part of Compoſition, which is Addition, *p.* 19. *l.* 8. *r.* Cyphers, *l.* 17. *r.* leaves, *p.* 21. *l.* 2. *r.* 40, *l.* 14. *r.* preceding, *p.* 22. *l.* 15. *r.* If of two, *p.* 23. *l. penult. r.* The, *p.* 25. *l.* 28. *r.* ſubſtracted, *p.* 27. *l.* 29. *r.* Rectangles, *p.* 29. *l.* 14. Marg. *for* Geometry, *r.* Grounded on, *p.* 30. *l.* 46. *r.* Multiplee, *p.* 31. *l.* 54. *r.* figures, *p.* 32. *l.* 14. *r.* comprehended, *p.* 36. ſet the figures over the Dividends a little more to the Right hand, *p.* 44. *l.* 17. *r.* Fractions of Fractions, *p.* 46. *l.* 42. *for* $\frac{16}{12}$. *r.* $\frac{13}{16}$, and tranſpoſe the Comma that is there ſet after [16 and 12] ſetting it after [Denominator], *l. antepenult.* for $\frac{16}{13}$. *r.* $\frac{16}{12}$, *p.* 48. *l.* 31 In the latter Dividend, *r.* $\frac{15}{4}$ *p.* 49. *l.* 1. *for*, in, *r.* is, *and for*, is, *r.* in, *p.* 50. *l.* 25. 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Groats, Col. 6. *for* 1$\frac{1}{2}$, *r.* 7$\frac{1}{2}$, *l. penult. r.* or 13 *s.* 4 *d.* *p.* 77. *l.* 34. *r.* 1633, *l.* 40. 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Col. *ult. for* 2$\frac{1}{4}$, *r.* 1$\frac{1}{4}$, *p.* 151. *l.* 28. *r.* Montferrat, *l.* 40. *r.* de minimis non curat, *p.* 152. *l.* 6. *r.* part or parts, From *p.* 152. to 172. the Running Title to be corrected by the Titles of the ſeveral Chapters, *p.* 153. in the Example B, *for* 6. *r.* 60, *p.* 154. *l.* 10. *r.* Shillings, *p.* 156. *l.* 3. *r.* Denomination, *l.* 6. *r.* then multiply, *p.* 158. *l.* 32. *for* 334 : 12 : *r.* 434 : 12, *p.* 159. *l.* 33. *for* 6906, *r.* 6960, *p.* 160. *l. ult. r.* 51 l. 13 s. 5 d. as at H, *p.* 161. *l.* 26. under Averdupois, *for* gr. *r.* qrs, *l.* 39. *for* 236. *r.* 364, *l. antepen. for* when, *r.* with, *p.* 162. *l.* 28. *for* leaſt *r.* laſt, *p.* 163. *l.* 31. for 6 s. *r.* 16 s. *p.* 164. *l.* 4. under Ton, *for* 29. *r.* 21, *l.* 21. dele, and Fractions, *l.* 22. *r.* of Integers and Fractions, *p.* 165. *l.* 27. At the end, add : And of the other, Book 1. Part 2. Chap. 5. *p.* 167. *l.* 23. *r.* both of one Nature, *l.* 32. Col. *ult. r.* 5. by $\frac{1}{2}$ or 5 half pounds—2 : 10 : 00, *l.* 33. Col. *ult. for* 8$\frac{1}{0}$, *r.* $\frac{1}{80}$, *p.* 169. *l.* 35. *for* $\frac{1}{4}$ *r.* $\frac{1}{4}$ l. *p.* 170. Example E, draw a line between 12. and 626, *l.* 35. *r.* Denominations, *p.* 172. *l.* 1. *r.* Page 39, *p.* 173. *l.* 9. Marg. *r.* contract, *p.* 174. *l.* 44. *r.* Iſoſceles, *l.* 45. *r.*

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Marg. *r.* Luna, *p.* 242. *l. ult.* ſave 3, *dele* the Comma, and ſet it 3 words after. In the Sexagenary Table, Col. 39. at Top, & 29. at Side, in the Common Angle, *for* $^{19.}{}_{51}$ *r.* $^{18.}{}_{51}$, Col. 36. at Top, and 36. at Side, in Com. Ang. *for* $^{2.}{}_{36}$ *r.* $^{21.}{}_{36}$, Col. 52. at Top, and 39. at Side, in Com. Ang. *for* $^{32.}{}_{48}$ *r.* $^{33.}{}_{48}$, *p.* 247. *l.* 19. *r.* $06'$, 0°_{5}, $20'$. *p.* 251. *l.* 20. *r.* Comma, *p.* 252. Log. of 46. *for* 583 &c. *r.* 783 &c. *p.* 253. Log. of 227. *for* 485 &c. *r.* 585 &c. *p.* 254. Log. of 502. *r.* 2,70070, Log. of 404. *for* 3, *r.* 2, Log. of 469. *for* 1, *r.* 2, For Num. 573. Col. 2. *r.* 473, *p.* 255. Log. of 749. *r.* 2,87448, *p.* 265. *l.* 38. in the Quotient of the Log. of $15,0'$. *dele* the Sign —, *l. ult.* in the Quotient of the Log. of 6,0. *dele* the Sign —, *p.* 267. *l.* 30. Tit. Cube, *r.* 00129, *p.* 269. *l.* 1. *r.* Ocular, *p.* 272. 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Names *i. e.* Parts, *p.* 357. *l.* 1. *for* and *r.* which, *l.* 5. *dele* mean, *l.* 38. *for* each of, *r.* it in, *l.* 42. upper Order, *i. e.* higher Rank, *l.* 44, 59. unequal *i. e.* odd, *l.* 45. equal *i. e.* even, *l.* 46. *r.* is made, *l.* 47. *dele* is, *p.* 358. *l.* 13. *r.* Concinnity, *p.* 359. *l.* 34. & 38. *dele* the Comma after [Quotient], *p.* 360. *l.* 3. wants a tittle under the laſt Figure of the Cube dividend, *p.* 361. *l.* 25. *r.* Examples, *p.* 363. *l.* 47. *r.* produces, *p.* 371. *l.* 1. *r.* 169 — 144. *p.* 375. *l.* 31. *for* 4Zq4. *r.* 4Zqq. *p.* 410. *l.* 26. *r.* Geodæticals, *p.* 411. *l.* 21. *for* $\frac{1}{7}\cdot\frac{1}{9}$ *r.* $\frac{1}{9}\cdot\frac{1}{7}$, *l.* 23. *for*, to the Third, *r.* to a Third, *p.* 412. *l.* 25. *r.* $\frac{2}{3}$ $\frac{2}{1}$ *p.* 416. *l.* 32. *for* Compounds, *r.* Compound, *p.* 419. *l. antepen. r.* compared to, *p.* 425. *l.* 29. *r.* Backward, *p.* 428. *l.* 10. *for* $\frac{5}{6}$. *r.* $\frac{5}{8}$. *p.* 432. Exam. 2. *l. ult.* of Caſe 2. *r.* 85 : 3 : 4. *p.* 435. *l.* 6. *for* 35 l. *r.* 35 ℔. *p.* 446. *l.* 24. *for* $^{2}\frac{07}{48}$. *r.* $^{2}\frac{07}{84}$. *p.* 447. *l.* 21. *for* $2\frac{1}{3}$. *r.* $2\frac{2}{3}$. *p.* 449. *l.* 33, 34. *r.* Suppoſititious, *p.* 453. *l.* 28. in the multipl. by 2, *for* 8. *r.* 84. *p.* 455. *l.* 36. in the Quot. *r.* 20 Crowns, *l.* 38. Divid. *for* 024. *r.* 240. *p.* 459. *l.* 18. *r.* abbreviate, *p.* 469. *l.* 2. *for* $\frac{11}{1}$. *r.* $\frac{11}{17}$. *l.* 10. *dele* the line between 1680. and 840. *l.* 21. *for* $\frac{}{2}$. *r.* $\frac{1}{2}$. *l.* 27. *r.* orderly by, *p.* 472. *l.* 37. *for* three Sexes, *r.* ſeveral Sexes, *p.* 477. Col. *ult. l.* 6. *for* 33. *r.* 13. *l.* 7. *for* 23. *r.* 33. *p.* 478. *l.* 8. *for* 38. *r.* 14. *p.* 482. *l.* 44. *r.* added, *p.* 486. *l.* 11. *r.* 6566(40. *l.* 16. *dele* the line between 29184. and 3648. *p.* 503. *l.* 40. *for* $2\frac{1}{1}$. *r.* $2\frac{1}{2}$. *p.* 525. *l.* 3. & Marg. *r.* Iſoperimeters, *l.* 8. *r.* Iſoperimetral, *l.* 45. *r.* Spheroide, *p.* 531. *l.* 20. *for* of *r.* or, *p.* 534. *l.* 32. *r.* := T. *p.* 541. *l.* 30. *r.* allotted, *p.* 551. *l.* 2. *r.* he hath, *p.* 553. *l.* 31. *r.* Numbers, *p.* 560. *l.* 21. place a Semicolon after [Integers], *l.* 28. place the Comma before [is], *p.* 566. *l.* 23. *r.* in the Third, *p.* 567. *l.* 40. *r.* till 3. *l.* 42. *dele* and, *p.* 568. *l.* 37. place a Comma after [ſought] and *dele* the latter [ſelf], *l.* 43. *r.* Quotients, *p.* 572. *l.* 16. *for* ⊔ *r.* ⊏ *p.* 573. *l.* 16. place I. over $\frac{3}{4}$. R. over $1\frac{1}{2}$ &c. *for* P. 577, 578. *r.* 578, 579. *p.* 578. *for* 579. *l.* 22. *r.* Pence in a, *p.* 585. *l.* 43. *r.* thereto, *p.* 591. *l.* 34. *r.* take, *p.* 595. *l.* 8. *dele* the negative Sign —. *p.* 613, &c. *for* Equations, *r.* Æquations, *p.* 613. *l.* 23. *r.* Andreas, *p.* 622. *l.* 11. *for* + 3. *r.* + 23. *p.* 630. *l.* 8. *r.* Ports, *p.* 631. *l.* 44. *r.* C to be, *p.* 637. *l.* 11. *r.* that is, *l.* 48. *for* Sign, *r.* Sign —, *p.* 643. *l.* 19. *for* Number *r.* Magnitude given. *for* the Rectangle *r.* a Rectangle, *l.* 20. *for* of *r.* under, *l.* 23. 25. manifeſt, ſhewn, *i. e.* expounded, *l.* 25. *for* denied *r.* negatively, *or* being negative. *for* affirmed *r.* affirmatively, *p.* 644. *l.* 10. *r.* 5 F. *l.* 25. *r.* 2. Let any Æquation be propounded : As this, &c. *l.* 31. *r.* Of the Root L. to be found out of theſe, there are 2. parts, *l.* 33. *r.* equal, *l.* 39. *r.* Coëfficient, *l.* 49. *for* ſquared *r.* the Square, *l.* 50. *dele* of, *l.* 51. *dele* the firſt [of], *p.* 645. *l.* 2. *dele* then, *l.* 21. *r.* in the propounded Æquation, *l.* 33. *dele* Figure of the, *p.* 646. *l.* 36. *for* alſo *r.* although, *p.* 647. *l.* 19. *for* his own *r.* that ſame, *l.* 32. *for* may be *r.* be much, *l.* 46. *r.* uſed mixtly, *l. penult.* and *p.* 648. *l.* 15, 16. *for* may *r.* are to, *l.* 26. Marg. *for* of *r.* by, *l.* 30. *dele* ſought, *l.* 36. *r.* 4879. *p.* 649. *l.* 2. *dele* ſought. After *l.* 23. Add, Therefore the true Root falls between 34 and 36. *l.* 31. *for* to be *r.* be, *l.* 39. *for* they leſſen *r.* leſſen it, *p.* 650. *l.* 21. *r.* Sign of R. *l.* 29. *for* Or *r.* And, *p.* 652. Ex. 3. *l.* 2. place a tittle over 7 q. *l.* 35. *dele* it, *p.* 654. *l. ult.* ſave 3, place the Comma after [multiplied], *p.* 658. *l. ult.* ſave 3, Col. 2. *r.* and the Coëff : *l. antepen. dele* the Comma, and ſet it in the Period's place, *p.* 663. *l.* 13. *r.* Sabbatary, *p.* 664. The Diagrams are ſet the wrong ſide upwards, *l.* 11. *dele* the Comma, and inſert it *l.* 12. after [half], *l.* 38. *for* from *r.* for.

THE Pointings of the Remains in Mr. Oughtred's 16. Examples of Affected Æquations (*in the Lines marked* R) *are omitted here. If any be deſirous to point them, they may have recourſe to his* Clavis, Edit. 3. Oxon. 1652. *which I eſteem the beſt.*

The Prefaces and Contents at the Beginning, and Table at the End of the Book; as alſo the preceding accompt of the Errata, *have all been examined and corrected by me at the Preſs: And I do not know of any miſtake in the Printing them.*

S. J.

MVSEVM
BRITAN
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